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New Practical Integral Variable Structure Controllers for Uncertain Nonlinear Systems

Jung-Hoon Lee
Gyeongsang National University
South Korea

1. Introduction

Stability analysis and controller design for uncertain nonlinear systems is open problem now(Vidyasagar, 1986). So far numerous design methodologies exist for the controller design of nonlinear systems(Kokotovic & Arcak, 2001). These include any of a huge number of linear design techniques(Anderson & More, 1990; Horowitz, 1991) used in conjuction with gain scheduling(Rugh & Shamma, 200); nonlinear design methodologies such as Lyapunov function approach(Vidyasagar, 1986; Kokotovic & Arcak, 2001; Cai et al., 2008; Gutman, 1979; Slotine & Li, 1991; Khalil, 1996), feedback linearization method(Hunt et al., 1987; Isidori, 1989; Slotine & Li, 1991), dynamics inversion(Slotine & Li, 1991), backstepping(Lijun & Chengkand, 2008), adaptive technique which encompass both linear adaptive(Narendra, 1994) and nonlinear adaptive control(Zheng & Wu, 2009), sliding mode control(SMC)(Utkin, 1978; Decarlo etal., 1988; Young et al., 1996; Drazenovic, 1969; Toledo & Linares, 1995; Bartolini & Ferrara, 1995; Lu & Spurgeon, 1997), and etc(Hu & Martin, 1999; Sun, 2009; Chen, 2003).

The sliding mode control can provide the effective means to the problem of controlling uncertain nonlinear systems under parameter variations and external disturbances(Utkin, 1978; Decarlo et. al., 1988; Young et al., 1996). One of its essential advantages is the robustness of the controlled system to variations of parameters and external disturbances in the sliding mode on the predetermined sliding surface, $s=0$(Drazenovic, 1969). In the VSS, there are the two main problems, i.e., the reaching phase at the initial stage(Lee & Youn, 1994) and chattering of the input (Chern & Wu, 1992). To remove the reaching phase, the two requirements are needed, i.e., the sliding surface must be determined from an any given initial state to the origin($s(x)|_{x=x(0)\ \&\ t=0}=0$) and the control input must satisfy the existence condition of the sliding mode on the pre-selected sliding surface for all time from the initial to the final time($s^T\dot{s}<0, \quad \text{for} \quad t\geq 0$).

In (Toledo & Linares, 1995), the sliding mode approach is applied to nonlinear output regulator schemes. The underlying concept is that of designing sliding submanifold which contains the zero tracking error sub-manifold. The convergence to a sliding manifold can be attained relying on a control strategy based on a simplex of control vectors for multi input uncertain nonlinear systems(Bartolini & Ferrara, 1995). A nonlinear optimal integral variable

structure controller with an arbitrary sliding surface without the reaching phase was proposed for uncertain linear plants(Lee, 1995). (Lu and Spurgeon, 1997) considered the robustness of dynamic sliding mode control of nonlinear system, which is in differential input-output form with additive uncertainties in the model. The discrete-time implementation of a second-order sliding mode control scheme is analyzed for uncertain nonlinear system in (Bartolini et al., 2001). (Adamy & Flemming, 2004) surveyed so called soft variable structure controls, compared them to others. The tracker control problem that is the regulation control problem from an arbitrary initial state to an arbitrary final state without the reaching phase is handled and solved for uncertain SISO linear plants in (Lee, 2004). For 2nd order uncertain nonlinear system with mismatched uncertainties, a switching control law between a first order sliding mode control and a second order sliding mode control is proposed to obtain the globally or locally asymptotic stability(Wang et al., 2007). The optimal SMC for nonlinear system with time-delay is suggested(Tang et al., 2008). The nonlinear time varying sliding sector is designed for continuous control of a single input nonlinear time varying input affine system which can be represented in the form of state dependent linear time variant systems with matched uncertainties(Pan et al., 2009). For uncertain affine nonlinear systems with mismatched uncertainties and matched disturbance, the systematic design of the SMC is reported(Lee, 2010a). The two clear proofs of the existence condition of the sliding mode with respect to the two transformations i.e., the two diagonalization methods are given for multi-input uncertain linear plants(Lee 2010b), while (Utkin, 1978) and (Decarlo et al., 1988) proved unclearly for uncertain nonlinear plants.

Until now, the integral action is not introduced to the variable structure system for uncertain nonlinear system with mismatched uncertainties and matched disturbance to improve the output performance by means of removing the reaching phase problems. And a nonlinear output feedback controller design for uncertain nonlinear systems with mismatched uncertainties and matched disturbance is not presented.

In this chapter, a systematic general design of new integral nonlinear full-state(output) feedback variable structure controllers based on state dependent nonlinear form is presented for the control of uncertain affine nonlinear systems with mismatched uncertainties and matched disturbances. After an affine uncertain nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of a new nonlinear full-state(output) feedback variable structure controller is presented. To be linear in the closed loop resultant dynamics, full-state(output) feedback (transformed) integral linear sliding surfaces are applied in order to remove the reaching phase, those are stemmed from the studys by (Lee & Youn, 1994; Lee, 2010b) which is the first time work of removing the reaching phase with the idea of introducing the initial condition for the integral state. The corresponding discontinuous (transformed) control inputs are proposed to satisfy the closed loop exponential stability and the existence condition of the sliding mode on the full-state(output) feedback integral sliding surfaces, which will be investigated in Theorem 1 and Theorem 2. For practical application to the real plant by means of removing the chattering problems, the implementation of the continuous approximation is essentially needed instead of the discontinuous input as the inherent property of the VSS. Using the saturation function, the different form from that of (Chern & Wu, 1992) for the continuous approximation is suggested. The two main problems of the VSS are removed and solved. Through the design examples and simulation studies, the usefulness of the proposed practical integral nonlinear VSS controller is verified.

2. Practical integral nonlinear variable structure systems

2.1 Descriptions of plants

Consider an affine uncertain nonlinear system

$$\dot{x} = f'(x,t) + g(x,t)u + \bar{d}(x,t), \qquad x(0) \tag{1}$$

$$y = C \cdot x, \qquad y(0) = C \cdot x(0) \tag{2}$$

where $x \in R^n$ is the state, $x(0)$ is its initial condition for the state, $y \in R^q$, $q \le n$ is the output, $y(0)$ is an initial condition of the output, $u \in R^1$ is the control to be determined, mismatched uncertainty $f'(x,t) \in C^k$ and matched uncertainty $g(x,t) \in C^k$, $k \ge 1$, $g(x,t) \ne 0$ for all $x \in R^n$ and for all $t \ge 0$ are of suitable dimensions, and $\bar{d}(x,t)$ implies bounded matched external disturbances.

Assumption (Pan et al., 2009)

A1: $f'(x,t) \in C^k$ is continuously differentiable and $f'(0,t) = 0$ for all $t \ge 0$.

Then, uncertain nonlinear system (1) can be represented in more affine nonlinear system of state dependent coefficient form(Pan et al., 2009; Hu & Martin, 1999; Sun, 2009)

$$\begin{aligned} \dot{x} &= f(x,t)x + g(x,t)u + \bar{d}(x,t), \qquad x(0) \\ &= [f_0(x,t) + \Delta f(x,t)]x + [g_0(x,t) + \Delta g(x,t)]u + \bar{d}(x,t) \\ &= f_0(x,t)x + g_0(x,t)u + d(x,t) \end{aligned} \tag{3}$$

$$y = C \cdot x, \tag{4}$$

$$d(x,t) = \Delta f(x,t)x + \Delta g(x,t)u + \bar{d}(x,t) \tag{5}$$

where $f_0(x,t)$ and $g_0(x,t)$ is each nominal value such that $f'(x,t) = [f_0(x,t) + \Delta f(x,t)]x$ and $g(x,t) = [g_0(x,t) + \Delta g(x,t)]$, respectively, $\Delta f(x,t)$ and $\Delta g(x,t)$ are mismatched or matched uncertainties, and $d(x,t)$ is the mismatched lumped uncertainty.

Assumption:

A2: The pair $(f_0(x,t), g_0(x,t))$ is controllable and $(f_0(x,t), C)$ is observable for all $x \in R^n$ and all $t \ge 0$ (Sun, 2009).

A3: The lumped uncertainty $d(x,t)$ is bounded.

A4: $\ddot{x}$ is bounded if $\dot{u}$ and $\dot{d}(x,t)$ is bounded.

2.2 Full sate feedback practical integral variable structure controller

2.2.1 Full-state feedback integral sliding surface

For use later, the integral term of the full-state is augmented as

$$x_0 = \int_0^t x(\tau)d\tau + \int_{-\infty}^0 x(\tau)d\tau = \int_0^t x(\tau)d\tau + x_0(0) \tag{6}$$

To control uncertain nonlinear system (1) or (3) with a linear closed loop dynamics and without reaching phase, the full-state feedback integral sliding surface used in this design is as follows:

$$S_f = L_1 x + L_1 x_0 = \begin{bmatrix} L_1 & L_0 \end{bmatrix} \cdot \begin{bmatrix} x \\ x_0 \end{bmatrix} (= 0) \tag{7}$$

where

$$x_0(0) = -L_0^- L_1 x(0) \tag{7a}$$

and $L_0^- = (L_0^T W L_0)^{-1} L_0^T W$, which is stemmed from the work by (Lee & Youn, 1994). At $t = 0$, the full-state feedback integral sliding surface is zero, Hence, the one of the two requirements is satisfied. Without the initial condition of the integral state, the reaching phase is not removed except the exact initial state on the sliding surface. With the initial condition (7a) for the integral state, the work on removing the reaching phase was reported by (Lee & Youn, 1994) for the first time, which is applied to the VSS for uncertain linear plants. In (7), L_1 is a non zero element as the design parameter such that the following assumption is satisfied.

Assumption

A5: $L_1 g(x,t)$ and $L_1 g_0(x,t)$ have the full rank, i.e. those are invertible

A6: $L_1 \Delta g(x,t) \left[L_1 g_0(x,t) \right]^{-1} = \Delta I$ and $|\Delta I| \leq \xi < 1$.

