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Integral Transform Method Versus Green Function Method in Electron, Hadron or Laser Beam - Water Phantom Interaction

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1. Introduction

Many practical applications require the detailed study of the thermal behavior of different systems. The difficulties arise when these systems are inhomogeneous with respect to the parameters involved in the heat diffusion process. Currently, the heat diffusion equation has no analytical solution in this case. There exists however a wide range of methods to approximate the solution of the heat diffusion equation in inhomogeneous systems, starting from the numerical methods and ending with the exact analytical solution for a few particular cases, each of them presenting specific advantages and disadvantages [1-6].

We assume that the source term in the heat equation has the form: $f(x,y,z,t) = f(x,y,z) \cdot [h(t) - h(t - t_0)]$ (where h is the step function) and the thermal conductivity can be expressed as: $k(x,y,z) = k(x)$. We developed under these assumptions a computing method for solving the diffusion equation describing the heat propagation in inhomogeneous materials.

The procedure for solving the heat diffusion equation in inhomogeneous systems, with a prescribed accuracy in respect to the thermal conductivity, is outlined. To this aim, the thermal conductivity was considered a discontinuous function having a linear value in each layer. This approach is nonetheless valid in many cases like thin films or welding technologies. In these cases there exists nevertheless an interface that ensures the continuity of the thermal conductivity function.

We believe that the programs of simulation which are used in the present paper can be help - full for medical staff.

2. One-dimensional mathematical model

In this section we introduce the procedure for approximating the exact solution of the heat diffusion equation (1) with respect to the thermal conductivity in inhomogeneous media. Our approach is one dimensional, but it can be rather easily extended to multi-dimensional equations.

We consider a parallelepiped volume of dimensions a, b and c. The equation describing the heat diffusion is :

$$\frac{\partial}{\partial x}\left(k(x)\frac{\partial T(x,t)}{\partial x}\right) - \rho c \frac{\partial T(x,t)}{\partial t} = -f(x,t), a \le x \le b \tag{1}$$

Here: ρ is the mass density; c is the heat capacity; k is the thermal conductivity; T is the local temperature and t is current time. In each sub-domain $[x_i, x_{i+1}]$, the thermal conductivity is considered to be a linear function of the coordinate x, $k_i(x) = k(x_i) + m_i(x - x_i)$, where m_i is a real constant number.

When: $x \in [x_i, x_{i+1}]$, we have:

$$\frac{\partial}{\partial x}\left(k_i(x)\frac{\partial T_i}{\partial x}\right) - \rho_i c_i \frac{\partial T_i}{\partial t} = -f_i(x,t), \tag{2}$$

with the boundary conditions :

$$T_{i-1,j,k,l}(x,y,z,t)\big|_{x=x_i} = T_{i,j,k,l}(x,y,z,t)\big|_{x=x_i} , \tag{3}$$

$$T_{i+1,j,k,l}(x,y,z,t)\big|_{x=x_{i+1}} = T_{i,j,k,l}(x,y,z,t)\big|_{x=x_{i+1}} ,$$

$$k_{i-1} \cdot T_{i-1,j,k,l}'(x,y,z,t)\big|_{x=x_i} = k_i \cdot T_{i,j,k,l}'(x,y,z,t)\big|_{x=x_i} ,$$

$$k_{i+1} \cdot T_{i+1,j,k,l}'(x,y,z,t)\big|_{x=x_{i+1}} = k_i \cdot T_{i,j,k,l}'(x,y,z,t)\big|_{x=x_{i+1}} .$$

At the margins of the sample, we assume:

$$k_0 \cdot T_{0,j,k,l}'(x,y,z,t)\big|_{x=x_0} = h_0 \cdot T_{0,j,k,l}(x,y,z,t)\big|_{x=x_0} ,$$

$$k_i \cdot T_{i,j,k,l}'(x,y,z,t)\big|_{y=y_2/2} = -h_i \cdot T_{i,j,k,l}(x,y,z,t)\big|_{y=y_2/2} ,$$

$$k_i \cdot T_{i,j,k,l}'(x,y,z,t)\big|_{y=-y_2/2} = h_i \cdot T_{i,j,k,l}(x,y,z,t)\big|_{y=-y_2/2} ,$$

$$k_i \cdot T_{i,j,k,l}'(x,y,z,t)\big|_{z=z_3/2} = -h_i \cdot T_{i,j,k,l}(x,y,z,t)\big|_{z=z_3/2} ,$$

$$k_i \cdot T_{i,j,k,l}'(x,y,z,t)\big|_{z=-z_3/2} = h_i \cdot T_{i,j,k,l}(x,y,z,t)\big|_{z=-z_3/2} ,$$

$$k_n \cdot T_{n,j,k,l}'(x,y,z,t)\big|_{x=x_n} = -h_n \cdot T_{n,j,k,l}(x,y,z,t)\big|_{x=x_n} ,$$

and we have: $x_0 = 0$, $x_n = a$, $y_2 = b$, $z_3 = c$.

We next applied the integral operator $K_{ij}(x,\lambda_j) = \dfrac{1}{C(\lambda_j)} \displaystyle\int_{x_i}^{x_{i+1}} \overline{K}_{ij}(x,\lambda_i)dx$ to equation (2) with

the kernel $\overline{K}_{ij}(x,\lambda_i)$. The kernel of this operator was derived by solving the partial differential equation :

$$\frac{\partial}{\partial x}\left(k_i(x)\frac{\partial \overline{K}_{ij}(x)}{\partial x}\right)+\lambda_j^2 \overline{K}_{ij}(x)=0 \qquad (4)$$

Equation (4) provides a series of positive eigenvalues λ_j, $j \in N$ and eigenfunctions $\overline{K}_{ij}(x,\lambda_i)$ of the differential operator: $\frac{\partial}{\partial x}\left(k(x)\frac{\partial}{\partial x}\right)$, [7]. The eigenfunctions were necessary for solving equation (2).

