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Control of Distributed Parameter Systems - Engineering Methods and Software Support in the MATLAB & Simulink Programming Environment

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1. Introduction

Distributed parameter systems (DPS) are systems with state/output quantities $X(x,t)$ / $Y(x,t)$ - parameters which are defined as quantity fields or infinite dimensional quantities distributed through geometric space, where x - in general is a vector of the three dimensional Euclidean space. Thanks to the development of information technology and numerical methods, engineering practice is lately modelling a wide range of phenomena and processes in virtual software environments for numerical dynamical analysis purposes such as ANSYS - www.ansys.com, FLUENT (ANSYS Polyflow) - www.fluent.com , ProCAST www.esi-group.com/products/casting/, COMPUPLAST - www.compuplast.com, SYSWELD - www.esi-group.com/products/welding, COMSOL Multiphysics - www.comsol.com, MODFLOW, MODPATH,... www.modflow.com , STAR-CD - www.cd-adapco.com, MOLDFLOW - www.moldflow.com, ... Based on the numerical solution of the underlying partial differential equations (PDE) these virtual software environments offer colorful, animated results in 3D. Numerical dynamic analysis problems are solved both for technical and non-technical disciplines given by numerical models defined in complex 3D shapes. From the viewpoint of systems and control theory these dynamical models represent DPS. A new challenge emerges for the control engineering practice, which is the objective to formulate control problems for dynamical systems defined as DPS through numerical structures over complex spatial structures in 3D.

The main emphasis of this chapter is to present a philosophy of the engineering approach for the control of DPS - given by numerical structures, which opens a wide space for novel applications of the toolboxes and blocksets of the MATLAB & Simulink software environment presented here.

The first monographs in the field of DPS control have been published in the second half of the last century, where mathematical foundations of DPS control have been established. This mathematical theory is based on analytical solutions of the underlying PDE (Butkovskij, 1965; Lions, 1971; Wang, 1964). That is the decomposition of dynamics into time and space components based on the eigenfunctions of the PDE. Recently in the field of mathematical control theory of DPS, publications on control of PDE have appeared (Lasiecka & Triggiani, 2000;...).

An engineering approach for the control of DPS is being developed since the eighties of the last century (Hulkó et al., 1981-2010). In the field of lumped parameters system (LPS) control, where the state/output quantities $\mathbf{x}(t)/\mathbf{y}(t)$ – parameters are given as finite dimensional vectors, the actuator together with the controlled plant make up a controlled LPS. In this sense the actuators and the controlled plant as a DPS create a controlled lumped-input and distributed-parameter-output system (LDS). In this chapter the general decomposition of dynamics of controlled LDS into time and space components is introduced, which is based on numerically computed distributed parameter transient and impulse characteristics given on complex shape definition domains in 3D. Based on this decomposition a methodical framework of control synthesis decomposition into space and time tasks will be presented where in space domain approximation problems are solved and in time domain synthesis of control is realized by lumped parameter control loops. For the software support of modelling, control and design of DPS, the **Distributed Parameter Systems Blockset for MATLAB & Simulink (DPS Blockset)** - www.mathworks.com/products/connections/ has been developed within the CONNECTIONS program framework of The MathWorks, as a Third-Party Product of The MathWorks Company (Hulkó et al., 2003-2010). When solving problems in the time domain, toolboxes and blocksets of the MATLAB & Simulink software environment such as for example the Control Systems Toolbox, Simulink Control Design, System Identification Toolbox, etc. are utilized. In the space relation the approximation task is formulated as an optimization problem, where the Optimization Toolbox is made use of. A web portal named **Distributed Parameter Systems Control** - www.dpscontrol.sk has been created for those interested in solving problems of DPS control (Hulkó et al., 2003-2010). This web portal features application examples from different areas of engineering practice such as the control of technological and manufacturing processes, mechatronic structures, groundwater remediation etc. Moreover this web portal offers the demo version of the **DPS Blockset** with the **Tutorial, Show, Demos** and **DPS Wizard** for download, along with the **Interactive Control** service for the interactive solution of model control problems via the Internet.

2. DPS – DDS – LDS

Generally in the control of lumped parameter systems the actuator and the controlled plant create a lumped parameter controlled system. In the field of DPS control the actuators together with the controlled plant - generally being a distributed-input and distributed-parameter-output system (DDS) create a controlled lumped-input and distributed-parameter-output system (LDS). Fig. 1.-3. and Fig. 6., 11., 14.

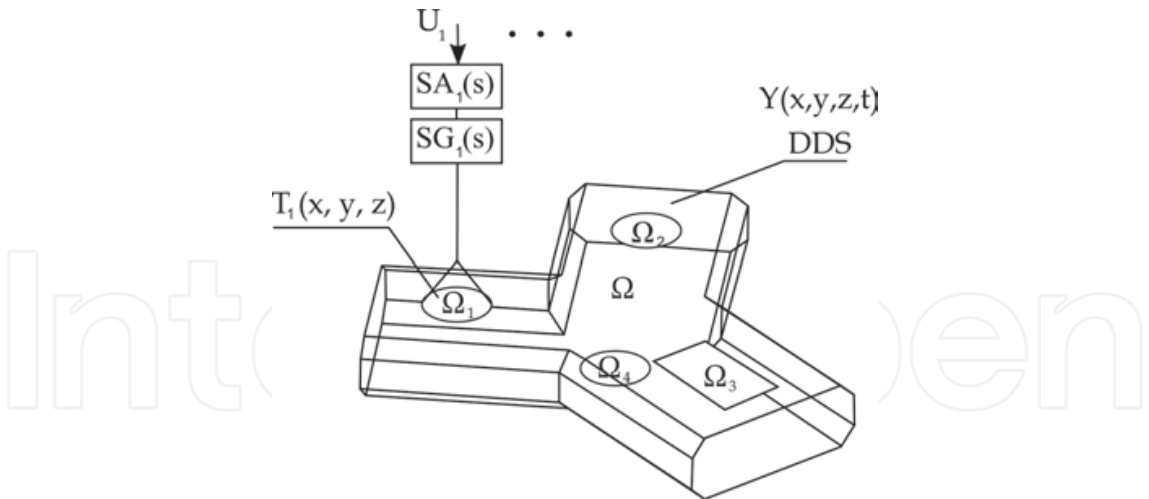


Fig. 1. Controlled DPS as LDS - heating of metal body of complex-shape

- $\{SA_i(s)\}_i, \{SG_i(s)\}_i, \{T_i(x, y, z)\}_i$ - models of actuators
- DDS - distributed-input and distributed-parameter-output system
- $\{U_i\}_i$ - lumped actuating quantities
- $\Omega / \{\Omega_i\}_i$ - complex-shape definition domain in 3D / actuation subdomains
- $Y(x, y, z, t)$ - temperature field - distributed output quantity



Fig. 2. Lumped-input and distributed-parameter-output system - LDS

- $\{U_i(t)\}_i$ - lumped input quantities
- $Y(x, t) = Y(x, y, z, t)$ - distributed output quantity

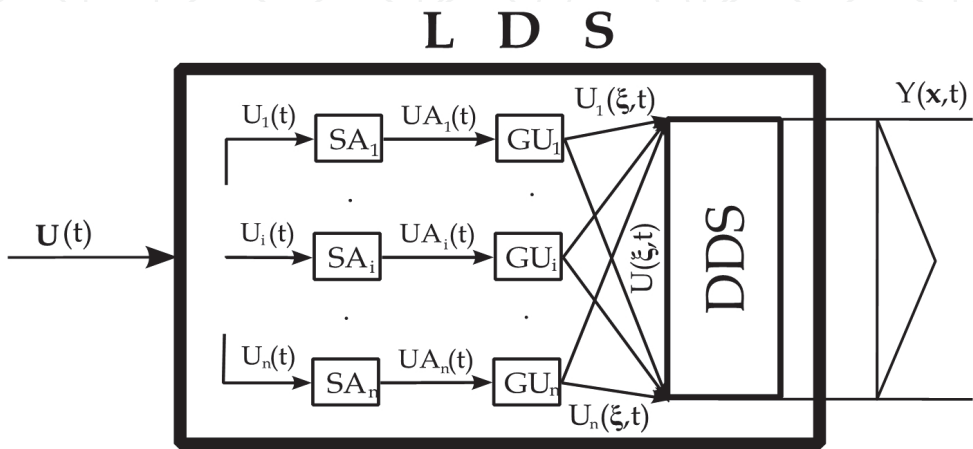


Fig. 3. General structure of lumped-input and distributed-parameter-output systems

LDS - lumped-input and distributed-parameter-output system

$\{SA_i\}_i$ - actuating members of lumped input quantities

$\{GU_i\}_i$ - generators of distributed input quantities

DDS - distributed-input and distributed-parameter-output system

$U(t) = \{U_i(t)\}_i$ - vector of lumped input quantities of LDS

$\{UA_i(t)\}_i$ - output quantities of lumped parameter actuators

$\{U_i(\xi, t)\}_i$ - distributed output quantities of generators $\{GU_i\}_i$

$U(\xi, t)$ - overall distributed input quantity for DDS

$Y(x, t) = Y(x, y, z, t)$ - distributed output quantity

Input-output dynamics of these DPS can be described, from zero initial conditions, by

$$Y(x, t) = \sum_{i=1}^n Y_i(x, t) = \sum_{i=1}^n G_i(x, t) \otimes U_i(t) = \sum_{i=1}^n \int_0^t G_i(x, \tau) U_i(t - \tau) d\tau \quad (1)$$

or in discrete form

$$Y(x, k) = \sum_{i=1}^n Y_i(x, k) = \sum_{i=1}^n \mathcal{G}H_i(x, k) \oplus U_i(k) = \sum_{i=1}^n \sum_{q=0}^k \mathcal{G}H_i(x, q) U_i(k - q) \quad (2)$$

where \otimes marks convolution product and \oplus marks convolution sum, $G_i(x, t)$ - distributed parameter impulse response of LDS to the i -th input, $\mathcal{G}H_i(x, k)$ - discrete time (DT) distributed parameter impulse response of LDS with zero-order hold units H - HLDS to the i -th input, $Y_i(x, t)$ - distributed output quantity of LDS to the i -th input, $Y_i(x, k)$ - DT distributed output quantity of HLDS to the i -th input. For simplicity in this chapter distributed quantities are considered mostly as continuous scalar quantity fields with unit sampling interval in time domain. Whereas DT distributed parameter step responses $\{\mathcal{H}H_i(x, k)\}_{i,k}$ of HLDS can be computed by common analytical or numerical methods then DT distributed parameter impulse responses can be obtained as

$$\{\mathcal{G}H_i(x, k) = \mathcal{H}H_i(x, k) - \mathcal{H}H_i(x, k-1)\}_{i,k} \quad (3)$$

3. Decomposition of dynamics

The process of dynamics decomposition shall be started from DT distributed parameter step and impulse responses of the analysed LDS. For an illustration, procedure of decomposition of dynamics and control synthesis will be shown on the LDS with zero-order hold units H - HLDS - distributed only on the interval $[0, L]$, with output quantity

$$Y(x, k) = \sum_{i=1}^n Y_i(x, k) \text{ discretised in time relation and continuous in space relation on this}$$

interval. Nevertheless the following results are valid in general both for continuous or discrete distributed quantities in space relation given on complex-shape definition domains over 3D as well.