In (7), the design parameters L_1 and L_0 satisfy the following relationship

$$L_1 \left[f_0(x,t) - g_0(x,t) K(x) \right] + L_0 = 0 \tag{8a}$$

$$L_0 = -L_1 \left[f_0(x,t) - g_0(x,t) K(x) \right] = -L_1 f_c(x,t) \tag{8b}$$

$$f_c(x,t) = \left[f_0(x,t) - g_0(x,t) K(x) \right] \tag{8c}$$

The equivalent control input is obtained using $\dot{S}_f = 0$ as(Decarlo et al., 1998)

$$u_{eq} = -\left[L_1 g(x,t) \right]^{-1} \left[L_1 f_0(x,t) + L_0 \right] x - \left[L_1 g(x,t) \right]^{-1} \Delta f(x,t) x - \left[L_1 g(x,t) \right]^{-1} \bar{d}(x,t) \tag{9}$$

This control input can not be implemented because of the uncertainties, but used to obtaining the ideal sliding dynamics. The ideal sliding mode dynamics of the sliding surface (7) can be derived by the equivalent control approach(Lee, 2010a) as

$$\dot{x}_s = \left[f_0(x_s,t) - g_0(x_s,t) \left[L_1 g(x_s,t) \right]^{-1} \left\{ L_1 f_0(x_s,t) + L_0 \right\} \right] x_s, \qquad x_s(0) = x(0) \tag{10}$$

$$\dot{x}_s = \left[f_0(x_s,t) - g_0(x_s,t) K(x_s) \right] x_s = f_c(x_s,t) x_s, \qquad x_s(0) = x(0) \tag{11}$$

$$K(x_s) = \left[L_1 g(x_s,t) \right]^{-1} \left\{ L_1 f_0(x_s,t) + L_0 \right\} \tag{12}$$

The solution of (10) or (11) identically defines the integral sliding surface. Hence to design the sliding surface as stable, this ideal sliding dynamics is designed to be stable, the reverse argument also holds. To choose the stable gain based on the Lyapunov stability theory, the ideal sliding dynamics (10) or (11) is represented by the nominal plant of (3) as

$$\begin{aligned} \dot{x} &= f_0(x,t) x + g_0(x,t) u, & u &= -K(x) x \\ &= f_c(x,t) x, & f_c(x,t) &= f_0(x,t) - g_0(x,t) K(x) \end{aligned} \tag{13}$$

To select the stable gain, take a Lyapunov function candidate as

$$V(x) = x^T Px, \qquad P > 0 \tag{14}$$

The derivative of (14) becomes

$$\dot{V}(x) = x^T \left[f_0^T(x,t)P + Pf_0(x,t) \right] x + u^T g_0^T(x,t)Px + x^T Pg_0(x,t)u \tag{15}$$

By the Lyapunov control theory(Slotine & Li, 1991), take the control input as

$$u = -g_0^T(x,t)Px \tag{16}$$

and $Q(x,t) > 0$ and $Q_c(x,t) > 0$ for all $x \in R^n$ and all $t \geq 0$ is

$$f_0^T(x,t)P + Pf_0(x,t) = -Q(x,t) \tag{17}$$

$$f_c^T(x,t)P + Pf_c(x,t) = -Q_c(x,t) \tag{18}$$

then

$$\begin{aligned} \dot{V}(x) &= -x^T Q(x,t)x - 2x^T Pg_0(x,t)g_0^T(x,t)Px \\ &= -x^T [Q(x,t) + 2Pg_0(x,t)g_0^T(x,t)P]x \\ &= -x^T [f_c^T(x,t)P + Pf_c(x,t)]x \\ &= -x^T Q_c(x,t)x \\ &\leq -\lambda_{\min}\{Q_c(x,t)\} \| x \|^2 \end{aligned} \tag{19}$$

where $\lambda_{\min}\{Q_c(x,t)\}$ means the minimum eigenvalue of $Q_c(x,t)$. Therefore the stable static nonlinear feedback gain is chosen as

$$K(x) = g_0^T(x,t)P \quad \text{or} \quad = \left[L_1 g_0(x,t) \right]^{-1} \left\{ L_1 f_0(x,t) + L_0 \right\} \tag{20}$$

2.2.2 Full-state feedback transformed discontinuous control input

The corresponding control input with the transformed gains is proposed as follows:

$$u_f = -K(x)x - \Delta Kx - K_1 S_f - K_2 sign(S_f) \tag{21}$$

where $K(x)$ is a static nonlinear feedback gain, ΔK is a discontinuous switching gain, K_1 is a static feedback gain of the sliding surface itself, and K_2 is a discontinuous switching gain, respectively as

$$\Delta K = \left[L_1 g_0(x,t) \right]^{-1} [\Delta k_i] \qquad i = 1,...,n \tag{22}$$

$$\Delta k_i = \begin{cases} \geq \dfrac{\max\left\{ L_1 \Delta f(x.t) - L_1 \Delta g(x,t)K(x) \right\}_i}{\min\left\{ I + \Delta I \right\}} & sign(S_f x_i) > 0 \\ \leq \dfrac{\min\left\{ L_1 \Delta f(x.t) - L_1 \Delta g(x,t)K(x) \right\}_i}{\min\left\{ I + \Delta I \right\}} & sign(S_f x_i) < 0 \end{cases} \tag{23}$$

$$K_1 = \left[L_1 g(x,t)\right]^{-1} K_1', \qquad K_1' > 0 \tag{24}$$

$$K_2 = \left[L_1 g(x,t)\right]^{-1} K_2', \qquad K_2' = \frac{\max\{|\, L_1 d(x,t)\,|\}}{\min\{I + \Delta I\}} \tag{25}$$

which is transformed for easy proof of the existence condition of the sliding mode on the chosen sliding surface as the works of (Utkin, 1978; Decarlo et al., 1988; Lee, 2010b). The real sliding dynamics by the proposed control with the linear integral sliding surface is obtained as follows:

$$\begin{aligned}
\dot{S}_f &= [L_1\dot{x} + L_0 x] \\
&= L_1[f_0(x,t)x + \Delta f(x,t)x + g(x,t)u_f + \bar{d}(x,t)] + L_0 x \\
&= L_1\left[f_0(x,t)x + \Delta f(x,t)x + g(x,t)\left\{-K(x)x - \Delta K x - K_1 S_f - K_2 sign(S_f)\right\} + \bar{d}(x,t)\right] + L_0 x \\
&= L_1 f_0(x,t)x - L_1 g_0(x,t)K(x)x + L_0 x + L_1\Delta f(x,t)x - L_1\Delta g(x,t)K(x)x \\
&\quad - L_1 g(x,t)\Delta K x - L_1 g(x,t)K_1 S_f + L_1\bar{d}(x,t) - L_1 g(x,t)K_2 sign(S_f) \\
&= L_1\Delta f(x,t)x - L_1\Delta g(x,t)K(x)x - [I + \Delta I]L_1 g_0(x,t)\Delta K x - [I + \Delta I]L_1 g_0(x,t)K_1 S_f \\
&\quad + L_1\bar{d}(x,t) - [I + \Delta I]L_1 g_0(x,t)K_2 sign(S_f)
\end{aligned} \tag{26}$$

The closed loop stability by the proposed control input with sliding surface together with the existence condition of the sliding mode will be investigated in next Theorem 1.

Theorem 1: *If the sliding surface (7) is designed in the stable, i.e. stable design of* $K(x)$*, the proposed input (21) with Assumption A1-A6 satisfies the existence condition of the sliding mode on the integral sliding surface and exponential stability.*

Proof(Lee, 2010b); Take a Lyapunov function candidate as

$$V(x) = \frac{1}{2} S_f^T S_f \tag{27}$$

Differentiating (27) with respect to time leads to and substituting (26) into (28)

$$\begin{aligned}
\dot{V}(x) &= S_f^T \dot{S}_f \\
&= S_f^T L_1\Delta f(x,t)x - S_f^T L_1\Delta g(x,t)K(x)x - S_f^T[I + \Delta I]L_1 g_0(x,t)\Delta K x \\
&\quad - S_f^T[I + \Delta I]L_1 g_0(x,t)K_1 S_f + S_f^T L_1\bar{d}(x,t) - S_f^T[I + \Delta I]L_1 g_0(x,t)K_2 sign(S_f) \\
&\le -\varepsilon K_1' |\,|S_f|\,|^2, \qquad \varepsilon = \min\{|\,|I + \Delta I|\,|\} \\
&= -\varepsilon K_1' S_f^T S_f \\
&= -2\varepsilon K_1' V(x)
\end{aligned} \tag{28}$$

The second requirement to remove the reaching phase is satisfied. Therefore, the reaching phase is completely removed. There are no reaching phase problems. As a result, the real output dynamics can be exactly predetermined by the ideal sliding output with the matched uncertainty. From (28), the following equations are obtained as

$$\dot{V}(x) + 2\varepsilon K_1' V(x) \le 0 \tag{29}$$

$$V(x(t)) \le V(x(0))e^{-2\varepsilon K_1 t} \tag{30}$$

And the second order derivative of $V(x)$ becomes

$$\ddot{V}(x) = \dot{S}_f^T \dot{S}_f + S_f^T \ddot{S}_f = \| \dot{S}_f \| ^2 + S_f (L_1 C\ddot{x} + L_0 \dot{x}) < \infty \tag{31}$$

and by Assumption A5 $\ddot{V}(x)$ is bounded, which completes the proof of Theorem 1.

2.2.3 Continuous approximation of full sate feedback discontinuous control input

The discontinuous control input (21) with (7) chatters from the beginning without reaching phase. The chattering of the discontinuous control input (21) may be harmful to the real dynamic plant. Hence using the saturation function for a suitable δ_f, one make the input be continuous for practical application as

$$u_{fc} = -K(x)x - K_1 S_f - \{\Delta K x + K_2 sign(S_f)\} \frac{S_f}{|S_f| + \delta_f} \tag{32}$$

which is different from that of (Chern & Wu, 1992) continuous approximation. For a first order system, this approximation is the same as that of (Chern & Wu, 1992) continuous approximation, but for a higher order system more than the first one, continuous approximation can be effectively made. The discontinuity of the control input can be dramatically improved without severe output performance deterioration.

2.3 Practical output feedback integral variable structure controller

For the implementation of the output feedback when full-state is not available, some additional assumptions are made

A7: The nominal input matrix $g_0(x,t)$ is constant, i.e, $g_0(x,t) = B$

A8: The unmatched $\Delta f(x,t)$, matched $\Delta g(x,t)$, and matched $d(x,t)$ are unknown and bounded and satisfied by the following conditions:

$$\Delta f(x,t) = \Delta f'(x,t) C^T C = \Delta f''(x,t) C \tag{33a}$$

$$\Delta g(x,t) = BB^T \Delta g'(x,t) = B\Delta I, \quad 0 \le |\Delta I| \le p < 1 \tag{33b}$$

$$d(x,t) = BB^T d'(x,t) = Bd''(x,t) \tag{33c}$$

2.3.1 Transformed output feedback integral sliding surface

Now, the integral of the output is augmented as follows:

$$\dot{y}_0(t) = A_0 \cdot y(t), \qquad y_0(0) \tag{34a}$$

$$y_0(t) = A_0 \cdot \int_0^t y(\tau) d\tau + y_0(0) \tag{34b}$$

where $y_0(t) \in R^r$, $r \le q$ is the integral of the output and $y_0(0)$ is the initial condition of the integral state determined later, and A_0 is appropriately dimensioned without loss of generality, $A_0 = I$.

