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# Robust Control of Active Vehicle Suspension Systems Using Sliding Modes and Differential Flatness with MATLAB

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## 1. Introduction

The main control objectives of active vehicle suspension systems are to improve the ride comfort and handling performance of the vehicle by adding degrees of freedom to the system and/or controlling actuator forces depending on feedback and feedforward information of the system obtained from sensors.

Passenger comfort is provided by isolating the passengers from undesirable vibrations induced from irregular road disturbances, and its performance is evaluated by the level of acceleration which vehicle passengers are exposed. Handling performance is achieved by maintaining a good contact between the tire and the road to provide guidance along the track.

The topic of active vehicle suspension control system, for linear and nonlinear models, in general, has been quite challenging over the years and we refer the reader to some of the fundamental work in the area which has been helpful in the preparation of this chapter. Control strategies like Linear Quadratic Regulator (LQR) in combination with nonlinear backstepping control techniques are proposed in (Liu et al., 2006). This strategy requires information about the state vector (vertical positions and speeds of the tire and car body). A reduced order controller is proposed in (Yousefi et al., 2006) to decrease the implementation costs without sacrificing the security and the comfort by using accelerometers for measurements of the vertical movement of the tire and car body. In (Tahboub, 2005) a controller of variable gain that considers the nonlinear dynamics of the suspension system is proposed. It requires measurements of the vertical positions of the car body and the tire, and the estimation of other states and of the road profile.

On the other hand, many dynamical systems exhibit a structural property called differential flatness. This property is equivalent to the existence of a set of independent outputs, called flat outputs and equal in number to the control inputs, which completely parameterizes every state variable and control input (Fliess et al., 1995). By means of differential flatness

the analysis and design of controller is greatly simplified. In particular, the combination of differential flatness with sliding modes, which is extensively used when a robust control scheme is required, e.g., parameter uncertainty, exogenous disturbances and un-modeled dynamics (see Utkin, 1978), qualifies as an adequate robust control design approach to get high vibration attenuation level in active vehicle suspension systems. Sliding mode control of a differentially flat system of two degrees of freedom, with vibration attenuation, is presented in (Enríquez-Zárate et al., 2000).

This chapter presents a robust active vibration control scheme based on sliding modes and differential flatness for electromagnic and hydraulic active vehicle suspension systems. Measurements of the vertical displacements of the car body and the tire are required for implementation of the proposed control scheme. On-line algebraic estimation of the states variables is used to avoid the use of sensors of acceleration and velocity. The road profile is considered as an unknown input disturbance that cannot be measured. Simulation results obtained from Matlab are included to show the dynamic performance and robustness of the active suspension systems with the proposed control scheme. This chapter applies the algebraic state estimation scheme proposed by Fliess and Sira-Ramírez (Fliess & Sira-Ramírez, 2004a, 2004b; Sira-Ramírez & Silva-Navarro, 2003) for control of nonlinear systems, which is based on the algebraic identification methodology of system parameters reported in (Fliess & Sira-Ramírez, 2003). The method is purely algebraic and involves the use of differential algebra. This method is applied to obtain an estimate of the time derivative from any signal, avoiding model reliance of the system at least in the estimation of states. Simulation and experimental results of the on-line algebraic estimation of states on a differentially flat system of two degrees of freedom are presented in (García-Rodríguez, 2005).

This chapter is organized as follows: Section 2 presents the linear mathematical models of vehicle suspension systems of a quarter car. The design of the controllers for the active suspension systems are introduced in Sections 3 and 4. Section 5 divulges the design of the algebraic estimator of states, while Section 6 shows the use of sensors for measuring the variables required by the controller. The simulation results are illustrated in Section 7. Finally, conclusions are brought out in Section 8.

## 2. Dynamic model of quarter-car suspension systems

### 2.1 Linear mathematical model of passive suspension system

A schematic diagram of a quarter-car suspension system is shown in Fig. 1(a). The mathematical model of the passive suspension system is given by

$$m_s \ddot{z}_s + c_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = 0 \tag{1}$$

$$m_u \ddot{z}_u - c_s(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) + k_t(z_u - z_r) = 0 \tag{2}$$

where $m_s$ represents the mass of a quarter car, $m_u$ represents the mass of one wheel with the suspension and brake equipment, $c_s$ is the damper coefficient of suspension, $k_s$ and $k_t$ are the spring coefficients of the suspension and the tire, $z_s$ and $z_u$ are the displacements of car body and the wheel and $z_r$ is the terrain input disturbance. This simplified linear mathematical model of a passive suspension system has been widely used in many previous works, such as (Liu et al., 2006; Yousefi et al., 2006).
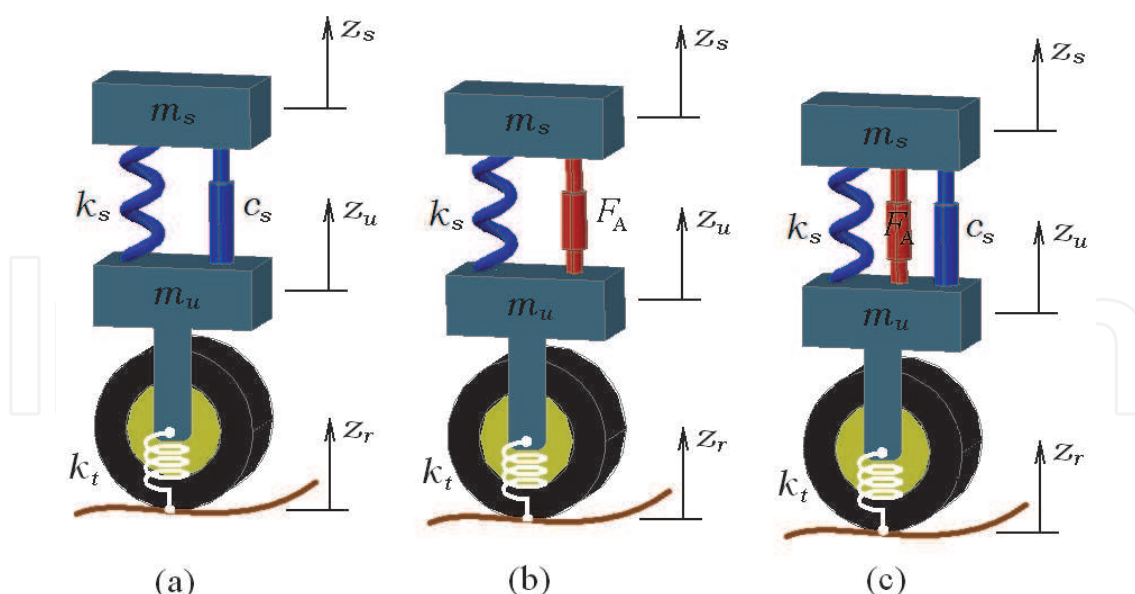
Fig. 1. Schematic diagram of a quarter-vehicle suspension system: (a) passive suspension system, (b) electromagnetic active suspension system and (c) hydraulic active suspension system.

## 2.2 Linear mathematical model of the electromagnetic active suspension system

A schematic diagram of a quarter-car active suspension system is illustrated in Fig.1 (b). The electromagnetic actuator replaces the damper, forming a suspension with the spring. The friction force of an electromagnetic actuator is neglected. The mathematical model of the electromagnetic suspension system, presented in (Martins et al., 2006), is given by:

$$m_s \ddot{z}_s + k_s(z_s - z_u) = F_A \tag{3}$$

$$m_u \ddot{z}_u - k_s(z_s - z_u) + k_t(z_u - z_r) = -F_A \tag{4}$$

where $m_s$, $m_u$, $k_s$, $k_t$, $z_s$, $z_u$ and $z_r$ represent the same parameters and variables shown in the passive suspension system. The electromagnetic actuator force is represented by $F_A$.

