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### Neural Control Toward a Unified Intelligent Control Design Framework for Nonlinear Systems

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#### 1. Introduction

There have been significant progresses reported in nonlinear adaptive control in the last two decades or so, partially because of the introduction of neural networks (Polycarpou, 1996; Chen & Liu, 1994; Lewis, Yesidirek & Liu, 1995; Sanner & Slotine, 1992; Levin & Narendra, 1993; Chen & Yang, 2005). The adaptive control schemes reported intend to design adaptive neural controllers so that the designed controllers can help achieve the stability of the resulting systems in case of uncertainties and/or unmodeled system dynamics. It is a typical assumption that no restriction is imposed on the magnitude of the control signal. Accompanied with the adaptive control design is usually a reference model which is assumed to exist, and a parameter estimator. The parameters can be estimated within a predesignated bound with appropriate parameter projection. It is noteworthy that these design approaches are not applicable for many practical systems where there is a restriction on the control magnitude, or a reference model is not available.

On the other hand, the economics performance index is another important objective for controller design for many practical control systems. Typical performance indexes include, for instance, minimum time and minimum fuel. The optimal control theory developed a few decades ago is applicable to those systems when the system model in question along with a performance index is available and no uncertainties are involved. It is obvious that these optimal control design approaches are not applicable for many practical systems where these systems contain uncertain elements.

Motivated by the fact that many practical systems are concerned with both system stability and system economics, and encouraged by the promising images presented by theoretical advances in neural networks (Haykin, 2001; Hopfield & Tank, 1985) and numerous application results (Nagata, Sekiguchi & Asakawa, 1990; Methaprayoon, Lee, Rasmiddatta, Liao & Ross, 2007; Pandit, Srivastava & Sharma, 2003; Zhou, Chellappa, Vaid & Jenkins, 1998; Chen & York, 2008; Irwin, Warwick & Hunt, 1995; Kawato, Uno & Suzuki, 1988; Liang 1999; Chen & Mohler, 1997; Chen & Mohler, 2003; Chen, Mohler & Chen, 1999), this chapter aims at developing an

intelligent control design framework to guide the controller design for uncertain, nonlinear systems to address the combining challenge arising from the following:

- The designed controller is expected to stabilize the system in the presence of uncertainties in the parameters of the nonlinear systems in question.
- The designed controller is expected to stabilize the system in the presence of unmodeled system dynamics uncertainties.
- The designed controller is confined on the magnitude of the control signals.
- The designed controller is expected to achieve the desired control target with minimum total control effort or minimum time.

The salient features of the proposed control design framework include: (a) achieving nearly optimal control regardless of parameter uncertainties; (b) no need for a parameter estimator which is popular in many adaptive control designs; (c) respecting the pre-designated range for the admissible control.

Several important technical aspects of the proposed intelligent control design framework will be studied:

- Hierarchical neural networks (Kawato, Uno & Suzuki, 1988; Zakrzewski, Mohler & Kolodziej, 1994; Chen, 1998; Chen & Mohler, 2000; Chen, Mohler & Chen, 2000; Chen, Yang & Moher, 2008; Chen, Yang & Mohler, 2006) are utilized; and the role of each tier of the hierarchy will be discussed and how each tier of the hierarchical neural networks is constructed will be highlighted.
- The theoretical aspects of using hierarchical neural networks to approximately achieve optimal, adaptive control of nonlinear, time-varying systems will be studied.
- How the tessellation of the parameter space affects the resulting hierarchical neural networks will be discussed.

In summary, this chapter attempts to provide a deep understanding of what hierarchical neural networks do to optimize a desired control performance index when controlling uncertain nonlinear systems with time-varying properties; make an insightful investigation of how hierarchical neural networks may be designed to achieve the desired level of control performance; and create an intelligent control design framework that provides guidance for analyzing and studying the behaviors of the systems in question, and designing hierarchical neural networks that work in a coordinated manner to optimally, adaptively control the systems.

This chapter is organized as follows: Section 2 describes several classes of uncertain nonlinear systems of interest and mathematical formulations of these problems are presented. Some conventional assumptions are made to facilitate the analysis of the problems and the development of the design procedures generic for a large class of nonlinear uncertain systems. The time optimal control problem and the fuel optimal control problem are analyzed and an iterative numerical solution process is presented in Section 3. These are important elements in building a solution approach to address the control problems studied in this paper which are in turn decomposed into a series of control problems that do not exhibit parameter uncertainties. This decomposition is vital in the proposal of the hierarchical neural network based control design. The details of the hierarchical neural control design methodology are given in Section 4. The synthesis of hierarchical neural controllers is to achieve (a) near optimal control (which can be timeoptimal or fuel-optimal) of the studied systems with constrained control; (b) adaptive control of the studied control systems with unknown parameters; (c) robust control of the studied control systems with the time-varying parameters. In Section 5, theoretical results are developed to justify the fuel-optimal control oriented neural control design procedures for the time-varying nonlinear systems. Finally, some concluding remarks are made.

#### 2. Problem formulation

As is known, the adaptive control design of nonlinear dynamic systems is still carried out on a per case-by-case basis, even though there have numerous progresses in the adaptive of linear dynamic systems. Even with linear systems, the conventional adaptive control schemes have common drawbacks that include (a) the control usually does not consider the physical control limitations, and (b) a performance index is difficult to incorporate. This has made the adaptive control design for nonlinear system even more challenging. With this common understanding, this Chapter is intended to address the adaptive control design for a class of nonlinear systems using the neural network based techniques. The systems of interest are linear in both control and parameters, and feature time-varying, parametric uncertainties, confined control inputs, and multiple control inputs. These systems are represented by a finite dimensional differential system linear in control and linear in parameters.

The adaptive control design framework features the following:

- The adaptive, robust control is achieved by hierarchical neural networks.
- The physical control limitations, one of the difficulties that conventional adaptive control can not handle, are reflected in the admissible control set.
- The performance measures to be incorporated in the adaptive control design, deemed as a technical challenge for the conventional adaptive control schemes, that will be considered in this Chapter include:
  - Minimum time resulting in the so-called time-optimal control
  - Minimum fuel resulting in the so-called fuel-optimal control
  - Quadratic performance index resulting in the quadratic performance optimal control.

Although the control performance indices are different for the above mentioned approaches, the system characterization and some key assumptions are common.

The system is mathematically represented by

$$\dot{x} = a(x) + C(x)p + B(x)u \tag{1}$$

where  $x \in G \subseteq \mathbb{R}^n$  is the state vector,  $p \in \Omega_p \subset \mathbb{R}^l$  is the bounded parameter vector,  $u \in \mathbb{R}^m$ is the control vector, which is confined to an admissible control set U,  $a(x) = \begin{bmatrix} a_1(x) & a_2(x) & \cdots & a_n(x) \end{bmatrix}^r$  is an n-dimensional vector function of x,  $C(x) = \begin{bmatrix} C_{11}(x) & C_{12}(x) & \cdots & C_{1l} \\ C_{21}(x) & C_{22}(x) & \cdots & C_{2l}(x) \\ \cdots & \cdots & \cdots & \cdots \\ C_{n1}(x) & C_{n2}(x) & \cdots & C_{nl}(x) \end{bmatrix}$  is an  $n \times l$ -dimensional matrix function of x, and  $B(x) = \begin{bmatrix} B_{11}(x) & B_{12}(x) & \cdots & B_{1m} \\ B_{21}(x) & B_{22}(x) & \cdots & B_{2m}(x) \\ \cdots & \cdots & \cdots & \cdots \\ B_{n1}(x) & B_{n2}(x) & \cdots & B_{nm}(x) \end{bmatrix}$  is an  $n \times m$ -dimensional matrix function of x.

