21

Error Separation Techniques Based on Telemetry and Tracking Data for Ballistic Missile

Huabo Yang, Lijun Zhang and Yuan Cao
National University of Defense Technology
China

1. Introduction

An intercontinental ballistic missile (ICBM) is a ballistic missile with a long range (some greater than 10000 km) and great firepower typically designed for nuclear weapons delivery, such as PeaceKeeper (PK) missile (Shattuck, 1992), Minutesman missile (Tony C. L., 2003). Due to the long-distance flight, the requirement for navigation system is rigorous and only gimbaled inertial navigation system (INS) is presently competent, such as the advanced inertial reference sphere (AIRS) used in the PK missile (John L., 1979), yet the strapdown inertial navigation system is generally not used on the intercontinental ballistic missile because of the poor accuracy (Titterton & Weston, 1997). The gimbaled inertial navigation system typically contains three single-degree-of-freedom rate integrating gyros, three mutually perpendicular single-axis accelerometers, a loop system and other auxiliary system, providing an orientation of the inertial navigation platform relative to inertial space. Due to system design and production technology there exist a lot of errors referred as guidance instrumentation systematic errors (IEEE Standards Committee, 1971; IEEE Standards Board, 1973), which have an important effect on impact accuracy of ballistic missile. Before the flight of ballistic missile, the guidance instrumentation systematic errors are need to calibrate, and then the calibration results are used to compensate the instrumental errors, which has been discussed in depth by Thompson (Thompson, 2000), Eduardo and Hugh (Eduardo & Hugh, 1999), Jackson (Jackson, 1973), Coulter and Meehan (Coulter & Meehan , 1981). Some content discussed has been issued as IEEE standard (IEEE Standards Committee, 1971; IEEE Standards Board, 1973).
However, the guidance instrumentation systematic errors cannot be completely compensated by using the calibration results. Therefore, flight test of ballistic missile is usually performed to qualify the performance. Because of different objectives of test or some other reasons specific testing trajectory is sometimes adopted, and herein the flight test cannot reflect the actual situation of ballistic missile in the whole trajectory. Consequently, it is necessary to analyze the landing errors resulted from guidance instrumentation systematic errors in the specific trajectory and convert them into those landing errors in the case of the whole trajectory.
In fact, there are many factors affecting the impact accuracy of ballistic missile, such as gravity anomaly, upper atmosphere, electromagnetic force, etc. Forsberg and Sideris has

taken into account the effect of gravity anomaly and presented the analysis method (Forsberg & Sideris, 1993). The effect of upper atmosphere and electromagnetic force is considered by Zheng (Zheng, 2006), but these error factors are so small compared to guidance instrumentation systematic errors that they are capable of not being considered when analyzing the impact accuracy. The analysis of guidance instrumentation systematic errors is generally performed using telemetry data and tracking data. Telemetry data are the angular velocity and acceleration information measured by inertial navigation system on the ballistic missile and transmitted by telemetric equipment, while tracking data are those information measured by radar and optoelectronic device in the test range. It is generally considered that the telemetry data contain instrumentation errors while tracking data contain systematic errors and random measurement errors of exterior measurement equipment, which is independent of instrumentation errors (Liu et al, 2000). Comparison of telemetry data and tracking data is used to obtain the velocity and position errors resulted from guidance instrumentation systematic errors. It is noticeable that the telemetry data are measured in the inertial coordinate system and exclude gravitational acceleration information while tracking data usually measured in the horizontal coordinate system. The conversion of two types of data into identical coordinate system is necessary.

Maneuvering launch manners are commonly adopted such as road-launched and submarine-launched manners to improve the viability and strike capacity for ballistic missile. Maneuvering launch ballistic missile especially for submarine-launched ballistic missile is often affected by ocean current, wave, and vibration environment, etc. Obviously, there are measurement errors in the initial launch parameters including location and orientation parameters as well as carrier’s velocity. Theoretical analysis and numerical simulation indicate that initial launch parameter errors are equivalent in magnitude to the guidance instrumentation systematic errors (Zheng, 2006; Gore,). Since the landing errors due to initial launch parameter errors and guidance instrumentation systematic errors are coupled, the error separation procedure for those two types of errors must be performed using telemetry and tracking data.

The error separation model can be simplified as a linear model using telemetry and tracking data (Yang et al, 2007). It is noted that the linear model is directly obtained by telemetry and tracking data and is independent of the flight of ballisitc missile. The remarkable features of this linear model is high dimension and collinearity, which is a severe problem when one wishes to perform certain types of mathematical treatment such as matrix inversion. These categories of problem can be treated many advanced methods, such as improved regression estimation (Barros & Rutledge, 1998; Cherkassky & Ma, 2005), partial least square (PLS) method (Wold et al, 2001), and support vector machines (SVM) (Cortes & Vapnik, 1995), however, these analysis methods are of no interest in this chapter. This chapter mainly focuses on the modeling of separation of instrumentation errors based on telemetry and tracking data and presents a novel error separation technique.

2. Calculation of difference between telemetry and tracking data

Telemetry and tracking data are known as important information sources in the error separation procedure. Two key problems are needed to be solved when computing the difference between telemetry and tracking data, since they are described in different coordinate systems. One is to convert the telemetry and tracking data into the same coordinate system, the other is to subtract the gravitational acceleration from tracking data or to add gravitational acceleration into telemetry data. The difference between telemetry and tracking data can be reckoned in either launch inertial coordinate system or launch coordinate system. A typical method is to convert the tracking data into launch inertial coordinate system and then to subtract the gravitational acceleration. In fact, guidance instrumentation systematic errors are contained in the telemetry data while initial launch parameter errors are generated in the case of the conversion for tracking data and the computation of gravity acceleration, so the sources of them are absolutely different.

The apparent velocity and position in the launch inertial coordinate system can be computed as follows.

1. Transformation matrix

The transformation matrix from geocentric coordinate system to launch coordinate system can be represented by

$$\begin{aligned}\mathbf{C}_e^g &= \mathbf{M}_2[-(90^0 + A_T)]\mathbf{M}_1[B_T]\mathbf{M}_3[-(90^0 - \lambda_T)] \\ &= \begin{bmatrix} -\sin A_T \sin\lambda_T - \cos A_T \sin B_T \cos\lambda_T & \sin A_T \cos\lambda_T - \cos A_T \sin B_T \sin\lambda_T & \cos A_T \cos B_T \\ \cos B_T \cos\lambda_T & \cos B_T \sin\lambda_T & \sin B_T \\ -\cos A_T \sin\lambda_T + \sin A_T \sin B_T \cos\lambda_T & \cos A_T \cos\lambda_T + \sin A_T \sin B_T \sin\lambda_T & -\sin A_T \cos B_T \end{bmatrix}\end{aligned} \tag{1}$$

where subscript e denotes geocentric coordinate system and superscript g denotes launch coordinate system; A_T , B_T , λ_T are astronomical azimuth, latitude and longitude, respectively. Also, the transformation matrix relating launch coordinate system to launch inertial coordinate system is given by

$$\mathbf{C}_g^a = \mathbf{A}^T\mathbf{B}^T\mathbf{A} \tag{2}$$

with

$$\mathbf{A} = \begin{bmatrix} \cos A_T \cos B_T & \sin B_T & -\sin A_T \cos B_T \\ -\cos A_T \sin B_T & \cos B_T & \sin A_T \sin B_T \\ \sin A_T & 0 & \cos A_T \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_e t & \sin\omega_e t \\ 0 & -\sin\omega_e t & \cos\omega_e t \end{bmatrix} \tag{3}$$

where superscript a denotes launch inertial coordinate system, ω_e is the earth rate, t is the in-flight time.

2. Radius vector from earth center to launch site

The radius of prime vertical circle of launch site is given by

$$N_0 = \frac{a_e(1-\alpha_e)}{\sqrt{1-(2\alpha_e - \alpha_e{}^2)\sin^2 B_0}} \tag{4}$$

where a_e is the earth semimajor axis, α_e is the earth flattening, B_0 is the geodetic latitude. Ignoring higher-order terms yields

$$N_0 = a_e(1 + \alpha_e \sin^2 B_0) \tag{5}$$

Thus, the components of launch site in the geocentric coordinate system are written as

$$\mathbf{R}_{0e} = \begin{bmatrix} (N_0 + H_0)\cos B_0 \cos\lambda_0 \\ (N_0 + H_0)\cos B_0 \sin\lambda_0 \\ \left(N_0\left(1-\alpha_e{}^2\right) + H_0\right)\sin B_0 \end{bmatrix} \tag{6}$$

where λ_0 , B_0 , H_0 are the geodetic longitude, geodetic latitude and geodetic height of launch site, respectively. Using coordinate transformation we can write the radius vector from earth center to launch site in the launch coordinate system as

$$\mathbf{R}_{0g} = \mathbf{C}_e^g \mathbf{R}_{0e} \tag{7}$$

3. Earth rate

The components of earth rate expressed in the launch coordinate system are given by

$$\boldsymbol{\omega}_{eg} = \mathbf{C}_e^g \begin{bmatrix} 0 \\ 0 \\ \omega_e \end{bmatrix} = \omega_e \begin{bmatrix} \cos B_T \cos A_T \\ \sin B_T \\ -\cos B_T \sin A_T \end{bmatrix} \tag{8}$$

The angular velocity of launch coordinate system with respect to launch inertial coordinate system is the earth rate, so earth rate expressed in the launch inertial coordinate system is given by

$$\boldsymbol{\omega}_{ea} = \mathbf{C}_g^a \cdot \boldsymbol{\omega}_{eg} \tag{9}$$

4. Gravitational acceleration

The radius vector from earth center to center of mass of missile in the launch coordinate system is given by

$$\mathbf{r}_g = \mathbf{R}_{0g} + \boldsymbol{\rho}_g \tag{10}$$

where $\boldsymbol{\rho}_g$ is the missile location provided by tracking data.

The gravitational acceleration taking into account the J_2 term in the launch coordinate system is given by

$$\mathbf{g}_g = g_r \cdot \frac{\mathbf{r}_g}{|\mathbf{r}_g|} + g_\omega \cdot \frac{\boldsymbol{\omega}_{eg}}{\omega_e} \tag{11}$$

where

$$g_r = -\frac{\mu}{r_g^2} \cdot [1 + J_2 \cdot (\frac{a_e}{r_g})^2 \cdot (1 - 5\sin^2\phi)] \tag{12}$$

$$g_\omega = -2\frac{\mu}{r_g^2} \cdot J_2 \cdot (\frac{a_e}{r_g})^2 \cdot \sin\phi \tag{13}$$

and the geocentric latitude ϕ can be computed as follows

$$\phi = \arcsin \frac{\mathbf{r}_g \cdot \boldsymbol{\omega}_e}{\left| \mathbf{r}_g \cdot \boldsymbol{\omega}_e \right|} \tag{14}$$

Hence, gravitational acceleration in the launch inertial coordinate system is written as

$$\mathbf{g}_a = \mathbf{C}_g^a \cdot \mathbf{g}_g \tag{15}$$

5. Calculation of apparent velocity and position of tracking data
The tracking apparent velocity is given by

$$\mathbf{W}_{\text{tra}}^a(t) = \mathbf{C}_g^a(t) \cdot \mathbf{V}_g(t) + \boldsymbol{\Omega}_\omega^a \cdot \mathbf{C}_g^a(t) \cdot \left(\mathbf{R}_0 + \boldsymbol{\rho}_g \right) - \mathbf{V}_{0a} - \int_0^t \mathbf{g}_a(\tau) d\tau \tag{16}$$

with

$$\boldsymbol{\Omega}_\omega^a = \begin{bmatrix} 0 & -\omega_{eaz} & \omega_{eay} \\ \omega_{eaz} & 0 & -\omega_{eax} \\ -\omega_{eay} & \omega_{eax} & 0 \end{bmatrix} \tag{17}$$

where $\omega_{eax}, \omega_{eay}, \omega_{eaz}$ are three components of $\boldsymbol{\omega}_{ea}$, respectively; $\mathbf{V}_g$ and $\boldsymbol{\rho}_g$ are the velocity and position of missile in the launch coordinate system provided by tracking data, respectively. $\mathbf{V}_{0a}$ is the initial velocity of launch site with respect to launch coordinate system due to earth rotation, written as

$$\mathbf{V}_{0a} = \boldsymbol{\omega}_{ea}(0) \times \mathbf{R}_{0a} \tag{18}$$

Likewise, the tracking apparent position is given by

$$\mathbf{W}_{\text{tra}}^a(t) = \mathbf{C}_g^a(t) \cdot \left(\mathbf{R}_0 + \boldsymbol{\rho}_g \right) - \mathbf{R}_{0a} - \mathbf{V}_{0a} \cdot t - \int_0^t \int_0^u \mathbf{g}_a(\tau) d\tau du \tag{19}$$

6. Calculation of apparent velocity and position of telemetry data
The telemetric apparent velocity can be obtained by the integration of telemetric apparent acceleration, given by

$$\mathbf{W}_{\text{tele}}^a(t) = \int_0^t \dot{\mathbf{W}}_{\text{tele}}^a(\tau) d\tau \tag{20}$$

Integrating Eq.(20) gives the telemetric apparent position

$$\mathbf{W}_{\text{tele}}^a(t) = \int_0^t \mathbf{W}_{\text{tele}}^a(\tau) d\tau \tag{21}$$

7. Calculation of difference between telemetry and tracking data
The difference between telemetry data and tracking data is obtained by subtracting synchronous tracking data and compensation from telemetry data, namely, we can have the difference between telemetry velocity and tracking velocity, $\delta \mathbf{X}_v(t)$, and the difference between telemetry velocity and tracking velocity, $\delta \mathbf{X}_r(t)$.