Assumption

A9: (H_1CB) has the inverse for some non zero row vector H_1.

Now, a transformed output feedback integral sliding surface is suggested be

$$S_0 = (H_1CB)^{-1} \cdot (H_1 \cdot y + H_0 \cdot y_0)(= 0) \tag{35}$$

$$y_0(0) = -H_0^- H_1 \cdot y(0) \tag{36}$$

where $H_0^- = (H_0^T W H_0)^{-1} H_0^T W$, which is transformed for easy proof of the existence condition of the sliding mode on the sliding surface as the works of (Decarlo et al., 1988) and (Lee, 2010b). In (35), non zero row vector H_0 and H_1 are the design parameters satisfying the following relationship

$$H_1C[f_0(x,t) - BG(y)C] + H_0C = H_1Cf_{0c}(x,t) + H_0C = 0 \tag{37}$$

where $f_{0c}(x,t) = f_0(x,t) - BG(y)C$ is a closed loop system matrix and $G(y)$ is an output feedback gain. At $t = 0$, this output feedback integral sliding surface is zero so that there will be no reaching phase(Lee & Youn, 1994). In (35), H_0 and H_1 are the non zero row vectors as the design parameters such that the following assumption is satisfied.

Assumption

A10: $H_1Cg(x,t)$ has the full rank and is invertible

The equivalent control input is obtained using as

$$u_{eq} = -[H_1Cg(x,t)]^{-1}[H_1Cf(x,t)x + H_0y_0(t)] - [H_1Cg(x,t)]^{-1}[H_1C\Delta f(x,t) + \bar{d}(x,t)] \tag{38}$$

This control input can not be implemented because of the uncertainties and disturbances. The ideal sliding mode dynamics of the output feedback integral sliding surface (35) can be derived by the equivalent control approach as (Decarlo et al., 1998)

$$\dot{x}_s = [f_0(x_s,t) - B(H_1CB)^{-1}H_1Cf_0(x_s,t) - B(H_1CB)^{-1}H_0C]x_s, \qquad x_s(0) = x(0) \tag{39}$$

$$y_s = C \cdot x_s \tag{40}$$

and from $\dot{S}_0 = 0$, the another ideal sliding mode dynamics is obtained as(Lee, 2010a)

$$\dot{y}_s = -H_1^- H_0 y_s, \qquad y_s(0) \tag{41}$$

where $H_1^- = (H_1^T W H_1)^{-1} H_1^T W$. The solution of (39) or (41) identically defines the output feedback integral sliding surface. Hence to design the output feedback integral sliding surface as stable, this ideal sliding dynamics (39) is designed to be stable. To choose the stable gain based on the Lyapunov stability theory, the ideal sliding dynamics (39) is represented by the nominal plant of (3) as

$$\begin{aligned} \dot{x} &= f_0(x,t)x + g_0(x,t)u, \qquad u = -G(y)y \\ &= f_{0c}(x,t)x \end{aligned} \tag{42}$$

To select the stable gain, take a Lyapunov function candidate as

$$V(x) = x^T Px, \qquad P > 0 \tag{43}$$

The derivative of (43) becomes

$$\dot{V}(x) = x^T [f_0(x,t)^T P + Pf_0(x,t)]x + u^T g_0^T(x,t)Px + x^T Pg_0(x,t)u \tag{44}$$

By means of the Lyapunov control theory(Khalil, 1996), take the control input as

$$u = -g_0^T(x,t)Py = -B^T Py \tag{45}$$

and $Q(x,t) > 0$ and $Q_c(x,t) > 0$ for all $x \in R^n$ and all $t \geq 0$ is

$$f_0^T(x,t)P + Pf_0(x,t) = -Q(x,t) \tag{46}$$

$$f_{0c}^T(x,t)P + Pf_{0c}(x,t) = -Q_c(x,t) \tag{47}$$

then

$$\begin{aligned} \dot{V}(x) &= -x^T Q(x,t)x - x^T C^T PBB^T Px - x^T PBB^T PCx \\ &= -x^T [Q(x,t) + C^T PBB^T P + PBB^T PC]x \\ &= -x^T [f_{0c}^T(x,t)P + Pf_{0c}(x,t)]x \\ &= -x^T Q_c(x,t)x \\ &\leq -\lambda_{\min}\{Q_c(x,t)\}x \end{aligned} \tag{48}$$

Therefore the stable gain is chosen as

$$G(y) = B^T P \quad \text{or} \quad = (H_1 CB)^{-1} H_1 Cf_0(x,t) \tag{49}$$

2.3.2 Output feedback discontinuous control input

A corresponding output feedback discontinuous control input is proposed as follows:

$$u_0 = -G(y)y - \Delta Gy - G_1 S_0 - G_2 sign(S_0) \tag{50}$$

where $G(y)$ is a nonlinear output feedback gain satisfying the relationship (37) and (49), ΔG is a switching gain of the state, G_1 is a feedback gain of the output feedback integral sliding surface, and G_2 is a switching gain, respectively as

$$\Delta G = [\Delta g_i] \qquad i = 1,...,q \tag{51}$$

$$\Delta g_i = \begin{cases} \geq \dfrac{\max\{(H_1 CB)^{-1} H_1 C\Delta f''(x.t) + \Delta I(H_1 CB)^{-1} H_1 f_0(x,t)\}_i}{\min\{I + \Delta I\}} & sign(S_0 y_i) > 0 \\ \leq \dfrac{\min\{(H_1 CB)^{-1} H_1 C\Delta f''(x.t) + \Delta I(H_1 CB)^{-1} H_1 f_0(x,t)\}_i}{\min\{I + \Delta I\}} & sign(S_0 y_i) < 0 \end{cases} \tag{52}$$

$$G_1 > 0 \tag{53}$$

$$G_2 = \frac{\max\{|\, d''(x,t)\,|\}}{\min\{I + \Delta I\}} \tag{55}$$

The real sliding dynamics by the proposed control (50) with the output feedback integral sliding surface (35) is obtained as follows:

$$\begin{aligned}
\dot{S}_0 &= (H_1CB)^{-1}[H_1\dot{y} + H_0 y] \\
&= (H_1CB)^{-1}[H_1Cf_0(x,t)x + H_1C\Delta f(x,t) + H_1C(B + \Delta g(x,t))u + H_1C\bar{d}(x,t) + H_0 y] \\
&= (H_1CB)^{-1}[H_1Cf_0(x,t)x - H_1CBG(y)y + H_0 y] \\
&\quad + (H_1CB)^{-1}[\, H_1C\Delta f(x,t) + H_1C\Delta g(x,t)K(y)y] \\
&\quad + (H_1CB)^{-1}[H_1C(B + \Delta g(x,t))(-\Delta Gy - G_1S_0 - G_2 sign(S_0) + H_1C\bar{d}(x,t)] \\
&= (H_1CB)^{-1}[H_1C\Delta f''(x,t)Cx + H_1C\Delta g(x,t)G(y)y] - (I + \Delta I)\Delta G(y)y \\
&\quad + [(I + \Delta I)(-G_1S_0 - G_2 sign(S_0)) + d''(x,t)] \\
&= (H_1CB)^{-1}H_1C\Delta f''(x,t)y + \Delta I(H_1CB)^{-1}H_1 f_0(x,t)y - (I + \Delta I)\Delta G(y)y \\
&\quad + (I + \Delta I)(-G_1S_0 - G_2 sign(S_0)) + d''(x,t)
\end{aligned} \tag{56}$$

The closed loop stability by the proposed control input with the output feedback integral sliding surface together with the existence condition of the sliding mode will be investigated in next Theorem 1.

Theorem 2: *If the output feedback integral sliding surface (35) is designed to be stable, i.e. stable design of* $G(y)$ *, the proposed control input (50) with Assumption A1-A10 satisfies the existence condition of the sliding mode on the output feedback integral sliding surface and closed loop exponential stability.*

Proof; Take a Lyapunov function candidate as

$$V(y) = \frac{1}{2}S_0^T S_0 \tag{57}$$

Differentiating (57) with respect to time leads to and substituting (56) into (58)

$$\begin{aligned}
\dot{V}(y) &= S_0{}^T\dot{S}_0 \\
&= S_0^T[(H_1CB)^{-1}H_1C\Delta f''(x,t) + \Delta I(H_1CB)^{-1}H_1 f_0(x,t)]y - S_0^T(I + \Delta I)\Delta G(y)y \\
&\quad + S_0^T(I + \Delta I)(-G_1S_0 - G_2 sign(S_0)) + S_0^T d''(x,t) \\
&\le -\varepsilon G_1 |\,|S_0|\,|^2, \qquad \varepsilon = \min\{|\,|I + \Delta I|\,|\} \\
&= -\varepsilon G_1 S_0^T S_0 \\
&= -2\varepsilon G_1 V(y)
\end{aligned} \tag{58}$$

From (58), the second requirement to get rid of the reaching phase is satisfied. Therefore, the reaching phase is clearly removed. There are no reaching phase problems. As a result, the real output dynamics can be exactly predetermined by the ideal sliding output with the matched uncertainty. Moreover from (58), the following equations are obtained as

$$\dot{V}(y) + 2\varepsilon G_1 V(y) \le 0 \tag{59}$$

$$V(y(t)) \le V(y(0))e^{-2\varepsilon G_1 t} \tag{60}$$

And the second order derivative of $V(x)$ becomes

$$\ddot{V}(y) = \dot{S}_0^T \dot{S}_0 + S_0^T \ddot{S}_0 = |\ |\dot{S}_0|\ |^2 + S_0(H_1 CB)^{-1}(H_1 C\ddot{x} + H_0 C\dot{x}) < \infty \tag{61}$$

and by Assumption A5 $\ddot{V}(x)$ is bounded, which completes the proof of Theorem 2.

2.3.3 Continuous approximation of output feedback discontinuous control input

Also, the control input (50) with (35) chatters from the beginning without reaching phase. The chattering of the discontinuous control input may be harmful to the real dynamic plant so it must be removed. Hence using the saturation function for a suitable δ_0, one make the part of the discontinuous input be continuous effectively for practical application as

$$u_{0c} = -G(y)y - G_1 S_0 - \{\Delta G y + G_2 sign(S_0)\}\frac{S_0}{|S_0| + \delta_0} \tag{62}$$

The discontinuity of control input of can be dramatically improved without severe output performance deterioration.