The control objective is to follow a theoretically sound control design methodology to design the controller such that the system is adaptively controlled with respect to parametric uncertainties and yet minimizing a desired control performance.

To facilitate the theoretical derivations, several conventional assumptions are made in the following and applied throughout the Chapter.

AS1: It is assumed that a(.), C(.) and B(.) have continuous partial derivatives with respect

to the state variables on the region of interest. In other words,  $a_i(x)$ ,  $C_{is}(x)$ ,  $B_{ik}(x)$ ,  $\frac{\partial a_i(x)}{\partial x_i}$ ,

$$\frac{\partial C_{is}(x)}{\partial x_j}$$
, and  $\frac{\partial B_{ik}(x)}{\partial x_j}$  for  $i, j = 1, 2, \dots, n$ ;  $k = 1, 2, \dots, m$ ;  $s = 1, 2, \dots, l$  exist and are continuous

and bounded on the region of interest.

It should be noted that the above conditions imply that a(.), C(.) and B(.) satisfy the Lipschitz condition which in turn implies that there always exists a unique and continuous solution to the differential equation given an initial condition  $x(t_0) = \xi_0$  and a bounded control u(t).

AS2: In practical applications, control effort is usually confined due to the limitation of design or conditions corresponding to physical constraints. Without loss of generality, assume that the admissible control set U is characterized by:

$$U = \{ u : | u_i | \le 1, i = 1, 2, \cdots, m \}$$
(2)

where  $u_i$  is u 's i th component.

AS3: It is assumed that the system is controllable.

AS4: Some control performance criteria J may relate to the initial time  $t_0$  and the final time  $t_f$ . The cost functional reflects the requirement of a particular type of optimal control.

AS5: The target set  $\theta_f$  is defined as  $\theta_f = \{x : \psi(x(t_f)) = 0\}$  where  $\psi_i$ 's ( $i = 1, 2, \dots, q$ ) are the components of the continuously differentiable function vector  $\psi(.)$ .

Remark 1: As a step of our approach to address the control design for the system (1), the above same control problem is studied with the only difference that the parameters in Eq. (1) are given. An optimal solution is sought to the following control problem:

The optimal control problem ( $P_0$ ) consists of the system equation (1) with fixed and known parameter vector p, the initial time  $t_0$ , the variable final time  $t_f$ , the initial state  $x_0 = x(t_0)$ , together with the assumptions AS1, AS2, AS3, AS4, AS5 satisfied such that the system state conducts to a pre-specified terminal set  $\theta_f$  at the final time  $t_f$  while the control performance index is minimized.

AS6: There do not exist singular solutions to the optimal control problem ( $P_0$ ) as described in Remark 1 (referenced as the control problem ( $P_0$ ) later on distinct from the original control problem (P)).

AS7:  $\frac{\partial x}{\partial p}$  is bounded on  $p \in \Omega_p$  and  $x \in \Omega_x$ .

Remark 2: For any continuous function f(x) defined on the compact domain  $\Omega_x \subset \mathbb{R}^n$ , there exists a neural network characterized by  $NN_f(x)$  such that for any positive number

 $\varepsilon_f^*$ ,  $|f(x) - NN_f(x)| < \varepsilon_f^*$ .

AS8: Let the sufficiently trained neural network be denoted by  $NN(x,\Theta_s)$ , and the neural network with the ideal weights and biases by  $NN(x,\Theta_*)$  where  $\Theta_s$  and  $\Theta_*$  designate the parameter vectors comprising weights and biases of the corresponding neural networks. The approximation of  $NN_f(x,\Theta_s)$  to  $NN_f(x,\Theta_*)$  is measured by  $\delta NN_f(x;\Theta_s;\Theta_*) = |NN_f(x,\Theta_s) - NN_f(x,\Theta_*)|$ . Assume that  $\delta NN_f(x;\Theta_s;\Theta_*)$  is bounded by a pre-designated number  $\varepsilon^s > 0$ , i.e.,  $\delta NN_f(x;\Theta_s;\Theta_*) < \varepsilon^s$ .

AS9: The total number of switch times for all control components for the studied fuel-

optimal control problem is greater than the number of state variables. Remark 3: AS9 is true for practical systems to the best knowledge of the authors. The assumption is made for the convenience of the rigor of the theoretical results developed in this Chapter.

#### 2.1 Time-optimal control

For the time-optimal control problem, the system characterization, the control objective, constraints remain the same as for the generic control problem with the exception that the control performance index reflected in the Assumption AS4 is replaced with the following:

AS4: The control performance criteria is  $J = \int_{t_0}^{t_f} 1 ds$  where  $t_0$  and  $t_f$  are the initial time and the

final time, respectively. The cost functional reflects the requirement of time-optimal control.

#### 2.2 Fuel-optimal control

For the fuel-optimal control problem, the system characterization, the control objective, constraints remain the same as for the time-optimal control problem with the Assumption AS4 replaced with the following:

AS4: The control performance criteria is  $J = \int_{t_0}^{t_f} \left[ e_0 + \sum_{k=1}^m e_k |u_k| \right] ds$  where  $t_0$  and  $t_f$  are the

initial time and the final time, respectively, and  $e_k$  ( $k = 0, 1, 2, \dots, m$ ) are non-negative constants. The cost functional reflects the requirement of fuel-optimal control as related to the integration of the absolute control effort of each control variable over time.

#### 2.3 Optimal control with quadratic performance index

For the quadratic performance index based optimal control problem, the system characterization, the control objective, constraints remain the same with the Assumption AS4 replaced with the following:

AS4: The control performance criteria is  

$$J = \frac{1}{2} (x(t_f) - r(t_f))^{\tau} S(t_f) (x(t_f) - r(t_f)) + \frac{1}{2} \int_{t_0}^{t_f} \left[ x^{\tau} Q x + (u - u_e)^{\tau} R(u - u_e) \right] ds \text{ where } t_0 \text{ and } t_f \text{ are}$$

the initial time and the final time, respectively; and  $S(t_f) \ge 0$ ,  $Q \ge 0$ , and  $R \ge 0$  with appropriate dimensions; and the desired final state  $r(t_f)$  is the specified as the equilibrium  $x_e$ , and  $u_e$  is the equilibrium control.

#### 3. Numerical solution schemes to the optimal control problems

To solve for the optimal control, mathematical derivations are presented below for each of the above optimal control problems to show that the resulting equations represent the Hamiltonian system which is usually a coupled two-point boundary-value problem (TPBVP), and the analytic solution is not available, to our best knowledge. It is worth noting that in the solution process, the parameter is assumed to be fixed.

#### 3.1 Numerical solution scheme to the time optimal control problem

By assumption AS4, the optimal control performance index can be expressed as

$$J(t_0) = \int_{t_0}^{t_f} 1dt$$

where  $t_0$  is the initial time, and  $t_f$  is the final time.

Define the Hamiltonian function as

$$H(x,u,t) = 1 + \lambda^{\tau}(a(x) + C(x)p + B(x)u)$$

where  $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \end{bmatrix}^{\tau}$  is the costate vector.

The final-state constraint is  $\psi(x(t_f)) = 0$  as mentioned before.