3. Separation model of guidance instrumentation systematic errors

There are many reasons influencing the landing errors of ICBM, which can be fallen into two categories: 1) guidance instrumentation systematic errors, and 2) initial launch parameter errors. Guidance instrumentation systematic errors primarily consist of accelerometer, gyroscope and platform systematic errors. Before the flight test ground calibration test is usually performed for inertial navigation system and then the estimates of instrumentation error coefficients are compensated in flight, which can reduce the landing errors and the difference between telemetry and tracking data effectively. However, because of the residual between the calibrated values and the actual values of instrumentation errors, the separation of the behaved values of the instrumentation error coefficients from telemetry and tracking data is need to perform.

3.1 Model of guidance instrumentation systematic errors

Since the determination of error model is correlated with the performance of inertial platform, there are many error coefficients required to separate for inertial platform with high accuracy while a minority of primary error terms for general inertial platform with poor accuracy. The gyroscope error model of inertial platform is given by

$$\begin{cases} \dot{\alpha}_x(t) = k_{g0x} + k_{g11x}\dot{W}_x(t) + k_{g12x}\dot{W}_z(t) \\ \dot{\alpha}_y(t) = k_{g0y} + k_{g11y}\dot{W}_y(t) + k_{g12y}\dot{W}_x(t) \\ \dot{\alpha}_z(t) = k_{g0z} + k_{g11z}\dot{W}_z(t) + k_{g12z}\dot{W}_y(t) \end{cases} \tag{22}$$

and accelerometer error model is given by

$$\begin{cases} \Delta_x(t) = k_{a0x} + k_{a1x}\dot{W}_x(t) \\ \Delta_y(t) = k_{a0y} + k_{a1y}\dot{W}_y(t) \\ \Delta_z(t) = k_{a0z} + k_{a1z}\dot{W}_z(t) \end{cases} \tag{23}$$

Where $\dot{\alpha}_x, \dot{\alpha}_y, \dot{\alpha}_z$ are angular velocity drifts of three gyroscopes, respectively; $\dot{W}_x, \dot{W}_y, \dot{W}_z$ are apparent accelerations of vehicle; $k_{g0x}, k_{g0y}, k_{g0z}$ are zero biases of three gyroscopes, $k_{g11x}, k_{g11y}, k_{g11z}$ are proportional error coefficients, $k_{g12x}, k_{g12y}, k_{g12z}$ are first-order error coefficients; $k_{a0x}, k_{a0y}, k_{a0z}$ are zero biases and $k_{a1x}, k_{a1y}, k_{a1z}$ are proportional error coefficients of three accelerometers. Model of guidance instrumentation systematic errors contains 15 error coefficients in total.

The accurate velocity, position and orientation information of ballistic missile are not available due to the errors resulted from maneuvering of ballistic missile and measurements, which generates the initial launch parameter errors. The initial launch parameter errors primarily consist of geodetic longitude, geodetic latitude, geodetic height, astronomical longitude, astronomical latitude and astronomical azimuth errors of launch site, and initial velocity errors of ballistic missile about three directions, amounting to 9 terms.

3.2 Separation model of instrumentation errors

Guidance instrumentation systematic errors can affect telemetric apparent acceleration so as to affect apparent velocity and position. Without regard to the calculation error of gravitational force, the velocity and position errors of trajectory are the errors of apparent velocity and position respectively. The apparent acceleration error arisen from guidance instrumentation systematic error is represented by

$$\delta\dot{\mathbf{W}} = \dot{\mathbf{W}}_p - \dot{\mathbf{W}}_a = \dot{\mathbf{W}}_p - \mathbf{M}_3(-\alpha_z)\mathbf{M}_2(-\alpha_y)\mathbf{M}_1(-\alpha_x)\cdot(\dot{\mathbf{W}}_p - \boldsymbol{\Delta}) \tag{24}$$

where $\dot{\mathbf{W}}_p$ is the apparent acceleration measured by inertial navigation platform, $\dot{\mathbf{W}}_a$ is the real apparent acceleration; $\mathbf{M}_3(\cdot)$, $\mathbf{M}_2(\cdot)$, $\mathbf{M}_1(\cdot)$ are the rotation matrices about z , y , x axis, respectively; α_x , α_y , α_z are the drift angles along the three directions, which are assumed as small values; $\boldsymbol{\Delta}$ is the error vector measured by accelerometer. Since the true value of $\dot{\mathbf{W}}_a$ is not available, the substitution of $\dot{\mathbf{W}}_a$ is generally obtained by converting the tracking data. Thereby $\delta\dot{\mathbf{W}}$ is the difference of apparent acceleration between telemetry and tracking data.
Neglecting the second-order term, Eq.(24) is changed to

$$\delta\dot{\mathbf{W}} = \dot{\mathbf{W}}_p - \begin{bmatrix} 1 & -\alpha_z & \alpha_y \\ \alpha_z & 1 & -\alpha_x \\ -\alpha_y & \alpha_x & 1 \end{bmatrix} \cdot (\dot{\mathbf{W}}_p - \boldsymbol{\Delta}) \tag{25}$$

Rearranging Eq.(25) and ignoring the second-order small values yield

$$\boldsymbol{\delta}\dot{\mathbf{W}} = \begin{bmatrix} 0 & -\dot{W}_{pz} & \dot{W}_{py} \\ \dot{W}_{pz} & 0 & -\dot{W}_{px} \\ -\dot{W}_{py} & \dot{W}_{px} & 0 \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} + \boldsymbol{\Delta} \tag{26}$$

where $\dot{W}_{px}$, $\dot{W}_{py}$, $\dot{W}_{pz}$ are the components of $\dot{\mathbf{W}}_p$; α_x , α_y , α_z are the drift angles of gyroscope and obtained by integrating Eq.(22)

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \int_0^t \begin{bmatrix} \dot{\alpha}_x \\ \dot{\alpha}_y \\ \dot{\alpha}_z \end{bmatrix} dt = \int_0^t \begin{bmatrix} k_{g0x} + k_{g11x}\dot{W}_{ax} + k_{g12x}\dot{W}_{ay} \\ k_{g0y} + k_{g11y}\dot{W}_{ay} + k_{g12y}\dot{W}_{ax} \\ k_{g0z} + k_{g11z}\dot{W}_{az} + k_{g12z}\dot{W}_{ay} \end{bmatrix} dt \tag{27}$$

By the accelerometer error model, we can have

$$\begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix} = \begin{bmatrix} k_{a0x} + k_{a1x}\dot{W}_{ax} \\ k_{a0y} + k_{a1y}\dot{W}_{ay} \\ k_{a0z} + k_{a1z}\dot{W}_{az} \end{bmatrix} \tag{28}$$

Note that $\dot{W}_{ax}, \dot{W}_{ay}, \dot{W}_{az}$ are the apparent accelerations in the launch inertial coordinate system, unfortunately we cannot obtain the measurements in practice. Since the values of $\dot{W}_{px}, \dot{W}_{py}, \dot{W}_{pz}$ are given by the telemetry data, so we can approximately substitute $\dot{W}_{px}, \dot{W}_{py}, \dot{W}_{pz}$ for $\dot{W}_{ax}, \dot{W}_{ay}, \dot{W}_{az}$ during the error separation process. Hence, Eqs.(27) and (28) can be rewritten respectively as

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \int_0^t \begin{bmatrix} \dot{\alpha}_x \\ \dot{\alpha}_y \\ \dot{\alpha}_z \end{bmatrix} dt = \int_0^t \begin{bmatrix} k_{g0x} + k_{g11x}\dot{W}_{px} + k_{g12x}\dot{W}_{py} \\ k_{g0y} + k_{g11y}\dot{W}_{py} + k_{g12y}\dot{W}_{px} \\ k_{g0z} + k_{g11z}\dot{W}_{pz} + k_{g12z}\dot{W}_{py} \end{bmatrix} dt \tag{29}$$

$$\begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix} = \begin{bmatrix} k_{a0x} + k_{a1x}\dot{W}_{px} \\ k_{a0y} + k_{a1y}\dot{W}_{py} \\ k_{a0z} + k_{a1z}\dot{W}_{pz} \end{bmatrix} \tag{30}$$

Herein we select $\mathbf{D} = \left[k_{g0x}\ k_{g0y}\ k_{g0z}\ k_{g11x}\ k_{g11y}\ k_{g11z}\ k_{g12x}\ k_{g12y}\ k_{g12z}\ k_{a0x}\ k_{a0y}\ k_{a0z}\ k_{a1x}\ k_{a1y}\ k_{a0z} \right]^T$, then apparent acceleration error $\boldsymbol{\delta}\dot{\mathbf{W}}$ and instrumentation error coefficients $\mathbf{D}$ are written in linear relation as

$$\boldsymbol{\delta}\dot{\mathbf{W}} = \mathbf{S}_a \cdot \mathbf{D} \tag{31}$$

where $\mathbf{S}_a$ is the environmental function matrix of apparent acceleration, given by

$$\mathbf{S}_a = \left[\mathbf{S}_e \cdot \mathbf{S}_{Ag} \quad \mathbf{S}_{Aa} \right] \tag{32}$$

where

$$\mathbf{S}_e = \begin{bmatrix} 0 & \dot{W}_{zp} & -\dot{W}_{yp} \\ -\dot{W}_{zp} & 0 & \dot{W}_{xp} \\ \dot{W}_{yp} & -\dot{W}_{xp} & 0 \end{bmatrix}, \ \mathbf{S}_{Aa} = \begin{bmatrix} 1 & 0 & 0 & \dot{W}_{px} & 0 & 0 \\ 0 & 1 & 0 & 0 & \dot{W}_{py} & 0 \\ 0 & 0 & 1 & 0 & 0 & \dot{W}_{pz} \end{bmatrix},$$
$$\mathbf{S}_{Ag} = \begin{bmatrix} t & 0 & 0 & W_{xp} & 0 & 0 & W_{yp} & 0 & 0 \\ 0 & t & 0 & 0 & W_{yp} & 0 & 0 & W_{xp} & 0 \\ 0 & 0 & t & 0 & 0 & W_{zp} & 0 & 0 & W_{yp} \end{bmatrix}$$

Integrating Eq.(31) gives the apparent velocity error

$$\boldsymbol{\delta}\mathbf{W}(t) = \int_0^t \mathbf{S}_a(\tau)dt \cdot \mathbf{D} = \mathbf{S}_v(t)\mathbf{D} \tag{33}$$

where $\mathbf{S}_v(t)$ is the environmental function matrix of instrumental error of apparent velocity. Taking the integration of Eq.(33) again gives the apparent position error

$$\boldsymbol{\delta}\underset{\cdot}{\mathbf{W}}(t) = \int_0^t \mathbf{S}_v(\tau)dt \cdot \mathbf{D} = \mathbf{S}_r(t)\mathbf{D} \tag{34}$$

where $\mathbf{S}_r(t)$ is the environmental function matrix of instrumental error of apparent position. In the actual situation, the apparent velocity and position error models with the consideration of random errors are represented by

$$\begin{aligned}\boldsymbol{\delta}\mathbf{W}(t) &= \mathbf{S}_v(t)\mathbf{D} + \boldsymbol{\varepsilon}_v \\ \boldsymbol{\delta}\underset{\cdot}{\mathbf{W}}(t) &= \mathbf{S}_r(t)\mathbf{D} + \boldsymbol{\varepsilon}_r\end{aligned} \tag{35}$$

where $\boldsymbol{\varepsilon}_v$ and $\boldsymbol{\varepsilon}_r$ are the random errors. It is seen from Eq.(35) that the separation model of instrumentation errors can be simplified as a linear model.

Actually, the apparent velocity and position errors are computed by the telemetry and tracking data. When taking no account of the random errors, the tracking data can be considered as the true values of ballistic data.

4. Error separation model of initial launch parameters

The initial launch parameter errors not only affect the apparent position and velocity and stress of ballistic missile, but also the airborne computer guidance calculation. The mechanism of initial errors is analyzed thereinafter.