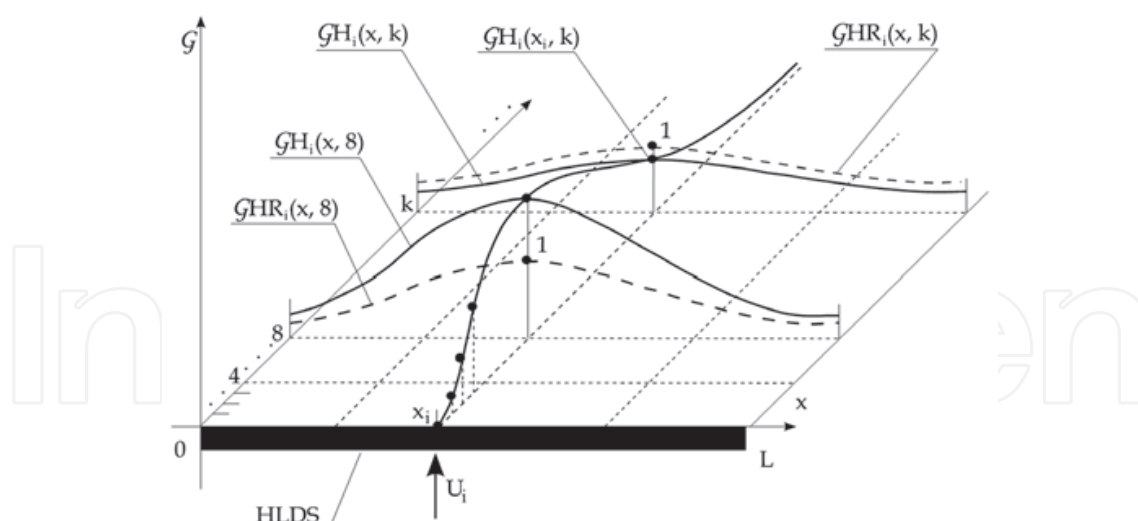


Fig. 4. i -th discrete distributed parameter impulse response of HLDS

$\mathcal{G}_{H_i}(x_i, k)$ - partial DT impulse response in time, t - relation to the i -th input considered as response with maximal amplitude in point $x_i \in [0, L]$

$\{\mathcal{G}_{H_i}(x, k)\}_{i,k}$ - partial DT impulse responses to the i -th input in space - x - relation

$\{\mathcal{G}_{HR_i}(x, k)\}_{i,k}$ - reduced partial DT impulse responses to the i -th input in space, x - relation for timesteps $\{k\}_k$

If the reduced DT partial distributed parameter impulse responses are defined as

$$\left\{ \mathcal{G}_{HR_i}(x, k) = \frac{\mathcal{G}_{H_i}(x, k)}{\mathcal{G}_{H_i}(x_i, k)} \right\}_{i,k} \quad (4)$$

for $\{\mathcal{G}_{H_i}(x_i, k) \neq 0\}_{i,k}$, then the i -th DT distributed output quantity in (2) can be rewritten by the means of the reduced characteristics as follows

$$Y_i(x, k) = \mathcal{G}_{H_i}(x_i, k) \mathcal{G}_{HR_i}(x, k) \oplus U_i(k) \quad (5)$$

At fixed x_i the partial DT distributed output quantity in time direction $Y_i(x_i, k)$ is given as the convolution sum $\mathcal{G}_{H_i}(x_i, k) \oplus U_i(k) = \sum_{q=0}^k \mathcal{G}_{H_i}(x_i, q) U_i(k-q)$, in case the relation

$\{\mathcal{G}_{HR_i}(x_i, q) = 1\}_{q=0,k}$ holds at the fixed point x_i . At fixed k , the partial discrete distributed output quantity in space direction $Y_i(x, k)$ is given as a linear combination of elements $\{\mathcal{G}_{HR_i}(x, q)\}_{q=0,k}$, where the reduced discrete partial distributed characteristics $\{\mathcal{G}_{HR_i}(x, q)\}_{q=0,k}$ are multiplied by corresponding elements of the set $\{\mathcal{G}_{H_i}(x_i, q) U_i(k-q)\}_{q=0,k}$, see Fig. 5.

This decomposition is valid for all given lumped input $\{U_i\}_i$ and corresponding output quantities $\{Y_i(x,k)\}_{i,k}$ - thus we obtain time and space components of HLDS dynamics in the following form:

Time Components of Dynamics $\{gH_i(x_i,k)\}_{i,k}$ - for given i and chosen x_i - variable k

Space Components of Dynamics $\{gHR_i(x,k)\}_{i,k}$ - for given i and chosen k - variable x

Also reduced components of single distributed output quantities are

$$\{YR_i(x,k)\}_i = \left\{ \frac{Y_i(x,k)}{Y_i(x_i,k)} \right\}_{i,k} \quad (6)$$

then $\{Y_i(x_i,k) \neq 0\}_{i,k}$ can be considered as time components and $\{YR_i(x,k)\}_{i,k}$ as space components of the output quantities.

When reduced steady-state distributed parameter transient responses are introduced $\{\mathcal{H}R_i(x,\infty)\}_i = \{\mathcal{H}H_i(x,\infty) / \mathcal{H}H_i(x_i,\infty)\}_i$ - for $\{\mathcal{H}H_i(x_i,\infty) \neq 0\}_i$ - and discrete transfer functions $\{SH_i(x_i,z)\}_i$ are assigned to partial distributed parameter transient responses with maximal amplitudes at points $\{x_i\}_i$ on the interval $[0,L]$, we obtain time and space components of HLDS dynamics for steady-state as:

Time Components of Dynamics $\{SH_i(x_i,z)\}_i$ - for given i and chosen x_i - variable z

Space Components of Dynamics $\{\mathcal{H}R_i(x,\infty)\}_i$ - for given i in ∞ - variable x

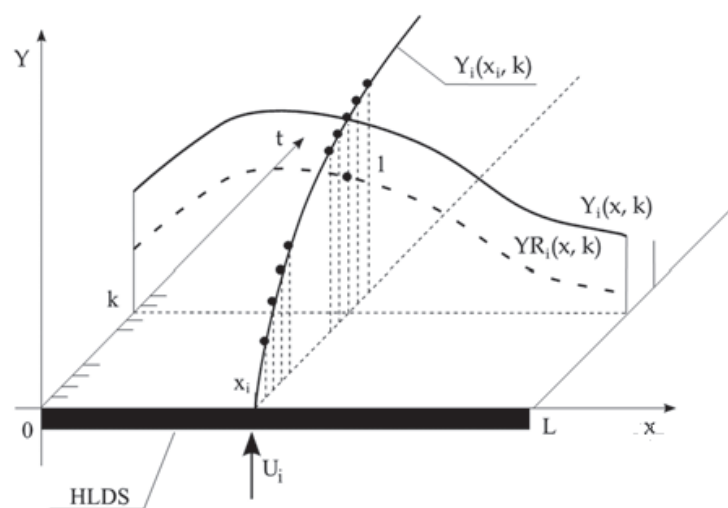


Fig. 5. Partial distributed output quantities in time and space direction

U_i - i -th DT lumped input quantity

$Y_i(x_i, k)$ - i -th partial DT distributed output quantity in time domain at chosen point x_i

$Y_i(x, k) / YR_i(x, k)$ - i -th partial distributed output/reduced output quantity in space direction

For the steady-state, when $k \rightarrow \infty$

$$\{YR_i(x, \infty) \rightarrow \mathcal{H}R_i(x, \infty)\}_{i,k} \quad (7)$$

then in the steady-state

$$Y(x, \infty) = \sum_{i=1}^n Y_i(x_i, \infty) YR_i(x, \infty) = \sum_{i=1}^n Y_i(x_i, \infty) \mathcal{H}R_i(x, \infty) \quad (8)$$

When distributed quantities are used in discrete form as finite sequences of quantities, the discretization in space domain is usually considered by the computational nodes of the numerical model of the controlled DPS over the complex-shape definition domain in 3D.

4. Distributed parameter systems of control

Based on decomposition of HLDS dynamics into time and space components, possibilities for control synthesis are also suggested by an analogous approach. In this section a methodical framework for the decomposition of control synthesis into space and time problems will be presented by select demonstration control problems. In the space domain control synthesis will be solved as a sequence of approximation tasks on the set of space components of controlled system dynamics, where distributed parameter quantities in particular sampling times are considered as continuous functions on the interval $[0, L]$ as elements of strictly convex normed linear space \mathbf{X} with quadratic norm. It is necessary to note as above that the following results are valid in general for DPS given on complex-shape definition domains in 3D both for continuous or discrete distributed quantities, in the space relation as well.

In the time domain, the control synthesis solutions are based on synthesis methods of DT lumped parameter systems of control.

