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# Stochastic and Deterministic Methods of Computing Graded Bedload Transport

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## 1. Introduction

Prediction of nonuniform sediment transport is an important issue for river engineers and morphologists. Presently available methods for sediment transport calculation involve empirical parameters and inexact ways of representing the graded materials and their threshold conditions in computations. The performances of the methods for predicting transport at various types of flows are uncertain. This reality entails verification and sometimes, adjustment of the parameters with reliable and correct data sets. On the other hand, majority of the equations have been developed from laboratory data of straight channels and, generally, verified either with simple straight flume data or field data from uncontrolled and, often poorly defined, natural environments. In this study, frequently used, as well as, recently developed bedload predictors are examined with data set from complex flows in strictly controlled laboratory experiments. The tests were carried out in the Flood Channel Facility, Hydraulic Research Wallingford, UK with graded materials. Local transport rates were sampled under complex flows in a meandering bend with bankfull and overbank conditions.

A meandering channel has straight reaches in the crossover region and curved reaches in the vicinity of the bend apex. Flow patterns in both reaches are complex due to formation of streamwise vortices and corresponding secondary flow cells. The velocity structure in the bend is fully three-dimensional. Consequently, sorting of grains and lateral transport occur in the vicinity of the bend which could persist up to the next crossover section (Dietrich and Smith, 1984; Parker and Andrews, 1985; Julien and Anthony, 2002; Bhuiyan and Hey, 2002). The present data of local graded sediment transport rate and corresponding hydraulic parameters are comprised of carefully carried out experiments under such complex flows. The performances of these methods are examined and issues related to prediction of graded sediment transport are presented in this paper.

## 2. Review of sediment transport predictors

Bed load transport predictors have a long history of development. Recent formulations are more oriented to incorporation of the probabilistic and stochastic concepts in representing flow turbulence and bed shear stress to compute the pickup rates of grains from channel beds composed of sediment mixtures. A brief review of the widely used classical as well as most recent methods and their limitations in applications are given here.

### 2.1 Meyer-Peter Muller (1948)

One of the old and widely used methods is the formula by Meyer-Peter and Muller (1948). This is based on extensive laboratory data with uniform and graded materials of various densities. The bed load transport is predicted from the effective bed shear stress taking into account of the form resistances. The equation is:

$$R(n' / n)^{3/2} S / \Delta d_m - 0.047 = 0.25 \rho^{1/3} \gamma_s q_b'^{2/3} / \Delta d_m \quad (1)$$

where  $R$  and  $S$  are the hydraulic radius and slope, respectively;  $n$  and  $n'$  are the Mannings coefficient related to total roughness and that due to grain roughness, respectively;  $\rho$ ,  $\gamma_s$ ,  $\Delta$ ,  $d_m$  are the density of water, unit weight of sediment, submerged specific weight of sediment and representative sediment diameter, respectively. The nondimensional shield shear stress may assume other values than 0.047, e.g., 0.042 by Habersack and Laronne (2002).

### 2.2 Einstein (1950)

The Einstein's method (1950) of predicting bed load transport is a complex procedure which introduced the probability of particle movement, variable hiding effects of finer particles and fractional transport rates for first time. The flow (shear) intensity parameter ( $\psi_i^*$ ) and transport intensity parameter ( $\phi_i^*$ ) are related as :

$$1 - 1 / \sqrt{\pi} \int_{-0.143\psi_i^{*-2}}^{0.143\psi_i^{*-2}} e^{-t^2} dt = 43.5\phi_i^* / (1 + 43.5\phi_i^*) \quad (2)$$

The transport intensity parameter is a function of the specific bed load transport rate for a size fraction. Contribution from various sediment fractions are summed up to obtain the required bed load transport. Although this method is a detailed one, the performance for predicting total bed load transport of nonuniform sediment is not satisfactory. The method considers only the effect of sheltering of finer particles by the coarser ones but the effect of enlarged exposure of the coarser particles is not taken care of. This is the main reason for poor performance of the method for prediction of individual fractions in a mixture (Misri et al., 1984). Again, the method may underpredict transport rate due to overestimation of form roughness in gravel rivers. Gomez and Church (1989) proposed a modified iterative procedure to incorporate known hydraulic parameters in calculation.