3. Design examples and simulation studies

3.1 Example 1: Full-state feedback practical integral variable structure controller

Consider a second order affine uncertain nonlinear system with mismatched uncertainties and matched disturbance

$$\dot{x}_1 = -x_1 + 0.1x_1 \sin^2(x_1) + x_2 + 0.02\sin(2.0x_1)u$$

$$\dot{x}_2 = x_2 + x_2 \sin^2(x_2) + (2.0 + 0.5\sin(2.0t))u + \bar{d}(x,t) \tag{63}$$

$$\bar{d}(x,t) = 0.7\sin(x_1) - 0.8\sin(x_2) + 0.2(x_1^2 + x_2^2) + 2.0\sin(5.0t) + 3.0 \tag{64}$$

Since (63) satisfy the Assumption A1, (63) is represented in state dependent coefficient form as

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 + 0.1\sin^2(x_1) & 1 \\ 0 & 1 + \sin^2(x_2)\end{bmatrix} \cdot \begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0.02\sin(x_1) \\ 2.0 + 0.5\sin(2.0t)\end{bmatrix} u + \begin{bmatrix}0 \\ \bar{d}(x,t)\end{bmatrix} \tag{65}$$

where the nominal parameter $f_0(x,t)$ and $g_0(x,t)$ and mismatched uncertainties $\Delta f(x,t)$ and $\Delta g(x,t)$ are

$$\begin{aligned} f_0(x,t) &= \begin{bmatrix}-1 & 1 \\ 0 & 1\end{bmatrix},\ g_0(x,t) = \begin{bmatrix}0 \\ 2.0\end{bmatrix},\ \Delta f(x,t) = \begin{bmatrix}0.1\sin^2(x_1) & 0 \\ 0 & \sin^2(x_2)\end{bmatrix} \\ \Delta g(x,t) &= \begin{bmatrix}0.02\sin(x_1) \\ 0.2\sin(2.0t)\end{bmatrix} \end{aligned} \tag{66}$$

To design the full-state feedback integral sliding surface, $f_c(x,t)$ is selected as

$$f_c(x,t) = f_0(x,t) - g_0(x,t)K(x) = \begin{bmatrix} -1 & 1 \\ -70 & -21 \end{bmatrix} \tag{67}$$

in order to assign the two poles at -16.4772 and -5.5228. Hence, the feedback gain $K(x)$ becomes

$$K(x) = \begin{bmatrix} 35 & 11 \end{bmatrix} \tag{68}$$

The P in (14) is chosen as

$$P = \begin{bmatrix} 100 & 17.5 \\ 17.5 & 5.5 \end{bmatrix} > 0 \tag{69}$$

so as to be

$$f_c^T(x,t)P + Pf_c(x,t) = \begin{bmatrix} -2650 & -670 \\ -670 & -196 \end{bmatrix} < 0 \tag{70}$$

Hence, the continuous static feedback gain is chosen as

$$K(x) = g_0^T(x,t)P = \begin{bmatrix} 35 & 11 \end{bmatrix} \tag{71}$$

Therefore, the coefficient of the sliding surface is determined as

$$L_1 = \begin{bmatrix} L_{11} & L_{12} \end{bmatrix} = \begin{bmatrix} 10 & 1 \end{bmatrix} \tag{72}$$

Then, to satisfy the relationship (8a) and from (8b), L_0 is selected as

$$L_0 = -L_1\left[f_0(x,t) - g_0(x,t)K(x)\right] = -L_1 f_c(x,t) = \begin{bmatrix} L_{11} + 70L_{12} & -L_{11} + 21L_{12} \end{bmatrix} = \begin{bmatrix} 80 & 11 \end{bmatrix} \tag{73}$$

The selected gains in the control input (21), (23)-(25) are as follows:

$$\Delta k_1 = \begin{cases} +4.0 & \text{if } S_f x_1 > 0 \\ -4.0 & \text{if } S_f x_1 < 0 \end{cases} \tag{74a}$$

$$\Delta k_2 = \begin{cases} +5.0 & \text{if } S_f x_2 > 0 \\ -5.0 & \text{if } S_f x_2 < 0 \end{cases} \tag{74b}$$

$$K_1 = 400.0 \tag{74c}$$

$$K_2 = 2.8 + 0.2(x_1^2 + x_2^2) \tag{74d}$$

The simulation is carried out under 1[msec] sampling time and with $x(0) = \begin{bmatrix} 10 & 5 \end{bmatrix}^T$ initial state. Fig. 1 shows four case x_1 and x_2 time trajectories (i)ideal sliding output, (ii) no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched

uncertainty and matched disturbance. The three case output responses except the case (iv) are almost identical to each other. The four phase trajectories (i)ideal sliding trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance are depicted in Fig. 2. As can be seen, the sliding surface is exactly defined from a given initial condition to the origin, so there is no reaching phase, only the sliding exists from the initial condition. The one of the two main problems of the VSS is removed and solved. The unmatched uncertainties influence on the ideal sliding dynamics as in the case (iv). The sliding surface $S_f(t)$ (i) unmatched uncertainty and matched disturbance is shown in Fig. 3. The control input (i) unmatched uncertainty and matched disturbance is depicted in Fig. 4. For practical application, the discontinuous input is made be continuous by the saturation function with a new form as in (32) for a positive $\delta_f = 0.8$. The output responses of the continuous input by (32) are shown in Fig. 5 for the four cases (i)ideal sliding output, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched uncertainty and matched disturbance. There is no chattering in output states. The four case trajectories (i)ideal sliding time trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance are depicted in Fig. 6. As can be seen, the trajectories are continuous. The four case sliding surfaces are shown in fig. 7, those are continuous. The three case continuously implemented control inputs instead of the discontinuous input in Fig. 4 are shown in Fig. 8 without the severe performance degrade, which means that the continuous VSS algorithm is practically applicable. The another of the two main problems of the VSS is improved effectively and removed.
From the simulation studies, the usefulness of the proposed SMC is proven.

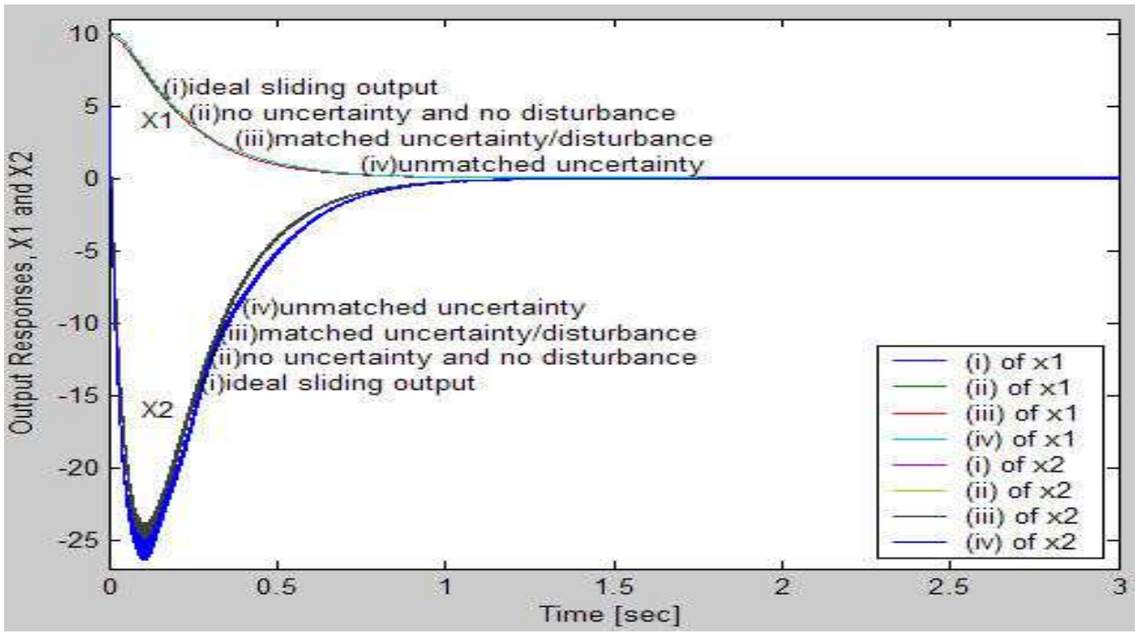


Fig. 1. Four case x_1 and x_2 time trajectories (i)ideal sliding output, (ii) no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched uncertainty and matched disturbance

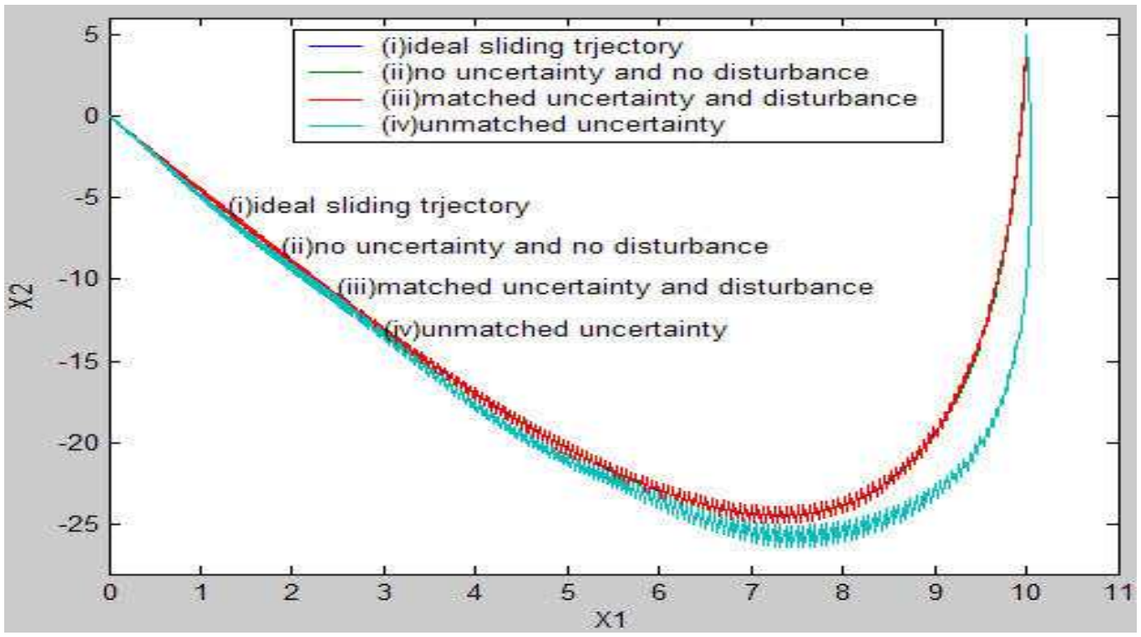


Fig. 2. Four phase trajectories (i)ideal sliding trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance

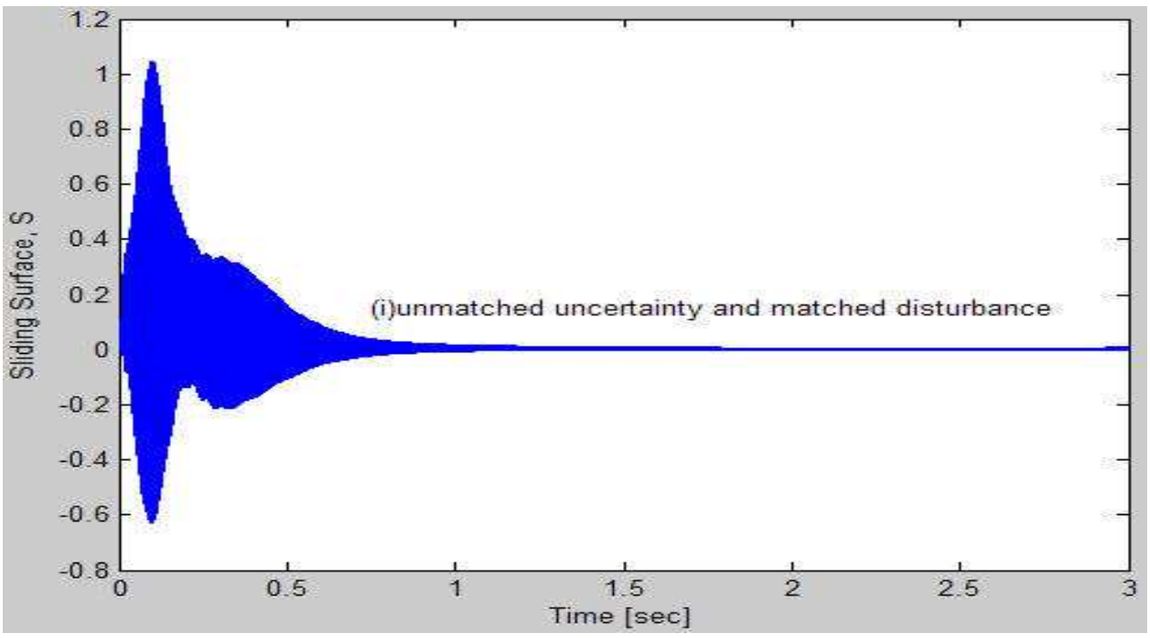


Fig. 3. Sliding surface $S_f(t)$ (i) unmatched uncertainty and matched disturbance

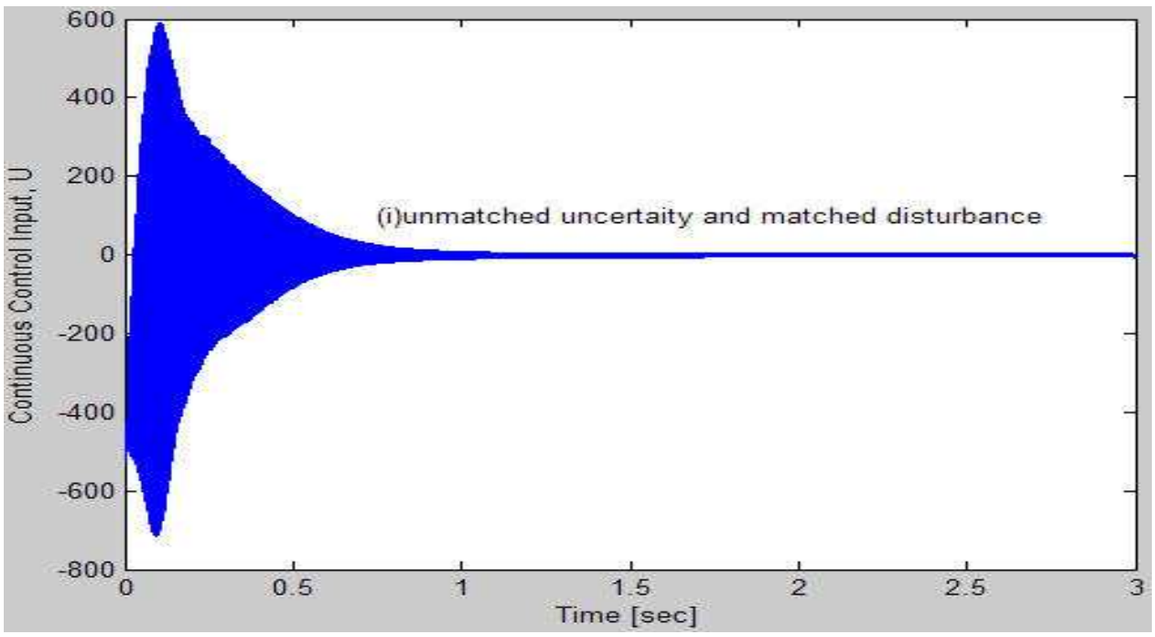


Fig. 4. Discontinuous control input (i) unmatched uncertainty and matched disturbance

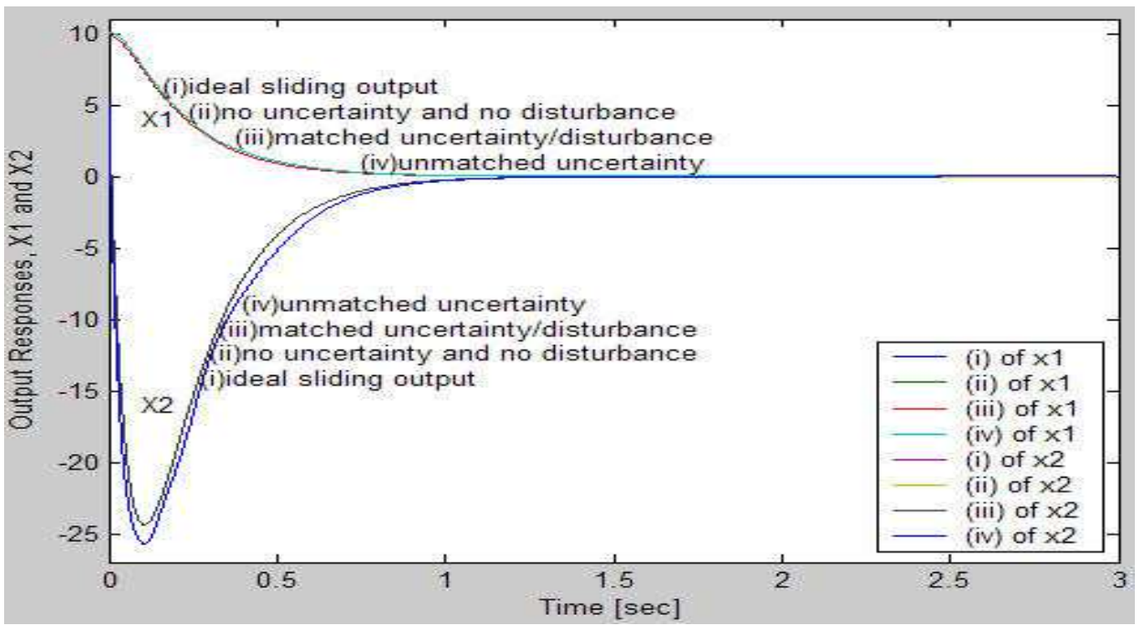


Fig. 5. Four case x_1 and x_2 time trajectories (i)ideal sliding output, (ii) no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched uncertainty and matched disturbance by the continuously approximated input for a positive $\delta_f = 0.8$

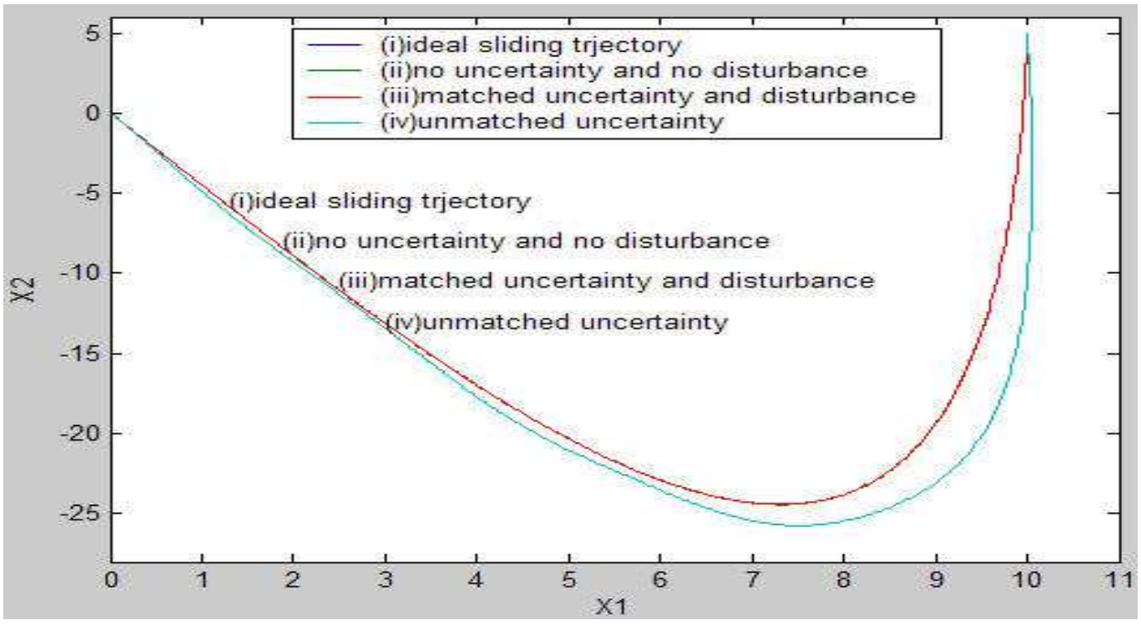


Fig. 6. Four phase trajectories (i)ideal sliding trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance by the continuously approximated input