4.1 Effect to landing error of ballistic missile caused by initial errors

1. Effect to trajectory in the geocentric coordinate system

The localization and orientation parameters directly determine the foundation of coordinate system. When the launch inertial coordinate system $O_a - x_a y_a z_a$ changes to $O'_a - x'_a y'_a z'_a$, the base of controlling the attitude motion will also change. At this point, the reference plane $O_a - x_a z_a$ controlled by pitch angle changes to $O'_a - x'_a z'_a$ plane, simultaneously the reference plane $O_a - x_a y_a$ controlled by yaw angle changes to $O'_a - x'_a y'_a$ plane. Due to the noncoincidence of the two pairs of planes, the shape and azimuth of the in-flight trajectory are not the same with respect to the "real earth". Also, the location of trajectory is determined by the initial localization and orientation parameters. Therefore, the position of landing point of ballistic missile in the geocentric coordinate system will offset the objective point when the parameters are not error-free, in despite of taking no account of other error factors.

2. Effect to the initial velocity of missile in the launch inertial coordinate system

The launch site coordinate $\boldsymbol{R}_{0a}$ and the earth rate $\boldsymbol{\omega}_{ea}$ are determined by the initial localization and orientation parameters, which affect the initial velocity and stress of ballistic missile.

In the case of maneuvering launch, the initial missile velocity in the launch inertial coordinate system is given by

$$\boldsymbol{V}_{0a} = \boldsymbol{\omega}_{ea} \times \boldsymbol{R}_{0a} + \boldsymbol{V}_s^a \tag{36}$$

where $\boldsymbol{V}_s^a$ is the carrier's instantaneous velocity with respect to the ground. Obviously, the initial velocity is largely related to the initial localization and orientation parameters and the velocity of carrier. When these parameters are with errors, the initial velocity of missile is in error.

3. Effect to the stress of missile

The acceleration of gravity of missile is determined by the angular velocity of the Earth and the coordinates of launch point in the launch inertial coordinate system and launch coordinate system. Due to the difference of stress of missile, the flight height and velocity are different, which indirectly causes the variation of thrust and aerodynamic forces. When computing the thrust forces, the effect of atmospheric pressure is considered, which is known as a function of height. At the same time, the calculation of thrust vector is related to the deflection angle of rudder, of which calculation is also affected by the height. In addition, the aerodynamic coefficients, velocity head and velocity are related to the height.

4. Effect to airborne guidance calculation

At present, the real velocity and position are commonly adopted for the calculation of guidance. Firstly, the integration of the apparent acceleration measured is performed to obtain apparent velocity; secondly, the real velocity and position are computed by the recursion formulas according to the computed apparent velocity and acceleration of gravity. When the true velocity and position satisfy the cut-off equations, the engines of missile shut down.

When there exist localization and orientation errors, on the one hand, the guidance coordinate system is different from the actual flight coordinate system, thereby the fact that the cut-off equations are satisfied cannot ensure that the missile hit the target; on the other hand, the initial values of recursion formulas involved real velocity and position and the calculation of gravitational acceleration are different from those of actual conditions, which induces that the computed real velocity and position don't agree with those under the actual situations.

For the closed-loop guidance case, the required commanded missile velocity is determined by the onboard computer in real time. Specifically, the required velocity is a function of current velocity and position of missile, location of launch point and target point, angular velocity of the Earth and orientation parameters, that is

$$\boldsymbol{V}_{aR} = \boldsymbol{V}_{aR}(\boldsymbol{V}_a, \boldsymbol{R}_a, \boldsymbol{R}_{obj}, \boldsymbol{R}_{0a}, \boldsymbol{\omega}_{ea}, \lambda_T, B_T, A_T) \tag{37}$$

It is obvious that the errors of localization and orientation parameters directly influence the calculation of required velocity and the cut-off of missile.

4.2 Sources of errors of initial localization and orientation parameters

In fact, the telemetry data should reflect the acceleration information of ballistic missile provided that the guidance instrumentation systematic errors are not taken into account. Tracking data are obtained in the horizontal coordinate system by measurement devices and then converted into geocentric coordinate system. Since the precise data in the local horizontal coordinate system are available, the tracking data measured in the geocentric coordinate system don't contain the initial errors and are precise.

The difference between telemetry and tracking data is generally reckoned in the launch inertial coordinate system. The launch inertial coordinate system is determined by the initial location and orientation parameters, and the launch inertial coordinate system is inaccurate if those parameters are with errors. It is necessary to convert the tracking data in the geocentric coordinate system into the launch inertial coordinate system. The location parameters are required for the calculation of initial velocity and position while orientation parameters are demanded for the calculation of the Euler angle mapping the geocentric coordinate system into launch inertial coordinate system, which generates the initial location and orientation parameter errors. The conversion of the tracking data is described as follows:

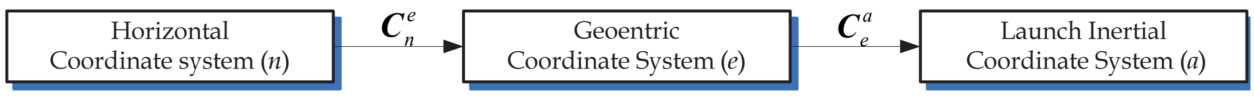


Fig. 1. The conversion of tracking data.

where $\boldsymbol{C}_n^e$ is the rotation matrix mapping horizontal coordinate system to geocentric coordinate system. The precise Euler angles are available since the geodetic coordinates of the observation station are accurate. However, there are errors in the Euler angles of rotation matrix $\boldsymbol{C}_e^a$ and then the orientation errors are introduced.

4.3 Relationship between initial orientation errors and alignment errors of platform

Before work the levelling and aligning are need to perform for inertial platform. For the maneuvering-launch-based missile, there may exist errors in the process of levelling and aligning for onboard platform system.

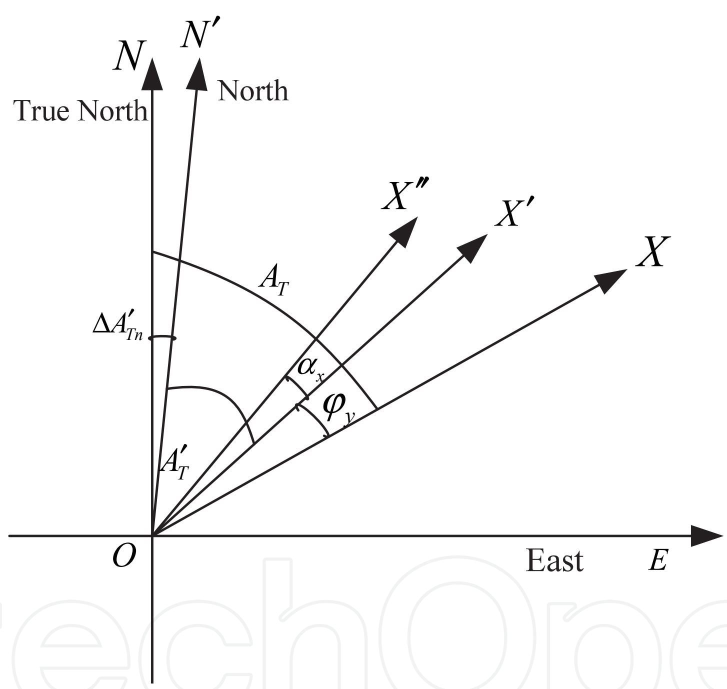


Fig. 2. The relationship between orientation errors and alignment errors of platform.

As shown in Fig.2, N is true north direction, N' is north direction measured by the vehicle, and $\Delta A'_{Tn}$ is the northing error. X is the ideal direction of fire, X' is the direction contaminated by alignment error φ_y , X'' is the actual direction provided by INS due to the platform drift angle α_x . In fact, telemetry data provides the apparent acceleration information measured in the frame involved in X'' axis while tracking data provides the information measured in the frame involved in X axis. Therefore, the azimuth from X direction to true north direction is given by

$$A_T = A'_T + \Delta A'_{Tn} + \varphi_y \tag{38}$$

and the initial azimuth error is defined as

$$\Delta A'_T = \Delta A'_{Tn} + \varphi_y \tag{39}$$

The above analysis gives an indication of linear correlation between the northing error and alignment errors of INS. Similarly, the relationship between astronomical latitude and levelling error is linear correlation.

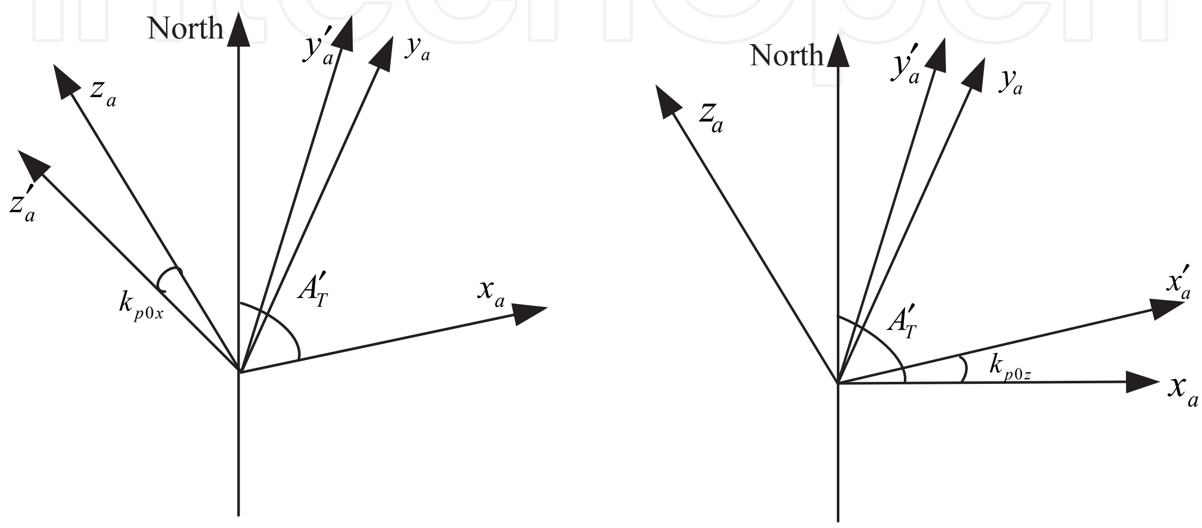


Fig. 3. The relationship between levelling errors and orientation parameters.

As can be seen in Fig.3, x_a, y_a, z_a are the coordinate axes of launch inertial frame, k_{p0x} and k_{p0z} are the levelling errors along x'_a and z'_a axes, respectively. Thus, the levelling errors can be converted into the astronomical latitude errors in the following form

$$\begin{aligned} \Delta B_p &= -k_{p0x} \sin A'_T - k_{p0z} \cos A'_T \\ \Delta \lambda_p &= k_{p0x} \cos A'_T - k_{p0z} \sin A'_T \end{aligned} \tag{40}$$

It is shown from the above analysis that the relationship between initial errors and levelling and alignment errors of guidance instrumentation systematic errors is linear correlation. Therefore, those errors cannot be separated merely using the telemetry and tracking data. Thereinafter the levelling and alignment errors are not included in the simulated cases.

4.4 Preliminary analysis of tracking data

In order to obtain the tracking data with sufficient precision, the incorporated measurement of multiple observation stations is generally used. It is pointed out in the previous section that the horizontal coordinate system of observation station is known exactly and the mapping relation with the geocentric coordinate system can be precisely described. To simplify the definition, the tracking velocity in the geocentric coordinate system is denoted by $\mathbf{V}_e$, and the position vector from the earth center expressed in the geocentric coordinate system is denoted as $\mathbf{r}_e$. Obviously, provided that the random errors of exterior devices are not taken into account, then both $\mathbf{V}_e$ and r_e are precise. The tracking velocity $\mathbf{V}_e$, consisting of three terms is written as

$$\mathbf{V}_e = \mathbf{V}_{eg} + \mathbf{V}_{es} + \mathbf{V}_{ew} \tag{41}$$

where $\mathbf{V}_{eg}$ is the incremental velocity due to gravitational acceleration, $\mathbf{V}_{es}$ is the velocity of maneuverable carrier, $\mathbf{V}_{ew}$ is the tracking apparent velocity which has removed the effect of the gravity forces and initial velocity of carrier.
The position vector $\mathbf{r}_e$ is given by

$$\mathbf{r}_e = \mathbf{r}_{eg} + \mathbf{r}_{es} + \mathbf{r}_{ew} + \mathbf{R}_{0e} \tag{42}$$

where $\mathbf{r}_{eg}$ is the incremental position due to gravitational acceleration, $\mathbf{r}_{es}$ is the incremental position due to the velocity of maneuverable carrier, $\mathbf{r}_{ew}$ is the apparent tracking position getting rid of the effect of gravity force and initial velocity of carrier, $\mathbf{R}_{0e}$ denotes the radius vector of origin of north-east-down coordinate system in the geocentric coordinate system.

