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# Mathematical Modelling and Numerical Simulation of the Dynamic Behaviour of Thermal and Hydro Power Plants

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## 1. Introduction

The global economic and social development process has brought about increasing capacities of electric energy production, transportation and distribution. This fact has required both the development of the national power systems and the interconnection of most of them, leading to real expanded continental power systems. But technological development and territorial expansion have generated new problems concerning their running and monitoring. Planning, designing, leading and running such huge systems has turned to very complex activities, indissolubly linked with their operating stability. The problem of power system stability has got new spatial and temporal dimensions implying the reconsideration of the means and methods of analysis through revising and expanding mathematical modelling in order to get a most accurate numerical simulation of the operating regime<sup>1</sup>. The operating experience shows that the synchronism of synchronous generators in networking power plants can be lost even at a few minutes after a disturbance has appeared. In this case the phenomena are much more complex and they refer to slow power oscillations on the interconnecting electric lines among large areas which lead to a decrease in frequency and loss of synchronism among these regions. Such phenomena can appear due to the poor performance of the frequency - exchange power control and the unsatisfactory answer of the slow action of the governing elements as, for example, those of the boilers, turbines, charging valves, feeding pumps, hydro units etc. In this respect, they speak about Long Term Dynamic (LTD) stability or slow phenomena stability<sup>2</sup>. From the point of view of time scale analysis, the phenomena which are manifest in Long Term Dynamic processes are minute long, comprising a part of the time allotted to the variation of consumers' electric loading and the values of the time constants of the boilers and steam turbines as well as those of the primary installations of the hydro units. Therefore it is necessary to increase the number of the system elements whose mathematical modelling has to be considered in simulation, so that the main components of the power system be included starting from the thermal, hydro and mechanical primary installations up to the consumers, including the characteristics of the respective elements and the functional relationships between the input and the output values and the assembly as a whole.

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<sup>1</sup> (Ernst et al., 2004)

<sup>2</sup> (Hongesombut et al., 2005)

## 2. The mathematical modelling of the primary installations of a thermal power plant

In Long Term Dynamics, due to the expansion of the time scale analysis, the slow thermal- mechanical processes will appear and influence the behaviour of the electric power system.

### 2.1 The mathematical modelling of the steam boiler

Two aspects have to be considered in this case: on one hand, it is the realization of the mathematical model, which automatically represents a storing element, inducing an important delay of the output signal and, on the other hand, the determination of the mathematical model of the boiler control system which, from the point of view of automation, is a chain of proportionally integrating and proportionally derivative elements with multiple delays and limitations<sup>3</sup>.

The boilers used in thermal power plants are drum type boilers and once-through boilers. In the case of the former, their dynamics is dominated by fuels and air, and for the latter the feed water is dominant on the main parameter, which changes in large limits and can influence the power of the corresponding turbine, namely the pressure of the live steam,  $p_t$ . The boiler aggregate, as a controlled object, having pressure as an output value is considered to be made of two cascaded elements: the combustion chamber and the steam generator (figure 1). In the case of boilers without coal dust hopper there appears a third element, the coal mill (figure 2).

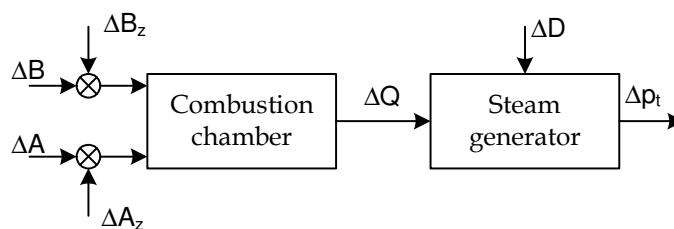


Fig. 1. The diagram of the steam boiler as a controlled object

The controlled value, pressure, changes with the variation of the heat quantity,  $\Delta Q$ , produced by the combustion chamber, as a result of the modification of the fuel flow rate,  $\Delta B$  and that of the air,  $\Delta A$ , at the entry into the combustion chamber. The main disturbances which operate on the steam pressure are the variation of the steam load required by the consumer,  $\Delta D$ , which acts as an external disturbance and the variation of the fuel flow rate,  $\Delta B_z$ , which acts as an internal disturbance. The variation of the air flow rate,  $\Delta A_z$ , although, from a quantitative point of view, having an effect similar with  $\Delta B_z$ , has got a much less effect on the steam pressure because the air excess in the combustion chamber leads to the increase of heat losses through the evacuated gases, and its excessive reduction determines the increase of losses due to imperfect chemical combustion. Steam pressure,  $p_t$ , remains constant if the thermal and mass balance are undisturbed.

<sup>3</sup> (Surianu, 2008)

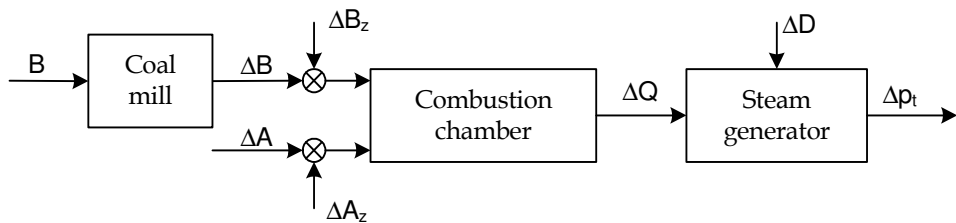


Fig. 2. The diagram of the steam boiler without coal dust hopper

If we consider that the mass balance between the feed water flow rate and steam flow rate is satisfied, the pressure variation process is described approximately through the differential equation:

$$T_a \frac{dp_t}{dt} = D_q - D ; \tag{1}$$

where:  $D_q$  - is the boiler thermal load;  $D$  - is the steam load;  $T_a$  [kg/at] - is the accumulation constant. By means of  $T_a$ , there can be calculated the boiler inertia time as following:

$$T_p = T_a \frac{p_{tn}}{D_{\max}} ; \tag{2}$$

For the modern boilers,  $T_p = (125 \div 300)$  seconds. If there are used per units related to the boiler nominal values, steam load,  $D$ , becomes  $d = p_t$  and from expressions (1) and (2) there follows:

$$T_p \frac{dp_t}{dt} = d_q - \dot{m}_t - p_t ; \tag{3}$$

where:  $d_q$  represents the boiler thermal load in per unit and  $\dot{m}_t$  is the steam flow rate, in per unit (p.u.). Expression (3) allows the representation of the steam generator through a transfer function having a first order delay, if the operational operator,  $s = \frac{d}{dt}$ , is introduced, namely:

$$H_c = \frac{1}{1 + s \cdot T_p} . \tag{4}$$

The mathematical model of the boiler, described by transfer function (4) has got for input value (figure 3) the algebraic sum of the signals of the load and the steam flow rate,  $\dot{m}_t$  (if there is no thermo-mechanical control system), or the algebraic sum between the load signal and the load reference signal of the synchronous generator (if there is a thermo-mechanical control system).

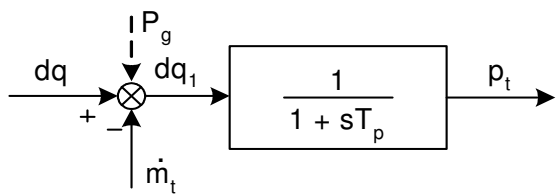


Fig. 3. The diagram of the mathematical model of the steam boiler.

The output value is represented by steam pressure,  $p_t$ . This mathematical model has the advantage of simplicity but it implies a more detailed representation of the control system. Though simple, it approximates fairly well the dynamic behaviour of any type of steam generator, both from a quantitative and from a qualitative point of view. As to the combustion chamber, its influence on the dynamic behaviour of the boiler depends on its construction and on the combustion method. The heat evolved by the combustion chamber is determined by the control system of the combustion process, which contains a fuel feeding system and a fuel control. The fuel controller reacts to the steam flow rate and steam pressure and to other parameters depending on its construction and it emits a modification impulse of the fuel flow rate. After emitting the impulse, after a certain period of time, depending on the delay in the fuel feeding system, there starts the modification of thermal load,  $D_q$ . In the case of drum type boilers, the modification process of the thermal load can be described by the differential equation:

$$T_c \frac{dD_q}{dt} + D_q = \mu_B \cdot e^{-\frac{t}{T_F}} ; \quad (5)$$

where:  $T_F = (20 \div 25)$  s is the time constant of the combustion chamber and  $T_c = (6 \div 60)$  s is the time constant of the fuel carriage which depends on the control type and on the type of the fuel that is used. The position of the control device of the fuel feeding system has been assigned to  $\mu_B$ . In the case of once-through boilers, as the dynamics of the boiler is determined by the rapid water-steam flow, the influence of the steam generator is much diminished versus the delay introduced by the water feed pumps, which can be represented by the following transfer function:

$$H_p = \frac{1}{1 + M \cdot s} \quad (6)$$

where:  $M = (20 \div 25)$  s represents the time constant of the boiler-water feed pumps<sup>4</sup>.

## 2.2 The mathematical modelling of the steam boiler automation

The automatic control of the steam boiler has to solve a set of problems connected with the synchronous control of more values: load control, combustion control, keeping constant the water level in the drum type boilers, keeping constant steam temperature and negative pressure in the combustion chamber. The control of these values means modelling the types of control equipments, namely the pressure of steam, the fuel and air, the feed water and the temperature. As temperature modifications and control are very slow, their modelling can be neglected. As to the fuel and air control and feed water control equipments, these will determine boiler load  $D_q$ . The main components which influence the boiler response being fuel dynamics and the dynamics of the air introduced in the ventilators as well as the response of the feed water pumps and of the associated control equipments, the dynamic dependences will be manifest through two distinct ways: a slow one (fuel-air) and a quick one (water-steam). The two ways have no direct physical correspondence, but they can be used together to simulate either drum type boilers (where the dominant effect on pressure belongs to fuel and air) or once-through boilers (where the boiler load dynamics is

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<sup>4</sup> (Surianu, 2009)

[illegible]

The pressure control equipments have been represented through a  $P-I$  type, followed by a differential control unit (figure 4). At the entrance into the control equipment there has been applied the signal of pressure error at the admission valve versus the pressure reference value, balanced with the load error of the generator. The output signal can be balanced with the generator load signal or with the signal of the turbine steam flow rate, when there is no coordinating thermo-mechanical control system and it has got both superior and inferior limits. Fuel-air dynamics (the case of the drum type boilers) is represented through a delay of  $(1/(1+T_F \cdot s))$  and  $(e^{-sT_c})$  type smoothness, due to fuel feed, smoothness that can be neglected if the boiler is fed with burning oil and gas. Water-steam dynamics (the case of the once-through boilers) is simulated through a  $P-I$  control unit described by expression  $(K_{W_1} + K_{W_2}/s)$ , with a corresponding limitation and by a first order delay function,  $I$ , due to the water feed pumps  $(1/(1+M \cdot s))$ , where  $M$  is the time constant of the water feed pumps. The action signal of the boiler load,  $D_q$ , has been got from adding up the output signals of the two ways (fuel-air and water-steam). The mathematical model described in figure 4 allows simulating automation for both types of boilers, drum type boilers and once-through boilers, through values 1, respectively, 0, assigned to weighting factor  $K$ .