## 2.3 Linear mathematical model of hydraulic active suspension system

A schematic diagram of an active quarter-car suspension system is shown in Fig. 1(c). The mathematical model of the hydraulic suspension system is given by

$$m_s \ddot{z}_s + c_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = -F_f + F_A \tag{5}$$

$$m_u \ddot{z}_u - c_s(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) + k_t(z_u - z_r) = F_f - F_A \tag{6}$$

where $m_s$, $m_u$, . $k_s$ ., $k_t$, $z_s$, $z_u$ and $z_r$ represent the same parameters and variables shown in the passive suspension system. The hydraulic actuator force is represented by $F_A$, and $F_f$ represents the friction force generated by the seals of the piston with the cylinder wall inside the actuator. This friction force has a significant magnitude $(> 200N)$ and cannot be ignored (Martins et al., 2006; Yousefi et al., 2006). The net force given by the actuator is the difference between the hydraulic force $F_A$ and the friction force $F_f$.

### 3. Control of electromagnetic suspension system

The mathematical model of the electromagnetic active suspension system illustrated in Fig. 1(b) is given by the equations (3) and (4). Defining the state variables $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_u$ and $x_4 = \dot{z}_u$ for the model of the equations mentioned, the representation in the state space form is,

$$\dot{x}(t) = Ax(t) + Bu(t) + Ez_r(t); \qquad x(t) \in \mathbb{R}^4, A \in \mathbb{R}^{4\times4}, B \in \mathbb{R}^{4\times1}, E \in \mathbb{R}^{4\times1},$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_s}{m_s} & 0 & \dfrac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & 0 & -\dfrac{k_s+k_t}{m_u} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m_s} \\ 0 \\ -\dfrac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix} z_r \qquad (7)$$

with $u = F_A$, the force provided by the electromagnetic actuator as control input.

### 3.1 Differential flatness

The system is controllable and hence, flat (Fliess et al., 1995; Sira-Ramírez & Agrawal, 2004), with the flat output being the positions of body car and wheel, $F = m_s x_1 + m_u x_3$, in (Chávez-Conde et al., 2009). For simplicity in the analysis of the differential flatness for the suspension system assume that $k_t z_r = 0$. In order to show the parameterization of all the state variables and control input, we firstly compute the time derivatives up to fourth order for $F$, resulting in

$$F = m_s x_1 + m_u x_3$$
$$\dot{F} = m_s x_2 + m_u x_4$$
$$\ddot{F} = -k_t x_3$$
$$F^{(3)} = -k_t x_4$$
$$F^{(4)} = \frac{k_t}{m_u} u - \frac{k_s k_t}{m_u}(x_1 - x_3) + \frac{k_t^2}{m_u} x_3$$

Then, the state variables and control input are differentially parameterized in terms of the flat output as follows

$$x_1 = \frac{1}{m_s}\left(F + \frac{m_u}{k_t}\ddot{F}\right)$$
$$x_2 = \frac{1}{m_s}\left(\dot{F} + \frac{m_u}{k_t}F^{(3)}\right)$$
$$x_3 = -\frac{1}{k_t}\ddot{F}$$
$$x_4 = -\frac{1}{k_t}F^{(3)}$$

$$u = \frac{m_u}{k_t} F^{(4)} + \left( \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \right) \ddot{F} + \frac{k_s}{m_s} F$$

## 3.2 Sliding mode and differential flatness control

The input $u$ in terms of the flat output and its time derivatives is given by

$$u = \frac{m_u}{k_t} F^{(4)} + \left( \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \right) \ddot{F} + \frac{k_s}{m_s} F \qquad (8)$$

where $F^{(4)} = v$ defines an auxiliary control input. This expression can be written in the following form:

$$u = d_1 F^{(4)} + d_2 \ddot{F} + d_3 F \qquad (9)$$

where $d_1 = \frac{m_u}{k_t}$, $d_2 = \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1$ and $d_3 = \frac{k_s}{m_s}$.

Now, consider a linear switching surface defined by

$$\sigma = F^{(3)} + \beta_2 \ddot{F} + \beta_1 \dot{F} + \beta_0 F \qquad (10)$$

Then, the error dynamics restricted to $\sigma = 0$ is governed by the linear differential equation

$$F^{(3)} + \beta_2 \ddot{F} + \beta_1 \dot{F} + \beta_0 F = 0 \qquad (11)$$

The design gains $\beta_2, \ldots, \beta_0$ are selected to verify that the associated characteristic polynomial $s^3 + \beta_2 s^2 + \beta_1 s + \beta_0$ be Hurwitz. As a consequence, the error dynamics on the switching surface $\sigma = 0$ is globally asymptotically stable. The sliding surface $\sigma = 0$ is made globally attractive with the continuous approximation to the discontinuous sliding mode controller as given in (Sira-Ramírez, 1993), i.e., by forcing to satisfy the dynamics,

$$\dot{\sigma} = -\mu [\sigma + \gamma \, sign(\sigma)] \qquad (12)$$

where $\mu$, $\gamma$ denote positive real constants and "sign" is the standard signum function. The sliding surface is globally attractive, $\sigma \dot{\sigma} < 0$ for $\sigma \neq 0$, which is a very well known condition for the existence of sliding mode presented in (Utkin, 1978). One then obtains the following sliding-mode controller:

$$u = d_1 v + d_2 \ddot{F} + d_3 F \qquad (13)$$

$$v = -\beta_2 F^{(3)} - \beta_1 \ddot{F} - \beta_0 \dot{F} - \mu [\sigma + \gamma \, sign(\sigma)]$$

This controller requires the measurement of all the state variables of the suspension system, $z_s$, $\dot{z}_s$, $z_u$ and $\dot{z}_u$, corresponding to the vertical positions and velocity of the body of the car and the wheel. The variables $\dot{z}_s$ and $\dot{z}_u$ are calculated through an online algebraic estimator, shown in Section 5.

## 4. Control control of hydraulic suspension system

The mathematical model of the hydraulic active suspension system shown in Fig. 1(c) is given by the equations (5) and (6). Defining the state variables $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_u$ and $x_4 = \dot{z}_u$ for the model of the equations mentioned, the representation in the state space form is, $\dot{x}(t) = Ax(t) + Bu(t) + Ez_r(t)$; $x(t) \in \mathbb{R}^4, A \in \mathbb{R}^{4 \times 4}, B \in \mathbb{R}^{4 \times 1}, E \in \mathbb{R}^{4 \times 1}$,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_s}{m_s} & -\dfrac{c_s}{m_s} & \dfrac{k_s}{m_s} & \dfrac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{c_s}{m_u} & -\dfrac{k_s + k_t}{m_u} & -\dfrac{c_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m_s} \\ 0 \\ -\dfrac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix} z_r \qquad (14)$$

with $u = F_A - F_f$, the net force provided by the hydraulic actuator as control input (the net force provided by the actuator is the difference between the hydraulic force $F_A$ and the frictional force $F_f$).

