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### Identification of Linearized Models and Robust Control of Physical Systems

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#### 1. Introduction

This chapter presents the design of a controller that ensures both the robust stability and robust performance of a physical plant using a linearized identified model . The structure of the plant and the statistics of the noise and disturbances affecting the plant are assumed to be unknown. As the design of the robust controller relies on the availability of a plant model, the mathematical model of the plant is first identified and the identified model, termed here the nominal model, is then employed in the controller design. As an effective design of the robust controller relies heavily on an accurately identified model of the plant, a reliable identification scheme is developed here to handle unknown model structures and statistics of the noise and disturbances. Using a mixed-sensitivity  $H_{\infty}$  optimization framework, a robust controller is designed with the plant uncertainty modeled by additive perturbations in the numerator and denominator polynomials of the identified plant model. The proposed identification and robust controller design are evaluated extensively on simulated systems as well as on two laboratory-scale physical systems, namely the magnetic levitation and two- tank liquid level systems. In order to appreciate the importance of the identification stage and the interplay between this stage and the robust controller design stage, let us first consider a model of an electro-mechanical system formed of a DC motor relating the input voltage to the armature and the output angular velocity. Based on the physical laws, it is a third-order closed-loop system formed of fast electrical and slow mechanical subsystems. It is very difficult to identify the fast dynamics of this system, and hence the identified model will be of a second-order while the true order remains to be three. Besides this error in the model order, there may also be errors in the estimated model parameters. Consider now the problem of designing a controller for this electro-mechanical system. A constant-gain controller based on the identified second-order model will be stable for all values of the gain as long the negative feedback is used. If, however, the constant gain controller is implemented on the physical system, the true closed-loop third-order system may not be stable for large values of the controller gain. This simple example clearly shows the disparity between the performance of the identified system and the real one and hence provides a strong motivation for designing a robust controller which factors uncertainties in the model.

A physical system, in general, is formed of cascade, parallel and feedback combinations of many subsystems. It may be highly complex, be of high order and its structure may be different from the one derived from physical laws governing its behavior. The identified model of a system is at best an approximation of the real system because of the many difficulties encountered and assumptions made in completely capturing its dynamical behavior. Factors such as the presence of noise and disturbances affecting the input and the output, the lack of persistency of excitation, and a finite number of input-output samples all contribute to the amount of uncertainty in the identified model. As a result of this, highfrequency behavior including fast dynamics may go un-captured in the identified model. The performance of the closed- loop system formed of a physical plant and a controller depends critically upon the quality of the identified model. Relying solely on the robustness of the controller to overcome the uncertainties of the identified plant will result in a poor performance. Generally, popular controllers such as proportional (P), proportional integral (PI) or proportional integral and derivative (PID) controllers are employed in practice as they are simple, intuitive and easy to use and their parameters can be tuned on line. When these controllers are designed using the identified model, and implemented on the physical system, there is no guarantee that the closed-loop system will be stable, let alone meeting the performance requirements. The design of controllers using identified models to ensure robust stability is becoming increasingly important in recent times. In (Cerone, Milanese, and Regruto, 2009), an interesting iterative scheme is proposed which consists of first identifying the plant and employing the identified model to design a robust controller, then implementing the designed controller on the real plant and evaluating its performance on the actual closed-loop system. However, it is difficult to establish whether the identifycontrol-implement-evaluate scheme will converge, and even if it does, whether it will converge to an optimal robust controller. In this work, each of these issues, namely the identification, the controller design and its implementation on an actual system, are all addressed separately with the clear objective of developing a reliable identification scheme so that the identified model will be close to the true model, hence yielding a reliable controller design scheme which will produce a controller that will be robust enough to ensure both stability and robust performance of the actual closed-loop system. Crucial issues in the identification of physical systems include the unknown order of the model, the partially or totally unknown statistics of the noise and disturbances affecting data, and the fact that the plant is operating in a closed-loop configuration. To tackle these issues, a number of schemes designed to (a) attenuate the effect of unknown noise and disturbances (Doraiswami, 2005), (b) reliably select the model order of the identified system (Doraiswami, Cheded, and Khalid, 2010) and (c) identify a plant operating in a closed-loop (Shahab and Doraiswami, 2009) have been developed and are presented here for completeness. The model uncertainty associated with the identified model is itself modeled as additive perturbations in both the plant numerator and the denominator polynomials so as to develop robust controllers using the mixed-sensitivity  $H_{\infty}$  controller design procedure (Kwakernaak, 1993). The mixed-sensitivity  $H_{\infty}$  control design procedure conservatively combines and simultaneously solves both problems of robust stability and robust performance using a single  $H_{\infty}$  norm.

This design procedure is sound, mature, focuses on handling the problem of controller design when the plant model is uncertain, and has been successfully employed in practice in recent years (Cerone, Milanese, and Regruto, 2009), (Tan, Marquez, Chen, and Gooden,

2001). The proposed scheme is extensively tested on both simulated systems and physical laboratory-scale systems namely, a magnetic levitation and two-tank liquid level systems. The key contribution herein is to demonstrate the efficacy of (a) the proposed model order selection criterion to reduce the uncertainty in the plant model structure, a criterion which is simple, verifiable and reliable (b) the two-stage closed-loop identification scheme which ensures quality of the identification performance, and (c) the mixed-sensitivity optimization technique in the  $H_{\infty}$ -framework to meet the control objectives of robust performance and robust stability without violating the physical constraints imposed by components such as actuators, and in the face of uncertainties that stem from the identified model employed in the design of the robust controller. It should be noted here that the identified model used in the design of the robust controller is the linearized model of the physical system at some operating point, termed the nominal model.

The chapter is structured as follows. Section 2 discusses the stability and performance of a typical closed-loop system. In Section 3, the robust performance and robust stability problems are considered in the mixed-sensitivity  $H_{\infty}$  framework. Section 4 discusses the problem of designing a robust controller using the identified model with illustrated examples. Section 5 gives a detailed description of the complete identification scheme used to select the model order, identify the plant in a closed-loop configuration and in the presence of unknown noise and disturbances. Finally, in Section 6, evaluations of the designed robust controllers on two-laboratory scale systems are presented.

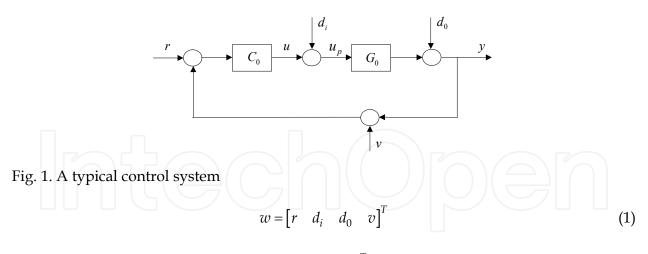
#### 2. Stability and performance of a closed-loop system

An important objective of the control system to ensure that the output of the system tracks a given reference input signal in the face of both noise and disturbances affecting the system, and the plant model uncertainty. A further objective of the control system is to ensure that the performance of the system meets the desired time-domain and frequency-domain specifications such as the rise time, settling time, overshoot, bandwidth, and peak of the magnitude frequency response while respecting the constraints on the control input and other variables. An issue of paramount practical importance facing the control engineer is how to design a controller which will both stabilize the plant when its model is uncertain and ensure that its performance specifications are all met. Put succinctly, we seek a controller that will ensure both stability and performance robustness in the face of model uncertainties. To achieve this dual purpose, we need to first introduce some analytical tools as described next.