The changes in the energetic status of a steam turbine can be generated by the following disturbing values:

- ### 2.3.1 The transfer function of the steam turbine

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$$P = M_T \Omega = \eta \cdot \eta_e \cdot D \cdot H ; \tag{7}$$

where:  $\eta$  is the real efficiency of the turbine,  $\eta_e$  is the electric energy efficiency,  $D$  is the steam flow rate at the entrance of the turbine and  $H$  is the enthalpy variation in the turbine<sup>5</sup>. The difference between the instantaneous values of mechanical torque  $M_T$ , of the turbine and the synchronous generator electric torque  $M_G$ , modifies the aggregate dynamic torque, according to the second principle of Newtonian mechanics:

$$M = M_T - M_G = J \frac{d\Delta\omega}{dt} . \tag{8}$$

In the transitory regimes of the electric power systems, the deviations of pulsation versus the synchronous pulsation ( $\Delta\omega = \omega - \omega_n$ ) do not exceed (20-30) %. In this scale, the characteristics turbine torque-pulsation can be approximated through tangent lines in the point corresponding to the rated speed. In figure 5 there is represented such a family of characteristics having  $\mu$  as position parameter of the steam admission valve of the speed control system. In this case, the equation of the characteristic turbine torque-pulsation is expressed in per units as following:

$$M_T = \alpha - \beta \cdot \omega_n ; \tag{9}$$

where:  $\alpha$  and  $\beta$  are constant coefficients which, according to figure 5, satisfy the expression:

$$\alpha = 1 + \beta ; \tag{10}$$

and  $\beta = (0.7 \div 1.2)$  for different types of turbines.

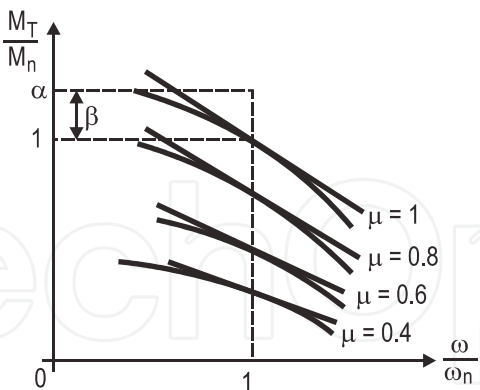


Fig. 5. The diagram of the characteristics turbine torque - pulsation

Experimentally there has been observed that, when the turbine power varies, the slope of the characteristics turbine torque-pulsation varies depending on the torque corresponding to the rated speed, namely:

$$M_T = M_n (\alpha - \beta \omega) . \tag{11}$$

<sup>5</sup> (Zhiyong et al., 2008)



In expression (11) coefficient  $M_n\beta = k\varphi$  is the turbine automatic control coefficient and it is equal with the slope of the characteristic. If the variation of the torque and pulsation in expression (8) are related to the corresponding nominal values, the following result is obtained:

$$\frac{\Delta M}{M_n} = J \frac{\omega_n}{M_n} \frac{d \frac{\Delta \omega}{\omega_n}}{dt}; \quad (12)$$

where: factor  $J\omega_n / M_n$  has a temporal dimension and represents the starting time constant or the launching time of the turbine - synchronous generator aggregate, namely:

$$T_l = J \frac{\omega_n}{M_n} = J \frac{\omega_n^2}{P_n}. \quad (13)$$

Launching time,  $T_l$ , does not modify proportionally with the machine power as inertia moment  $J$ , due to constructive reasons, does not increase proportionally with the increase in the machine power.

Equation (12) can be rewritten in an operational form, as a transfer function  $H(s)$ , considering the operator  $s = \frac{d}{dt}$ , namely:

$$H(s) = \frac{1}{T_l \cdot s}. \quad (14)$$

Relation (14) shows that the aggregate behaves like an integral unit whose output value has a linear variation in time, at a step variation of the input value.

### 2.3.2 The mathematical modelling of the boiler with steam re-heater

The presence of the steam re-heater modifies the behaviour in the case of turbines built with three pressure units. The use of steam re-heater in this case allows increasing the aggregate power and efficiency but it leads to a worsened dynamic behaviour because the steam re-heater follows with some delay the power variations generated by the control valve of the intermediate and low pressure units (IPU+LPU), introducing a dead time in the transmission of the variations of the steam flow rate at the entrance<sup>6</sup>. At the same time, the large volume of steam leads, through pressure release in (IPU+LPU), to large super-adjustments of speed and rapid power variations. This is due to the fact that the contribution to power of (IPU+LPU) is about (70÷80) % of the nominal value while that of the high pressure unit (HPU) is only (20÷30) %. A simplified representation of the turbine-synchronous generator aggregate, having a steam re-heater is given in figure 6.

The steam generated by the boiler flows through HPU, where it releases part of its thermal energy, determining a torque  $MT_1$  at the turbine axis, then flows through the connecting pipe and enters the steam re-heater where it receives an excess of thermal

<sup>6</sup> (Dimo et al., 1980)



energy and from here it flows through the connecting pipe with IPU+LPU, where determines torque  $MT_2$  at the turbine axis. The rotors of units HPU, IPU and LPU being rigidly coupled, the total torque will be the sum of the torques developed in each unit. In the studies of Long Term Dynamic stability the mathematical modelling of the turbine with three pressure units and steam re-heater can be described through the block diagram in figure 7.

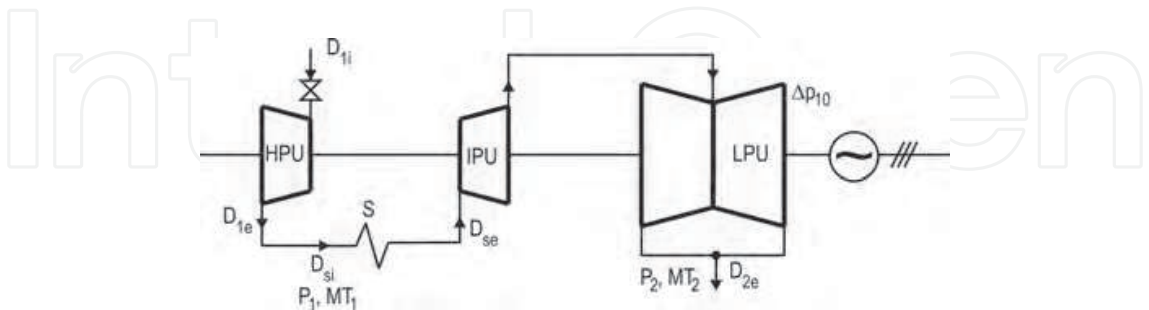


Fig. 6. Block diagram of the boiler with steam re-heater

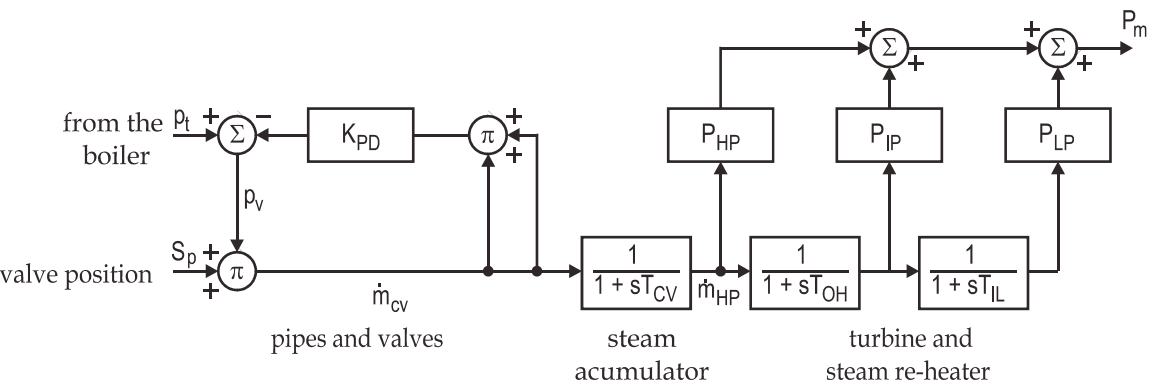


Fig. 7. The block diagram of the steam turbine aggregate.

The input value of the mathematical model of the steam turbine in figure 7 is steam flow rate  $\dot{m}_{CV}$ , at the output of the assembly of pipes – admission valves – speed governor. The steam flow rate,  $\dot{m}_{CV}$ , is calculated in per units, multiplying the position signal of the admission valve,  $S_p$ , by the pressure at admission valve,  $p_v$ , ( $p_v$  being calculated through the algebraic summing of the steam pressure given by the boiler with the output value of the proportional reaction given by the assembly of high pressure pipes). There follows:

$$\dot{m}_{CV} = S_p \left( p_t - K_{PD} \cdot \dot{m}_{CV}^2 \right); \tag{15}$$

where:  $K_{PD}$  is the amplifying factor corresponding to the drop pressure in the high pressure pipes. To the steam flow rate,  $\dot{m}_{CV}$ , there is applied a delay described by a transfer function with a first order delay  $1/(1+sT_{CV})$ , where  $T_{CV}$  is the steam time constant through pipes and valves, resulting in steam flow rate  $\dot{m}_{HP}$  at the entrance of the high pressure unit of the turbine. The steam turbine is modelled through two serially connected first order delay elements representing the delays given by the steam re-heater having time constant  $T_{OH}$  and by the steam transfer between the intermediate and low pressure units with time constant  $T_{IL}$ , as well as three proportional elements corresponding to the mechanical power weight of the

high, intermediate and low pressure units. The generally accepted values<sup>7</sup> of the constants of the mathematical model are given in Table 2. The amplifying factors are given in per units and the time constants, in seconds. The output value of the mathematical model is the mechanical power at turbine axle,  $P_m$ , expressed in per units, too.

