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A PI^{2}D Feedback Control Type for Second Order Systems

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1. Introduction

Second order autonomous systems are key systems in the study of non linear systems because their solution trajectories can be represented by curves in the plane (Khalil, 2002), which helps in the development of control strategies through the understanding of their dynamical behaviour. Such autonomous systems are often obtained when considering feedback control strategies, because the closed loop system might be rewritten in terms of the state system and perturbation terms, which are function of the state as well. Thus, analyses of stability properties of second order autonomous systems and their convergence are areas of interest on the control community.

Moreover, several applications consider nonlinear second order systems; there are various examples of this:

1. In mechanical systems the pendulum, the inverted pendulum, the translational oscillator with rotational actuator (TORA) and the mass-spring systems;
2. In electrical systems there are examples such as the tunnel diode circuit, some electronic oscillators as the negative-resistance twin-tunnel-diode circuit; and finally
3. Other type of these systems are mechanical-electrical-electronic combinations, for example a two degree of freedom (DOF) robot arm or a mobile planar robot and among every degree of freedom on a robotic structure can be represented by a second order nonlinear system.

Therefore, due to the wide applications in second order nonlinear systems, several control laws have been proposed, which comprises from simple ones, like linear controllers, to the more complex, like sliding mode, backstepping approach, output-input feedback linearization, among others (Khalil, 2002).

Despite the development of several control strategies for nonlinear second order systems, it is not surprising that for several years and even nowadays the classical PID controllers have been widely used in technical and industrial applications and even on research fields. This is due to the good understanding that engineers have of them. Moreover, the PID controllers have several important functions: provide feedback, has the ability to eliminate steady state offset through integral action, and it can anticipate the future through derivative action.

PID controllers are sufficient for many control problems, particularly when system dynamics are favourable and the performance requirements are moderate. These types of

controllers are important elements of distributed control system. Many useful features of PID control are considered trade secrets, (Astrom and Hagglund, 1995). To build complicated automation systems in widely production systems as energy, transportation and manufacturing, PID control is often combined with logic, sequential machines, selectors and simple function blocks. And even advanced techniques as model predictive control is encountered to be organized in hierarchically, where PID control is used in the lower level. Therefore, it can be inferred that PID control is a key ingredient in control engineering.

For the above reasons several authors have developed PID control strategies for nonlinear systems, this is the case of (Ortega, Loria and Kelly, 1995) that designed an asymptotically stable proportional plus integral regulator with position feedback for robots with uncertain payload that results in a PI²D regulator. In the work of (Kelly, 1998), the author proposed a simple PD feedback control plus integral action of a nonlinear function of position errors of robot manipulators, that resulted effective on the control of this class of second nonlinear systems and it is known as PD control with gravity compensation. Also PID modifications for control of robot manipulators are proposed at the work of (Loria, Lefeber and Nijmeijer, 2000), where global asymptotic stability is proven. In process control a kind of PI² compensator was developed in the work of (Belanger and Luyben, 1997) as a low frequency compensator, due to the additional double integral compensation rejects the effects of ramp-like disturbances; and in the work of (Monroy-Loperena, Cervantes, Morales and Alvarez-Ramirez, 1999), a parametrization of the PI² controller in terms of a nominal closed-loop and disturbance estimation constants is obtained, despite both works are on the process control field, their analysis comprises second order plants.

In the present work a class of nonlinear second order system is consider, where the control input can be consider as result of state feedback, that in the case of second order systems is equivalent to a PD controller, meanwhile double integral action is provided when the two state errors are consider, both regulation and tracking cases are considered.

Stability analysis is developed and tuning gain conditions for asymptotic convergence are provided. A comparison study against PID type controller is presented for two examples: a simple pendulum and a 2 DOF robot arm. Simulation results confirm the stability and convergence properties that are predicted by the stability analysis, which is based on Lyapunov theory. Finally, the chapter closes with some conclusions.

2. Problem formulation

Two cases are considered in this work, first regulation to a constant reference is boarded, second tracking a time varying reference is studied; in both cases stability and tuning gain conditions are provided.

2.1 Regulation

Consider the following type of second order system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -f(x) + g(x)u \end{aligned} \tag{1}$$

Where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^n$ is the control input, such that fully actuated systems are considered, $g(x) \in \mathbb{R}^{n \times n}$ is a non linear function that maps the input to the system dynamics, and it is assumed that such function is known and invertible along all solutions of the

system, $f(x)$ is a nonlinear function that is continuously differentiable, and locally Lipschitz. It is assumed that the state is measurable and that $f(x)$ is known.
The control objective is to regulate the state $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ to a constant value $x_{ref} = \begin{bmatrix} x_{1ref} & 0 \end{bmatrix}^T$.
The proposed dynamic control considers full cancellation of the system dynamics, and it is given by

$$u = g(x)^{-1}\left(f(x) + u_n\right) \tag{2}$$

where u_n represents a nominal feedback control that would be designed to ensure the regulation of (1) to x_{ref}.
The nominal control is designed as a feedback state control plus a type of double integral control and is provided in the following equation

$$u_n = -K_P\left(x_1 - x_{1ref}\right) - K_D x_2 - K_I \int \left(\left(x_1 - x_{1ref}\right) + x_2\right) dt \tag{3}$$

Control (3) provides an extra integral action with the integration of the state x_2. The constant gains are K_P, K_D and K_I and must be positive. The integral action provides an augmented state, therefore system (1) in closed loop with control (2) and (3) is re-written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -K_P\left(x_1 - x_{1ref}\right) - K_D x_2 - K_I x_3 \\ \dot{x}_3 &= \left(x_1 - x_{1ref}\right) + x_2 \end{aligned} \tag{4}$$

The closed-loop system (4) has a unique equilibrium point in $x_{ref} = \begin{bmatrix} x_{1ref} & 0 & 0 \end{bmatrix}^T$.
In the following a stability analysis for the regulation case is determined.

2.1.1 Stability analysis for the regulation case

Consider the following position error vector $e = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^T$, with $e_1 = x_1 - x_{1ref}$, $e_2 = x_2$, $e_3 = \int (e_1 + e_2) dt$, such that the closed loop error dynamics (4), which corresponds to an autonomous system, might be rewritten as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -K_P e_1 - K_D e_2 - K_I e_3 \\ \dot{e}_3 &= e_1 + e_2 \end{aligned} \tag{5}$$

Provided that the gains K_P, K_D and K_I are different from zero and positive, it is immediate to obtain that the equilibrium of system (5) corresponds to $e^* = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. On the following stability conditions and tuning guidelines for the control gains K_P, K_D and K_I will be presented.

Theorem 1

Consider the autonomous dynamic second order system given by (5), which represents the closed loop error dynamics obtained from system (1) with the control law (2), and the nominal PI²D controller given by (3). The autonomous dynamic system (5) converge

asymptotically to its equilibrium point $e^* = [0 \quad 0 \quad 0]^T$, if the positive control gains K_P, K_D and K_I satisfy the following conditions

$$\begin{aligned} &K_I > 8 \\ &K_D > 3K_I + 2\sqrt{2K_I{}^2 - K_I - 1} \\ &K_P > K_I + K_D \end{aligned} \tag{6}$$

Proof:

