We are IntechOpen,
the world's leading publisher of
Open Access books
Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

154
Countries delivered to

Our authors are among the

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

CLARIVATE ANALYTICS
BOOK CITATION INDEX
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Heat Transfer in Complex Fluids

Mehrdad Massoudi

U.S. Department of Energy, National Energy Technology Laboratory (NETL), Pittsburgh
USA

1. Introduction

Amongst the most important constitutive relations in Mechanics, when characterizing the behaviour of complex materials, one can identify the (Cauchy) stress tensor **T**, the heat flux vector **q** (related to heat conduction) and the radiant heating (related to the radiation term in the energy equation). Of course, the expression 'complex materials' is not new. In fact, at least since the publication of the paper by Rivlin & Ericksen (1955), who discussed fluids of complexity *n* (Truesdell & Noll, 1992)], to the recently published books (Deshpande et al., 2010), the term complex fluids refers in general to fluid-like materials whose response, namely the stress tensor, is 'non-linear' in some fashion. This non-linearity can manifest itself in variety of forms such as memory effects, yield stress, creep or relaxation, normal-stress differences, etc. The emphasis in this chapter, while focusing on the constitutive modeling of complex fluids, is on granular materials (such as coal) and non-linear fluids (such as coal-slurries).

One of the main areas of interest in energy related processes, such as power plants, atomization, alternative fuels, etc., is the use of slurries, specifically coal-water or coal-oil slurries, as the primary fuel (Papachristodoulou & Trass, 1987; Roh et al., 1995; Tsai & Knell, 1986). Some studies indicate that the viscosity of coal-water mixtures depends not only on the volume fraction of solids, and the mean size and the size distribution of the coal, but also on the shear rate, since the slurry behaves as shear-rate dependent fluid (Gupta & Massoudi, 1993). There are also studies which indicate that preheating the fuel results in better performance (Tsai, et al., 1988; Saeki & Usui, 1995), and as a result of such heating, the viscosity changes. Constitutive modeling of these non-linear fluids, commonly referred to as non-Newtonian fluids, has received much attention (Schowalter, 1978; Larson 1999). Most of the naturally occurring and synthetic fluids are non-linear fluids, for example, polymer melts, suspensions, blood, coal-water slurries, drilling fluids, mud, etc… It should be noted that sometimes these fluids show Newtonian (linear) behavior for a given range of parameters or geometries; there are many empirical or semi-empirical constitutive equations suggested for these fluids (Massoudi & Vaidya, 2008). There have also been many non-linear constitutive relations which have been derived based on the techniques of continuum mechanics (Batra, 2006; Haupt, 2002). The non-linearities oftentimes appear due to higher gradient terms or time derivatives. When thermal and or chemical effects are also important, the (coupled) momentum and energy equations can give rise to a variety of interesting problems, such as instability, for example the phenomenon of double-diffusive convection in a fluid layer (Straughan, 1998, 2008).

When a coal stockpile is stored in the presence of air, slow oxidation of the carbonaceous materials occurs and heat is released. The self-heating of coal stockpiles has a long history of posing significant problems to coal producers because it lowers the quality of coal and may result in hazardous thermal runaway. The prediction of the self-heating process is, therefore, necessary in order to identify and evaluate control measures and strategies for safe coal mining, storage and transportation; this requires an accurate estimate of the various processes associated with the self-heating which are impossible unless the appropriate phenomenological coefficients are known. In such storage-type problems, the critical ignition temperature θ_{cr} also known as the critical storage temperature, is an important design and control parameter, since at higher temperatures than this θ_{cr}, thermal ignition occurs, possibly giving rise to a variety of instabilities and problems.

Most complex fluids are multi-component mixtures. In many applications, these fluids are treated as a single continuum suspension with non-linear material properties and the techniques and models used in rheology or mechanics of non-linear fluids can generally be used to study such problems (Larson, 1999). In this case, global or macroscopic information about the variables such as the velocity or temperature fields for the whole suspension can be obtained. In many other applications, however, there is a need to know the details of the field variables such as velocity, concentration, temperature, etc., of each component and in such cases one needs to resort to the multi-component modeling approaches (Rajagopal & Tao, 1995; Massoudi, 2008, 2010). Examples of complex fluids whereby both approaches can be used are coal-slurries, many of the biological fluids such as blood and the synovial fluid, and many chemically- reacting fluids.

Granular materials exhibit non-linear phenomena like yield stress and normal stress differences, the latter usually being referred to as *dilatancy* (Reynolds, 1885, 1886). The normal-stress phenomenon is a characteristic of non-linear fluids and non-linear elastic solids. The central role played by this phenomenon in determining the character of granular materials was recognized early in the development of the theories for modeling granular materials. Interestingly, a constitutive model that was proposed for wet sand (Reiner, 1945), enjoyed a good bit of popularity as a model for non-Newtonian fluids before losing its appeal. One approach in the modeling of granular materials is to treat it as a *continuum,* which assumes that the material properties of the ensemble may be represented by continuous functions so that the medium may be divided infinitely without losing any of its defining properties. Since granular materials conform to the shape of the vessel containing them, they can be considered fluid-like. However, unlike fluids, they can be heaped. Characterizing bulk solids is difficult mainly because small variations in some of the primary properties such as size, shape, hardness, particle density, and surface roughness can result in very different behavior. Furthermore, secondary factors (such as the presence or absence of moisture, and ambient temperature) that are not directly associated with the particles, but are associated with the environment can have a significant effect on the behavior of the bulk solids (Massoudi, 2004).

Recently, Mehrabadi et al., (2005) have derived a set of conservation laws and constitutive relations of a density-gradient-dependent viscous fluid as a multipolar continuum where the connection between their model and the materials of Korteweg type (Truesdell & Noll, 1992) is also discussed. To replace the classical theory of capillarity, which specifies a jump condition at the surface separating homogeneous fluids possessing different densities, Korteweg proposed smooth constitutive equations for the stresses that depend on density

gradient[1]. Massoudi & Phuoc (2008) studied the flow of a compressible (density-gradient-dependent) non-linear fluid down an inclined plane, subject to radiation boundary condition. They assumed that the heat of reaction appears as a source term in the energy equation; in a sense they did not allow for a chemical reaction to occur and thus the conservation equation for the chemical species was ignored. A more general approach is, for example, that of Straughan & Tracey (1999) where the density is assumed to be not only a function of temperature, but also of (salt) concentration and there is an additional balance equation (the diffusion equation). In recent years, chemically reacting flows have been studied in a variety of applications. A class of problems where for example heat and salinity concentration compete with each other, and as a result the density distribution can be affected, has received much attention (Straughan, 2002, 2007). The phenomenon of double-diffusive convection in a fluid layer presents a challenging problem not only to engineers, but also to mathematicians since the stability of the flow is also of concern and interests.

In this Chapter, we provide a brief discussion of the important constitutive aspects of heat transfer in flows of complex fluids. We study the flow of a compressible (density gradient type) non-linear fluid down an inclined plane, subject to radiation boundary condition (For a full treatment of this problem, we refer the reader to Massoudi & Phuoc, 2008). The convective heat transfer is also considered where a source term, similar to the Arrhenius type reaction, is included. The non-dimensional forms of the equations are solved numerically and the competing effects of conduction, dissipation, heat generation and radiation are discussed.

2. Nomenclature

Symbol	Explanation
b	body force vector
D	Symmetric part of the velocity gradient
g	acceleration due to gravity
H	characteristic length
l	identity tensor
k	thermal conductivity
K_0	reaction rate
L	gradient of the velocity vector
N	dimensionless average volume fraction
q	heat flux vector
t	time
T	Cauchy stress tensor
u_0	reference velocity
x	spatial position occupied at time t

---

[1] Korteweg's equation in modern notation is (Truesdell & Noll, 1992, p. 514):

$$T_{ij} = (-p + \alpha \rho_{,k} \rho_{,k} + \beta \rho_{,kk}) \delta_{ij} + \gamma \rho_{,i} \rho_{,j} + \nu \rho_{,ij} + \lambda D_{kk} \delta_{ij} + 2\mu D_{ij},$$

where p, α, β, γ, ν, λ, and μ, are functions of ρ and θ alone. It is interesting to note that if we replace the density ρ by the temperature θ, we obtain equations very similar to those proposed by Maxwell (1876), who proposed constitutive equations for stresses in rarified gases that arise from inequalities of temperature.

y	direction normal to the inclined plane
(Y) or $\bar{y}$	dimensionless y
β_i	granular material constitutive coefficients, i = 0 to 5
v	volume fraction
ρ	bulk density
ρ_0	reference density
θ	temperature
div	divergence operator
∇	gradient symbol
$\otimes$	outer product

3. Governing equations

The balance laws, in the absence of chemical and electromagnetic effects, are the conservation of mass, linear momentum, angular momentum and energy (Truesdell & Noll, 1992). The conservation of mass in the Eulerian form is given by:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0 \tag{1}$$

where $\mathbf{u}$ is the velocity, ρ is the density, and $\partial/\partial t$ is the partial derivative with respect to time. The balance of linear momentum is

$$\rho \frac{d\mathbf{u}}{dt} = \text{div}\mathbf{T} + \rho \mathbf{b} \tag{2}$$

where d/dt is the total time derivative, given by

$$\frac{d(.)}{dt} = \frac{\partial(.)}{\partial t} + [\text{grad}(.)]\mathbf{u}, \tag{3}$$

where $\mathbf{b}$ is the body force, and $\mathbf{T}$ is the Cauchy stress tensor. The balance of angular momentum (in the absence of couple stresses) yields the result that the Cauchy stress is symmetric. The energy equation in general can have the form:

$$\rho \frac{d\varepsilon}{dt} = \mathbf{T}.\mathbf{L} - \text{div}\mathbf{q} + \rho r + QC_0 K_0 \tag{4}$$

where ε denotes the specific internal energy, $\mathbf{q}$ is the heat flux vector, r is the radiant heating, Q is the heat of reaction, C_0 is the initial concentration of the reactant species, K_0 is the reaction rate expression which is a function of temperature, and $\mathbf{L}$ is the velocity gradient. For most common applications where there are no chemical reactions or heat generation, the last term on the right hand side is ignored. It should also be noted that the form of the energy equation for a complex fluid is in general not the same as the standard energy equation given in many books and articles, where the substantial (or total) time derivative of the temperature appears on the left hand side of Eqn (4) instead of the internal energy. For the detailed derivation of Eqn (4) and the assumptions implicit in obtaining this form using the First Law of Thermodynamics, see Appendix A.