4.1 Open-loop control

Assume the open-loop control of a distributed parameter system, where dynamic characteristics give an ideal representation of controlled system dynamics and $V(x, t) = 0$, that is with zero initial steady-state, in which all variables involved are equal to zero – see Fig. 6 for reference. Let us consider a step change of distributed reference quantity – $W(x, k) = W(x, \infty)$, see Fig. 7. For simplicity let the goal of the control synthesis is to generate a sequence of control inputs $\bar{U}(k)$ in such fashion that in the steady-state, for $k \rightarrow \infty$, the control error $E(x, k) = W(x, \infty) - Y(x, k)$ will approach its minimal value $\check{E}(x, \infty)$ in the quadratic norm:

$$\min \|E(x, \infty)\| = \min \|W(x, \infty) - Y(x, \infty)\| = \|\check{E}(x, \infty)\| \quad (9)$$

First, an approximation problem will be solved in the space synthesis (SS) block:

$$\min \left\| W(x, \infty) - \sum_{i=1}^n W_i(x_i, \infty) \mathcal{H}R_i(x, \infty) \right\| \quad (10)$$

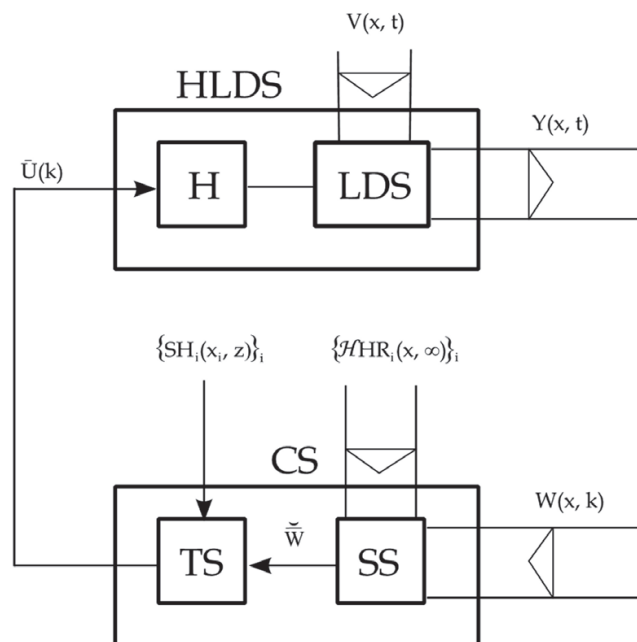


Fig. 6. Distributed parameter open-loop system of control

LDS - lumped-input and distributed-parameter-output system

H - zero-order hold units

HLDS - controlled system with zero-order hold units

CS - control synthesis

TS - time part of control synthesis

SS - space part of control synthesis

$Y(x, t) / W(x, k) = W(x, \infty)$ - distributed controlled/reference quantity

$V(x, t)$ - distributed disturbance quantity

$\tilde{W} = \{\tilde{W}_i(x_i, \infty)\}_i$ - vector of lumped reference quantities

$\bar{U}(k)$ - vector of lumped control quantities

$\{SH_i(x_i, z) / \mathcal{H}HR_i(x, \infty)\}_i$ - time/space components of controlled system dynamics

where $\{\mathcal{H}HR_i(x, \infty)\}_i$ are steady-state reduced distributed parameter transient responses of the controlled system – HLDS and $\{W_i(x_i, \infty)\}_i$ are parameters of approximation. Functions

$\{\mathcal{H}HR_i(x, \infty)\}_i$ form a finite-dimensional subspace of approximation functions F_n in the

strictly convex normed linear space of distributed parameter quantities X on $[0, L]$ with quadratic norm, where the approximation problem is to be solved, see Fig. 8. for reference.

From approximation theory in this relation is known the theorem:

Let F_n be a finite-dimensional subspace of a strictly convex normed linear space X . Then, for each $f \in X$, there exists a unique element of best approximation.

(Shadrin, 2005). So the solution of the approximation problem (10) is guaranteed as a unique

best approximation $\tilde{W}O(x, \infty) = \sum_{i=1}^n \tilde{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty)$, where $\tilde{W} = \{\tilde{W}_i(x_i, \infty)\}_i$ is the

vector of optimal approximation parameters. Hence:

$$\begin{aligned} \min & \left\| W(x, \infty) - \sum_{i=1}^n W_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\| = \\ & \left\| W(x, \infty) - \sum_{i=1}^n \check{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\| = \left\| W(x, \infty) - \check{W}O(x, \infty) \right\| \end{aligned} \tag{11}$$

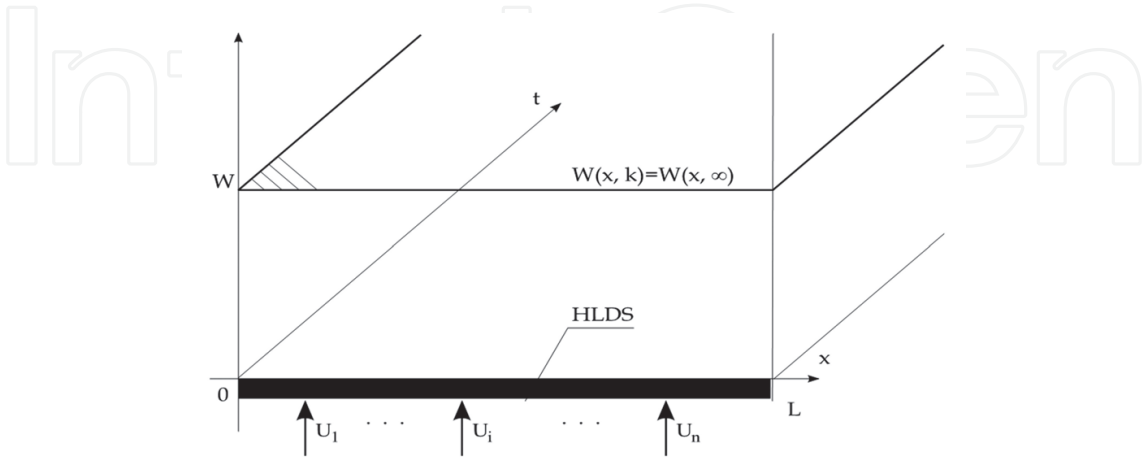


Fig. 7. Step change of distributed reference quantity

- HLDS - controlled system with zero-order hold units
- $\{U_i\}_i$ - lumped control quantities
- $W(x,k) = W(x,\infty)$ - step change of distributed reference quantity

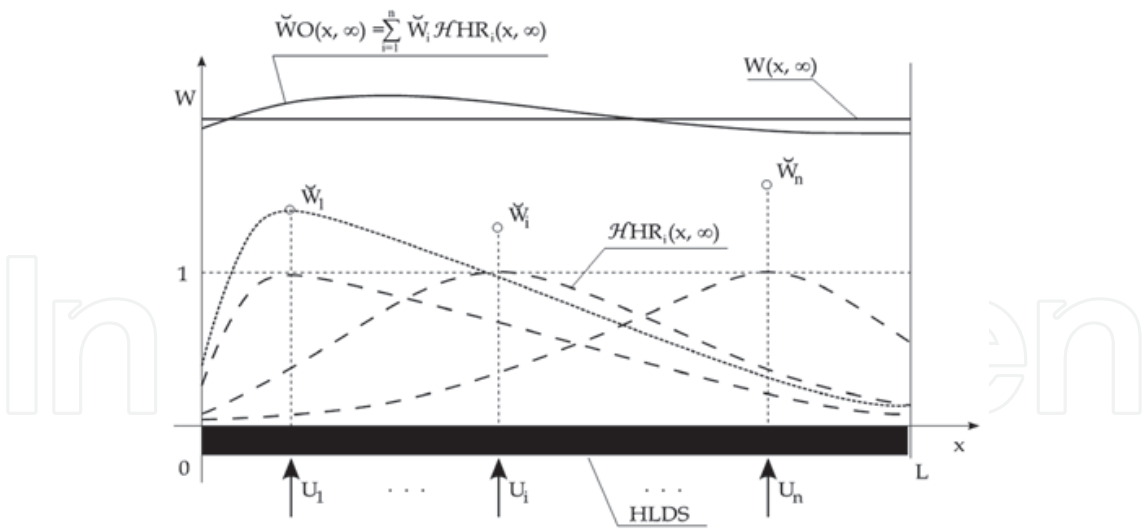


Fig. 8. Solution of the approximation problem

- HLDS - controlled system with zero-order hold units
- $\{U_i\}_i$ - lumped control quantities
- $\{\check{W}_i\}_i = \{\check{W}_i(x_i, \infty)\}_i$ - optimal approximation parameters, lumped references
- $\{\mathcal{H}HR_i(x, \infty)\}_i$ - reduced steady-state distributed parameter transient responses

$W(x, \infty)$ - distributed reference quantity
 $\check{W}O(x, \infty)$ - unique best approximation of reference quantity

Let us assume vector $\check{\check{W}}$ enters the block of time synthesis (TS). In this block, there are „n“ single-input /single-output (SISO) lumped parameter control loops: $\{SH_i(x_i, z), R_i(z)\}_i$, see Fig. 9. for reference. The controlled systems of these loops are lumped parameter systems assigned to HLDS as time components of dynamics: $\{SH_i(x_i, z)\}_i$. Controllers, $\{R_i(z)\}_i$, are to be chosen such that for $k \rightarrow \infty$ the following relation holds:

$$\left\{ \lim_{k \rightarrow \infty} \check{E}_i(x_i, k) = \lim_{k \rightarrow \infty} [\check{W}_i(x_i, \infty) - Y_i(x_i, k)] = 0 \right\}_{i,k}$$

(12)