Consider the position error vector $e = [e_1 \quad e_2 \quad e_3]^T$ and the Lyapunov function

$$V_e = \tfrac{1}{2} e^T M(K_P, K_D, K_I) e \tag{7}$$

where $M(K_P, K_D, K_I) \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix, with all entries $m_{i,j}$ real and positive for all i, j; in order to simplify the Lyapunov function computation the following conditions are introduced

$$\begin{aligned} m_{1,3} &= m_{3,1} = 0 \\ m_{1,2} &= m_{2,3} \end{aligned}$$

The time derivative of the Lyapunov function (7) is function of the closed loop error dynamics (5), and it is given by

$$\begin{aligned} \dot{V}(e) &= e^T M \dot{e} \\ &= e_1 e_2 \left(m_{1,1} + m_{1,2} - m_{2,2} K_P - m_{1,2} K_D \right) + e_1 e_3 \left(m_{3,3} - m_{1,2} K_P - m_{1,2} K_I \right) + \\ &+ e_2 e_3 \left(m_{3,3} - m_{1,2} K_D - m_{2,2} K_I \right) - e_1{}^2 m_{1,2} K_P + e_2{}^2 \left(2m_{1,2} - m_{2,2} K_D \right) - e_3{}^2 m_{1,2} K_I \end{aligned}$$

Thus, a straightforward simplification of the time derivative of the Lyapunov function is to cancel the crossed error terms $e_1 e_2$, $e_1 e_3$, $e_2 e_3$, which results in $\dot{V}(e)$ given by quadratic error terms. First, in order to cancel the crossed term $e_1 e_3$ conditions on $m_{3,3}$ can be obtained, and then to cancel the crossed term $e_2 e_3$ the matrix entry $m_{2,2}$ is defined appropriately, finally by defining $m_{1,1}$ the crossed error term $e_1 e_2$ is eliminated. So far the conditions on matrix $M(K_P, K_D, K_I)$ are summarized as follows

$$\begin{aligned} m_{1,3} &= m_{3,1} = 0 \\ m_{1,2} &= m_{2,3} \\ m_{1,1} &= \frac{m_{1,2}}{K_I} \left(K_P{}^2 + K_P (K_I - K_D) + K_I (K_D - 1) \right) \\ m_{2,2} &= \frac{m_{1,2}}{K_I} \left(K_P + K_I - K_D \right) \\ m_{3,3} &= m_{12} \left(K_P + K_I \right) \end{aligned} \tag{8}$$

On the other hand, to guarantee that $m_{1,1}$, $m_{2,2}$ and $m_{3,3}$ of the matrix $M(K_P, K_D, K_I)$ are positive, it is necessary to satisfy the following conditions. For the matrix entry $m_{3,3}$ to be

positive, it is enough to have that $K_P > 0$ and $K_I > 0$ which are satisfied by conditions (6) of Theorem 1. For $m_{2,2} > 0$ it follows that $K_P > K_D - K_I$, that is satisfy by the conditions $K_D > 0$ and $K_P > K_D + K_I$ as stated at (8). Finally, for $m_{1,1} > 0$ it follows that ${K_P}^2 + K_P(K_I - K_D) + K_I(K_D - 1) > 0$, which implies conditions on K_P and K_D, to find out such conditions, the solutions of equation ${K_P}^2 + K_P(K_I - K_D) + K_I(K_D - 1) = 0$ are computed as follows

$$K_P = \frac{(K_D - K_I) \pm \sqrt{(K_I - K_D)^2 - 4K_I(K_D - 1)}}{2}$$

For K_P to be positive, it is required that $K_D > K_I$, that is satisfied by conditions (6). Then for K_P to be purely real, it is required that the argument of the squared root being positive, i.e. $(K_I - K_D)^2 - 4K_I(K_D - 1) > 0$, which when it is equal to zero implies the solutions $K_D = 3K_I \pm \sqrt{2{K_I}^2 - K_I}$. Thus, by taking the positive part of the solution and considering that $K_D > 3K_I + \sqrt{2{K_I}^2 - K_I}$ it is guaranteed that K_P is real, and condition $K_I > 1/2$ implies that K_D is real. Notice that all these conditions are satisfied by those stated at Theorem 1, equations (6).

At this point, it is guaranteed that the solutions K_P are real and positive, those to ensure that $m_{1,1} > 0$ it is considered that K_P must satisfied

$$K_P > \frac{(K_D - K_I) + \sqrt{(K_I - K_D)^2 - 4K_I(K_D - 1)}}{2}$$

Such a condition is clearly over satisfied by the condition $K_P > K_D + K_I$ given at Theorem 1, equations (6).

Therefore, if conditions given by (6) at Theorem 1 are satisfied, it implies that $m_{1,1}$, $m_{2,2}$ and $m_{3,3}$ of the matrix $M(K_P, K_D, K_I)$ are positive.

Furthermore, the definition of $m_{1,1}$, $m_{2,2}$ and $m_{3,3}$ stated by (8), yield a time derivative Lyapunov function given by quadratic error terms. Nonetheless it is necessary to check positive definitiveness of the Lyapunov function (7), which after conditions (8) are considered is given by

$$V(e) = m_{1,2}\left\{\frac{{e_1}^2}{K_I}\left[{K_P}^2 + K_P(K_I - K_D) + K_I(K_D - 1)\right] + 2e_1e_2 + {e_2}^2\frac{K_P + K_I - K_D}{K_I} + 2e_2e_3 + {e_3}^2(K_P + K_I)\right\}$$

Notice that the positive condition on the coefficient of the term e_1^2, i.e. ${K_P}^2 + K_P(K_I - K_D) + K_I(K_D - 1) > 0$ has already been considered for positive definitiveness of $m_{1,1}$. Therefore, the conditions on K_P, K_D and K_I given by (6) imply that all coefficients of the quadratic terms of $V(e)$ are positive. Now to guarantee $V(e) > 0$ note that the cross error terms $2e_1e_2$ and $2e_2e_3$ can be rewritten as part of a quadratic form, for that it is required

$$K_P{}^2 + K_P(K_I - K_D) + K_I(K_D - 1) > 1$$
$$\frac{K_P + K_I - K_D}{K_I} > 2$$
$$K_P + K_I > 1$$

The last two conditions imply that $K_P > 1 - K_I$ and $K_P > K_I + K_D$ which are satisfied by conditions (6). And the first condition implies to solve the equation $K_P{}^2 + K_P(K_I - K_D) + K_I(K_D - 1) = 1$ for K_P. Similar to the way in which conditions for positive value of the entries of matrix $M(K_P, K_D, K_I)$, it follows that

$$K_P > \frac{(K_D - K_I) + \sqrt{(K_I - K_D)^2 - 4\left[K_I(K_D - 1) - 1\right]}}{2}$$

That is conservatively satisfied by the condition $K_P > K_D + K_I$ given at Theorem 1, equation (6). On the other hand for K_P to be real, it is necessary that $K_D > 3K_I + 2\sqrt{2K_I{}^2 - K_I - 1}$, and for K_D to be real it is required that $K_I > 1$; all these conditions are clearly satisfied by those stated at Theorem 1, equations (6).

Therefore, if the conditions given by (6) are satisfied, the Lyapunov function results on a sum of quadratic terms

$$V(e) = m_{1,2}\left\{(e_1 + e_2)^2 + (e_2 + e_3)^2 + k_1 e_1{}^2 + k_2 e_2{}^2 + k_3 e_3{}^2\right\} \tag{9}$$

for positive parameters k_1, k_2, k_3; thus concluding that $V(e) > 0$ for $e \neq 0$, and $V(e) = 0$ for $e = 0$.