The state equation can be expressed as

$$\dot{x} = \frac{\partial H}{\partial \lambda} = a(x) + C(x)p + B(x)u, t \ge t_0$$

The costate equation can be written as

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = \frac{\partial (a(x) + C(x)p + B(x)u)^{\tau}}{\partial x} \lambda, t \leq T$$

The Pontryagin minimum principle is applied in order to derive the optimal control (Lee & Markus, 1967). That is,

$$H(x^*, u^*, \lambda^*, t) \le H(x^*, u^*, \lambda^*, t)$$

for all admissible u.

where  $u^*$ ,  $x^*$  and  $\lambda^*$  correspond to the optimal solution. Consequently,

$$\lambda^{*\tau} \sum_{k=1}^m B_k(x^*) u_k^* \leq \lambda^{\tau} \sum_{k=1}^m B_k(x) u_k$$

where  $B_k(x)$  is the *k* th column of the B(x).

Since the control components  $u_k$  's are all independent, the minimization of  $\lambda^{\tau} \sum_{k=1}^{m} B_k(x) u_k$  is equivalent to the minimization of  $\lambda^{\tau} B_k(x) u_k$ .

The optimal control can be expressed as  $u_k^* = -\text{sgn}(s_k^*(t))$ , where sgn(.) is the sign function defined as sgn(t) = 1 if t > 0 or sgn(t) = -1 if t < 0; and  $s_k(t) = \lambda^{\tau} B_k(x)$  is the *k* th component of the switch vector  $S(t) = B(x)^{\tau} \lambda$ .

It is observed that the resulting Hamiltonian system is a coupled two-point boundary-value problem, and its analytic solution is not available in general.

With assumption AS6 satisfied, it is observed from the derivation of the optimal time control that the control problem ( $P_0$ ) has bang-bang control solutions.

Consider the following cost functional:

$$J = \int_{t_0}^{t_f} 1 dt + \sum_{i=1}^{q} \rho_i \psi_i^2(x(t_f))$$

where  $\rho_i$ 's are positive constants, and  $\psi_i$ 's are the components of the defining equation of the target set  $\theta_f = \{x : \psi(x(t_f)) = 0\}$  to the system state is transferred from a given initial state by means of proper control, and q is the number of components in  $\psi$ .

It is observed that the system described by Eq. (1) is a nonlinear system but linear in control. With assumption AS6, the requirements for applying the Switching-Time-Varying-Method (STVM) are met. The optimal switching-time vector can be obtained by using a gradient-based method. The convergence of the STVM is guaranteed if there are no singular solutions.

Note that the cost functional can be rewritten as follows:

$$J = \int_{t_0}^{t_f} \left[ (a'_0(x) + \langle b'_0(x), u \rangle) \right] dt$$

where  $a'_0(x) = 1 + 2\sum_{i=1}^q \rho_i \psi_i < \frac{\partial \psi_i}{\partial x}, a(x) + C(x)p >$ ,  $b'_0(x) = 2\sum_{i=1}^q \rho_i \psi_i [\frac{\partial \psi_i}{\partial x}]^{\tau} B(x)$ , and a(x), C(x), p and B(x) are as given in the control problem ( $P_0$ ). Define a new state variable  $x_0(t)$  as follows:

$$x_0(t) = \int_{t0}^t [(a_0'(x) + \langle b_0'(x), u \rangle)] dt$$

Define the augmented state vector  $\underline{x} = \begin{bmatrix} x_0 & x^{\tau} \end{bmatrix}^{\tau}$ ,  $\underline{a}(\underline{x}) = \begin{bmatrix} a_0'(x) & (a(x) + C(x)p)^{\tau} \end{bmatrix}^{\tau}$ , and  $\underline{B}(\underline{x}) = \begin{bmatrix} b_0'(x) & (B(x))^{\tau} \end{bmatrix}^{\tau}$ .

The system equation can be rewritten in terms of the augmented state vector as

$$\underline{\dot{x}} = \underline{a}(\underline{x}) + \underline{B}(\underline{x})u$$
 where  $\underline{x}(t_0) = \begin{bmatrix} 0 & x(t_0)^{\tau} \end{bmatrix}^{\tau}$ 

A Hamiltonian system can be constructed for the above state equation with the costate equation given by

$$\dot{\lambda} = -\frac{\partial}{\partial \underline{x}} (\underline{a}(\underline{x}) + \underline{B}(\underline{x})u)^{\tau} \lambda \text{ where } \lambda(t_f) = \frac{\partial J}{\underline{\partial \underline{x}}} |\underline{x}(t_f)|.$$

It has been shown (Moon, 1969; Mohler, 1973; Mohler, 1991) that the number of the optimal switching times must be finite provided that no singular solutions exist. Let the zeros of  $-s_k(t)$  be  $\tau_{k,j}^+$  ( $j = 1, 2, \dots, 2N_k^+$ ,  $k = 1, 2, \dots, m$ ; and  $\tau_{k,j_1}^+ < \tau_{k,j_2}^+$  for  $1 \le j_1 < j_2 \le 2N_k^+$ ).

$$u_{k}^{*}(t) = \sum_{j=1}^{N_{k}^{+}} [\operatorname{sgn}(t - \tau_{k,2j-1}^{+}) - \operatorname{sgn}(t - \tau_{k,2j}^{+})].$$

Let the switch vector for the *k* th component of the control vector be  $\underline{\tau}^{N_k} = \underline{\tau}^{N_k^+}$  where  $\underline{\tau}^{N_k^+} = \begin{bmatrix} \tau_{k,1}^+ & \cdots & \tau_{k,2N_k^+}^+ \end{bmatrix}^{\tau}$ . Let  $N_k = 2N_k^+$ . Then  $\underline{\tau}^{N_k}$  is the switching vector of  $N_k$  dimensions.

Let the vector of switch functions for the control variable  $u_k$  be defined as  $\phi^{N_k} = \begin{bmatrix} \phi_1^{N_k} & \cdots & \phi_{2N_k^+}^{N_k} \end{bmatrix}^{\tau}$  where  $\phi_j^{N_k} = (-1)^{j-1} s_k(\tau_{k,j}^+)$   $(j = 1, 2, \cdots, 2N_k^+)$ .

The gradient that can be used to update the switching vector  $\underline{\tau}^{N_k}$  can be given by

$$\nabla^J_{\underline{\tau}^{N_k}} = -\underline{\phi}^{N_k}$$

The optimal switching vector can be obtained iteratively by using a gradient-based method.

$$\underline{\tau}^{N_k,i+1} = \underline{\tau}^{N_k,i} + K^{k,i} \phi^{N_k}$$

where  $K^{k,i}$  is a properly chosen  $N_k \times N_k$ -dimensional diagonal matrix with non-negative entries for the *i* th iteration of the iterative optimization process; and  $\underline{\tau}^{N_k,i}$  represents the *i* th iteration of the switching vector  $\underline{\tau}^{N_k}$ .

Remark 4: The choice of the step sizes as characterized in the matrix  $K^{k,i}$  must consider two facts: if the step size is chosen too small, the solution may converge very slowly; if the step size is chosen too large, the solution may not converge. Instead of using the gradient descent method, which is relatively slow compared to other alternative such as methods based on Newton's method and inversion of the Hessian using conjugate gradient techniques.

When the optimal switching vectors are determined upon convergence, the optimal control trajectories and the optimal state trajectories are computed. This process will be repeated for all selected nominal cases until all needed off-line optimal control and state trajectories are obtained. These trajectories will be used in training the time-optimal control oriented neural networks.

#### 3.2 Numerical solution scheme to the fuel optimal control problem

By assumption AS4, the optimal control performance index can be expressed as

$$J(t_0) = \int_{t_0}^{t_f} \left[ e_0 + \sum_{k=1}^m e_k \mid u_k \mid \right] dt$$

where  $t_0$  is the initial time, and  $t_f$  is the final time.