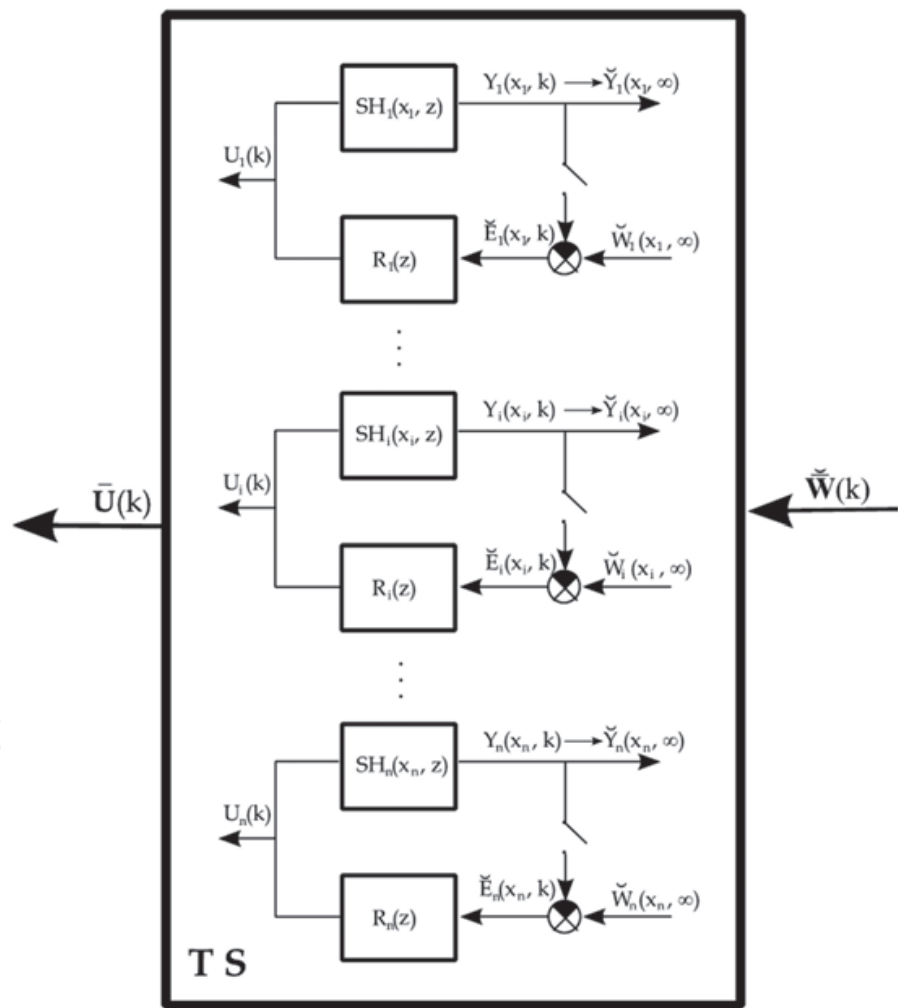


Fig. 9. SISO lumped parameter control loops in the block TS

TS - time part of control synthesis
 $\{SH_i(x_i, z)\}_i$ - time components of HLDS dynamics
 $\{R_i(z)\}_i$ - lumped parameter controllers

$\{Y_i(x_i, k)\}_{i,k} / \{\tilde{W}_i\}_i = \{\tilde{W}_i(x_i, \infty)\}_i$ - controlled/reference quantities

$\{\tilde{E}_i(x_i, k)\}_{i,k}$ - lumped control errors

$\{U_i(k)\}_{i,k}$ - lumped control quantities

$\tilde{W}(k) / \bar{U}(k)$ - vector of lumped reference/control quantities

When the individual components of the vector $\tilde{W} = \{\tilde{W}_i(x_i, \infty)\}_i$ are input in the particular control loops: $\{SH_i(x_i, z), R_i(z)\}_i$, the control processes take place. The applied control laws result in the sequences of control inputs: $\{U_i(k)\}_{i,k}$, and respectively the output quantities, for $k \rightarrow \infty$ converging to reference quantities

$$\{Y_i(x_i, k) \rightarrow Y_i(x_i, \infty) = \tilde{W}_i(x_i, \infty)\}_{i,k} \quad (13)$$

Values of these lumped controlled quantities in new steady-state will be further denoted as

$$\{\tilde{Y}_i(x_i, \infty) = \tilde{W}_i(x_i, \infty)\}_i \quad (14)$$

see Fig. 9. – 10. for reference.

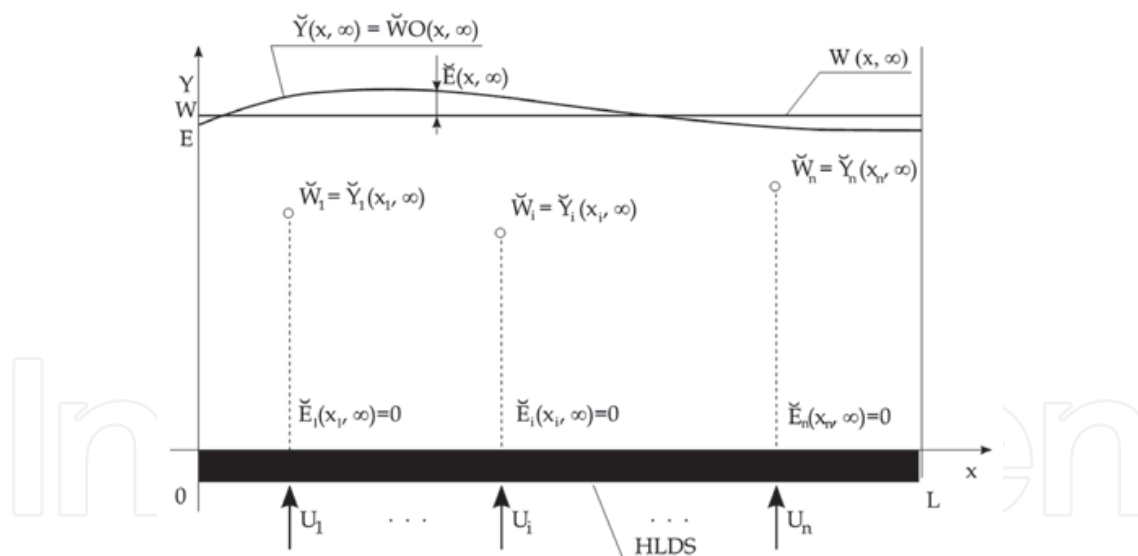


Fig. 10. Quantities of distributed parameter open-loop control in new steady-state

HLDS - controlled system with zero-order hold units

$\{U_i\}_i$ - lumped control quantities

$\{\tilde{Y}_i(x_i, \infty) / \tilde{W}_i = \tilde{W}_i(x_i, \infty)\}_i$ - controlled/reference quantities in new steady-state

$\{\tilde{E}_i(x_i, \infty)\}_i$ - lumped control errors

$\tilde{Y}(x, \infty)$ - controlled distributed quantity in new steady-state

$W(x, \infty)$ - distributed reference quantity

$\tilde{W}O(x, \infty)$ - unique best approximation of reference quantity

$\tilde{E}(x, \infty)$ - distributed control error with minimal norm

Then according to equations (12-14) for the new steady-state it holds: $\{\tilde{Y}_i(x_i, \infty) \mathcal{H}R_i(x, \infty) = \tilde{W}_i(x_i, \infty) \mathcal{H}R_i(x, \infty)\}_i$, which implies that the overall distributed output quantity at the time $k \rightarrow \infty$: $\tilde{Y}(x, \infty)$ gives the unique best approximation of the distributed reference variable: $W(x, \infty)$

$$\tilde{Y}(x, \infty) = \sum_{i=1}^n \tilde{Y}_i(x_i, \infty) \mathcal{H}R_i(x, \infty) = \sum_{i=1}^n \tilde{W}_i(x_i, \infty) \mathcal{H}R_i(x, \infty) = \tilde{W}O(x, \infty) \quad (15)$$

Therefore the control error has a unique form as well, with minimal quadratic norm

$$\begin{aligned} \|\tilde{E}(x, \infty)\| &= \|W(x, \infty) - \tilde{Y}(x, \infty)\| = \left\| W(x, \infty) - \sum_{i=1}^n \tilde{Y}_i(x_i, \infty) \mathcal{H}R_i(x, \infty) \right\| = \\ &= \left\| W(x, \infty) - \sum_{i=1}^n \tilde{W}_i(x_i, \infty) \mathcal{H}R_i(x, \infty) \right\| = \|W(x, \infty) - \tilde{W}O(x, \infty)\| \end{aligned} \quad (16)$$

Thus the control task, defined at equation (9), is accomplished with $\{\tilde{E}_i(x_i, \infty) = 0\}_i$ - see Fig. 10. for reference. As the conclusion of this section we may state that the control synthesis in open-loop control system is realized as:

Time Tasks of Control Synthesis - in lumped parameter control loops

Space Tasks of Control Synthesis - as approximation task.

When mathematical models cannot provide an ideal representation of controlled DPS dynamics and disturbances are present with an overall effect on the output in steady-state expressed by $EY(x, \infty)$ then

$$\begin{aligned} \|W(x, \infty) - \tilde{Y}(x, \infty) \pm EY(x, \infty)\| &\leq \|W(x, \infty) - \tilde{Y}(x, \infty)\| + \|EY(x, \infty)\| = \\ \|W(x, \infty) - \tilde{W}O(x, \infty)\| + \|EY(x, \infty)\| &= \|\tilde{E}(x, \infty)\| + \|EY(x, \infty)\| \end{aligned} \quad (17)$$

Finally at the design stage of a control system, for a given desired quality of control δ in space domain, it is necessary to choose appropriate number and layout of actuators for the fulfillment of this requirement

$$\|\tilde{E}(x, \infty)\| + \|EY(x, \infty)\| \leq \delta \quad (18)$$

4.2 Closed-loop control with block RHLDS

Let us consider now a distributed parameter feedback control loop with initial conditions identical as the case above, see Fig. 11. In blocks SS1 a SS2 approximation problems are solved while in block RHLDS reduced distributed output quantities $\{YR_i(x, k)\}_{i,k}$ are

generated. Block TS in Fig. 12., contains the controllers $\{R_i(z)\}_i$ designed as the controllers for SISO lumped parameter control loops $\{SH_i(x_i,z),R_i(z)\}_i$ with respect to the request formulated by equation (12). In the k-th step in block SS2 at approximation of $Y(x,k)$ on the subspace of $\{YR_i(x,k)\}_{i,k}$

$$\min \left\| Y(x,k) - \sum_{i=1}^n Y_i(x_i,k) YR_i(x,k) \right\| \tag{19}$$

time components of partial output quantities $\{\tilde{Y}_i(x_i,k)\}_i$ are obtained, in block SS1 reference quantities $\{\tilde{W}_i(x_i,\infty)\}_i$ are computed. Then on the output of the algebraic block is $\{\tilde{E}_i(x_i,k) = \tilde{W}_i(x_i,\infty) - \tilde{Y}_i(x_i,k)\}_i$. These sequences $\{\tilde{E}_i(x_i,k)\}_i$ enter into the TS on $\{R_i(z)\}_i$ and give $\{U_i(k)\}_i$, which enter into HLDS with $\{\tilde{Y}_i(x_i,k)\}_i$ on the SS2 output - among $\{U_i(k)\}_i$ and $\{\tilde{Y}_i(x_i,k)\}_i$ there are relations $\{SH_i(x_i,z)\}_i$. - This analysis of control synthesis process shows that synthesis in time domain is realized on the level of one parameter control loops $\{SH_i(x_i,z),R_i(z)\}_i$, Fig. 9.