Since the definition of the matrix entries (8) allows cancellation of all cross error terms on the time derivative of the Lyapunov function (7), then along the position error solutions, it follows that

$$\dot{V}(e) = e^T M\dot{e} = -m_{1,2}\left[K_P e_1{}^2 + K_I e_3{}^2 + \frac{K_P K_D + K_I(K_D - 2) - K_D{}^2}{K_I} e_2{}^2\right] \tag{10}$$

To ensure that $\dot{V}(e) < 0$, it is required that $K_P K_D + K_I(K_D - 2) - K_D{}^2 > 0$, which implies that

$$K_P > \frac{2K_I - K_D K_I + K_D{}^2}{K_D}$$

which is satisfied by the condition $K_P > K_D + K_I$ given at Theorem 1, equations (6). Nonetheless to guaranteed that K_P is real, it follows that $2K_I - K_D K_I + K_D{}^2 > 0$, that implies when considering equal to zero, that the solutions are

$$K_D = \frac{K_I \pm \sqrt{K_I(K_I - 8)}}{2}$$

Thus for K_D to be real it is required that $K_I > 8$ and finally the condition on K_D results on

$$K_D > \frac{K_I + \sqrt{K_I(K_I - 8)}}{2}$$

Such that, the above conditions are satisfied by considering those of Theorem 1, equation (6). Therefore, by satisfying conditions (6) it can be guaranteed that all coefficients of the derivative of the Lyapunov function $\dot{V}(e)$ are positive, such that $\dot{V}(e) < 0$ for $e \neq 0$, and $\dot{V}(e) = 0$ for $e = 0$.

Thus, it can be concluded that the closed loop system dynamic (5) is stable and the error vector e converges globally asymptotically to its equilibrium $e^* = [0 \quad 0 \quad 0]^T$.

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Remark 1

The conditions stated at Theorem 1, equations (6) are rather conservative in order to guarantee stability and asymptotic convergence of the closed loop errors. The conditions (6) are only sufficient but not necessary to guarantee the stability of the system.

Remark 2

Because full cancellation of the system dynamics function $f(x)$ in (1) is assumed by the control law (2), in order to obtain the closed loop error dynamics (5), then the auxiliary polynomial $P(s) = s^3 + s^2 K_D + s(K_P + K_I) + K_I$ can be considered to obtain a Hurwitz polynomial, and to characterize some properties of the closed loop system.

2.1.2 Stability analysis for the regulation case with non vanishing perturbation

In case that no full cancellation of $f(x)$ in (1) can be guaranteed, either because of uncertainties on $f(x)$, $g(x)$, or in the system parameters, convergence of the system to the equilibrium point $e^* = [0 \quad 0 \quad 0]^T$ is not guaranteed. Nonetheless, the Lipschitz condition on $f(x)$, and assuming that $f(x)$ is bounded in terms of x, i.e $\|f(x)\| \leq \gamma \|x\|$ for positive γ, then locally uniformly ultimate boundedness might be proved for large enough control gains K_P, K_D and K_I, see (Khalil, 2002).

2.2 Tracking

In the case of tracking, the problem statement is now to ensure that the sate vector $x = [x_1 \quad x_2]^T$ follows a time varying reference $x_{ref}(t) = \left[x_{1ref}(t) \quad \dot{x}_{1ref}(t) \right]^T$; this trajectory is at least twice differentiable, smooth and bounded. For this purpose the control proposed in (2) is considered, but with the nominal controller u_n given by

$$u_n = -K_P \left(x_1 - x_{1ref} \right) - K_D \left(x_2 - \dot{x}_{1ref} \right) - K_I \int \left(\left(x_1 - x_{1ref} \right) + \left(x_2 - \dot{x}_{1ref} \right) \right) dt + \ddot{x}_{1ref} \tag{11}$$

2.2.1 Stability analysis for the tracking case

Similar to the regulation case, the following position error vector $e = [e_1 \quad e_2 \quad e_3]^T$ is defined, with $e_1 = x_1 - x_{1ref}$, $e_2 = x_2 - \dot{x}_{1ref}$, $e_3 = \int \left(\left(x_1 - x_{1ref} \right) + \left(x_2 - \dot{x}_{1ref} \right) \right) dt$, such that the closed loop error dynamics of system (1), with the controller (2) and (11) results in the same dynamic systems given by (5), such that Theorem 1 applies for the tracking case.

Remark 3

The second integral action proposed in the nominal controllers, (3) for regulation, and (11) for tracking case, can be interpreted as a composed measured output function, such that this action helps the controller by integrating the velocity errors. When all non linearity is cancelled the integral action converges to zero, yielding asymptotic stability of the complete state of the system. If not all nonlinear dynamics is cancelled, or there is perturbation on the system, which depends on the state, then it is expected that the integral action would act as estimator of such perturbation, and combined with suitable large control gains, it would render ultimate uniformly boundedness of the closed loop states.

3. Results

In this section two systems are consider, a simple pendulum with mass concentrated and a 2 DOF planar robot. First the pendulum system results are showed.

3.1 Simple pendulum system at regulation

Consider the dynamic model of a simple pendulum, with mass concentrated at the end of the pendulum and frictionless, given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -f(x) + c\,u \end{aligned} \tag{12}$$

where $f(x) = a\sin(x_1) + bx_2$ with $a = g/l > 0$, $b = k/m > 0$ and $c = 1/ml^2 > 0$, with the notation m for the mass, k for the spring effects, l the length of the pendulum, and g the gravity acceleration. The values of the model parameters are presented at Table 1, and the initial condition of the pendulum is $x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$.

The proposed PI²D is applied and compared against a PID control that also considers full dynamic compensation, i.e. the classical PID is programmed as follows

$$\begin{aligned} u &= g(x)^{-1}\left(f(x) + u_n\right) \\ u_n &= -K_P\left(x_1 - x_{1ref}\right) - K_D\left(x_2 - \dot{x}_{1ref}\right) - K_I\int\left(x_1 - x_{1ref}\right)dt \end{aligned}$$

The comparative results are shown in Figure 1. The control gains were tuned accordingly to conditions given by (6), see Table 1, such that it was considered that: $K_I > 8$, thus for the selected K_I value, it was obtained that $K_D > 57.49$, and after selection of K_D, it was finally obtained that $K_P > 70$. For the tuned gains listed at Table 1, it follows that the eigenvalues of the closed loop system (5) are the roots of the characteristic polynomial $P(s) = s^3 + s^2K_D + s(K_P + K_I) + K_I$, such that $s_1 = -0.1208$, $s_2 = -1.4156$, and $s_3 = -58.4635$. Therefore, the closed loop system behaves as an overdamped system as shown in Figure 1. The behaviour of the closed-loop system for the PID and PI²D controllers is shown in Figure 1; the performance of the double integral action on the PID proposed by the nominal controller (3) shows faster and overdamped convergence to the reference $x_{ref} = \begin{bmatrix} \pi/4 & 0 \end{bmatrix}^T$ than the PID controller, in which performance it is observed overshoot. Notice however that both input controls are similar in magnitude and shape; this implies better performance of the PI²D controller without increasing the control action significantly.

$a = 10$	$K_I = 10$
$b = 0.1$	$K_D = 60$
$c = 10$	$K_P = 80$

Table 1. Pendulum parameters and control gains.

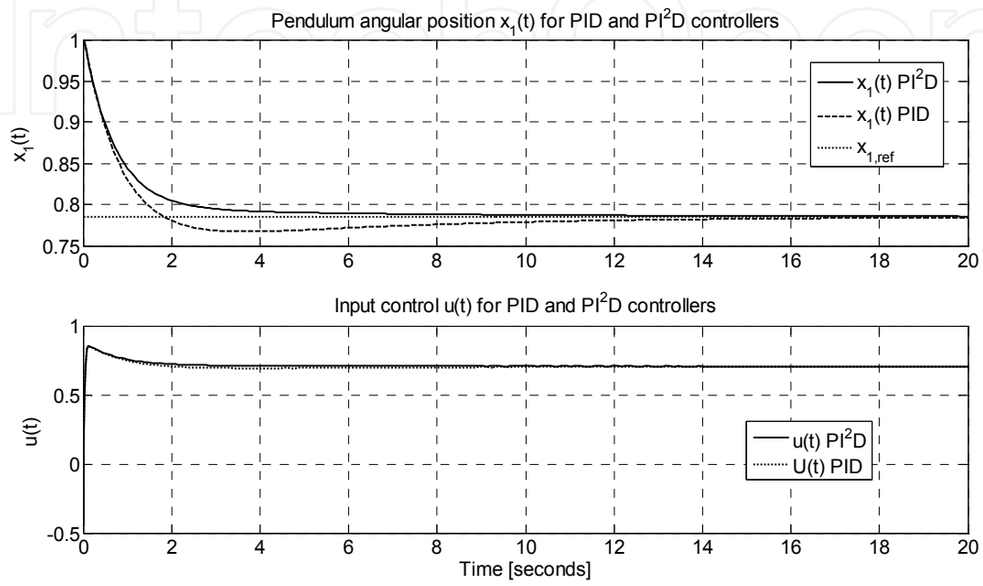

Fig. 1. Comparison study for PID vs PI²D controllers for a simple pendulum system.