4.4.1 Analysis of tracking data in the launch coordinate system

The tracking missile position in the launch coordinate system can be written in vector form

$$\boldsymbol{\rho}_g = \mathbf{C}_e^g \cdot \mathbf{r}_e - \mathbf{R}_{0g} \tag{43}$$

with

$$\mathbf{C}_e^g = \mathbf{M}_2(-\frac{\pi}{2} - A'_T)\mathbf{M}_1(B'_T)\mathbf{M}_3(-\frac{\pi}{2} + \lambda'_T) \tag{44}$$

and

$$\mathbf{R}_{0g} = \mathbf{C}_e^g(\lambda'_T, B'_T, A'_T)\begin{bmatrix} (N_0 + H'_0)\cos B'_0 \cos\lambda'_0 \\ (N_0 + H'_0)\cos B'_0 \sin\lambda'_0 \\ \left[N_0\left(1-e^2\right) + H'_0\right]\sin B' \end{bmatrix} \tag{45}$$

where λ'_T, B'_T, A'_T are the orientation parameters contaminated by random errors, H'_0, B'_0, λ'_0 are localization parameters contaminated by random errors.
The tracking velocity expressed in the launch coordinate system is represented by

$$\mathbf{V}_g = \mathbf{C}_e^g \cdot \mathbf{V}_e \tag{46}$$

The initial errors are introduced due to the localization and orientation parameters contaminated by random errors when computing transformation matrix $\mathbf{C}_e^g$ and position vector $\mathbf{R}_{0g}$, although precise $\mathbf{V}_e$ and $\mathbf{r}_e$ are available.

4.4.2 Effect of maneuverable carrier's velocity

The carrier's velocity is generally expressed in the body frame and the measurement is denoted as $\mathbf{V}_s'$, which is represented in the north-east-down (NED) coordinate system by

$$\mathbf{V}_s^n = \mathbf{M}_2(A_s)\mathbf{M}_3(-\varphi_s)\mathbf{M}_1(-\gamma_s)\mathbf{V}_s' \tag{47}$$

where A_s is the flight-path angle, which is measured from the north and is clockwise about the body axes , is positive; φ_s is the pitch angle, upward direction is positive; γ_s is the roll angle, and is clockwise about the body axes, is positive. Herein assume A_s , φ_s and γ_s are known exactly. Letting $\mathbf{C}_n^g = \mathbf{M}_2(-A_T')$, thus,

$$\mathbf{V}_s^a = \mathbf{C}_g^a \cdot \mathbf{V}_s^g = \mathbf{C}_g^a \cdot \mathbf{C}_n^g \cdot \mathbf{V}_s^n \tag{48}$$

where $\mathbf{C}_n^g$ is coordinate transformation matrix relating horizontal coordinate system to launch coordinate system. It is seen that the carrier's velocity is related to the launch azimuth. The carrier's velocity is known as a portion of initial velocity of missile, yet the tracking velocity and position reflect the real velocity and position of missile if the random errors are not taken into account, therefore, the tracking velocity contains the information of the carrier's velocity.

The position variation of missile due to the initial velocity is represented by

$$\mathbf{r}_s^a = \mathbf{C}_g^a \cdot \mathbf{r}_s^g = \mathbf{C}_g^a \cdot \mathbf{V}_s^g \cdot t = \mathbf{C}_g^a \cdot \mathbf{C}_n^g \cdot \mathbf{V}_s^n \cdot t \tag{49}$$

It follows from Eq.(36) that the carrier's velocity is contained in the initial velocity of missile and the incurred position variation is also contained in the tracking data.

4.5 Separation model of initial errors

It follows from the foregoing analysis that the guidance instrumentation systematic errors are contained in the telemetry data while the initial errors are primarily introduced during the data processing for tracking data. Therefore, the separation of these two types of errors can be performed respectively. The difference between telemetry and tracking data is written in the following form

$$\boldsymbol{\delta}\mathbf{X} = \begin{bmatrix} \mathbf{W}_{\text{tele}}^a(t) - \mathbf{W}_{\text{tra}}^a(t) \\ \underset{\cdot}{\mathbf{W}}_{\text{tele}}^a(t) - \underset{\cdot}{\mathbf{W}}_{\text{tra}}^a(t) \end{bmatrix} = \begin{bmatrix} \left(\mathbf{W}_{\text{tele}}^a(t) - \mathbf{W}_{\text{tra0}}^a(t)\right) - \left(\mathbf{W}_{\text{tra}}^a(t) - \mathbf{W}_{\text{tra0}}^a(t)\right) \\ \left(\underset{\cdot}{\mathbf{W}}_{\text{tele}}^a(t) - \underset{\cdot}{\mathbf{W}}_{\text{tra0}}^a(t)\right) - \left(\underset{\cdot}{\mathbf{W}}_{\text{tra}}^a(t) - \underset{\cdot}{\mathbf{W}}_{\text{tra0}}^a(t)\right) \end{bmatrix} \tag{50}$$

where $\mathbf{W}_{\text{tele}}^a$ and $\underset{\cdot}{\mathbf{W}}_{\text{tele}}^a$ are apparent velocity and position provided by telemetry data, respectively; $\mathbf{W}_{\text{tra}}^a$ and $\underset{\cdot}{\mathbf{W}}_{\text{tra}}^a$ are apparent velocity and position provided by tracking data, respectively; $\mathbf{W}_{\text{tra0}}^a$ and $\underset{\cdot}{\mathbf{W}}_{\text{tra0}}^a$ are the tracking information which don't contain the initial errors. The term on the right-hand side of Eq.(50) comprises two parts of information, one is the effect of guidance instrumentation systematic errors, and the other is the effect of initial errors. Thus, the difference between the telemetry data and tracking data can be rewritten as

$$\boldsymbol{\delta}\mathbf{X} = \begin{bmatrix} \mathbf{W}_{\text{tele}}^{a}(t) - \mathbf{W}_{\text{tra0}}^{a}(t) \\ \underset{\cdot}{\mathbf{W}}_{\text{tele}}^{a}(t) - \underset{\cdot}{\mathbf{W}}_{\text{tra0}}^{a}(t) \end{bmatrix} - \begin{bmatrix} \mathbf{W}_{\text{tra}}^{a}(t) - \mathbf{W}_{\text{tra0}}^{a}(t) \\ \underset{\cdot}{\mathbf{W}}_{\text{tra}}^{a}(t) - \underset{\cdot}{\mathbf{W}}_{\text{tra0}}^{a}(t) \end{bmatrix} \equiv \boldsymbol{\delta}\mathbf{X}_I - \boldsymbol{\delta}\mathbf{X}_P \tag{51}$$

where $\boldsymbol{\delta}\mathbf{X}_I$ is the difference of telemetry data and tracking data due to guidance instrumentation systematic errors, and $\boldsymbol{\delta}\mathbf{X}_P$ is the difference of telemetry data and tracking data due to initial errors.
Define the initial errors as

$$\mathbf{P}_a = \mathbf{P}' - \mathbf{P} \tag{52}$$

where $\mathbf{P}'$ are the known binding values of initial launch parameters consisting of 9 terms mentioned above, $\mathbf{P}$ is the unknown true value.
Recalling Eqs.(16) and (19) gives $\mathbf{W}_{tra}^{a}(t)$ and $\underset{\cdot}{\mathbf{W}}_{tra}^{a}(t)$

$$\mathbf{W}_{\text{tra}}^{a}(t) = \mathbf{C}_g^a(t) \cdot \mathbf{V}_g(t) + \boldsymbol{\Omega}_{\omega}^{a} \cdot \mathbf{C}_g^a(t) \cdot \left(\mathbf{R}_0 + \boldsymbol{\rho}_g\right) - \mathbf{V}_{0a} - \int_0^t \mathbf{g}_a(\tau) d\tau \tag{53}$$

$$\underset{\cdot}{\mathbf{W}}_{\text{tra}}^{a}(t) = \mathbf{C}_g^a(t) \cdot \left(\mathbf{R}_0 + \boldsymbol{\rho}_g\right) - \mathbf{R}_{0a} - \mathbf{V}_{0a} \cdot t - \int_0^t \int_0^u \mathbf{g}_a(\tau) d\tau du \tag{54}$$

The tracking position in the launch inertial coordinate system can be written as

$$\boldsymbol{\rho}_a = \mathbf{C}_g^a(t) \cdot (\mathbf{R}_0 + \boldsymbol{\rho}_g) - \mathbf{R}_{0a} = \mathbf{C}_g^a(t) \cdot \mathbf{r}_g - \mathbf{R}_{0a} \tag{55}$$

and the tracking velocity expressed in the launch inertial coordinate system is given by

$$\mathbf{V}_a = \mathbf{C}_g^a(t) \cdot \mathbf{V}_g = \mathbf{C}_g^a(t) \cdot \mathbf{C}_e^g \cdot \mathbf{V}_e \tag{56}$$

where $\mathbf{r}_e$ and $\mathbf{V}_e$ are the error-free tracking position and velocity expressed in the geocentric coordinate system.
By the definition of transformation matrix, we can have

$$\mathbf{C}_g^a \cdot \mathbf{C}_e^g = \left(\mathbf{M}_3(B_T')\mathbf{M}_2(A_T')\right)^T \mathbf{M}_1(-\omega_{0e}t)\mathbf{M}_3(B_T')\mathbf{M}_2(-\frac{\pi}{2})\mathbf{M}_1(B_T')\mathbf{M}_3(-\frac{\pi}{2} + \lambda_T') \tag{57}$$

Simplifying the Eq.(57) results in

$$\mathbf{C}_e^a = \mathbf{C}_g^a \cdot \mathbf{C}_e^g = \mathbf{M}_2(-A_T')\mathbf{M}_3(-B_T')\mathbf{M}_2(-\frac{\pi}{2})\mathbf{M}_3(-\frac{\pi}{2} + \lambda_T' - \omega_{0e}t) \tag{58}$$

Substituting Eq.(58) into Eqs. (55) and (56) yields the tracking apparent velocity

$$\mathbf{W}_{\text{tra}}^{a}(t) = \mathbf{C}_e^a(t) \cdot \mathbf{V}_e + \boldsymbol{\omega}_e^a \times \left(\mathbf{C}_e^a(t) \cdot \mathbf{r}_e\right) - \mathbf{V}_{0a} - \int_0^t \mathbf{g}_a(\tau) d\tau \tag{59}$$

and the tracking apparent position

$$\underset{\cdot}{\mathbf{W}}_{\text{tra}}^{a}(t) = \mathbf{C}_e^a(t) \cdot \mathbf{r}_e - \mathbf{R}_{0a} - \mathbf{V}_{0a} \cdot t - \int_0^t \int_0^u \mathbf{g}_a(\tau) d\tau du \tag{60}$$

Taking the total differentiation of Eq.(59), thus apparent velocity error is given by

$$\begin{aligned}\delta\mathbf{X}_{Pv} &= \Delta\left(\mathbf{C}_e^a(t)\cdot\mathbf{V}_e(t)+\boldsymbol{\omega}_e^a\times\left(\mathbf{C}_e^a(t)\cdot\mathbf{r}_e\right)-\mathbf{V}_{0a}-\int_0^t\mathbf{g}_a(\tau)d\tau\right)\\ &= \Delta\left(\mathbf{C}_e^a(t)\cdot\mathbf{V}_e(t)\right)+\Delta\left(\boldsymbol{\omega}_e^a\times\left(\mathbf{C}_e^a(t)\cdot\mathbf{r}_e\right)\right)-\Delta\mathbf{V}_{0a}-\Delta\int_0^t\mathbf{g}_a(\tau)d\tau\end{aligned} \tag{61}$$

Similarly, taking the total differentiation of Eq.(60) gives apparent position error

$$\begin{aligned}\delta\mathbf{X}_{\mathbf{Pr}} &= \Delta\left(\mathbf{C}_e^a(t)\cdot\mathbf{r}_e-\mathbf{R}_{0a}-\mathbf{V}_{0a}\cdot t-\int_0^t\int_0^u\mathbf{g}_a(\tau)d\tau du\right)\\ &= \Delta\left(\mathbf{C}_e^a(t)\cdot\mathbf{r}_e\right)-\Delta\mathbf{R}_{0a}-\Delta\mathbf{V}_{0a}\cdot t-\Delta\int_0^t\int_0^u\mathbf{g}_a(\tau)d\tau du\end{aligned} \tag{62}$$

4.5.1 Error analysis of apparent velocity

It follows From Eq.(61) that the tracking apparent velocity is related to initial localization and orientation parameters, initial velocity and the calculation of attraction. To separate the initial errors, the relationship between them is needed to be analyzed. Four terms contained in Eq.(61) are taken into account as follows.