### 2.3 White and Day (1982)

White and Day (1982) modified the Ackers and White (1973) formula for fractional bed load transport:

$$q_{bi} = G_{gri} \gamma_s U (U / u_* )^{n_w} d_{gr} \quad (3)$$

where  $G_{gri}$  is the transport parameter of the grain size class  $i$ ;  $U$  and  $u_*$  are the mean velocity and shear velocity, respectively;  $d_{gr}$  is the grain size parameter on which other exponents of the equations depend.

#### 2.4 van Rijn (1984, 1993, 2002)

The van Rijn's method (1984) is based on detailed physical process of grain transport. The transport for a particular size fraction is given by:

$$q_{bi} = 0.1 \gamma_s \Delta^{0.5} g^{-0.5} d_i^{1.5} p_i T_i^{1.5} D_*^{-0.3} \quad (4)$$

where  $T_i$  is the transport parameter counting relative excess grain shear stress;  $D_*$  is the Bonnefille grain parameter;  $p_i$  is the percentage of size class  $i$  in the bed material. Later on van Rijn (1993) developed a stochastic method for predicting sediment transportation. Kleinhans and van Rijn (2002) used this method with the aid of laboratory experimental data and modified classical deterministic bed load equations (e.g., Mayer-Peter and Muller (1948) and van Rijn (1984) to stochastic predictors. The classical concept of turbulence is incorporated in this method and the shear stress responsible for grain movement is assumed to have a stochastic nature. The effective instantaneous bed shear stress ( $\tau'$ ) is assumed to have a normal distribution with a standard deviation  $\sigma$  and a mean  $\bar{\tau}'$  as:

$$F(\tau') = 1 / \sqrt{2\pi\sigma} \exp\left(-1/2((\tau' - \bar{\tau}') / \sigma)^2\right) \quad (5a)$$

The probability that  $\tau'$  will exceed critical shear stress for grain movement ( $\tau_{ci}$ ) in upstream or downstream direction can be obtained by integrating the probability density function for  $|\tau'| > |\tau_{ci}|$  along these directions. Thus dimensionless bed load transport is expressed as:

$$q_{bi} / (\rho_s d \sqrt{\Delta d g}) = \int_{-\infty}^{-\tau_{ci}} P_{\tau'} \phi_{\tau'_i} d\tau' + \int_{\tau_{ci}}^{\infty} P_{\tau'} \phi_{\tau'_i} d\tau' \quad (5b)$$

where  $P_{\tau'}$  is the time fraction at which instantaneous shear is active for grain movement; and  $\phi_{\tau'_i}$  is the dimensionless sediment transport of fraction  $i$  due to this instantaneous effective shear. The relative standard deviation of the instantaneous shear with respect to  $\bar{\tau}'$  is normally taken as 0.4 in the above computation. A finite lower and upper bound of the above integration is chosen such that most of the distribution is covered (e.g.,  $6\sigma$ ).

#### 2.5 Patel and Raju (1996)

Patel and Raju (1996) modified the previous work by Misri et al. (1984) for predicting nonuniform bed load transport. Both extensive laboratory data as well as field data have been used to develop an empirical relationship between transport rate of individual size fraction and various parameters including hiding-exposure effects and effective grain shear stress. The final relationship between the product  $\xi_B \theta'_i$  (i.e. the product of hiding-exposure parameter and dimensionless effective shear stress) and  $\phi_B$  (dimensionless bed load transport) is presented in graphical format. In logarithmic scale this can be approximated with straight line for  $\phi_B < 10^{-2}$  and for higher  $\phi_B$  by cubic spline.

#### 2.6 Sun and Donahue (2000)

Similar to Einstein (1950), the probability concept has been used by Sun and Donahue (2000) to develop bed load transport functions for nonuniform sediment. A theoretical attempt has been made by the authors to combine stochastic process with mechanics and measured data.