For the sake of comparison another simulation is developed considering imperfect model cancellation, in this case due to pendulum parameters uncertainty considered for the definition of the controller (2). The nominal model parameters are those of Table 1, while the control parameters are $\bar{a} = 11.5$, $\bar{b} = 0.01$, $\bar{c} = 11$. The control gains and initial conditions are the same as for the case of perfect cancellation.

The obtained simulation results are shown in Figure 2, where also a change in reference signal is considered from $x_{ref} = \left[\pi/4 \quad 0 \right]^T$ [rad] in $0 \le t \le 30$ seconds to $x_{ref} = \left[\pi/3 \quad 0 \right]^T$ in $30 < t \le 60$ seconds. In the case of non complete dynamic cancellation due to uncertain parameters, it can be seen that the PI²D controller proposed by (2) and (3) also responds faster that the classical PID with dynamic cancellation, besides the control actions are similar in magnitude and shape as shown in Figure 2.

3.2 Simple pendulum system at tracking

A periodic reference given by $x_{1ref} = \sin\left(\pi t/5\right)$ [rad] is considered. The simulation results are shown in Figure 3; the control gains are the same as listed at Table 1. In Figure 3 is depicted both behaviour of the PID and PI²D with perfect dynamic compensation, the PI²D controller shows faster convergence to the desired trajectory than the PID control, nonetheless both control actions are similar in magnitude and shape, this shows that a small change on the control action might render better convergence performance, in such a case the double

integral action of the PI²D controller plays a key role in improving the closed loop system performance

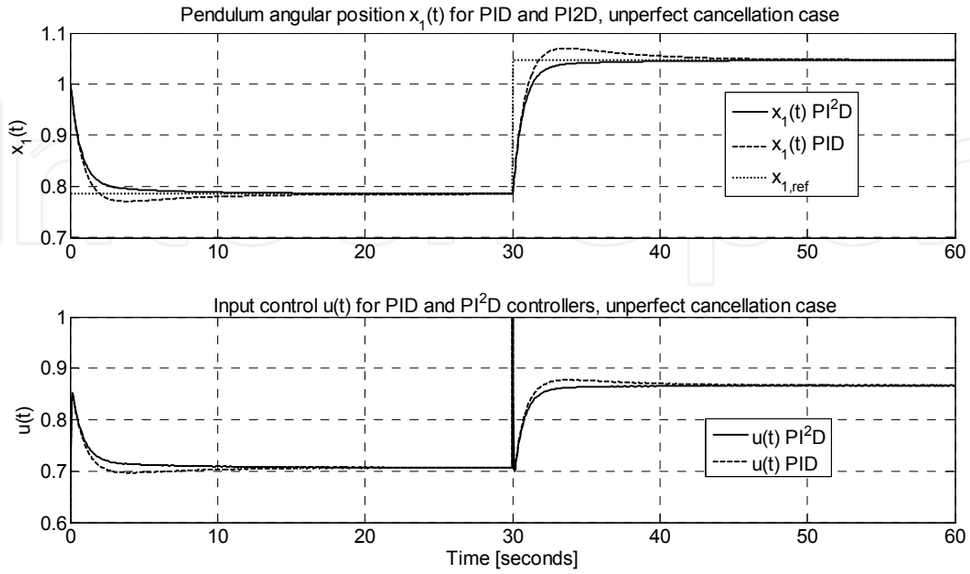

Fig. 2. Comparison study for PID vs PI²D controllers for a simple pendulum system with model parameter uncertainty.

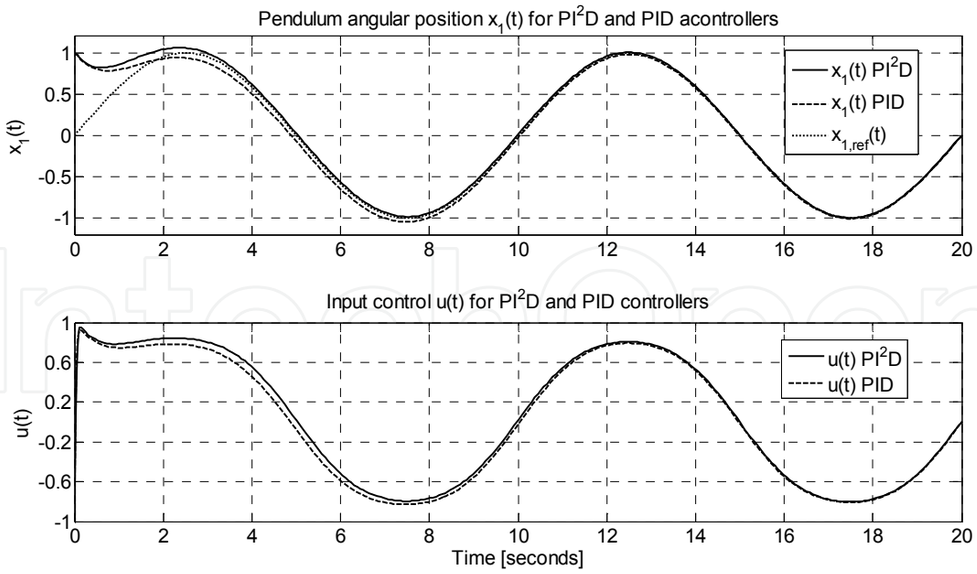

Fig. 3. Tracking response of pendulum system (1) for PID and PI²D controllers.

To close with the pendulum example, uncertainty on the parameters is considered, such that there is no cancellation of the function $f(x) = a\sin(x_1) + bx_2$, i.e. the parameters of the controller $u(t)$ given by (2) are set as $\bar{a} = 0$, $\bar{b} = 0$, and $\bar{c} = 1$; and the controller gains are the

same as listed at Table 1. In Figure 4 the comparison results are showed, despite there is no model cancellation, the PI²D controller shows better performance that the PID case, i.e faster convergence (less than 4 seconds), requiring minimum changes on the control action magnitude and shape, as shown on the below plot of Figure 4, where the control actions are similar to those of Figure 3, which implies that the control gains absorbed the model parameter uncertainties on parameter $\bar{c} = 1$ as well as the non model cancellation. Notice that the control actions present a sort of chattering that is due to the effort to compensate the no model cancellation

3.3 A 2 DOF planar robot at regulation

The dynamic model of a 2 DOF serial rigid robot manipulator without friction is considered, and it is represented by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{13}$$

Where $q,\ \dot{q},\ \ddot{q} \in \mathbb{R}^2$ are respectively, the joint position, velocity and acceleration vectors in generalized coordinates, $D(q) \in \mathbb{R}^{2\times 2}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{2\times 2}$ is the Coriolis and centrifugal matrix, $g(q) \in \mathbb{R}^2$ is the gravity vector and $\tau \in \mathbb{R}^2$ is the input torque vector. The system (13) presents the following properties (Spong and Vidyasagar, 1989).