1. First term

The first term on the right-hand side of Eq.(61) can be written in expended form

$$\delta\mathbf{X}_{pv1} = \Delta\mathbf{C}_e^a(t)\cdot\mathbf{V}_e(t) = \left(\frac{\partial\mathbf{C}_e^a}{\partial\lambda_T'}\Delta\lambda_T'+\frac{\partial\mathbf{C}_e^a}{\partial B_T'}\Delta B_T'+\frac{\partial\mathbf{C}_e^a}{\partial A_T'}\Delta A_T'\right)\cdot\mathbf{V}_e(t) \tag{63}$$

where

$$\frac{\partial C_e^a}{\partial\lambda_T'} = \begin{bmatrix} -\cos(\lambda_T'-\omega_e t)\sin A_T'+\sin(\lambda_T'-\omega_e t)\sin B_T'\cos A_T' & -\cos(\lambda_T'-\omega_e t)\sin B_T'\cos A'-\sin(\lambda_T'-\omega_e t)\sin A_T' & 0\\ -\sin(\lambda_T'-\omega_e t)\cos B_T' & \cos(\lambda_T'-\omega_e t)\cos B_T' & 0\\ -\cos(\lambda_T'-\omega_e t)\cos A_T'-\sin(\lambda_T'-\omega_e t)\sin B_T'\sin A_T' & \cos(\lambda_T'-\omega_e t)\sin B_T'\sin A_T'-\sin(\lambda_T'-\omega_e t)\cos A_T' & 0 \end{bmatrix}$$

$$\frac{\partial C_e^a}{\partial B_T'} = \begin{pmatrix} -\cos(\lambda_T'-\omega_e t)\cos B_T'\cos A_T' & -\sin(\lambda_T'-\omega_e t)\cos B_T'\cos A_T' & -\sin B_T'\cos A_T'\\ -\cos(\lambda_T'-\omega_e t)\sin B_T' & -\sin(\lambda_T'-\omega_e t)\sin B_T' & \cos B_T'\\ \cos(\lambda_T'-\omega_e t)\cos B_T'\sin A_T' & \sin(\lambda_T'-\omega_e t)\cos B_T'\sin A_T' & \sin B_T'\sin A_T' \end{pmatrix}$$

$$\frac{\partial C_e^a}{\partial A_T'} = \begin{bmatrix} -\sin(\lambda_T'-\omega_e t)\cos A_T'+\cos(\lambda_T'-\omega_e t)\sin B_T'\sin A_T' & \sin(\lambda_T'-\omega_e t)\sin B_T'\sin A_T'+\cos(\lambda_T'-\omega_e t)\cos A_T' & -\cos B_T'\sin A_T'\\ 0 & 0 & 0\\ \sin(\lambda_T'-\omega_e t)\sin A_T'+\cos(\lambda_T'-\omega_e t)\sin B_T'\cos A_T' & \sin(\lambda_T'-\omega_e t)\sin B_T'\cos A_T'-\cos(\lambda_T'-\omega_e t)\sin A_T' & -\cos B_T'\cos A_T' \end{bmatrix}$$

Therefore, $\delta\mathbf{X}_{pv1}$ can be rewritten as follows

$$\delta\mathbf{X}_{pv1} = \begin{bmatrix}\dfrac{\partial\mathbf{C}_e^a}{\partial\lambda_T'}\cdot\mathbf{V}_e & \dfrac{\partial\mathbf{C}_e^a}{\partial B_T'}\cdot\mathbf{V}_e & \dfrac{\partial\mathbf{C}_e^a}{\partial A_T'}\cdot\mathbf{V}_e\end{bmatrix}\cdot\mathbf{P}_t \tag{64}$$

where $\mathbf{P}_t \equiv \begin{bmatrix}\Delta\lambda_T' & \Delta B_T' & \Delta A_T'\end{bmatrix}^T$.

2. Second term

The second term on the right-hand side of Eq.(61) can be written in expended form

$$\delta\mathbf{X}_{pv2}=\Delta\left(\boldsymbol{\omega}_e^a\times(\mathbf{C}_e^a(t)\mathbf{r}_e)\right)=\Delta\boldsymbol{\omega}_e^a\times(\mathbf{C}_e^a(t)\mathbf{r}_e)+\boldsymbol{\omega}_e^a\times\Delta(\mathbf{C}_e^a(t)\mathbf{r}_e)$$
$$=\left(\frac{\partial\boldsymbol{\omega}_e^a}{\partial\lambda_T'}\Delta\lambda_T'+\frac{\partial\boldsymbol{\omega}_e^a}{\partial B_T'}\Delta B_T'+\frac{\partial\boldsymbol{\omega}_e^a}{\partial A_T'}\Delta A_T'\right)\times(\mathbf{C}_e^a(t)\mathbf{r}_e)+\boldsymbol{\omega}_e^a\times\left(\frac{\partial\mathbf{C}_e^a}{\partial\lambda_T'}\Delta\lambda'+\frac{\partial\mathbf{C}_e^a}{\partial B_T'}\Delta B_T'+\frac{\partial\mathbf{C}_e^a}{\partial A_T'}\Delta A_T'\right)\mathbf{r_e} \quad (65)$$

where

$$\frac{\partial\boldsymbol{\omega}_e^a}{\partial\lambda_T'}=0\,,\ \frac{\partial\boldsymbol{\omega}_e^a}{\partial B_T'}=\omega_e\begin{pmatrix}-\sin B_T'\cos A_T'\\ \cos B_T'\\ \sin B_T'\sin A_T'\end{pmatrix},\text{ and }\frac{\partial\boldsymbol{\omega}_e^a}{\partial A_T'}=\omega_e\begin{pmatrix}-\cos B_T'\sin A_T'\\ 0\\ -\cos B_T'\cos A_T'\end{pmatrix}.$$

Thus, Eq.(65) can be rewritten as follows

$$\delta\mathbf{X}_{pv2}=\left[\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{C}_e^a}{\partial\lambda_T'}\mathbf{r}_e\quad \frac{\partial\boldsymbol{\omega}_e^a}{\partial B_T'}\times(\mathbf{C}_e^a(t)\mathbf{r}_e)+\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{C}_e^a}{\partial B_T'}\mathbf{r}_e\quad \frac{\partial\boldsymbol{\omega}_e^a}{\partial A_T'}\times(\mathbf{C}_e^a(t)\mathbf{r}_e)+\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{C}_e^a}{\partial A_T'}\mathbf{r}_e\right]\mathbf{P}_t \quad (66)$$

3. Third term

At launch, launch coordinate system coincides with launch inertial coordinate system, so the initial velocity expressed in the launch inertial coordinate system can be substituted for the initial velocity expressed in the launch coordinate system.

The third term on the right-hand side of Eq.(61) can be written in expended form

$$\begin{aligned}\delta\mathbf{X}_{Pv3}&=-\Delta\mathbf{V}_{0a}=-\Delta\left(\boldsymbol{\omega}_e^a(0)\times\mathbf{R}_{0a}\right)-\Delta\left(\mathbf{C}_n^a(0)\cdot\mathbf{V}_s^n\right)\\&=-\left(\frac{\partial\boldsymbol{\omega}_e^a}{\partial\lambda_T'}\Delta\lambda_T'+\frac{\partial\boldsymbol{\omega}_e^a}{\partial B_T'}\Delta B_T'+\frac{\partial\boldsymbol{\omega}_e^a}{\partial A_T'}\Delta A_T'\right)\times\mathbf{R}_{0a}\\&-\boldsymbol{\omega}_e^a\times\left(\frac{\partial\mathbf{R}_{0a}}{\partial\lambda_T'}\Delta\lambda_T'+\frac{\partial\mathbf{R}_{0a}}{\partial B_T'}\Delta B_T'+\frac{\partial\mathbf{R}_{0a}}{\partial A_T'}\Delta A_T'+\frac{\partial\mathbf{R}_{0a}}{\partial\lambda_0'}\Delta\lambda_0'+\frac{\partial\mathbf{R}_{0a}}{\partial B_0'}\Delta B_0'+\frac{\partial\mathbf{R}_{0a}}{\partial H_0'}\Delta H_0'\right)\\&-\left(\frac{\partial\mathbf{C}_n^a(0)}{\partial\lambda_T'}\Delta\lambda_T'+\frac{\partial\mathbf{C}_n^a(0)}{\partial B_T'}\Delta B_T'+\frac{\partial\mathbf{C}_n^a(0)}{\partial A_T'}\Delta A_T'\right)\mathbf{V}_s^n-\mathbf{C}_n^a(0)\cdot\Delta\mathbf{V}_s^n\end{aligned} \quad (67)$$

Similarly, Eq.(67) can be rewritten in the form

$$\begin{aligned}\delta\mathbf{X}_{Pv3}&=-\left[\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{R}_{0a}}{\partial\lambda_0'}\quad\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{R}_{0a}}{\partial B_0'}\quad\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{R}_{0a}}{\partial H_0'}\right]\cdot\mathbf{P}_s\\&-\left[\frac{\partial\boldsymbol{\omega}_e^a}{\partial\lambda_T'}\times\mathbf{R}_{0a}+\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{R}_{0a}}{\partial\lambda_T'}\quad\frac{\partial\boldsymbol{\omega}_e^a}{\partial B_T'}\times\mathbf{R}_{0a}+\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{R}_{0a}}{\partial B_T'}\quad\frac{\partial\boldsymbol{\omega}_e^a}{\partial A_T'}\times\mathbf{R}_{0a}+\boldsymbol{\omega}_e^a\times\frac{\partial\mathbf{R}_{0a}}{\partial A_T'}\right]\mathbf{P}_t\\&-\left[\frac{\partial\mathbf{C}_n^a(0)}{\partial\lambda_T'}\mathbf{V}_s^n\quad\frac{\partial\mathbf{C}_n^a(0)}{\partial B_T'}\mathbf{V}_s^n\quad\frac{\partial\mathbf{C}_n^a(0)}{\partial A_T'}\mathbf{V}_s^n\right]\mathbf{P}_t-\mathbf{C}_n^a(0)\cdot\mathbf{P}_v\end{aligned} \quad (68)$$

where $\mathbf{C}_n^a(0)=\mathbf{C}_n^g(0)$, $\mathbf{P}_s\equiv\left[\Delta\lambda_0'\quad\Delta B_0'\quad\Delta H_0'\right]^T$, $\mathbf{P}_v\equiv\left[\Delta V_{sx}\quad\Delta V_{sy}\quad\Delta V_{sz}\right]^T$.

4. Fourth term

Because the telemetry data don't contain the effect of gravitational acceleration, the effect of gravitational acceleration of tracking data is necessary to drop when computing the difference between telemetry data and tracking data. Integrating gravitational acceleration one can obtain the velocity and perform the integration again to obtain the position. It is noted that the tracking data is used to calculate the gravitational acceleration. It follows from the previous section that the gravitational acceleration in the launch inertial coordinate system is given by

$$\mathbf{g}_a = g_r \frac{\mathbf{C}_e^a \cdot \mathbf{r}_e}{r} + g_\omega \frac{\mathbf{C}_e^a \cdot \boldsymbol{\omega}_{e0}}{\omega_e} = \mathbf{C}_e^a \cdot \left(g_r \frac{\mathbf{r}_e}{r} + g_\omega \frac{\boldsymbol{\omega}_{e0}}{\omega_e} \right) \tag{69}$$

By examining Eq.(69) we can find that the main reason introducing the computational error of gravitational acceleration is that there exist errors in the Euler angles of transformation matrix $\mathbf{C}_e^a$, whereas the bracketed term on the right-hand side of Eq.(69) is error-free. It is noted that

$$\sin\varphi_e = \frac{\mathbf{r}_a \cdot \boldsymbol{\omega}_{ea}}{r\omega_e} = \frac{(\mathbf{C}_e^a \mathbf{r}_e)\cdot(\mathbf{C}_e^a \boldsymbol{\omega}_{e0})}{r\omega_e} = \frac{(\mathbf{C}_e^a \mathbf{r}_e)^T (\mathbf{C}_e^a \boldsymbol{\omega}_{e0})}{r\omega_e} = \frac{\mathbf{r}_e \cdot \boldsymbol{\omega}_{e0}}{r\omega_e} \tag{70}$$

which can be computed exactly, thus, the error of gravitational acceleration is given by

$$\Delta\mathbf{g}_a = \Delta\mathbf{C}_e^a \cdot \left(g_r \frac{\mathbf{r}_e}{r} + g_\omega \frac{\boldsymbol{\omega}_{e0}}{\omega_e} \right) = \left[\frac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \mathbf{g}_e \quad \frac{\partial \mathbf{C}_e^a}{\partial B_T'} \mathbf{g}_e \quad \frac{\partial \mathbf{C}_e^a}{\partial A_T'} \mathbf{g}_e \right] \mathbf{P}_t = \mathbf{G}_g \cdot \mathbf{P}_t \tag{71}$$

where $\mathbf{g}_e = \left(g_r \frac{\mathbf{r}_e}{r} + g_\omega \frac{\boldsymbol{\omega}_{e0}}{\omega_e} \right)$.