The solution of equation (4) has the form:

$$\overline{K}_{ij}(x,\lambda_i)=A_i J_0\left(\frac{2\lambda_j \sqrt{\mu_i(k(x_i)+m_i x)}}{|m_i|}\right)+B_i Y_0\left(\frac{2\lambda_j \sqrt{\mu_i(k(x_i)+m_i x)}}{|m_i|}\right),\, j \in N \qquad (5)$$

Where: $\mu_i = \rho_i c_i$ and J_0 and Y_0 are the Bessel and Weber functions respectively.

After the application of the integral operator $K_i(x)$ equation (4) becomes:

$$-\lambda_j^2 \overline{u}_i(\lambda_j,t)-\rho_i c_i \frac{\partial \overline{T}_i(\lambda_j,t)}{\partial t}=-\overline{f}_i(\lambda_j,t) \qquad (6)$$

where:

$$\overline{u}_i(\lambda_j,t)=\frac{1}{C(\lambda_j)}\int_{x_i}^{x_{i+1}} T(x,t)\overline{K}_i(x,\lambda_j)dx$$

$$\overline{f}(\lambda_j,t)=\frac{1}{C(\lambda_j)}\int_{x_i}^{x_{i+1}} f(x,t)\overline{K}_j(x,\lambda_j)dx$$

and $C(\lambda_j)$ is a normalization factor.

Here we have [7]:

$$C(\lambda_j)=\sum_{i=0}^{n-1}\int_{x_i}^{x_{i+1}} \overline{K}_{ij}^2(x_i,\lambda_j)dx \,. \qquad (7)$$

In the same manner, one can apply the functions: $\overline{K}_k(\mu_k,y)$ and $\overline{K}_l(\varepsilon_l,z)$, which satisfy the equations:

$$\frac{\partial^2 \overline{K}_k(\mu_k,y)}{\partial y^2}+\mu_k^2 \overline{K}_k(\mu_k,y)=0$$
$$\frac{\partial^2 \overline{K}_l(\varepsilon_l,z)}{\partial z^2}+\varepsilon_l^2 \overline{K}_l(\varepsilon_l,z)=0 \qquad (8)$$

This next gives: $\overline{K}_l(\varepsilon_l,z)=\cos(\varepsilon_l \cdot z)+(h/k\varepsilon_l)\cdot \sin(\varepsilon_l z)$, with, k being the thermal conductivity and h the heat transfer coefficient .

Then, the following equation is inferred:

$$\lambda_j^2 \hat{T}_i(\lambda_j,\mu_k,\varepsilon_l,t) + \mu_k^2 \hat{T}_i(\lambda_j,\mu_k,\varepsilon_l,t) + \varepsilon_l^2 \hat{T}_i(\lambda_j,\mu_k,\varepsilon_l,t) +$$
$$+ \frac{\partial \hat{T}_i(\lambda_j,\mu_k,\varepsilon_l,t)}{\partial t} = \frac{\hat{f}_i(x,y,z,t)}{\rho_i c_i} \tag{9}$$

Where from it follows:

$$\hat{T}_i(\lambda_j,\mu_k,\varepsilon_l,t) = \frac{1}{C(\lambda_j)C(\mu_k)C(\varepsilon_l)} \times$$
$$\times \int_{x_i}^{x_{i+1}} \int_{-y_2/2}^{y_2/2} \int_{-z_3/2}^{z_3/2} T(x,y,z,t) \cdot \overline{K}_{ij}(\lambda_j,x) \cdot \overline{K}_k(\mu_k,y) \cdot \overline{K}_l(\varepsilon_l,z) dx dy dz \tag{10}$$

In order to eliminate the time parameter t, we apply the direct and inverse Laplace transform to equation (9).

If we have, like in most cases: $f(x,y,z,t) = f(x,y,z) \cdot [h(t) - h(t-t_0)]$, one can get the solution:

$$T_i(x,y,z,t) = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \frac{1}{(\lambda_j^2 + \varepsilon_l^2 + \mu_k^2)} [1 - e^{-\gamma \cdot (\lambda_j^2 + \varepsilon_l^2 + \mu_k^2)t} - (1 - e^{-\gamma \cdot (\lambda_j^2 + \varepsilon_l^2 + \mu_k^2)(t-t_0)})h(t-t_0)] \times$$
$$g(\lambda_j,\mu_k,\varepsilon_l) \times \overline{K}_{ij}(\lambda_j,x) \times \overline{K}_k(\mu_k,y) \times \overline{K}_l(\varepsilon_l,z) \tag{11}$$

where:

$$g(\lambda_j,\mu_k,\varepsilon_l) = \frac{1}{C(\lambda_j)C(\mu_k)C(\varepsilon_l)} \times$$
$$\sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} \int_{-y_2/2}^{y_2/2} \int_{-z_3/2}^{z_3/2} f_i(x,y,z,t) \cdot \overline{K}_{ij}(\lambda_j,x) \cdot \overline{K}_k(\mu_k,y) \cdot \overline{K}_l(\varepsilon_l,z) dx dy dz \tag{12}$$

γ stands here for the thermal diffusivity. We point out that our semi-analytical solution becomes analytical, if we observe the, after 10 iterations, the solution becomes convergent (we have values of temperature less than 10^{-2}K for: i>10;j>10 and k>10). Under these conditions equation (10) becomes:

$$T_i(x,y,z,t) = \sum_{j=1}^{10} \sum_{k=1}^{10} \sum_{l=1}^{10} \frac{1}{(\lambda_j^2 + \varepsilon_l^2 + \mu_k^2)} [1 - e^{-\gamma \cdot (\lambda_j^2 + \varepsilon_l^2 + \mu_k^2)t} - (1 - e^{-\gamma \cdot (\lambda_j^2 + \varepsilon_l^2 + \mu_k^2)(t-t_0)})h(t-t_0)] \times$$
$$g(\lambda_j,\mu_k,\varepsilon_l) \times \overline{K}_{ij}(\lambda_j,x) \times \overline{K}_k(\mu_k,y) \times \overline{K}_l(\varepsilon_l,z) \tag{13}$$

3. Application of the theory: Laser-assisted hadron and electron beams therapy

It is well known [8] that the hadrons therapy (e.g. with protons) is much more suitable and efficient compared to electrons therapy, because the absorption curve is (in this case) a Dirac function. We can solve easily the heat equation for this case, and we obtain the temperature field in Fig.1. - (The Dirac absorption function is at 4 cm from the surface). Here dT is the temperature variation (dT=T_f -T_i), rather than the absolute temperature.