Thermodynamical considerations require the application of the second law of thermodynamics or the entropy inequality. The local form of the entropy inequality is given by (Liu, 2002, p. 130):

$$\rho\dot{\eta} + \text{div}\,\boldsymbol{\varphi} - \rho s \geq 0 \tag{5}$$

where $\eta(\mathbf{x}, t)$ is the specific entropy density, $\phi(\mathbf{x}, t)$ is the entropy flux, and s is the entropy supply density due to external sources, and the dot denotes the material time derivative. If it is assumed that $\boldsymbol{\varphi} = \dfrac{1}{\theta}\mathbf{q}$, and $s = \dfrac{1}{\theta}r$, where θ is the absolute temperature, then Equation (5) reduces to the Clausius-Duhem inequality

$$\rho\dot{\eta} + \text{div}\,\frac{\mathbf{q}}{\theta} - \rho\frac{r}{\theta} \geq 0 \tag{6}$$

Even though we do not consider the effects of the Clausius-Duhem inequality in this Chapter, for a complete thermo-mechanical study of a problem, the Second Law of Thermodynamics has to be considered (Müller, 1967; Ziegler, 1983; Truesdell & Noll, 1992; Liu, 2002).

Constitutive relations for complex materials can be obtained in different ways, for example, by using: (a) continuum mechanics, (b) physical and experimental models, (c) numerical simulations, (d) statistical mechanics approaches, and (e) ad-hoc approaches. In the next section, we provide brief description of the constitutive relations that will be used in this Chapter. In particular, we mention the constitutive relation for the stress tensor $\mathbf{T}$ and the heat flux vector $\mathbf{q}$. A look at the governing equation (1-4) reveals that constitutive relations are required for $\mathbf{T}$, $\mathbf{q}$, Q, ε, and r. Less obvious[2] is the fact that in many practical problems involving competing effects such as temperature and concentration, the body force $\mathbf{b}$, which in problems dealing with natural convection oftentimes depends on the temperature and is modeled using the Boussinesq assumption (Rajagopal et al., 2009), now might have to be modeled in such a way that it is also a function of concentration [see for example, Equation (2.2) of Straughan and Walker (1997)].

4. Stress tensor and viscous dissipation

One of the most widely used and successful constitutive relations in fluid mechanics is the Navier-Stokes model, where the stress $\mathbf{T}$ is explicitly and linearly related to the symmetric

---

[2] In certain applications, such as the flow of chemically reactive fluids, the conservation of concentration $\dfrac{\partial c}{\partial t} + \text{div}\,(c\mathbf{u}) = f$ where c is the concentration and f is a constitutive function also needs to be considered. This equation is also known as the convection-reaction-diffusion equation. For example, for the concentration flux, Bridges & Rajagopal (2006) suggested $f = -\text{div}\,\mathbf{w}$ where $\mathbf{w}$ is a flux vector, related to the chemical reactions occurring in the fluid and is assumed to be given by a constitutive relation similar to the Fick's assumption, namely $\mathbf{w} = -K_1 \nabla c$, where K_1 is a material parameter which is assumed to be a scalar-valued function of (the first Rivlin-Ericksen tensor) $\mathbf{A}_1$, $K_1 = K_1(\mathbf{A}_1) = \kappa\left\|\mathbf{A}_1^2\right\|$, where κ is constant, and $\|.\|$ denotes the trace-norm. Clearly K_1 can also depend on the concentration and temperature as well as other constitutive variables.

part of the velocity gradient **D**. From a computational point of view, it is much easier and less cumbersome to solve the equations for the explicit models[3]. For many fluids such as polymers, slurries and suspensions, some generalizations have been made to model shear dependent viscosities. These fluids are known as the power-law models or the generalized Newtonian fluid (GNF) models, where

$$\mathbf{T} = -p\mathbf{1} + \mu_0 \left(tr\mathbf{A}_1^2 \right)^m \mathbf{A}_1 \tag{7}$$

where p is the indeterminate part of the stress due to the constraint of incompressibility, and **1** is the identity tensor, μ_0 is the coefficient of viscosity, m is a power-law exponent, a measure of non-linearity of the fluid related to the shear-thinning or shear-thickening effects, tr is the trace operator and $\mathbf{A}_1$ is related to the velocity gradient. A sub-class of the GNF models is the chemically reacting fluids which offer many technological applications ranging from the formation of thin films for electronics, combustion reactions, catalysis, biological systems, etc. (Uguz & Massoudi, 2010). Recently, Bridges & Rajagopal (2006) have proposed constitutive relations for chemically reacting fluids where

$$\mathbf{T} = -\pi\mathbf{1} + \mu(c, \mathbf{A}_1)\mathbf{A}_1 \tag{8}$$

where π is the constraint due to incompressibility and

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T \; ; \mathbf{L} = grad\mathbf{u} \; ; c = \frac{\rho_r}{\rho_r + \rho} \tag{9}$$

where ρ is the density of the fluid and ρ_r denotes the density of the (coexisting) reacting fluid. Furthermore they assumed

$$\mu(c, \mathbf{A}_1) = \mu^*(c)[1 + \alpha tr(\mathbf{A}_1^2)]^n \tag{10}$$

where n determines whether the fluid is shear-thinning (n<0), or shear-thickening (n>0). A model of this type where μ^* is constant, i.e., when μ does not depend on c, has been suggested by Carreau et al., (1997) to model the flows of polymeric liquids. The viscosity is assumed to depend on the concentration c ; depending on the form of $\mu^*(c)$ the fluid can be either a chemically-thinning or chemically-thickening fluid, implying a decrease or an increase in the viscosity, respectively, as c increases. The second law of thermodynamics requires the constant $\alpha \geq 0$ [Bridges & Rajagopal, 2006]. Clearly, in general, it is possible to define other sub-classes of the GNF models where the viscosity can also be function of temperature, pressure, electric or magnetic fields, etc.

Another class of non-linear materials which in many ways and under certain circumstances behave as non-linear fluids is granular materials which exhibit two unusual and peculiar characteristics: (i) normal stress differences, and (ii) yield criterion. The *first* was observed by

---

[3] There are, however, cases [such as Oldroyd (1984) type fluids and other rate-dependent models] whereby it is not possible to express **T** explicitly in terms of **D** and other kinematical variables. For such cases, one must resort to implicit theories, for example, of the type (Rajagopal, 2006) $\mathbf{f}(\mathbf{T}, \mathbf{D}, \theta) = \mathbf{0}$, where θ is the temperature.

Reynolds (1885, 1886) who called it 'dilatancy.' Dilatancy is described as the phenomenon of expansion of the voidage that occurs in a tightly packed granular arrangement when it is subjected to a deformation. Reiner (1945, 1948) proposed and derived a constitutive relation for wet sand whereby the concept of dilatancy is given a mathematical structure. This model does not take into account how the voidage (volume fraction) affects the stress. Using this model, Reiner showed that application of a non-zero shear stress produces a change in volume. The constitutive relation of the type $S_{lm} = F_0\delta_{lm} + 2\eta D_{lm} + 4\eta_c D_{lj}D_{jm}$ describing the rheological behavior of a non-linear fluid was named by Truesdell (Truesdell & Noll, 1992) as the Reiner–Rivlin (Rivlin, 1948) fluid, where in modern notation the stress tensor **T** is related to **D** (Batra, 2006, p. 221):

$$\mathbf{T} = -p(\rho)\mathbf{I} + f_1\mathbf{D} + f_2\mathbf{D}^2 \tag{11}$$

where f's are function of $\rho, \mathrm{tr}\mathbf{D},$ and $\mathrm{tr}\mathbf{D}^2$. Perhaps the simplest model which can predict the normal stress effects (which could lead to phenomena such as 'die-swell' and 'rod-climbing', which are manifestations of the stresses that develop orthogonal to planes of shear) is the second grade fluid, or the Rivlin-Ericksen fluid of grade two (Rivlin & Ericksen, 1955; Truesdell & Noll, 1992). This model has been used and studied extensively and is a special case of fluids of differential type. For a second grade fluid the Cauchy stress tensor is given by:

$$\mathbf{T} = -p\mathbf{1} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{12}$$

where p is the indeterminate part of the stress due to the constraint of incompressibility, μ is the coefficient of viscosity, α_1 and α_2 are material moduli which are commonly referred to as the normal stress coefficients. The kinematical tensor $\mathbf{A}_2$ is defined through

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1\mathbf{L} + (\mathbf{L})^\mathbf{T}\mathbf{A}_1 \tag{13}$$

where $\mathbf{A}_1$ and **L** are given by Eqn (9). The thermodynamics and stability of fluids of second grade have been studied in detail by Dunn & Fosdick (1974). They show that if the fluid is to be thermodynamically consistent in the sense that all motions of the fluid meet the Clausius-Duhem inequality and that the specific Helmholtz free energy of the fluid be a minimum in equilibrium, then

$$\begin{aligned} \mu &\geq 0, \\ \alpha_1 &\geq 0, \\ \alpha_1 + \alpha_2 &= 0 \end{aligned} \tag{14}$$

It is known that for many non-Newtonian fluids which are assumed to obey Equation (12), the experimental values reported for α_1 and α_2 do not satisfy the restriction $(14)_{2,3}$. In an important paper, Fosdick & Rajagopal (1979) show that irrespective of whether $\alpha_1 + \alpha_2$ is positive, the fluid is unsuitable if α_1 is negative. It also needs to be mentioned that second grade fluids (or higher order models) raise the order of differential equations by introducing higher order derivates into the equations. As a result, in general, one needs additional

boundary conditions; for a discussion of this issue, see Rajagopal (1995), and Rajagopal & Kaloni (1989).