Aggregate type	$K_{PD}$	$P_{HP}$	$P_{IP}$	$P_{LP}$	$T_{CV}$ [s]	$T_{OH}$ [s]	$T_{IL}$ [s]
Turbine with 3 units of pressure and steam re-heater	$0 \div 1$	0.3	0.4	0.3	$0.1 \div 0.4$	$4 \div 11$	$0.3 \div 0.5$

Table 2. The values of the constants of the steam turbine

The mathematical model of the steam turbine represented in the block diagram in figure 7 is described by the following set of algebraic and differential equations:

$$\dot{m}_{CV} = S_p \left( p_t - K_{PD} \cdot \dot{m}_{CV}^2 \right);$$
$$\frac{d\dot{m}_{HP}}{dt} = T_{CV}^{-1} \left( \dot{m}_{CV} - \dot{m}_{HP} \right);$$
$$\frac{d\dot{m}_{IP}}{dt} = T_{OH}^{-1} \left( \dot{m}_{HP} - \dot{m}_{IP} \right);$$
$$\frac{d\dot{m}_{LP}}{dt} = T_{IL}^{-1} \left( \dot{m}_{IP} - \dot{m}_{LP} \right);$$
$$P_m = P_{HP}\dot{m}_{HP} + P_{IP}\dot{m}_{IP} + P_{LP}\dot{m}_{LP}.$$

(16)

The initial values of the variables are obtained from the pre-disturbance steady state, which results from annulling the derivatives in expression (16), this fact leading to the system of above mentioned algebraic equations. Though, there has to be mentioned that the steam turbine, as an assembly, behaves like a derivative element of first order delay, the modelling described as a transfer function  $K_T / (1 + sT_T)$ , is legitimate if constants  $K_T$  and  $T_T$  are chosen appropriately. But we consider that such a representation is too general to be valid in the case of interconnecting the steam turbine with the steam boiler upstream and the synchronous generator downstream.

3. The mathematical modelling of the primary installations of a hydro power plant

The theoretical study of the behaviour and stability of hydro power plants is complex and highly difficult due to the large number of variables, to the fact that hydro units cannot be standardized (each of them depending on the geographical situation of the area where it is placed) and to the non-linearity of the hydro power system<sup>8</sup>. That is why the first research works in the field have been directed to realizing the linearity of the system around the

<sup>7</sup> (Surianu, 2009)

<sup>8</sup> (Fraile-Ardanuy et al., 2006)

operating point. In order to be able to analyze the stability of a hydro power plant, this should be theoretically divided into two subsystems: the hydro subsystem (from the reservoir to the turbine) and the electro-mechanical one (comprising the turbine, the admission valves control system and the speed governor). The assembly of the two subsystems is represented in figure 8.

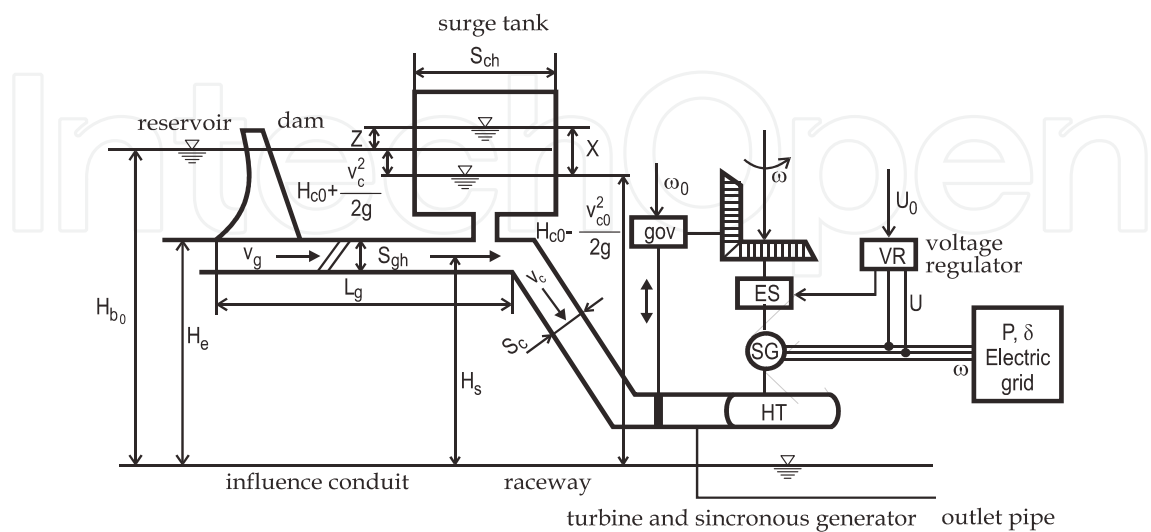


Fig. 8. The block diagram of a hydro power plant.

3.1 The simple mathematical modelling of a hydro power plant

The hydro power plant comprises the reservoir, the influent conduit, the surge tank, the raceway and the hydro turbine. The mathematical modelling of the hydro power plant implies writing the characteristic operating equations for each unit separately, established for common operating conditions and assembling the equations in a mathematical system which is solved at each simulation step. The problem is highly difficult and that is why, for the largest number of the power system stability studies, the modelling of the hydro power plant is reduced to modelling the hydro turbine whose transitory characteristics are deduced from the dynamics of the water in the raceway. The following mathematical model is obtained and it is presented in figure 9.

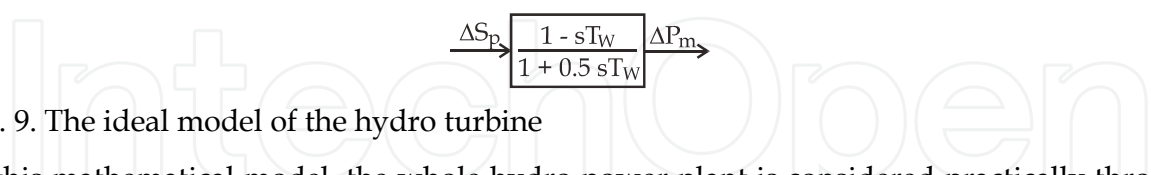


Fig. 9. The ideal model of the hydro turbine

In this mathematical model, the whole hydro power plant is considered practically through the value attributed to constant,  $T_W$ , which represents the water launching time or the water time constant. It is associated with the water acceleration time in the raceway between the dam and the turbine and it can be determined with the following expression:

$$T_W = \frac{L \cdot v}{g \cdot H_t} ; \tag{17}$$

where:  $L$  is the length of the raceway,  $v$  is the water speed,  $g$  is the gravity acceleration and  $H_t$  is the net head of water in the raceway. The longer the net head of water is the shorter the water launching time and, usually,  $T_W = (0.5 \div 7)$  s. This mathematical model is simple and easy to handle but it is too general and cannot be applied to long raceways.

3.2 The complex mathematical model of the hydro power plant

In order to obtain the complex mathematical model of a hydro power plant<sup>9</sup> we need some explanations and preliminary calculations. Thus:

- a. All the values are expressed in per units related to the absolute values corresponding to the operating point in the pre-disturbance steady state.
- b. Considering the problem of a hydro unit stability implies that with a relatively short time variation ,  $\Delta t$ , all the values vary only in the vicinity of their operating point in steady state, fact accepted physically due to the big inertia of the system and of the corresponding large time constants. This way, the infinitesimal quantities of a rank higher than 2 are neglected and only the first terms in the series development around the steady state point are kept, namely the equations describing the behaviour in time of the different elements of the hydro power plants are smoothed.
- c. There are defined the following values for the hydro turbine and hydro unit according to the turbine mechanical, hydro and geometrical parameters. These values are presented in Table 3:

Physical values	Mathematical formula	Observations
Turbine energy value	$\varepsilon = \frac{2 \cdot g \cdot H_n}{R^2 \cdot n^2}$	R- turbine radius; n- turbine speed.
Turbine water flow rate value	$\gamma = \frac{Q_n}{S \cdot R^3 \cdot n}$	$Q_n$ - nominal flow rate; S - turbine section.
Turbine power value	$\psi = \frac{2 \cdot P_{mn}}{\rho \cdot S \cdot R^5 \cdot n^3}$	$P_{mn}$ -nominal mechanical power; $\rho$ - water density.
Turbine efficiency value	$\eta = \frac{P_{mn}}{\rho \cdot g \cdot H_n \cdot Q_n}$	$H_n$ - the net head of water.
Relationship among the 4 values	$\psi = \varepsilon \cdot \gamma \cdot \eta$	
Reference section of the turbine	$s_r = \frac{S}{R^2}$ where: $S = \pi(R^2 - R_n^2)$ $S = \pi R_e^2$	S - turbine section; R - turbine radius (Francis, Kaplan turbines); $R_e$ - Pelton turbines.
The maximum water level in the surge tank	$Z = v_{g0} \sqrt{\frac{L_g \cdot S_g}{g \cdot S_{ch}}}$	$S_{ch}$ -surge tank section; $L_g, S_g$ - geometry of the influent conduit; $v_{g0}$ - water speed.
The time constant of the raceway	$T_g = \sqrt{\frac{L_g \cdot S_g}{g \cdot S_{ch}}}$	

Table 3. Basic values for the turbine and the hydro power plant.