The near-bed exchange of graded materials is modeled by employing a Markov process with continuous time. The dimensionless average velocity and time of single-step motion are considered to be the functions of the flow intensity parameter which reflect the physical properties of the bed load transport rather than as constants expressed by Einstein. The probability of fractional incipient motion for nonuniform sediment  $\alpha_i$  is derived considering random properties of forces and their moment arms as:

$$\alpha_i = 1 - \frac{1}{\sqrt{2\pi}} \int_{-2.7(\sqrt{0.0822\psi_i-1})}^{2.7(\sqrt{0.0822\psi_i-1})} e^{-0.5t^2} dt \quad (6a)$$

where flow intensity ( $\psi_i$ ) is defined as the inverse of the non-dimensional grain shear stress corrected for the shelter-exposure effect of the nonuniform sediment mixture. The bed load formula for the  $i$ th class (percentage of this class on bed =  $P_{bi}$ ) of the nonuniform sediment is:

$$\phi_i = 0.3P_{bi}\alpha_i / (\psi_i^{3/4}(1 - \alpha_i)) \quad (6b)$$

The limitations of this method have been explained by Wu and Yang (2004) while developing their own model (see later). The main conceptual limitation is that although they used continuous-time Markov process of the two-state model (static and moving), the transitional probabilities used in this model are only suitable for discrete-time Markov process. Secondly, the rolling probability has been used for entrainment of particles but only saltation mode has been considered to calculate mean particle velocities. Although this method has been developed to apply for both fully mobilized and partial transport, mainly full transport data were used to calibrate various parameters.

### 2.7 Wu et al. (2000)

Wu et al. (2000) derived a semi-theoretical relation for calculating critical shear stress for incipient motion of size fractions in nonuniform sediment. The hiding-exposure probabilities of particles are related to the size and gradation of the bed material based on stochastic assumption. The exposure height of a particle on bed is taken as a random variable which follows a uniform probability distribution. An empirical relation between excess shear stress,  $T_i$  and non-dimensional fractional bed load transport is deduced as:

$$\phi_{bi} = 0.0053 \left( (n' / n)^{1.5} (\tau_b / \tau_{ci}) - 1 \right)^{2.2} \quad (7a)$$

where  $\tau_{ci}$  is a function of the hiding and exposure probabilities ( $p_{hi}$  and  $p_{ei}$ ) of the particle  $d_i$  which are calculated summing over all fractions:

$$p_{hi} = \sum P_{bj}d_j / (d_i + d_j) \quad \text{and} \quad p_{ei} = \sum P_{bj}d_i / (d_i + d_j) \quad (7b)$$

### 2.8 Wu and Yang (2004)

Wu and Yang updated the concept of the stochastic method by Sun and Donahue (2000). The movements of bed sediment particles are represented by the pseudo four-state

continuous-time Markov process. Both rolling and lifting modes have been considered to calculate probabilities of sediment entrainment. The final equation of this method is:

$$q_{bi} = \frac{\pi}{6} \rho_s P_{bi} \sqrt{(\rho_s - \rho) g d_i^{1.5} Y_i P_{mi} V_{pi} (\Delta_i Y_i^{1.8})} \quad (8)$$

where  $Y_i$  and  $V_{pi}$  are the fractional mobility and dimensionless mean particle velocity, respectively;  $P_{mi}$  is the limiting probability (long-run moving probability) that a bed load particle of size class  $i$  is in the moving state. The last two terms in parenthesis of the above equation are the correction factors for the fully and the partially mobilized fractions. Laboratory and field data of both partial transport and fully mobilized transport have been used in developing this method. The fractional mobility  $Y_i$  is approximated by a cumulative lognormal distribution of effective dimensionless shear stress ( $\theta'_i$ ) with its mean and standard deviation affected by the sand content of the sediment mixture.

### 3. Results and discussion

The transport rates are predicted with section-averaged parameters as well as local parameters (spaced at  $y/B = 0.0625$ ;  $y$  and  $B$  are the lateral distance and width of channel, respectively). Figure 1a and Figure 1b show comparison of observed and predicted transport rates. Transport rates in five cross-sections in the experimental river bend are shown in the first figure. Observed transport rate by Halley-Smith sampler showed a low rate in the apex section which may be attributed to the complex sorting processes around the bend. Predictions are much scattered in that section, mainly underestimated. Local transport rates and velocities were sampled at various lateral locations spaced at  $y/B = 0.0625$ . For each cross-section, predictions were carried out using local data.

Figure 1(b) shows an example of results for the cross-over section. Predictions with both the original sediment mixture and the local sediment properties were examined. The predicted rates from the former shows large scatter indicating the importance of using local sediment characteristics for local transport prediction. In some cases, although predictions for total transport of the section appeared to be closer to the observed rates, local transport distribution across the section were not satisfactory. Again, the calculated local transport rates were much lower than observed values in the apex section.