The *second* peculiarity is that for many granular materials there is often a yield stress. This yield condition is often related to the angle of repose, friction, and cohesion among other things. Perhaps the most popular yield criterion for granular materials is the Mohr-Coulomb one, although by no means the only one (Massoudi & Mehrabadi, 2001). Overall, it appears that many of the bulk solids behave as visco-plastic fluids. Bingham (1922, p. 215) proposed a constitutive relation for a visco-plastic material in a simple shear flow where the relationship between the shear stress (or stress **T** in general), and the rate of shear (or the symmetric part of the velocity gradient **D**) is given by (Prager, 1989, p. 137)

$$2\mu D_{ij} = \begin{cases} 0 & \text{for} \quad F < 0 \\ FT'_{ij} & \text{for } F \geq 0 \end{cases} \tag{15}$$

where T'_{ij} denotes the stress deviator and F, called the yield function, is given by

$$F = 1 - \frac{K}{II'^{1/2}_2} \tag{16}$$

where II'_2 is the second invariant of the stress deviator, and in simple shear flows it is equal to the square of the shear stress and K is called yield stress (a constant). For one dimensional flow, these relationships reduce to the ones proposed by Bingham (1922), i.e.

$$F = 1 - \frac{K}{|T_{12}|} \tag{17}$$

And

$$2\mu D_{12} = \begin{cases} 0 & \text{for} F < 0 \\ FT_{12} & \text{for} F \geq 0 \end{cases} \tag{18}$$

The constitutive relation given by Eqn (15) is known as Bingham model (Zhu et al., 2005).

We now provide a brief description of a model due to Rajagopal & Massoudi (1990) which will be used in this Chapter; this model is capable of predicting both of the above mentioned non-linear effects, namely possessing a yield stress and being capable of demonstrating the normal-stress differences. The Cauchy stress tensor **T** in a flowing granular material may depend on the manner in which the material is distributed, *i.e.*, the volume fraction ν and possibly also its gradient, and the symmetric part of the velocity gradient tensor **D**. Based on this observation, Rajagopal & Massoudi (1990) derived a constitutive model that predicts the possibility of both normal stress-differences and is properly frame invariant (Cowin, 1974; Savage, 1979):

$$\begin{aligned} \mathbf{T} = [\beta_o(\rho) + \beta_1(\rho) \nabla\rho \cdot \nabla\rho + \beta_2(\rho) \text{ tr } \mathbf{D}] \mathbf{1} \\ + \beta_3(\rho)\mathbf{D} + \beta_4(\rho, \nabla\rho) \nabla\rho \otimes \nabla\rho + \beta_5(\rho) \mathbf{D}^2 \end{aligned} \tag{19}$$

where β's are material properties, and $\mathbf{D} = \dfrac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}\right]$. In what follows we will use the concept of volume fraction[4] ν, and use this instead of the density ρ, as a variable, where ν is represented as a continuous function of position and time $0 \le \nu(\mathbf{x}, t) < 1$ and is related to the classical mass density or bulk density ρ, through $\rho = \rho_{10}\nu$ where ρ_{10} is the reference density (a constant value). That is, in some sense we have normalized the density through the introduction of volume fraction. Now if the material is flowing, the following representations are proposed for the β's:

$$\beta_o = f\nu; \quad f < 0 \quad \begin{cases} \beta_1 = \beta_1^*\left(1 + \nu + \nu^2\right) \\ \beta_2 = \beta_2^*\left(\nu + \nu^2\right) \\ \beta_3 = \beta_3^*\left(\nu + \nu^2\right) \\ \beta_4 = \beta_4^*\left(1 + \nu + \nu^2\right) \\ \beta_5 = \beta_5^*\left(\nu + \nu^2\right) \end{cases} \tag{20}$$

The above representation can be viewed as Taylor series approximation for the material parameters [Rajagopal, et al (1994)]. Such a quadratic dependence, at least for the viscosity β_3, is on the basis of dynamic simulations of particle interactions (Walton & Braun, 1986a, 1986b). Furthermore, it is assumed (Rajagopal & Massoudi, 1990) that

$$\beta_{30} = \beta_{20} = \beta_{50} \tag{21}$$

In their studies, Rajagopal et al. (1992) proved existence of solutions, for a selected range of parameters, when, $\beta_1 + \beta_4 > 0$, and f <0. Rajagopal & Massoudi (1990) gave the following rheological interpretation to the material parameters: β_0 is similar to pressure in a compressible fluid or the yield stress and is to be given by an equation of state, β_2 is like the second coefficient of viscosity in a compressible fluid, β_1 and β_4 are the material parameters connected with the distribution of the granular materials, β_3 is the viscosity of the granular materials, and β_5 is similar to what is referred to as the 'cross-viscosity' in a Reiner-Rivlin fluid. The distinct feature of this model is its ability to predict the normal stress differences which are often related to the dilatancy effects. The significance of this model is discussed by Massoudi (2001), and Massoudi & Mehrabadi (2001). If the material is just about to yield, then

---

[4] The volume fraction field $\nu(\mathbf{x}, t)$ plays a major role in many of the proposed continuum theories of granular materials. In other words, it is assumed that the material properties of the ensemble are continuous functions of position. That is, the material may be divided indefinitely without losing any of its defining properties. A distributed volume, $V_t = \int \nu dV$ and a distributed mass, $M = \int \rho_s \nu dV$ can be defined, where the function ν is an independent kinematical variable called the volume distribution function and has the property $0 \le \nu(\mathbf{x}, t) \le \nu_m < 1$. The function ν is represented as a continuous function of position and time; in reality, ν in such a system is either one or zero at any position and time, depending upon whether one is pointing to a granule or to the void space at that position. That is, the real volume distribution content has been averaged, in some sense, over the neighborhood of any given position. The classical mass density or bulk density, ρ is related to ρ_s and ν through $\rho = \rho_s \nu$.

Massoudi & Mehrabadi (2001) indicate that if the model is to comply with the Mohr-coulomb criterion, the following representations are to be given to the material parameters in Eqn (19):

$$\beta_o = c \cot \phi$$
$$\beta_1 = \frac{\beta_4}{2} \left(\frac{1}{\sin \phi} - 1 \right)$$

(22)

where ϕ is the internal angle of friction and c is a coefficient measuring cohesion. Rajagopal et al., (2000) and Baek et al., (2001) discuss the details of experimental techniques using orthogonal and torsional rheometers to measure the material properties β_1 and β_4. Rajagopal et al., (1994) showed that by using an orthogonal rheometer, and measuring the forces and moments exerted on the disks, one can characterize the material moduli $\beta's$.
Finally, we can see from Eqn (4) that the term usually referred to as the viscous dissipation is given by the first term on the right hands side of Eqn (4), that is

$$\zeta = \mathbf{T}.\mathbf{L}$$

(23)

Thus, there is no need to model the viscous dissipation term independently, since once the stress tensor for the complex fluid is derived or proposed, ζ can be obtained from the definition given in Eqn (23).

5. Heat flux vector (Conduction)

For densely packed granular materials, as particles move and slide over each other, heat is generated due to friction and therefore in such cases the viscous dissipation should be included. Furthermore, the constitutive relation for the heat flux vector is generally assumed to be the Fourier's law of conduction where

$$\mathbf{q} = -k\nabla\theta$$

(24)

where k is an effective or modified form of the thermal conductivity. In general, k can also depend on concentration, temperature, etc., and in fact, for anisotropic material, k becomes a second order tensor. There have been many experimental and theoretical studies related to this issue and in general the flux $\mathbf{q}$ could also include additional terms such as the Dufour and Soret effects. Assuming that $\mathbf{q}$ can be explicitly described as a function of temperature, concentration, velocity gradient, etc., will make the problem highly non-linear. Kaviany (1995, p.129) presents a thorough review of the appropriate correlations for the thermal conductivity of packed beds and the effective thermal conductivity concept in multiphase flows. Massoudi (2006a, 2006b) has recently given a brief review of this subject and has proposed and derived a general constitutive relation for the heat flux vector for a flowing granular media. It is important to recognize that in the majority of engineering applications, the thermal conductivity of the material is assumed (i) *a priori* to be based on the Fourier's heat conduction law, and (ii) is a measurable quantity (Narasimhan, 1999). Jeffrey (1973) derived an expression for the effective thermal conductivity which includes the second order effects in the volume fraction (Batchelor & O'Brien, 1977):

$$k = \kappa_M[1 + 3\xi\nu + \hat{\xi}\nu^2] + O(\nu^3)$$

(25)

where

$$\hat{\xi} = 3\xi^2 + \frac{3\xi^3}{4} + \frac{9\xi^3}{16}\left(\frac{\omega+2}{2\omega+3}\right) + \frac{3\xi^4}{26} + ... \qquad (26)$$