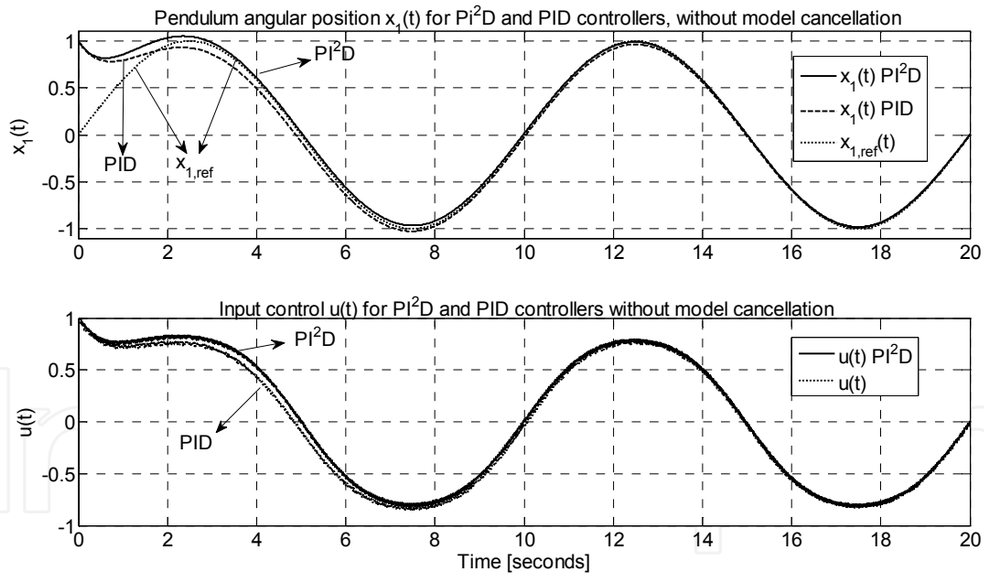

Fig. 4. Tracking response of pendulum system (1) for PID and PI²D controllers without model cancellation.

Property 1.- The inertia matrix is a positive symmetric matrix satisfying $\lambda_{\min} I \le D(q) \le \lambda_{\max} I$, for all $q \in \mathbb{R}^2$, and some positive constants $\lambda_{\min} \le \lambda_{\max}$, where I is the 2-dimensional identity matrix.

Property 2.- The gravity vector $g(q)$ is bounded for all $q \in \mathbb{R}^2$. That is, there exist $n = 2$ positive constants γ_i such that $\sup_{q\in\mathbb{R}^2} |g_i(q)| \le \gamma_i$ for all $i = 1,\ldots,n$.

From the generalized 2 DOF dynamic system, eq. (13), each DOF is rewritten as a nonlinear second order system as follows.

$$\begin{aligned} \dot{x}_{1,i} &= x_{2,i} \\ \dot{x}_{2,i} &= -f_i(x) + g_i(x)\, u_i \end{aligned} \tag{14}$$

With $f_i(x)$ and $g_i(x)$ obtained from rewritten system (13), solving for the acceleration vector and considering the inverse of the inertia matrix. As for the pendulum case a PI²D controller of the form given by (2) and (3) is designed and compared against a PID, similar to section 3.1, for both regulation and tracking tasks.

From Figure (5) to Figure (7), the closed loop with dynamic compensation is presented, where the angular position, the regulation error and the control input, are depicted. The PI²D controller shows better behaviour and faster response than the PID. The controller gains for both DOF of the robot are listed at Table 1. The desired reference is $x_d = \left[\pi/2 \quad \pi/4 \right]^T$.

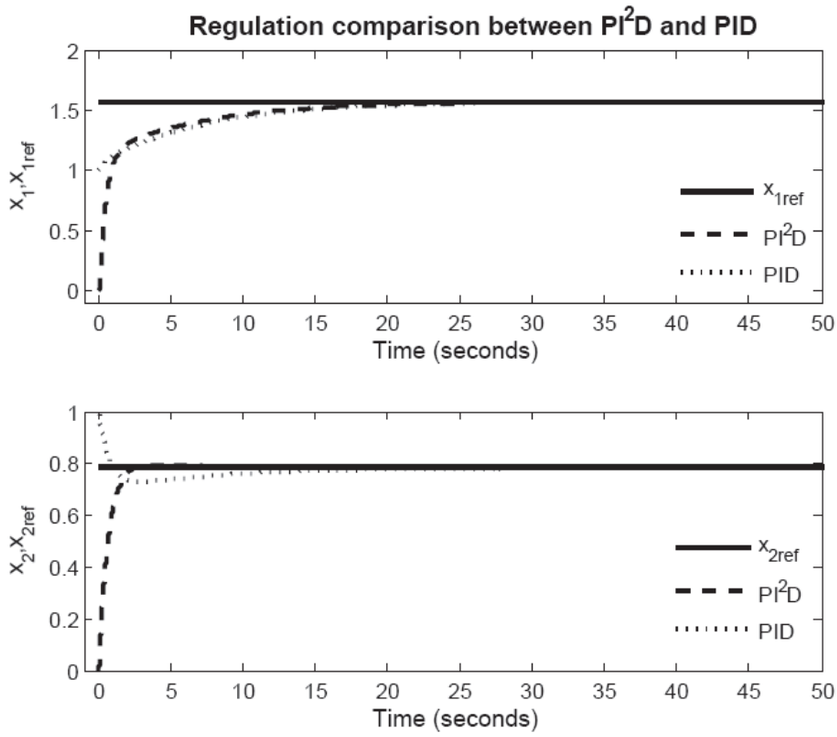

Fig. 5. Robot angular position for PI²D and PID controllers with perfect cancellation.

To test the proposed controller robustness against model and parameter uncertainty, it was considered unperfected dynamic compensation, for both links a sign change on the inertia terms corresponding to the function $g(x)$ is considered and no gravitational compensation was made, meaning that $f(x) = 0$ at the controller. The control gains remained the same as for all previous cases. Figures (8) to (10) show the simulation results. Although the inexact compensation, the proposed PI²D controller behaves faster and with a smaller control effort than the PID control.

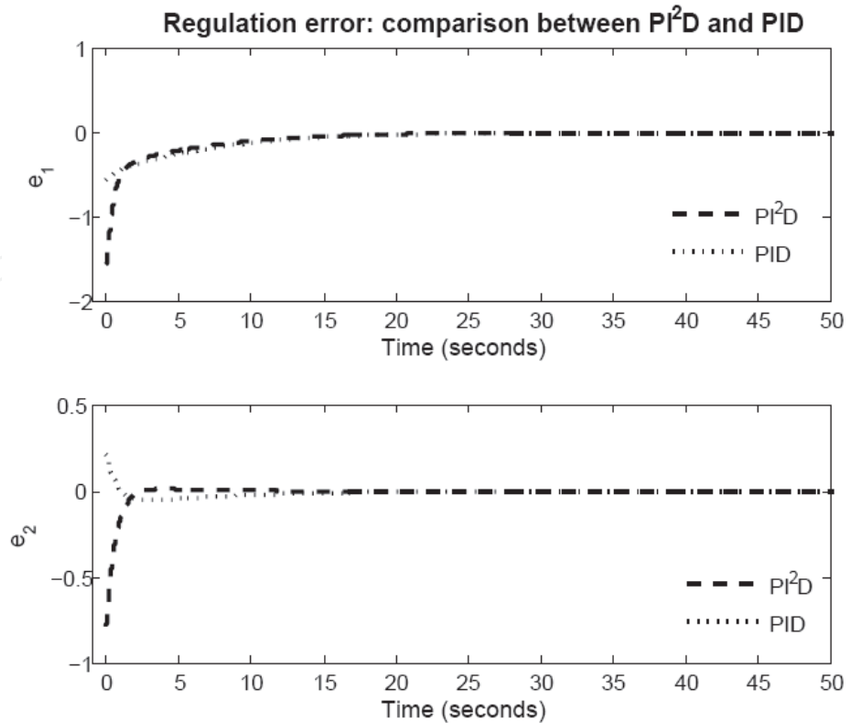

Fig. 6. Robot regulation error for PI²D and PID controllers with perfect cancellation.