Define the Hamiltonian function as

$$H(x, u, t) = e_0 + \sum_{k=1}^{m} e_k |u_k| + \lambda^{\tau} (a(x) + C(x)p + B(x)u)$$

where  $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \end{bmatrix}^r$  is the costate vector. The final-state constraint is  $\psi(x(t_f)) = 0$  as mentioned before. The state equation can be expressed as

$$\dot{x} = \frac{\partial H}{\partial \lambda} = a(x) + C(x)p + B(x)u, t \ge t_0$$

The costate equation can be written as

$$-\lambda = \frac{\partial H}{\partial x} = \frac{\partial (a(x) + C(x)p + B(x)u)^{\tau}}{\partial x}\lambda + \frac{\partial (e_0 + \sum_{k=1}^m e_k |u_k|)}{\partial x} = \frac{\partial (a(x) + C(x)p + B(x)u)^{\tau}}{\partial x}\lambda, t \le T$$

The Pontryagin minimum principle is applied in order to derive the optimal control (Lee & Markus, 1967). That is,

 $H(x^*, u^*, \lambda^*, t) \le H(x^*, u, \lambda^*, t)$  for all admissible u, where  $u^*$ ,  $x^*$  and  $\lambda^*$  correspond to the optimal solution.

Consequently,

$$\sum_{k=1}^{m} e_k |u_k^*| + \lambda^{*\tau} \sum_{k=1}^{m} B_k(x^*) u_k^* \leq \sum_{k=1}^{m} e_k |u_k| + \lambda^{\tau} \sum_{k=1}^{m} B_k(x) u_k$$

where  $B_k(x)$  is the *k* th column of the B(x).

Since the control components  $u_k$ 's are all independent, the minimization of  $\sum_{k=1}^{m} e_k |u_k| + \lambda^{\tau} \sum_{k=1}^{m} B_k(x) u_k$  is equivalent to the minimization of  $e_k |u_k| + \lambda^{\tau} B_k(x) u_k$ .

Since  $e_k \neq 0$ , define  $s_k = \lambda^{\tau} B_k(x) / e_k$ . The fuel-optimal control satisfies the following condition:

$$u_{k}^{*} = \begin{cases} -\operatorname{sgn}(s_{k}^{*}(t)), | s_{k}^{*}(t) | > 1 \\ 0, | s_{k}^{*}(t) | < 1 \\ undefined, | s_{k}^{*}(t) | = 1 \end{cases}$$

where  $k = 1, 2, \cdots, m$ .

Note that the above optimal control can be written in a different form as follows:

$$u_k^* = u_k^{*+} + u_k^{*-}$$

where  $u_k^{*+} = \frac{1}{2} \left[ \text{sgn}(-s_k^{*}(t) - 1) + 1 \right]$ , and  $u_k^{*-} = \frac{1}{2} \left[ \text{sgn}(-s_k^{*}(t) + 1) - 1 \right]$ .

It is observed that the resulting Hamiltonian system is a coupled two-point boundary-value problem, and its analytic solution is not available in general. With assumption AS6 satisfied, it is observed from the derivation of the optimal fuel control that the control problem ( $P_0$ ) only has bang-off-bang control solutions. Consider the following cost functional:

$$J = \int_{t_0}^{t_f} \left[ e_0 + \sum_{k=1}^m e_k \mid u_k \mid \right] dt + \sum_{i=1}^q \rho_i \psi_i^2(x(t_f))$$

where  $\rho_i$ 's are positive constants, and  $\psi_i$ 's are the components of the defining equation of the target set  $\theta_f = \{x : \psi(x(t_f)) = 0\}$  to the system state is transferred from a given initial state by means of proper control, and q is the number of components in  $\psi$ .

It is observed that the system described by Eq. (1) is a nonlinear system but linear in control. With assumption AS6, the requirements for the STVM's application are met. The optimal switching-time vector can be obtained by using a gradient-based method. The convergence of the STVM is guaranteed if there are no singular solutions.

Note that the cost functional can be rewritten as follows:

$$J = \int_{t_0}^{t_f} \left[ (a_0'(x) + \langle b_0'(x), u \rangle) + \sum_{k=1}^m e_k \mid u_k \mid \right] dt$$

where  $a_0'(x) = e_0 + 2\sum_{i=1}^q \rho_i \psi_i < \frac{\partial \psi_i}{\partial x}$ , a(x) + C(x)p >,  $b_0'(x) = 2\sum_{i=1}^q \rho_i \psi_i [\frac{\partial \psi_i}{\partial x}]^{\tau} B(x)$ , and a(x), C(x), p and B(x) are as given in the control problem ( $P_0$ ).

Define a new state variable  $x_0(t)$  as follows:

$$x_0(t) = \int_{t0}^t \left[ (a_0'(x) + \langle b_0'(x), u \rangle) + \sum_{k=1}^m e_k |u_k| \right] dt$$

Define the augmented state vector  $\underline{x} = \begin{bmatrix} x_0 & x^{\tau} \end{bmatrix}^{\tau}$ ,

$$\underline{a}(\underline{x}) = \begin{bmatrix} a'_0(x) & (a(x) + C(x)p)^{\tau} \end{bmatrix}^{\tau} \text{ and } \underline{B}(\underline{x}) = \begin{bmatrix} b'_0(x) & (B(x))^{\tau} \end{bmatrix}^{\tau}$$

The system equation can be rewritten in terms of the augmented state vector as

$$\underline{\dot{x}} = \underline{a}(\underline{x}) + \underline{B}(\underline{x})u$$
 where  $\underline{x}(t_0) = \begin{bmatrix} 0 & x(t_0)^{\tau} \end{bmatrix}^{\tau}$ 

The adjoint state equation can be written as

$$\dot{\lambda} = -\frac{\partial}{\partial \underline{x}} (\underline{a}(\underline{x}) + \underline{B}(\underline{x})u)^{\tau} \lambda \text{ where } \lambda(t_f) = \frac{\partial J}{\underline{\partial \underline{x}}} |\underline{x}(t_f)|$$

It has been shown (Moon, 1969; Mohler, 1973; Mohler, 1991) that the number of the optimal switching times must be finite provided that no singular solutions exist. Let the zeros of  $-s_k(t)-1$  be  $\tau_{k,j}^+$  ( $j=1,2,\dots,2N_k^+$ ,  $k=1,2,\dots,m$ ; and  $\tau_{k,j_1}^+ < \tau_{k,j_2}^+$  for  $1 \le j_1 < j_2 \le 2N_k^+$ ) which represent the switching times corresponding to positive control  $u_k^{*+}$ , the zeros of  $-s_k(t)+1$  be  $\tau_{k,j}^-$  ( $j=1,2,\dots,2N_k^-$ ,  $k=1,2,\dots,m$ ; and  $\tau_{k,j_1}^- < \tau_{k,j_2}^-$  for  $1 \le j_1 < j_2 \le 2N_k^-$ ) which represent the switching times corresponding to negative control  $u_k^{*-}$ . Altogether  $\tau_{k,j}^+$ 's and  $\tau_{k,j_j}^-$ 's represent the switching times which uniquely determine  $u_k^*$  as follows:

$$u_{k}^{*}(t) = \frac{1}{2} \{ \sum_{j=1}^{N_{k}^{+}} [\operatorname{sgn}(t - \tau_{k,2j-1}^{+}) - \operatorname{sgn}(t - \tau_{k,2j}^{+})]$$
  
$$\sum_{j=1}^{N_{k}^{-}} [\operatorname{sgn}(t - \tau_{k,2j-1}^{-}) - \operatorname{sgn}(t - \tau_{k,2j}^{-})] \}.$$

Let the switch vector for the *k* th component of the control vector be  $\underline{\tau}^{N_k} = \left[ (\underline{\tau}^{N_k^+})^{\tau} \quad (\underline{\tau}^{N_k^-})^{\tau} \right]^{\tau}$  where  $\underline{\tau}^{N_k^+} = \left[ \tau_{k,1}^+ \quad \cdots \quad \tau_{k,2N_k^+}^+ \right]^{\tau}$  and  $\underline{\tau}^{N_k^-} = \left[ \tau_{k,1}^- \quad \cdots \quad \tau_{k,2N_k^-}^- \right]^{\tau}$ . Let  $N_k = 2N_k^+ + 2N_k^-$ . Then  $\underline{\tau}^{N_k}$  is the switching vector of  $N_k$  dimensions.