<sup>9</sup> (Huimin & Chao, 2006)

### 3.2.1 The determination of the basic hydro parameters of the hydro turbine

In steady state, if the cavitation is neglected, the hydro behaviour of a turbine is determined by the following expressions:

$$F(\varepsilon, \gamma, \eta) = 0 \text{ and } G(\varepsilon, \gamma, A) = 0 ; \quad (18)$$

which in the spatial Cartesian system represent two surfaces. But, if plan representation is used, we get functions  $\varepsilon = f(\gamma)$ , with  $\eta$  parameter and  $\varepsilon = f(\gamma)$ , with  $A$  parameter, representing the position of the wicket and the turbine blades. These functions can be represented in plan  $(\varepsilon, \gamma)$  according to figure 10. If we consider point P, as the operating point in steady state, the behaviour of the turbine, from the point of view of stability, is wholly determined by two tangent plans in point P, to the surfaces described in expressions (18). But the orientation of each plan is determined by two slopes, then, theoretically, it suffices to know 4 of the slopes to approach any stability problem of the turbine around the point corresponding to the steady state<sup>10</sup>. Practically, these 4 slopes are obtained in per units, as following:

$$t_1 = \left. \frac{\delta \gamma}{\delta \varepsilon} \right|_{A_0} ; t_2 = \left. \frac{\partial \gamma}{\partial A} \right|_{\varepsilon_0} ; t_3 = \left. \frac{\delta \eta}{\delta \varepsilon} \right|_{A_0} ; t_4 = \left. \frac{\partial \eta}{\partial A} \right|_{\varepsilon_0} . \quad (19)$$

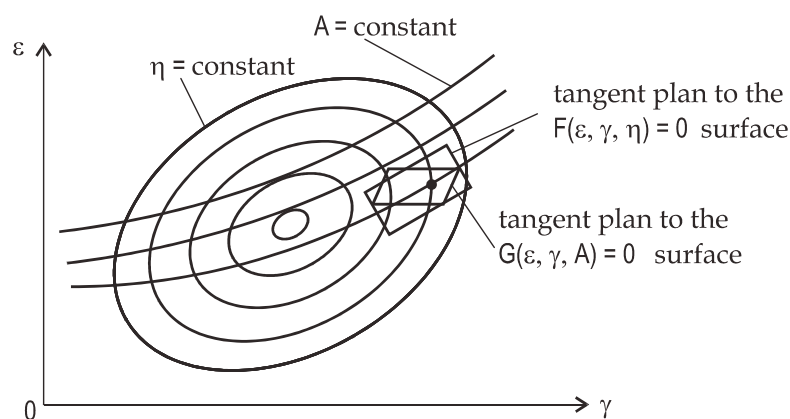


Fig. 10. The diagram of the operating characteristics of the hydro turbine.

The 4 slopes represent the basic hydro values of a hydro turbine. But their values cannot be calculated unless the surfaces which characterize the hydro behaviour of the turbine are expressed analytically. That is why for solving this problem we use statistical data obtained from a large number of turbines of all types (Pelton, Francis, spiral and Kaplan), resulting in variation curves of the basic hydro values depending on the speed value, given in Figure 11. The turbine speed value is defined by the expression:

$$v = n \sqrt{\frac{Q}{s_r \cdot (g \cdot H_n)^{3/2}}} ; \quad (20)$$

where:  $Q$  – the turbine water flow rate;  $n$  – the turbine speed;  $s_r$  – the reference section;  $H_n$  – the net head of water and  $g$  – gravity acceleration.

<sup>10</sup> (Surianu & Dilertea, 2004)

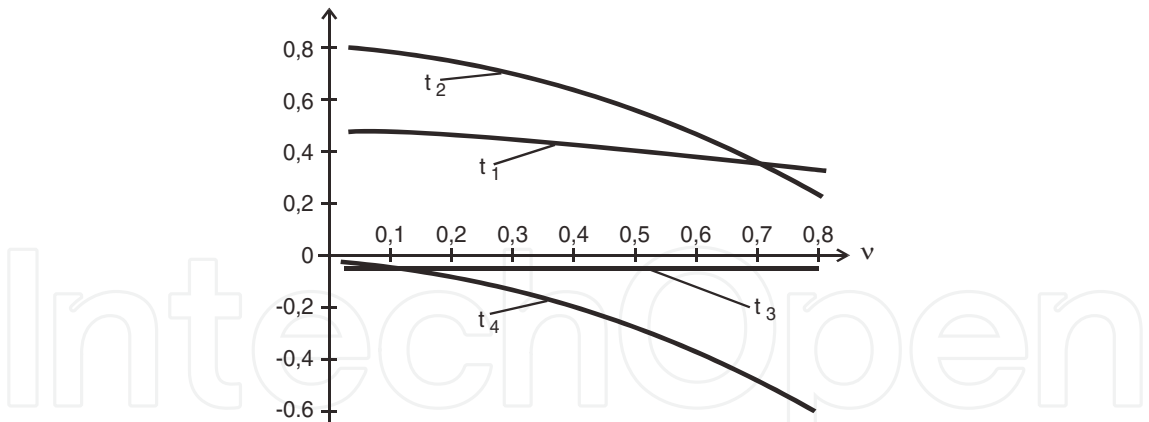


Fig. 11. The statistical relationships among the basic hydro values and speed value  $v$  .

By means of the basic hydro values, the following auxiliary hydro values are defined:

$t_5 = 1 + t_1 + t_3; t_6 = t_2 + t_4; t_7 = 1 - 2t_1; t_8 = t_2 \cdot t_5; t_9 = 1 - 2t_1 - 2t_3; t_{11} = -2t_3.$  (21)

These values, together with the basic hydro ones define completely the hydro turbine behaviour in terms of stability, the differentials of functions  $\gamma_r = f(\epsilon_r, a); \eta_r = f(\epsilon_r, a);$  and  $\psi_r = \epsilon_r \cdot \gamma_r \cdot \eta_r$  are expressed as following:

$d\gamma_r = t_1 \cdot d\epsilon_r + t_2 \cdot da; d\eta_r = t_3 \cdot d\epsilon_r + t_4 \cdot da; d\psi_r = t_5 \cdot d\epsilon_r + t_6 \cdot da.$  (22)

3.2.2 The operating equations of the hydro power plant

The operating equations of the hydro power plant are presented in Table 4, in the order corresponding to the water flow sense, from the water reservoir to the hydro turbine.

Equation type	Mathematical expression
Equation of losses in the influent conduit	$\Delta h_g = 2\Delta q_{V_g}^2$ $\Delta h_g$ - variation of the water height losses; $\Delta q_{V_g}$ - variation of the water flow rate.
Equation of the kinetic energy in the point of the surge tank insertion	$\Delta e_{ch} = 2 \cdot \Delta q_{V_g}$
Equation of surge tank filling	$S_{ch} \frac{dX}{dt} = Q_{ch}$ $S_{ch}$ - surge tank section; $dX$ - water height variation; $Q_{ch}$ - water flow rate.
Surge tank equation	$T_{ch} \frac{dX}{dt} = Q_{ch},$ where: $T_{ch} = \frac{S_{ch}(H_{bo} - H_{go})}{Q_{V_{go}}}$ $T_{ch}$ - surge tank time constant; $H_{bo}$ - water height in the reservoir; $H_{go}$ - water height in the influent conduit.

Equation type	Mathematical expression
Water flow rate equation	$Q_{ch} = Q_{Vg} - Q_c$ <p><math>Q_{ch}</math> - water flow rate at the basis of the surge tank; <math>Q_{Vg}</math> - water flow rate in the influent conduit <math>Q_c</math> - water flow rate in the raceway.</p>
Influent conduit equation	$T_{ch} \cdot T_{gi} \frac{d^2 X}{dt^2} + 2(c_2 + c_4) T_{ch} \frac{dX}{dt} + X =$ $= -T_{gi} \frac{d\Delta q_c}{dt} - 2(c_2 + c_4) \Delta q_c + (1 + c_2) g \cdot \Delta h_b$ <p>with: <math>c_2 = \frac{p_0}{h_0}</math> and <math>c_4 = \frac{v_{ch}^2}{2g(H_{b0} - H_{g0})}</math></p> <p>where: <math>p_0 = \frac{H_{g0}}{Z}</math> and <math>h_0 = \frac{H_{b0} - H_{g0}}{Z}</math></p> <p><math>T_{gi}</math> - time constant of the influent conduit; <math>c_2</math> - load loss in the influent conduit in steady state; <math>c_4</math> - kinetic energy in the point of surge tank insertion; <math>\Delta h_b = H_{b0} - H_{g0}</math>, is the water height variation in the reservoir (p.u.).</p>
Equation of specific energy related to the mass in the point of surge tank insertion	$T_{gi} \frac{d\Delta e_a}{dt} + 2(c_2 + c_4) \Delta e_a =$ $= T_{gi} \frac{dX}{dt} + 2c_2 X + 2c_4 (1 + c_2) g \Delta h_b$ <p><math>\Delta e_a</math> - variation of specific energy related to the mass.</p>
Equation of load losses in the raceway	$\Delta e_c = 2\Delta q_c$ <p><math>\Delta e_c</math> - variation of the water energy in the raceway; <math>\Delta q_c</math> - variation of water flow rate in the raceway.</p>
Water hammer equation in the raceway	$e_p = -T_c \frac{d\Delta q_c}{dt}, \text{ where: } T_c = \frac{\int_0^{L_c} v_{c0} dL_c}{gH_{no}}$ <p><math>e_p</math> - specific energy related to mass (p.u.) due to the water hammer; <math>T_c</math> - hydro inertia time constant of the raceway.</p>
Equation of the net specific energy related to mass	$\Delta e_k = (1 + h_2) \Delta e_a - 2h_2 \Delta q_c - T_c \frac{d\Delta q_c}{dt}$ <p><math>\Delta e_k</math> - variation of net specific energy related to mass; <math>h_2</math> - load loss in the raceway in steady state.</p>
Equation of the turbine water flow rate	$\Delta q = t_7 \Delta n_r + t_1 \Delta e_k + t_2 \Delta a$ <p>where: <math>\Delta q_c = \Delta q</math> (continuity law)</p>
Equation of the hydro turbine efficiency	$\Delta \eta_r = t_{11} \Delta n_r + t_3 \Delta e_k + t_4 \Delta a$
Equation of the hydro turbine mechanical power	$\Delta p_m = t_9 \Delta n_r + t_5 \Delta e_k + t_6 \Delta a$

Table 4. Operating equations of the hydro power plant in dynamic regime



### 3.2.3 The expression of the complex mathematical model of the hydro power plant

Starting from the above mentioned equations, there can be written a system of differential and algebraic equations to synthesize the mathematical models of the different elements within the hydro power plant equipped with influent conduit and surge tank and which, together with the equation of rotor (turbine & generator) movement and the equations of the speed control system of the power generating unit characterize completely the behaviour of a hydro power plant in dynamic stability<sup>11</sup>.

The set of differential and algebraic equations consists of:

- the equation of the water level in the surge tank;
- the equation of the net specific energy in the point of surge tank insertion;
- the equation of the net specific energy;
- the equation of the hydro turbine flow rate;
- the equation of the hydro turbine mechanical power.

