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# Induced Acceleration Analysis of Three-Dimensional Multi-Joint Movements and Its Application to Sports Movements

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## 1. Introduction

A knowledge of how muscle forces produce joint rotations is fundamental in all fields of human movement science, including rehabilitation and sports biomechanics. Such knowledge is necessary for improving the diagnosis and treatment of persons with movement disabilities and analyzing the techniques used by high-performance athletes (Zajac and Gordon, 1989). However, it is difficult to intuitively understand how muscle forces produce joint rotations in multi-joint movements because of the complexity of inter-joint interactions, especially in three-dimensional (3D) movements. Therefore, although muscle activities, joint torques, and joint rotations themselves have been examined extensively in many sports movements (Barrentine et al., 1998; Elliott et al., 2003; Feltner and Dapena, 1986; Fleisig et al., 1995; Fleisig et al., 1996b; Glousman et al., 1988; Hirashima et al., 2002; Marshall and Elliott, 2000; Matsuo et al., 2001; Nunome et al., 2002; Putnam, 1991; Sakurai et al., 1993; Sakurai and Ohtsuki, 2000; Sprigings et al., 1994), the knowledge about the cause-and-effect relationships between these variables is insufficient.

The purpose of this chapter is to provide the framework to properly understand the cause-and-effect relationship between joint torques and rotations during sports movements. In fast sports movements such as baseball pitching and soccer kicking, joint rotations occur sequentially from proximal joints to distal joints. This kinematic sequence itself or the underlying kinetic mechanism is often called the “proximal-to-distal sequence,” “kinetic chain,” or “whip-like effect” (Atwater, 1979; Feltner, 1989; Fleisig et al., 1996a; Kibler, 1995; Kindall, 1992; Putnam, 1991). However, the kinetic mechanism has not been properly understood, because previous studies on sports movements have not focused on the fact that a joint rotation is caused by *two different mechanisms*: instantaneous and cumulative effects. The instantaneous effect is an instantaneous angular acceleration induced by a joint torque at that instant, whereas the cumulative effect is an angular acceleration induced by the entire joint torque and gravity torque history until that instant (Hirashima et al., 2008; Zajac et al., 2002). Because the mechanical causes are clearly different between the two effects in terms of time, clear differentiation is necessary to understand the original cause of each joint rotation and develop effective training programs.

In section 2, I will explain the instantaneous effects produced by a joint torque by systematically presenting examples of single-joint movements, multi-joint movements in a two-dimensional (2D) space, and multi-joint movements in a three-dimensional (3D) space. I

will not only introduce a mathematical method called “induced acceleration analysis” to derive the joint rotations induced by a joint torque but also provide concrete examples with effective graphics so that the meaning of the mathematical formulations can be easily understood by patients, athletes, and coaches, as well as biomechanics researchers. In section 3, I will explain the difference between instantaneous and cumulative effects. Then, in section 4, by applying the induced acceleration analysis to baseball pitching motions, I will demonstrate that such a distinction is helpful to gain insight into the chain of causation for sports movements and the control strategy adopted by high-performance athletes.

## 2. Instantaneous effect

### 2.1 Single-joint movements

I begin the explanation with a single-joint movement in which only one joint is free to rotate around a single axis (e.g., elbow flexion-extension movement in a horizontal plane). Because only one joint angle ( $\theta$ ) is necessary to specify the posture of the system, the number of degrees of freedom (DOF) is one. In such a 1-DOF system, the effect of a torque on the joint rotation is very simple because there is only one equation of motion. The angular acceleration ( $\ddot{\theta}$ ) produced by the joint torque ( $\tau$ ) is determined by the magnitude of that torque and the moment of inertia ( $I$ ) about the joint axis as follows:

$$\ddot{\theta} = \frac{\tau}{I} \quad (1)$$

### 2.2 Multi-joint movements in 2D

However, the relation between the torque and acceleration become more complex when multiple joints are involved in the movement. For example, we now consider a 2-DOF (e.g.,  $\theta_1$ , shoulder joint angle;  $\theta_2$ , elbow joint angle) movement in a vertical plane (Fig. 1). The equations of motion for the upper-arm (Eq. 2) and forearm (Eq. 3) can be written as follows:

$$\begin{aligned} \ddot{\theta}_1 [I_1 + I_2 + m_2 l_1^2 + m_1 r_1^2 + m_2 r_2^2 + 2m_2 l_1 r_2 \cos \theta_2] + \ddot{\theta}_2 [I_2 + m_2 r_2^2 + m_2 l_1 r_2 \cos \theta_2] \\ = \tau_1 + \dot{\theta}_2^2 [m_2 l_1 r_2 \sin \theta_2] + \dot{\theta}_1 \dot{\theta}_2 [2m_2 l_1 r_2 \sin \theta_2] - g[(m_2 l_1 + m_1 r_1) \cos \theta_1 + m_2 r_2 \cos(\theta_1 + \theta_2)] \end{aligned} \quad (2)$$

$$\begin{aligned} \ddot{\theta}_1 [I_2 + m_2 r_2^2 + m_2 l_1 r_2 \cos \theta_2] + \ddot{\theta}_2 [I_2 + m_2 r_2^2] \\ = \tau_2 - \dot{\theta}_1^2 [m_2 l_1 r_2 \sin \theta_2] - g[m_2 r_2 \cos(\theta_1 + \theta_2)] \end{aligned} \quad (3)$$

where  $I_i$  = moment of inertia about the center of gravity,  $r_i$  = distance to center of mass from proximal joint of the segment,  $l_i$  = length,  $m_i$  = mass,  $\tau_i$  = joint torque ( $i = 1$ : upper arm, 2: forearm).

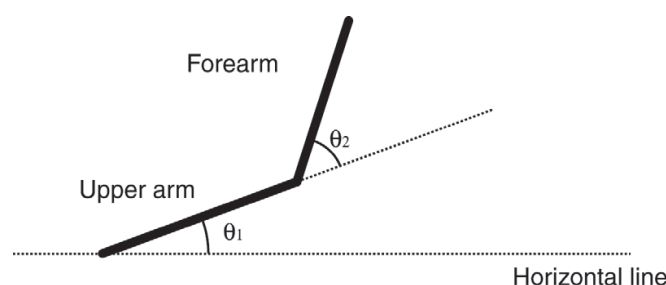


Fig. 1. Two-joint system in a vertical plane.