### 4.1 Differential flatness

The system is controllable and hence, flat (Fliess et al., 1995; Sira-Ramírez & Agrawal, 2004), with the flat output being the positions of body car and wheel, $F = m_s x_1 + m_u x_3$, in (Chávez-Conde et al., 2009). For simplicity in the analysis of the differential flatness for the suspension system assume that $k_t z_r = 0$. In order to show the parameterization of all the state variables and control input, we firstly compute the time derivatives up to fourth order for $F$, resulting in

$$\begin{aligned} F &= m_s x_1 + m_u x_3 \\ \dot{F} &= m_s x_2 + m_u x_4 \\ \ddot{F} &= -k_t x_3 \\ F^{(3)} &= -k_t x_4 \\ F^{(4)} &= \frac{k_t}{m_u} u - \frac{c_s k_t}{m_u}(x_2 - x_4) - \frac{k_s k_t}{m_u}(x_1 - x_3) + \frac{k_t^2}{m_u} x_3 \end{aligned}$$

Then, the state variables and control input are parameterized in terms of the flat output as follows

$$x_1 = \frac{1}{m_s}\left(F + \frac{m_u}{k_t}\ddot{F}\right)$$

$$x_2 = \frac{1}{m_s}\left(\dot{F} + \frac{m_u}{k_t}F^{(3)}\right)$$

$$x_3 = -\frac{1}{k_t}\ddot{F}$$

$$x_4 = -\frac{1}{k_t}F^{(3)}$$

$$u = \frac{m_u}{k_t}F^{(4)} + \left(\frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t}\right)F^{(3)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1\right)\ddot{F} + \frac{c_s}{m_s}\dot{F} + \frac{k_s}{m_s}F$$

## 4.2 Sliding mode and differential flatness control

The input $u$ in terms of the flat output and its time derivatives is given by

$$u = \frac{m_u}{k_t}v + \left(\frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t}\right)F^{(3)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1\right)\ddot{F} + \frac{c_s}{m_s}\dot{F} + \frac{k_s}{m_s}F \qquad (15)$$

where $F^{(4)} = v$ defines the auxiliary control input. The expression can be written in the following form:

$$u = \eta_1 v + \eta_2 F^{(3)} + \eta_3 \ddot{F} + \eta_4 \dot{F} + \eta_5 F \qquad (16)$$

where $\eta_1 = \frac{m_u}{k_t}$, $\eta_2 = \frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t}$, $\eta_3 = \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1$, $\eta_4 = \frac{c_s}{m_s}$ and $\eta_5 = \frac{k_s}{m_s}$.

Now, consider a linear switching surface defined by

$$\sigma = F^{(3)} + \beta_2 \ddot{F} + \beta_1 \dot{F} + \beta_0 F \qquad (17)$$

Then, the error dynamics restricted to $\sigma = 0$ is governed by the linear differential equation

$$F^{(3)} + \beta_2 \ddot{F} + \beta_1 \dot{F} + \beta_0 F = 0 \qquad (18)$$

The design gains $\beta_2, \ldots, \beta_0$ are selected to verify that the associated characteristic polynomial $s^3 + \beta_2 s^2 + \beta_1 s + \beta_0$ be Hurwitz. As a consequence, the error dynamics on the switching surface $\sigma = 0$ is globally asymptotically stable. The sliding surface $\sigma = 0$ is made globally attractive with the continuous approximation to the discontinuous sliding mode controller as given in (Sira-Ramírez, 1993), i.e., by forcing to satisfy the dynamics

$$\dot{\sigma} = -\mu[\sigma + \gamma sign(\sigma)] \qquad (19)$$

where $\mu$, $\gamma$ denote positive real constants and "sign" is the standard signum function. The sliding surface is globally attractive, $\sigma\dot{\sigma} < 0$ for $\sigma \neq 0$, which is a very well known condition for the existence of sliding mode presented in (Utkin, 1978). One then obtains the following sliding-mode controller:

$$u = \eta_1 v + \eta_2 F^{(3)} + \eta_3 \ddot{F} + \eta_4 \dot{F} + \eta_5 F \qquad (20)$$

$$v = -\beta_2 F^{(3)} - \beta_1 \ddot{F} - \beta_0 \dot{F} - \mu[\sigma + \gamma sign(\sigma)]$$

This controller requires the measurement of all the variables of state of suspension system, $z_s$, $\dot{z}_s$, $z_u$ and $\dot{z}_u$ corresponding to the vertical positions and velocity of the body of the car and the tire, respectively. The variables $\dot{z}_s$ and $\dot{z}_u$ are calculated through an online algebraic estimator, shown in Section 5.

## 5. On-line algebraic state estimation of active suspension system

### 5.1 First time derivative algebraic estimation

The algebraic methodology proposed in (Fliess & Sira-Ramírez, 2003) allows us to estimate the derivatives of a smooth signal considering a signal model of $n-th$ order, thus it is not necessary to design the time derivative estimator from a specific dynamic model of the plant. Through valid algebraic manipulations of this approximated model in the frequency domain, and using the algebraic derivation with respect to the complex variable $s$, we neglect the initial conditions of the signal. The resulting equation is multiplied by a negative power $s^{n-1}$ and returned to the time domain. This last expression is manipulated algebraically in order to find a formula to estimate the first time derivative of $y(t)$.

Consider a fourth order approximation of a smooth signal $y(t)$,

$$\frac{d^4 y(t)}{dt^4} = 0 \tag{21}$$

This model indicates that $y(t)$ is a signal whose behavior can be approximated by a family of polynomials of third order, thus the fourth time derivative is assumed close to zero. The order of this approximation can be increased to enhance the estimation quality of the algebraic estimator. From (21) it is possible to design a time derivative algebraic estimator. First, we apply Laplace transform to (21),

$$s^4 Y(s) - s^3 Y(0) - s^2 \dot{Y}(0) - s\ddot{Y} - Y^{(3)} = 0 \tag{22}$$

Now, taking successive derivatives until a number of three with respect to the complex variable $s$, we obtain a expression which is free of initial conditions,

$$\frac{d^4 \left(s^4 Y\right)}{ds^4} = 0 \tag{23}$$

Expanding this expression and multiplying by $s^{-3}$,

$$24s^{-3} Y + 96s^{-2} \frac{dY}{ds} + 72s^{-1} \frac{d^2 Y}{ds^2} + 16 \frac{d^3 Y}{ds^3} + s \frac{d^4 Y}{ds^4} \tag{24}$$

Returning to the time domain,

$$\frac{d}{dt}(t^4 z(t)) - 16t^3 z(t) + 72 \int_0^t \lambda_1^2 z(\lambda_1) d\lambda_1$$

$$- 96 \int_0^t \int_0^{\lambda_1} \lambda_2 z(\lambda_2) d\lambda_2 d\lambda_1 + 24 \int_0^t \int_0^{\lambda_1} \int_0^{\lambda_2} z(\lambda_3) d\lambda_3 d\lambda_2 d\lambda_1 = 0$$

From the last equation is possible to obtain the following algebraic estimator,

$$\frac{dz}{dt} = \frac{12t^3 z + 96 \int_0^t \int_0^{\lambda_1} \lambda_2 z(\lambda_2) d\lambda_2 d\lambda_1 - 24 \int_0^t \int_0^{\lambda_1} \int_0^{\lambda_2} z(\lambda_3) d\lambda_3 d\lambda_2 d\lambda_1}{t^4} \tag{25}$$

This formula is valid for $t > 0$. Since (25) provides an approximated value of the first derivative, this is only valid during a period of time. So the state estimation should be calculated periodically as follows,

$$\left. \frac{dz}{dt}\right|_{t_i}^{t} = \frac{12(t-t_i)^3 z + 96\int_{t_i}^{t}\int_{t_i}^{\lambda_1}\lambda_2 z(\lambda_2)d\lambda_2 d\lambda_1}{-24\int_{t_i}^{t}\int_{t_i}^{\lambda_1}\int_{t_i}^{\lambda_2}z(\lambda_3)d\lambda_3 d\lambda_2 d\lambda_1}{(t-t_i)^4} , \quad \forall (t-t_i) > 0 \tag{26}$$

where $(t_i, t)$ is the estimation period.

In order to obtain a better and smoother estimated value of the vertical velocity, we have implemented simultaneously two algebraic estimators for each velocity to estimate. Proceeding with an out-of-phase policy for one of these algebraic estimators, the outputs of both are combined properly to obtain a final estimated value.