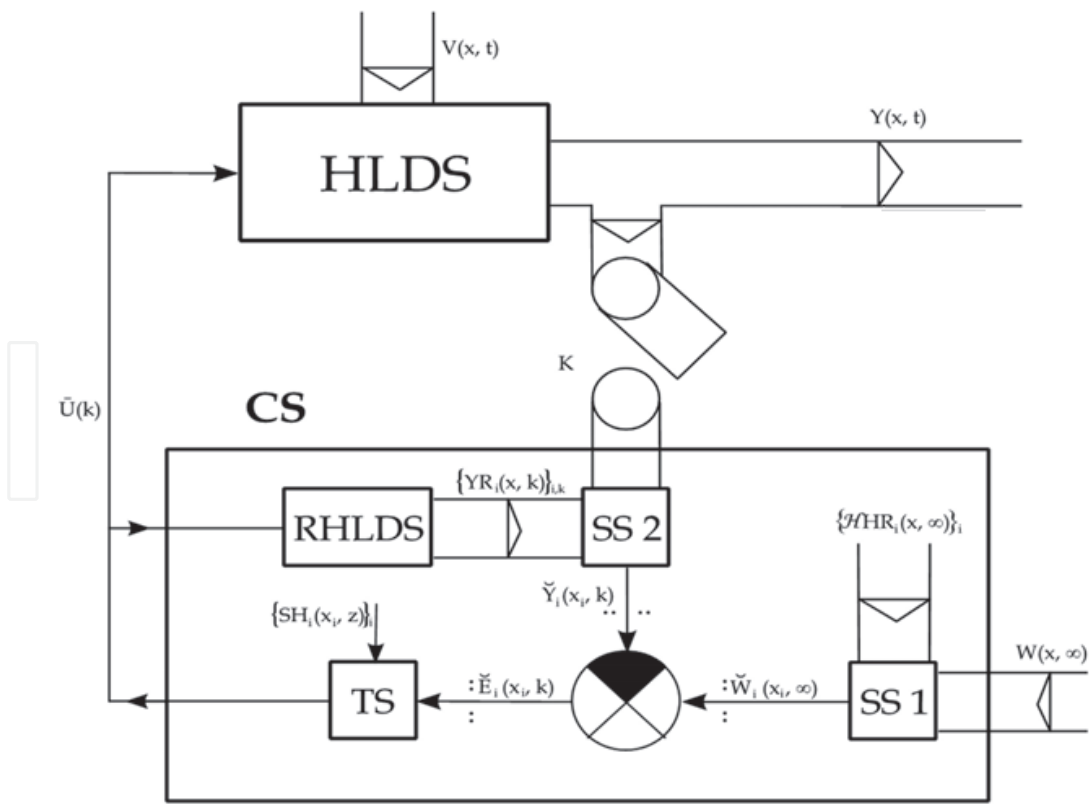


Fig. 11. Distributed parameter closed-loop system of control with reduced space components of output quantity

HLDS - controlled system with zero-order hold units

RHLDS - model for reduced space components: $\{YR_i(x, k)\}_{i,k}$

CS - control synthesis

TS/SS1, SS2 - time/space parts of control synthesis

K - time/space sampling

$Y(x, t)$ - distributed output quantity

$W(x, \infty), V(x, t)$ - reference and disturbance quantities

$\{Y_i(x_i, k)\}_{i,k} = \{\tilde{Y}_i(x_i, k)\}_{i,k}$ - time components of output quantity

$\{\tilde{W}_i = \tilde{W}_i(x_i, \infty)\}_i / \{\tilde{E}_i(x_i, k)\}_{i,k}$ - reference quantities/control errors

$\bar{U}(k)$ - vector of lumped control quantities

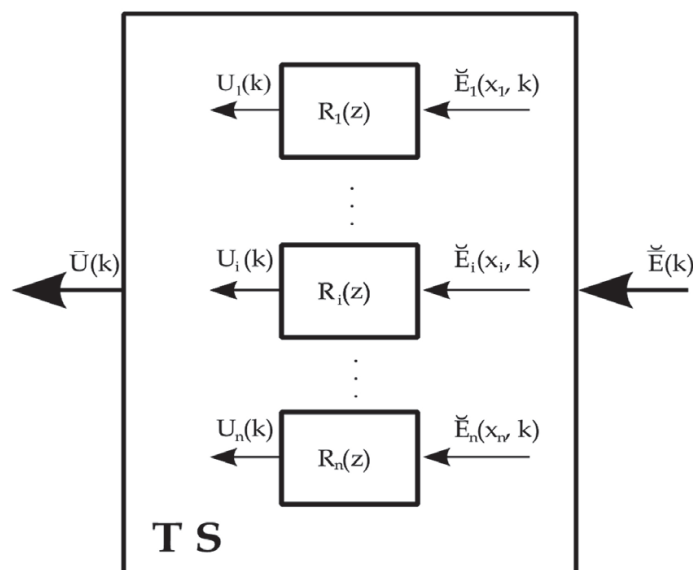


Fig. 12. The block of time synthesis

TS - time part of control synthesis

$\{R_i(z)\}_i$ - lumped parameter controllers

$\tilde{E}(k) = \{\tilde{E}_i(x_i, k)\}_{i,k}$ - vector of lumped control errors

$\bar{U}(k) = \{U_i(k)\}_{i,k}$ - vector of lumped control quantities

For $k \rightarrow \infty$ $\{YR_i(x, k) \rightarrow \mathcal{H}R_i(x, \infty)\}_{i,k}$, $\{\tilde{Y}_i(x_i, k)\}_{i,k} \rightarrow \{\tilde{Y}_i(x_i, \infty) = \tilde{W}_i(x_i, \infty)\}_i$ along with

$\{\tilde{E}_i(x_i, k) \rightarrow \tilde{E}_i(x_i, \infty) = 0\}_{i,k}$. Thus the control task, defined by equation (9) is accomplished

as given by relation (16). In case of the uncertainty of the control process relations similar to (17-18) are also valid.

Let's consider now the approximation of $W(x, \infty)$ in the block SS1 in timestep k , on the set of $\{YR_i(x, k)\}_{i,k}$. Then in the control process sequences of quantities $\{\tilde{W}_i(x_i, k)\}_{i,k}$ are obtained, as desired quantities of SISO control loops $\{SH_i(x_i, z), R_i(z)\}_i$ which are closed throughout the blocks TS, HLDS and SS2, see Fig. 13.

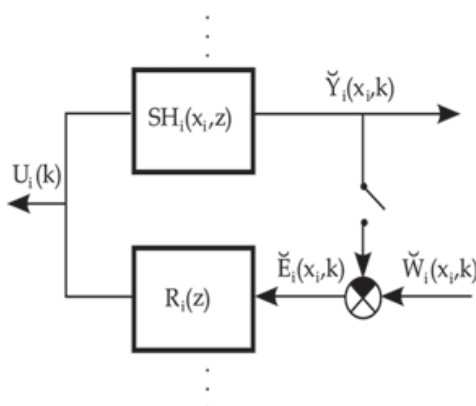


Fig. 13. Lumped parameter SISO control loops – i-th control loop

$SH_i(x_i, z)$ - i-th time component of HLDS dynamics

$R_i(z)$ - i-th lumped parameter controller

$\check{Y}_i(x_i, k) / U_i(k)$ - i-th controlled/control quantity

$\check{W}_i(x_i, k) / \check{E}_i(x_i, k)$ - i-th desired quantity/control error

If for $k \rightarrow \infty \{YR_i(x, k) \rightarrow \mathcal{H}R_i(x, \infty)\}_{i,k}, \{Y_i(x_i, k) = \check{Y}_i(x_i, k)\}_{i,k} \rightarrow \{\check{Y}_i(x_i, \infty) = \check{W}_i(x_i, \infty)\}_i$

along with $\{\check{E}_i(x_i, k) \rightarrow \check{E}_i(x_i, \infty) = 0\}_{i,k}$ - this actually means that the control task defined in equation (9), is accomplished as given by relation (16).

Finally we may state as a summary, that in closed-loop control with RHLDS the control synthesis is realized as:

Time Tasks of Control Synthesis – on the level of lumped parameter control loops

Space Tasks of Control Synthesis – as approximation tasks.

At the same time the solution of the approximation problem in block SS1 on the approximation set $\{YR_i(x, k)\}_{i,k}$

$$\min \left\| W(x, \infty) - \sum_{i=1}^n W_i(x_i, k) YR_i(x, k) \right\| \quad (20)$$

in timestep k is obtained

$$W(x, \infty) = \sum_{i=1}^n \check{W}_i(x_i, k) YR_i(x, k) + \check{E}(x, k) \quad (21)$$

where $\check{E}(x, k)$ is the unique element at the best approximation of $W(x, \infty)$ on the set of approximate functions $\{YR_i(x, k)\}_i$. Similarly by the solution of approximation problem in the block SS2 - $\{\check{Y}_i(x_i, k)\}_i$ in the timestep k distributed output quantity $Y(x, k)$ is given as

$$Y(x, k) = \sum_{i=1}^n \check{Y}_i(x_i, k) YR_i(x, k) \quad (22)$$

4.3 Closed-loop control

Let us now consider a distributed parameter feedback loop as featured in Fig. 14. with initial conditions as above, where in timestep k an approximation problem is solved

$$\min \left\| E(x, k) - \sum_{i=1}^n E_i(x_i, k) YR_i(x, k) \right\| \quad (23)$$

and as a result in timestep k a vector $\tilde{\tilde{E}}(k) = \{\tilde{\tilde{E}}_i(x_i, k)\}_i$ is obtained. By relations (20-22) the further equations are valid

$$\begin{aligned} \min \left\| E(x, k) - \sum_{i=1}^n E_i(x_i, k) YR_i(x, k) \right\| &= \\ &= \min \left\| W(x, \infty) - Y(x, k) - \sum_{i=1}^n E_i(x_i, k) YR_i(x, k) \right\| = \\ &= \min \left\| W(x, \infty) - \sum_{i=1}^n [\tilde{Y}_i(x_i, k) + E_i(x_i, k)] YR_i(x, k) \right\| = \\ &= \left\| W(x, \infty) - \sum_{i=1}^n [\tilde{Y}_i(x_i, k) + \tilde{\tilde{E}}_i(x_i, k)] YR_i(x, k) \right\| \end{aligned} \quad (24)$$

The problem solution $W(x, \infty) = \sum_{i=1}^n [\tilde{Y}_i(x_i, k) + \tilde{\tilde{E}}_i(x_i, k)] YR_i(x, k) + \tilde{\tilde{E}}(x, k)$ is obtained by approximation.