By using some substitutions, these can be written in the following form:

$$I_{11}\ddot{\theta}_1 + I_{12}\ddot{\theta}_2 = \tau_1 + V_1(\theta, \dot{\theta}) + g_1(\theta) \quad (4)$$

$$I_{21}\ddot{\theta}_1 + I_{22}\ddot{\theta}_2 = \tau_2 + V_2(\theta, \dot{\theta}) + g_2(\theta) \quad (5)$$

where  $\theta = (\theta_1 \ \theta_2)^T$  is the joint angle vector,  $\dot{\theta} = (\dot{\theta}_1 \ \dot{\theta}_2)^T$  is the angular velocity vector,  $V_i(\theta, \dot{\theta})$  is the angular-velocity-dependent torque, and  $g_i(\theta)$  is the gravity torque. Now consider the question of how large the angular accelerations ( $\ddot{\theta}_1$  and  $\ddot{\theta}_2$ ) are if joint torques ( $\tau_1$  and  $\tau_2$ ) are applied at the two joints when the system is in the state of  $\theta$  and  $\dot{\theta}$ . Because unknown variables ( $\ddot{\theta}_1$  and  $\ddot{\theta}_2$ ) are present in both equations (Eq. 4 and Eq. 5), we must solve them as simultaneous equations in the following way:

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} + \begin{pmatrix} V_1(\theta, \dot{\theta}) \\ V_2(\theta, \dot{\theta}) \end{pmatrix} + \begin{pmatrix} g_1(\theta) \\ g_2(\theta) \end{pmatrix} \right\} \quad (6)$$

Accordingly, the angular accelerations of the joints caused by the joint torques can be obtained as follows:

$$\begin{aligned} \ddot{\theta}_1 &= A_{11}(\tau_1 + V_1(\theta, \dot{\theta}) + g_1(\theta)) + A_{12}(\tau_2 + V_2(\theta, \dot{\theta}) + g_2(\theta)) \\ \ddot{\theta}_2 &= A_{21}(\tau_1 + V_1(\theta, \dot{\theta}) + g_1(\theta)) + A_{22}(\tau_2 + V_2(\theta, \dot{\theta}) + g_2(\theta)) \end{aligned} \quad (7)$$

where  $A_{11}$ ,  $A_{12}$ ,  $A_{21}$ , and  $A_{22}$  are the component of the matrix  $\mathbf{I}(\theta)^{-1}$ :

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix}^{-1} = \mathbf{I}(\theta)^{-1} \quad (8)$$

### 2.2.1 Joint torque accelerates all joints in multi-joint system

The first important point shown by Eq. 7 is that the angular accelerations of both joints depend on both joint torques. This indicates that, for example, a shoulder joint torque ( $\tau_1$ ) induces angular accelerations not only at the shoulder joint but also at the elbow joint. Figure 2a shows a simulation result where only the shoulder flexor torque is exerted on the 2-DOF (shoulder and elbow) system in a horizontal plane. It is intuitively understandable that the shoulder flexor torque induces the shoulder flexion acceleration, but it should be noted that it also induces the elbow extension acceleration, even though the elbow joint torque is zero. Similarly, an elbow joint torque ( $\tau_2$ ) induces angular accelerations at both joints. Figure 2b shows a simulation result where only the elbow flexor torque is exerted; the elbow flexor torque induces not only elbow flexion acceleration but also shoulder extension acceleration.

Why does a joint torque accelerate not only its own joint but also another remote joint? In fact, interaction forces between body segments occur at the same time as a joint torque is exerted at a certain joint if the body segments are linked by the joints. These interaction forces are the causes of the angular accelerations arising at the remote joints. Because the interaction forces arise instantaneously at the exertion of the joint torque, the angular acceleration at the remote joint also arises at that instant. This phenomenon arising from the mechanical constraint of the joint is called "dynamic coupling" (Zajac and Gordon, 1989).

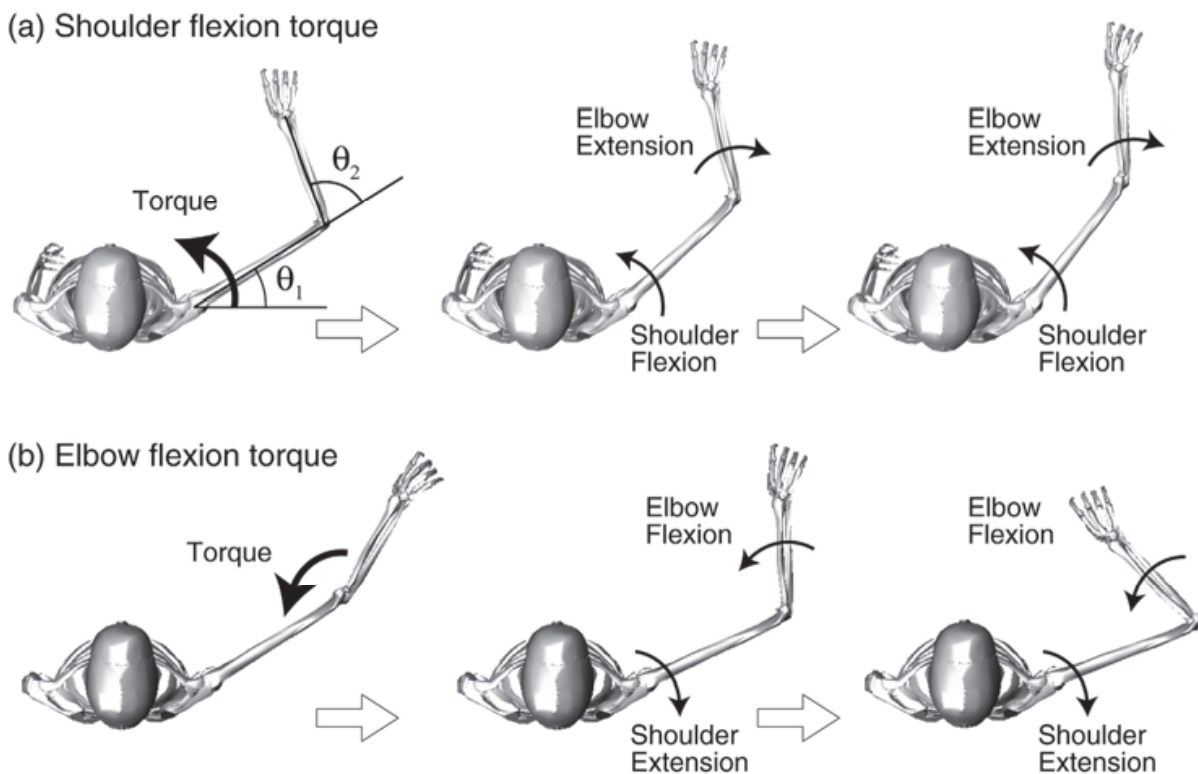


Fig. 2. Simulated motions of two-joint (shoulder and elbow) system in a horizontal plane. Fig. 2a shows angular accelerations induced only by the shoulder flexion torque. Fig. 2b shows angular accelerations induced only by the elbow flexion torque.

### 2.2.2 Joint torque action depends on system posture

The second important point shown by Eq. 7 is that the angular accelerations produced by a joint torque depend on the system inertia matrix ( $\mathbf{I}(\boldsymbol{\theta})$ ), namely the posture of the system. This indicates that even though the same joint torque is exerted, the angular acceleration induced by the joint torque will vary based on the system's posture when the joint torque is exerted. Figure 3 shows the simulation results when the shoulder flexor torque is exerted on two different postures. The shoulder flexor torque induces the shoulder flexion and elbow *extension* for posture (a), whereas it induces the shoulder flexion and elbow *flexion* for posture (b).

Thus, in order to estimate the angular accelerations induced by joint torques in multi-joint movements, we need to pay attention, not only to the joint torques, but also to the posture of the system. Mathematically speaking, the instantaneous accelerations ( $\ddot{\boldsymbol{\theta}}^T = (\ddot{\theta}_1 \ \ddot{\theta}_2)^T$ ) induced by a joint torque can be calculated by multiplying the inverse of the system inertia matrix ( $\mathbf{I}(\boldsymbol{\theta})^{-1}$ ) to the torque vector ( $\boldsymbol{\tau} = (\tau_1 \ \tau_2)^T$ ) as follows:  $\ddot{\boldsymbol{\theta}}^T = \mathbf{I}(\boldsymbol{\theta})^{-1} \boldsymbol{\tau}$ . This method is often called "induced acceleration analysis." This induced acceleration analysis has been mainly used in gait analysis to determine how individual muscle forces contribute to the forward and vertical acceleration of the body during walking (Anderson and Pandy, 2003; Fox and Delp, 2010; Kepple et al., 1997).