## 6. Instrumentation of the active suspension system

The only variables required for the implementation of the proposed controllers are the vertical displacements of the body of the car $z_s$ and the vertical displacement of the wheel $z_u$. These variables are needed to measure by some sensor. In (Chamseddine et al., 2006) the use of sensors in experimental vehicle platforms, as well as in commercial vehicles is presented. The most common sensors used for measuring the vertical displacement of the body of the car and the wheel are laser sensors. This type of sensors could be used to measure the variables $z_s$ and $z_s$ needed for controller implementation. An accelerometer or another type of sensor is not needed to measure the variables $\dot{z}_s$ and $\dot{z}_u$, these variables are estimated with the use of algebraic estimators from knowledge of the variables $z_s$ and $z_u$. Fig. 2 shows a schematic diagram of the instrumentation for the active suspension system.
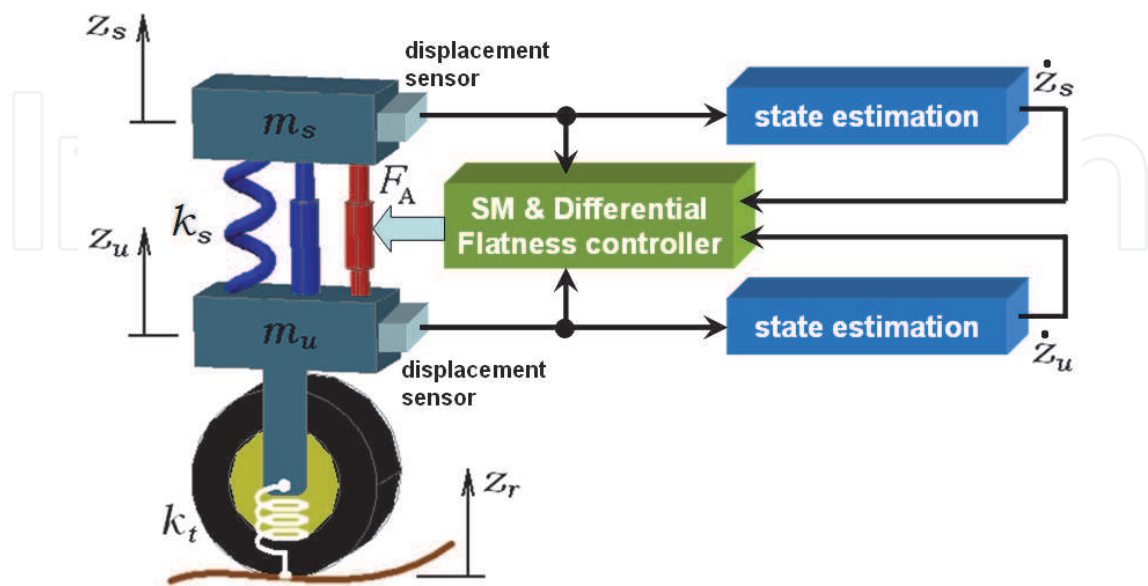


Fig. 2. Schematic diagram of the instrumentation of the active suspension system.

## 7. Simulation results

The simulation results were obtained by means of MATLAB/Simulink$^{®}$, with the Runge-Kutta numerical method and a fixed integration step of $1ms$. The numerical values of the quarter-car model parameters (Sam & Hudha, 2006) are presented in Table 1.

| Parameter | Value |
|---|---|
| Sprung mass ($m_s$) | 282 $[kg]$ |
| Unsprung mass ($m_u$) | 45 $[kg]$ |
| Spring stiffness ($k_s$) | 17900 $\left[\dfrac{N}{m}\right]$ |
| Damping constant ($c_s$) | 1000 $\left[\dfrac{N \cdot s}{m}\right]$ |
| Tire stiffness ($k_t$) | 165790 $\left[\dfrac{N}{m}\right]$ |

Table 1. Quarter-car model parameters

In this simulation study, the road disturbance is shown in Fig. 3 and set in the form of (Sam & Hudha, 2006):

$$z_r = a\frac{1 - cos(8\pi t)}{2}$$

with $a = 0.11$ [m] for $0.5 \leq t \leq 0.75$, $a = 0.55$ [m] for $3.0 \leq t \leq 3.25$ and 0 otherwise.
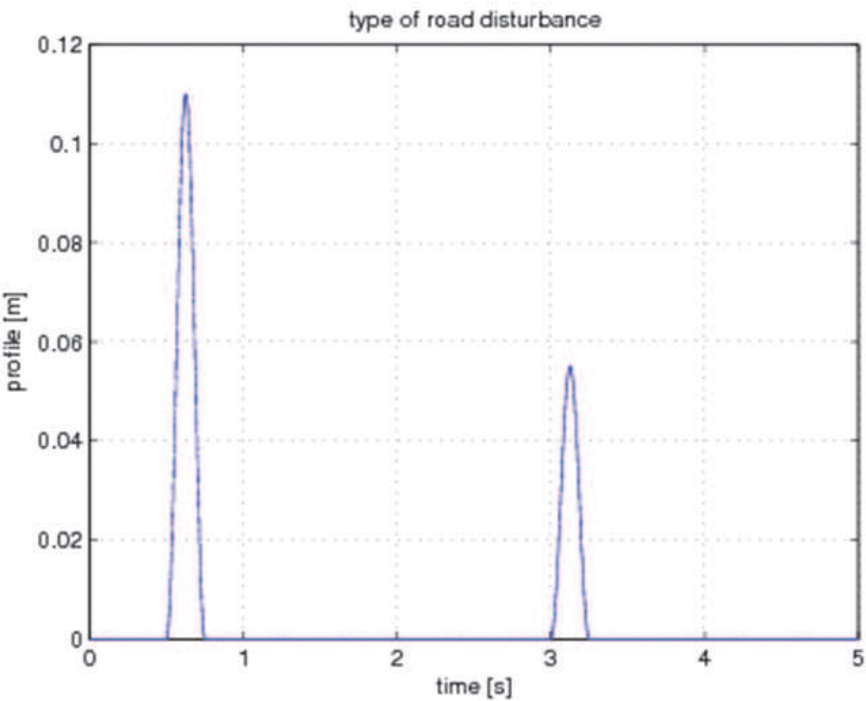


Fig. 3. Type of road disturbance.

It is desired to stabilize the suspension system at the positions $z_s = 0$ and $z_u = 0$. The gains of both electromagnetic and hydraulic suspension controllers were obtained by forcing their closed loop characteristic polynomials to be given by the following Hurwitz polynomial: $(s + p)(s^2 + 2\zeta\omega_n s + \omega_n^2)$ with $p = 100$, $\zeta = 0.5$, $\omega_n = 90$, $\mu = 95$ y $\gamma = 95$.