T_f and T_i are the final and initial temperatures respectively.
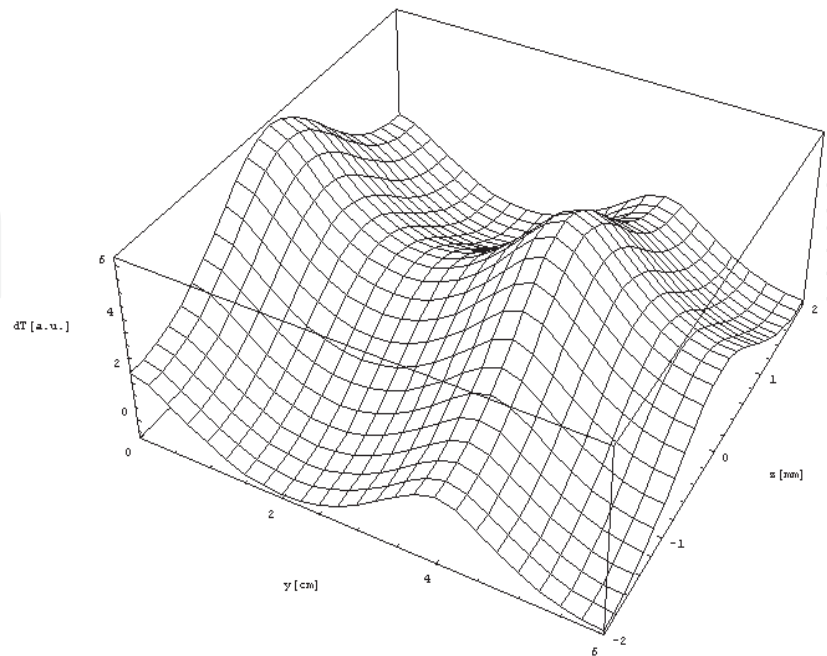

Fig. 1. Thermal field distribution in case of 1 MeV proton beam irradiation of a water phantom, for 120 sec.
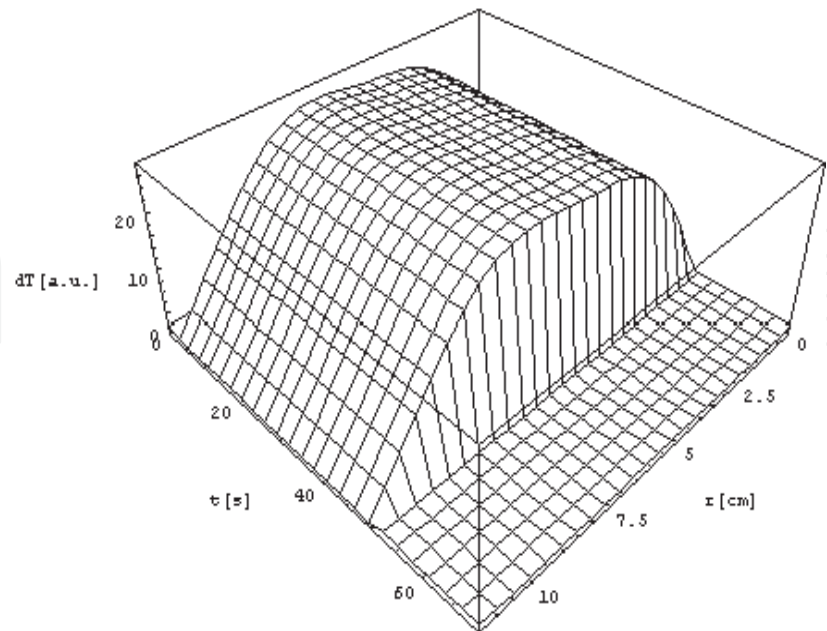

Fig. 2. Thermal field in water submitted to cw CO_2 laser irradiation for 50 sec.

The power of the cw CO_2 laser beam was P= 1W.

It is known from experience [8] that proton therapy is more efficient in the "presence" of a laser beam. We plotted in figure 2 the thermal gradient in water produced by cw CO_2 laser irradiation for 50 sec. (P = 1W). In Fig. 3 we presented the temperature field in water produced by an electron beam, when the "steady - state" is achieved. The white color corresponds to an increase of temperature, and the black color represents a decrease of temperature. We have use sub-domains of 0.25 cm. The thickness of the water phantom was 0.25 cm, and was contained in a   plastic cube with a mass density close to 1 g/cm³. Figure 3 was obtained using eq. (13).
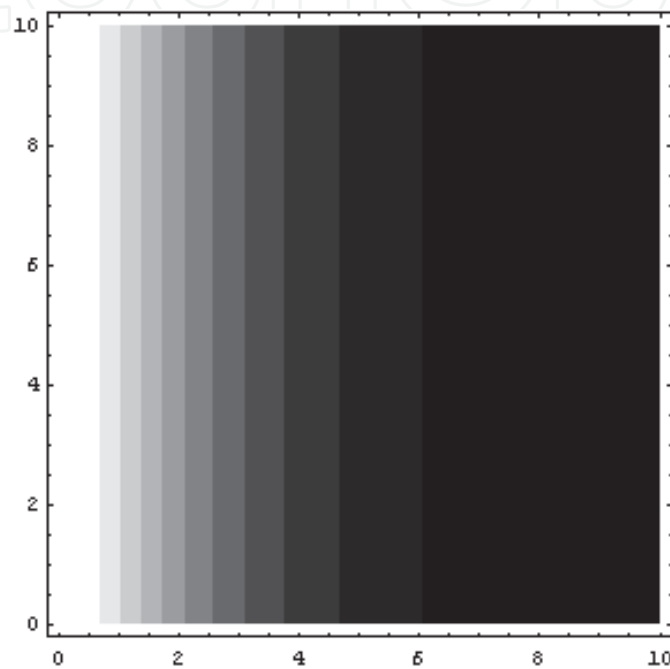

Fig. 3. Temperature field in water produce by an electron beam, when the "steady- state" is achieved.