### 2.3 n-DOF system in 2D

The two properties described above (2.2.1 and 2.2.2) are applicable to a system with more than 2 DOFs. Although the equation of motion with more than 2 DOFs is much more

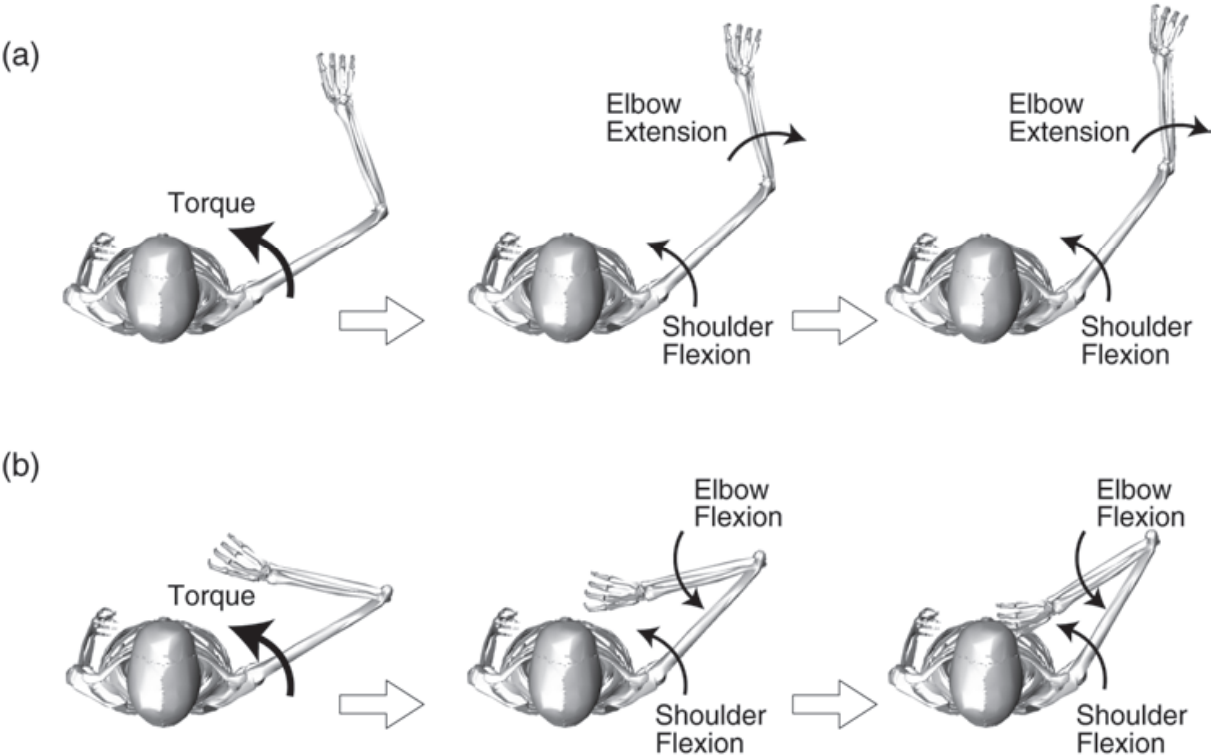


Fig. 3. Simulated motions of two-joint (shoulder and elbow) system in a horizontal plane when the same shoulder flexor torque is exerted on two different postures.

complex than Eq. 2 and Eq. 3, the equation for an n-DOF system ( $n \geq 3$ ) can always be written in the following form:

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \vdots \\ \ddot{\theta}_n \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} & \cdots & I_{1n} \\ I_{21} & I_{22} & \cdots & I_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ I_{n1} & I_{n2} & \cdots & I_{nn} \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{pmatrix} + \begin{pmatrix} V_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ V_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ \vdots \\ V_n(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \end{pmatrix} + \begin{pmatrix} g_1(\boldsymbol{\theta}) \\ g_2(\boldsymbol{\theta}) \\ \vdots \\ g_n(\boldsymbol{\theta}) \end{pmatrix} \right\} \tag{9}$$

It can be written more simply as follows:

$$\ddot{\boldsymbol{\theta}} = \mathbf{I}(\boldsymbol{\theta})^{-1} \{ \boldsymbol{\tau} + \mathbf{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta}) \} \tag{10}$$

Because this equation has the same form as Eq. 6 in the 2-DOF system, we can discuss the cause-and-effect relation between joint torques and accelerations in a way similar to the discussion for the 2-DOF system. In general, the mechanism by which joint torques induce angular accelerations can be graphically summarized as shown in Fig. 4. This figure indicates that:

1. A joint torque at each joint induces acceleration not only at its own joint (i.e., direct effect, solid line) but also at the other joints (i.e., remote effect, dotted line).
2. The effects induced by a joint torque depend on the posture of the whole system.

Thus, for example, in the case of a 3-DOF system (shoulder, elbow, and wrist joints), a shoulder joint torque instantaneously accelerates not only the shoulder but also the elbow and wrist joints.



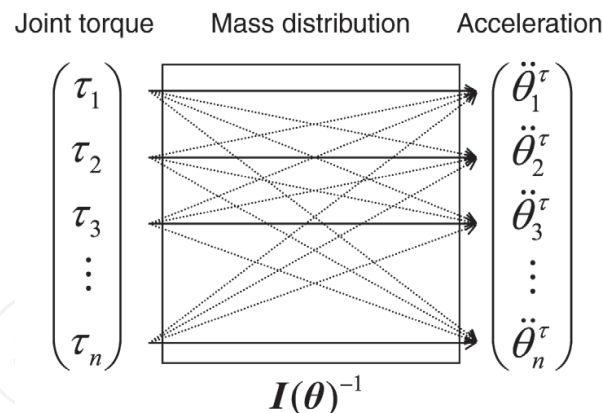


Fig. 4. Schematic figure of how joint torque at each joint induces angular accelerations at all joints in n-DOF system.

## 2.4 n-DOF system in 3D

The equation of motion for a multi-joint system in 3D is much more complicated than the equation in 2D. Fortunately, however, the equations for 3D movements can also be written in the same form as the equations for 2D movements (Eqs. 9 and 10). The mathematical derivation is described in a reference (Hirashima and Ohtsuki, 2008). It indicates that, similar to 2D multi-joint movements, a joint torque at a certain DOF induces accelerations at all DOFs in 3D multi-DOF movements. Now, let us consider 3D movements, including 3-DOF shoulder joint rotations. In this case,  $\tau_1$ ,  $\tau_2$ , and  $\tau_3$  in Fig. 4 can be regarded as the shoulder joint torques about the internal-external rotation axis, adduction-abduction axis, and horizontal flexion-extension axis, respectively, while  $\ddot{\theta}_1$ ,  $\ddot{\theta}_2$ , and  $\ddot{\theta}_3$  can be regarded as the angular accelerations about the three axes, respectively. Thus, the 3D version of Fig. 4 indicates that, when only the joint torque about the internal-external rotation axis ( $\tau_1$ ) is exerted, it can induce angular accelerations not only about the internal-external rotation axis ( $\ddot{\theta}_1$ ) but also about the other two axes ( $\ddot{\theta}_2$  and  $\ddot{\theta}_3$ ). This might seem counterintuitive to some readers, because people tend to intuitively and wrongly believe that the “shoulder internal rotation torque” can only produce a shoulder internal rotation. Actually, however, such a 1-to-1 causal relationship occurs in only a few cases (see below). In almost every case, a joint torque accelerates all of the joint rotations.