The Simulink model of the sliding mode and differential flatness controller of the active suspension system is shown in Fig. 4. For the electromagnetic active suspension system, it is assumed that $c_z = 0$. In Fig. 5 is shown the Simulink model of the sliding mode and differential flatness controller with algebraic state estimation.
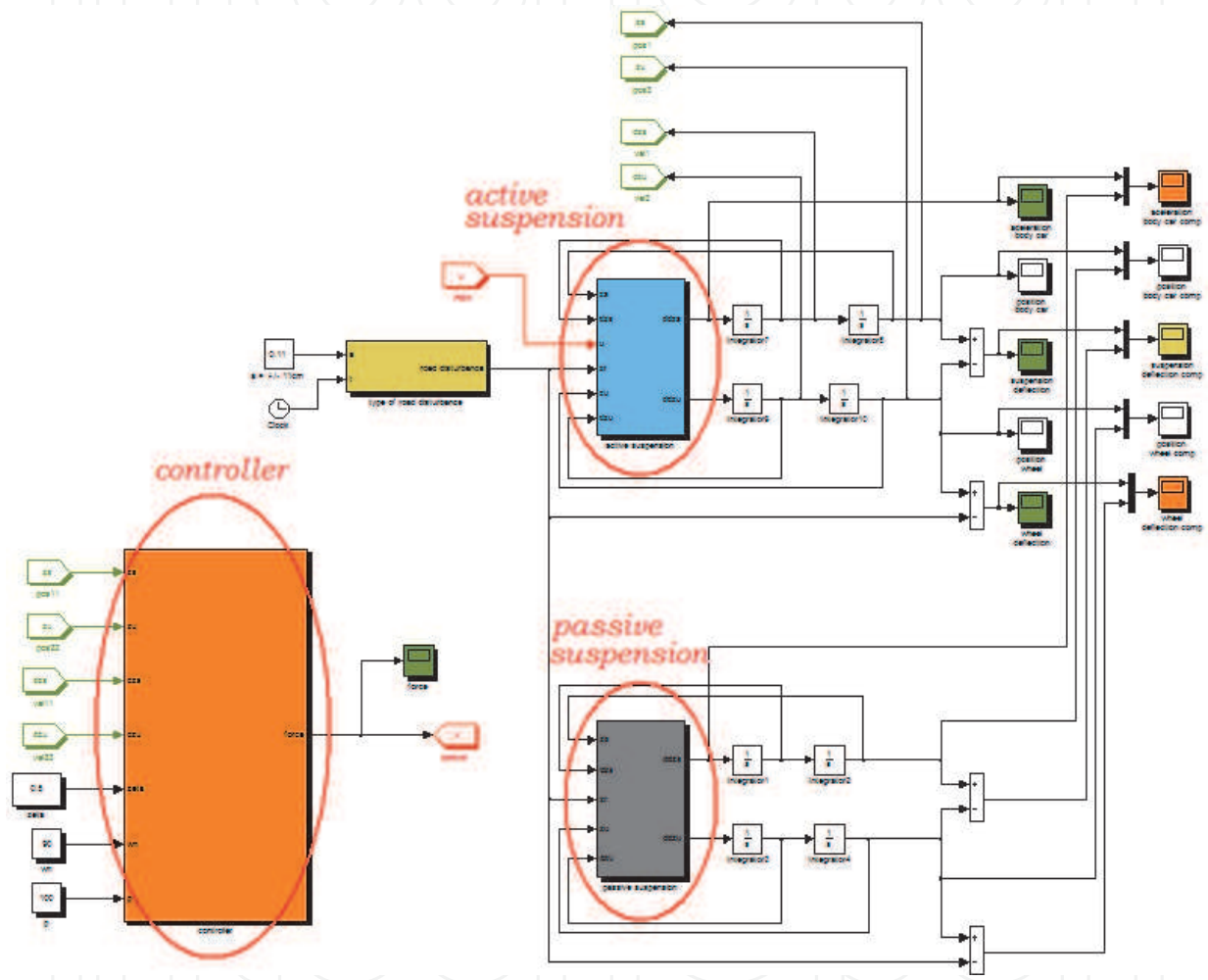


Fig. 4. Simulink model of the sliding mode and differential flatness controller.

In Fig. 6 is depicted the robust performance of the electromagnetic suspension controller. It can be seen the high vibration attenuation level of the active vehicle suspension system compared with the passive counterpart. Similar results on the implementation of the hydraulic suspension controller are depicted in Fig. 7.

In Fig. 8 is presented the algebraic estimation process of the velocities of the car body and the wheel. There we can observe a good and fast estimation. In Figs. 9 and 10 are shown the simulation results on the performance of the electromagnetic and hydraulic suspension controllers using the algebraic estimators of velocities. These results are quite similar to those gotten by the controllers using the real velocities.
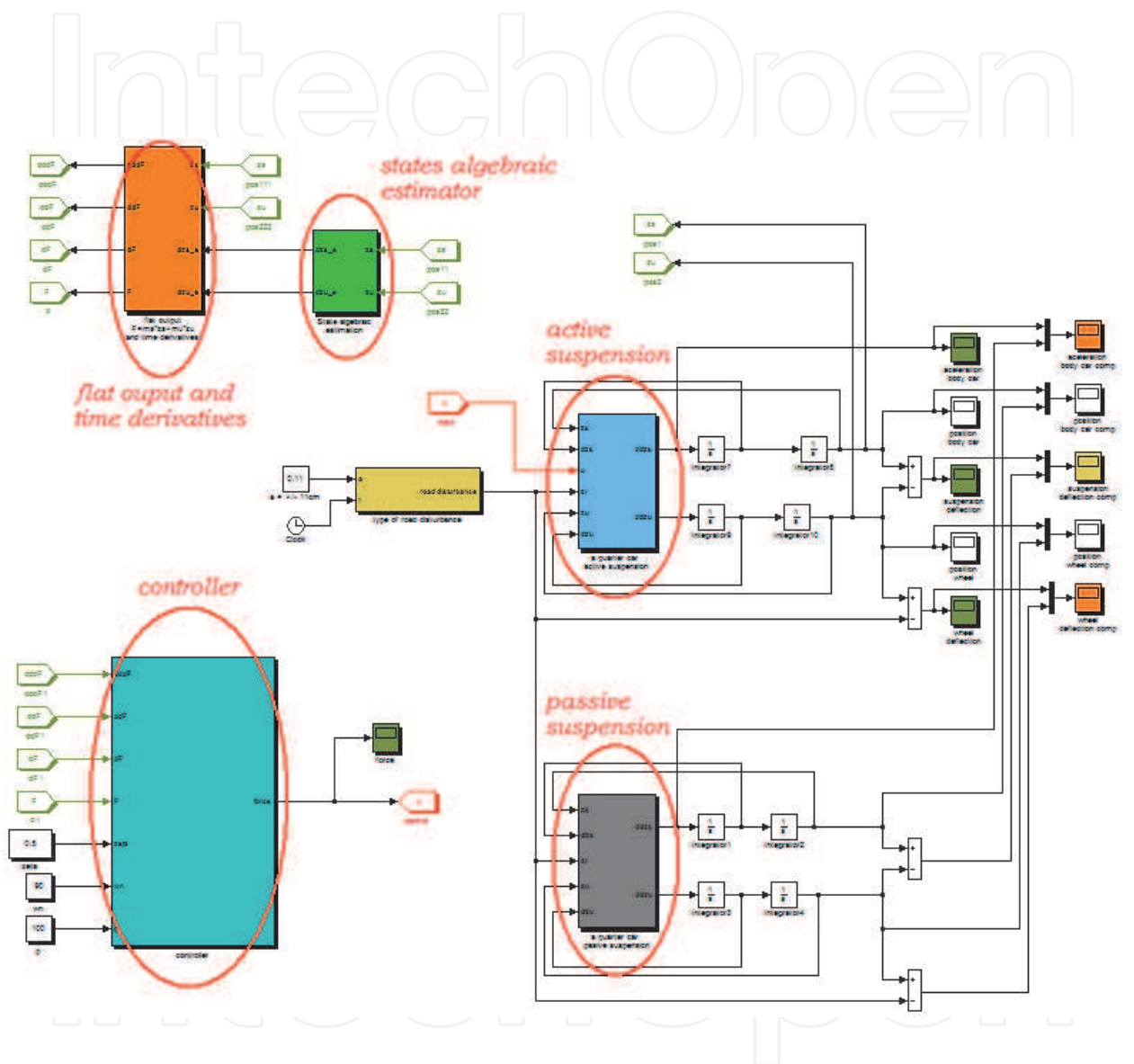
Fig. 5. Simulink model of the sliding mode and differential flatness controller with state estimation.
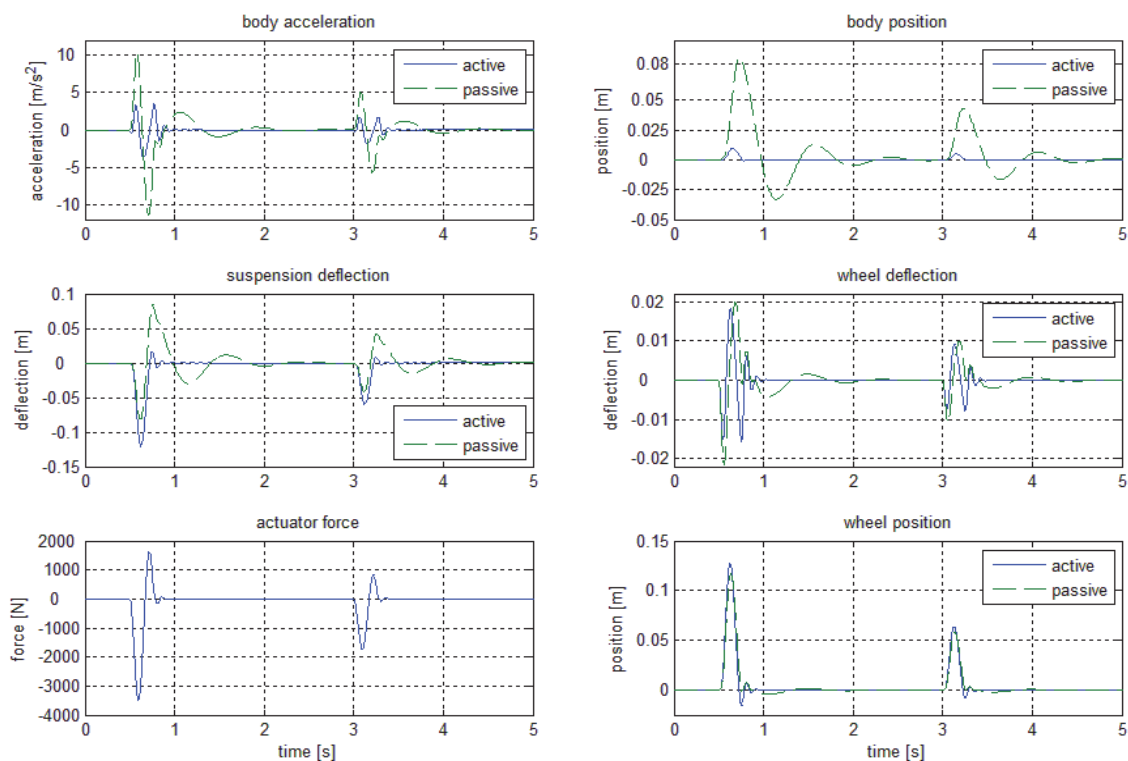
Fig. 6. Electromagnetic active vehicle suspension system responses with sliding mode and differential flatness based controller.
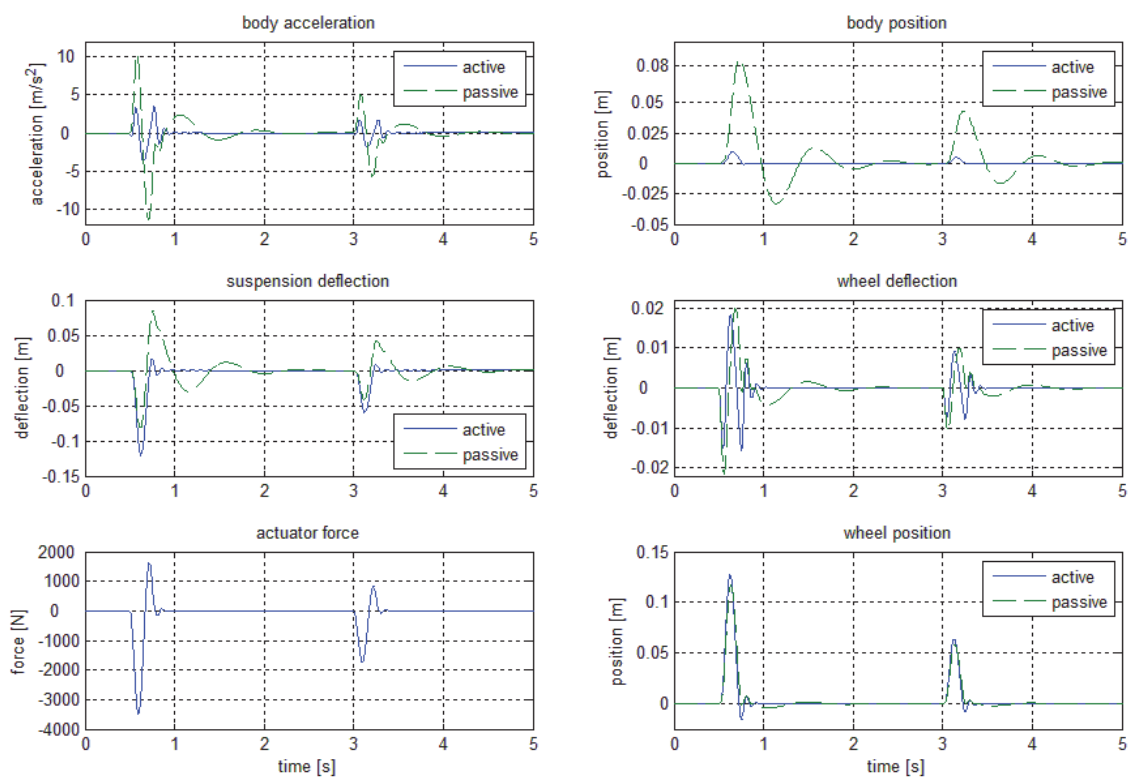