Let the vector of switch functions for the control variable  $u_k$  be defined as  $\phi^{N_k} = \begin{bmatrix} \phi_1^{N_k} & \cdots & \phi_{2N_k^+}^{N_k} & \phi_{2N_k^++1}^{N_k} & \cdots & \phi_{2N_k^++2N_k^-}^{N_k} \end{bmatrix}$  where  $\phi_j^{N_k} = (-1)^{j-1} e_k(s_k(\tau_{k,j}^+)+1)$  $(j = 1, 2, \cdots, 2N_k^+)$ , and  $\phi_{j+2N_k^+}^{N_k} = (-1)^j e_k(s_k(\tau_{k,j}^-)-1)$   $(j = 1, 2, \cdots, 2N_k^-)$ .

The gradient that can be used to update the switching vector  $\underline{\tau}^{N_k}$  can be given by

The optimal switching vector can be obtained iteratively by using a gradient-based method.

 $\nabla^J_{\tau^{N_k}} = -\underline{\phi}^{N_k}$ 

$$\underline{\tau}^{N_k,i+1} = \underline{\tau}^{N_k,i} + K^{k,i} \phi^{N_k}$$

where  $K^{k,i}$  is a properly chosen  $N_k \times N_k$ -dimensional diagonal matrix with non-negative entries for the *i* th iteration of the iterative optimization process; and  $\underline{\tau}^{N_k,i}$  represents the *i* th iteration of the switching vector  $\underline{\tau}^{N_k}$ .

When the optimal switching vectors are determined upon convergence, the optimal control trajectories and the optimal state trajectories are computed. This process will be repeated for

all selected nominal cases until all needed off-line optimal control and state trajectories are obtained. These trajectories will be used in training the fuel-optimal control oriented neural networks.

#### **3.3 Numerical solution scheme to the quadratic optimal control problem** The Hamiltonian function can be defined as

 $H(x, u, t) = \frac{1}{2} (x^{\tau}Qx + (u - u_e)^{\tau}R(u - u_e)) + \lambda^{\tau}(a + Cp + Bu)$ The state equation is given by

 $\dot{x} = \frac{\partial H}{\partial \lambda} = a + Cp + Bu$ 

The costate equation can be given by

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = \frac{\partial (a + Cp + Bu)^{\tau}}{\partial x} \lambda + Qx$$

The stationarity equation gives

$$0 = \frac{\partial H}{\partial u} = \frac{\partial (a + Cp + Bu)^{\tau}}{\partial u} \lambda + R(u - u_e)$$

*u* can be solved out as

$$u = -R^{-1}B^{\tau}\lambda + u_{\rho}$$

The Hamiltonian system becomes

$$\begin{cases} \dot{x} = a(x) + C(x)p + B(x)(-R^{-1}B^{\tau}\lambda + u_e) \\ -\dot{\lambda} = \frac{\partial(a(x) + C(x)p + B(x)(-R^{-1}B^{\tau}\lambda + u_e))^{\tau}}{\partial x}\lambda + Qx \end{cases}$$

Furthermore, the boundary condition can be given by

$$\lambda(t_f) = S(t_f)(x(t_f) - r(t_f))$$

Notice that for the Hamiltonian system which is composed of the state and costate equations, the initial condition is given for the state equation, and the constraints on the costate variables at the final time for the costate equation.

It is observed that the Hamiltonian system is a set of nonlinear ordinary differential equations in x(t) and  $\lambda(t)$  which develop forward and back in time, respectively. Generally, it is not possible to obtain the analytic closed-form solution to such a two-point boundary-value problem (TPBVP). Numerical methods have to be employed to solve for the Hamiltonian system. One simple method, called shooting method may be used. There are other methods like the "shooting to a fixed point" method, and relaxation methods, etc.

The idea for the shooting method is as follows:

- 1. First make a guess for the initial values for the costate.
- 2. Integrate the Hamiltonian system forward.
- 3. Evaluate the mismatch on the final constraints.
- 4. Find the sensitivity Jacobian for the final state and costate with respect to the initial costate value.
- 5. Using the Newton-Raphson method to determine the change on the initial costate value.
- 6. Repeat the loop of steps 2 through 5 until the mismatch is close enough to zero.

#### 4. Unified hierarchical neural control design framework

Keeping in mind that the discussions and analyses made in Section 3 are focused on the system with a fixed parameter vector, which is the control problem ( $P_0$ ). To address the original control problem (P), the parameter vector space is tessellated into a number of sub-regions. Each sub-region is identified with a set of vertexes. For each of the vertexes, a different control problem ( $P_0$ ) is formed. The family of control problems ( $P_0$ ) are combined together to represent an approximately accurate characterization of the dynamic system behaviours exhibited by the nonlinear systems in the control problem (P). This is an important step toward the hierarchical neural control design framework that is proposed to address the optimal control of uncertain nonlinear systems.

#### 4.1 Three-layer approach

While the control problem (P) is approximately equivalent to the family of control problems ( $P_0$ ), the solutions to the respective control problems ( $P_0$ ) must be properly coordinated in order to provide a consistent solution to the original control problem (P). The requirement of consistent coordination of individual solutions may be mapped to the hierarchical neural network control design framework proposed in this Chapter that features the following:

- For a fixed parameter vector, the control solution characterized by a set of optimal state and control trajectories shall be approximated by a neural network, which may be called a nominal neural network for this nominal case. For each nominal case, a nominal neural network is needed. All the nominal neural network controllers constitute the nominal layer of neural network controllers.
- For each sub-region, regional coordinating neural network controllers are needed to coordinate the responses from individual nominal neural network controllers for the sub-region. All the regional coordinating neural network controllers constitute the regional layer of neural network controllers.
- For an unknown parameter vector, global coordinating neural network controllers are needed to coordinate the responses from regional coordinating neural network controllers. All the global coordinating neural network controllers constitute the global layer of neural networks controllers.

The proposed hierarchical neural network control design framework is a systematic extension and a comprehensive enhancement of the previous endeavours (Chen, 1998; Chen & Mohler & Chen, 2000).

#### 4.2 Nominal layer

Even though the hierarchical neural network control design methodology is unified and generic, the design of the three layers of neural networks, especially the nominal layer of neural networks may consider the uniqueness of the problems under study. For the time optimal control problems, the role of the nominal layer of neural networks is to identify the switching manifolds that relate to the bang-bang control. For the fuel optimal problems, the role of the nominal layer of neural networks is to identify the switching manifolds that relate to the bang-bang control. For the fuel optimal problems, the role of the nominal layer of neural networks is to identify the switching manifolds that relate to the bang-off-bang control. For the quadratic optimal control problems, the role of the nominal layer of neural networks is to approximate the optimal control based on the state variables.