### 2.4.1 Inertial property of 3D objects

To understand this from a mechanical point of view, it is necessary to deepen our understanding of the inertial property of 3D objects. Any object has three orthogonal principal axes of inertia (Zatsiorsky, 2002). If the inertia matrix is expressed in the orthogonal coordinate system (X,Y,Z) whose axes correspond to the principal axes of inertia (Fig. 5a), it can be written as follows:

$$\mathbf{I}_A = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \quad (11)$$

Then, the angular accelerations induced by the torque about the X axis can be expressed as follows:

$$\begin{pmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{pmatrix} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}^{-1} \begin{pmatrix} \tau_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix} \begin{pmatrix} \tau_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} A_{xx}\tau_x \\ 0 \\ 0 \end{pmatrix} \quad (12)$$

Thus, in a case where the axis of the torque vector corresponds to the principal axis of inertia, the torque vector induces angular acceleration about this axis alone. For a human upper limb, this situation occurs only in an ideal simple case where the upper arm, forearm, and hand are linked in line (Fig. 5a) and one of the principal axes of inertia for the entire kinematic chain exactly corresponds to the longitudinal axis of the upper arm. In this case, a torque about the longitudinal axis of the upper arm causes internal-external rotation alone. On the other hand, when the elbow is at a flexed position (Fig. 5b), the principal axes of inertia for the entire kinematic chain distal to the shoulder deviate from the shoulder joint coordinate system (X,Y,Z). When the inertia matrix is expressed in a coordinate system whose axes do not correspond to the principal axes of inertia, it is written as follows:

$$\mathbf{I}_B = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \quad (13)$$

Then, the angular accelerations induced by the torque about the X axis can be expressed as follows:

$$\begin{pmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}^{-1} \begin{pmatrix} \tau_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \begin{pmatrix} \tau_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} A_{xx}\tau_x \\ A_{yx}\tau_x \\ A_{zx}\tau_x \end{pmatrix} \quad (14)$$

Thus, when the axis of the joint torque does not correspond to the principal axis of inertia, the torque induces angular accelerations about all three joint axes.

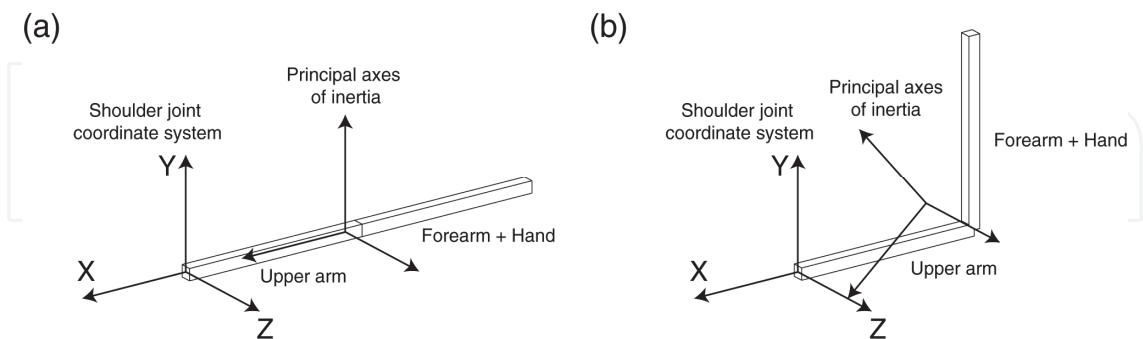


Fig. 5. Schematic drawings of upper arm, forearm and hand with elbow angle at 0° (A) and 90° (B). The shoulder joint coordinate axes (XYZ) and the principal axes of inertia for the entire kinematic chain are drawn [Adapted from (Hirashima et al., 2007a)].

**2.4.2 Examples of 3D multi-DOF movements**

Computer simulations were used to observe these two situations (Eqs. 12 and 14). Figure 6a shows the simulation results when a shoulder internal rotation torque is exerted on the ideal



simple case in which the principal axes of inertia correspond to the shoulder joint coordinate axes. For simplicity, the simulation was conducted in a zero gravity environment. In this case, the shoulder internal rotation torque produces only the shoulder internal rotation. However, when the shoulder internal rotation torque is exerted on a system whose principal axes of inertia do not correspond to the shoulder joint coordinate axes (Fig. 6b), the shoulder internal rotation torque produces not only the internal rotation but also abduction and horizontal extension.

Figure 7 shows another example of exerting the shoulder horizontal flexion torque, which approximately simulates the situation where the anterior deltoid is activated. In a case where the principal axes of inertia correspond to the shoulder joint coordinate axes (Fig. 7a), the shoulder horizontal flexion torque produces only horizontal flexion. However, in the case of the posture shown in Fig. 7b, the shoulder horizontal flexion torque produces not only horizontal flexion but also external rotation. This might be surprising for some readers, because anatomy textbooks never describe the external rotation as a function of the anterior deltoid. Here, I would like to call attention to the fact that anatomy textbooks describe the *torque vector* produced by each muscle, not the *rotation* produced by the muscle: this is a blind spot for novices in the field of biomechanics and motor control. It should be kept in mind that, in order to know how each muscle force produces angular accelerations, we need to consider both the torque vector produced by the muscle ( $\tau$ ) and the inertial property of the system  $\mathbf{I}(\theta)$  to which the torque is applied.

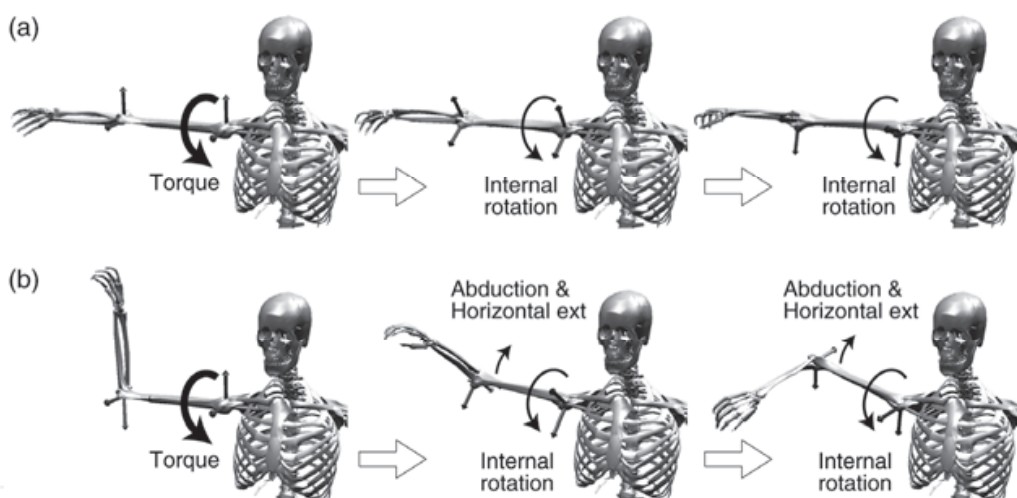


Fig. 6. Simulated motions in zero gravity environment when shoulder internal rotation torque is exerted on two different postures.