where

$$\xi = \frac{\omega-1}{\omega+2} \qquad (27)$$

$$\omega = \frac{k_2}{k_1} \qquad (28)$$

where ω is the ratio of conductivity of the particle to that of the matrix, κ the effective conductivity of the suspension, κ_M the conductivity of the matrix, and ν is the solid volume fraction (Bashir & Goddard, 1990). Massoudi (2006a, 2006b) has conjectured, based on arguments in mechanics, that the heat flux vector for a 'reasonably' dense assembly of granular materials where the media is assumed to behave as a continuum in such a way that as the material moves and is deformed, through the distribution of the voids, the heat flux is affected not only by the motion but also by the density (or volume fraction) gradients. To keep things simple, it was assumed that the interstitial fluid does not play a major role (some refer to this as 'dry' granular medium), and as a result a frame-indifferent model for the heat flux vector of such a continuum was derived to be:

$$\mathbf{q} = a_1\mathbf{n} + a_2\mathbf{m} + a_3\mathbf{Dn} + a_4\mathbf{Dm} + a_5\mathbf{D^2n} + a_6\mathbf{D^2m} \qquad (29)$$

where

$$\mathbf{m} = \mathrm{grad}\rho \qquad (30)$$

$$\mathbf{n} = \mathrm{grad}\theta \qquad (31)$$

where the a's in general have to be measured experimentally; within the context of the proposed theory they depend on the invariants and appropriate material properties. It was shown that (i) when $a_2 = a_3 = a_4 = a_5 = a_6 = 0$, and a_1 =constant=-k, then we recover the standard Fourier's Law:

$$\mathbf{q} = -k\nabla\theta \qquad (32)$$

And (ii) when a_1 =constant=-k, and a_2 =constant, we have

$$\mathbf{q} = -k\nabla\theta + a_2\nabla\nu \qquad (33)$$

Soto et al., (1999) showed that based on molecular dynamics (MD) simulations of inelastic hard spheres (IHS), the basic Fourier's law has to be modified for the case of fluidized granular media. It is noted that Wang (2001) also derived a general expression for the heat flux vector for a fluid where heat convection is also important; he assumed that

$\mathbf{q} = \mathbf{f}(\theta, \nabla\theta, \mathbf{v}, \mathbf{L}, X)$ where $\mathbf{f}$ is a vector-valued function, θ temperature, $\nabla\theta$ is the gradient of temperature, $\mathbf{v}$ the velocity vector, $\mathbf{L}$ its gradient, and X designates other scalar-valued thermophysical parameters.

6. A brief discussion of other constitutive parameters

Looking at Eqn (4), it can be seen that constitutive relations are also needed for K_0, r, and ε. As shown by Dunn & Fosdick (1974), the specific internal energy ε, in general, is related to the specific Helmholtz free energy ψ through:

$$\varepsilon = \psi + \theta\eta = \varepsilon(\theta, \mathbf{A_1}, \mathbf{A_2}) = \hat{\varepsilon}(y) \tag{34}$$

where η is the specific entropy and θ is the temperature. In the problem considered in this chapter, due to the nature of the kinematical assumption about $\mathbf{u}$ and θ, it can be seen that $\dfrac{d\varepsilon}{dt} = 0$. We now discuss briefly the constitutive modeling of K_0 and r.

We assume that the heat of reaction appears as a source term in the energy equation; in a sense we do not allow for a chemical reaction to occur and thus the conservation equation for the chemical species is ignored. This is only to be considered as a first approximation; a more general approach is, for example, that of Straughan & Tracey (1999) where the density is assumed to be not only a function of temperature, but also of (salt) concentration and there is an additional balance equation (the diffusion equation). We assume that K_0 is given by (Boddington et al., 1983)

$$K_0(\theta) = A_0 \left(\frac{\bar{k}\theta}{\bar{v}h}\right)^m \exp\left(\frac{-E}{R\theta}\right) \tag{35}$$

where A_0 is the rate constant, E is the activation energy, R is the universal gas constant, h is the Planck's number, $\bar{k}$ is the Boltzmann's constant, $\bar{v}$ is the vibration frequency and m is an exponent related to the type of reaction; for example, $m \in \{0.5, 0, -2\}$ correspond to Bimolecular temperature dependence, Arrhenius or zero order reaction and sensitized temperature dependence. As indicated by Boddington et al., (1977), "Even when reactions are kinetically simple and obey the Arrhenius equation, the differential equations for heat balance and reactant consumption cannot be solved explicitly to express temperatures and concentrations as functions of time unless strong simplifications are made." One such simplification is to assume that there is no reactant consumption, which as mentioned earlier, is the approach that we have taken in the present study. Furthermore, although in this paper we assume A_0 to be constant and K_0 to be a function of temperature only, in reality, we expect K_0 (and/or A_0) to be function of volume fraction (density).

Combined heat transfer processes, such as convection-radiation, play a significant role in many chemical processes (Siegel & Howell, 1981) involving combustion, drying, fluidization, MHD flows, etc (Zel'dovich & Raizer, 1967; Pomraning, 1973). In general, the radiative process either occurs at the boundaries or as a term in the energy equation. The latter case is usually accomplished by a suggestion due to Rosseland (Clouet, 1997) where the radiative term is approximated as a flux in such a way that the term corresponding to radiation in the heat transfer (energy) equation now appears as a gradient term similar to

the Fourier's conduction term. This method has found much favor among many researchers especially those working with viscoelastic fluids and MHD flows. Alternatively, radiation effects can be incorporated at the boundaries through appropriate (constitutive) assumption, such as the Stefan-Boltzmann condition. Free surface flows present a challenging problem to engineers as the combined convection-radiation at the boundaries has major applications in many industries (Miller & Weaver, 2003). At the free surface, the no-traction boundary condition is imposed on the stress tensor, and as a result we obtain two expressions for the velocity gradient, and for the temperature we apply the Stefan-Boltzmann condition (Saldanha da Gama, 2004)

$$q = \gamma\sigma(\theta_h^4 - \theta_\infty^4) \tag{36}$$

where γ is surface emissivity, σ is the Stefan–Boltzmann constant, the surrounding temperature is designated as θ_∞ and the temperature at the free surface is θ_h. Finally, in order to study the phenomenon of self-ignition or spontaneous combustion in a coal stockpile, a multiphase flow approach [see for example, Salinger et al., (1994) or Akgun & Essenhigh, 2001) should be used where the effects of moisture content, oxidation, etc., are also considered.

7. Flow down an inclined plane

Flow down an inclined plane occurs naturally as in the cases of avalanches and mudslides; it is also used for transporting and drying of bulk solids (such as agricultural and pharmaceutical products). It is a viscometric flow (Truesdell, 1974) and one which amends itself to fundamental theoretical and experimental studies. Studies have shown that if the effects of viscous dissipation are included, strong non-linearities in the temperature profiles are observed (Massoudi & Anand, 2004). It is assumed that the flow is fully developed [see Figure 1]. The free surface is exposed to high ambient temperature and as a result a modified Stefan-Boltzmann correlation is used at that surface (Fuchs 1996, p.331).
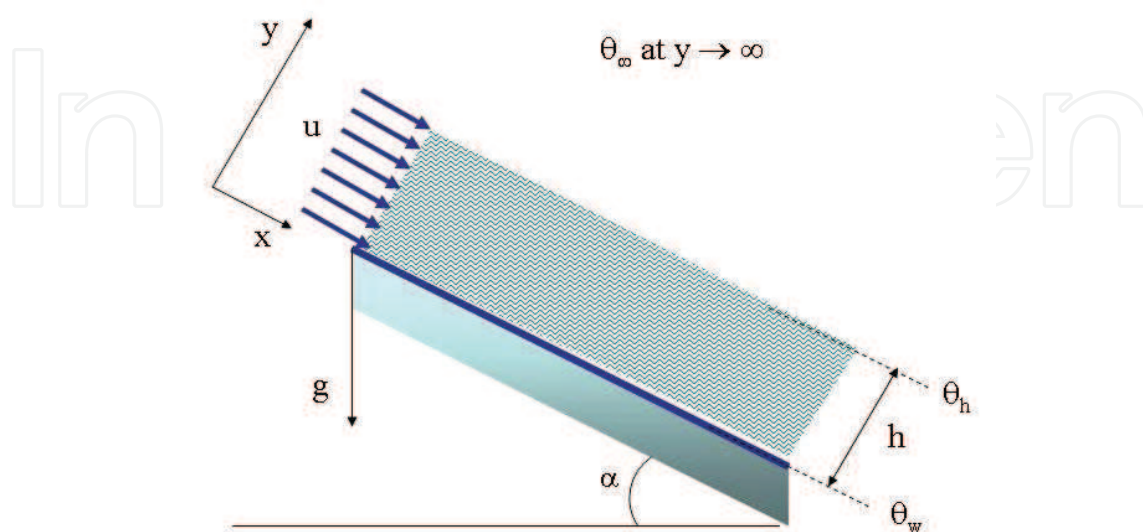

Fig. 1. Flow down an inclined plane

For the problem under consideration, we make the following assumptions:

i.   the motion is steady,
ii.   the effects of radiant heating 'r' are imposed at the free surface,
iii. the constitutive equation for the stress tensor is given by Equation (19) and the constitutive equation for the heat flux vector is that of Fourier's law, given by Equations (24)-(25);
iv.   the heat of reaction is given by equation (35)
v.   the density (or volume fraction), velocity and temperature fields are of the form

$$\left\{ \begin{array}{l} v = v(y) \\ \mathbf{u} = u(y)\mathbf{i} \\ \theta = \theta(y) \end{array} \right\} \tag{38}$$