#### 2.1 Key sensitivity functions

Consider the typical closed-loop system shown in Fig. 1 where  $G_0$  is the nominal plant,  $C_0$  the controller that stabilizes the nominal plant  $G_0$ ; r and y the reference input, and output, respectively;  $d_i$  and  $d_0$  the disturbances at the plant input and plant output, respectively, and v the measurement or sensor noise. The nominal model, heretofore referred to as the identified model, represents a mathematical model of a physical plant obtained from physical reasoning and experimental data.

Let w and z be, respectively, a (4x1) input vector comprising r,  $d_0$ ,  $d_i$  and v, and a (3x1) output vector formed of the plant output y, control input u, and the tracking error e, as given below by:



$$z = \begin{bmatrix} e & u & y \end{bmatrix}^T$$
(2)

The four key closed-loop transfer functions which play a significant role in the stability and performance of a control system are the four sensitivity functions for the nominal plant and nominal controller. They are the system's sensitivity  $S_0$ , the input-disturbance sensitivity  $S_{i0}$ , the control sensitivity  $S_{u0}$  and the complementary sensitivity  $T_0$ , given by:

$$S_0 = \frac{1}{1 + G_0 C_0}, S_{i0} = \frac{G_0}{1 + G_0 C_0} = S_0 G_0, S_{u0} = \frac{C_0}{1 + G_0 C_0} = S_0 C_0, T_0 = \frac{G_0 C_0}{1 + G_0 C_0}$$
(3)

The performance objective of a control system is to regulate the tracking error e = r - y so that the steady-state tracking error is acceptable and its transient response meets the timeand frequency-domain specifications respecting the physical constraints on the control input so that, for example, the actuator does not get saturated. The output to be regulated, namely *e* and *u*, are given by:

$$e = S_0(r - d_0) + T_0 v - S_{i0} d_i \tag{4}$$

$$u = S_{u0}(r - v - d_0) - T_0 d_i$$
(5)

The transfer matrix relating w to z is then given by:

$$\begin{bmatrix} e \\ u \end{bmatrix} = \begin{bmatrix} S_0 & -S_{i0} & -S_0 & T_0 \\ S_{u0} & -T_0 & -S_{u0} & -S_{u0} \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d_0 \\ v \end{bmatrix}$$
(6)

#### 2.2 Stability and performance

One cannot reliably assert the stability of the closed-loop by merely analyzing only one of the four sensitivity functions such as the closed-loop transfer function  $T_0(s)$  because there may be an implicit pole/zero cancellation process wherein the unstable poles of the plant (or the controller) may be cancelled by the zeros of the controller (or the plant). The cancellation of unstable poles may exhibit unbounded output response in the time domain.

In order to ensure that there is no unstable pole-zero cancellation, a more rigorous definition of stability, termed internal stability, needs to be defined. The closed-loop system is internally stable if and if all the eight transfer function elements of the transfer matrix of Equation (6) are stable. Since there are only four distinct sensitivity functions,  $S_0$ ,  $S_{i0}$ ,  $S_{u0}$  and  $T_0$ , the closed-loop system is therefore internally stable if and only if these four sensitivity functions  $S_0$ ,  $S_{i0}$ ,  $S_{u0}$  and  $T_0$  are all stable. Since all these sensitivity functions have a common denominator  $(1+G_0C_0)$ , the characteristic polynomial  $\varphi_0(s)$  of the closed-loop system is:

$$\varphi_0(s) = N_{p0}(s)D_{c0}(s) + D_{p0}(s)N_{c0}(s)$$

where  $N_{p0}(s)$ ,  $D_{p0}(s)$  and  $N_{c0}(s)$ ,  $D_{c0}(s)$  are the numerator and the denominator polynomials of  $G_0(s)$  and  $C_0(s)$ , respectively. One may express internal stability in terms of the roots of the characteristic polynomial as follows.

**Lemma 1** (Goodwin, Graeb, and Salgado, 2001): The closed-loop system is internally stable if and only if the roots of  $\varphi_0(s)$  all lie in the open left-half of the s-plane.

We will now focus on the performance of the closed-loop system by analyzing the closed-loop transfer matrix given by Equation (6). We will focus on the tracking error e for performance, and the control input u for actuator saturation:

- The tracking error *e* is small if (a) S<sub>0</sub> is small in the frequency range where *r* and d<sub>0</sub> are large, (b) S<sub>u0</sub> is small in the frequency range where d<sub>i</sub> is large and (c) T<sub>0</sub> and is small in the frequency range where *v* is large.
- The control input u is small if (a)  $S_{u0}$  is small in the frequency range where r,  $d_0$  and v are large, and (b)  $T_0$  is small in the frequency range where  $d_i$  is large.

Thus the performance requirement must respect the physical constraint that imposes on the control input to be small so that the actuator does not get saturated.

#### 3. Robust stability and performance

Model uncertainty stems from the fact that it is very difficult to obtain a mathematical model that can capture completely the behavior of a physical system and which is relevant for the intended application. One may use physical laws to obtain the structure of a mathematical model of a physical system, with the parameters of this model obtained using system identification techniques. However, in practice, the structure as well as the parameters need to be identified from the input-output data as the structure derived from the physical laws may not capture adequately the behavior of the system or, in the extreme case, the physical laws may not be known. The "true" model is a more comprehensive model that contains features not captured by the identified model, and is relevant to the application at hand, such as controller design, fault diagnosis, and condition monitoring. The difference between the nominal and true model is termed as the modeling error which includes the following:

- The structure of the nominal model which differs from that of the true model as a result of our inability to identify features such as high-frequency behavior, fast subsystem dynamics, and approximation of infinite-dimensional system by a finite- dimensional ones.
- Errors in the estimates of the numerator and denominator coefficients, and in the estimate of the time delay

(7)

• The deliberate negligence of fast dynamics to simplify sub-systems' models. This will yield a system model that is simple, yet capable enough to capture the relevant features that would facilitate the intended design.

#### 3.1 Co-prime factor-based uncertainty model

The numerator-denominator perturbation model considers the perturbation in the numerator and denominator polynomials separately, instead of lumping them together as a single perturbation of the overall transfer function. This perturbation model is useful in applications where an estimate of the model is obtained using system identification methods such as the best least-squares fit between the actual output and its estimate obtained from an assumed mathematical model. Further, an estimate of the perturbation on the numerator and denominator coefficients may be computed from the data matrix and the noise variance. Let  $G_0$  and G be respectively the nominal and actual SISO rational transfer functions. The normalized co-prime factorization in this case is given by

$$G_0 = N_0 D_0^{-1}$$

$$G = N D^{-1}$$
(8)

where  $N_0$  and N are the numerator polynomials, and both  $D_0$  and D the denominator polynomials. In terms of the nominal numerator and denominator polynomials, the transfer function *G* is given by:

$$G = (N_0 + \Delta_N)(D_0 + \Delta_D)^{-1}$$
<sup>(9)</sup>

where  $\Delta_N$  and  $\Delta_D \in RH_{\infty}$  are respectively the frequency-dependent perturbation in the numerator and denominator polynomials (Kwakernaak, 1993). Fig. 2 shows the closed- loop system driven by a reference input *r* with a perturbation in the numerator and denominator polynomials. The three relevant signals are expressed in equations (10-12).