### 3. Cumulative effect

So far, I have explained how a joint torque at a certain instant induces angular accelerations on the system at that instant. However, the effect of the joint torque is not limited to these instantaneous accelerations because the accelerations accumulate and remain in the system as a velocity. The velocity-dependent torques ( $\mathbf{V}(\theta, \dot{\theta})$ ) such as Coriolis and centrifugal torques can also induce angular accelerations. The angular acceleration induced by velocity-dependent torques at a certain instant is expressed as follows:

$$\ddot{\theta}^V = \mathbf{I}(\theta)^{-1} \mathbf{V}(\theta, \dot{\theta}) \quad (15)$$

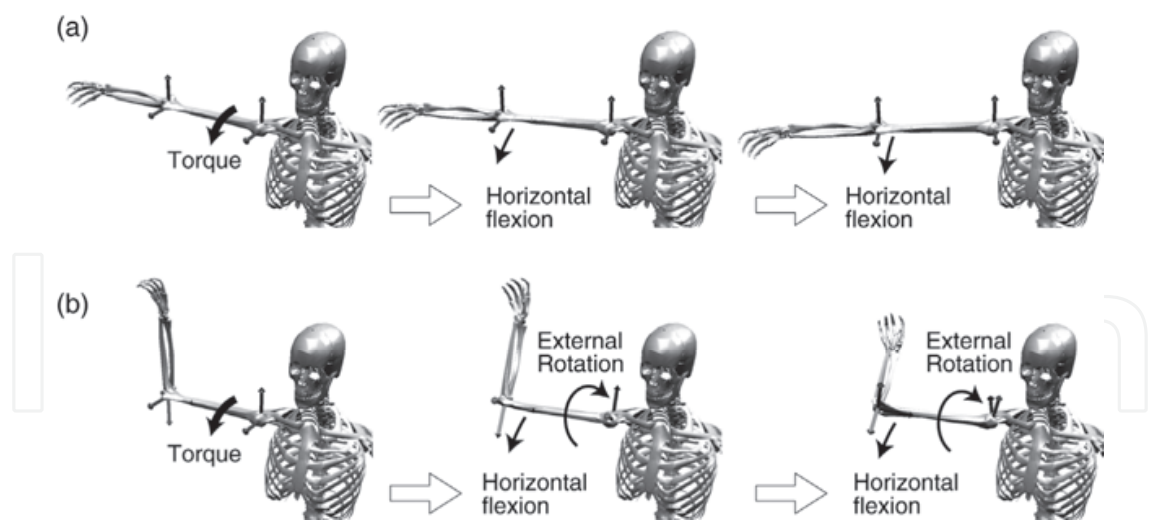


Fig. 7. Simulated motions in zero gravity environment when shoulder horizontal flexion torque is exerted on two different postures.

It should be noted that because the angular velocity is integrated from the angular acceleration, the acceleration induced by the velocity-dependent torque at a certain instant (Eq. 15) reflects the cumulative effects from the history of all the muscles and gravity torques that have been applied to the system until that instant.

In summary, the induced acceleration analysis suggests that the angular acceleration at a certain instant is the sum of 1) the instantaneous effect induced by the joint torques and gravity torque at that instant and 2) the cumulative effect from the history of the joint torques and gravity torque applied to the system until that instant (Fig. 8).

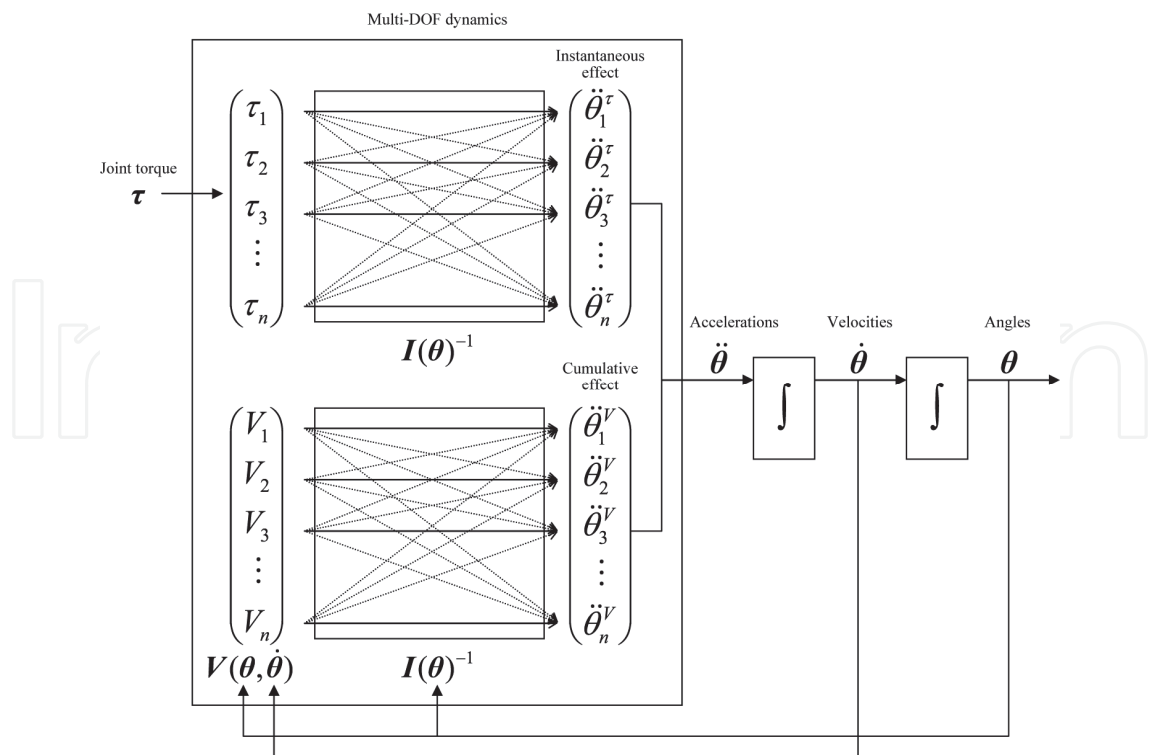


Fig. 8. Block diagram of generation mechanism for multi-DOF human motion [Adapted from (Hirashima et al., 2008)].

## 4. Application to sports movements

Understanding the cause-and-effect relation between the kinetics (e.g., muscle force, joint torque) and kinematics (e.g., joint rotation) is difficult in multi-joint movements because of the complexity of inter-joint interactions. The induced acceleration analysis provides the framework to show that joint angular acceleration is induced by two different mechanisms (i.e., instantaneous and cumulative effects). Determining whether a joint angular acceleration is caused by the instantaneous or cumulative effect is very important because the original causes of these accelerations are clearly different in terms of time. The cause of the instantaneous effect is the joint torque and gravity torque at that instant, whereas the cause of the cumulative effect is the entire history of the joint torques and gravity torque applied to the system until that instant. However, previous studies on fast sports movements such as baseball pitching and soccer kicking have not clearly differentiated these two effects, and hence the causes of the joint rotations have not been properly understood. In section 4, I apply the induced acceleration analysis to the baseball pitching motion to understand the kinetic mechanism used by skilled baseball players to generate a large ball velocity.