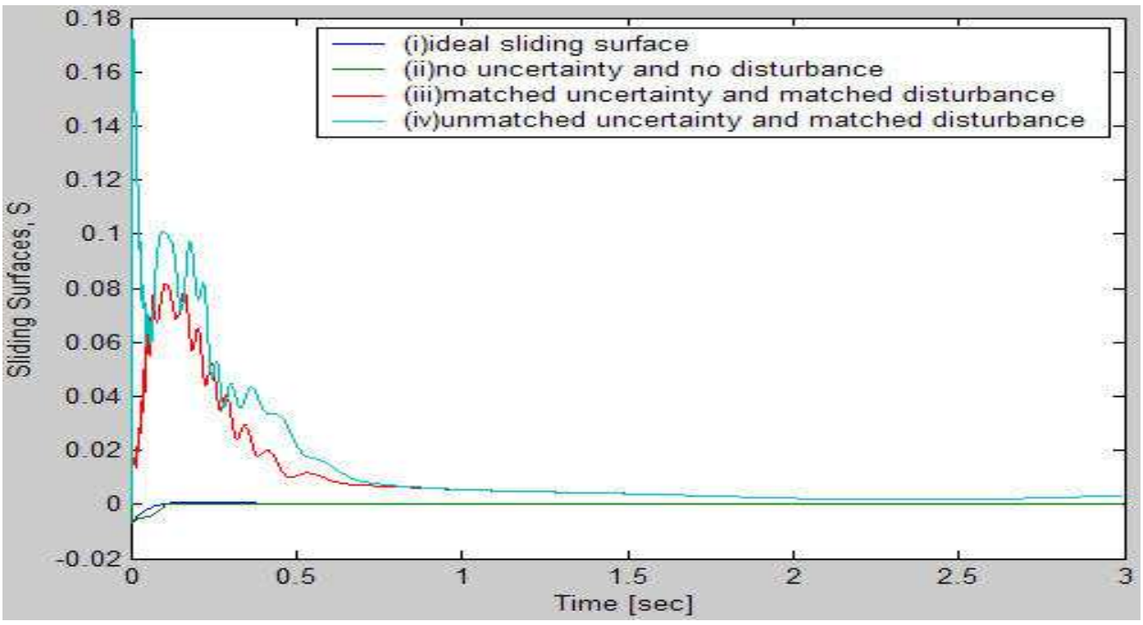


Fig. 7. Four sliding surfaces (i)ideal sliding surface, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance by the continuously approximated input

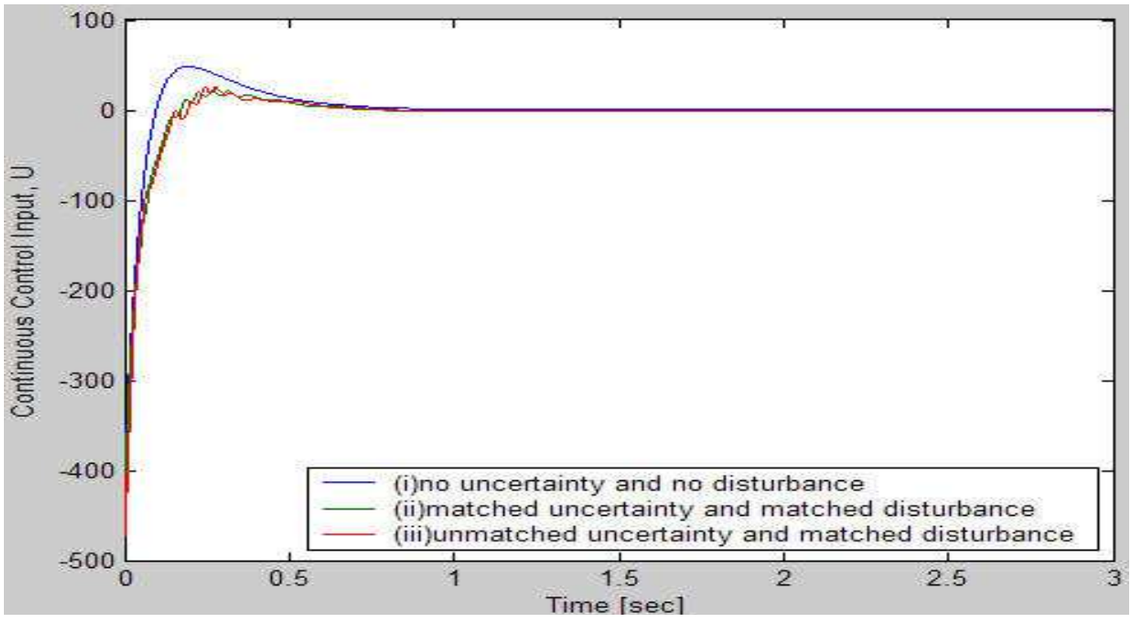


Fig. 8. Three case continuous control inputs u_{fc} (i)no uncertainty and no disturbance (ii)matched uncertainty/disturbance, and (iii) unmatched uncertainty and matched

3.2 Example 2: Output feedback practical integral variable structure controller

Consider a third order uncertain affine nonlinear system with unmatched system matrix uncertainties and matched input matrix uncertainties and disturbance

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-3-3\sin^2(x_1) & 1 & 0\\ 0 & -1 & 1\\ 1+0.5\sin^2(x_2) & 0 & 2+0.4\sin^2(x_3)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 2+0.3\sin(2t)\end{bmatrix}u+\begin{bmatrix}0\\ 0\\ \bar{d}_1(x,t)\end{bmatrix} \tag{75}$$

$$y=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \tag{76}$$

$$\bar{d}_1(x,t)=0.7\sin(x_1)-0.8\sin(x_2)+0.2(x_1^2+x_3^2)+1.5\sin(2t)+1.5 \tag{77}$$

where the nominal matrices $f_0(x,t)$, $g_0(x,t)=B$ and C, the unmatched system matrix uncertainties and matched input matrix uncertainties and matched disturbance are

$$f_0(x,t)=\begin{bmatrix}-3 & 1 & 0\\ 0 & -1 & 1\\ 1 & 0 & 2\end{bmatrix},\ B=\begin{bmatrix}0\\ 0\\ 2\end{bmatrix},\ C=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\ \Delta f=\begin{bmatrix}-3\sin^2(x_1) & 0 & 0\\ 0 & 0 & 0\\ 0.5\sin^2(x_2) & 0 & 0.4\sin^2(x_3)\end{bmatrix}$$

$$\Delta g(x,t)=\begin{bmatrix}0\\0\\0.3\sin(2t)\end{bmatrix},\quad \bar{d}(x,t)=\begin{bmatrix}0\\0\\\bar{d}_1(x,t)\end{bmatrix}. \tag{78}$$

The eigenvalues of the open loop system matrix $f_0(x,t)$ are -2.6920, -2.3569, and 2.0489, hence $f_0(x,t)$ is unstable. The unmatched system matrix uncertainties and matched input matrix uncertainties and matched disturbance satisfy the assumption A3 and A8 as

$$\Delta f''=\begin{bmatrix}-3\sin^2(x_1) & 0\\0 & 0\\0.5\sin^2(x_2) & 0.4\sin^2(x_3)\end{bmatrix},\quad \Delta I=0.15\sin(2t)\le 0.15<1,\quad d''(x,t)=\frac{1}{2}\bar{d}_1(x,t) \tag{79}$$

disturbance by the continuously approximated input for a positive $\delta_f=0.8$
To design the output feedback integral sliding surface, $f_c(x,t)$ is designed as

$$f_{0c}(x,t)=f_0(x,t)-BG(y)C=\begin{bmatrix}-3 & 1 & 0\\0 & -1 & 1\\-19 & 0 & -30\end{bmatrix} \tag{80}$$

in order to assign the three stable pole to $f_c(x,t)$ at -30.0251 and $-2.4875\pm i0.6636$. The constant feedback gain is designed as

$$G(y)C=2^{-1}\left\{\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}-19 & 0 & 30\end{bmatrix}\right\} \tag{81}$$

$$\therefore G(y)=\begin{bmatrix}10 & 16\end{bmatrix} \tag{82}$$

Then, one find $H_1=\begin{bmatrix}h_{11} & h_{12}\end{bmatrix}$ and $H_0=\begin{bmatrix}h_{01} & h_{02}\end{bmatrix}$ which satisfy the relationship (37) as

$$h_{11}=0,\qquad h_{01}=19h_{12},\qquad h_{02}=30h_{12} \tag{83}$$

One select $h_{12}=1$, $h_{01}=19$, and $h_{02}=30$. Hence $H_1CB=2h_{12}=2$ is a non zero satisfying A4. The resultant output feedback integral sliding surface becomes

$$S_0=\frac{1}{2}\left\{\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}+\begin{bmatrix}19 & 30\end{bmatrix}\begin{bmatrix}y_{01}\\y_{02}\end{bmatrix}\right\} \tag{84}$$

where

$$y_{01}=\int_0^t y_1(\tau)d\tau \tag{85}$$

$$y_{02}=\int_0^t y_2(\tau)d\tau-y_2(0)/30 \tag{86}$$

The output feedback control gains in (50), (51)-(55) are selected as follows:

$$\Delta g_1 = \begin{cases} +1.6 & \text{if} \quad S_0 y_1 > 0 \\ -1.6 & \text{if} \quad S_0 y_1 < 0 \end{cases} \tag{87a}$$

$$\Delta g_2 = \begin{cases} +1.7 & \text{if} \quad S_0 y_2 > 0 \\ -1.7 & \text{if} \quad S_0 y_2 < 0 \end{cases} \tag{87b}$$

$$G_1 = 500.0 \tag{87c}$$

$$G_2 = 3.2 + 0.2(y_1^2 + y_2^2) \tag{87d}$$

The simulation is carried out under 1[msec] sampling time and with $x(0) = \begin{bmatrix} 10 & 0.0 & 5 \end{bmatrix}^T$ initial state. Fig. 9 shows the four case two output responses of y_1 and y_2 (i)ideal sliding output, (ii) with no uncertainty and no disturbance, (iii)with matched uncertainty and matched disturbance, and (iv) with ummatched uncertainty and matched disturbance. The each two output is insensitive to the matched uncertainty and matched disturbance, hence is almost equal, so that the output can be predicted. The four case phase trajectories (i)ideal sliding trajectory, (ii) with no uncertainty and no disturbance, (iii)with matched uncertainty and matched disturbance, and (iv) with ummatched uncertainty and matched disturbance are shown in Fig. 10. There is no reaching phase and each phase trajectory except the case (iv) with ummatched uncertainty and matched disturbance is almost identical also. The sliding surface is exactly defined from a given initial condition to the origin. The output feedback integral sliding surfaces (i) with ummatched uncertainty and matched disturbance is depicted in Fig. 11. Fig. 12 shows the control inputs (i)with unmatched uncertainty and matched disturbance. For practical implementation, the discontinuous input can be made continuous by the saturation function with a new form as in (32) for a positive $\delta_0 = 0.02$. The output responses by the continuous input of (62) are shown in Fig. 13 for the four cases (i)ideal sliding output, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched uncertainty and matched disturbance. There is no chattering in output responses. The four case trajectories (i)ideal sliding time trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance are depicted in Fig. 14. As can be seen, the trajectories are continuous. The four case sliding surfaces are shown in fig. 15, those are continuous also. The three case continuously implemented control inputs instead of the discontinuous input in Fig. 12 are shown in Fig. 16 without the severe performance loss, which means that the chattering of the control input is removed and the continuous VSS algorithm is practically applicable to the real dynamic plants. From the above simulation studies, the proposed algorithm has superior performance in view of the no reaching phase, complete robustness, predetermined output dynamics, the prediction of the output, and practical application. The effectiveness of the proposed output feedback integral nonlinear SMC is proven.