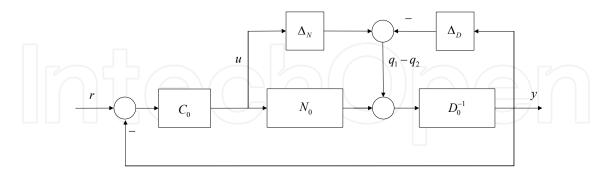


Fig. 2. Co-prime factor-based uncertainty model for a SISO plant

$$u = \frac{C_0}{1 + G_0 C_0} r - \frac{D_0^{-1} C_0}{1 + G_0 C_0} (q_1 - q_2) = S_{u0} r - D_0^{-1} S_{u0} (q_1 - q_2)$$
(10)

$$y = T_0 r + \frac{D_0^{-1}}{1 + G_0 C_0} (q_2 - q_1) = T_0 r + D_0^{-1} S_0 (q_2 - q_1)$$
(11)

$$q_1 - q_2 = \begin{bmatrix} \Delta_N & -\Delta_D \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}$$
(12)

#### 3.2 Robust stability and performance

Since the reference input does not play any role in the stability robustness, it is set equal to zero and the robust stability model then becomes as given in Fig. 3

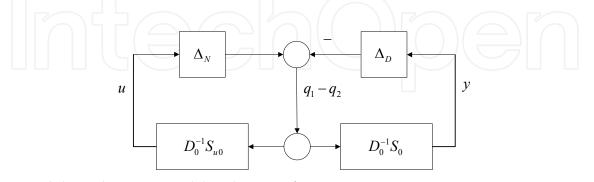


Fig. 3. Stability robustness model with zero reference input

The robust stability of the closed-loop system with plant model uncertainty is established using the small gain theorem.

**Theorem 1:** Assume that  $C_0$  internally stabilizes the nominal plant  $G_0$ . Hence  $S_0 \in RH_{\infty}$  and  $S_{u0} \in RH_{\infty}$ . Then the closed-loop system stability problem is well posed and the system is internally stable for all allowable numerator and denominator perturbations, i.e.:

$$\left\| \begin{bmatrix} \Delta_N & \Delta_D \end{bmatrix} \right\| \le 1 / \gamma_0 \tag{13}$$

If and only if

$$\| \begin{bmatrix} S_0 & S_{u0} \end{bmatrix} D_0^{-1} \|_{\infty} < \gamma_0 \tag{14}$$

**Proof:** The SISO robust stability problem considered herein is a special case of the MIMO case proved in (Zhou, Doyle, & Glover, 1996).

Thus to ensure a robustly-stable closed-loop system, the nominal sensitivity  $S_0$  should be made small in frequency regions where the denominator uncertainty  $\Delta_D$  is large, and the nominal control input sensitivity  $S_{u0}$  should be made small in frequency regions where the numerator uncertainty  $\Delta_N$  is large.

Our objective here is to design a controller  $C_0$  such that robust performance and robust stability of the system are both achieved, that is, both the performance and stability hold for all allowable plant model perturbations  $\|[\Delta_N \quad \Delta_D]\| \le 1 / \gamma_0$  for some  $\gamma_0 > 0$ . Besides these requirements, we need also to consider physical constraints on some components such as actuators, for example, that especially place some limitations on the control input. From Theorem 1 and Equation (6), it is clear that the requirements for robust stability, robust performance and control input limitations are inter-related, as explained next:

• Robust performance for tracking with disturbance rejection as well as robust stability in the face of denominator perturbations require a small sensitivity function *S*<sub>0</sub> in the low-frequency region and,

• Control input limitations and robust stability in the face of numerator perturbations require a small control input sensitivity function  $S_{u0}$  in the relevant frequency region.

With a view to addressing these requirements, let us select the regulated outputs to be a frequency-weighted tracking error  $e_w$ , and a weighted control input  $u_w$  to meet respectively the requirements of performance, and control input limitations.

$$z_w = \begin{bmatrix} e_w & u_w \end{bmatrix}^T \tag{15}$$

where  $z_w$  is a (2x1) vector output to be regulated,  $e_w$ , and  $u_w$  are defined by their respective Fourier transforms:  $e_w(j\omega) = e(j\omega)W_S(j\omega)$  and  $u_w(j\omega) = u(j\omega)W_u(j\omega)$ . The frequency weights involved,  $W_S(j\omega)$  and  $W_u(j\omega)$ , are chosen such that their inverses are the upper bounds of the respective sensitive functions so that weighted sensitive functions become normalized, i.e.:

$$|W_{\rm S}(j\omega)S_0(j\omega)| \le 1 , |W_u(j\omega)S_{u0}(j\omega)| \le 1$$

$$\tag{16}$$

The map relating the frequency weighted output  $z_w$  and the reference input r is shown in Fig. 4:

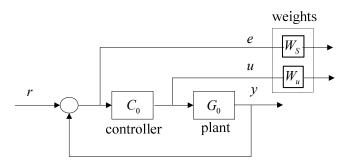


Fig. 4. Nominal closed-loop system relating the reference input and the weighted outputs

The weighting functions  $W_s(j\omega)$ , and  $W_{su}(j\omega)$  provide the tools to specify the trade-off between robust performance and robust stability for a given application. For example, if performance robustness (and stability robustness to the denominator perturbation  $\Delta_D$ ) is more important than the control input limitation, then the weighting function  $W_S$  is chosen to be larger in magnitude than  $W_{u0}$ . On the other hand, to emphasize control input limitation (and stability robustness to the numerator perturbation  $\Delta_N$ ), the weighting function  $W_{u0}$  is chosen to be larger in magnitude than  $W_S$ . For steady-state tracking with disturbance rejection, one may include in the weighting function  $W_S$  an approximate but stable 'integrator' by choosing its pole close to zero for continuous-time systems or close to unity for discrete-time systems so as to avoid destabilizing the system (Zhou, Doyle, and Glover, 1996). Let  $T_{rz}$  be the nominal transfer matrix (when the plant perturbation  $\Delta_0 = 0$ ) relating the reference input to the frequency-weighted vector output  $z_w$ , which is a function of  $G_0$  and  $C_0$ , be given by:

$$T_{rz} = D_0^{-1} \begin{bmatrix} \overline{W}_S S_0 & \overline{W}_u S_{u0} \end{bmatrix}^T$$
(17)

where  $\overline{W}_s = D_0 W_s$  and  $\overline{W}_u = D_0 W_u$  so that the  $D_0^{-1}$  term appearing in the mixed sensitivity measure  $T_{rz}$  is cancelled, thus yielding the following simplified measure  $T_{rz} = \begin{bmatrix} W_s S_0 & W_u S_{u0} \end{bmatrix}^T$ . The mixed-sensitivity optimization problem for robust performance and stability in the  $H_\infty$  – framework is then reduced to finding the controller  $C_0$  such that :

$$\left\|T_{rz}\left(C_{0},G_{0}\right)\right\|_{\infty} \le \gamma < 1 \tag{18}$$

It is shown in (McFarlane & Glover, 1990) that the minimization of  $||T_{rz}||_{\infty}$  as given by Equation (18), guarantees not only robust stability but also robust performance for all allowable perturbations satisfying  $||[\Delta_N \quad \Delta_D]||_{\infty} \leq 1/\gamma$ .