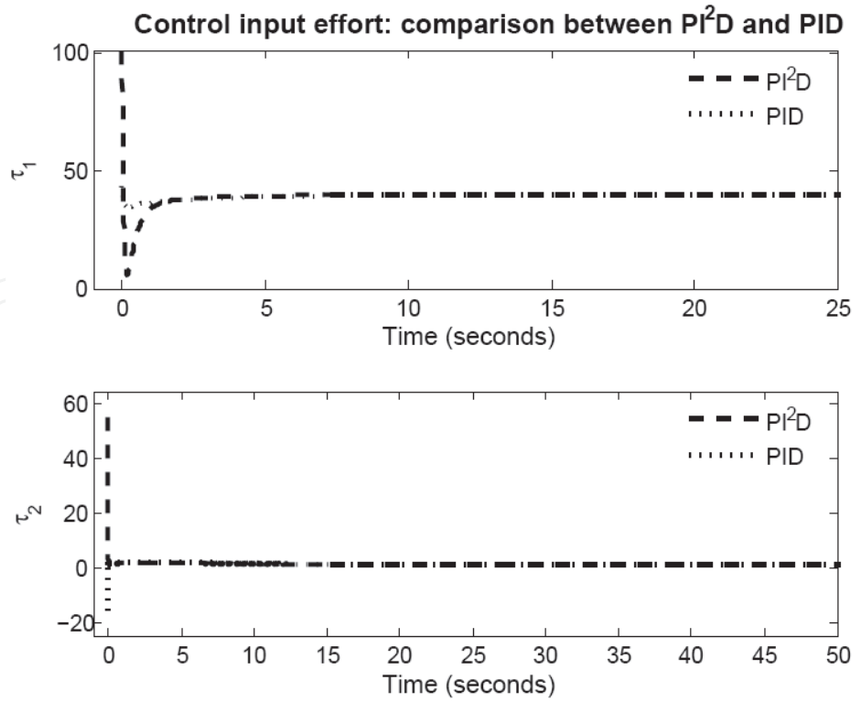

Fig. 7. Robot input torque for PI²D and PID controllers with perfect cancellation.

3.4 A 2 DOF planar robot at tracking

For the tracking case study a simple periodical signal given by $x_d(t) = \left[\sin\left(\pi t/40\right) \quad \sin\left(\pi t/20\right) \right]$ is tested. First perfect cancellation is considered, and then unperfected cancellation of the robot dynamics is taken into account. The control gains are the same as those listed at Table 1. Figures (11) to (13) show the system closed loop performance with perfect dynamic compensation, where the angular position, the regulation error and the control input, respectively, are depicted. The PI²D controller shows a better behaviour and faster response than the PID, both with dynamical compensation.

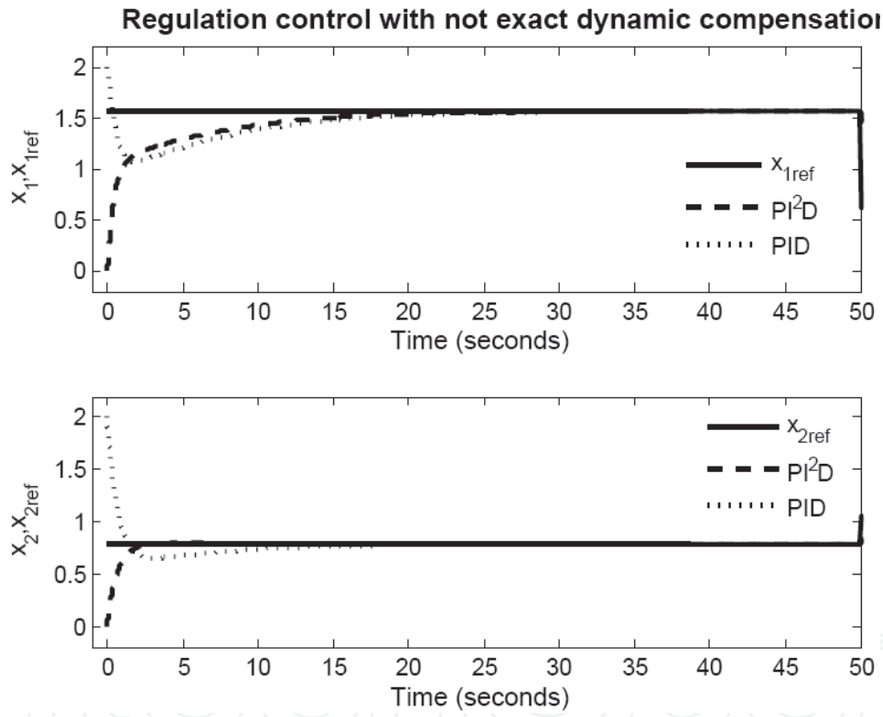

Fig. 8. Robot angular position for PI²D and PID controllers without perfect cancellation.

To test the proposed controller robustness against model and parameter uncertainty, it was considered imperfect dynamic compensation considering as in the regulation case a sign change in $g(x)$, and no compensation on $f(x)$. The control gains remained the same as for all previous cases. Figures (14) to (16) show the simulation results. Although the inexact compensation, the proposed PI²D controller behaves faster and with a smaller control effort than the PID control.

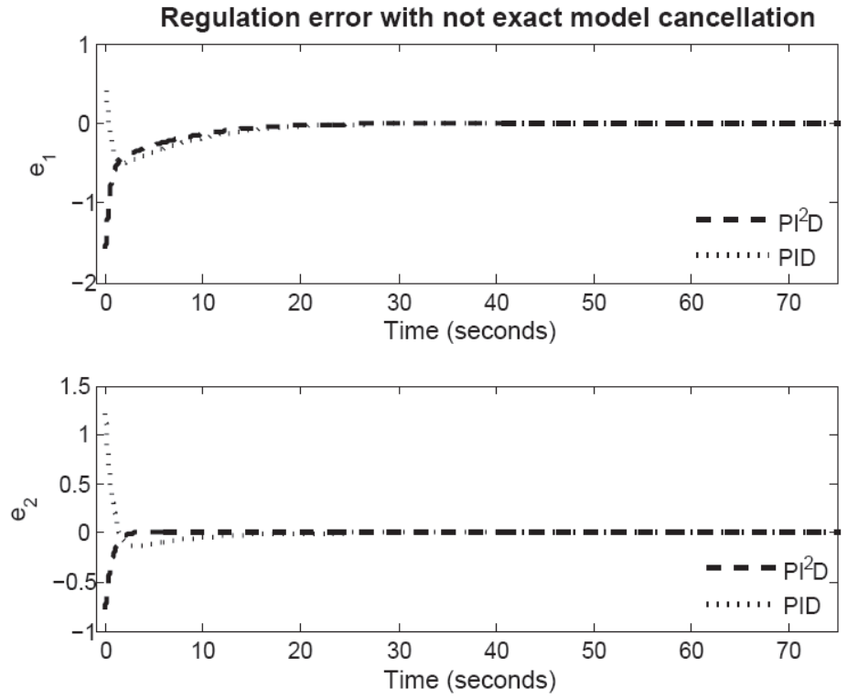

Fig. 9. Robot regulation error for PI²D and PID controllers without perfect cancellation.