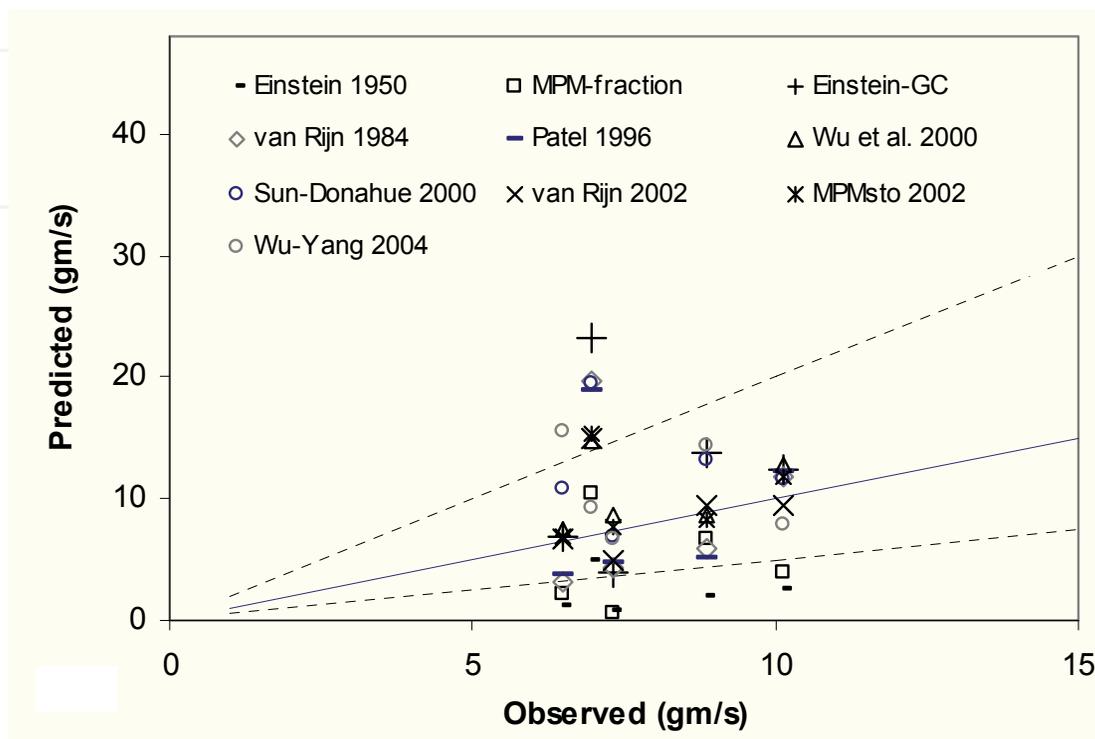
To examine the relative performance of the methods, several parameters has been used. The discrepancy ratio, weighted discrepancy ratio and their geometric mean are used (Habersack and Laronne, 2002). The uniformity coefficient ( $\eta$ ), root mean square error ( $rmse$ ) and relative error ( $\varepsilon_r$ ) are also used :

$$\eta = rmse / \left( \left( 1/n \sum q_{oi}^2 \right)^{1/2} + \left( 1/n \sum q_{pi}^2 \right)^{1/2} \right) \quad (9a)$$