#### 4. $H_{\infty}$ controller design using the identified model

Consider the problem of designing a controller for an unknown plant *G*. We will assume however that the system *G* is linear and admits a rational polynomial model. A number of identification experiments are performed off-line under various operating regimes that includes assumptions on the model and its environment, such as :

- The model order
- The length of the data record
- The type of rich inputs
- Noise statistics
- The plant operates in a closed-loop, thus making the plant input correlated with both the measurement noise and disturbances
- Combinations of any the above

Let  $\hat{G}_i$  be the identified model from the  $i^{th}$  experiment based on one or more of the above stated assumptions. Let  $\hat{C}_i$  be the corresponding controller which stabilizes all the plants in the neighborhood of  $\hat{G}_i$  within a ball of radius  $1 / \hat{\gamma}_i$ . Given an estimate of the plant model  $\hat{G}_i$ , the controller  $\hat{C}_i$  is then designed using the mixed-sensitivity  $H_{\infty}$  optimization scheme, with both the identified model  $\hat{G}_i$  and the controller  $\hat{C}_i$  based on it, now effectively replacing the nominal plant  $G_0$  and nominal controller  $C_0$ , respectively. Let the controller  $\hat{C}_i$ 

stabilize the identified plant  $\hat{G}_i$  for all  $\|\hat{\Delta}_i\|_{\infty} \leq 1/\hat{\gamma}_i$  where  $\hat{\Delta}_i$  is formed of the perturbations

in the numerator and denominator of  $\hat{G}_i$ . To illustrate the identification-based  $H_{\infty}$ -optimization scheme, let us consider the following example. Let the true order of the system *G* be 2 and assume the noise to be colored. Let  $\hat{G}_i : i = 1, 2, 3$  be the estimates obtained assuming the model order to be 2, 3, and 4, respectively and let the noise be a zero-mean white noise process;  $\hat{G}_4$  is obtained assuming the model order to be 2, the noise to be colored but the input not to be rich enough; Let  $\hat{G}_5$  be an estimate based on correct assumptions regarding model order, noise statistics, richness of excitation of the input and other factors as pointed out above. Clearly the true plant *G* may not be in the neighborhood of  $\hat{G}_i$ , i.e.  $G \notin \hat{S}_i$  for all  $i \neq 5$  where

$$\hat{S}_i = \left\{ \hat{G}_i : \left\| \hat{\Delta}_i \right\|_\infty \le 1 / \hat{\gamma}_i \right\}$$
(19)

The set  $\hat{S}_i$  is a ball of radius  $(1/\hat{\gamma}_i)$  centered at  $\hat{G}_i$ . Fig. 5 below shows the results of performing a number of experiments under different assumptions on the model order, types

of rich inputs, length of the data record, noise statistics and their combinations. The true plant *G*, its estimates  $\hat{G}_i$  and the set  $\hat{S}_i$  are all indicated by a circle of radius  $(1/\hat{\gamma}_i)$  centered at  $\hat{G}_i$  in Figure 5. The true plant *G* is located at the center of the set  $\hat{S}_5$ .

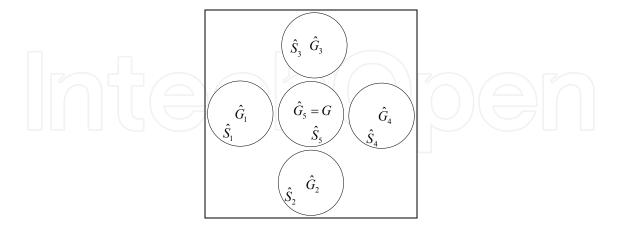


Fig. 5. The set  $\hat{S}_i$  is a ball of radius  $1 / \hat{\gamma}_i$  centered at  $\hat{G}_i$ 

#### 4.1 Illustrative example: $H_{\infty}$ controller design

A plant is first identified and then the identified model is employed in designing an  $H_{\infty}$  controller using the mixed sensitivity performance measure. As discrete-time models and digital controllers are commonly used in system identification and controller implementation, a discrete-time equivalent of the continuous plant is used here to design a discrete-time  $H_{\infty}$  controller. The plant model is given by:

$$G_0(z) = \frac{0.5335(1-z^{-1})}{1-0.7859z^{-1}+0.3679z^{-2}}$$
(20)

The weighting function for the sensitivity and control input sensitivity functions were chosen to be  $W_s = \frac{0.01}{1 - 0.99z^{-1}}$ ,  $W_u = 0.1$ . The weighting function for the sensitivity is chosen

to have a pole close to the unit circle to ensure an acceptable small steady-state error. The controller will have a pole at 0.99 approximating a stable integrator. The plant is identified for (a) different choices of model orders ranging from 1 to 10 when the true order is 2, and (b) different values of the standard deviation of the colored measurement noise  $\sigma_v$ . Fig. 6 shows the step and the magnitude response of the sensitivity function. The closed-loop system is unstable when the selected order is 1 and for some realizations of the noise, and hence these cases are not included in the figures shown here. When the model order is selected to be less than the true order, in this case 1, and when the measurement noise's standard deviation  $\sigma_v$  is large, the set of identified models does not contain the true model. Consequently the closed-loop system will be unstable.

**Comments:** The robust performance and the stability of the closed-loop system depend upon the accuracy of the identified model. One cannot simply rely on the robustness of the  $H_{\infty}$  controller to absorb the model uncertainties. The simulation results clearly show that the model error stems from an improper selection of the model order and the Signal-to-Noise Ratio (SNR) of the input-output data. The simulation results show that there is a need for an appropriate identification scheme to handle colored noise and model order selection to ensure a more robust performance and stability.

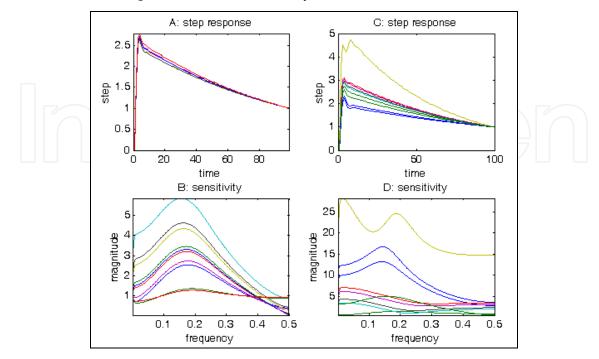


Fig. 6. Figures A and B on the left show the Step responses (top) and Magnitude responses of sensitivity (bottom) when the model order is varied from 2 to 10 when the noise standard deviation is  $\sigma_v = 0.001$ . Similarly figures C and D on the right-hand show when the noise standard deviation  $\sigma_v$  is varied in the range  $\sigma_v \in [0.02 \ 0.11]$ .