Through design examples and simulation studies, the usefulness of the proposed practical integral nonlinear variable structure controllers is verified. The continuous approximation VSS controllers without the reaching phase in this chapter can be practically applicable to the real dynamic plants.

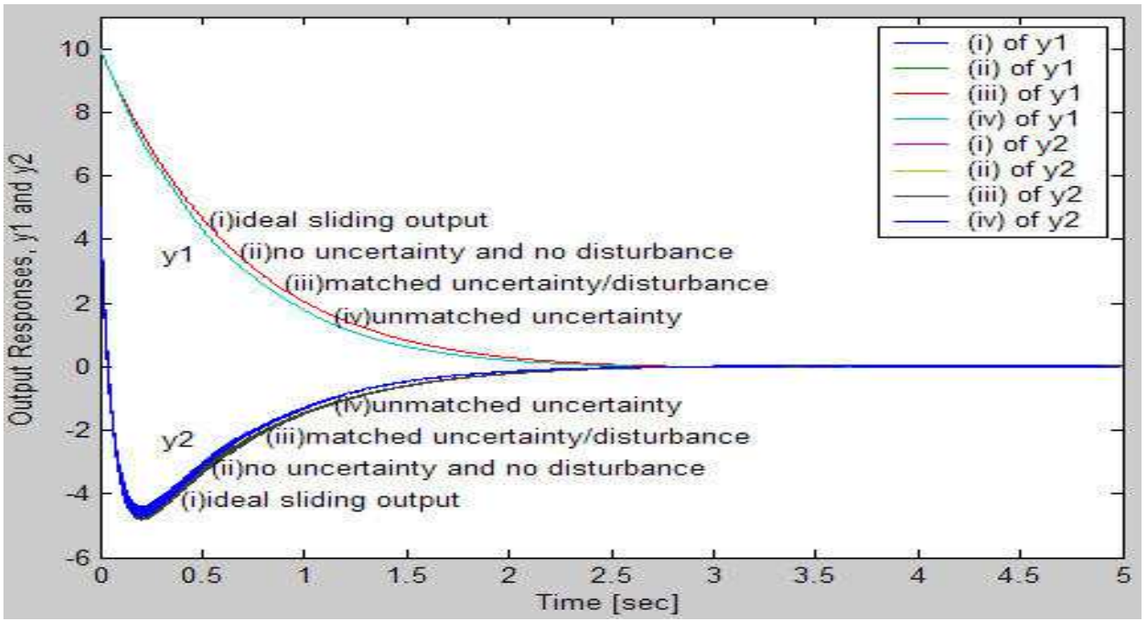


Fig. 9. Four case two output responses of y_1 and y_2 (i)ideal sliding output, (ii) with no uncertainty and no disturbance, (iii)with matched uncertainty and matched disturbance, and (iv) with ummatched uncertainty and matched disturbance

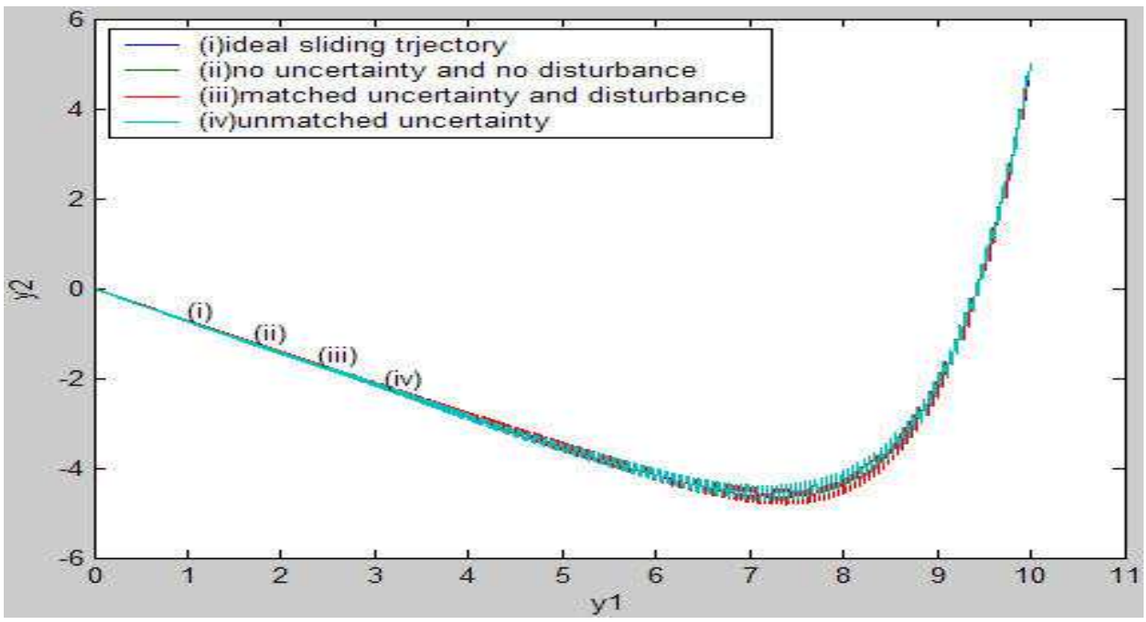


Fig. 10. Four phase trajectories (i)ideal sliding trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched uncertainty and matched disturbance

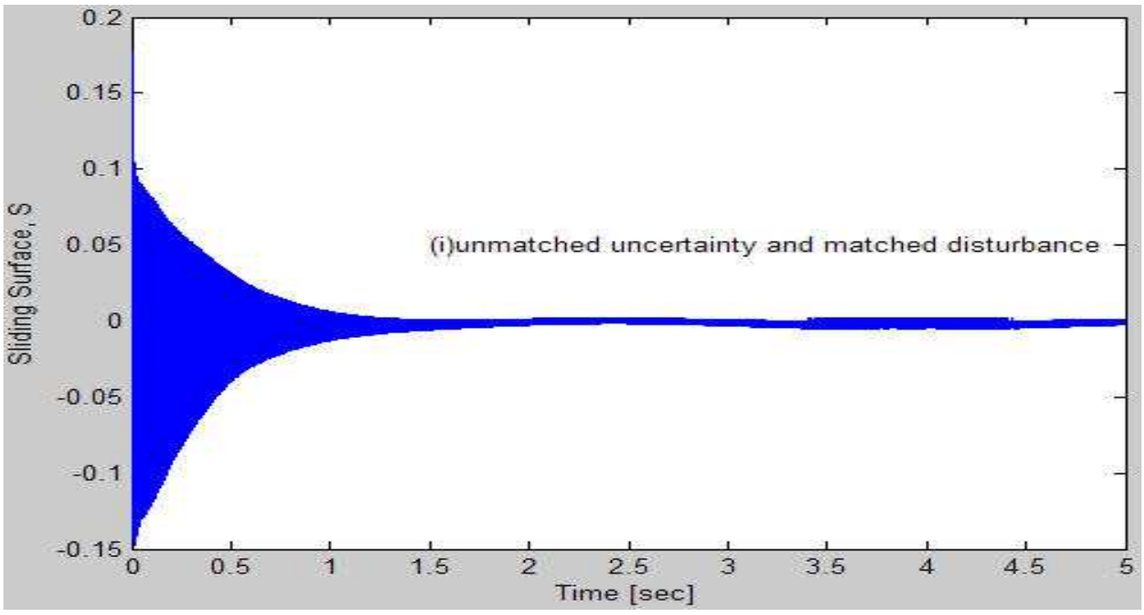


Fig. 11. Sliding surface $S_0(t)$ (i) unmatched uncertainty and matched disturbance

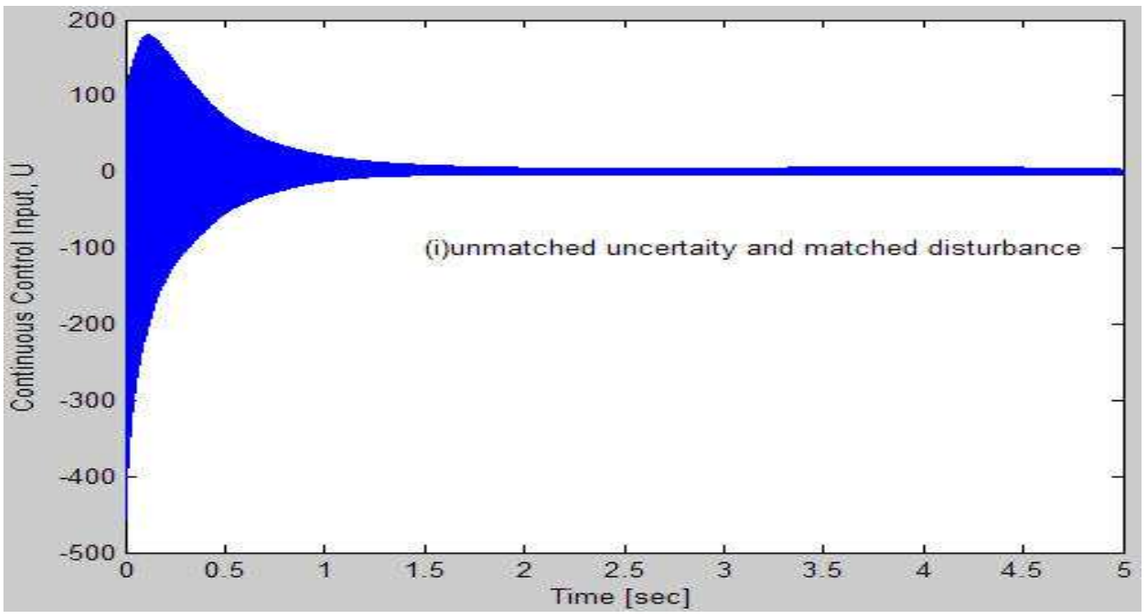


Fig. 12. Discontinuous control input (i) unmatched uncertainty and matched disturbance