With the above assumptions, the conservation of mass is automatically satisfied and by defining the dimensionless distance $\bar{y}$, the velocity $\bar{u}$ and the temperature $\bar{\theta}$ by the following equations (Na, 1979):

$$\bar{y} = \frac{y}{H}; \quad \bar{u} = \frac{u}{u_0}; \quad \bar{\theta} = \frac{\theta}{\theta_w} \tag{39}$$

where u_0 is a reference velocity, θ_w is the wall temperature[5], and H is the constant height to the free surface, the dimensionless form of the equations are:

$$R_1 \frac{dv}{d\bar{y}} + R_2 \left(1 + v + v^2\right) \frac{dv}{d\bar{y}} \frac{d^2 v}{d\bar{y}^2} + R_2 (1 + 2v) \left(\frac{dv}{d\bar{y}}\right)^3 + R_5 (1 + 2v) \frac{dv}{d\bar{y}} \left(\frac{d\bar{u}}{d\bar{y}}\right)^2$$
$$+ 2R_5 v(1 + v) \left(\frac{d\bar{u}}{d\bar{y}}\right) \left(\frac{d^2 \bar{u}}{d\bar{y}^2}\right) = v \cos\alpha \tag{40}$$

$$R_3 v (1 + v) \frac{d^2 \bar{u}}{d\bar{y}^2} + R_3 (1 + 2v) \frac{dv}{d\bar{y}} \frac{d\bar{u}}{d\bar{y}} = -v \sin\alpha \tag{41}$$

$$\left(1 + av + bv^2\right) \frac{d^2 \bar{\theta}}{d\bar{y}^2} + (a + 2bv) \frac{dv}{d\bar{y}} \frac{d\bar{\theta}}{d\bar{y}} + R_7 \bar{\theta}^m e^{\frac{-1}{\zeta \bar{\theta}}} = -R_4 v (1 + v) \left(\frac{d\bar{u}}{d\bar{y}}\right)^2 \tag{42}$$

_____

[5] It needs to be mentioned that it is possible to use another reference temperature for this problem, namely a proper dimensionless quantity as it appears in the exponent of the heat of reaction. For example, as Boddington et al. (1983, p. 249) show, if $\theta = \dfrac{T - T_a}{(RT_a^2 / E)}$ and $\varepsilon = \dfrac{RT_a}{E}$ are used, then "the concepts of explosion and criticality are appropriate so long as the activation energy E is not too low or the ambient temperature T_a is not too high, i.e., so that $\varepsilon = \dfrac{RT_a}{E} < \varepsilon_{tr}$". If one uses this, then one obtains a relationship which expresses the balance between the heat generated and the heat conducted, as for example in the Frank-Kamenetskii (1969) approach. However, since in our problem we also have radiation at the free surface, we have decided to use the wall temperature as the reference temperature.

where

$$R_1 = \frac{f}{\rho_{10}gH} ; \quad R_2 = \frac{2\left(\beta_1^* + \beta_4^*\right)}{\rho_{10}gH^3} ; \quad R_3 = \frac{\beta_3^* u_0^2}{2H^2\rho_{10}g} ; \quad R_4 = \frac{\beta_3^* u_w^2}{K_M\theta_w} ;$$

$$R_5 = \frac{\beta_5^* u_w^2}{\rho_{10}gH^3} ; \quad R_6 = \frac{\gamma\sigma\theta_w^3 H}{K_M} ; R_7 = \frac{H^2 Q C_0 A_0 \bar{k}^m \theta_w^{m-1}}{K_M h^m \bar{v}^m} ; \zeta = \frac{R\theta_w}{E}$$

(43)

and the boundary conditions become [At the wall surface of the inclined we assume the no-slip condition for the velocity and a constant temperate θ_w.]:

$$\text{at } \bar{y} = 0 : \begin{cases} \bar{u} = 0 \\ \bar{\theta} = 1 \end{cases}$$

(44)

At the free surface, the no-traction boundary condition is imposed on the stress tensor, and as a result we obtain two expressions for the velocity gradient and the volume fraction [see below], and for the temperature we apply the Stefan-Boltzmann[6] condition (Saldanha da Gama, 2004; Wright, 2007;Wright et al., 2002; Agudelo & Cortes, 2010) when the surrounding temperature is designated as θ_∞ and the temperature at the free surface is θ_H.

$$\text{at } \bar{y} = 1 : \begin{cases} \dfrac{d\bar{u}}{d\bar{y}} = 0 \\ R_1 v + \dfrac{R_2}{2}\left(1 + v + v^2\right)\left(\dfrac{dv}{d\bar{y}}\right)^2 + \dfrac{R_5}{4}\left(1 + v + v^2\right)\left(\dfrac{d\bar{u}}{d\bar{y}}\right)^2 = 0 \\ \dfrac{d\bar{\theta}}{d\bar{y}} = \dfrac{-R_6}{1 + av + bv^2}\left(\bar{\theta}_{\bar{y}=1}^4 - \bar{\theta}_\infty^4\right) \end{cases}$$

(45)

We can see that we still need an additional boundary condition for v. Two options are available to us: (i) we can impose a distribution function, which could be a constant number for v at the wall, based on experimental results (practically this may mean gluing particles), or (ii) we can give an average value for v integrated over the cross section (a measure of the amount of particles in the system).

Case a

$$\begin{cases} v = v_o & \text{at } \bar{y} = 0 \\ v = v_H & \text{at } \bar{y} = 1 \end{cases}$$

(46)

or
Case b:

$$\begin{cases} v = v_o & \text{at } \bar{y} = 0 \\ N = \int_0^1 v \, d\bar{y} \end{cases}$$

(47)

---

[6] Equation (36) is really our first approximation (Wu & Chu, 1999; Miller & Weaver, 2003) and a more appropriate one for the case of granular materials might be to introduce into the equation a function for the dependence of the volume fraction, for example, $q = f(v)\varepsilon\sigma\left(\theta_H^4 - \theta_s^4\right)$

And

$$\overline{\theta}_\infty = \frac{\theta_\infty}{\theta_w} \tag{48}$$

These dimensionless parameters can be given the following physical interpretations: R_1 could be thought of as the ratio of the pressure force to the gravity force, R_2 is the ratio of forces developed in the material due to the distribution of the voids to the force of gravity, R_4 is a measure of viscous dissipation, which is the product of the Prandtl number and the Eckert number, R_5 is related to the normal stress coefficient, R_6 is a measure of the emissivity of the particles to the thermal conductivity, and R_7 is a measure of the heat generation due to chemical reaction. It follows from Rajagopal & Massoudi (1990) that R_1, must always be less than zero for the solution to exist and all the other non-dimensional parameters, i.e., R_3 and R_4 must be greater than zero, since the viscosity is positive. In addition to these dimensionless numbers, the value of the volume fraction at the lower plate, N a, b, and α are also parameters which need to be specified.

8. Numerical solution and concluding remarks

The volume fraction equation, Eqn. (40) and the velocity equation, Eqn. (41) are coupled and must be solved simultaneously. Once these solutions are obtained the temperature distribution can be found by integrating equation (42). For equation (40) the boundary conditions $v = v_o$ at $\overline{y} = 0$ and $N_o = \int_0^1 v d\overline{y}$ are assumed *a priori*, but $dv/d\overline{y}$ at $\overline{y} = 0$ is unknown. For equation (41) the boundary conditions are $\overline{u} = 0$ at $\overline{y} = 0$ and $(d\overline{u}/d\overline{y})_{\overline{y}=1} = 0$ but $(d\overline{u}/d\overline{y})_{\overline{y}=0}$ is unknown. The unknown boundary conditions must be assumed so that the integration can be performed. That is, it is necessary to correct the initial guesses to be such that the derivative of the velocity at the edge of the solution domain and the distribution of volume fraction satisfy the corresponding known conditions given by equations (33) and (34). Procedures for such corrections are given by Massoudi & Phuoc (2005, 2007) and they will be used here.

The effect of R_7 on the distribution of the dimensionless temperature is shown in Fig. 2. The results indicate that the temperature is always higher inside the domain than the inclined surface temperature and the free surface temperature. Since R_7 is the measure of the heat generation due to chemical reaction, when the reaction is frozen ($R_7 = 0.0$) the temperature distribution would depend only on R_1, and R_2, representing the effects of the pressure force developed in the material due to the distribution, R_3 and R_4 viscous dissipation, R_5 the normal stress coefficient, R_6 the measure of the emissivity of the particles to the thermal conductivity, etc. When the flow is not frozen ($R_7 > 0$) the temperature inside the flow domain is much higher than those at the inclined and free surfaces. As a result, heat is transferred away from the flow toward both the inclined surface and the free surface with a rate that increases as R_7 increases.