#### 5. Identification of the plant

The physical system is in general complex, high-order and nonlinear and therefore an assumed linear mathematical model of such a system is at best an approximation of the 'true model'. Nevertheless a mathematical model linearized at a given operating point can be identified and the identified model successfully used in the design of the required controller, as explained below. Some key issues in the identification of a physical system include (a) the unknown statistics of the noise and disturbance affecting the input-output data (b) the proper selection of an appropriate structure of the mathematical model, especially its order and (c) the plants operating in a closed-loop configuration.

For the case **(a)** a two-stage identification scheme, originally proposed in (Doraiswami, 2005) is employed here. First a high-order model is selected so as to capture both the system dynamics and any artifacts (from noise or other sources). Then, in the second stage, lower-order models are derived from the estimated high-order model using a frequency-weighted estimation scheme. To handle the model order selection, and the identification of the plant, especially an unstable one, approaches proposed in (Doraiswami, Cheded, and Khalid, 2010) and (Shahab and Doraiswami, 2009) are employed respectively.

#### 5.1 Model order selection

For mathematical tractability, the well-known criteria based on information-theoretic criteria such as the famous Akaike Information Criterion (Stoica and Selen, 2004), when applied to

a physical system, may require simplified assumptions such as long and uncorrelated data records, linearized models and a Gaussian probability distribution function (PDF) of the residuals. Because of these simplifying assumptions, the resulting criteria may not always give the correct model order. Generally, the estimated model order may be large due to the presence of artifacts arising from noise, nonlinearities, and pole-zero cancellation effects. The proposed model order selection scheme consists of selecting only the set of models, which are identified using the scheme proposed in (Doraiswami, 2005), and for which all the poles are in the right-half plane (Doraiswami, Cheded, and Khalid, 2010). The remaining identified models are not selected as they consist of extraneous poles.

**Proposed Criterion:** The model order selection criterion hinges on the following Lemma established in (Doraiswami, Cheded, and Khalid, 2010).

**Lemma:** If the sampling frequency is chosen in the range  $2f_c \le f_s < 4f_c$ , then the complexconjugate poles of the equivalent discrete-time equivalent of a continuous-time system will all lie on the right-half of the z-plane, whereas the real ones will all lie on the positive real line.

This shows that the discrete-time poles lie on the right-half of the z-plane if the sampling rate ( $f_s$ ) is more than twice the Nyquist rate ( $2f_c$ ). Thus, to ensure that the system poles are located on the right-half and the noise poles on the left-half of the z-plane, the sampling rate  $f_s$  must be larger than four times the maximum frequency  $f_{max}^s$  of the system, and less than four times the minimum frequency of the noise,  $f_{min}^v$ .

$$4f_{\max}^s \le f_s < 4f_{\min}^v \tag{21}$$

#### 5.2 Identification of a plant operating in closed loop

In practice, and for a variety of reasons (for e.g. analysis, design and control), it is often necessary to identify a system that must operate in a closed-loop fashion under some type of feedback control. These reasons could also include safety issues, the need to stabilize an unstable plant and /or improve its performance while avoiding the cost incurred through downtime if the plant were to be taken offline for test. In these cases, it is therefore necessary to perform closed-loop identification. There are three basic approaches to closed-loop identification, namely a direct, an indirect and a two-stage one. A direct approach to identifying a plant in a closed-loop identification scheme using the plant input and output data is fraught with difficulties due to the presence of unknown and generally inaccessible noise, the complexity of the model or a combination of both. Although computationally simple, this approach can lead to parameter estimates that may be biased due mainly to the correlation between the input and the noise, unless the noise model is accurately represented or the signal-to-noise ratio is high (Raol, Girija, & Singh, 2004). The conventional indirect approach is based on identifying the closed-loop system using the reference input and the system (plant) output. Given an estimate of the system open-loop transfer function, an estimate of the closed-loop transfer function can be obtained from the algebraic relationship between the system's open-loop and closed-loop transfer functions. The desired plant transfer function can then be deduced from the estimated closed-loop transfer function. However, the derivation of the plant transfer function from the closedloop transfer function may itself be prone to errors due to inaccuracies in the model of the subsystem connected in cascade with the plant. The two-stage approach, itself a form of an indirect method, is based on first identifying the sensitivity and the complementary

sensitivity functions using a subspace Multi-Input, Multi-Output (MIMO) identification scheme (Shahab & Doraiswami, 2009). In the second stage, the plant transfer function is obtained from the estimates of the plant input and output generated by the first stage.

#### 5.2.1 Two-stage identification

In the first stage, the sensitivity function S(z) and the complementary sensitivity functions T(z) are estimated using all the three available measurements, namely the reference input, r, plant input, u and the plant output, y, to ensure that the estimates are reliable. In other words, a Multiple-Input, Multiple-Output (MIMO) identification scheme with one input (the reference input r), and two outputs (the plant input u and the plant output y) is used here rather than a Single-Input, Single-Output (SISO) scheme using one input u and one output y. The MIMO identification scheme is based on minimizing the performance measure, J, as:

$$\min_{x} J = \|z - \hat{z}\|^2 \tag{22}$$

where  $z = \begin{bmatrix} y & u \end{bmatrix}^T$  and  $\hat{z} = \begin{bmatrix} \hat{y} & \hat{u} \end{bmatrix}^T$ ,  $\hat{u}$  is the estimated plant input and  $\hat{y}$  is the estimated plant output. The plant input *u*, and the plant output *y* are related to the reference input *r* and the disturbance *w* by:

$$u(z) = S(z)r(z) + S(z)w(z)$$
(23)

$$y(z) = T(z)r(z) + T(z)w(z) + v(z)$$
(24)

As pointed out earlier, the proposed MIMO identification scheme will ensure that the estimates of the sensitivity and the complementary sensitivity functions are consistent (i.e. they have identical denominators), and hence will also ensure that the estimates of the plant input *u* and the plant output *y*, which are both employed in the second stage, are reliable. Note here that the reference signal *r* is uncorrelated with the measurement noise *w* and the disturbance *v*, unlike in the case where the plant is identified using the direct approach. This is the main reason for using the MIMO scheme in the first stage. In the second stage, the plant *G*(*z*) is identified from the estimated plant input,  $\hat{u}$ , and plant output,  $\hat{y}$ , obtained from the stage 1 identification scheme, i.e.:

$$\hat{u}(z) = \hat{S}(z)r(z)$$
(25)

$$\hat{y}(z) = T(z)r(z) \tag{26}$$

Note that here the input  $\hat{u}$  and the output  $\hat{y}$  are not correlated with the noise w and disturbance term v. Treating  $\hat{u}$  as the input and  $\hat{y}$  as the output of the plant, and  $\hat{y}$  as the estimate of the plant output estimate,  $\hat{y}$ , the identification scheme is based on minimizing the weighted frequency-domain performance measure

$$\min_{\hat{\hat{y}}_{j}} \left\| W(j\omega) \left( \hat{y}(j\omega) - \hat{\hat{y}}(j\omega) \right) \right\|^{2}$$
(27)

where  $W(j\omega)$  is the weighting function. Furthermore, it is shown that:

**Lemma:** If the closed-loop system is stable, then

- The unstable poles of the plant must be cancelled exactly by the zeros of the sensitivity function if the reference input is bounded.
- The zeros of the plants form a subset of the zeros of the complementary transfer function

This provides a cross-checking of the estimates of the poles and the zeros of the plant estimated in the second stage with the zeros of the sensitivity and complementary functions in the first stage, respectively.