To make the set of equations easier to approach through integrating the differential equations and solving the algebraic ones, the equations are ranked and displayed in a form to allow applying Runge-Kutta integration methods. Thus, the following system is obtained:

$$\begin{aligned}\frac{dB}{dt} &= -\frac{2(c_2 + c_4)}{T_{gi}}B - \frac{1}{T_{gi}T_{ch}}X - \frac{2(c_2 + c_4)}{T_{gi}T_{ch}}\Delta q - \frac{1}{T_{ch}}\frac{d\Delta q}{dt} + \frac{1+c_2}{T_{gi}T_{ch}}g\Delta h_b; \\ \frac{dX}{dt} &= B; \quad \frac{d\Delta e_a}{dt} = B + \frac{2c_2}{T_{gi}}X - \frac{2(c_2 + c_4)}{T_{gi}}\Delta e_a + \frac{2c_4(1+c_2)}{T_{gi}}g\Delta h_b; \\ \frac{d\Delta q}{dt} &= \frac{(1+h_2)}{T_c}\Delta e_a - \frac{2h_2}{T_c}\Delta q - \frac{1}{T_c}\Delta e_k; \quad \Delta e_k = \frac{1}{t_1}(\Delta q - t_7 \cdot \Delta n_r - t_2 \cdot \Delta a) \\ \Delta p_m &= t_9\Delta n_r + t_5\Delta e_k + t_6\Delta a.\end{aligned}\tag{23}$$

To the set of equations (23) we have added the movement equation of the assembly of rotors (turbine & synchronous generator) and the equations of the speed governor system (SG), which give the values of speed variation  $\Delta n_r$  and the value of the variation of valve position,  $\Delta a$ . Figure 12 presents the block diagram of an operating hydro-mechanical installation equipped with influent conduit and surge tank. Equations (23) and the corresponding block diagram in figure 12 have a general character describing the behaviour of the whole hydro-mechanic installation around the steady state point. If the hydro power plant has no influent conduit and surge tank, equations (23) stay valid but they are particularized through annulling the constants corresponding to these elements, and, in figure 12, the corresponding blocks disappear from the diagram.

### 3.3 The mathematical modelling of the speed governor

Frequency, as a unique parameter of the electric power system, plays a special role in its reliable and economic operation. If the reactive current component is neglected (for  $\cos \varphi = 0.8$ ) there can be stated that the active power losses in the electric power systems are proportional to frequency increased to the power of four:

<sup>11</sup> (Surianu & Barbulescu, 2008)

$$\Delta P_{EPS} \cong K_P \cdot f^4 \ ;$$

(24)

and then an increase in frequency leads to increasing power losses, and a decrease in frequency leads to a diminished consumers` productivity. There has also to be mentioned that keeping frequency at a constant level is a “must” in the case of the interconnection of large power systems, frequency variations being incompatible between partners.

In real circumstances, frequency is not kept strictly constant, but variable in pre-established limits, slightly depending on disturbance, which in this case is represented by the active power variation ( $\Delta P$ ) consumed in electric power systems. That is why there are realized static characteristics of the speed to allow a univocal distribution of the disturbing values on the power units set in parallel, either in the same power plant or in different points of the power system.

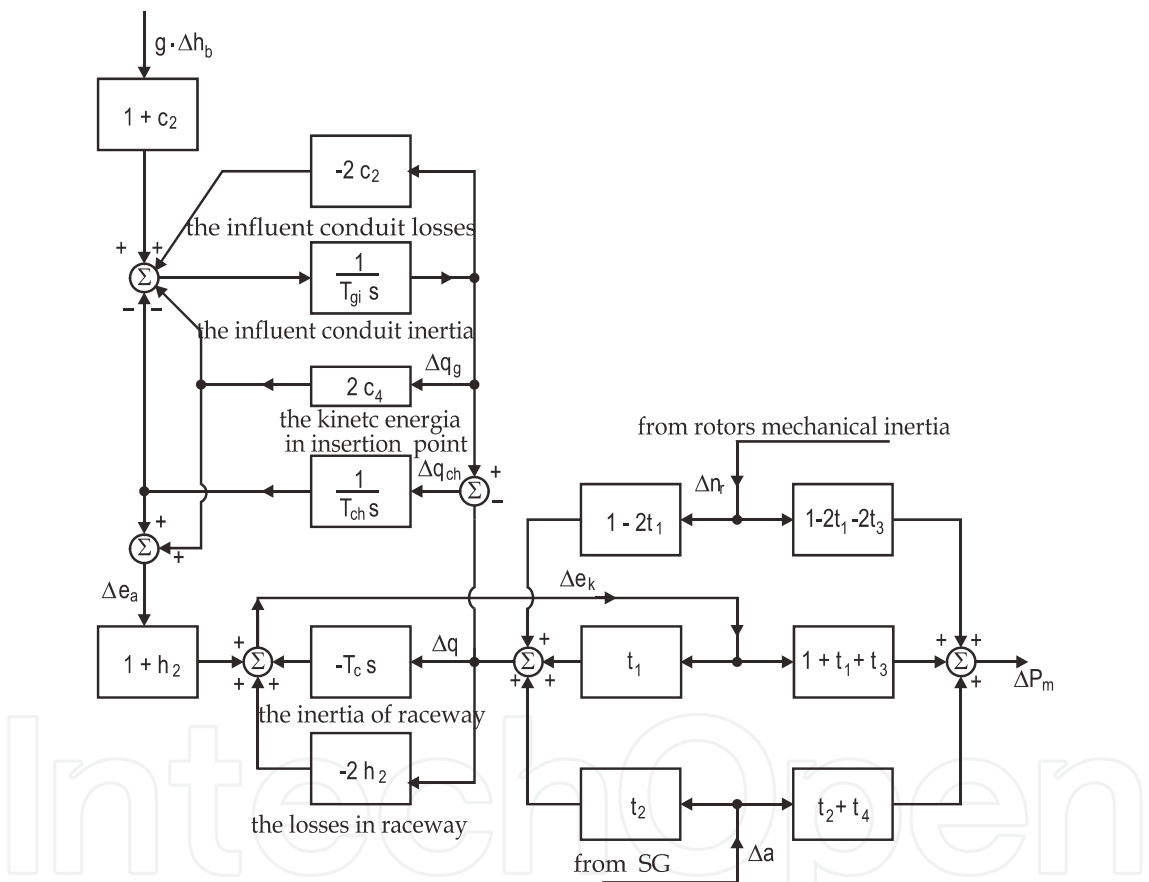
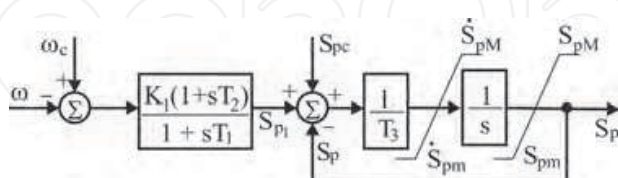


Fig. 12. The block diagram of a hydro-mechanical installation equipped with influent conduit and surge tank.

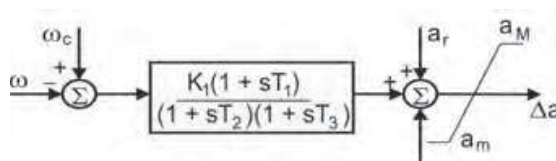
3.3.1 The block diagram and the mathematical models for the speed governors of the thermal and hydro power plants

The speed control of the power units is realized by means of turbine automatic control systems, namely speed governors (SG). To make possible univocal distribution and necessity based modification of the disturbing values distribution, the automatic control of speed is a static one, with offset characteristics between (1 – 7) %. There are a large number of SG types, with mechanical, electrical, electronic elements with which turbines are

equipped<sup>12</sup>, but no matter the type, SG's mainly consist of the following elements: a measuring element for speed (or frequency), amplifying elements (servo-engines) which take over the shifting of the pendulum and shift the heavy control units of the turbine, and reaction devices which insure the control stability and quality of transitory processes. From Long Term Dynamics studies there have been chosen two different general models to describe SG behaviour for thermal, respectively hydro-mechanical installations, these types being described in the block diagram in figure 13.



a) thermal installations;



b) hydro installations

Fig. 13. The diagrams of general SG models:

The equations which describe SG behaviour in figure 13 are:

for the thermal model:

$$\frac{dS_{p1}}{dt} = \frac{1}{T_1} \left( K_1 \omega_c - K_1 \omega - S_{p1} - K_1 T_2 \frac{d\omega}{dt} \right); \quad \frac{dS_p}{dt} = \frac{1}{T_3} (S_{pc} + S_{p1} - S_p); \quad (25)$$

with :

$$S_{pm} \leq S_p \leq S_{pM}; \quad \dot{S}_{pm} \leq \dot{S}_p \leq \dot{S}_{pM}.$$

for the hydro model:

$$\frac{dz}{dt} = \frac{1}{T_1} \left( \omega_c - \omega - z - T_2 \frac{d\omega}{dt} \right); \quad \frac{da_1}{dt} = \frac{1}{T_3} \left( K_1 \cdot z - a_1 + K_1 T_2 \frac{dz}{dt} \right); \quad (26)$$

with:

$$\Delta a = a_r + a_1; \quad a_m \leq \Delta a \leq a_M.$$

To equations (25) and (26) the movement equation of rotors is added.

As to the values of the coefficients in equations (25) and (26), these can have the following values:  $T_1 = (0.2 - 2.8) \text{ s}$ ;  $T_2 = (0 - 1.0) \text{ s}$ ;  $T_3 = (0.025 - 0.15) \text{ s}$ ;  $K_1 = (10; 15; 25)$ ;  $\dot{S}_{pm} = -0.1 \text{ p.u./s}$ ;

<sup>12</sup> (Hanmandlu et al., 2006)

$\dot{S}_{PM} = 0.1 \text{ p.u./s}; S_{pm} = 0.0 \text{ p.u.}; S_{PM} = 1.0 \text{ p.u.}; a_m = 0.1 \text{ p.u.}; a_M = 1.1 \text{ p.u.}$

3.4 Numerical simulations of the primary installations of the power plants

For the numerical simulation of power plants in dynamic regimes, the mathematical models of the components of the primary installations have to be assembled according to their causal links and there have to be written the corresponding systems of equations. Conceiving the algorithms and writing the calculation programs for each type of power plant and, finally applying them to real operating power plants turns hypotheses to certainty<sup>13</sup>.

3.4.1 The model of the primary installations of a thermal power plant

Based on the mathematical models of the elements of a thermal power plant, there has been conceived an assembly operating block diagram of the thermal unit of a power plant. It is represented in figure 14. The diagram allows modelling both types of primary installations, either those provided with drum type boilers or those with once-through boilers.