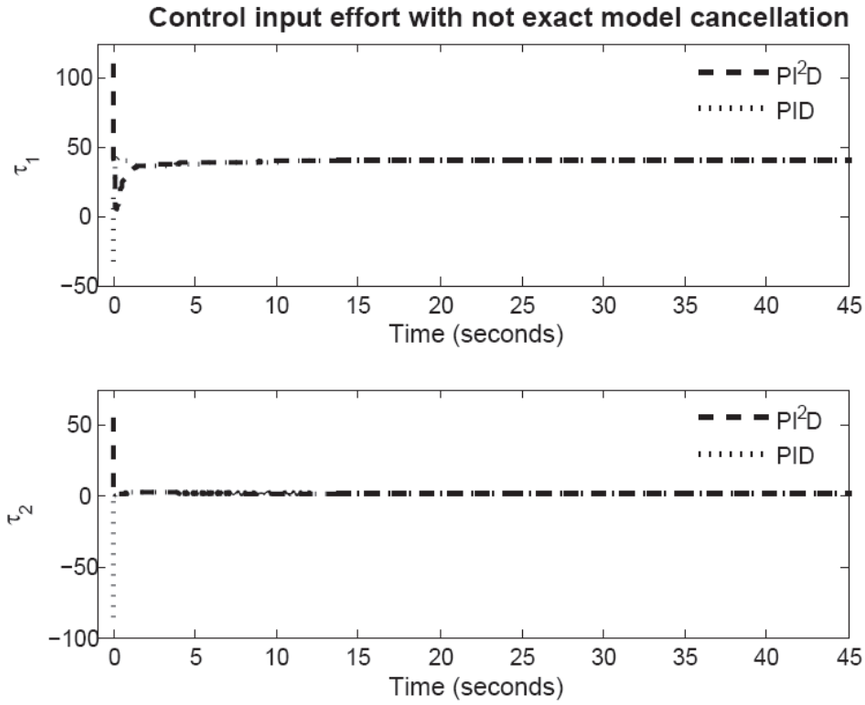

Fig. 10. Robot input torque for PI²D and PID controllers without perfect cancellation.

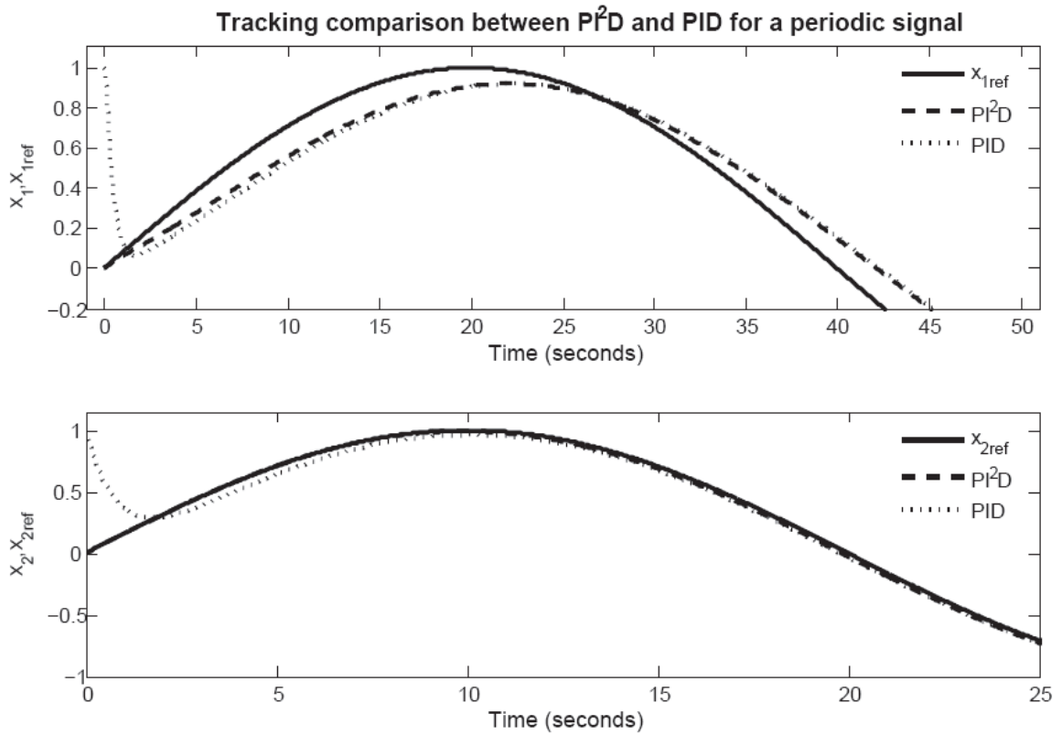

Fig. 11. Robot angular position for PI²D and PID controllers with perfect cancellation.

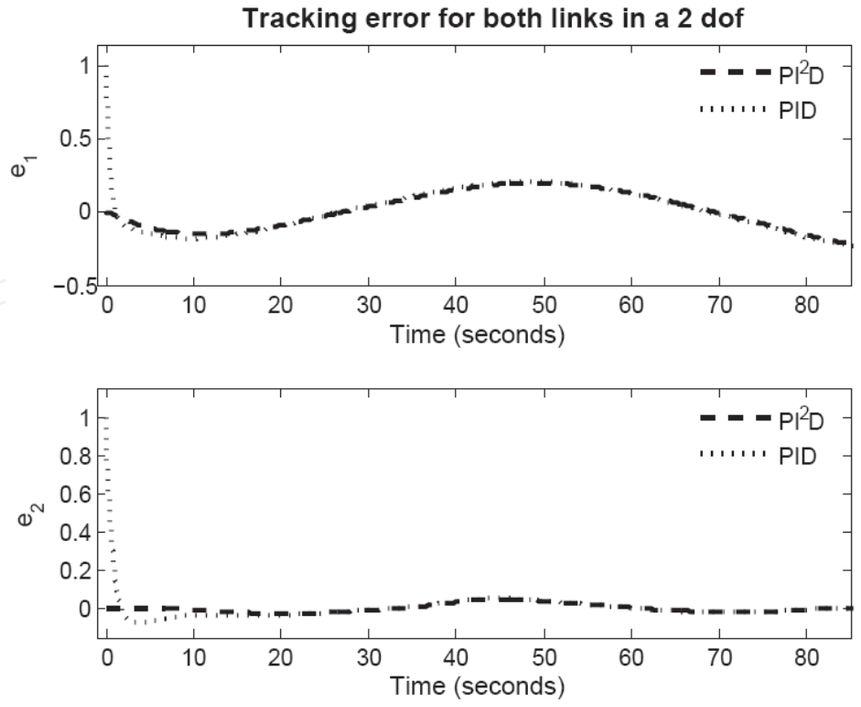

Fig. 12. Robot tracking error for PI²D and PID controllers with perfect cancellation.

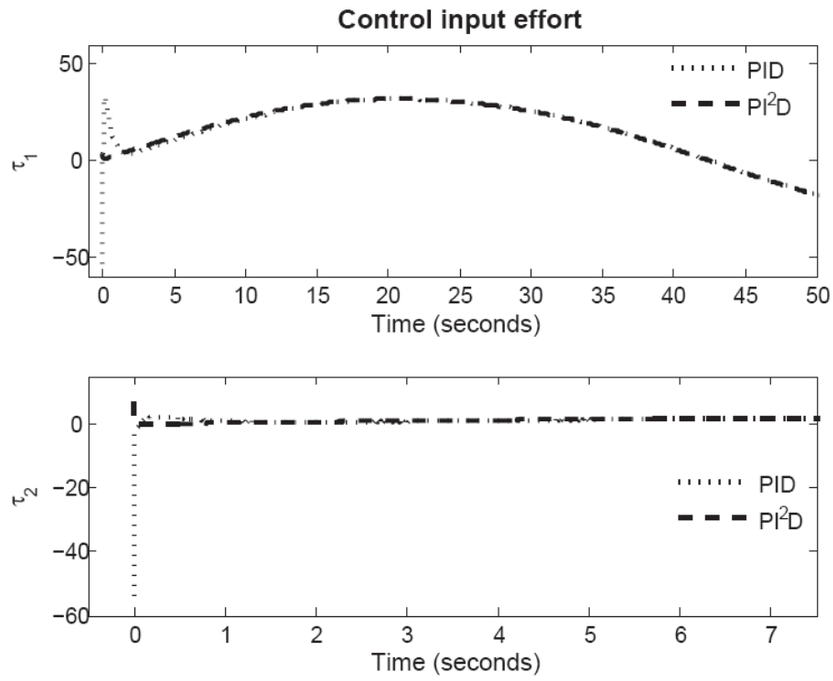

Fig. 13. Robot input torque for PI²D and PID controllers with perfect cancellation.