Fig. 7. Hydraulic active vehicle suspension system responses with sliding mode and differential flatness based controller.
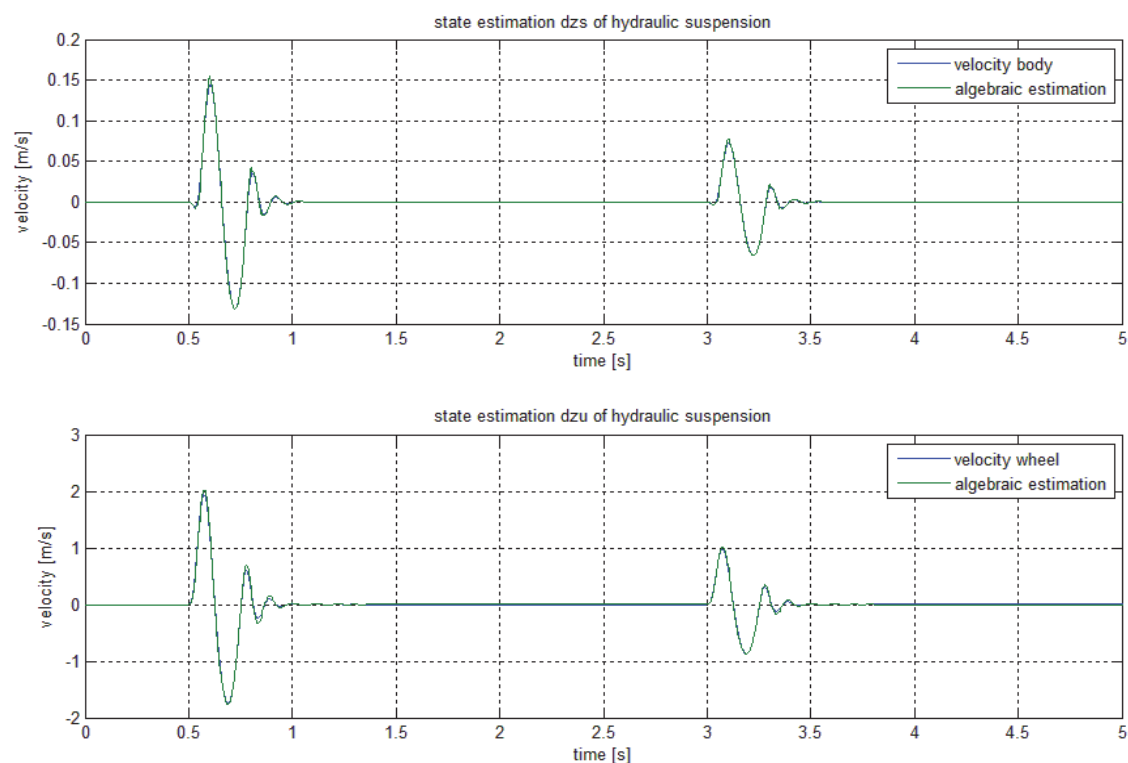
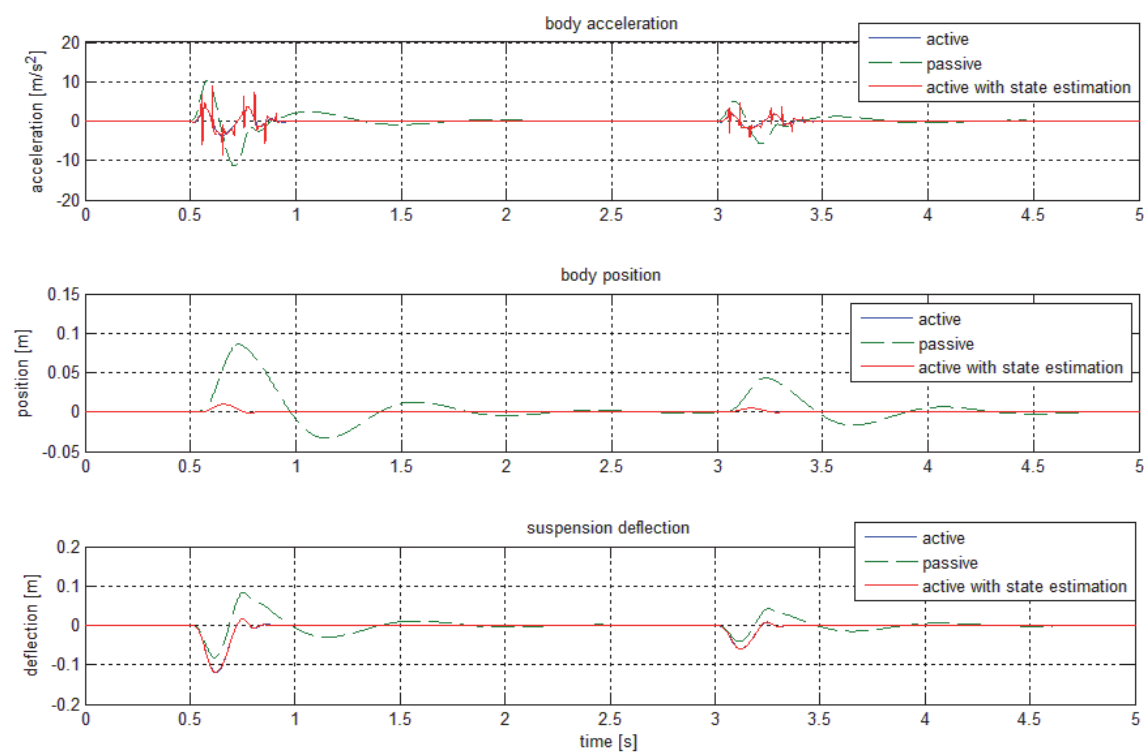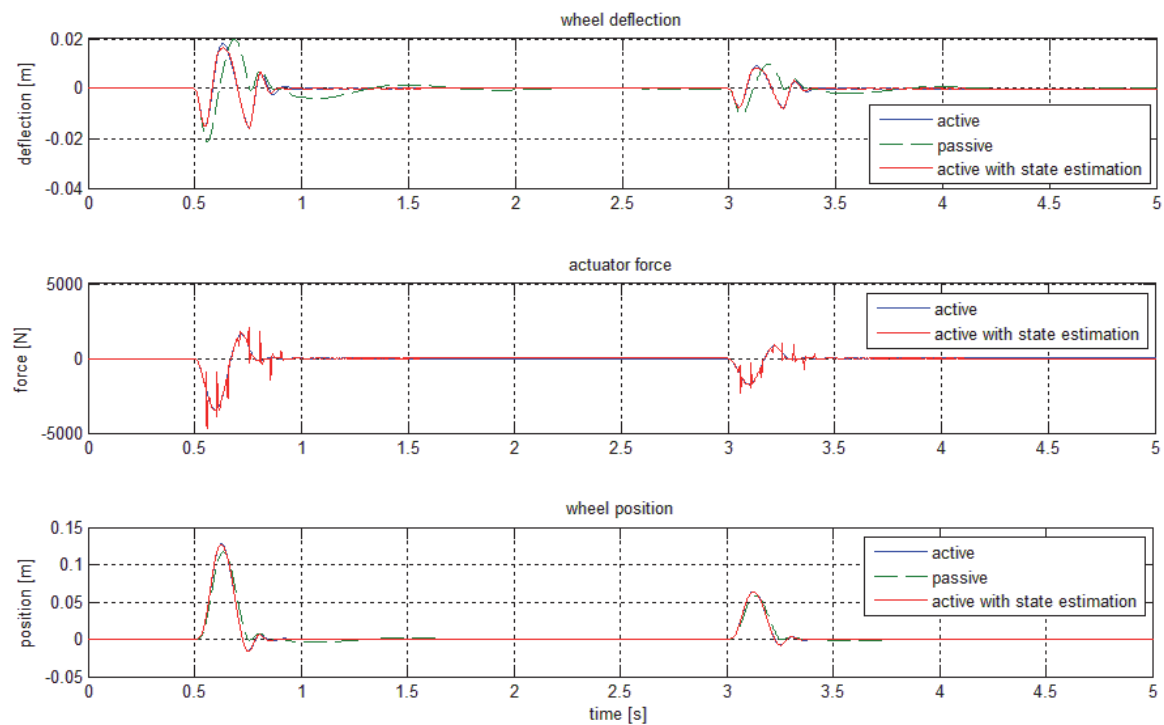Fig. 8. On-line algebraic state estimates of the hydraulic active suspension system.