The error of tracking apparent velocity is given by

$$\delta\mathbf{X}_{Pv4} = -\int_0^t \Delta\mathbf{g}_a(\tau) d\tau \cdot \mathbf{P}_t = -\int_0^t \mathbf{G}_g(\tau) d\tau \cdot \mathbf{P}_t \tag{72}$$

4.5.2 Error analysis of apparent position

Recalling Eq.(62) gives apparent position error

$$\begin{aligned} \delta\mathbf{X}_{\mathbf{Pr}} &= \Delta\left(\mathbf{C}_e^a(t) \cdot \mathbf{r}_e - \mathbf{R}_{0a} - \mathbf{V}_{0a} \cdot t - \int_0^t \int_0^u \mathbf{g}_a(\tau) d\tau du \right) \\ &= \Delta\left(\mathbf{C}_e^a(t) \cdot \mathbf{r}_e \right) - \Delta\mathbf{R}_{0a} - \Delta\mathbf{V}_{0a} \cdot t - \Delta\int_0^t \int_0^u \mathbf{g}_a(\tau) d\tau du \end{aligned} \tag{73}$$

In the similar manner four terms contained in Eq.(73) are analyzed as follows.

1. First term

The first term on the right-hand side of Eq.(73) can be written in expended form

$$\delta\mathbf{X}_{Pr1} = \Delta\mathbf{C}_e^a \cdot \mathbf{r}_e = \left(\frac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \Delta\lambda' + \frac{\partial \mathbf{C}_e^a}{\partial B_T'} \Delta B_T' + \frac{\partial \mathbf{C}_e^a}{\partial A_T'} \Delta A_T' \right) \cdot \mathbf{r}_e \tag{74}$$

Rearranging Eq.(74) gives

$$\delta\boldsymbol{X}_{Pr1} = \left[\frac{\partial \boldsymbol{C}_e^a}{\partial \lambda_T'} \cdot \boldsymbol{r}_e \quad \frac{\partial \boldsymbol{C}_e^a}{\partial B_T'} \cdot \boldsymbol{r}_e \quad \frac{\partial \boldsymbol{C}_e^a}{\partial A_T'} \cdot \boldsymbol{r}_e \right] \cdot \boldsymbol{P}_t \tag{75}$$

2. Second term

The second term on the right-hand side of Eq.(73) can be written in expended form

$$\begin{aligned}\delta\mathbf{X}_{Pr2} &= -\Delta\mathbf{C}_e^a\cdot\mathbf{R}_{0e}-\mathbf{C}_e^a\cdot\Delta\mathbf{R}_{0e}\\ &= -\left(\frac{\partial\mathbf{C}_e^a}{\partial\lambda_T'}\Delta\lambda'+\frac{\partial\mathbf{C}_e^a}{\partial B_T'}\Delta B_T'+\frac{\partial\mathbf{C}_e^a}{\partial A_T'}\Delta A_T'\right)\cdot\mathbf{R}_{0e}\\ &\quad-\mathbf{C}_e^a\cdot\left(\frac{\partial\mathbf{R}_{0e}}{\partial\lambda_0'}\Delta\lambda_0'+\frac{\partial\mathbf{R}_{0e}}{\partial B_0'}\Delta B_0'+\frac{\partial\mathbf{R}_{0e}}{\partial H_0'}\Delta H_0'\right)\end{aligned}\tag{76}$$

It follows from the previous section that

$$\mathbf{R}_{0e}=\begin{bmatrix}(N_0+H_0')\cos B_0'\cos\lambda_0'\\(N_0+H_0')\cos B_0'\sin\lambda_0'\\\left[N_0\left(1-e^2\right)+H_0'\right]\sin B_0'\end{bmatrix}\tag{77}$$

Therefore, we can have that

$$\frac{\partial\mathbf{R}_{0e}}{\partial\lambda_0'}=\begin{pmatrix}-\cos B_0'\sin\lambda_0'[H_0'+a_e(1+\alpha_e\sin^2 B_0')]\\\cos B_0'\cos\lambda_0'[H_0'+a_e(1+\alpha_e\sin^2 B_0')]\\0\end{pmatrix}\tag{78}$$

$$\frac{\partial\mathbf{R}_{0e}}{\partial B_0'}=\begin{pmatrix}-\cos\lambda_0'\sin B_0'[a_e+H_0'-\frac{1}{2}a_e\alpha_e(1+3\cos 2B_0')]\\-\sin\lambda_0'\sin B_0'[a_e+H_0'-\frac{1}{2}a_e\alpha_e(1+3\cos 2B_0')]\\\cos B_0'[a_e+H_0'-2a_e\alpha_e-\frac{3}{2}a_e\alpha_e(-1+2\alpha_e)(1-\cos 2B_0')]\end{pmatrix}\tag{79}$$

$$\frac{\partial\mathbf{R}_{0e}}{\partial H_0'}=\begin{pmatrix}\cos B_0'\cos\lambda_0'\\\cos B_0'\sin\lambda_0'\\\sin B_0'\end{pmatrix}\tag{80}$$

Thus, Eq.(76) can be rewritten as

$$\delta\mathbf{X}_{Pr2}=-\begin{bmatrix}\frac{\partial\mathbf{C}_e^a}{\partial\lambda_T'}\mathbf{R}_{0e} & \frac{\partial\mathbf{C}_e^a}{\partial B_T'}\mathbf{R}_{0e} & \frac{\partial\mathbf{C}_e^a}{\partial A_T'}\mathbf{R}_{0e}\end{bmatrix}\cdot\mathbf{P}_t-\begin{bmatrix}\mathbf{C}_e^a\frac{\partial\mathbf{R}_{0e}}{\partial\lambda_0'} & \mathbf{C}_e^a\frac{\partial\mathbf{R}_{0e}}{\partial B_0'} & \mathbf{C}_e^a\frac{\partial\mathbf{R}_{0e}}{\partial H_0'}\end{bmatrix}\cdot\mathbf{P}_s\tag{81}$$

3. Third term

Similarly, launch coordinate system coincides with launch inertial coordinate system at launch moment, so the radius of earth center in the launch inertial coordinate system can be represented by that in the launch coordinate system, thus,

$$\delta\mathbf{X}_{Pr3}=-\Delta\left(\boldsymbol{\omega}_e^a\times\mathbf{R}_{0a}\right)t-\Delta\left(\mathbf{C}_n^a(0)\cdot\mathbf{V}_s^n\right)\cdot t\tag{82}$$

Combing the analysis of apparent velocity gives

$$\delta\mathbf{X}_{Pr3} = \delta\mathbf{X}_{Pv3} \cdot t \tag{83}$$

4. Fourth term

The fourth term is the gravitational acceleration term, which can be obtained by integrating the error of apparent tracking velocity, written as

$$\delta X_{Pr4} = -\int_0^t \delta X_{Pv4} d\tau = -\int_0^t \int_0^u G_g(\tau) d\tau du \tag{84}$$

4.5.3 Relationship of the difference between telemetry data, tracking data and initial errors

According to the above analysis, the relationship of the difference between telemetry velocity and tracking velocity and initial errors can be concluded as follows

$$\begin{aligned} \delta\mathbf{X}_{Pv} &= \delta\mathbf{X}_{Pv1} + \delta\mathbf{X}_{Pv2} + \delta\mathbf{X}_{Pv3} + \delta\mathbf{X}_{Pv4} \\ &= \mathbf{G}_{vt} \cdot \mathbf{P}_t + \mathbf{G}_{vs} \cdot \mathbf{P}_s + \mathbf{G}_{vv} \cdot \mathbf{P}_v + \Delta\mathbf{v}_g \end{aligned} \tag{85}$$

where

$$\mathbf{G}_{vt1} = \frac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \cdot \mathbf{V}_e + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \mathbf{r}_e - \left(\frac{\partial \boldsymbol{\omega}_e^a}{\partial \lambda_T'} \times \mathbf{R}_{0\text{a}} + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial \lambda_T'} \right) - \frac{\partial \mathbf{C}_n^a(0)}{\partial \lambda_T'} \mathbf{V}_s^n \tag{86}$$

$$\mathbf{G}_{vt2} = \frac{\partial \mathbf{C}_e^a}{\partial B_T'} \cdot \mathbf{V}_e + \frac{\partial \boldsymbol{\omega}_e^a}{\partial B_T'} \times (\mathbf{C}_e^a \mathbf{r}_e) + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{C}_e^a}{\partial B_T'} \mathbf{r}_e - \left(\frac{\partial \boldsymbol{\omega}_e^a}{\partial B_T'} \times \mathbf{R}_{0a} + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial B_T'} \right) - \frac{\partial \mathbf{C}_n^a(0)}{\partial B_T'} \mathbf{V}_s^n \tag{87}$$

$$\mathbf{G}_{vt3} = \frac{\partial \mathbf{C}_e^a}{\partial A_T'} \cdot \mathbf{V}_e + \frac{\partial \boldsymbol{\omega}_e^a}{\partial A_T'} \times (\mathbf{C}_e^a \mathbf{r}_e) + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{C}_e^a}{\partial A_T'} \mathbf{r}_e - \left(\frac{\partial \boldsymbol{\omega}_e^a}{\partial A_T'} \times \mathbf{R}_{0a} + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial A_T'} \right) - \frac{\partial \mathbf{C}_n^a(0)}{\partial A_T'} \mathbf{V}_s^n \tag{88}$$

$$\mathbf{G}_{vs} = \begin{bmatrix} -\boldsymbol{\omega}_e^a \times \dfrac{\partial \mathbf{R}_{0a}}{\partial \lambda_0'} & -\boldsymbol{\omega}_e^a \times \dfrac{\partial \mathbf{R}_{0a}}{\partial B_0'} & -\boldsymbol{\omega}_e^a \times \dfrac{\partial \mathbf{R}_{0a}}{\partial H_0'} \end{bmatrix} \tag{89}$$

$$\mathbf{G}_{vv} = -\mathbf{C}_n^a(0) \tag{90}$$

$$\Delta\dot{\mathbf{v}}_g = -\Delta\mathbf{g}_a = -\begin{bmatrix} \dfrac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \mathbf{g}_e & \dfrac{\partial \mathbf{C}_e^a}{\partial B_T'} \mathbf{g}_e & \dfrac{\partial \mathbf{C}_e^a}{\partial A_T'} \mathbf{g}_e \end{bmatrix} \mathbf{P}_t \tag{91}$$

In the same manner the relationship of the difference between telemetry position and tracking position and initial errors can be concluded as follows

$$\begin{aligned} \delta\mathbf{X}_{Pr} &= \delta\mathbf{X}_{Pr1} + \delta\mathbf{X}_{Pr2} + \delta\mathbf{X}_{Pr3} + \delta\mathbf{X}_{Pr4} \\ &= \mathbf{G}_{st} \cdot \mathbf{P}_t + \mathbf{G}_{ss} \cdot \mathbf{P}_s + \mathbf{G}_{sv} \cdot \mathbf{P}_v + \Delta\mathbf{s}_g \end{aligned} \tag{92}$$

where

$$\mathbf{G}_{st1} = \frac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \cdot \mathbf{r}_e - \frac{\partial \mathbf{C}_e^a}{\partial \lambda_T'} \mathbf{R}_{e0} - \left(\frac{\partial \boldsymbol{\omega}_e^a}{\partial \lambda_T'} \times \mathbf{R}_{0a} + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial \lambda_T'} \right) \cdot t - \frac{\partial \mathbf{C}_n^a(0)}{\partial \lambda_T'} \mathbf{V}_s^n \cdot t \tag{93}$$

$$\mathbf{G}_{st2} = \frac{\partial \mathbf{C}_e^a}{\partial B_T'} \cdot \mathbf{r}_e - \frac{\partial \mathbf{C}_e^a}{\partial B_T'} \mathbf{R}_{e0} - \left(\frac{\partial \boldsymbol{\omega}_e^a}{\partial B_T'} \times \mathbf{R}_{0a} + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial B_T'} \right) \cdot t - \frac{\partial \mathbf{C}_n^a(0)}{\partial B_T'} \mathbf{V}_s^n \cdot t \tag{94}$$

$$\mathbf{G}_{st3} = \frac{\partial \mathbf{C}_e^a}{\partial A_T'} \cdot \mathbf{r}_e - \frac{\partial \mathbf{C}_e^a}{\partial A_T'} \mathbf{R}_{e0} - \left(\frac{\partial \boldsymbol{\omega}_e^a}{\partial A_T'} \times \mathbf{R}_{0a} + \boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial A_T'} \right) \cdot t - \frac{\partial \mathbf{C}_n^a(0)}{\partial A_T'} \mathbf{V}_s^n \cdot t \tag{95}$$

$$\boldsymbol{G}_{ss} = \left[-\boldsymbol{C}_e^a \frac{\partial \boldsymbol{R}_{0e}}{\partial \lambda_0'} - \boldsymbol{\omega}_e^a \times \frac{\partial \boldsymbol{R}_{0a}}{\partial \lambda_0'} t \quad -\boldsymbol{C}_e^a \frac{\partial \boldsymbol{R}_{0e}}{\partial B_0'} - \boldsymbol{\omega}_e^a \times \frac{\partial \boldsymbol{R}_{0a}}{\partial B_0'} t \quad -\boldsymbol{C}_e^a \frac{\partial \boldsymbol{R}_{0e}}{\partial H_0'} - \boldsymbol{\omega}_e^a \times \frac{\partial \boldsymbol{R}_{0a}}{\partial H_0'} t \right] \tag{96}$$

$$\mathbf{G}_{sv} = -\mathbf{C}_n^a(0) \cdot t \,, \ \Delta \dot{\mathbf{s}}_g = \Delta \mathbf{v}_g \tag{97}$$

Let $\delta \mathbf{X}_P = \left[\delta \mathbf{X}_{Pv} \ \ \delta \mathbf{X}_{Pr} \right]^T$, $\mathbf{P}_a = \left[\mathbf{P}_t \quad \mathbf{P}_s \quad \mathbf{P}_v \right]^T$, then the difference between the telemetry data and tracking data can be written in matrix form

$$\delta \mathbf{X}_P = \begin{bmatrix} \mathbf{G}_{vt} - \int_0^t \mathbf{G}_g dt & \mathbf{G}_{vs} & \mathbf{G}_{vv} \\ \mathbf{G}_{st} - \int_0^t \int_0^u \mathbf{G}_g d\tau du & \mathbf{G}_{ss} & \mathbf{G}_{sv} \end{bmatrix} \cdot \mathbf{P}_a = \begin{bmatrix} \mathbf{G}_v \\ \mathbf{G}_s \end{bmatrix} \cdot \mathbf{P}_a \tag{98}$$

By examining the above model, we can find that the correlation of the environmental function column corresponding to the geodetic latitude and height in the velocity domain, namely, $-\boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial B_0'}$ and $-\boldsymbol{\omega}_e^a \times \frac{\partial \mathbf{R}_{0a}}{\partial H_0'}$ in the $\mathbf{G}_{vs}$ matrix, is large and the separation between them is not easy. But in the position domain, the property of initial error environmental function matrix is good therefore, the separation of initial errors is needed to perform in the position domain or velocity-position domain.