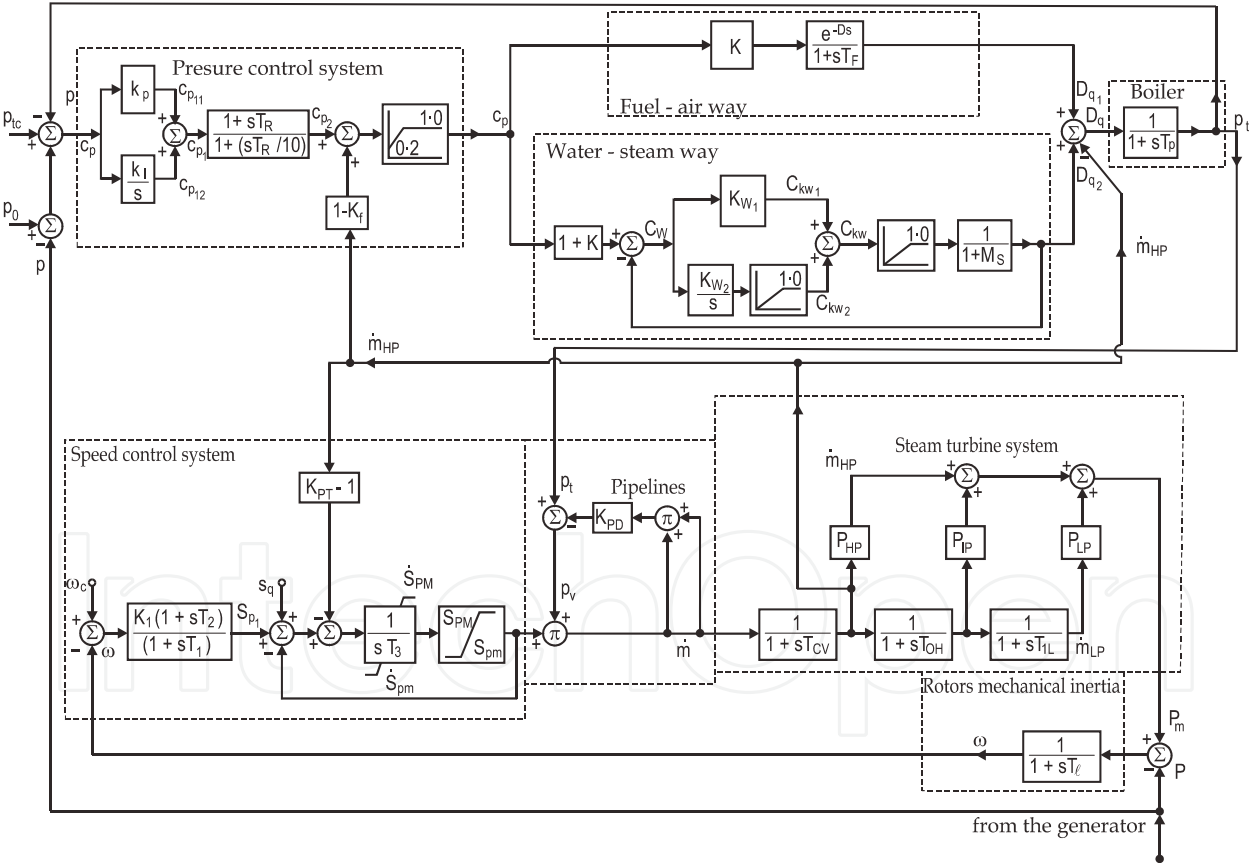


Fig. 14. The operating block diagram of the primary installations of a thermal power plant

The assembly operation is described by a set of algebraic and differential equations which are added to the inequalities of the corresponding limitations, as following:

<sup>13</sup> (Surianu, 2009)

$$\begin{aligned}
C_p &= P_0 + p_{tc} - P - p_t; \quad C_{p11} = K_p \cdot C_p; & \frac{dp_t}{dt} &= \frac{1}{T_p} D_q - \frac{1}{T_p} p_t; \\
\frac{dC_{p12}}{dt} &= K_I C_p; & \frac{d\omega}{dt} &= \frac{1}{T_L} P_m - \frac{1}{T_L} P; \\
C_{p1} &= C_{p11} + C_{p12}; & \frac{dS_{p1}}{dt} &= \frac{1}{T_1} \left( K_1 \omega_c - K_1 \omega - S_{p1} - K_1 T_2 \frac{d\omega}{dt} \right); \\
\frac{dG}{dt} &= -\frac{10}{T_R} G + \frac{10}{T_R} C_{p1}; & \frac{dS_p}{dt} &= \frac{1}{T_3} (S_{pc} - S_{p1} - S_p) - (K_{PT} - 1) \dot{m}_{HP}; \\
C_{p2} &= G + T_R \frac{dG}{dt}; \\
C_{pm} &= C_{p2} + (1 - K_f) \dot{m}_{HP}; \\
0.2 \leq C_{p2} &\leq 1.0; & \dot{S}_{pm} &\leq \dot{S}_p \leq \dot{S}_{pM}; \\
\frac{dD_{q1}}{dt} &= \frac{K(t - T_c)}{T_F} C_{pm} - \frac{1}{T_F} D_{q1}; & S_{pm} &\leq S_p \leq S_{pM}; \\
C_w &= (1 - K) C_{pm} - D_{q2}; & p_V &= p_t - K_{PD} \cdot \dot{m}_{CV}^2; \\
C_{kw1} &= K_{w1} C_w; & \dot{m}_{CV} &= S_p \cdot p_V; \\
\frac{dC_{kw2}}{dt} &= K_{w2} C_w; & \frac{d\dot{m}_{HP}}{dt} &= \frac{1}{T_{CV}} (\dot{m}_{CV} - \dot{m}_{HP}); \\
0.0 \leq C_{kw2} &\leq 1.0; & \frac{d\dot{m}_{IP}}{dt} &= \frac{1}{T_{OH}} (\dot{m}_{HP} - \dot{m}_{IP}); \\
C_{kw} &= C_{kw1} + C_{kw2}; & \frac{d\dot{m}_{LP}}{dt} &= \frac{1}{T_{IL}} (\dot{m}_{IP} - \dot{m}_{LP}); \\
0.0 \leq C_{kw} &\leq 1.0; & P_m &= P_{HP} \cdot \dot{m}_{HP} + P_{IP} \cdot \dot{m}_{IP} + P_{LP} \cdot \dot{m}_{LP}; \\
\frac{dD_{q2}}{dt} &= \frac{1}{M} C_{kw} - \frac{1}{M} D_{q2}; \\
D_q &= D_{q1} + D_{q2} - \dot{m}_{HP};
\end{aligned} \tag{27}$$

### 3.4.2 The model of the primary installations of the hydro power plant

The operational block diagram for the primary installations of a hydro power plant has been presented in figure 12 and the equations corresponding to the mathematical model are expressions (23). If to this block diagram there is added the general representation of SG and the block corresponding to the mechanical inertia of the assembly of the turbine & synchronous generator rotors, we get the complete operational block diagram in figure 15. The equations describing the operation of the primary installations of the hydro power plants equipped with speed governors are:

$$\begin{aligned}
\frac{dB}{dt} &= -\frac{2(c_2 + c_4)}{T_{gi}} B - \frac{1}{T_{gi} T_{ch}} X - \frac{2(c_2 + c_4)}{T_{gi} T_{ch}} \Delta q - \frac{1}{T_{ch}} \frac{d\Delta q}{dt} + \frac{1 + c_2}{T_{gi} T_{ch}} g \Delta h_b; \\
\frac{dX}{dt} &= B; \quad \frac{d\Delta e_a}{dt} = B + \frac{2c_2}{T_{gi}} X - \frac{2(c_2 + c_4)}{T_{gi}} \Delta e_a + \frac{2c_4(1 + c_2)}{T_{gi}} g \Delta h_b; \\
\frac{d\Delta q}{dt} &= \frac{(1 + h_2)}{T_c} \Delta e_a - \frac{2h_2}{T_c} \Delta q - \frac{1}{T_c} \Delta e_k;
\end{aligned}$$

$$Q = Q + \Delta q; \quad \Delta e_k = \frac{1}{t_1}(\Delta q - t_7 \Delta n_r - t_2 \Delta a);$$
$$E_k = E_k + \Delta e_k; \quad \Delta p_m = t_5 \Delta e_k + t_9 \Delta n_r + t_6 \Delta a; \quad (28)$$
$$P_m = P_m + \Delta p_m; \quad \frac{d\omega}{dt} = \frac{1}{T_l} \Delta p_m - \frac{1}{T_l} \Delta p;$$
$$\Delta n_r = \omega_c - \omega; \quad \frac{dz}{dt} = \frac{1}{T_1} \left( \omega_c - \omega - z - T_2 \frac{d\omega}{dt} \right);$$
$$\frac{da_1}{dt} = \frac{1}{T_3} \left( K \cdot z - a_1 + K T_2 \frac{dz}{dt} \right);$$
$$\Delta a = a_r + a_1; \quad a_m \leq \Delta a \leq a_M.$$

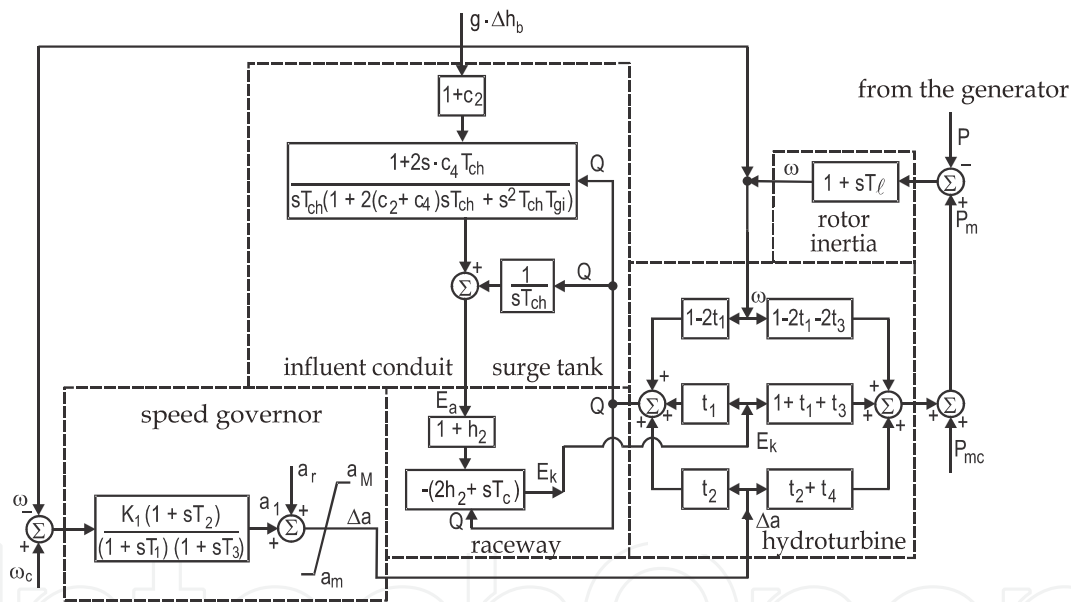


Fig. 15. The operating block diagram of the primary installations of a hydro power plant

3.4.3 Applications and the interpretation of the results of the computer simulation of thermal and hydro primary installations

For simulating the dynamic processes in power plants, using the mathematical models of the primary thermal and hydro installations, there have been conceived two calculation programs, named THERMO and HYDRO, whose flow charts are described in figure 16, a and b. The programs have been written in DELPHI. They have aimed at studying the way in which the systems of equations satisfy the initial conditions corresponding to a pre-disturbance steady state and they allow the calculus of the initial values of variables. We have studied the way in which the models respond to a given disturbance, the adjustment of the models according to the response to disturbances as well as checking the stability of the mathematical models having in view the possibility of linking them to the mathematical

models of the synchronous generators and electric networks<sup>14</sup>. The validity of the mathematical models has been demonstrated by applying them to the operating parameters of two real big power installations of the electric power system of Romania. These installations belong to Thermal Power Plant Mintia, equipped with six power units of 210 MW each and Hydro Power Plant Raul Mare, equipped with two power units of 167.5 MW each. The analysis of the components has been made for a single power unit of each type.