Fig. 1. Nominal neural network for time optimal control

Consequently a nominal neural network for the time optimal control takes the form of a conventional neural network with continuous activation functions cascaded by a two-level stair case function which itself may viewed as a discrete neural network itself, as shown in Fig. 1. For the fuel optimal control, a nominal neural network takes the form of a conventional neural network with continuous activation functions cascaded by a three-level stair case function, as shown in Fig. 2.



Fig. 2. Nominal neural network for fuel optimal control

For the quadratic optimal control, no switching manifolds are involved. A conventional neural network with continuous activation functions is sufficient for a nominal case, as shown in Fig. 3.



#### 4.3 Overall architecture

The overall architecture of the multi-layered hierarchical neural network control framework, as shown in Fig. 4, include three layers: the nominal layer, the regional layer, and the global layer. These three layers play different roles and yet work together to attempt to achieve desired control performance.

At the nominal layer, the nominal neural networks are responsible to compute the near optimal control signals for a given parameter vector. The post-processing function block is necessary for both time optimal control problem and fuel optimal control problems while indeed it may not be needed for the quadratic optimal control problems. For time optimal control problems, the post-processing function is a sign function as shown in Fig. 2. For the fuel optimal control problems, the post-processing is a slightly more complicated stair-case function as shown in Fig. 3.

At the regional layer, the regional neural networks are responsible to compute the desired weighting factors that are in turn used to modulate the control signals computed by the nominal neural networks to produce near optimal control signals for an unknown parameter vector situated at the know sub-region of the parameter vector space. The post-processing function block is necessary for all the three types of control problems studied in this Chapter. It is basically a normalization process of the weighting factors produced by the regional neural networks for a sub-region that is enabled by the global neural networks.

At the global layer, the global neural networks are responsible to compute the possibilities of the unknown parameter vector being located within sub-regions. The post-processing function block is necessary for all the three types of control problems studied in this Chapter. It is a winner-take-all logic applied to all the output data of the global neural networks. Consequently, only one sub-regional will be enabled, and all the other subregions will be disabled. The output data of the post-processing function block is used to turn on only one of the sub-regions for the regional layer.

To make use of the multi-layered hierarchical neural network control design framework, it is clear that the several key factors such as the number of the neural networks for each layer, the size of each neural network, and desired training patterns, are important. This all has to do with the determination of the nominal cases. A nominal case designates a group of system conditions that reflect one of the typical system behaviors. In the context of control of a dynamic system with uncertain parameters, which is the focus of this Chapter, a nominal case may be designated as corresponding to the vertexes of the sub-regions when the parameter vector space is tessellated into a number of non-overlapping sub-regions down to a level of desired granularity.



Fig. 4. Multi-layered hierarch neural network architecture

Once the nominal cases are identified, the numbers of neural networks for the nominal layer, the regional layer and the global layer can be determined accordingly. Each nominal neural network corresponds to a nominal case identified. Each regional neural network corresponds to a nominal neural neural network corresponds to a sub-region.

With the numbers of neural networks for all the three layers in the hierarchy determined, the size of each neural network is dependent upon the data collected for each nominal case. As shown in the last Section, the optimal state trajectories and the optimal control trajectories for each of the control problems ( $P_0$ ) can be obtained through use of the STVM approach for time optimal control and for fuel optimal control or the shooting method for the quadratic optimal control. For each of the nominal cases, the optimal state trajectories and optimal control trajectories may be properly utilized to form the needed training patterns.

#### 4.4 Design procedure

Below is the design procedure for multi-layered hierarchical neural networks:

- Identify the nominal cases. The parameter vector space may be tessellated into a number of non-overlapping sub-regions. The granualarity of the tessellation process is determined by how sensitive the system dynamic behaviors are to the changes of the parameters. Each vertext of the sub-regions identifies a nominal case. For each nominal case, the optimal control problem may be solved numerically and the nuermical solution may be obtained.
- Determine the size of the nominal layer, the regional layer and the global layer of the hierarchy.
- Determine the size of the neural networks for each layer in the hierarchy.
- Train the nominal neural networks. The numerically obtained optimal state and control trajectories are acquired for each nominal case. The training data pattern for the nominal neural networks is composed of the state vector as input and the control signal as the output. In other words, the nominal layer is to establish and approximate a state feedback control. Finish training when the training performance is satisfactory. Repeat this nominal layer training process for all the nominal neural networks.
- Training the regional neural networks. The input data to the nominal neural networks is also part of the input data to the regional neural networks. In addition, for a specific regional neural network, the ideal output data of the corresponding nominal neural network is also part of its input data. The ideal output data of the regional neural neural network can be determined as follows:
  - If the data presented to a given regional neural network reflects a nominal case that corresponds to the vertex that this regional neural network is to be trained for, then assign 1 or else 0.
- Training the global neural networks. The input data to the nominal neural networks is also part of the input data to the global neural networks. In addition, for a specific global neural network, the ideal output data of the corresponding nominal neural network is also part of its input data. The ideal output data of the global neural network can be determined as follows:
  - If the data presented to a given global neural network reflects a nominal case that corresponds to the sub-region that this global neural network is to be trained for, then assign 1 or else 0.

#### 5. Theoretical justification

This Section provides theoretical support for the adoption of the hierarchical neural networks.

As shown in (Chen, Yang & Moher, 2006), the desired prediction or control can be achieved by a properly designed hierarchical neural network.

Proposition 1 (Chen, Yang & Mohler, 2006): Suppose that an ideal system controller can be characterized by function vectors  $f_i^u$  and  $f_i^l$  ( $1 \le i \le n_l = n_u$ ) which are continuous mappings from a compact support  $\Omega \subset \mathbb{R}^{n_x}$  to  $\mathbb{R}^{n_y}$ , such that a continuous function vector f also defined on  $\Omega$  can be expressed as  $f_j(x) = \sum_{i=1}^{n_l} f_{i,j}^u(x) \times f_{i,j}^l(x)$  on the point-wise basis

 $(x \in \Omega; \text{ and } f_{i,j}^u(x) \text{ and } f_{i,j}^l(x) \text{ are the } j\text{th component of } f_i^u$  and  $f_i^l$ ). Then there exists a hierarchical neural network, used to approximate the ideal system controller or system identifier, that includes lower level neural networks  $nn_i^l$ 's and upper level neural networks

 $nn_i^u$   $(1 \le i \le n_l = n_u)$  such that for any  $\varepsilon_j > 0$ ,  $\sup_{x \in \Omega} |f_j - \sum_{i=1}^{n_l} nn_{i,j}^l \times nn_{i,j}^u | < \varepsilon_j$  where  $nn_{i,j}^u(x)$  and  $nn_{i,j}^l(x)$  are the *j*th component of  $nn_i^u$  and  $nn_i^l$ .

The following proposition is to show that the parameter uncertainties can also be handled by the hierarchical neural networks.

Proposition 2: For the system (1) and the assumptions AS1-AS9, with the application of the hierarchical neural controller, the deviation of the resuting state trajectory for the unknow parameter vector from that of the optimal state trajectory is bounded.