### 4.1 Kinematic analysis

The first step in understanding how skilled baseball players generate a large ball velocity at ball release is to determine the joint angular velocities that are mostly responsible for the translational velocity of the ball. Here, we consider a serially linked four-segment model of the trunk and upper limb that has 13 degrees of freedom (DOFs) (Fig. 9a). There are three translational DOFs and three rotational DOFs for the trunk, three rotational DOFs for the shoulder, two rotational DOFs for the elbow, and two rotational DOFs for the wrist (for details see the legend of Fig. 9). Suppose the joint rotates only about the  $i$ -th DOF axis at an angular velocity of  $\dot{\theta}_i$  (Fig. 9b). The translational velocity of the fingertip ( $\dot{\mathbf{r}}_i$ ) produced by this rotation is expressed as

$$\dot{\mathbf{r}}_i = \dot{\theta}_i \times \mathbf{p}_i \quad (16)$$

where  $\dot{\theta}_i (= \dot{\theta}_i \mathbf{u}_i)$  is the angular velocity vector ( $\mathbf{u}_i$  is the unit vector of the joint axis) and  $\mathbf{p}_i$  is the vector from the joint center to the fingertip. Because joints actually rotate about all DOFs, the actual translational velocity of the fingertip ( $\dot{\mathbf{r}}$ ) is expressed as the sum of their effects.

$$\dot{\mathbf{r}} = \sum_{i=1}^{13} \dot{\mathbf{r}}_i \quad (17)$$

We can determine the contribution of each angular velocity to the magnitude of the fingertip translational velocity by projecting each vector ( $\dot{\mathbf{r}}_i$ ) onto the unit vector ( $\dot{\mathbf{r}}_u$ ) of the translational velocity of the fingertip (Feltner and Nelson, 1996; Miyanishi et al., 1996; Sprigings et al., 1994).

$$|\dot{\mathbf{r}}| = \sum_{i=1}^{13} (\dot{\mathbf{r}}_i \cdot \dot{\mathbf{r}}_u) \quad (18)$$

Figure 9c shows the contributions of the 13 DOF velocities to the fingertip velocity at the time of ball release in skilled baseball players (Hirashima et al., 2007b). This analysis reveals

that it is mainly produced by the leftward rotation of the trunk, internal rotation of the shoulder, elbow extension, and wrist flexion.

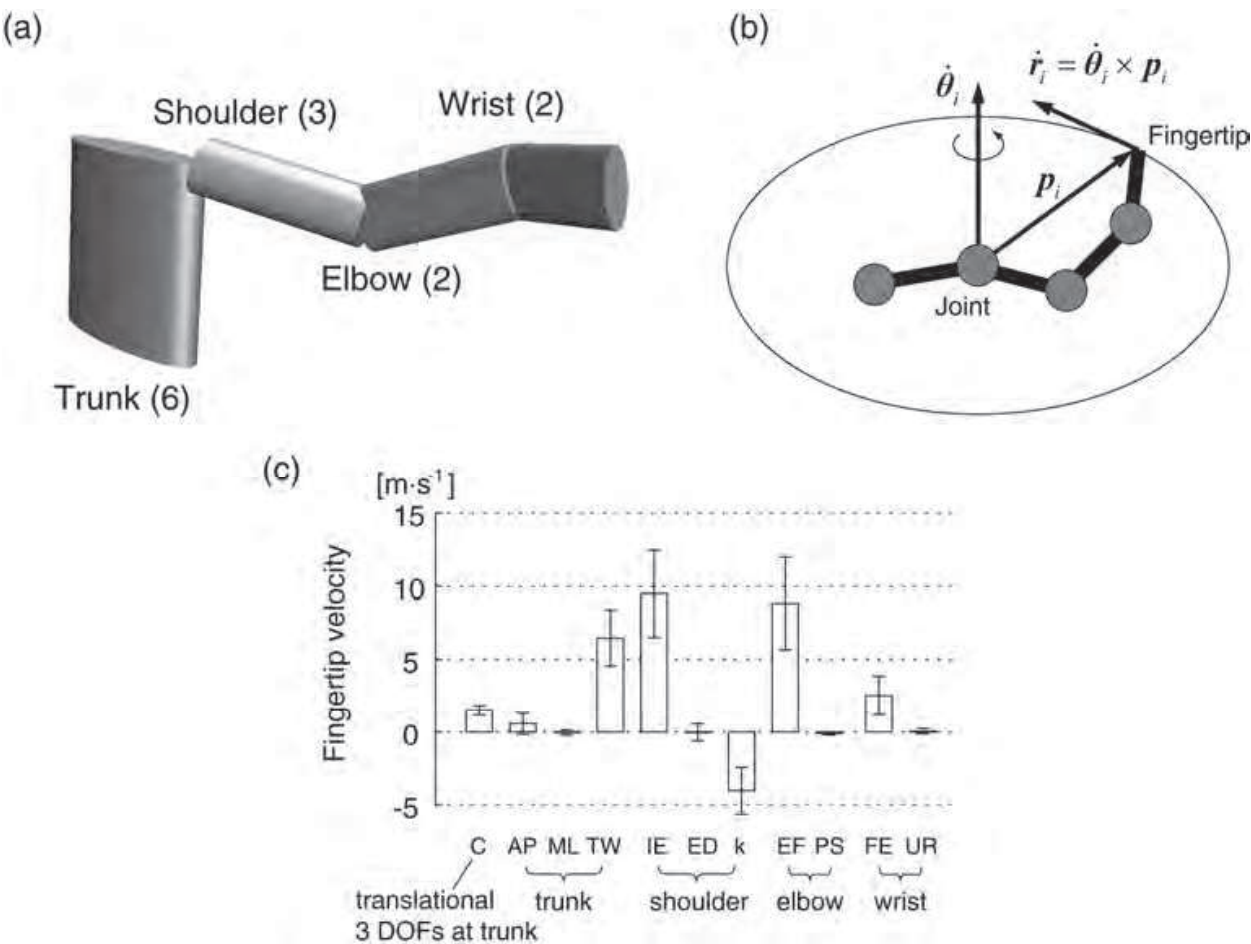


Fig. 9. (a) 13-DOF model. (b) Schematic figure of the relation between joint angular velocity vector  $\dot{\theta}_i$  at one joint coordinate axis and the translational velocity vector of the fingertip ( $\dot{r}_i$ ). (c) Contributions to the translational velocity of the fingertip at the time of ball release. There are 13 contributors to the fingertip velocity: C, translation of the trunk including three degrees of freedom (DOFs) (forward-backward, upward-downward, and right-left); AP, anterior (+)-posterior (-) lean; ML, medial (+)-lateral (-) tilt; TW, left (+)-right (-) twist of the trunk; IE, internal (+)-external (-) rotation; ED, elevation (+)-depression (-); k, third-axis rotation of the shoulder; EF, extension (+)-flexion (-); PS, pronation (+)-supination (-) of the elbow; FE, flexion (+)-extension (-); and UR, ulnar (+)-radial (-) deviation of the wrist [Adapted from (Hirashima et al., 2007b)].