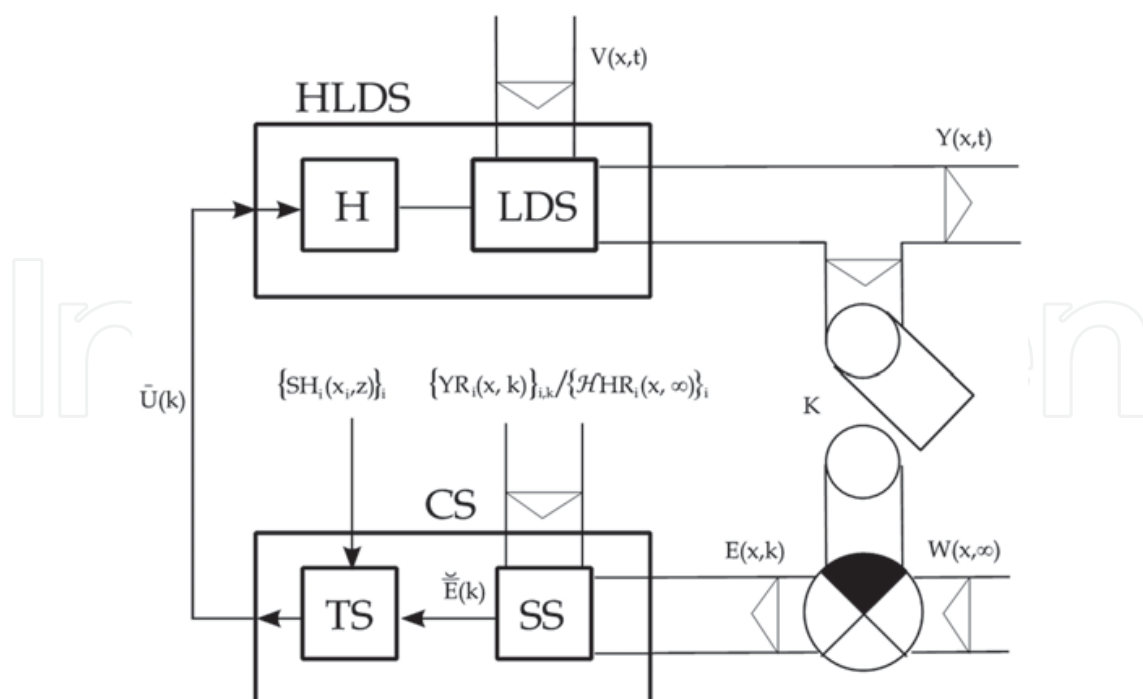


Fig. 14. Distributed parameter closed-loop system of control

HLDS - LDS with zero-order hold units

CS – control synthesis

TS /SS – time/space control synthesis

K – time/space sampling

$Y(x, t) / W(x, \infty)$ – distributed controlled/reference quantity

$\{SH_i(x_i, z)\}_i$ – transfer functions - dynamic characteristics of HLDS in time domain

$\{YR_i(x, k)\}_i / \{HHR_i(x, \infty)\}_i$ – reduced characteristics in space domain

$E(x, k)$ – distributed control error

$V(x, t)$ – distributed disturbance quantity

$\tilde{E}(k)$ – vector of control errors

$\bar{U}(k)$ – vector of control quantities

Comparison of relation (21) and result of the approximation problem (24) gives

$$\sum_{i=1}^n \tilde{W}_i(x_i, k) YR_i(x, k) + \tilde{E}(x, k) = \sum_{i=1}^n [\tilde{Y}_i(x_i, k) + \tilde{E}_i(x_i, k)] YR_i(x, k) + \tilde{E}(x, k) \quad \text{and then}$$

$$\sum_{i=1}^n \tilde{W}_i(x_i, k) YR_i(x, k) = \sum_{i=1}^n [\tilde{Y}_i(x_i, k) + \tilde{E}_i(x_i, k)] YR_i(x, k), \quad \text{finally}$$

$\{\tilde{W}_i(x_i, k) = \tilde{Y}_i(x_i, k) + \tilde{E}_i(x_i, k)\}_{i,k}$ is obtained. Now in vector form this means

$$\tilde{W}(k) = \tilde{Y}(k) + \tilde{E}(k) \Rightarrow \tilde{E}(k) = \tilde{W}(k) - \tilde{Y}(k) = \{\tilde{E}_i(x_i, k) = \tilde{W}_i(x_i, k) - \tilde{Y}_i(x_i, k)\}_{i,k} \quad (25)$$

Then sequences $\{\tilde{E}_i(x_i, k)\}_{i,k}$ enter into the block TS on $\{R_i(z)\}_i$ and give $\{U_i(k)\}_i$, among $\{U_i(k)\}_i$ and $\{\tilde{Y}_i(x_i, k)\}_i$ there are relations $\{SH_i(x_i, z)\}_i$. Finally this analysis of control synthesis process shows that synthesis in time domain is realized on the level of one parameter control loops $\{SH_i(x_i, z), R_i(z)\}_i$, Fig. 13. - closed throughout the structure of distributed parameter control loop, Fig. 14. If for $k \rightarrow \infty$ $\{YR_i(x, k) \rightarrow HHR_i(x, \infty)\}_i$ and $\{\tilde{Y}_i(x_i, k)\}_{i,k} \rightarrow \{\tilde{Y}_i(x_i, \infty) = \tilde{W}_i(x_i, \infty)\}_i$ along with $\{\tilde{E}_i(x_i, k) \rightarrow \tilde{E}_i(x_i, \infty) = 0\}_{i,k}$ then

$$\begin{aligned} \min \left\| E(x, \infty) - \sum_{i=1}^n \tilde{E}_i(x_i, \infty) HHR_i(x, \infty) \right\| &= \left\| W(x, \infty) - \tilde{Y}(x, \infty) - \sum_{i=1}^n \tilde{E}_i(x_i, \infty) HHR_i(x, \infty) \right\| = \\ &= \left\| \sum_{i=1}^n \tilde{W}_i(x_i, \infty) HHR_i(x, \infty) + \tilde{E}(x, \infty) - \sum_{i=1}^n \tilde{Y}_i(x_i, \infty) HHR_i(x, \infty) - \sum_{i=1}^n \tilde{E}_i(x_i, \infty) HHR_i(x, \infty) \right\| = (26) \\ &= \left\| \sum_{i=1}^n [\tilde{W}_i(x_i, \infty) - \tilde{Y}_i(x_i, \infty)] HHR_i(x, \infty) + \tilde{E}(x, \infty) - \sum_{i=1}^n \tilde{E}_i(x_i, \infty) HHR_i(x, \infty) \right\| = \left\| \tilde{E}(x, \infty) \right\| \end{aligned}$$

is valid, so the above given control task (9) is accomplished - whereas in the steady-state $\{\tilde{W}_i(x_i, \infty) - \tilde{Y}_i(x_i, \infty) = \tilde{E}_i(x_i, \infty) = 0\}_i$. By concluding the above presented discussion, the control synthesis in closed-loop control is realized as:

Time Tasks of Control Synthesis – on the level of lumped parameter control loops

Space Tasks of Control Synthesis – as approximation tasks.

When mathematical models cannot provide an ideal description of controlled DPS dynamics and disturbances are present with an overall effect on the output in steady-state, expressed by $EY(x, \infty)$ then the relations similar to (17-18) are also valid here.

In practice mostly only reduced distributed parameter transient responses in steady-state $\{HHR_i(x, \infty)\}_i$ are considered for the solution of the approximation tasks in the block SS of the scheme in Fig. 14. along with robustification of controllers $\{R_i(z)\}_i$.

For simplicity problems of DPS control have been formulated here for the distributed desired quantity $W(x, \infty)$. In case of $W(x, k)$ is assumed, the control synthesis is realized similarly:

- *In Space Domain* - as problem of approximation in particular sampling intervals
- *In Time Domain* - as control synthesis in lumped parameter control loops, closed throughout structures of the distributed parameter control loop.

The solution of the presented problems of control synthesis an assumption is used, that in the framework of the chosen control systems the prescribed control quality can be reached both in the in space and time domain. However in the design of actual control systems for the given distributed parameter systems, usually the

- optimization of the number and layout of actuators
- optimization of dynamical characteristics of lumped/distributed parameter actuators
- optimization of dynamical characteristics of lumped parameter control loops

is required and necessary.

5. Distributed Parameter Systems Blockset for MATLAB & Simulink

As a software support for DPS modelling, control and design of problems in MATLAB & Simulink the programming environment **Distributed Parameter Systems Blockset for MATLAB & Simulink (DPS Blockset)** - a Third-Party Product of The MathWorks www.mathworks.com/products/connections/ - Fig. 15., has been developed within the program CONNECTIONS of The MathWorks Corporation by the Institute of Automation, Measurement and Applied Informatics of Mechanical Engineering Faculty, Slovak University of Technology in Bratislava (IAMAI-MEF-STU) (Hulkó et al., 2003-2010). Fig. 16. shows The library of **DPS Blockset**. The **HLDS** and **RHLDS** blocks model controlled DPS dynamics described by numerical structures as LDS with zero-order hold units - H. **DPS Control Synthesis** provides feedback to distributed parameter controlled systems in control loops with blocks for discrete-time **PID, Algebraic, State-Space and Robust Synthesis**. The block **DPS Input** generates distributed quantities, which can be used as distributed reference quantities or distributed disturbances, etc. **DPS Display** presents distributed quantities with many options including export to AVI files. The block **DPS Space Synthesis** performs space synthesis as an approximation problem.

As a demonstration, some results of the discrete-time PID control of complex-shape metal body heating by the **DPS Blockset** are shown in Fig. 17.-19., where the heating process was modelled by finite element method in the COMSOL Multiphysics virtual software environment - www.comsol.com.