#### 6.1 Evaluation on a physical system: magnetic levitation system (MAGLEV)

The physical system is a feedback magnetic levitation system (MAGLEV) (Galvao, Yoneyama, Marajo, & Machado, 2003). Identification and control of the magnetic levitation system has been a subject of research in recent times in view of its applications to transportation systems, magnetic bearings used to eliminate friction, magnetically-levitated micro robot systems, magnetic levitation-based automotive engine valves. It poses a challenge for both identification and controller design.

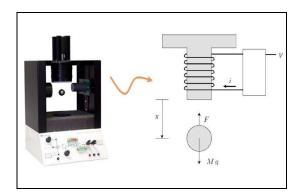


Fig. 7. Laboratory-scale MAGLEV system

The model of the MAGLEV system, shown in Fig. 7, is unstable, nonlinear and is modeled by:

$$\frac{y(s)}{u(s)} = \frac{\beta}{s^2 - \alpha}$$
(28)

where *y* is the position, and *u* the voltage input. The poles, *p*, of the plant are real and are symmetrically located about the imaginary axis, i.e.:  $p = \pm \sqrt{\alpha}$ . The linearized model of the system was identified in a closed-loop configuration using LABVIEW data captured through both A/D and D/A devices. Being unstable, the plant was identified in a closed-loop configuration using a controller which was a lead compensator. The reference input was a rich persistently-exciting signal consisting of a random binary sequence. An appropriate sampling frequency was determined by analyzing the input-output data for different choices of the sampling frequencies. A sampling frequency of 5msec was found to be the best as it proved to be sufficiently small to capture the dynamics of the system but not the noise artifacts. The physical system was identified using the proposed two-stage MIMO identification scheme. First, the sensitivity and complementary sensitivity functions of the

closed-loop system were identified. The estimated plant input and output were employed in the second stage to estimate the plant model. The model order for identification was selected to be second order using the proposed scheme. Figure 8 below gives the pole-zero maps of both the plant and the sensitivity function on the left-hand side, and, on the righthand side, the comparison between the frequency response of the identified model  $\hat{G}(j\omega)$ , obtained through non-parametric identification, i.e. estimated by injecting various sinusoidal inputs of different frequencies applied to the system, and the estimate of the transfer function obtained using the proposed scheme.

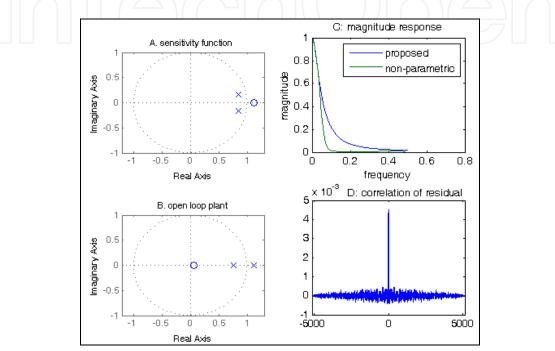


Fig. 8. A and B show pole-zero maps of the plant and of the sensitivity function (left) while C and D (right) show the comparison of the frequency response of the identified model with the non-parametric model estimate, and the correlation of the residual, respectively

The nominal closed-loop input sensitivity function was identified as:

$$S_0(z) = \frac{1.7124 z^{-1} \left(1 - 1.116 z^{-1}\right)}{1 - 1.7076 z^{-1} + 0.7533 z^{-2}}$$
(29)  
and the nominal plant model as:

$$G_0(z) = \frac{N_0}{D_0} = \frac{0.0582z^{-1} \left(1 - 0.0687z^{-1}\right)}{\left(1 - 1.116z^{-1}\right) \left(1 - 0.7578z^{-1}\right)}$$
(30)

#### 6.1.1 Model validation

The identified model was validated using the following criteria:

• The proposed model-order selection was employed. The identifications in stages I and II were performed for orders ranging from 1 to 4. A second-order model was selected in both stages since all the poles of the identified model were located in the right-half of

the z-plane. Note here that the dynamics of the actuator (electrical subsystem) was not captured by the model as it is very fast compared to that of the mechanical subsystem.

- A 4<sup>th</sup> order model was employed in stage I to estimate the plant input and the output for the subsequent stage II identification.
- The plant has one stable pole located at 0.7580 and one unstable pole at 1.1158. The reciprocity condition is not exactly satisfied as, theoretically, the stable pole should be at 0.8962 and not at 0.7580.
- The zeros of the sensitivity function contain the unstable pole of the plant, i.e. the unstable pole of the plant located at 1.1158 is a zero of the sensitivity function.
- The frequency responses of the plant, computed using two entirely different approaches, should be close to each other. In this case, a non-parametric approach was employed and compared to the frequency response obtained using the proposed model-based scheme, as shown on the right-hand side of Fig. 8. The non-parametric approach gives an inaccurate estimate at high frequencies due to correlation between the plant input and the noise.
- The residual is zero mean white noise with very small variance.

#### 6.1.2 $H_{\infty}$ Mixed sensitivity $H_{\infty}$ controller design

The weighting functions are selected by giving more emphasis on robust stability and less on robust performance:  $W_s(j\omega) = 0.001$  and  $W_u(j\omega) = 0.1$ . To improve the robustness of the closed-loop system, a feed-forward control of the reference input is used, instead of the inclusion of an integrator in the controller. The  $H_{\infty}$  controller is given by:

$$C_0(z) = \frac{2.5734 \left(1 + 1.113 z^{-1}\right) \left(1 - 0.7578 z^{-1}\right)}{\left(1 - 0.2044 z^{-1}\right) \left(1 + 0.7457 z^{-1}\right)}$$
(31)

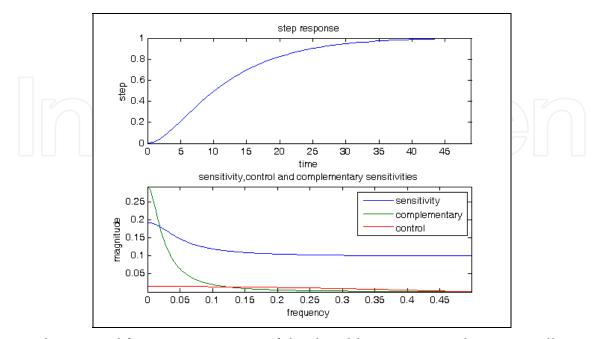


Fig. 9. The step and frequency responses of the closed-loop system with  $H_{\infty}$  controller

It is interesting to note here that there is a pole-zero cancelation between the nominal plant and the controller since a plant pole and a controller zero are both equal to 0.7578. In this case, the  $H_{\infty}$  norm is  $\gamma = 0.1513$  and hence the performance and stability measure is  $\|[W_S S_0 \quad W_u S_{u0}]\|_{\infty} = \gamma = 0.1513$  with  $\|[\Delta_N \quad \Delta_D]\|_{\infty} \le 1/\gamma = 6.6087$ . The step response and magnitude responses of the weighted sensitivity, complementary sensitivity and the control input sensitivity of the closed-loop control system are all shown above in Fig. 9.