The white color corresponds to the temperature increase while, the black color represents the temperature decrease. We have used sub-domains of 0.25 cm length.

4. The green function method

We start from the heat equation:

$$\left\{\frac{\partial}{\partial x}[K_x(T)\frac{\partial T}{\partial X}]+\frac{\partial}{\partial y}[K_y(T)\frac{\partial T}{\partial y}]+\frac{\partial}{\partial z}[K_z(T)\frac{\partial T}{\partial z}]\right\}=-S(x,y,z) \tag{14}$$

where S(x, y, z) is proportional with the absorbed dose. We consider [9], the case of a 10 MeV electron beam interactions with water. We have:

$$S(x,y,z)=K(y,z).D_{10}(x) \tag{15}$$

where according to experimental data from our laboratory:

$$D_{10}(x)=83.2337+18.6522\cdot x^2+15.1080\cdot x^3-4.1417\cdot x^4+0.3506\cdot x^5 \tag{16}$$

Here x stands for the direction of electron propagation. We will consider the radiation (electron beam) normal to water surface.

From the standard theory of Green function applied to multi-layer structures, we have:

$$K_\perp = \frac{l_1 + l_2 + \cdots + l_n}{l_1/k_1 + l_2/k_2 + \cdots + l_n/k_n} = \frac{l}{l_1/k_1 + l_2/k_2 + \cdots + l_n/k_n} \tag{17}$$

where l_i is the length and k_i is the thermal conductivity of the i-th layer.

We introduce the area of the layer A_i:

$$K_{||} = \frac{K_1 A_1 + K_2 A_2 + \cdots + K_n A_n}{A_1 + A_2 + \cdots + A_n} = \frac{K_1 A_1 + K_2 A_2 + \cdots + K_n A_n}{A} \tag{18}$$

We define the "linear" temperature

$$\theta(T) = \theta(T_0) + (1/K(T_0)) \int_{T_0}^{T} K(T') dT' \tag{19}$$

and we can write:

$$\theta_\perp = \frac{P[1 - R_\perp(T)]}{(\alpha)^{1/2} (\pi)^{3/2} K(T_0)} \int_0^\infty f_\perp(\xi) d\xi \tag{20}$$

where:

$$\alpha = \frac{K_\perp}{K_{||}}$$

and:

$$K_{||} = K(T) \tag{21}$$

The function $f_\perp$ is given by:

$$f_\perp(\xi) = \frac{\exp -[[X^2(\xi^2 + 1)] + [Y^2/(\xi^2 + 1)] + (Z^2/\alpha\xi^2)]}{\{(\xi^2 + 1)\}} \tag{22}$$

We plotted in Fig.4 the analytical results obtained with the Green function method.

The white color corresponds to temperature increase, and the black color represents a decrease of temperature. We have used sub-domains of 0.25 cm length. Figs. 3 and 4 allow for a direct comparison between the temperature fields in water computed with the integral transform technique and Green function method under identical conditions.

5. The thermal fields when we have multiple sources irradiations

We consider a parallelepiped sample with dimensions a, b, and c. The sample is irradiated by three laser beams which propagate along the Cartesian coordinate axes. The model is also valid for electron or hadrons beam irradiations.

Let us considering the following relations:

$$A(x,y,z,t) = A_1(x,y,z,t) + A_2(x,y,z,t) + A_3(x,y,z,t) \tag{23}$$

Therefore:

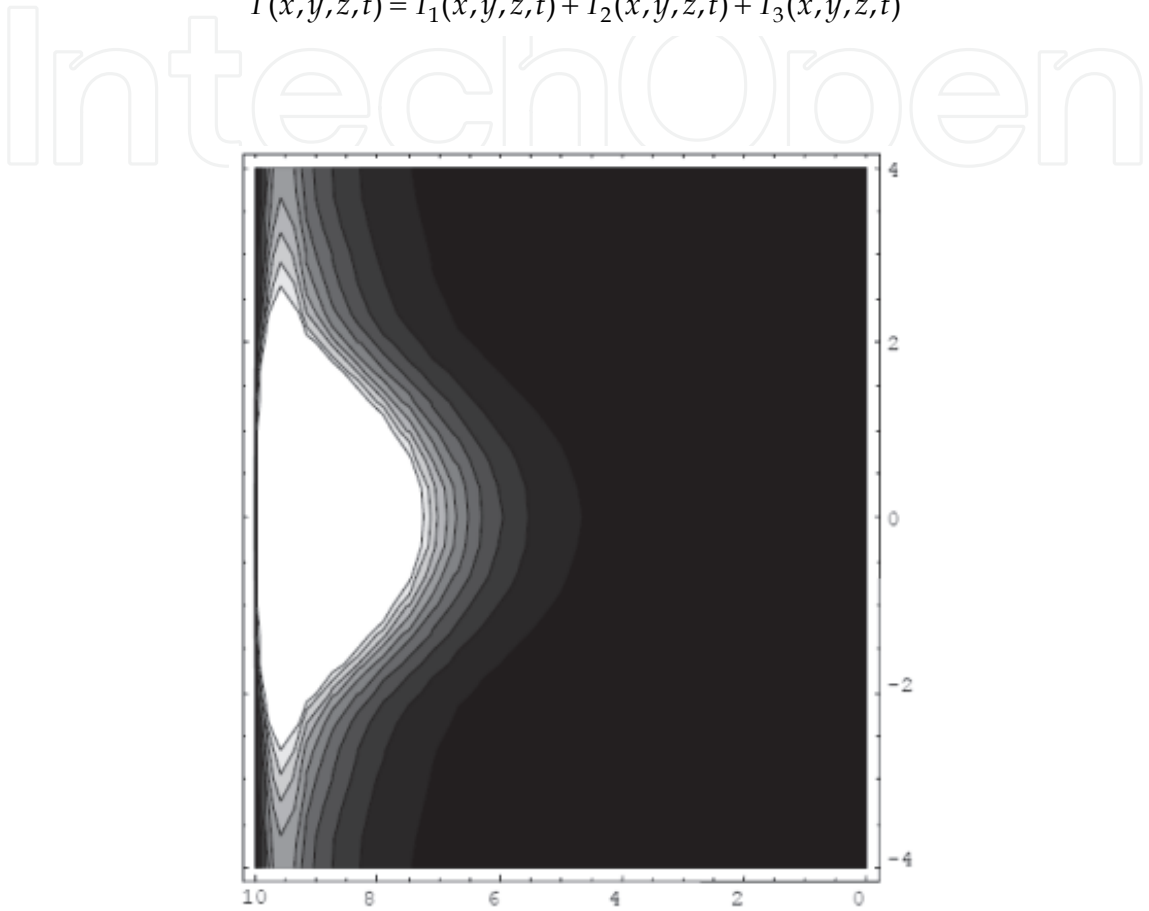$$T(x,y,z,t) = T_1(x,y,z,t) + T_2(x,y,z,t) + T_3(x,y,z,t) \tag{24}$$

Fig. 4. The temperature field in water produced by a 10 MeV electron beam, when the "steady- state" is achieved.

We suppose that for the heat transfer coefficients: $h_1 = h_2 = h_3 = h_4 = h_5 = h_6 = h$. If we consider a linear heat transfer at the sample surface (the "radiation" boundary condition [11]), we have:
for the first laser beam , direction of propagation along x axis:

$$\left[\frac{\partial K_x}{\partial x} - \frac{h}{K}K_x\right]_{x=-\frac{a}{2}} = 0 \;\; ; \;\; \left[\frac{\partial K_x}{\partial x} + \frac{h}{K}K_x\right]_{x=\frac{a}{2}} = 0 \;\; ; \;\; \left[\frac{\partial K_y}{\partial y} + \frac{h}{K}K_y\right]_{y=\frac{b}{2}} = 0 \;\; ;$$

$$\left[\frac{\partial K_y}{\partial y} + \frac{h}{K}K_y\right]_{y=-\frac{b}{2}} = 0 \;\; ; \;\; \left[\frac{\partial K_z}{\partial z} + \frac{h}{K}K_z\right]_{z=\frac{c}{2}} = 0 \;\; ; \;\; \left[\frac{\partial K_z}{\partial z} + \frac{h}{K}K_z\right]_{z=-\frac{c}{2}} = 0 \tag{25}$$

for the second laser beam, direction of propagation along y axis:

$$\left[\frac{\partial L_x}{\partial x}+\frac{h}{K}L_x\right]_{x=-\frac{a}{2}}=0 \;\; ; \;\; \left[\frac{\partial L_x}{\partial x}+\frac{h}{K}L_x\right]_{x=\frac{a}{2}}=0 \;\; ; \;\; \left[\frac{\partial L_y}{\partial y}+\frac{h}{K}L_y\right]_{y=\frac{b}{2}}=0 \;\; ;$$

$$\left[\frac{\partial L_y}{\partial y}-\frac{h}{K}L_y\right]_{y=-\frac{b}{2}}=0 \;\; ; \;\; \left[\frac{\partial L_z}{\partial z}+\frac{h}{K}L_z\right]_{z=\frac{c}{2}}=0 \;\; ; \;\; \left[\frac{\partial L_z}{\partial z}+\frac{h}{K}L_z\right]_{z=-\frac{c}{2}}=0 \tag{26}$$

for the third laser beam, direction of propagation along z axis:

$$\left[\frac{\partial M_x}{\partial x}+\frac{h}{K}M_x\right]_{x=-\frac{a}{2}}=0 \;\; ; \;\; \left[\frac{\partial M_x}{\partial x}+\frac{h}{K}M_x\right]_{x=\frac{a}{2}}=0 \;\; ; \;\; \left[\frac{\partial M_y}{\partial y}+\frac{h}{K}M_y\right]_{y=\frac{b}{2}}=0 \;\; ;$$

$$\left[\frac{\partial M_y}{\partial y}+\frac{h}{K}M_y\right]_{y=-\frac{b}{2}}=0 \;\; ; \;\; \left[\frac{\partial M_z}{\partial z}+\frac{h}{K}M_z\right]_{z=\frac{c}{2}}=0 \;\; ; \;\; \left[\frac{\partial M_z}{\partial z}-\frac{h}{K}M_z\right]_{z=-\frac{c}{2}}=0 \tag{27}$$