The effect of the reaction can also be investigated in terms of the activation energy parameter ζ. To see this, we integrate the energy equation with ζ varying from 0.5 to 20 while keeping all other parameters constant. The temperature distributions are shown in Fig. 3. For a given temperature, an increase in ζ implies that the activation energy is smaller and thus, the reaction rate is increased leading to an increase in the heat of the reaction. As a

result the flow is chemically heated and its temperature increase. The results shown here indicate that for all values of ζ used the chemical effects are significant and the temperature is always higher than both the surface temperature and the free surface temperature. The heat transfer is always from the flow toward both the inclined surface and the free stream.
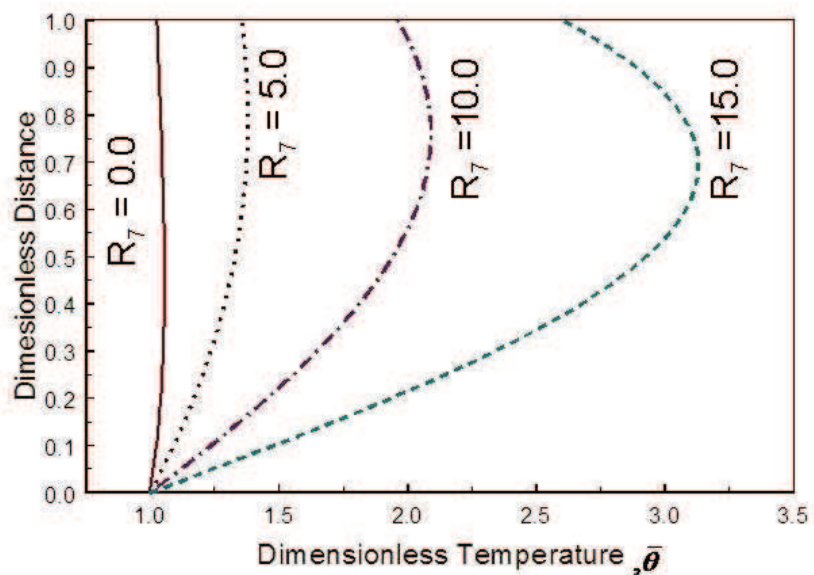

Fig. 2. Effect of R_7 on the dimensionless temperature profiles (a = 0.75, b = 1.0, R_1 = -3.0; R_2 = 10.0; R_3 = 0.05, R_4 = 0.1, R_5 = 0.01, R_6 = 0.1, v_w = 0.4; N = 0.3; α = 30°, m = 0.5, ζ = 0.5, and $\overline{\theta}_\infty = 0.5$)
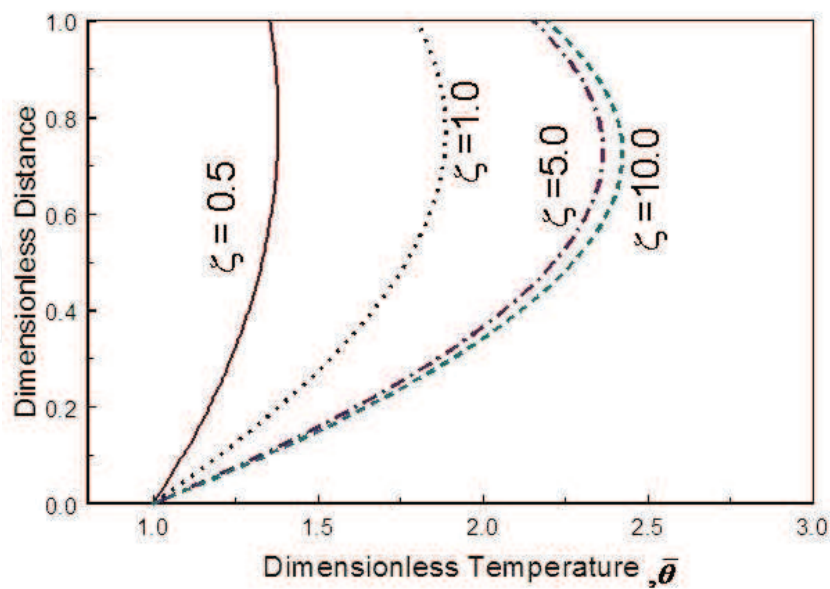
Fig. 3. Effect of ζ on the dimensionless temperature profiles (a = 0.75, b = 1.0, R_1 = -3.0; R_2 = 10.0; R_3 = 0.05, R_4 = 0.1, R_5 = 0.01, R_6 = 0.1, R_7 =5.0, v_w = 0.4; N = 0.3; α = 30°, m = 0.5, and $\overline{\theta}_\infty = 0.5$)

The effects of the reaction order m on the distribution of the dimensionless temperature and the heat transfer rate at the inclined and free surfaces are shown in Fig 4. For all values of m chosen here, the temperature is higher than the surface and the free stream temperature. The heat transfer at the inclined surface and at the free stream increase slowly for negative values of m to about m=0.5, but it begins to significantly increase for m greater than 0.5
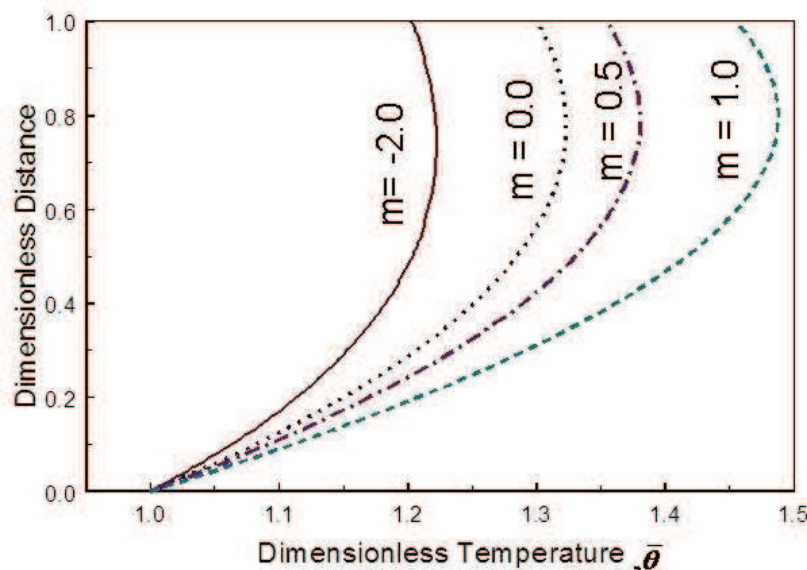

Fig. 4. Effect of the reaction order m on the dimensionless temperature profiles (a = 0.75, b = 1.0, R_1 = -3.0; R_2 = 10.0; R_3 = 0.05, R_4 = 0.1, R_5 = 0.01, R_6 = 0.1, R_7 =5.0, ζ = 0.5, v_w = 0.4; N = 0.3; α = 30º, and $\bar{\theta}_\infty$ = 0.5)

In Conclusion, we have studied the flow of a compressible (density gradient type) non-linear fluid down an inclined plane, subject to radiation boundary condition. The heat transfer is also considered where a source term, similar to the Arrhenius type reaction, is included. The non-dimensional forms of the equations are solved numerically and the competing effects of conduction, dissipation, heat generation and radiation are discussed. It is observed that the velocity increases rapidly in the region near the inclined surface and is slower in the region near the free surface. Since R_7 is a measure of the heat generation due to chemical reaction, when the reaction is frozen (R_7=0.0) the temperature distributions would depend only on R_1, and R_2, representing the effects of the pressure force developed in the material due to the distribution, R_3 and R_4 viscous dissipation, R_5 the normal stress coefficient, R_6 the measure of the emissivity of the particles to the thermal conductivity, etc. When the flow is not frozen ($R_7 > 0$) the temperature inside the flow domain is much higher than those at the inclined and free surfaces. As a result, heat is transferred away from the flow toward both the inclined surface and the free surface with a rate that increases as R_7 increases. For a given temperature, an increase in ζ implies that the activation energy is smaller and thus, the reaction rate is increased leading to an increase in the heat of the reaction. As a result the flow is chemically heated and its temperature increase. The results shown here indicate that for all values of ζ used the chemical effects are significant and the temperature is always higher than both the surface temperature and the free surface temperature. The heat transfer is always from the flow toward both the inclined surface and

the free stream. It is also noticed that for all values of m chosen in this study, the temperature is higher than the surface and the free stream temperature. The heat transfer at the inclined surface and at the free stream increase slowly for negative values of m to about m=0.5, but it begins to significantly increase for m greater than 0.5.

9. Appendix A

In order to show that the primary focus of the formulation for heat conduction in complex materials should be on the derivation of the heat flux vector and not on the thermal conductivity, we start with the general statement of the balance of energy, also known as the first law of thermodynamics. The second law of thermodynamics, also known as the entropy law, entropy inequality, or in its various forms as Clausius-Duhem inequality, the dissipation inequality, etc., is very much a subject matter of controversy[7] as there does not seem to be a consensus whether this law should be applied locally, i.e., to certain parts of the system, or to the whole system; there is, however, no disagreement on the generality, acceptance, and applicability of the first law of thermodynamics (Truesdell, 1984, p.67) which states:

$$\dot{E} = W + Q \qquad (A\text{-}1)$$

where E is the internal energy of a body, W is the net working on it, and Q is the heating associated with it. By definition the net working W at time t on the body is given by

$$W \equiv P - \dot{K} \qquad (A\text{-}2)$$

where P is the mechanical power and K is the kinetic energy. Furthermore, according to a theorem proved by Stokes (Truesdell, 1984, p.109)

$$W = \int_B w\,dV \qquad (A\text{-}3)$$

where w is called the stress power and is given by

$$w = \mathbf{T} \cdot \mathrm{grad}\,\mathbf{u} = \mathbf{T.L} \qquad (A\text{-}4)$$

where $\mathbf{T}$ is the Cauchy stress tensor and $\mathbf{u}$ is the velocity vector. It is also assumed that Q is composed of two types of heating: one associated with the body and its interactions with the outside environment, Q_b , and the other , Q_c , is concerned with the contacts between different parts of the body:

$$\begin{aligned} Q &= Q_b + Q_c \\ &= \int_{\partial B} h\,dA + \int_B r\,dM \end{aligned} \qquad (A\text{-}5)$$

where Q_b and Q_c are often described as radiation and conduction, h is called the influx of heating (related to the heat flux vector $\mathbf{q}$), and r is the heating supply (related to the

---

[7] This is in fact exacerbated further for the granular materials as the concept of entropy has been given different meanings by different people (Mehta, (1994).

radiation effect). If one assumes that E is given by a continuous additive set function ε (called the specific internal energy):

$$E = \int_B \varepsilon dM \tag{A-6}$$

then it can be shown, by using the divergence theorem, that for sufficiently smooth fields, using Equations (A-4) –(A-6), Equation (A-1) can be written as:

$$\rho\dot{\varepsilon} = w\text{-div}\mathbf{q} + \rho r \tag{A-7}$$

where

$$h = -\mathbf{q.n} \tag{A-8}$$

where $\mathbf{q}$ is the heat flux vector and $\mathbf{n}$ is the outward unit normal vector to the surface. Note that in the notation used in this Chapter $w = \xi$. Equation (A-7) is often regarded as the local or the differential form of the first law of thermodynamics, whereas, we can see that it is derived based on a series of definitions along with using Cauchy's laws of motion: $\rho\dfrac{d\mathbf{u}}{dt} = \text{div}\mathbf{T} + \rho\mathbf{b}$ and $\mathbf{T} = \mathbf{T}^T$, where $\mathbf{b}$ is the body force vector.