4.2 Induced acceleration analysis

The next step is to understand how baseball players generate a large angular velocity at these joint rotations by coordinating the joint torque and velocity-dependent torque. As described in the above section (2.4), angular accelerations can be calculated by the following equation:

$$\ddot{\theta} = I(\theta)^{-1} \{ \tau + V(\theta, \dot{\theta}) + g(\theta) \}$$

(19)

where  $\ddot{\theta}$ ,  $\tau$ ,  $V(\theta, \dot{\theta})$ , and  $g(\theta)$  are the 13-dimensional vectors, and  $I(\theta)$  is the 13-by-13 matrix for the 13-DOF model used here. This equation tells us that the  $j$ -th angular acceleration ( $\ddot{\theta}_j$ ) is produced by the instantaneous direct effect ( $\ddot{\theta}_j^{\tau j}$ ), instantaneous remote effect ( $\ddot{\theta}_j^{\tau k} (k \neq j)$ ), cumulative effect ( $\ddot{\theta}_j^V$ ), and instantaneous effect from gravity torque ( $\ddot{\theta}_j^g$ ) as expressed in the following:

$$\begin{aligned}\ddot{\theta}_j &= \sum_{i=1}^{13} A_{ji} \tau_i + \sum_{i=1}^{13} A_{ji} V_i + \sum_{i=1}^{13} A_{ji} g_i \\ &= (\ddot{\theta}_j^{\tau 1} + \ddot{\theta}_j^{\tau 2} + \dots + \ddot{\theta}_j^{\tau 13}) + \ddot{\theta}_j^V + \ddot{\theta}_j^g\end{aligned}\quad (20)$$

where  $A_{ji}$  is the  $(j, i)$  component of the matrix,  $I(\theta)^{-1}$ .

In baseball pitching, the fastest possible speed at the hand is required, and eventually large angular accelerations are required. Because the instantaneous accelerations induced by the muscle force are limited by the muscle force-producing capacity (i.e., physical capacity), additional utilization of the velocity-dependent torque is a very effective strategy for producing larger angular accelerations than the muscle torque could produce on its own. To what extent do skilled players utilize the velocity-dependent torque? Do they utilize the velocity-dependent torque at all joints?

Figure 10 shows the contributions of the 13 joint torques, velocity-dependent torque, and gravity torque to the trunk forward motion, trunk leftward rotation, shoulder internal rotation, shoulder horizontal flexion, elbow extension, and wrist flexion. This figure indicates that the motions of the trunk (Figs. 10a and b) and shoulder (Figs. 10c and d) are produced by respective joint torques (i.e., instantaneous direct effect): for example, the trunk leftward rotation is accelerated by the trunk leftward rotation torque (Fig. 10b). In contrast, the elbow extension and wrist flexion are mainly produced by the velocity-dependent torque (Figs. 10e and f).

I further examined which segment motion is the source of the velocity-dependent torque acting on the elbow and wrist accelerations by decomposing the velocity-dependent torque into some kinematic sources. The results show that the velocity-dependent torques at the shoulder, elbow, and wrist were produced by the forearm angular velocity that was originally produced by the trunk and shoulder joint torques in an earlier phase (for more details, see Hirashima et al., 2008).

## 5. Discussion

### 5.1 Kinetic chain

Taken together, the kinetic chain of baseball pitching can be summarized as follows. First, the trunk forward motion and leftward rotation are accelerated by respective joint torques (instantaneous direct effect) produced by relatively large muscles located at the lower extremity and trunk (Fig. 11a). The shoulder horizontal flexion torque and internal rotation torque during this phase prevent the upper arm from lagging behind relative to the trunk. As a result, the angular velocity of the upper arm also increases with that of the trunk (Fig. 11b). Thus, the motions of the trunk and upper arm in the early phase are produced by the instantaneous direct effect from large proximal muscles. The angular velocities of the trunk and upper arm produced by the above mechanism are the sources of the velocity-dependent torque acting for the elbow extension (Figs. 11c and d). As a result, the elbow joint angular velocity increases, and concurrently, the forearm angular velocity relative to the ground also increases. The forearm angular velocity subsequently accelerates the elbow extension



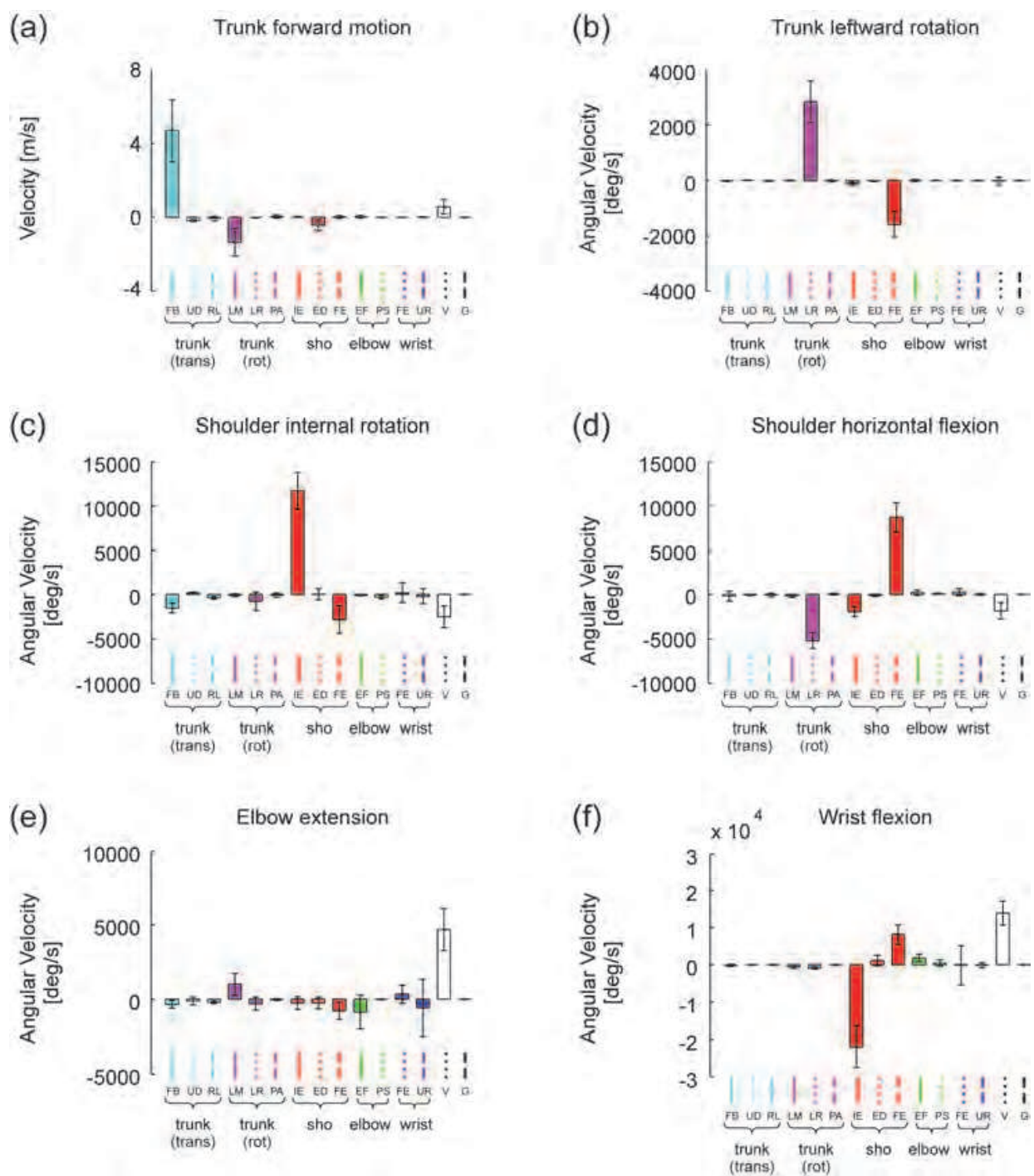


Fig. 10. Contributions of 13 joint torques, velocity-dependent torque, and gravity torque to the trunk forward motion (a), trunk leftward rotation (b), shoulder internal rotation (c), shoulder horizontal flexion (d), elbow extension (e), and wrist flexion (f) during baseball pitching [Adapted from (Hirashima et al., 2008)].