The block **Tutorial** presents methodological framework for formulation and solution of control problems for DPS. The block **Show** contains motivation examples such as: *Control of temperature field of 3D metal body* (the controlled system was modelled in the virtual software environment COMSOL Multiphysics); *Control of 3D beam of „smart“ structure* (the controlled system was modelled in the virtual software environment ANSYS); *Adaptive control of glass furnace* (the controlled system was modelled by Partial Differential Equations Toolbox of the MATLAB), and *Groundwater remediation control* (the controlled system was modelled in the virtual software environment MODFLOW). The block **Demos** contains examples oriented at the methodology of modelling and control synthesis. The **DPS Wizard** gives an automatized guide for arrangement and setting distributed parameter control loops in step-by-step operation.

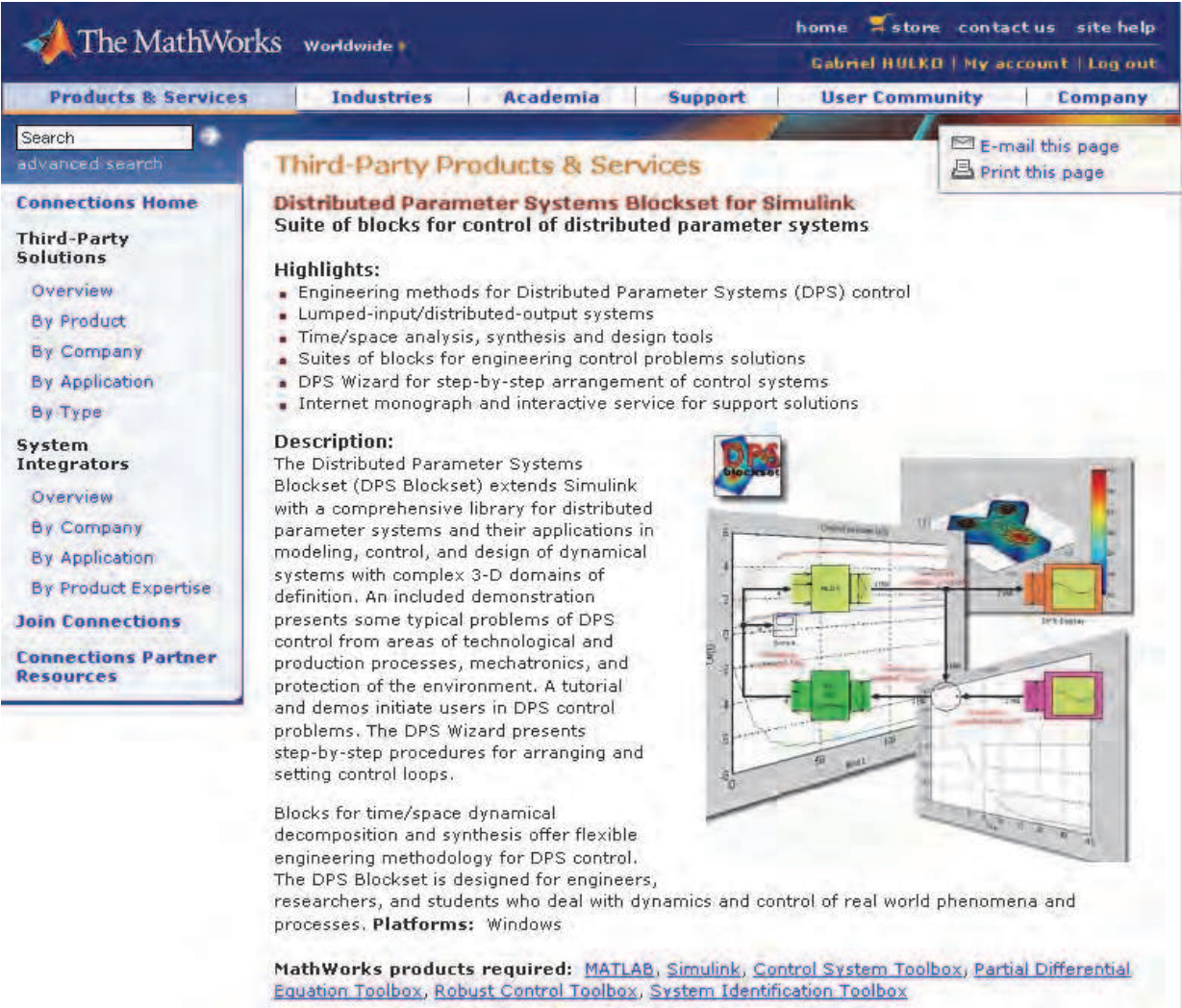


Fig. 15. Distributed Parameter Systems Blockset on the web portal of The MathWorks Corporation

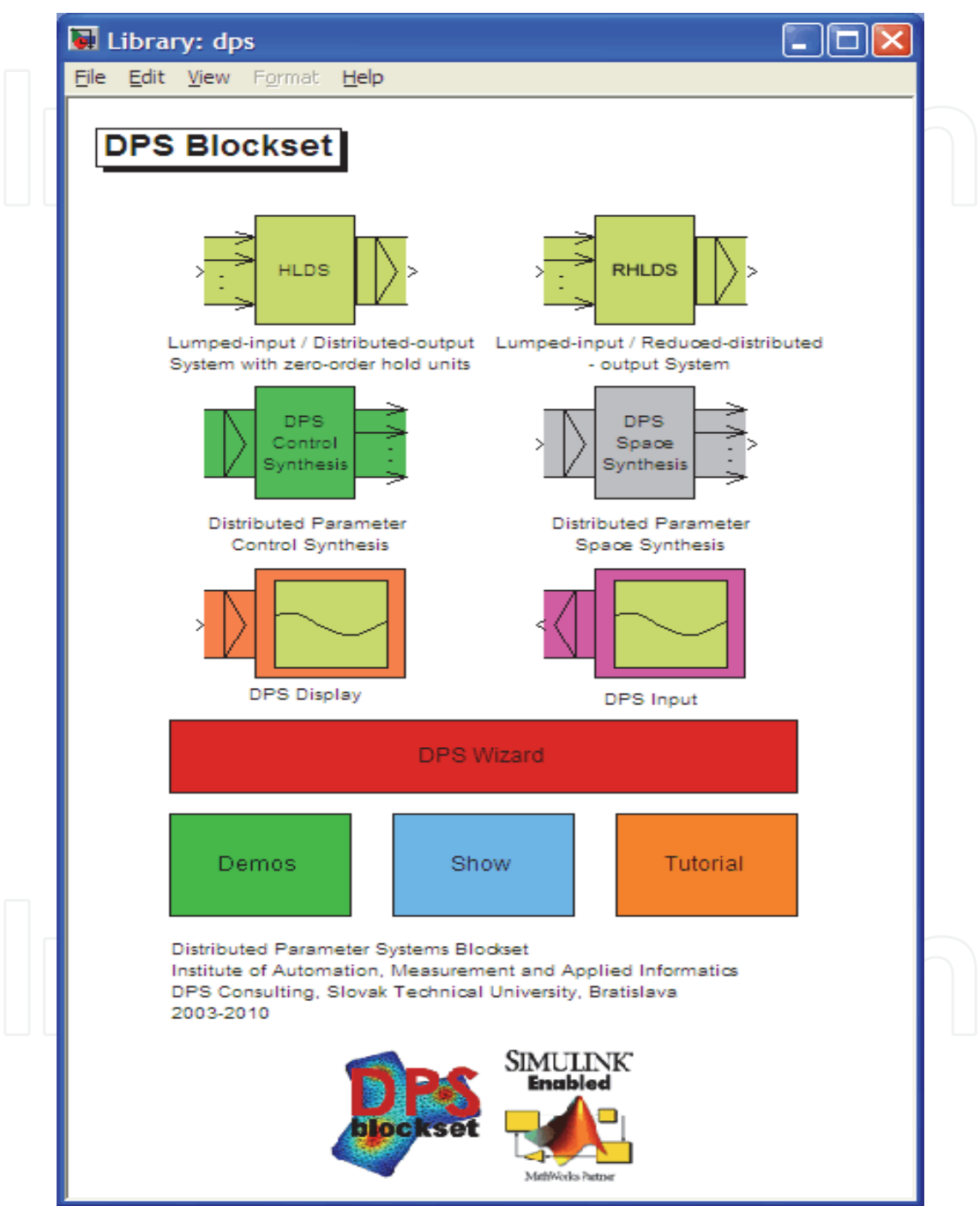


Fig. 16. The library of Distributed Parameter Systems Blockset for MATLAB & Simulink – Third-Party Product of The MathWorks

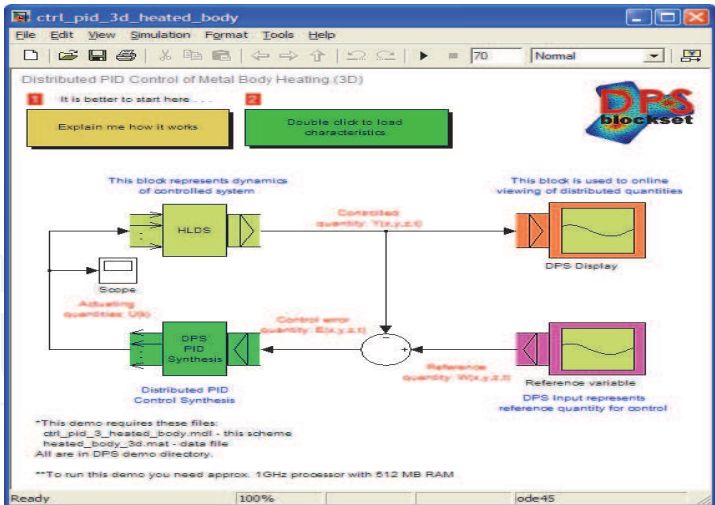


Fig. 17. Distributed parameter control loop for discrete-time PID control of heating of metal body in DPS Blockset environment