4.6 Separation model of instrumentation errors and initial errors

It is pointed out in the previous section that the guidance instrumentation systematic errors are contained in the telemetry data and the initial errors are primarily introduced during the data processing of tracking data. Consequently, in addition to the alignment errors and levelling errors of inertial platform and initial error parameters, the other error coefficients are separated. It follows from Eqs.(51) and (98) that the relationship involved in instrumentation error coefficients and initial errors as well as the difference between telemetry data and tracking data, which can be described as follows

$$\boldsymbol{\delta}\mathbf{X} = \mathbf{S} \cdot \mathbf{D} - \mathbf{G} \cdot \mathbf{P}_a + \boldsymbol{\varepsilon} \tag{99}$$

where $\mathbf{S}$ is the environmental function matrix of instrumentation errors and $\mathbf{G}$ is the environmental function matrix of initial errors. This model is known as the separation model of instrumentation errors and initial errors and it is a linear model.

5. Simulated cases

In the previous section, the separation model of initial errors based on telemetry and tracking data and the separation model of instrumentation errors and initial errors are deduced in detail. In this section, numerical examples are given to verify the separation model of initial errors and instrumentation errors and initial errors.

5.1 Verification of separation model of initial errors

The telemetry and tracking data are obtained using the six-degree-of-freedom ballistic program. For the certain trajectory with 10000 kilometers of range, the initial errors are listed in Table 1.

Parameter	Error Value	Parameter	Error Value	Parameter	Error Value
Astronomical Longitude λ_T	30 arcsec	Geodetic Longitude λ_0	-20 arcsec	Initial Velocity V_x	-0.1m/s
Astronomical Latitude B_T	30 arcsec	Geodetic Latitude B_0	-20 arcsec	Initial Velocity V_y	-0.05m/s
Astronomical Azimuth A_T	120 arcsec	Geodetic Height H_0	-5 m	Initial Velocity V_z	0.1m/s

Table 1. The true values of initial errors.

During the simulation process, all the guidance instrumentation systematic errors are set to zero therefore, the difference between telemetry data and tracking data merely contain initial errors. Herein, define $\mathbf{Y} = \mathbf{G} \cdot \mathbf{P}_{a0}$, namely, $\mathbf{Y}$ is the difference between telemetry data and tracking data, which is calculated using the product of environmental function matrix of initial errors $\mathbf{G}$ and true values of initial errors $\mathbf{P}_{a0}$. Define $\mathbf{\delta X}_P$ is the difference between telemetry data and tracking data obtained by the simulation data. Now, define $\mathbf{\delta Y} = \mathbf{\delta X}_P - \mathbf{Y}$ is the residual of the difference between telemetry data and tracking data. Simulation results are shown in the following figures, Fig.4 shows the difference between telemetry velocity and tracking velocity, $\boldsymbol{\delta X}_{Pv}$; Fig.5 shows the difference between telemetry position and tracking position, $\mathbf{\delta X}_{Ps}$; Fig.6 shows the residual of the difference between telemetry velocity and tracking velocity, $\mathbf{\delta Y}_v$; and Fig.7 shows the residual of the difference between telemetry position and tracking position, $\mathbf{\delta Y}_s$.

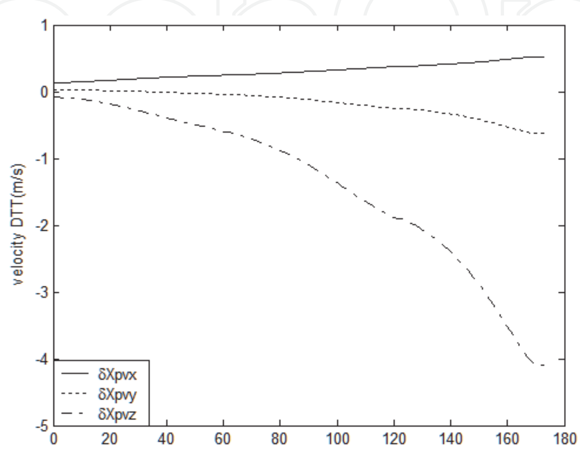


Fig. 4. The difference between telemetry and tracking velocity.

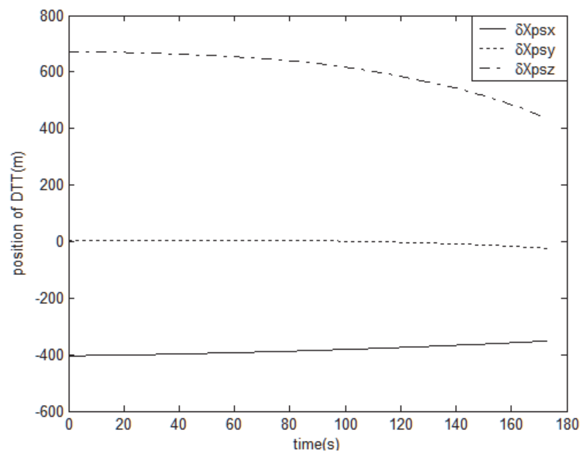


Fig. 5. The difference between telemetry and tracking position.

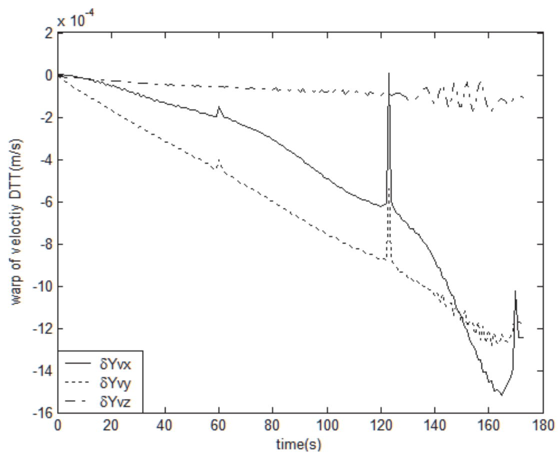


Fig. 6. The residual of the difference between telemetry and tracking velocity.

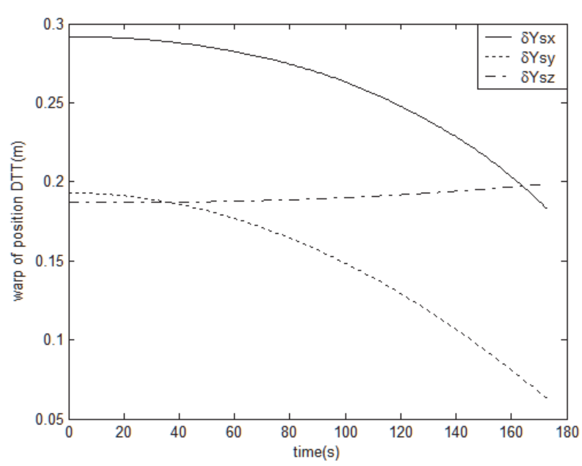


Fig. 7. The residual of the difference between telemetry and tracking position.

It is clearly seen from Figs. 4 and 6 that the differences between telemetry velocity and tracking velocity obtained by the two methods agree well. When the third stage engine shut down, the difference between telemetry velocity and tracking velocity is $(0.52, -0.53, -4.1)\mathrm{m/s}$, while the largest residual computed by the two methods is $(-0.0015, -0.0013, -0.0002)\mathrm{m/s}$, which is quite smaller than the difference between telemetry velocity and tracking velocity. Similarly, as seen in Figs. 5 and 7, when the third stage engine shut down, the difference between telemetry and tracking position is $(-400, -25, 672)\mathrm{m}$, while the largest residual computed by the two methods is $(0.29, 0.19, 0.2)\mathrm{m}$, which is quite smaller than the difference between telemetry position and tracking position. It follows that the separation model of initial errors are exact and the accuracy is fine.

Therefore, the initial errors can be estimated by using the computed difference between telemetry data and tracking data and the environmental function matrix of initial errors. In position domain, using the least-square estimation method we can have

$$\hat{\mathbf{P}}_a = (\mathbf{G}_s{}^T \cdot \mathbf{G}_s)^{-1}\mathbf{G}_s{}^T\boldsymbol{\delta}\mathbf{X}_p \tag{100}$$

The estimates of initial errors are given in Table 2.

	λ_T (arcsec)	B_T (arcsec)	A_T (arcsec)	λ_0 (arcsec)	B_0 (arcsec)	H_0 (m)	V_x (m/s)	V_y (m/s)	V_z (m/s)
True	30	30	120	-20	-20	-5	-0.1	-0.05	0.1
Est	30.013	29.987	120.01	-19.988	-20.007	-5.210	-0.0993	-0.049	0.1001

Table 2. The estimates of initial errors. Notes: True denotes the true value of parameter and Est denotes the estimates of parameter.

As it is seen from Table 2, the estimated accuracy of initial errors is high, such as astronomical longitude and latitude, azimuth, geodetic longitude and latitude, among others, of which the estimated relative error is smaller than 0.1%. Simultaneously, the estimated relative error of initial velocity is smaller than 1% and the estimated relative error of geodetic height is 4.2%.

It is necessary to point out that the variation of apparent acceleration due to the uncertainty of initial parameters in the error separation model mentioned above is not taken into consideration. In effect, the state of missile will change as the initial launch parameters change, subsequently the thrust and aerodynamic forces acting on the missile will vary. Simulation results indicate that the assumption that the factor is neglected is rational in the most cases. However, if there are errors in the position of vertical direction, then the large errors may be caused, for example, the variation of geodetic height will affect the shape of the trajectory severely. Although the error of geodetic height affect the apparent acceleration, this deviation of apparent acceleration can be measured onboard and reflected in both telemetry data and tracking data, which can be offset when computing the difference between telemetry data and tracking data. In the practical project, the ballistic missile is generally equipped with guidance system. Under the ideal situation, the error of apparent acceleration due to the initial errors can be completely offset therefore, the error of apparent acceleration will do no effect on the impact point.

5.2 Verification of separation model of instrumentation errors and initial errors

In the same manner the telemetry data and tracking data are generated by using the six-degree-of-freedom ballistic program with 10000 kilometers of range, and the initial errors

are seen in Table 1. The model of guidance instrumentation systematic errors are given by Eqs.(22) and (23) and the levelling and alignment errors are not included.
Similarly, environmental function matrix of instrumentation error, $\mathbf{S}$, and environmental function matrix of initial errors, $\mathbf{G}$, are obtained to compute the difference between telemetry data and tracking data, $\mathbf{Y}$, which is defined as $\mathbf{Y} = \mathbf{S} \cdot \mathbf{D}_0 - \mathbf{G} \cdot \mathbf{P}_{a0}$. Simultaneously, $\boldsymbol{\delta}\mathbf{X}$ is the difference between telemetry and tracking data obtained by the simulation data. Likewise, define $\boldsymbol{\delta}\mathbf{Y} = \boldsymbol{\delta}\mathbf{X} - \mathbf{Y}$ is the residual. Simulation results are shown in the following figures, Fig.8 shows the difference between telemetry velocity and tracking velocity, $\boldsymbol{\delta}\mathbf{X}_v$; Fig.9 shows the difference between telemetry position and tracking position, $\boldsymbol{\delta}\mathbf{X}_s$; Fig.10 shows the residual of the difference between telemetry velocity and tracking velocity, $\boldsymbol{\delta}\mathbf{Y}_v$, and Fig.11 shows the residual of the difference between telemetry position and tracking position, $\boldsymbol{\delta}\mathbf{Y}_s$.