The solution of the heat equation subjected to boundary conditions (25), (26) and (27) is:

$$\Delta T(x,y,z,t)=\sum_{m,n}\left\{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{o=1}^{\infty}a(\alpha_i,\beta_j,\chi_o)b(\alpha_i,\beta_j,\chi_o,t)K_x(\alpha_i,x)K_y(\beta_j,y)K_z(\chi_o,z)\right\}$$

$$+\sum_{v,\varphi}\left\{\sum_{p=1}^{\infty}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}c(\delta_p,\varepsilon_r,\eta_s)d(\delta_v,\varepsilon_r,\eta_s,t)L_x(\delta_v,x)L_y(\varepsilon_r,y)L_z(\eta_s,z)\right\} \tag{28}$$

$$+\sum_{r,\varphi}\left\{\sum_{t=1}^{\infty}\sum_{v=1}^{\infty}\sum_{w=1}^{\infty}e(\mu_t,\sigma_v,\omega_w)f(\mu_t,\sigma_v,\omega_w,t)M_x(\mu_t,x)M_y(\sigma_v,y)M_z(\omega_w,z)\right\}$$

We have:

$$I_{mn}(x,y)=I_{0mn}\left[H_m(\tfrac{\sqrt{2}x}{w})H_n(\tfrac{\sqrt{2}y}{w})\times\exp\left[-\left(\frac{x^2+y^2}{w^2}\right)\right]\right]^2 \tag{29}$$

Here w is the width of the laser beam.

$$a(\alpha_i,\beta_j,\chi_o)=\frac{\sum\limits_{m,n}I_{0x,m,n}}{KC_iC_jC_o}\int_{-\frac{a}{2}}^{+\frac{a}{2}}(\alpha_i e^{-\alpha_i x}(1-r_S)K_x(\alpha_i,x)+$$

$$+r_S\cdot\delta_S(x))dx\int_{-\frac{b}{2}}^{+\frac{b}{2}}\int_{-\frac{c}{2}}^{+\frac{c}{2}}I_{m,n}K_y(\beta_j,y)K_z(\chi_o,z)dydz \tag{30}$$

where:

$$b(\alpha_i,\beta_j,\chi_o,t)=\frac{1}{\alpha_i^2+\beta_j^2+\chi_o^2}[1-e^{-\gamma_{ijo}^2 t}-(1-e^{-\gamma_{ijo}^2(t-t_o)})h(t-t_o)] \tag{31}$$

and

$$\gamma_{ijo}^2 = \gamma(\alpha_i^2 + \beta_j^2 + \chi_o^2) \,.$$

We have:

$$K_x(\alpha_i, x) = \cos(\alpha_i \cdot x) + (h / k\lambda_i) \cdot \sin(\alpha_i \cdot x) \qquad (32)$$

The other formulas can be easily obtains by "rotations" of the indices. t -is the time and t_o the exposure time.

We have: r_S is the parameter which take care of the surface absorption and which make sense only for one photon absorption.

Here: $\alpha_i, \alpha_i^2, \delta_p, \delta_p^2, \mu_t, \mu_t^2$ are the eigenvalues corresponding to the eigenfunctions: $K_x, K_y, K_z, P_x, P_y, P_z, L_x, L_y, L_z, T_x, T_y, T_z, M_x, M_y, M_z, N_x, N_y, N_z$ [7].

$C_i C_j, C_o, C_p, C_r, C_s, C_t, C_v$ and C_w as well similarly formulas for two photon absorption.

$h(t - t_0)$ is the step function [7].

We can generalize formula (28) taking into account the one and two absorption coefficient. In this case we have the following solution:

$$
\begin{aligned}
\Delta T(x,y,z,t) = \sum_{m,n} &\left\{ \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{o=1}^{\infty} a(\alpha_i, \theta_j, \chi_o) b(\alpha_i, \theta_j, \chi_o, t) K_x(\alpha_i, x) K_y(\theta_j, y) K_z(\chi_o, z) \right\} \\
+ \sum_{m,n} &\left\{ \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{o=1}^{\infty} \beta(\alpha_i^2, \theta_j^2, \chi_o^2) b(\alpha_i^2, \theta_j^2, \chi_o^2, t) P_x(\alpha_i^2, x) P_y(\theta_j^2, y) P_z(\chi_o^2, z) \right\} \\
+ \sum_{v,q} &\left\{ \sum_{p=1}^{\infty}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} c(\delta_p, \varepsilon_r, \eta_s) d(\delta_v, \varepsilon_r, \eta_s, t) L_x(\delta_v, x) L_y(\varepsilon_r, y) L_z(\eta_s, z) \right\} \\
+ \sum_{v,q} &\left\{ \sum_{p=1}^{\infty}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \pi(\delta_p^2, \varepsilon_r^2, \eta_s^2) d(\delta_v^2, \varepsilon_r^2, \eta_s^2, t) T_x(\delta_v^2, x) T_y(\varepsilon_r^2, y) T_z(\eta_s^2, z) \right\} \\
+ \sum_{r,\varphi} &\left\{ \sum_{t=1}^{\infty}\sum_{v=1}^{\infty}\sum_{w=1}^{\infty} e(\mu_t, \sigma_v, \omega_w) f(\mu_t, \sigma_v, \omega_w, t) M_x(\mu_t, x) M_y(\sigma_v, y) M_z(\omega_w, z) \right\} \\
+ \sum_{r,\varphi} &\left\{ \sum_{t=1}^{\infty}\sum_{v=1}^{\infty}\sum_{w=1}^{\infty} \psi(\mu_t^2, \sigma_v^2, \omega_w^2) f(\mu_t^2, \sigma_v^2, \omega_w^2, t) N_x(\mu_t^2, x) N_y(\sigma_v^2, y) N_z(\omega_w^2, z) \right\}
\end{aligned}
\qquad (33)
$$

In formula (33) the upper index 2 means that the corresponding values are connected with two photon absorption. The eigenfunctions and the eigenvalues for two absorption phenomena can be calculated in the same way like in the case of one photon absorption with the only change that we have another absorption formula. It make no sense to take into account three or more photons absorption phenomena because in this situations the cross sections are very small.

In the next pages we will present three simulations, using the developed "multiple beam irradiation".