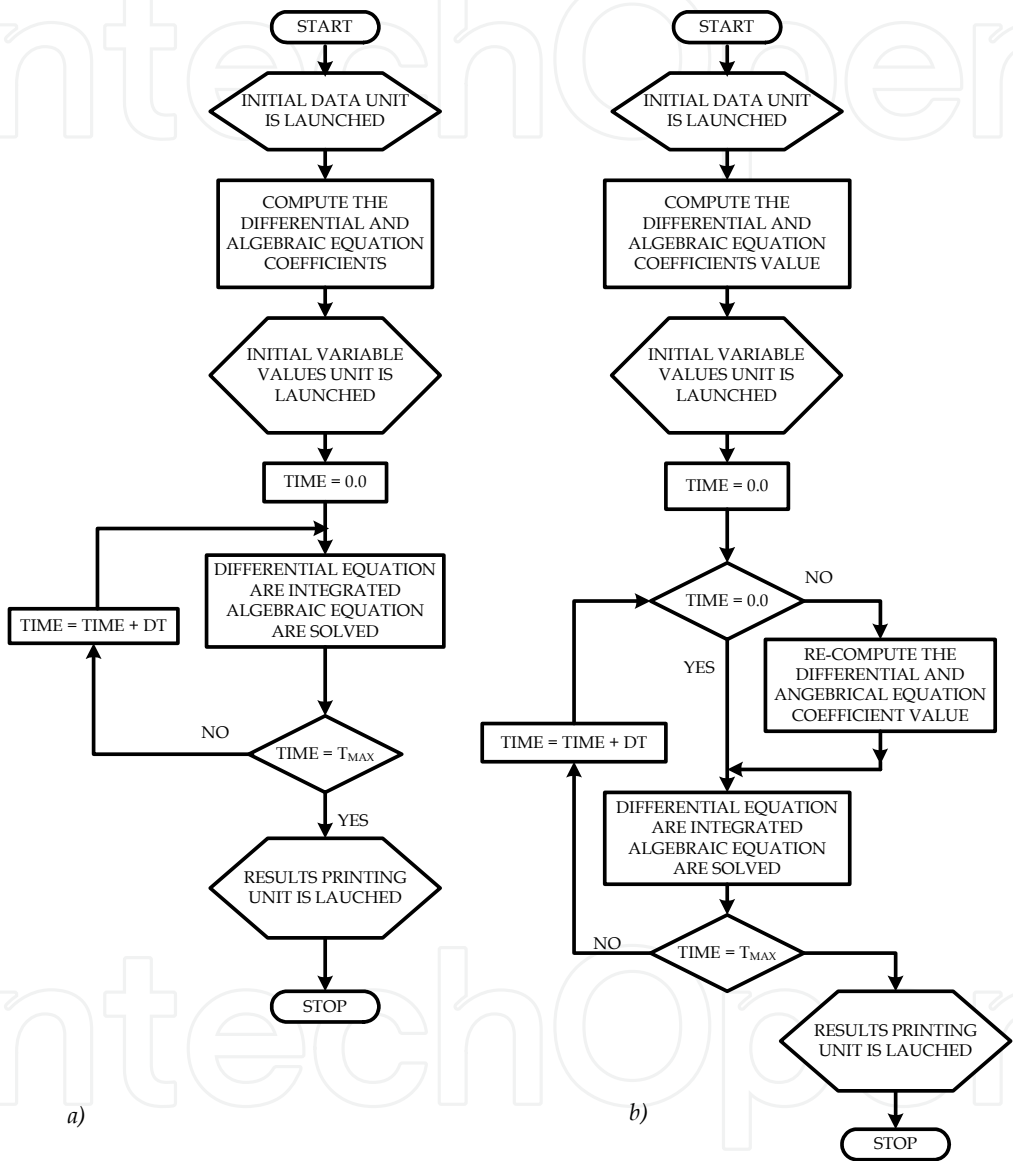


Fig. 16. The flow charts of the calculation programs. a) THERMO; b) HYDRO

By means of THERMO there has been analyzed the response in time of a thermal-mechanical installation equipped with a once-through boiler at a sudden electric load increase of 5 % at the synchronous generator terminals. There has been considered the boiler having a nominal steam pressure of  $p_t = 140 \text{ at}$  and a normal steam flow rate of  $\dot{m}_t = 630 \text{ t / h}$ , supplying with steam an assembly turbine & synchronous generator of a power

<sup>14</sup> (Surianu, 2009)



unit having a nominal electric power  $P_n = 210\text{ MW}$  and operating at an electric power  $P = 0.8 \cdot P_n$ . The launching time of the assembly is  $T_l = 6.5\text{ s}$  at the nominal frequency,  $f = 50\text{ Hz}$ . The boiler inertia time constant is  $T_p = 300\text{ s}$ , and the principal automation constants of the boiler on the feed water way are:  $K_{w1} = 1.5$ ;  $K_{w2} = 0.5$ ;  $M = 5\text{ s}$ . The turbine with three pressure units and re-heater has got the following parameters:  $T_{CV} = 0.5\text{ s}$ ,  $T_{OH} = 7\text{ s}$ ;  $T_{IL} = 0.4\text{ s}$ ;  $P_{HP} = 0.3$ ;  $P_P = 0.4$ ;  $P_{LP} = 0.3$ . The analysis of the dynamic evolution of the thermal-mechanical system has been made for a time of  $100\text{ s}$ , with an increasing step of  $\Delta t = 1\text{ s}$ . All the values have been written in per units. In Figure 17, there have been presented synthetically the main thermal - mechanical installations of Thermal Power Plant Mintia, Romania to which dynamic simulation has been applied and the results of the dynamic behaviour analysis are represented in Figure 18, having an operating once-through boiler.

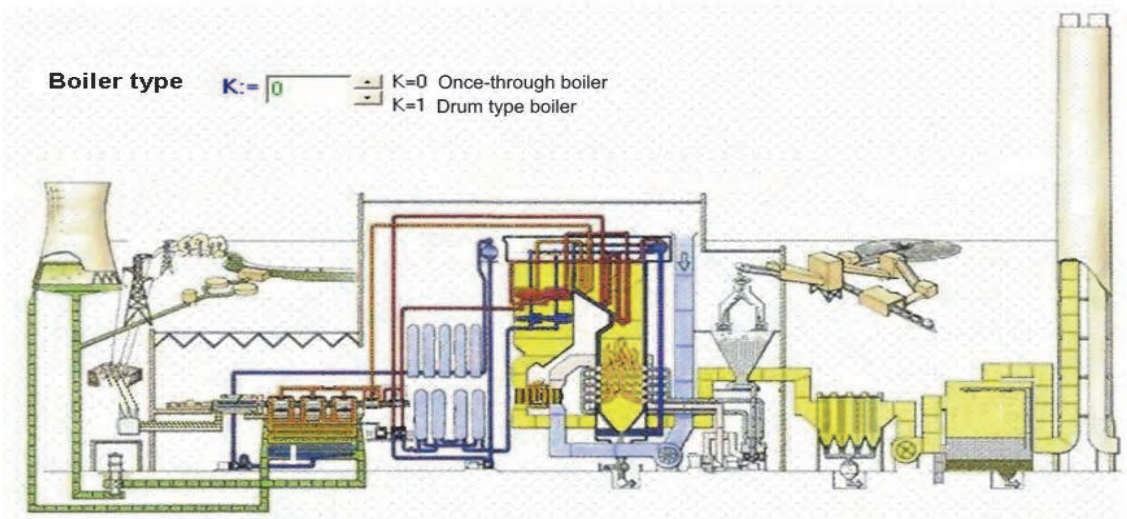


Fig. 17. Representation of the main thermal-mechanical installations of the power plant

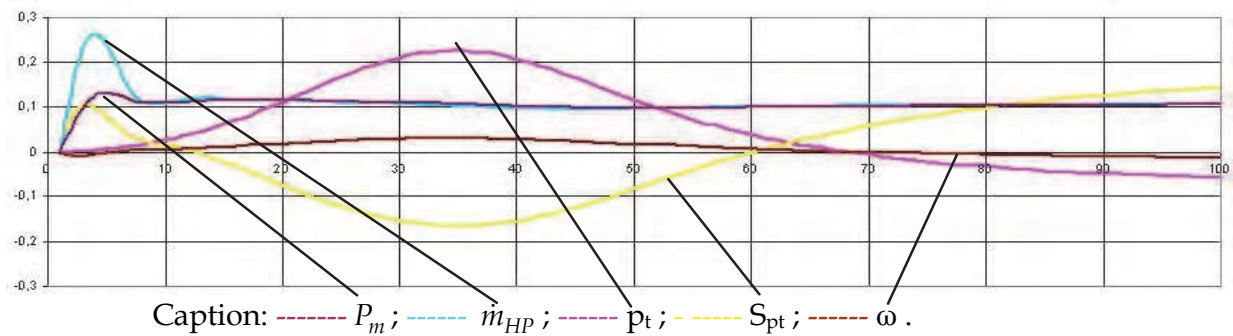


Fig. 18. The diagram of the simulation of the dynamic behaviour of the thermo-mechanical installations of Thermal Power Plant Mintia, Romania

Analyzing the curves obtained, there has been observed that at a sudden increase in the power necessity of the power system, there starts opening the steam admission valves,  $S_{pt}$ . Thus the steam flow rate at the turbine increases rapidly,  $\dot{m}_{HP}$  and the mechanical power,  $P_m$ , starts increasing. Steam pressure  $p_t$  varies slowly due to the oscillatory movement of the steam admission valve and the big inertia of the primary fuel feeding

installation. The slow pressure oscillation at the admission valve is longer than 100 seconds. In the first five seconds after the disturbance moment, the pulsation,  $\omega$ , slightly decreases due to the increase in electric power, which determines a retarding torque at the synchronous generator axle coupled to the steam turbine. After this decreasing tendency, the pulsation recovers slowly at a relatively constant value, a little bit higher than the initial one, and after 60 seconds, due to the action of the speed governor, the initial speed is reached, simultaneously with reaching the balance of the mechanical power with the electric one. The analysis of the dynamics of the thermo-mechanic installations for a period of 100 seconds points out the stabilization tendency of the thermo-mechanic operating system through a damped oscillatory process of all the thermal and mechanical values.