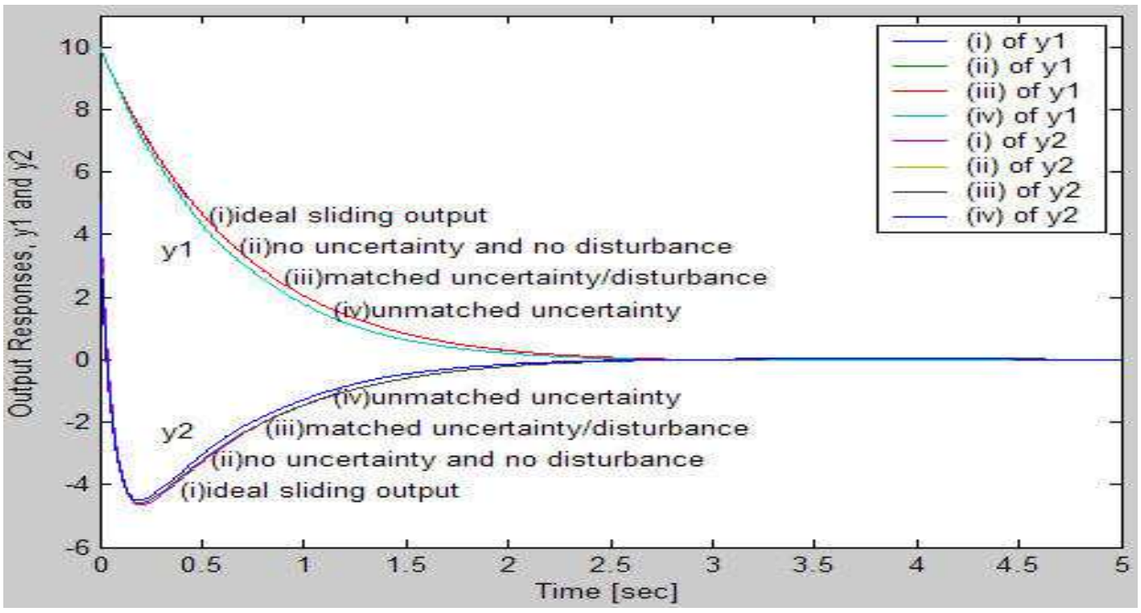


Fig. 13. Four case y_1 and y_2 time trajectories (i)ideal sliding output, (ii) no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv)unmatched uncertainty and matched disturbance by the continuously approximated input for a positive $\delta_0 = 0.02$

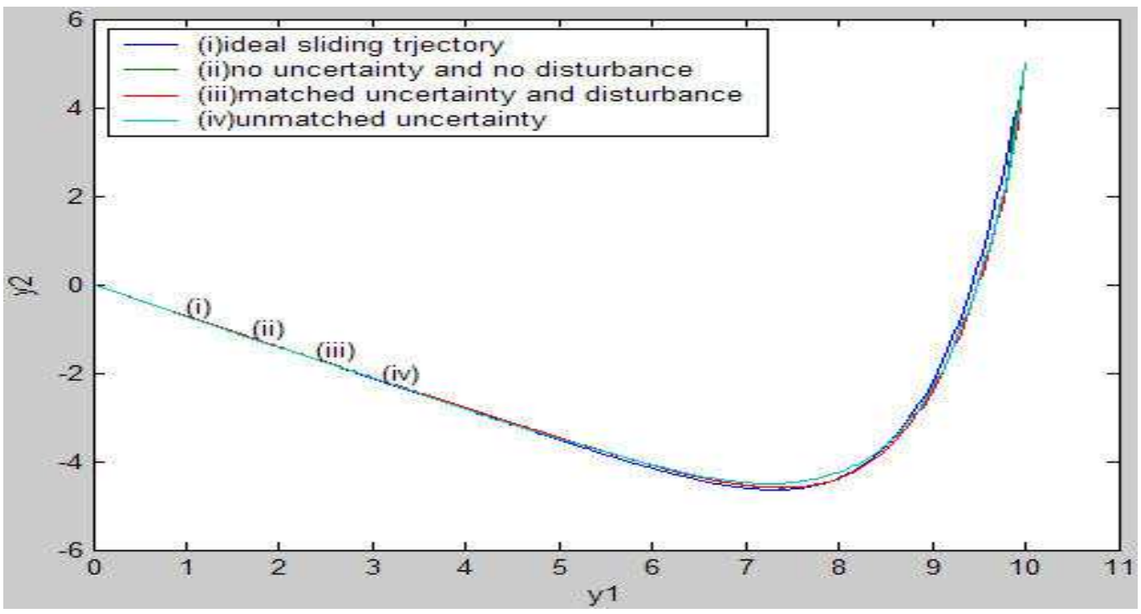


Fig. 14. Four phase trajectories (i)ideal sliding trajectory, (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance by the continuously approximated input

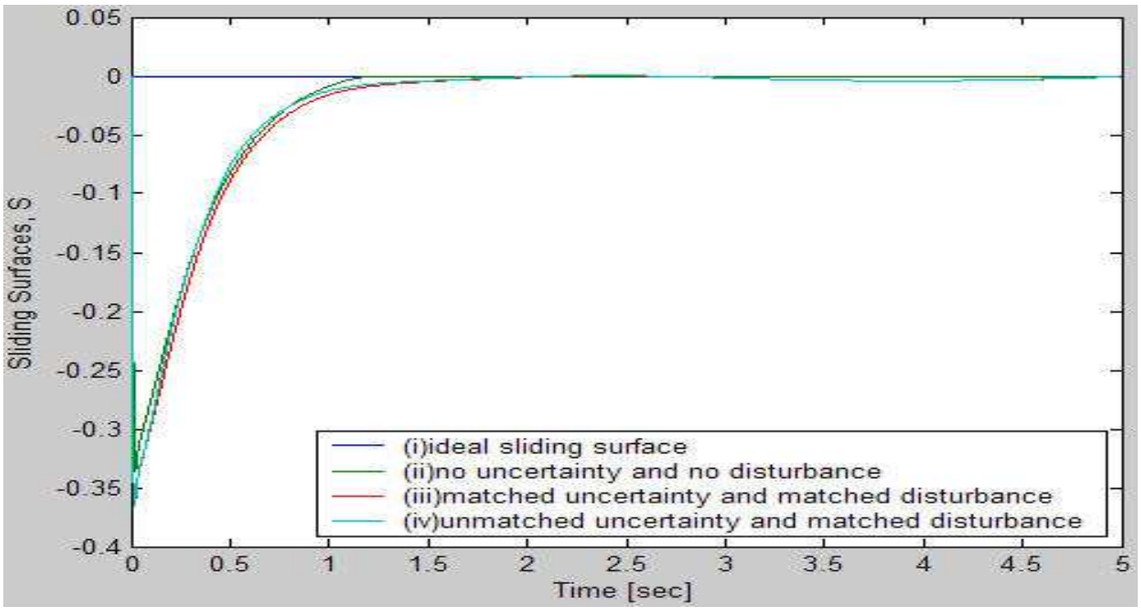


Fig. 15. Four sliding surfaces (i)ideal sliding surface , (ii)no uncertainty and no disturbance (iii)matched uncertainty/disturbance, and (iv) unmatched uncertainty and matched disturbance by the continuously approximated input

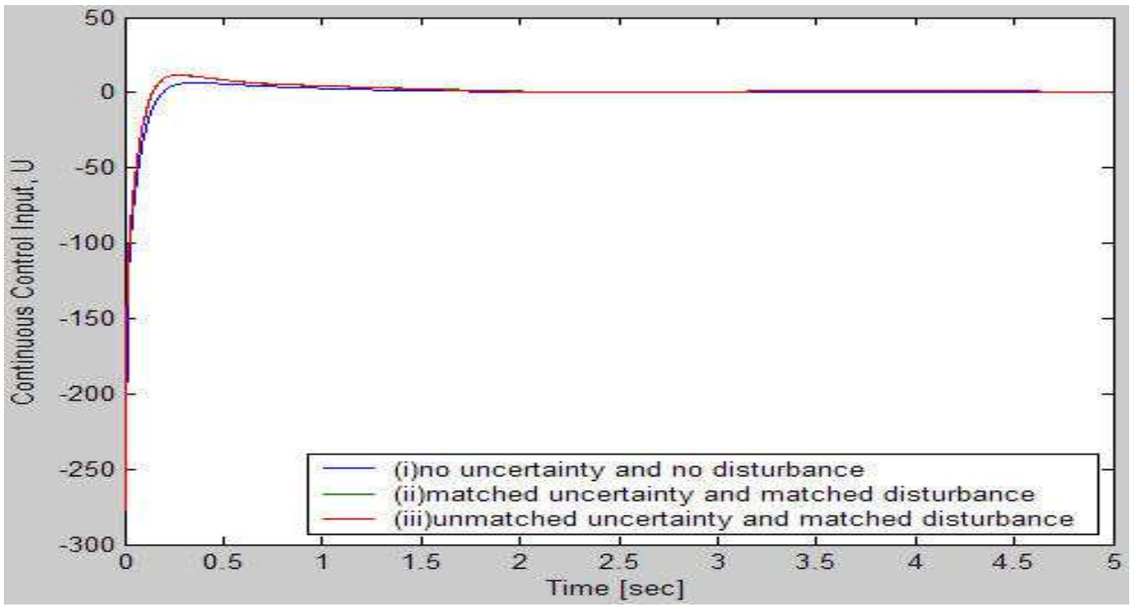


Fig. 16. Three case continuous control inputs u_{0c} (i)no uncertainty and no disturbance (ii)matched uncertainty/disturbance, and (iii) unmatched uncertainty and matched disturbance by the continuously approximated input for a positive $\delta_0 = 0.02$

4. Conclusion

In this chapter, a new practical robust full-state(output) feedback nonlinear integral variable structure controllers with the full-state(output) feedback integral sliding surfaces are presented based on state dependent nonlinear form for the control of uncertain more affine nonlinear systems with mismatched uncertainties and matched disturbance. After an affine uncertain nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of the new robust integral nonlinear variable structure controllers with the full-state(output) feedback (transformed) integral sliding surfaces are suggested for removing the reaching phase. The corresponding (transformed) control inputs are proposed. The closed loop stabilities by the proposed control inputs with full-state(output) feedback integral sliding surface together with the existence condition of the sliding mode on the selected sliding surface are investigated in Theorem 1 and Theorem 2 for all mismatched uncertainties and matched disturbance. For practical application of the continuous discontinuous VSS, the continuous approximation being different from that of (Chern & Wu, 1992) is suggested without severe performance degrade. The two practical algorithms, i.e., practical full-state feedback integral nonlinear variable structure controller with the full-state feedback transformed input and the full-state feedback sliding surface and practical output feedback integral nonlinear variable structure controller with the output feedback input and the output feedback transformed sliding surface are proposed. The outputs by the proposed inputs with the suggested sliding surfaces are insensitive to only the matched uncertainty and disturbance. The unmatched uncertainties can influence on the ideal sliding dynamics, but the exponential stability is satisfied. The two main problems of the VSS, i.e., the reaching phase at the beginning and the chattering of the input are removed and solved.

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Robust control has been a topic of active research in the last three decades culminating in H_2/H_\infty and \mu design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
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InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
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