The different characteristics of dielectrics under one laser beam irradiation have been very well studied in literature. We will take the case of a ZnSe sample (all characteristics of the material can be found in reference [11]).
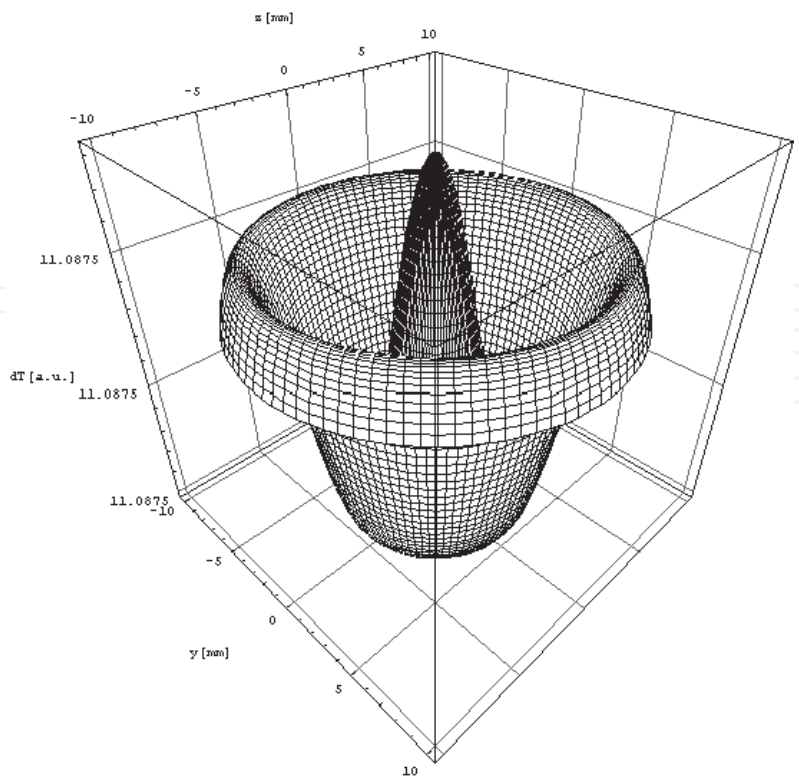
The sample is a cube with the dimension about 2 cm.

Fig. 5. Temperature field in the plane x=0, during a 100s irradiation with a 10 W CO_2 laser beam.
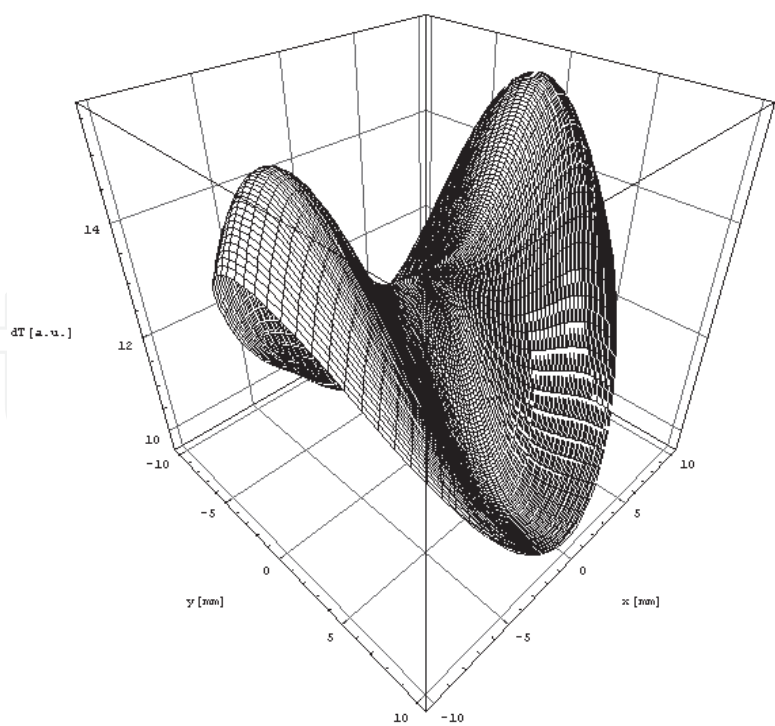

Fig. 6. Temperature field plotted during 100s irradiation with a 50 W CO_2 laser beam, operating in the TEM_{03}.

Our study indicates that for a sample under one, two or three laser irradiation, the heat equation has an exact semi-analytical solution. In fact it can be considered an analytical solution because the eigenvalues with index higher than 10 does not contribute to the solution of heat equation. This solution it is not simply the sum of solutions from three one-dimensional heat equations, because $T_1(x,y,z,t), T_2(x,y,z,t)$ and $T_3(x,y,z,t)$ are coupled via boundary conditions. Our model can be easily generalized for the cases when: $h_1 \neq h_2 \neq h_3 \neq h_4 \neq h_5 \neq h_6$ or $\alpha_x \neq \alpha_y \neq \alpha_z$ .The model could be applied to any laser-solid system whose interaction can be described by Beer law.

The integral transform technique has proved once again it's "power" in resolving heat equation problems [14-17].
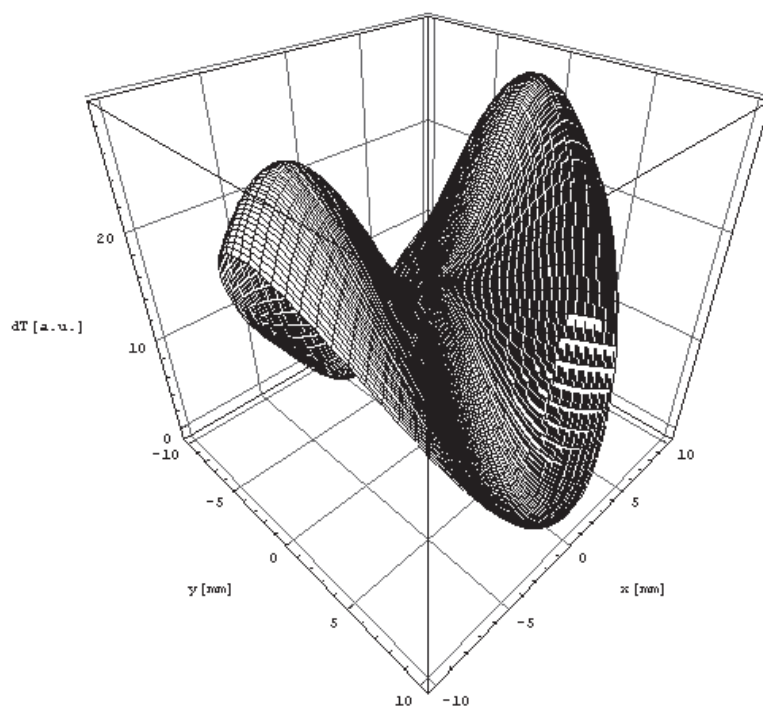