Fig. 9. a. Electromagnetic active vehicle suspension system responses with sliding mode and differential flatness based controller using algebraic state estimation.
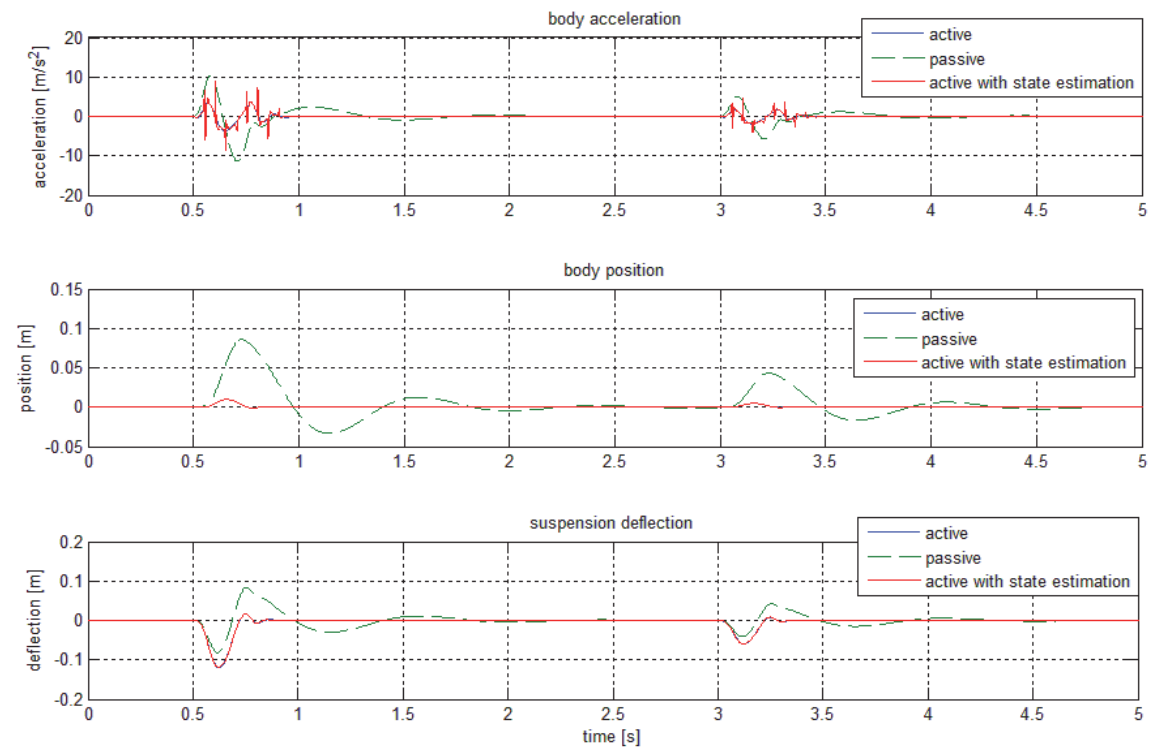
Fig. 9. b. Electromagnetic active vehicle suspension system responses with sliding mode and differential flatness based controller using algebraic state estimation.