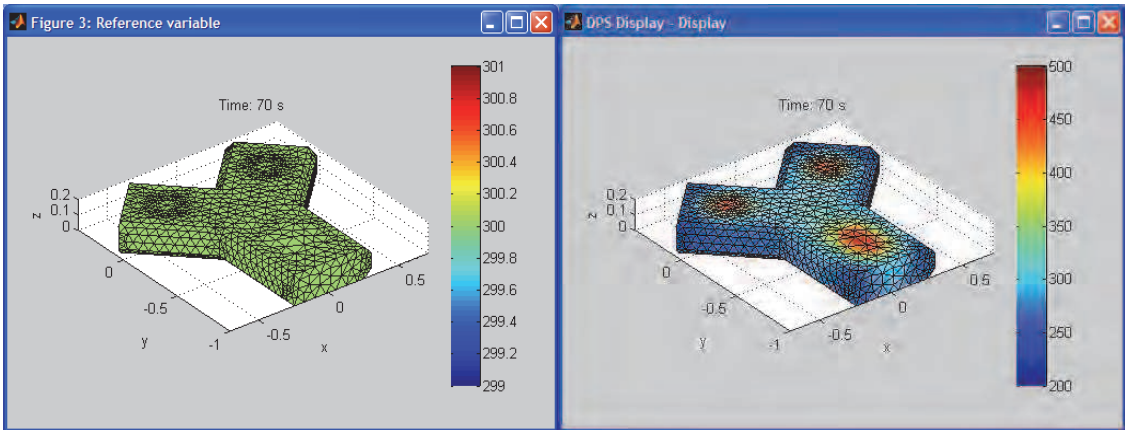


Fig. 18. Distributed reference and controlled quantities of metal body heating over the numerical net

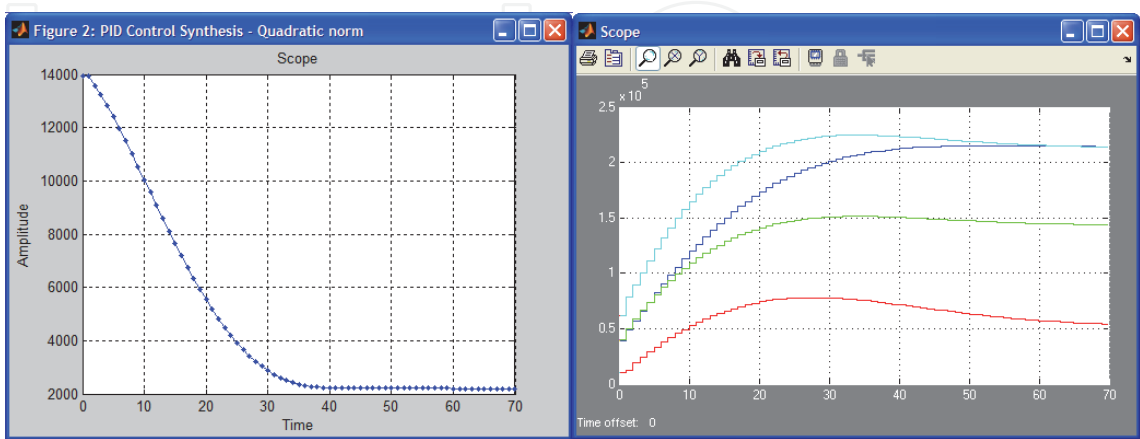


Fig. 19. Quadratic norm of distributed control error and discrete lumped actuating quantities at discrete-time PID control of heating of metal body in DPS Blockset environment

6. Interactive control via the Internet

For the interactive formulation and solution of DPS demonstration control problems via the Internet, an **Interactive Control** service has been started on the web portal **Distributed Parameter Systems Control** - www.dpscontrol.sk of the IAMAI-MEF-STU (Hulkó, 2003-2010) – see Fig. 20. for a screenshot of the site. In the framework of the problem formulation, first the computational geometry and mesh are chosen in the complex 3D shape definition domain, then DT distributed transient responses are computed in virtual software environments for numerical dynamical analysis of machines and processes. Finally, the distributed reference quantity is specified in points of the computational mesh - Fig. 18. Representing the solution to those interested animated results of actuating quantities, quadratic norm of control error, distributed reference and controlled quantity are sent in the form of **DPS Blockset** outputs – see Fig. 17-19. for illustration.

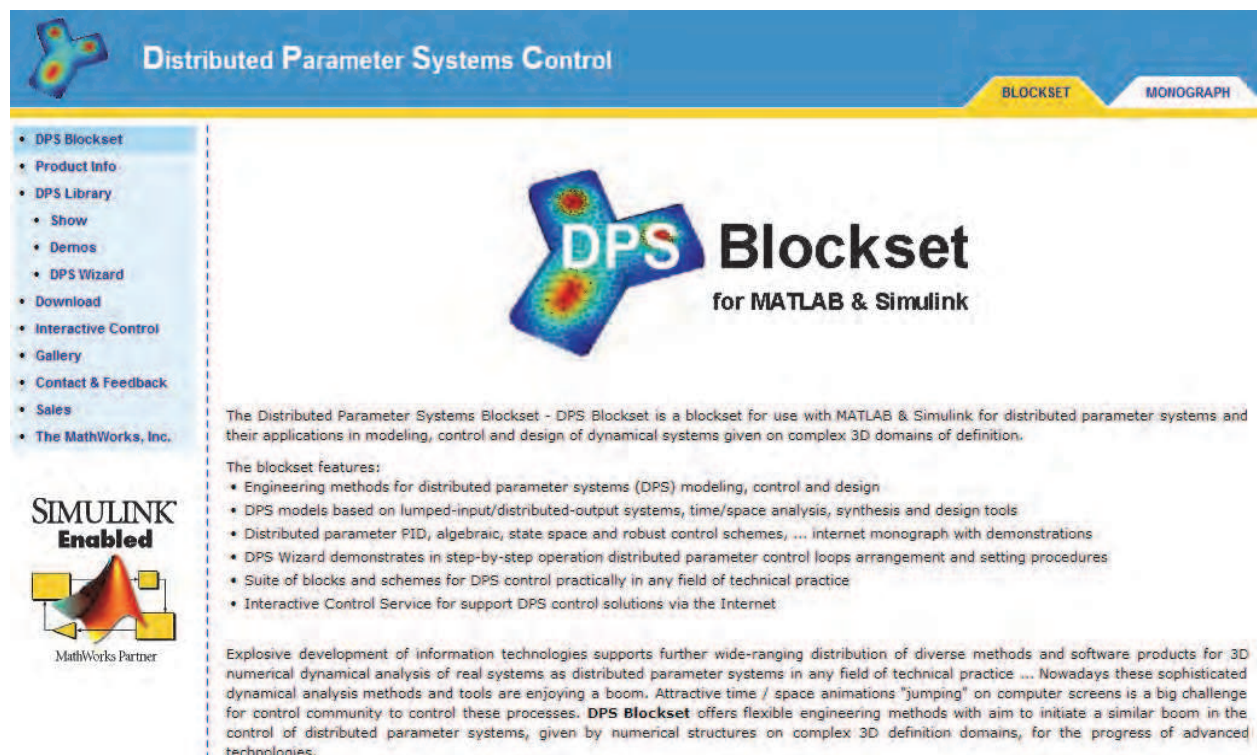


Fig. 20. Web portal Distributed Parameter Systems Control with monograph Modeling, Control and Design of Distributed Parameter Systems with Demonstrations in MATLAB and service Interactive Control

7. Conclusion

The aim of this chapter is to present a philosophy of the engineering approach for the control of DPS – given by numerical structures, which opens a wide space for novel applications of the toolboxes and blocksets of the MATLAB & Simulink software environment. This approach is based on the general decomposition into time and space components of controlled DPS dynamics represented by numerically computed distributed parameter transient and impulse characteristics, given on complex shape definition domains in 3D. Starting out from this dynamics decomposition a methodical framework is presented

for the analogous decomposition of control synthesis into the space and time subtasks. In space domain approximation problems are solved, while in the time domain control synthesis is realized by lumped parameter SISO control loops (Hulkó et al., 1981-2010). Based on these decomposition a software product named **Distributed Parameter Systems Blockset for MATLAB & Simulink** - a Third-Party software product of The MathWorks - www.mathworks.com/products/connections/ has been developed within the program CONNECTIONS of The MathWorks Corporation, (Hulkó et al., 2003-2010), where time domain toolboxes and blocksets of software environment MATLAB & Simulink as Control Systems Toolbox, Simulink Control Design, System Identification Toolbox,... are made use of. In the space domain approximation problems are solved as optimization problems by means of the Optimization Toolbox.

For the further support of research in this area a web portal named **Distributed Parameter Systems Control** - www.dpscontrol.sk was realized (Hulkó et al., 2003-2010), see Fig. 20. for an illustration. On the above mentioned web portal, the online version of the monograph titled **Modeling, Control and Design of Distributed Parameter Systems with Demonstrations in MATLAB** - www.mathworks.com/support/books/ (Hulkó et al., 1998), is presented along with application examples from different disciplines such as: control of technological and production processes, control and design of mechatronic structures, groundwater remediation control, etc. This web portal also offers for those interested the download of the demo version of the **Distributed Parameter Systems Blockset for MATLAB & Simulink** with **Tutorial**, **Show**, **Demos** and **DPS Wizard**. This portal also offers the **Interactive Control** service for interactive solution of model control problems of DPS via the Internet.

8. Acknowledgment

This work was supported by the Slovak Scientific Grant Agency VEGA under the contract No. 1/0138/11 for project „Control of dynamical systems given by numerical structures as distributed parameter systems“ and the Slovak Research and Development Agency under the contract No. APVV-0160-07 for project „Advanced Methods for Modeling, Control and Design of Mechatronical Systems as Lumped-input and Distributed-output Systems“ also the project No. APVV-0131-10 „High-tech solutions for technological processes and mechatronic components as controlled distributed parameter systems“.

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MATLAB for Engineers - Applications in Control, Electrical Engineering, IT and Robotics

Edited by Dr. Karel Perutka

ISBN 978-953-307-914-1

Hard cover, 512 pages

Publisher InTech

Published online 13, October, 2011

Published in print edition October, 2011

The book presents several approaches in the key areas of practice for which the MATLAB software package was used. Topics covered include applications for: -Motors -Power systems -Robots -Vehicles The rapid development of technology impacts all areas. Authors of the book chapters, who are experts in their field, present interesting solutions of their work. The book will familiarize the readers with the solutions and enable the readers to enlarge them by their own research. It will be of great interest to control and electrical engineers and students in the fields of research the book covers.

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<http://www.intechopen.com/books/matlab-for-engineers-applications-in-control-electrical-engineering-it-and-robotics/control-of-distributed-parameter-systems-engineering-methods-and-software-support-in-the-matlab-simu>

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