10. References

Agudelo, A., & C. Cortes. (2010). Thermal radiation and the second law. *Energy*, Vol. 35, pp. 670-691.

Akgun, F.& R. H. Essenhigh (2001). Self-ignition characteristics of coal stockpiles: theoretical prediction from   a two-dimensional unsteady-state model. *Fuel*, Vol. 80, pp. 409-415.

Baek, S.; K. R. Rajagopal, & A. R. Srinivasa (2001). Measurements related to the flow of granular materials in a torsional rheometer. *Particulate Sci. Tech.*, Vol. 19, pp. 175-186.

Bashir, Y. H., & J. D. Goddard (1990). Experiments on the conductivity of suspensions of ionically-conductive spheres. *AIChE J.*, Vol. 36, pp. 387-396.

Batra, Romesh C. (2006) *Elements of Continuum Mechanics*. American Institute of Aeronautics and Astronautics (AIAA) Inc., Reston, VA.

Batchelor, G. K., & R. W. O'Brien (1977). Thermal or electrical conduction through a granular material. *Proc. R. Soc. Lond.* A. Vol. 355, pp. 313-333.

Bingham, E. C. (1922). *Fluidity and Plasticity.* McGraw Hill. New York.

Boddington, T.; Gray, P. & Wake, G. C. (1977) Criteria for thermal explosions with and without reactant consumption, *Proc. Royal Society London* A, Vol. 357, pp. 403-422.

Boddington, T.; Feng, C. G., & Gray, P. (1983). Thermal explosions, criticality and the disappearance of criticality in systems with distributed temperatures .1. Arbitrary Biot-number and general reaction-rate laws, *Proc. Royal Society London* A, Vol. 390, pp. 247-264.

Bridges, C., & Rajagopal, K. R. (2006). Pulsatile flow of a chemically-reacting nonlinear fluid. *Computers and Mathematics with Applications*, Vol. 52, pp. 1131-1144.

Carreau, P. J.; D. De Kee, & R. J. Chhabra. (1997). *Rheology of Polymeric Systems.* Hanser/Gardner Publications. Cincinnati, OH.

Clouet, J. -F. (1997). The Rosseland approximation for radiative transfer problems in heterogeneous media. *Journal of Quantitative Spectroscopy and Radiative Transfer,* Vol. 58, pp. 33-43.

Cowin, S.C. (1974). A theory for the flow of granular material. *Powder Tech.,* Vol. 9, pp. 61-69.

Deshpande, A. P.; Krishnan, J. M., & P. B. Sunil (Eds.). (2010) *Rheology of Complex Fluids.* 1st Edition. Springer, ISBN: 978-1-4419-6493-9.

Dunn, J. E., & R. L. Fosdick. (1974). Thermodynamics, stability, and boundedness of fluids of complexity 2 and fluids of second grade, *Arch. Rational Mech. Anal.,* Vol. 56 , pp. 191-252.

Fosdick, R. L., & K. R. Rajagopal. (1979). Anomalous features in the model of "second order fluids". *Arch. Rational Mech. Anal.,* Vol. 70, pp.145-152.

Frank-Kamenetskii, D. (1969). A. *Diffusion and Heat Transfer in Chemical Kinetics.* 2nd Edition. New York: Plenum Press.

Fuchs, H. U. (1996). *The Dynamics of Heat.* Springer-Verlag, Inc., New York.

Gupta, G. & M. Massoudi. (1993). Flow of a generalized second grade fluid between heated plates. *Acta Mechanica,* Vol. 99, pp. 21-33.

Haupt, P. (2002). *Continuum Mechanics and Theory of Materials.* 2nd edition. Springer. Berlin.

Jeffrey, D. J. (1973). Conduction through a random suspension of spheres. *Proc. R. Soc. Lond.* A. Vol. 335, pp. 355-367.

Kaviany, M. (1995). *Principles of Heat Transfer in Porous Media.* Second edition. Springer-Verlag, New York.

Larson, R. G. (1999). *The Structure and Rheology of Complex Fluids.* Oxford University Press. New York.

Liu, I. S. (2002). *Continuum Mechanics.* Springer-Verlag. Berlin.

Massoudi, M. (2001). On the flow of granular materials with variable material properties. *Int. J. Non-Linear Mech.,* Vol. 36, pp. 25-37.

Massoudi, M. (2004). Constitutive Modelling of Flowing Granular Materials: A Continuum Approach. In *Granular Materials: Fundamentals and Applications.* Edited by S. J. Antony, W. Hoyle, and Y. Ding. The Royal Society of Chemistry. Cambridge, UK, 2004, pp.63-107.

Massoudi, M. (2006a). On the heat flux vector for flowing granular materials, Part I: Effective thermal conductivity and background. *Math. Methods Applied Sci.,* Vol. 29, pp. 1585-1598.

Massoudi, M. (2006b) On the heat flux vector for flowing granular materials, Part II: Derivation and special cases. *Math. Methods Applied Sci.,* Vol. 29, pp. 1599-1613.

Massoudi, M. (2008). A note on the meaning of mixture viscosity using the classical continuum theories of mixtures. *International Journal of Engineering Science,* Vol. 46, pp.677-689

Massoudi, M. (2010). A Mixture Theory formulation for hydraulic or pneumatic transport of solid particles. *International Journal of Engineering Science.* Vol. 48, pp. 1440-1461.

Massoudi, M. & M. M. Mehrabadi. (2001). A continuum model for granular materials: Considering dilatancy, and the Mohr-Coulomb criterion. *Acta Mech.,* Vol. 152, pp. 121-138.

Massoudi, M. & N. K. Anand. (2004). A theoretical study of heat transfer to flowing granular materials. *International Journal of Applied Mechanics and Engineering*, Vol. 9, pp. 383-398.

Massoudi, M. & Phuoc, T. X. (2005). Heat transfer in flowing granular materials: The effects of radiation boundary condition. *Int. J. of Applied Mechanics and Engineering*, Vol. 10, pp. 489-503.

Massoudi, M. & T. X. Phuoc. (2007). Conduction and dissipation in the shearing flow of granular materials modeled as non-Newtonian fluids. *Powder Technology*, Vol. 175, pp.146-162.

Massoudi, M. & A. Vaidya. (2008). On some generalizations of the second grade fluid model, *Nonlinear Analysis, Part II, Real World Applications*, Vol. 9, pp.1169-1183.

Massoudi M. & T. X. Phuoc. (2008). Flow of a non-linear (density-gradient-dependent) viscous fluid with heat generation, viscous dissipation and radiation. *Mathematical Methods in the Applied Sciences*, Vol. 31, pp.1685-1703.

Maxwell, J. C. (1876). On stresses in rarified gases arising from inequalities of temperature. *Phil. Trans. Roy. Soc. Lond.*, Vol. 170, pp. 231-256.

Mehrabadi, M. M.; S. C. Cowin, & M. Massoudi. (2005). Conservation laws and constitutive relations for density-gradient-dependent viscous fluids. *Continuum Mechanics and Thermodynamics*, Vol. 17, pp. 183-200.

Mehta, A. (Editor). (1994). *Granular matter*. Springer-Verlag. New York.

Miller, J. R., & P. M. Weaver. (2003). Temperature profiles in composites plates subject to time-dependent complex boundary conditions. Vol. 59, pp. 267-278.

Müller, I. (1967). On the entropy inequality. *Arch. Rat. Mech. and Anal.*, Vol. 26, pp. 118-14.

Na, T. Y. (1979). *Computational Methods in Engineering Boundary Value Problems.* Academic Press, New York.

Narasimhan, T. N. (1999). Fourier's heat conduction equation: History, influence, and connections. *Rev. Geophys.*, Vol. 37, pp.151-172.

Oldroyd, J. G. (1984). An approach to non-Newtonian fluid mechanics. *J. Non-Newtonian Fluid Mech.*, 14, pp. 9-46.

Papachristodoulou, G. & Trass, O. Coal slurry fuel technology. (1987). *Canadian J. Chem. Engng*, Vol. 65, pp. 177-201.

Pomraning, G. C. (1973). *The Equations of Radiative Hydrodynamics*. Dover Publications, Inc., Mineola, NY.

Prager, W. (2004). *Introduction to Mechanics of Continua.* Dover Publications, Inc., Mineola, NY.

Rajagopal, K. R. (1995). On boundary conditions for fluids of differential type, in: A. Sequeira (Ed.) *Navier-Stokes Equations and Related Nonlinear Problems*. Plenum Press, New York, 1995, pp. 273-278.