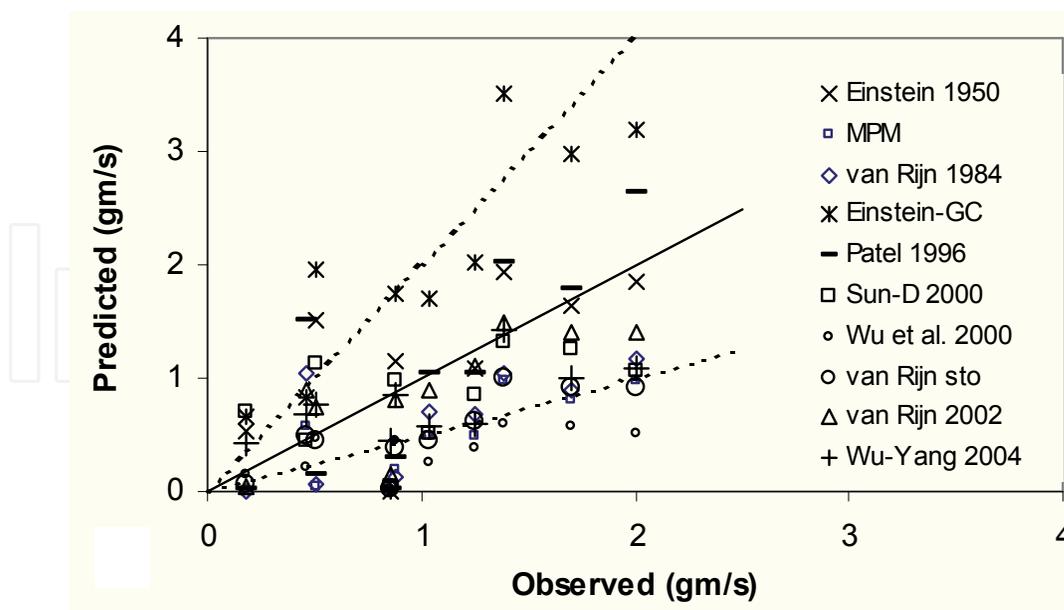
$$rmse = \left( \sum_{i=1}^n 1/n (q_{oi} - q_{pi})^2 \right)^{1/2} ; \text{ and } \varepsilon_r = 1/n \sum_{i=1}^n (q_{oi} - q_{pi}) / q_{oi} \quad (9b)$$

$q_{oi}$  and  $q_{pi}$  are the observed and predicted transport rate of the fraction  $i$  and  $n$  is the total number of fractions for calculation. Based on such calculations, ranking or the performance matrix of selected predictors are shown in Table 1. Here the numbers indicate the position in

the list (e.g., 1 = first, indicating the best performance) depending on certain parameter. The first column is based on the number of predicted data lying in the range 50% to 200% of the observed values. It is obvious that the ranking changes depending on the selected parameter.



(a)



(b)

Fig. 1. Comparison of predicted and observed transport rates (a) Section-averaged rates (b) Local transport rates (solid line: perfect agreement; broken lines: 50% and 200% of the observed values)

	50 % to 200%	Uniformity coefficient	Mean discrepancy ratio, s	Weighted s	Root mean square error, rmse	Relative error
Van Rijn 2002	1	1	1	1	1	1
Wu-Yang	2	3	2	2	2	3
Sun-Donahue	3	4	3	3	3	2
Einstein-GC	4	6	9	8	10	10
Patel and Raju	5	2	4	4	4	6
van Rijn 1984	6	5	6	5	5	7
van Rijn (sto)	7	7	5	6	6	4
MPM	8	8	7	7	7	5
Wu et al.	9	9	8	9	8	8
White-Day	10	10	10	10	9	9

Table 1. Ranking of selected predictors

#### 4. Conclusion

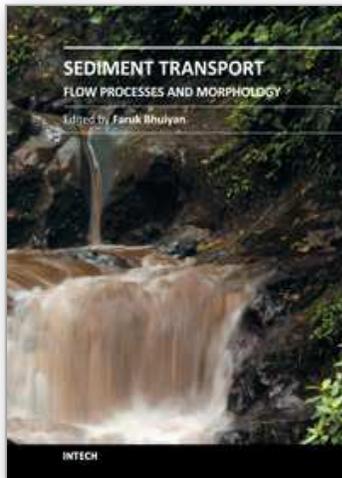
Nonuniform sediment transport prediction capabilities of classical as well as recently developed bed load transport predictors are evaluated. Data from controlled laboratory experiments of graded material transport in complex flows of meander bends have been used for this purpose. Predicting capabilities of majority of the methods are reduced in the vicinity of the bend apex. This indicates the complex flow pattern and sorting process around the bend. The methods based on probability concept appeared to be superior for predicting local transport of bedload. Although several deterministic methods show comparable performance for predicting total sectional transport rate, their performances are significantly reduced for predicting lateral variation of local transportation rates.

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The purpose of this book is to put together recent developments on sediment transport and morphological processes. There are twelve chapters in this book contributed by different authors who are currently involved in relevant research. First three chapters provide information on basic and advanced flow mechanisms including turbulence and movement of particles in water. Examples of computational procedures for sediment transport and morphological changes are given in the next five chapters. These include empirical predictions and numerical computations. Chapters nine and ten present some insights on environmental concerns with sediment transport. Last two contributions deal with two large-scale case studies related to changes in the transport and provenance of glacial marine sediments, and processes involving land slides.

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