Fig. 10. a. Hydraulic active vehicle suspension system responses with sliding mode and differential flatness based controller using algebraic state estimation.
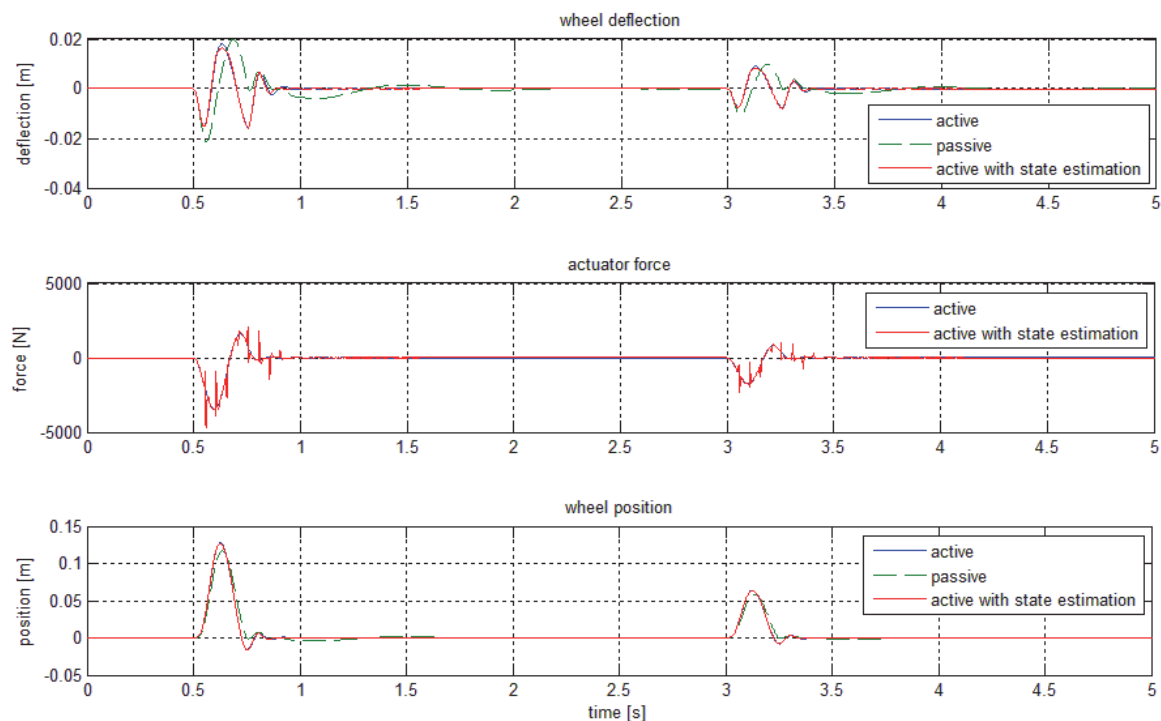
Fig. 10. b. Hydraulic active vehicle suspension system responses with sliding mode and differential flatness based controller using algebraic state estimation.

## 8. Conclusions

The stabilization of the vertical position of the quarter of car is obtained in a time much smaller to that of the passive suspension system. The sliding mode based differential flatness controller requires the knowledge of all the state variables. Nevertheless the fast stabilization with amplitude in acceleration and speed of the body of the car very remarkable is observed. On-line state estimation is obtained successfully, however when it is used into the controller one can observe a deterioration of the control signal. This can significantly improve with a suitable interpolation between the estimated values at each restart of the integrations. In addition, the simulations results show that the stabilization of the system is obtained before the response of the passive suspension system, with amplitude of acceleration and speed of the body of the car very remarkable. Finally, the robustness of the controllers is observed to take to stabilize to the system before the unknown disturbance.

# 9. References

Chamseddine, Abbas; Noura, Hassan; Raharijaona, Thibaut "Control of Linear Full Vehicle Active Suspension System Using Sliding Mode Techniques",  2006 IEEE International Conference on Control Applications. pp. 1306-1311, Munich, Germany, October 4-6, 2006.

Chávez-Conde, E.; Beltrán-Carbajal, F.; Blanco-Ortega, A.; Méndez-Azúa, H. "Sliding Mode and Generalized PI Control of Vehicle Active Suspensions",  2009 IEEE International Conference on Control Applications. Saint Petersburg, Russia, July 8-10, 2009.

Enríquez-Zárate, J.; Silva-Navarro, G.; Sira-Ramírez, H. "Sliding Mode Control of a Differentially Flat Vibrational Mechanical System: Experimental Results", 39th IEEE Conference on Decision and Control. pp. 1679-1684, Sydney, Australia, December 2000.

Flies, M.; Lévine, J.; Martin, Ph.; Rouchon, P. "Flatness and defect of nonlinear systems: introductory theory and examples", Int. Journal of Control. Vol. 61, pp. 1327-1361, 1995.

Fliess, M.; Sira-Ramírez, H. "Reconstructeurs d'états".  C.R. Acad. Sci. Paris, I-338, pp. 91-96, 2004.

Fliess, M.; Sira-Ramírez, H. "Control via state estimations of some nonlinear systems", 4th IFAC Nolcos Conference. Germany, 2004.

Fliess, M.; Sira-Ramírez, H. "An Algebraic Framework For Linear Identification", ESAIM Control, Optimization and Calculus of Variations. Vol 9, pp. 151-168, January. 2003.

García-Rodríguez, C. "Estimación de estados por métodos algebraicos", Master of Thesis. CINVESTAV-IPN, México, D.F., México, 2005.

Liu, Zhen; Luo, Cheng; Hu, Dewen, "Active Suspension Control Design Using a Combination of LQR and Backstepping", 25th IEEE Chinese Control Conference, pp. 123-125, Harbin, Heilongjiang, August 7-11, 2006.

Martins, I.; Esteves, J.; Marques, D. G.; Da Silva, F. P. "Permanent-Magnets Linear Actuators Applicability in Automobile Active Suspensions", IEEE Trans. on Vehicular Technology. Vol. 55, No. 1, pp. 86-94, January 2006.

Sam, Y. M.; Hudha, K. "Modeling and Force Tracking Control of Hydraulic Actuator for an Active Suspension System", IEEE ICIEA, 2006.

Sira-Ramírez, H. "A dynamical variable structure control strategy in asymptotic output tracking problems", IEEE Trans. on Automatic Control. Vol. 38, No. 4, pp. 615-620, April 1993.

Sira-Ramírez, H.; Silva-Navarro, G. "Algebraic Methods in Flatness, Signal Processing and State Estimation", Innovación Editorial Lagares de México, November 2003.

Sira-Ramírez, H.; Agrawal, Sunil K. "Differentially Flat Systems", Marcel Dekker, N.Y., 2004.

Tahboub, Karim A. "Active Nonlinear Vehicle-Suspension Variable-Gain Control",  13th IEEE Mediterranean Conference on Control and Automation, pp. 569-574, Limassol, Cyprus, June 27-29, 2005.

Utkin,V. I. "Sliding Modes and Their Applications in Variable Structure Systems". Moscow: MIR, 1978.

Yousefi, A.; Akbari, A and Lohmann, B., "Low Order Robust Controllers for Active Vehicle Suspensions", IEEE International Conference on Control Applications, pp. 693-698, Munich, Germany, Octuber 4-6, 2006.

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