Rajagopal, K. R. (2006). On implicit constitutive theories for fluids. *J. Fluid Mech.*, Vol. 550, pp. 243-249.

Rajagopal, K. R. & P. N. Kaloni. (1989). Some remarks on boundary conditions for flows of fluids of the differential type. In: *Continuum Mechanics and its Applications*, Hemisphere Press, 1989, pp. 935-942.

Rajagopal, K. R. & M. Massoudi. (1990). *A Method for Measuring Material Moduli of Granular Materials: Flow in an Orthogonal Rheometer.* Topical Report, DOE/PETC/TR-90/3, 1990.

Rajagopal, K. R., W.C. Troy & M. Massoudi. (1992). Existence of solutions to the equations governing the flow of granular materials. *European J. Mech., B/Fluids*, Vol. 11, pp. 265-276.

Rajagopal, K. R., M. Massoudi, & A. S. Wineman. (1994). Flow of granular materials between rotating disks. *Mech. Research Comm.*, Vol. 21, pp. 629-634.

Rajagopal, K. R., & L. Tao. (1995). *Mechanics of Mixtures.* New Jersey: World Scientific Publishing.

Rajagopal, K. R., G. Gupta, & R. C. Yalamanchili. (2000). A rheometer for measuring the properties of granular materials. *Particulate Sci. Tech.*, Vol. 18, pp.39-55.

Rajagopal, K. R., G. Saccomandi, & L. Vergori. (2009). On the Oberbeck-Boussinesq approximation for fluids with pressure dependent viscosities. *Nonlinear Anal. Real World Applcs.*, Vol. 10, pp. 1139-1150.

Reiner, M. (1945). A mathematical theory of dilatancy. *American J. Math.*, Vol. 67, pp. 350-362.

Reiner, M. (1948). Elasticity beyond the elastic limit. *Amer. J. Math.*, Vol. 70, pp. 433-466.

Reynolds, O. (1885). On the dilatancy of media composed of rigid particles in contact with experimental illustrations. *Phil. Mag., Series 5*, Vol. 20, pp. 469-481.

Reynolds, O. (1886). Experiments showing dilatancy, a property of granular material, possibly connected with gravitation. *Proc. Roy. Inst. of Gr. Britain*, Vol. 11, pp.354-363.

Rivlin, R. S. (1948). The hydrodynamics of non-Newtonian fluids, I. *Proc. Royal Soc. Lond.*, Vol. 193, pp. 260-281.

Rivlin, R. S. & Ericksen, J. L. (1955). Stress deformation relations for isotropic materials, *J. Rat. Mech. Anal.* Vol. 4, pp.323-425.

Roh, N.S., Shin, D. Y., Kim, D. C., & Kim, J. D. (1995). Rheological behavior of coal-water mixtures: 1. Effects of coal type, loading and particle size. *Fuel*, Vol. 74, pp. 1220-1225.

Saeki, T, & Usui, H. (1995). Heat transfer characteristics of coal-water mixtures. *Canadian J. Chem. Engng.*, Vol. 73, pp. 400-404.

Saldanha da Gama, R. M. (2004). On the conduction/radiation heat transfer problem in a body with wavelength-dependent properties. *Appl. Math. Modelling*, Vol. 28, pp. 795-816.

Salinger, A. G., Aris, R., & Derby, J. J. (1994). Modeling the spontaneous ignition of coal stockpiles , *AICHE Journal*, Vol. 40, pp. 991-1004.

Savage, S.B. (1979). Gravity Flow of Cohesionless Granular Materials in Chutes and Channels. *J. Fluid Mech.*, Vol. 92, pp. 53-96.

Schowalter, E. R. (1978). *Mechanics of Non-Newtonian Fluids*. Pergamon Press, New York.

Siegel, R., & J. R. Howell. (1981). *Thermal Radiation Heat Transfer*. 2nd edition. Hemisphere Publishing Company. New York.

Soto, R., M. Mareschal, & D. Risso. (1999). Departures from Fourier's law for fluidized granular media. *Phys. Rev. Lett.*, Vol. 83, pp. 5003-5006.

Straughan, B. (1998). *Explosive Instabilities in Mechanics*. Springer Verlag. New York.

Straughan, B. (2002). Global stability for convection induced absorption of radiation. *Dyn. Atmos. Oceans*, Vol. 35, pp.351-361.

Straughan, B. (2007). A note on convection with nonlinear heat flux. *Ricerche mat.*, Vol. 56, pp. 229-239.

Straughan. B. (2008). *Stability and wave motion in porous media.* Springer-Verlag, New York.

Straughan, B., & D. W. Walker. (1997). Multi-component diffusion and penetrative convection. *Fluid Dyn. Research*, Vol. 19, pp. 77-89.

Straughan, B., & J. Tracey. (1999). Multi-component convection-diffusion with internal heating or cooling. *Acta Mech.*, Vol. 111, pp.219-238.

Truesdell, C. (1976). The meaning of viscometry in fluid mechanics. *Annual Rev. Fluid Mech.*, Vol. 6, pp. 111-146.

Truesdell, C. & W. Noll. (1992). *The Non-Linear Field Theories of Mechanics*. Springer-Verlag, New York.

Tsai, C. Y., Navack, M., & Roffe, G. (1988). *Rheological and heat transfer characteristics of flowing coal-water mixtures.* DOE Report, DOE/MC/23255-2763, December 1988.

Tsai, S. C., & Knell, E. W. (1986). Viscosity and rheology of coal water slurry. *Fuel*, Vol. 65, pp. 566-571.

Uguz, A. K., & M. Massoudi. (2010). Heat transfer and Couette flow of a chemically-reacting non-linear fluid. *Mathematical Methods in the Applied Sciences*, Vol. 33, pp.1331-1341.

Walton, O.R. & R.L. Braun. (1986a). Stress calculations for assemblies of inelastic spheres in uniform shear. *Acta Mechanica*, Vol. 63, pp. 73-86.

Walton, O.R. & R.L. Braun. (1986b). Viscosity, granular-temperature, and stress calculations for shearing assemblies of inelastic, frictional disks. *J. Rheology*, Vol. 30, pp.949-980.

Wang, L. (2001). Vector-field theory of heat flux in convective heat transfer. *Nonlinear Anal.*, Vol. 47, pp. 5009-5020.

Wright, S. E. (2007). The Clausius inequality corrected for heat transfer involving radiation. *Int. J. Eng. Sci.*, Vol. 45, pp. 1007-1016.

Wright, S. E., M. A. Rosen, D. S. Scott, & J. B. Haddow. (2002). The exergy flux of radiative heat transfer with an arbitrary spectrum. *Exergy*, Vol. 2, pp. 69-77.

Wu, J. W., & H. S. Chu. (1999). Combined conduction and radiation heat transfer in plane-parallel packed beds with variable porosity. *J. Quant. Spectrosc. Radiat. Transfer*, Vol. 61, pp. 443-452.

Zhu, H., Y. D. Kim, & D. De Kee. (2005). Non-Newtonian fluids with a yield stress. *J. Non-Newtonian Fluid Mech.*, Vol. 129, pp.177-181.

Zel'dovich, Ya. B., & Yu. P. Raizer. (1967). *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena*. Dover Publications, Inc., Mineola, NY.

Ziegler, H. (1983). *An Introduction to Thermomechanics*. Second Revised Edition. North-Holland Publishing Company. Amsterdam.
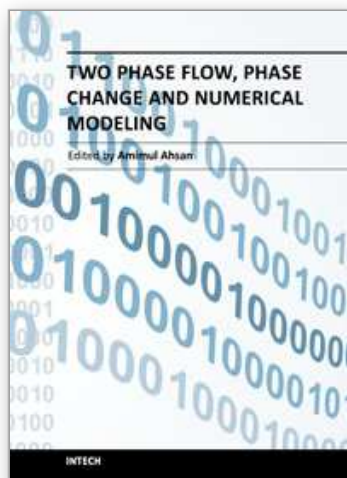
Two Phase Flow, Phase Change and Numerical Modeling

Edited by Dr. Amimul Ahsan

The heat transfer and analysis on laser beam, evaporator coils, shell-and-tube condenser, two phase flow, nanofluids, complex fluids, and on phase change are significant issues in a design of wide range of industrial processes and devices. This book includes 25 advanced and revised contributions, and it covers mainly (1) numerical modeling of heat transfer, (2) two phase flow, (3) nanofluids, and (4) phase change. The first section introduces numerical modeling of heat transfer on particles in binary gas-solid fluidization bed, solidification phenomena, thermal approaches to laser damage, and temperature and velocity distribution. The second section covers density wave instability phenomena, gas and spray-water quenching, spray cooling, wettability effect, liquid film thickness, and thermosyphon loop. The third section includes nanofluids for heat transfer, nanofluids in minichannels, potential and engineering strategies on nanofluids, and heat transfer at nanoscale. The forth section presents time-dependent melting and deformation processes of phase change material (PCM), thermal energy storage tanks using PCM, phase change in deep CO_2 injector, and thermal storage device of solar hot water system. The advanced idea and information described here will be fruitful for the readers to find a sustainable solution in an industrialized society.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Mehrdad Massoudi (2011). Heat Transfer in Complex Fluids, Two Phase Flow, Phase Change and Numerical Modeling, Dr. Amimul Ahsan (Ed.), ISBN: 978-953-307-584-6, InTech, Available from: http://www.intechopen.com/books/two-phase-flow-phase-change-and-numerical-modeling/heat-transfer-in-complex-fluids

INTECH
open science | open minds