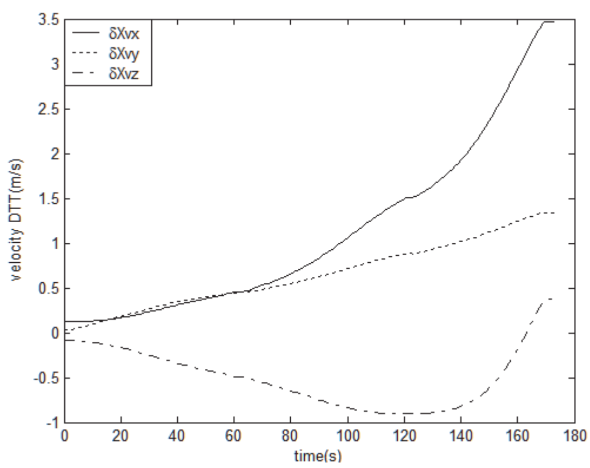


Fig. 8. The difference between telemetry and tracking velocity.

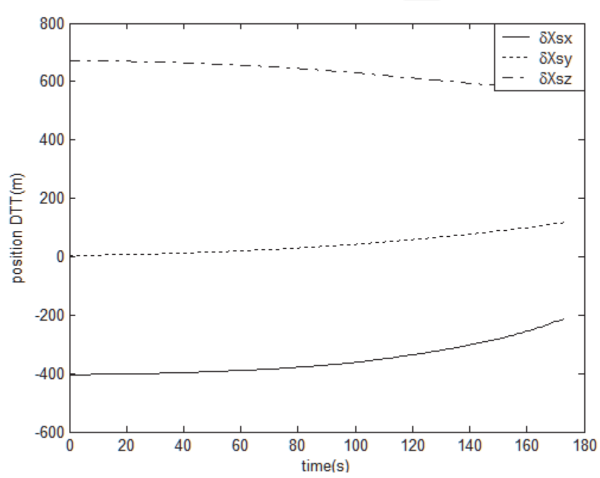


Fig. 9. The difference between telemetry and tracking position.

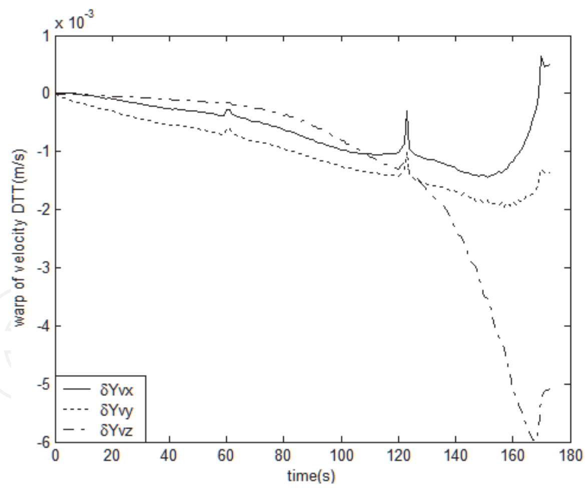


Fig. 10. The residual of the difference between telemetry and tracking velocity.

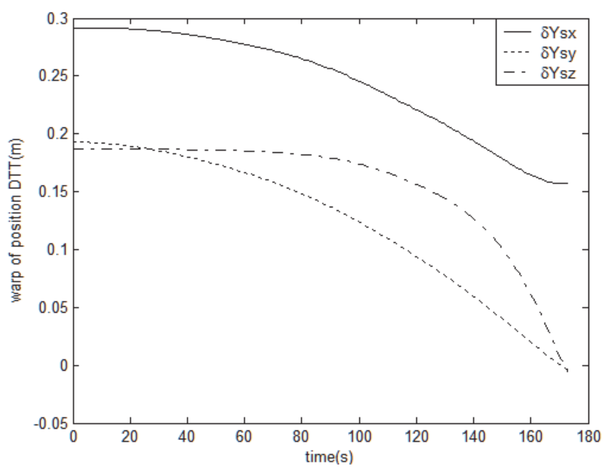


Fig. 11. The residual of the difference between telemetry and tracking position.

It is clearly seen from Figs. 8 and 10 that the differences between telemetry velocity and tracking velocity obtained by the two methods agree well. When the third stage engine shut down, the difference between telemetry velocity and tracking velocity is $(3.46, 1.34, -0.90)\mathrm{m/s}$, while the largest residual computed by the two methods is $(-0.0015, -0.0019, -0.006)\mathrm{m/s}$, which is quite smaller than the difference between telemetry velocity and tracking velocity. Similarly, as seen in Figs. 5 and 7, when the third stage engine shut down, the difference between telemetry position and tracking position is $(-400, -25, 672)\mathrm{m}$, while the largest residual computed by the two methods is $(0.29, 0.19, 0.19)\mathrm{m}$, which is quite smaller than the difference between telemetry position and tracking position. It follows that the separation model of instrumentation errors and initial errors are exact and precise.

The instrumentation errors and initial errors are estimated by using the above data. Selecting vector $\mathbf{K}=\begin{bmatrix}\mathbf{D}^T & \mathbf{P}_a^T\end{bmatrix}^T$ and letting $\mathbf{H}_s=\begin{bmatrix}\mathbf{S}_s{}^T & -\mathbf{G}_s{}^T\end{bmatrix}^T$, the in the position domain, the

$$\hat{\mathbf{K}} = (\mathbf{H}_s{}^T \mathbf{H}_s)^{-1} \mathbf{H}_s{}^T \boldsymbol{\delta}\mathbf{X} \tag{101}$$

The estimates of error coefficients of gyroscope and accelerometer are given in Tables 3 and 4, respectively. The estimates of initial errors are given in Table 5.

	k_{g0x}	k_{g0y}	k_{g0z}	k_{g11x}	k_{g11y}	k_{g11z}	k_{g12x}	k_{g12y}	k_{g12z}
True	0.5	0.3	-0.5	-0.01	0.01	0.01	-0.01	0.02	0.02
Est	0.632	0.368	-0.503	-0.0139	0.0051	-0.0172	-0.0113	0.0200	0.0201

Table 3. The estimates of gyroscope error coefficients. (Units: deg/hour)

	k_{a0x} ($\mathrm{m/s^2}$)	k_{a0y} ($\mathrm{m/s^2}$)	k_{a0z} ($\mathrm{m/s^2}$)
True	-2.0×10^{-3}	-2.0×10^{-3}	1.0×10^{-3}
Est	-2.0374×10^{-3}	-2.0086×10^{-3}	1.0826×10^{-3}
	k_{a1x}	k_{a1y}	k_{a1z}
True	5.0×10^{-4}	5.0×10^{-4}	5.0×10^{-4}
Est	5.0042×10^{-4}	5.0055×10^{-4}	3.8726×10^{-4}

Table 4. The estimates of accelerometer error coefficients.

	λ_T (arcsec)	B_T (arcsec)	A_T (arcsec)	λ_0 (arcsec)	B_0 (arcsec)	H_0 (m)	V_x (m/s)	V_y (m/s)	V_z (m/s)
True	30	30	120	-20	-20	-5	-0.1	-0.05	0.1
Est	30.586	30.198	116.25	-19.989	-20.007	-5.195	-0.0998	-0.049	0.1002

Table 5. The estimates of initial errors.

It is seen from Tables 3 through 5 that the instrumentation error coefficients and initial errors are well estimated in the position domain by using the separation model of instrumentation errors and initial errors mentioned above.

6. Conclusions

In this chapter, the separation model of initial launch parameter errors and guidance instrumentation systematic errors are formulated based on telemetry and tracking data. The calculation of difference between telemetry and tracking data is discussed in detail. It is generally considered that the telemetry data contain instrumentation errors while tracking data contain systematic errors and random measurement errors of exterior measurement equipment. Numerical examples are given for the verification of the separation by using six-degree-of-freedom trajectory program. Simulation results indicate that the separation model of initial errors and guidance instrumentation systematic errors can estimate the error coefficient well and is exact.

7. References

Barros A. S. & Rutledge D. N. (1998). Genetic Algorithm Applied to the Selection of Principal Components, Chemometrics and Intelligent Laboratory Systems, pp.65-81.

Cherkassky V. & Ma Y.Q. (2005). Multiple Model Regression Estimation, IEEE Trans on Neural Networks, Vol. 16, No. 4, 785-798.

Cortes C. & Vapnik V., (1995). Support-Vector Networks, Machine Learning, Vol(20), pp. 273～297.

Coulter J. E. & Meehan J. J. (1981). Test Methodology for Evaluation of the Input Axis of a Pendulous Integrating Gyro Accelerometer. AIAA81-1971.

Eduardo N. & Hugh D.W. (1999). Initial Calibration and Alignment of Low Cost Inertial Navigation Units for Land Vehicle Applications, Journal of Robotics Systems, Vol. 16, No. 2, pp. 81-92.

Forsberg R. & Sideris M. G. (1993). Geoid Computation by the Multi-band Spherical FFT Approach, manuscripta geodaetica, Vol. l8,No. 2, pp. 82－90.

Gore R. C. The Effect of Geophysical and Geodetic Uncertainties at Launch Area on Ballistic Missile Impact Accuracy, AD602214.

Jackson A. D. (1973). Continuous Calibration and Alignment Techniques for an All-attitude Inertial Platform, AIAA1973-865.

IEEE Standards Committee (1971). IEEE Standard Specification Format Guide and Test Procedure for Linear, Single-Axis, Pendulous, Analog Torque Balance Accelerometer, IEEE Std337-1972, , New York.

IEEE Standards Board (1973). IEEE Standard Specification Format Guide and Test Procedure for Single-Degree-of-Freedom Rate-Integrating Gyros, IEEE Std517-1974, Approved December 13.

John L. (1979). Characterization Testing of the MX AIRS. AIAA79-1888.

Liu Y. C., Fang H.R. and Zhang F. (2000). Telemetry and Telecommand System, National Defense Industry Press, Beijing, P.R.China.

Shattuck C.J.W. (1992). Technology and the PeaceKeeper, AIAA1992-1326.

Tony C. L. (2003). Development of U.S. Air Force Intercontinental Ballistic Missile Weapon Systems, Journal of Spacecraft and Rockets, Vol. 40, No. 4, pp. 491-509.

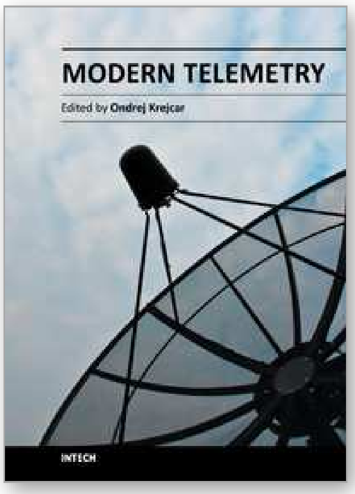
Titterton D. H. & Weston J. L. (1997). Strapdown Inertial Navigation Technology. London:United Kingdom, Peter Peregrinus.

Thompson A. A. (2000). Calibration of Inertial Sensors, AD20001121-025.

Wold S., Trygg J., Berglund A. et al. (2001). Some recent developments in PLS modeling. Chemometrics and Intelligent Laboratory Systems.

Yang H.B., Zhang S.F. and Cai H. (2007). Modeling and Parameters Estimation of Guidance Instrumentation Systematic Error and Initial Launched Parameters Error for Marine Missile, Journal of Astronautics, China, Vol. 28, No. 6, pp. 1638-1641.

Zheng W. (2006). Research on Effect of Geophysical Disturbance Factors on Hit Accuracy of Long-range Ballistic Missile and the Compensation Method for it, Graduate School of National University of Defense Technology Changsha，Hunan，P.R.China.



Modern Telemetry
Edited by Dr. Ondrej Krejcar

ISBN 978-953-307-415-3
Hard cover, 470 pages
Publisher InTech
Published online 05, October, 2011
Published in print edition October, 2011

Telemetry is based on knowledge of various disciplines like Electronics, Measurement, Control and Communication along with their combination. This fact leads to a need of studying and understanding of these principles before the usage of Telemetry on selected problem solving. Spending time is however many times returned in form of obtained data or knowledge which telemetry system can provide. Usage of telemetry can be found in many areas from military through biomedical to real medical applications. Modern way to create a wireless sensors remotely connected to central system with artificial intelligence provide many new, sometimes unusual ways to get a knowledge about remote objects behaviour. This book is intended to present some new up to date accesses to telemetry problems solving by use of new sensors conceptions, new wireless transfer or communication techniques, data collection or processing techniques as well as several real use case scenarios describing model examples. Most of book chapters deals with many real cases of telemetry issues which can be used as a cookbooks for your own telemetry related problems.

INTECH
open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821