Proof: Let the estiamte of the parameter vector be denoted by  $\hat{p}$ . The counterpart of system (1) for the estimated parameter vector  $\hat{p}$  can be given by

$$\dot{x} = a(x) + C(x)\hat{p} + B(x)u$$

Integrating of the above equation and system (1) from  $t_0$  to t leads to the following two equations:

$$x_{1}(t) = x_{1}(t_{0}) + \int_{t_{0}}^{t} [a(x_{1}(s)) + C(x_{1}(s))\hat{p} + B(x_{1}(s))u(s)]ds$$
$$x_{2}(t) = x_{2}(t_{0}) + \int_{t_{0}}^{t} [a(x_{2}(s)) + C(x_{2}(s))p + B(x_{2}(s))u(s)]ds$$

By noting that  $x_1(t_0) = x_2(t_0) = x_0$ , subtraction of the above two equations yields

$$x_{1}(t) - x_{2}(t) = \int_{t_{0}}^{t} \{a(x_{1}(s)) - a(x_{2}(s)) + [B(x_{1}(s)) - B(x_{2}(s))]u(s)]\}ds + \int_{t_{0}}^{t} \{C(x_{1}(s))(\hat{p} - p) + [C(x_{1}(s)) - C(x_{2}(s))]p\}ds$$

Note that, by Taylor's theorem,  $a(x_1(s)) - a(x_2(s)) = a_T(x_1(s) - x_2(s))$ ,  $B(x_1(s)) - B(x_2(s)) = B_T(x_1(s) - x_2(s))$ , and  $C(x_1(s)) - C(x_2(s)) = C_T(x_1(s) - x_2(s))$ .

Define  $\Delta x(t) = x_1(t) - x_2(t)$ , and  $\Delta p = p - \hat{p}$ . Then we have

$$\Delta x(t) = \int_{t_0}^t \{a_T \Delta x(s) + B_T \Delta x(s)u(s)\} + C_T \Delta x(s)p\} ds + \int_{t_0}^t C(x_1(s))\Delta p ds$$

If the both sides of the above equation takes an appropriate norm and the triangle inequality is applied, the following is obtained:

$$||\Delta x(t)|| \leq ||\int_{t_0}^t \{a_T \Delta x(s) + B_T \Delta x(s)u(s)] + C_T \Delta x(s)p\} ds|| + \int_{t_0}^t ||C(x_1(s))\Delta p|| ds$$

Note that  $||C(x_1(s)\Delta p)||$  can be made uniformly bounded by  $\varepsilon$  as long as the estimate of p is made sufficiently close to p (which can be controlled by the granularity of tessellation), and p is bounded;  $|u(t)| \le 1$ ;  $||a_T|| = \sup_{x \in \Omega} a_T(x) < \infty$ ,  $||B_T|| = \sup_{x \in \Omega} B_T(x) < \infty$  and  $||C_T|| = \sup_{x \in \Omega} C_T(x) < \infty$ .

It follows that

$$||\Delta x(t)|| \le \varepsilon(t - t_0) + (||a_T|| + ||B_T|| + ||C_T||| + ||p||) \int_{t_0}^t \Delta x(s) ds$$

Define a constant  $K_0 = (||a_T||+||B_T||+||C_T||||p||)$ . Applying the Gronwall-Bellman Inequality to the above inequality yields

$$\begin{split} ||\Delta x(t)|| &\leq \varepsilon(t-t_0) + \int_{t_0}^t K_0 \varepsilon(s-t_0) \exp\{\int_s^t K_0 d\sigma\} ds \\ &\leq \varepsilon(t-t_0) + \varepsilon K_0 \frac{(t-t_0)^2}{2} \exp(K_0(t-t_0)) \leq K\varepsilon \end{split}$$
  
where  $K = (t-t_0)(1+K_0 \frac{(t-t_0)}{2} \exp(K_0(t-t_0)))$ , and  $K < \infty$ .  
This completes the proof.

#### 6. Simulation

Consider the single-machine infinity-bus (SMIB) model with a thyristor-controlled seriescapacitor (TCSC) installed on the transmission line (Chen, 1998) as shown in Fig. 5, which may be mathematically described as follows:

$$\begin{bmatrix} \dot{\delta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega_b(\omega - 1) \\ \frac{1}{M}(P_m - P_0 - D(\omega - 1) - \frac{V_t V \infty}{X_d + (1 - s)X_e} \sin \delta) \end{bmatrix}$$

where  $\delta$  is rotor angle (rad),  $\omega$  rotor speed (p.u.),  $\omega_b = 2\pi \times 60$  synchronous speed as base (rad/sec),  $P_m = 0.3665$  is mechanical power input (p.u.),  $P_0$  is unknown fixed load (p.u.), D = 2.0 damping factor, M = 3.5 system inertia referenced to the base power,  $V_t = 1.0$  terminal bus voltage (p.u.),  $V_{\infty} = 0.99$  infinite bus voltage (p.u.),  $X_d = 2.0$  transient reactance of the generator (p.u.),  $X_e = 0.35$  transmission reactance (p.u.),  $s \in [s_{\min}, s_{\max}] = [0.2, 0.75]$  series compensation degree of the TCSC, and  $(\delta_e, 1)$  is system equilibrium with the series compensation degree fixed at  $s_e = 0.4$ .

The goal is to stabilize the system in the near optimal time control fashion with an unknown load  $P_0$  ranging 0 and 10% of  $P_m$ . Two nominal cases are identified. The nominal neural networks have 15 and 30 neurons in the first and second hidden layer with log-sigmoid and tan-sigmoid activation functions for these two hidden layers, respectively. The input data to regional neural networks is the rotor angle, its two previous values, the control and its previous value, and the outputs are the weighting factors. The regional neural networks have 15 and 30 neurons in the first and second hidden layers, respectively. The global neural networks are really not necessary in this simple case of parameter uncertainty.

Once the nominal and regional neural networks are trained, they are used to control the system after a severe short-circuit fault and with an unknown load (5% of  $P_m$ ). The resulting trajectory is shown in Fig. 6. It is observed that the hierarchical neural controller stabilizes the system in a near optimal control manner.



Fig. 5. The SMIB system with TCSC



Fig. 6. Control performance of hierarchical neural controller. Solid - neural control; dashed - optimal control.

#### 7. Conclusion

Even with remarkable progress witnessed in the adaptive control techniques for the nonlinear system control over the past decade, the general challenge with adaptive control of nonlinear systems has never become less formidable, not to mention the adaptive control of nonlinear systems while optimizing a pre-designated control performance index and respecting restrictions on control signals. Neural networks have been introduced to tackle the adaptive control of nonlinear systems, where there are system uncertainties in parameters, unmodeled nonlinear system dynamics, and in many cases the parameters may be time varying. It is the main focus of this Chapter to establish a framework in which general nonlinear systems will be targeted and near optimal, adaptive control of uncertain, time-varying, nonlinear systems is studied. The study begins with a generic presentation of the solution scheme for fixed-parameter nonlinear systems. The optimal control solution is presented for the purpose of minimum time control and minimum fuel control, respectively. The parameter space is tessellated into a set of convex sub-regions. The set of parameter vectors corresponding to the vertexes of those convex sub-regions are obtained. Accordingly, a set of optimal control problems are solved. The resulting control trajectories and state or output trajectories are employed to train a set of properly designed neural networks to establish a relationship that would otherwise be unavailable for the sake of near optimal controller design. In addition, techniques are developed and applied to deal with the time varying property of uncertain parameters of the nonlinear systems. All these pieces

come together in an organized and cooperative manner under the unified intelligent control design framework to meet the Chapter's ultimate goal of constructing intelligent controllers for uncertain, nonlinear systems.