Fig. 7. Temperature field when the sample is irradiated simultaneously with the two laser beams, mentioned above (Fig.5 and Fig.6)

6. Discussions and conclusions

We developed a method for solving the heat diffusion equation- based on dividing the whole domain into small intervals, the length of each depending on the required accuracy of the final solution. The theory is applicable to laser, electrons and hadrons beams interaction with human tissues (which are simulated by a water phantom). In each of the obtained intervals the thermal conductivity function is approximated by a linear function. This function is introduced in the heat equation associated to each interval. At the interface between intervals, the continuity of temperature function and its first derivative are ensured, these conditions providing the values for the coefficients obtained in the final solution.

In order to solve such a system, the formalism of the integral operators with respect to the space and temporal dimensions was applied and the initial system becomes an algebraic one [7]. After solving the system, inverse transformations were applied and the final solution for each interval was obtained as a series of Bessel and Weber functions depending on the space coordinate.

We had thus developed a semi-analytical model for describing the beam – inhomogeneous medium interaction. It can be applied to beam-target interaction where the temperature variation is not very large. This experimental restraint is required because the model does not take into consideration the variation of the thermal parameters with the temperature.

From a practical point of view, the eigenvalues can be obtained from the boundary conditions. Also the constants A_i, B_i can be obtained easily from the same boundary conditions.

Here follows a few examples of the model applications: electron beam-water phantom interaction, proton-water phantom interaction, laser-optical components interaction and, in general, laser-solid media interaction (with the condition that the absorption coefficient keeps small).

We also made simulations using the Green function method. The results represented in Figures 3 and 4 are similar, with the exception of the edges temperature, where we believe that the Green function method is more close to the reality. In fact, the Green function method takes more into account that at the edges of the sample the heat transfer coefficients are higher and in consequence the temperatures get lower.

In previous papers different models (which were in fact particular cases of the present model) were applied to describe the interaction between a multi-mode cw CO_2 laser beam with multi-layered structures (of the type thin films substrate) [10] or with optical components [11].

The actual strength of the model is that it can take into account any form of the beam spatial distribution and any stationary type of interaction. That was the starting point for developing the semi-classical heat equation solution, which included the multi-photons laser-sample interaction [12]. The particular case $m_i = 0$ (i.e. when $k_i (x)=k (x_i)$) was analyzed in Ref. - [13].

The "power" of integral transform technique was emphasized in references [14-17]; both in classic and quantum physics.

Finally: a remark about figures 1-4. We mention that: dT (x, y, z, t) is in general proportional with S (x, y, z, t). This is not always true, but in our case is valid because the small values of the heat transfer coefficient. For a comprehensive discussion of the importance of heat transfer coefficient, see Appendix B in reference [11].

Our model offers a first simple approximation of the temperature field in (electron, proton, laser) beam (liquid, solid) target interaction.

The model can also describe the thermal fields for three different beams (electron, proton and laser), which act simultaneously onto a sample along the three Cartesian coordinates axes.

Figure 3 is illustrative for the strength of our model. The simulations performed using sub-domains of 0.25 cm were indeed in good agreement with the solutions given by the Green function method.

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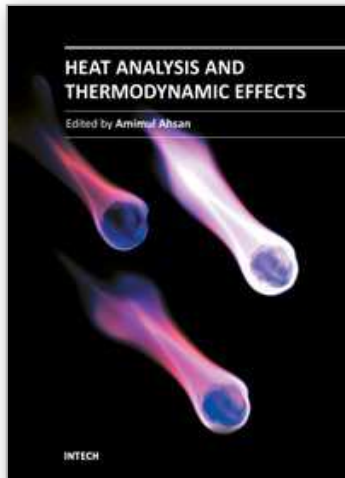
Heat Analysis and Thermodynamic Effects

Edited by Dr. Amimul Ahsan

The heat transfer and analysis on heat pipe and exchanger, and thermal stress are significant issues in a design of wide range of industrial processes and devices. This book includes 17 advanced and revised contributions, and it covers mainly (1) thermodynamic effects and thermal stress, (2) heat pipe and exchanger, (3) gas flow and oxidation, and (4) heat analysis. The first section introduces spontaneous heat flow, thermodynamic effect of groundwater, stress on vertical cylindrical vessel, transient temperature fields, principles of thermoelectric conversion, and transformer performances. The second section covers thermosyphon heat pipe, shell and tube heat exchangers, heat transfer in bundles of transversely-finned tubes, fired heaters for petroleum refineries, and heat exchangers of irreversible power cycles. The third section includes gas flow over a cylinder, gas-solid flow applications, oxidation exposure, effects of buoyancy, and application of energy and thermal performance index on energy efficiency. The forth section presents integral transform and green function methods, micro capillary pumped loop, influence of polyisobutylene additions, synthesis of novel materials, and materials for electromagnetic launchers. The advanced ideas and information described here will be fruitful for the readers to find a sustainable solution in an industrialized society.

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