#### 6.2 Evaluation on a physical sensor network: a two-tank liquid level system

The physical system under evaluation here is formed of two tanks connected by a pipe. A dc motor-driven pump supplies fluid to the first tank and a PI controller is used to control the fluid level in the second tank by maintaining the liquid height at a specified level, as shown in Fig. 10. This system is a cascade connection of a dc motor and a pump relating the input to the motor, u, and the flow  $Q_i$ . It is expressed by the following first-order time-delay system:

$$\dot{Q}_i = -a_m Q_i + b_m \phi(u) \tag{32}$$

where  $a_m$  and  $b_m$  are the parameters of the motor-pump subsystem and  $\phi(u)$  is a dead-band and saturation-type of nonlinearity. The Proportional and Integral (PI) controller is given by:

where  $k_p$  and  $k_I$  are the PI controller's gains and r is the reference input.

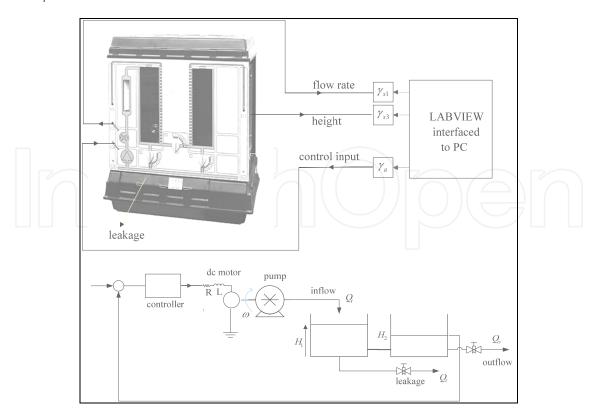


Fig. 10. Two-tank liquid level system

With the inclusion of the leakage, the liquid level system is now modeled by :

$$A_{1} \frac{dH_{1}}{dt} = Q_{i} - C_{12}\varphi(H_{1} - H_{2}) - C_{\ell}\varphi(H_{1})$$

$$A_{2} \frac{dH_{2}}{dt} = C_{12}\varphi(H_{1} - H_{2}) - C_{0}\varphi(H_{2})$$
(34)

where  $\varphi(.) = sign(.)\sqrt{2g(.)}$ ,  $Q_{\ell} = C_{\ell}\varphi(H_1)$  is the leakage flow rate,  $Q_0 = C_0\varphi(H_2)$  is the output flow rate,  $H_1$  is the height of the liquid in tank 1,  $H_2$  the height of the liquid in tank 2,  $A_1$ and  $A_2$  the cross-sectional areas of the 2 tanks,  $g=980 \text{ cm} / \sec^2$  the gravitational constant, and  $C_{12}$  and  $C_o$  the discharge coefficients of the inter-tank and output valves, respectively. The linearized model of the entire system formed by the motor, pump, and the tanks is given by:

$$\dot{x} = Ax + Br$$

$$y = Cx$$
(35)

where *x*, *A*, *B* and *C* are given by:

$$x = \begin{bmatrix} h_1 \\ h_2 \\ x_3 \\ q_i \end{bmatrix}, A = \begin{bmatrix} -a_1 - \alpha & a_1 & 0 & b_1 \\ a_2 & -a_2 - \beta & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -b_m k_p & 0 & b_m k_I & -a_m \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 & b_m k_p \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

 $q_i$ ,  $q_\ell$ ,  $q_0$ ,  $h_1$  and  $h_2$  are respectively the increments in  $Q_i$ ,  $Q_\ell$ ,  $Q_o$ ,  $H_1^0$  and  $H_2^0$ , whereas  $a_1$ ,  $a_2$ ,  $\alpha$  and  $\beta$  are parameters associated with the linearization process,  $\alpha$  is the leakage flow rate,  $q_\ell = \alpha h_1$ , and  $\beta$  is the output flow rate, and  $q_o = \beta h_2$ . The dual-tank fluid system structure can be cast into that of an interconnected system with a sensor network, composed of 3 subsystems  $G_{eu}$ ,  $G_{uq}$ , and  $G_{qh}$  relating the measured signals, namely the error e, control input u, flow rate Q and the height h, respectively. The proposed two-stage identification scheme is employed to identify these subsystems. It consists of the following two stages:

- In Stage 1, the MIMO closed-loop system is identified using data formed of the reference input *r*, and the subsystems' outputs measured by the 3 available sensors.
- In Stage 2, the subsystems  $G_{eu}$   $G_{uq}$ , and  $G_{qh}$  are then identified using the subsystem's estimated input and output measurements obtained from the first stage.

Figure 11 shows the estimation of the 4 key signals e, u, Q and h in our two-tank experiment, that are involved in the MIMO transfer function in stage I identification. Stage I identification yields the following MIMO closed-loop transfer function given by:

$$\begin{bmatrix} \hat{e}(z) & \hat{u}(z) & \hat{f}(z) & \hat{h}(z) \end{bmatrix}^T = D^{-1}(z)N(z)r(z)$$
(36)

456

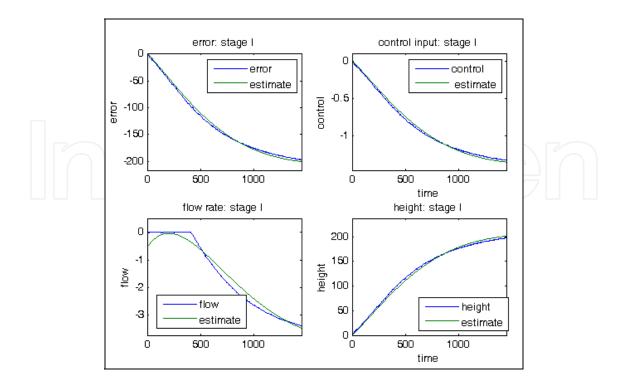


Fig. 11. (Left) The error and flow rate and their estimates and (Right) the control input and height and their estimates.

where 
$$N = \begin{bmatrix} 1.9927 & -191.5216z^{-1} & 380.4066z^{-2} & -190.8783z^{-3} \\ 0.0067 & -1.2751z^{-1} & 2.5526z^{-2} & -1.2842z^{-3} \\ -183.5624 & 472.5772z^{-1} & -394.4963z^{-2} & 105.4815z^{-3} \\ -0.9927 & 189.1386z^{-1} & -378.6386z^{-2} & 190.4933z^{-3} \end{bmatrix}$$
  
 $D = 1.0000 = 2.3830z^{-1} + 1.7680z^{-2} = 0.3850z^{-3}$ 

The zeros of the sensitivity function, relating the reference input r to the error e, are located at 1.02 and 1.0.

Fig. 12 below shows the combined plots of the actual values of the height, flow rate and control input, and their estimates from both stages 1 and 2. From this figure, we can conclude that the results are on the whole excellent, especially for both the height and control input.