(Fig. 11e) and wrist flexion (Fig. 11f). Thus, the angular velocities of the trunk and upper arm produced by the proximal muscles in the early phase remain in the cumulative effect loop and accelerate the distal joint rotations in the later phase.



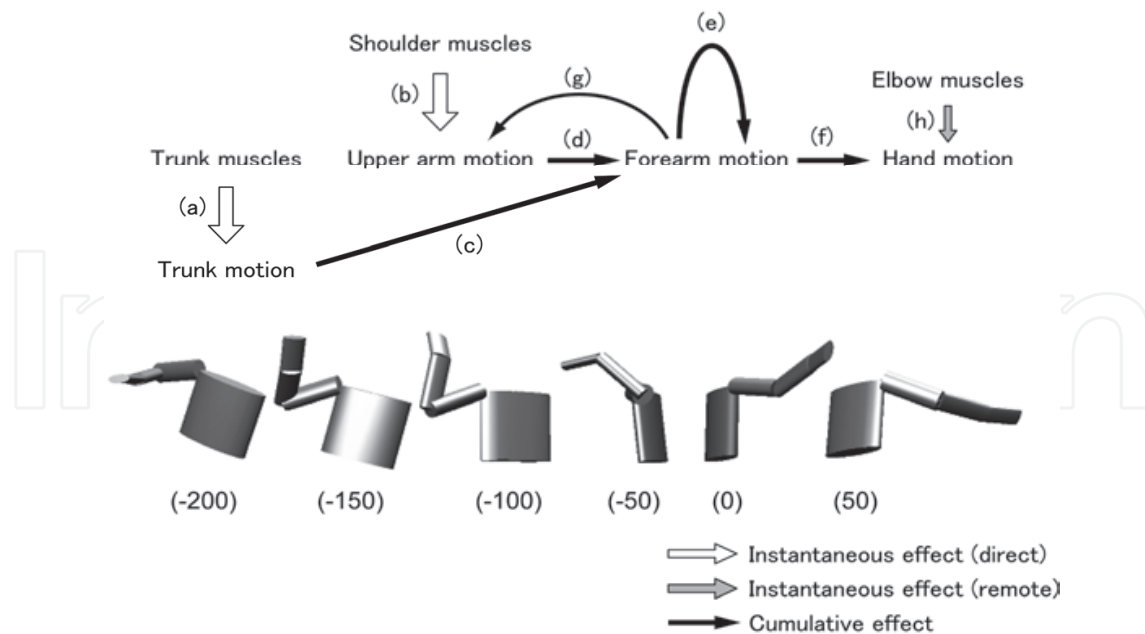


Fig. 11. Kinetic chain of baseball pitching. The number in parentheses indicate the time [msec].

### 5.2 Instantaneous remote effect vs. cumulative effect

It should be noted that the mechanism for the cumulative effect is different from the mechanism for the “torque reversal” examined by (Chowdhary and Challis, 2001) and (Herring and Chapman, 1992). They demonstrated, using simulations, that the braking of a proximal segment accelerated the distal segment and was effective at generating the fastest throw. In baseball pitching, the elbow joint torque reversed its direction from extension to flexion at about -40 ms (Hirashima et al., 2008). While the elbow flexion torque decelerated the elbow extension, it accelerated the distal wrist flexion (Fig. 11h). Thus, the positive effect of the torque reversal is one of the instantaneous effects from the remote joint torques (dotted line in Fig. 4). Therefore, this effect is not influenced by the proximal joint torques in the early phase, in contrast to the cumulative effect. Until now, however, these two effects have been considered similar because they are caused by torques *other than the direct* joint torque. Because the original cause of the instantaneous remote effects is clearly different from that of the cumulative effect in terms of time, in order to develop effective training programs, coaches and athletes should clearly discriminate between these two effects.

### 5.3 Sequence that does not obey proximal-to-distal sequence

In baseball pitching, almost all of the joint rotations obey the proximal-to-distal sequence. However, one exceptional sequence does not obey the rule. The shoulder internal rotation occurs after the elbow extension. Although this distal-to-proximal sequence has been observed for 20 years, it has been unclear why this sequence is adopted by skilled throwers. One possible reason is that extending the elbow before the shoulder internal rotation can decrease the moment of inertia about the shoulder internal rotation axis. The other reason is that the velocity-dependent torque arising from the elbow extension accelerates the shoulder internal rotation (Fig. 11g): cumulative effects do not always have the proximal-to-distal sequence. Although it has long been believed that the throwing motion is controlled by the role of the proximal-to-distal sequence, here we propose that the throwing motion is controlled by the role of utilizing the cumulative effect. This idea can explain the sequence of joint rotations during the throwing movement without exception.

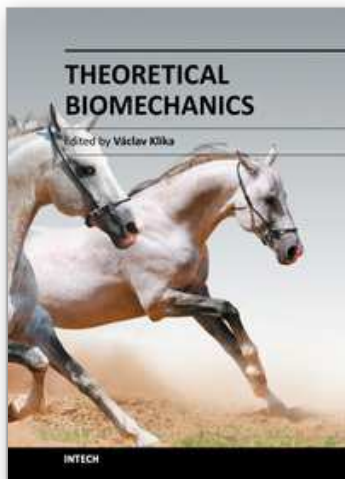
## 6. Conclusions

Until now, the kinetic aspect of sports movements has primarily been examined using the *inverse dynamics technique*, which calculates the joint torques that are required to achieve a focused movement. These findings are helpful for specifying the body parts receiving excessive loads and understanding the mechanism underlying sports disorders (Fleisig et al., 1995). However, the cause-and-effect relationship for how these kinetic parameters (joint torque) accelerate joint rotations and how joint rotations affect each other has been unclear. In this chapter, I have shown that the “induced acceleration analysis” method can greatly assist us in understanding the cause-and-effect relationship between joint torques and rotations. In particular, clearly discriminating between the instantaneous direct effect, instantaneous remote effect, and cumulative effect is very important for coaches and athletes, as well as biomechanics researchers, because their causes are clearly different in terms of time and space.

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## **Theoretical Biomechanics**

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During last couple of years there has been an increasing recognition that problems arising in biology or related to medicine really need a multidisciplinary approach. For this reason some special branches of both applied theoretical physics and mathematics have recently emerged such as biomechanics, mechanobiology, mathematical biology, biothermodynamics. This first section of the book, General notes on biomechanics and mechanobiology, comprises from theoretical contributions to Biomechanics often providing hypothesis or rationale for a given phenomenon that experiment or clinical study cannot provide. It deals with mechanical properties of living cells and tissues, mechanobiology of fracture healing or evolution of locomotor trends in extinct terrestrial giants. The second section, Biomechanical modelling, is devoted to the rapidly growing field of biomechanical models and modelling approaches to improve our understanding about processes in human body. The last section called Locomotion and joint biomechanics is a collection of works on description and analysis of human locomotion, joint stability and acting forces.

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