For modelling the dynamic behaviour of a hydro power station using the HYDRO program, we have considered Hydro Power Plant Raul Mare, Romania, equipped with a Francis turbine defined by the following parameters: nominal mechanical power,  $P_{mn} = 167.5 \text{ MW}$ ,  $f = 50 \text{ Hz}$ , nominal pulsation  $n_0 = 52.36 \text{ rad/s}$ , reference radius  $R = 1.425 \text{ m}$  and unit launching time,  $T_l = 7.5 \text{ s}$ . For the nominal speed value,  $v_0 = 0.408 \text{ m/s}$ , obtained from nominal water flow rate  $Q = 127 \text{ m}^3/\text{s}$ , there have been obtained the turbine angular parameters operating in nominal regime,  $t_1 = 0.47$ ;  $t_2 = 0.6$ ;  $t_3 = -0.03$ ;  $t_4 = -0.23$ . The hydro power plant has the following structure: an influent conduit of length  $L_g = 18.400 \text{ m}$  and section  $S_g = 18.8 \text{ m}^2$ , a surge tank of section  $S_{ch} = 109 \text{ m}^2$  and a raceway of length  $L_c = 812 \text{ m}$  and section  $S_c = 31.2 \text{ m}^2$ . The water height in the reservoir is  $H_{b0} = 980 \text{ m}$  and the water height in the influent conduit,  $H_{g0} = 13.37 \text{ m}$ , resulting the head of the water in the surge tank,  $H_{k0} = 966.63 \text{ m}$ , a net head of water in the raceway,  $H_k = 474.5 \text{ m}$  and a load loss in raceway,  $h_2 = 16.33 \text{ m}$ . The water speed in the influent conduit, in nominal regime is  $v_{g0} = 3.52 \text{ m/s}$  and in the raceway,  $v_{c0} = 4.07 \text{ m/s}$ . The elements of the hydro assembly have the following inertia time constants:  $T_{gi} = 6.7 \text{ s}$ ;  $T_{ch} = 99 \text{ s}$  and  $T_c = 1.15 \text{ s}$ . The steady state pre-disturbance consists of the hydro power plant operating at electric power  $P = 134.05 \text{ MW}$  and a net head of water in the raceway of  $H_n = 474.5 \text{ m}$ .

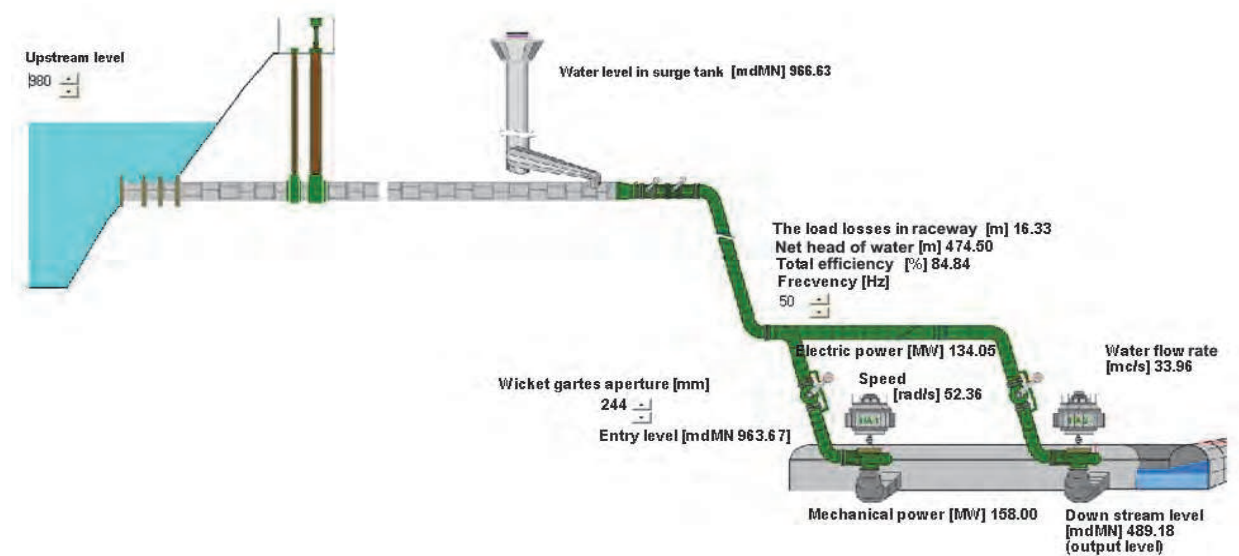


Fig. 19. The diagram of the main hydro-mechanic installations of the hydro power plant

The dynamic behaviour of the hydro-mechanic installation has been analyzed for a disturbance consisting of 10% increase of electric power versus the steady state at the synchronous generator's terminals, represented by the hydro power station operating with a single generating unit loaded with 80 % of the nominal power. There has been studied the evolution in time of the mechanic and hydro values on an interval of 200 seconds, with a time incremental step of  $\Delta t = 1 \text{ s}$ . In Figure 19 there are presented synthetically the main hydro-mechanic installations of the hydro power plant to which the dynamic simulation is applied. The results of the dynamic behaviour are represented in Figure 20. There has been observed that a sudden increase in the electric power required at the terminals of the synchronous generator results in a rapid decrease in speed,  $n$ , due to the appearance of a strong braking couple. The speed governor registers the speed decrease and initiates the opening of the wicket gates. At the beginning, the wicket gates open very quickly and mechanical power  $P_m$  increases, surpassing the value of the electric power, this fact leading to the appearance of an accelerating mechanical torque and, this way, speed  $n$  starts increasing. But as soon as the wicket gates open an increase in water flow rate  $Q$  of the turbine is registered and this cannot be compensated by the water leaking through the dam, through the influent conduit and water level,  $H_k$ , in the surge tank starts decreasing, leading to a diminishing of the net head of water and, correspondingly, to a decrease in the net specific energy,  $E_k$ .

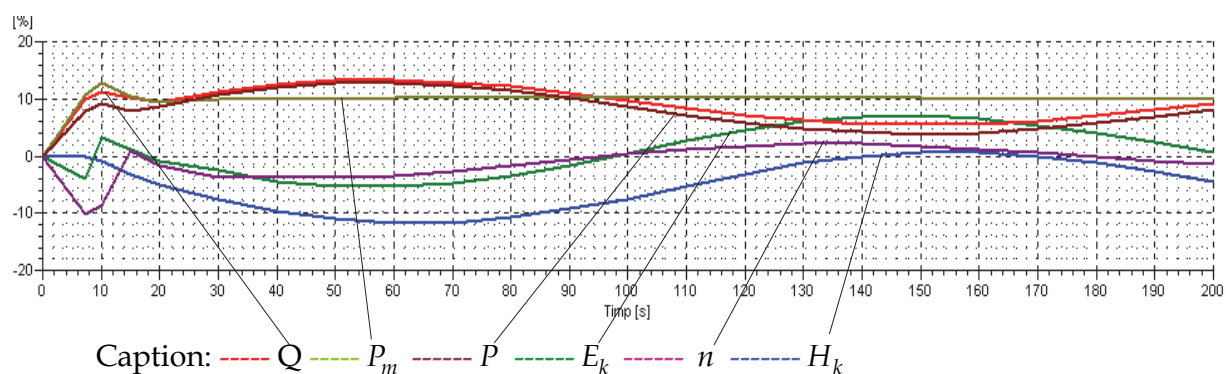


Fig. 20. Mathematical simulation of the dynamic behaviour the hydro-mechanic installations of Hydro Power Plant Raul Mare, Romania

This dynamic process in the hydro power plant, due to its big inertia, is produced much more slowly than the control dynamic process given by the speed governor which now, registering the speed increase, initiates the closing of the wicket gates and reduces the mechanical power under the value of the electric one. The speed decreases and re-starts opening the wicket gates but, as the water level in the surge tank is diminished and the net head of water is shorter, in order to get the corresponding mechanical power, there is needed a bigger water flow rate on a longer period of time. After having balanced the powers, the closing of the wicket gates is initiated, this leading to an increase in the water level of the surge tank and in the corresponding net specific energy.

#### 4. Conclusions

The analysis of the simulation results has shown concordance with the evolution of the dynamics of thermal and hydro-mechanic primary installations in real circumstances. The

simulation represents realistically the physical phenomena both in pre- disturbance steady state and in the dynamic processes following the disturbances in the electric power system. These models have proved to be useful for experts to draw up contingencies leading to optimum operating regimes and appreciate the necessary measures to be taken in critical circumstances. They also provide instruments for the operating regimes and for further studies concerning the expansion of the existing electric power systems.

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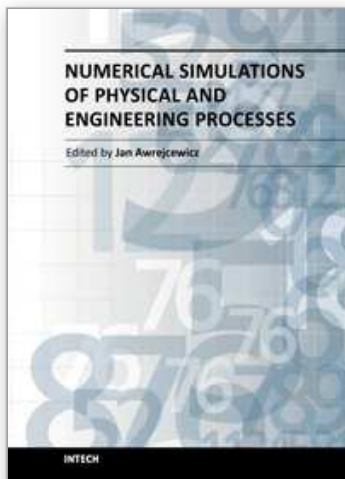
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