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#### 9. References

- Chen, D. (1998). *Nonlinear Neural Control with Power Systems Applications*, Doctoral Dissertation, Oregon State University, ISBN 0-599-12704-X.
- Chen, D. & Mohler, R. (1997). Load Modelling and Voltage Stability Analysis by Neural Network, Proceedings of 1997 American Control Conference, pp. 1086-1090, ISBN 0-7803-3832-4, Albuquerque, New Mexico, USA, June 4-6, 1997.
- Chen, D. & Mohler, R. (2000). Theoretical Aspects on Synthesis of Hierarchical Neural Controllers for Power Systems, *Proceedings of 2000 American Control Conference*, pp. 3432 – 3436, ISBN 0-7803-5519-9, Chicago, Illinois, June 28-30, 2000.
- Chen, D. & Mohler, R. (2003). Neural-Network-based Loading Modeling and Its Use in Voltage Stability Analysis. *IEEE Transactions on Control Systems Technology*, Vol. 11, No. 4, pp. 460-470, ISSN 1063-6536.
- Chen, D., Mohler, R., & Chen, L. (1999). Neural-Network-based Adaptive Control with Application to Power Systems, *Proceedings of 1999 American Control Conference*, pp. 3236-3240, ISBN 0-7803-4990-3, San Diego, California, USA, June 2-4, 1999.
- Chen, D., Mohler, R., & Chen, L. (2000). Synthesis of Neural Controller Applied to Power Systems. *IEEE Transactions on Circuits and Systems I*, Vol. 47, No. 3, pp. 376 – 388, ISSN 1057-7122.
- Chen, D. & Yang, J. (2005). Robust Adaptive Neural Control Applied to a Class of Nonlinear Systems, *Proceedings of 17th IMACS World Congress: Scientific Computation, Applied Mathematics and Simulation,* Paper T5-I-01-0911, pp. 1-8, ISBN 2-915913-02-1, Paris, July 2005.
- Chen, D., Yang, J., & Mohler, R. (2006). Hierarchical Neural Networks toward a Unified Modelling Framework for Load Dynamics. *International Journal of Computational Intelligence Research*, Vol. 2, No. 1, pp. 17-25, ISSN 0974-1259.
- Chen, D., Yang, J., & Mohler, R. (2008). On near Optimal Neural Control of Multiple-Input Nonlinear Systems. *Neural Computing & Applications*, Vol. 17, No. 4, pp. 327-337, ISSN 0941-0643.
- Chen, D., Yang, J., & Mohler, R. (2006). Hierarchical Neural Networks toward a Unified Modelling Framework for Load Dynamics. *International Journal of Computational Intelligence Research*, Vol. 2, No. 1, pp. 17-25, ISSN 0974-1259.
- Chen, D. & York, M. (2008). Neural Network based Approaches to Very Short Term Load Prediction, *Proceedings of 2008 IEEE Power and Energy Society General Meeting*, pp. 1-8, ISBN 978-1-4244-1905-0, Pittsbufgh, PA, USA, July 20-24, 2008.

- Chen, F. & Liu, C. (1994). Adaptively Controlling Nonlinear Continuous-Time Systems Using Multilayer Neural Networks. *IEEE Transactions on Automatic Control*, Vol. 39, pp. 1306–1310, ISSN 0018-9286.
- Haykin, S. (2001). *Neural Networks: A Comprehensive Foundation*, Prentice-Hall, ISBN 0132733501, Englewood Cliffs, New Jersey.
- Hebb, D. (1949). *The Organization of Behavior*, John Wiley and Sons, ISBN 9780805843002, New York.
- Hopfield, J. J., & Tank, D. W. (1985). Neural Computation of Decisions in Optimization Problems. *Biological Cybernetics*, Vol. 52, No. 3, pp. 141-152.
- Irwin, G. W., Warwick, K., & Hunt, K. J. (1995). *Neural Network Applications in Control,* The Institution of Electrical Engineers, ISBN 0906048567, London.
- Kawato, M., Uno, Y., & Suzuki, R. (1988). Hierarchical Neural Network Model for Voluntary Movement with Application to Robotics. *IEEE Control Systems Magazine*, Vol. 8, No. 2, pp. 8-15.
- Lee, E. & Markus, L. (1967). Foundations of Optimal Control Theory, Wiley, ISBN 0898748070, New York.
- Levin, A. U., & Narendra, K. S. (1993). Control of Nonlinear Dynamical Systems Using Neural Networks: Controllability and Stabilization. *IEEE Transactions on Neural Networks*, Vol. 4, No. 2, pp. 192-206.
- Lewis, F., Yesidirek, A. & Liu, K. (1995). Neural Net Robot Controller with Guaranteed Tracking Performance. *IEEE Transactions on Neural Networks*, Vol. 6, pp. 703-715, ISSN 1063-6706.
- Liang, R. H. (1999). A Neural-based Redispatch Approach to Dynamic Generation Allocation. *IEEE Transactions on Power Systems*, Vol. 14, No. 4, pp. 1388-1393.
- Methaprayoon, K., Lee, W., Rasmiddatta, S., Liao, J. R., & Ross, R. J. (2007). Multistage Artificial Neural Network Short-Term Load Forecasting Engine with Front-End Weather Forecast. *IEEE Transactions Industry Applications*, Vol. 43, No. 6, pp. 1410-1416.
- Mohler, R. (1991). *Nonlinear Systems Volume I, Dynamics and Control,* Prentice Hall, Englewood Cliffs, ISBN 0-13-623489-5, New Jersey.
- Mohler, R. (1991). *Nonlinear Systems Volume II, Applications to Bilinear Control,* Prentice Hall, Englewood Cliffs, ISBN 0-13- 623521-2, New Jersey.
- Mohler, R. (1973). Bilinear Control Processes, Academic Press, ISBN 0-12-504140-3, New York.
- Moon S. (1969). *Optimal Control of Bilinear Systems and Systems Linear in Control,* Ph.D. dissertation, The University of New Mexico.
- Nagata, S., Sekiguchi, M., & Asakawa, K. (1990). Mobile Robot Control by a Structured Hierarchical Neural Network. *IEEE Control Systems Magazine*, Vol. 10, No. 3, pp. 69-76.
- Pandit, M., Srivastava, L., & Sharma, J. (2003). Fast Voltage Contingency Selection Using Fuzzy Parallel Self-Organizing Hierarchical Neural Network. *IEEE Transactions on Power Systems*, Vol. 18, No. 2, pp. 657-664.
- Polycarpou, M. (1996). Stable Adaptive Neural Control Scheme for Nonlinear Systems. *IEEE Transactions on Automatic Control*, Vol. 41, pp. 447-451, ISSN 0018-9286.

- Sanner, R. & Slotine, J. (1992). Gaussian Networks for Direct Adaptive Control. *IEEE Transactions on Neural Networks*, Vol. 3, pp. 837-863, ISSN 1045-9227.
- Yesidirek, A. & Lewis, F. (1995). Feedback Linearization Using Neural Network. *Automatica*, Vol. 31, pp. 1659-1664, ISSN.
- Zakrzewski, R. R., Mohler, R. R., & Kolodziej, W. J. (1994). Hierarchical Intelligent Control with Flexible AC Transmission System Application. *IFAC Journal of Control Engineering Practice*, pp. 979-987.
- Zhou, Y. T., Chellappa, R., Vaid, A., & Jenkins B. K. (1988). Image Restoration Using a Neural Network. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. 36, No. 7, pp. 1141-1151.





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Robust control has been a topic of active research in the last three decades culminating in H\_2/H\_\infty and \mu design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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