Stage II identification yields the following three open-loop transfer functions that are identified using their respective input/output estimates generated by the stage-1 identification process:

$$\hat{G}_{eu}(z) = \frac{u(z)}{e(z)} = 0.0067 + \frac{0.4576z^{-1}}{1 - z^{-1}}$$
(37)

$$G_{uq}(z) = \frac{Q(z)}{u(z)} = \frac{0.0104z^{-1}}{1 - 0.9968z^{-1}}$$
(38)

457

$$Q(z) \quad 1 - 1.0039z^{-1}$$

1000

1000

1000

1200

1200

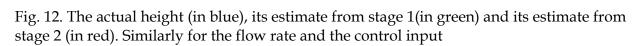
1200

1400

1400

1400

$$G_{qh}(z) = \frac{h(z)}{Q(z)} = \frac{0.7856z^{-1}}{1 - 1.0039z^{-1}}$$
(39)



#### **Comments:**

- The two-tank level system is highly nonlinear as can be clearly seen especially from the flow rate profile located at the top right corner of Fig. 11. There is a saturation-type nonlinearity involved in the flow process.
- The subsystems  $G_{eu}$  and  $G_{qh}$  representing respectively the PI controller and the transfer function relating the flow rate to the tank height are both unstable with a pole at unity representing an integral action. The estimated transfer functions  $\hat{G}_{eu}$  and  $\hat{G}_{qh}$  have captured these unstable poles. Although the pole of  $\hat{G}_{eu}$  is exactly equal to unity, the pole of  $\hat{G}_{qh}$ , located at 1.0039, is very close to unity. This slight deviation from unity may be due to the nonlinearity effects on the flow rate
- The zeros of the sensitivity function have captured the unstable poles of the open-loop unstable plant with some error. The values of the zeros of the sensitivity function are 1.0178, and 1.0002 while those of the subsystem poles are 1 and 1.0039.

#### **6.2.1** Mixed-sensitivity $H_{\infty}$ controller design

0. 0

0 -1 flow -2 -3

Π

0

0 -0.5 control -1

200

200

200

400

400

400

600

600

ഞ

800

800

800

flow rate and its estimate: stage II

control input and its estumate: stage II

time

The identified plant is the cascade combination of the motor, pump and the two tanks, which is essentially the forward path transfer function formed of the cascade combinations of  $G_{uq}$  and  $G_{qh}$ , that relates the control input *u* to the tank height *h*, and which is given by:

$$G_0(z) = \frac{N_0(z)}{D_0(z)} = \frac{z^{-2}}{1 - 1.997z^{-1} + 0.9968z^{-2}}$$
(40)

The weighting functions are selected by giving more emphasis on robust stability and less on robust performance:  $W_s(z) = \left[ 0.01 / \left( 1 - 0.99z^{-1} \right) \right]$  and  $W_u(z) = 1$  where  $z = e^{j\omega}$ . The  $H_{\infty}$  controller is then given by:

$$C_0(z) = \frac{0.044029(1+z^{-1})(1-1.98z^{-1}+0.9804z^{-2})}{(1-0.99z^{-1})(1-0.6093z^{-1})(1+0.6008z^{-2})}$$
(41)

The controller has an approximate integral action for steady-state tracking with disturbance rejection and a pole at 0.99 which is very close to unity. In this case, the  $H_{\infty}$  norm is  $\gamma = 0.0663$ . The step response and the magnitude responses of the sensitivity, complementary sensitivity and the control input sensitivity of the closed-loop control system are all shown in Fig. 13.

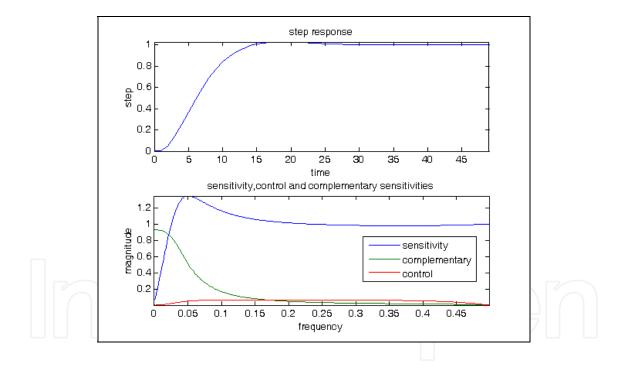


Fig. 13. Step and magnitude freq. res1ponses of the closed-loop system with  $H_{\infty}$  controller

#### 6.2.2 Remarks on the mixed-sensitivity $H_{\infty}$ control design

The sensitivity is low in the low frequency regions where the denominator perturbations are large, the control sensitivity is small in the high frequency regions of the numerator perturbations, and the complementary sensitivity is low in the high frequency region where the overall multiplicative model perturbations are high. As the robustness is related to

performance, this will ensure robust performance for steady-state tracking with disturbance rejection, controller input limitations and measurement noise attenuation. When tight performance bounds are specified, the controller will react strongly but may be unstable when implemented on the actual physical plant. For safety reasons, the controller design is started with very loose performance bounds, resulting in a controller with very small gains to ensure stability of the controller on the actual plant. Then, the performance bounds are made tighter to gradually increase the performance of the controller. The design method based on the mixed-sensitivity criterion generalizes some classical control design techniques such as the classical loop-shaping technique, integral control to ensure tracking, performance and specified high frequency roll-off, and direct control over the closed-loop bandwidth and time response by means of pole placement.

#### 7. Conclusion

This chapter illustrates, through analysis, simulation and practical evaluation, how the two key objectives of control system design, namely robust stability and robust performance, can be achieved. Specifically, it shows that in order to ensure both robust performance and robust stability of a closed-loop system where the controller is designed based on an identified model of the plant, it is then of paramount importance that both the identification scheme as well as the controller design strategy be selected appropriately, as the tightness of the achieved robustness bound depends on the magnitude of the modeling error produced by the selected identification scheme. In view of this close dependence, a comprehensive closed-loop identification scheme was proposed here that greatly mitigates the effects of measurement noise and disturbances and relies on a novel model order selection scheme. More specifically, the proposed identification consists of (a) a two-stage scheme to overcome the unknown noise and disturbance by first obtaining a high-order model, and then deriving from it a reduced-order model, (b) a novel model-order selection criterion based on verifying the location of the poles and (c) a two-stage scheme to identify first the closed-loop transfer functions of subsystems, and then obtain the plant model using the estimates on the input and output from the first stage. The controller design was based on the well-known mixed-sensitivity  $H_{\infty}$  controller design technique that achieves simultaneously robust stability and robust performance. This technique is able to handle plant uncertainties modeled as additive perturbations in the numerator and denominator of the identified model, and provides tools to achieve a trade-off between robust stability, robust performance and control input limitations. The identification and controller design were both successfully evaluated on a number of simulated as well practical physical systems including the laboratory-scale unstable magnetic levitation and two-tank liquid level systems. This study has provided us with ample encouragement to replicate the use of the powerful techniques used in this chapter, on different systems and to enrich the overall approach with other identification and robust controller design.

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Robust control has been a topic of active research in the last three decades culminating in H\_2/H\_\infty and \mu design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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