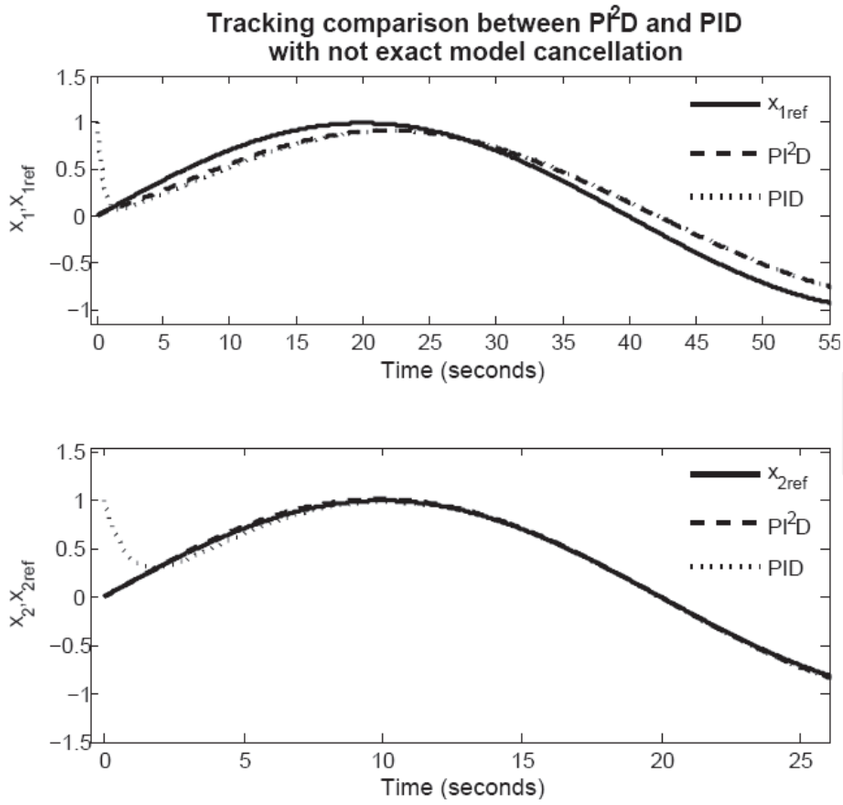

Fig. 14. Robot angular position for PI²D and PID controllers without perfect cancellation.

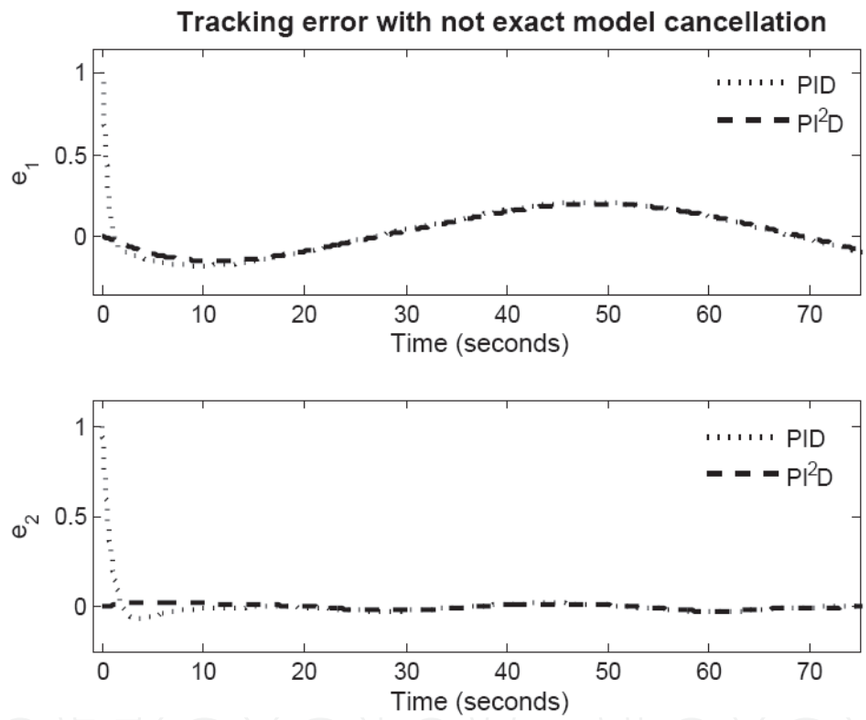

Fig. 15. Robot tracking error for PI²D and PID controllers without perfect cancellation.

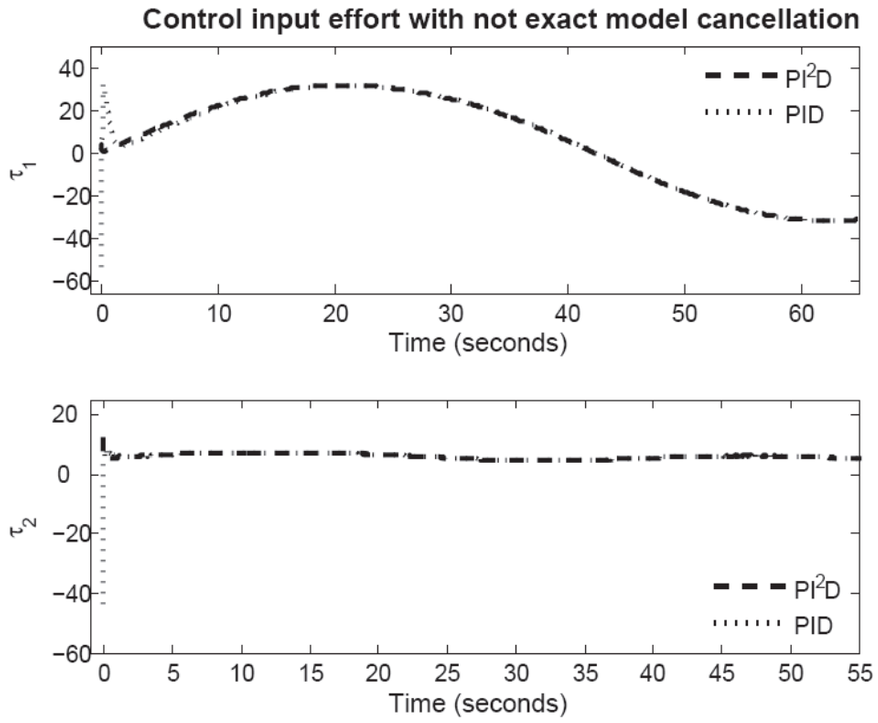

Fig. 16. Robot input torque for PI²D and PID controllers without perfect cancellation.

4. Conclusions

The proposed controller represents a version of the classical PID controller, where an extra feedback signal and integral term is added. The proposed PI²D controller shows better performance and convergence properties than the PID. The stability analysis yields easy and direct control gain tuning guidelines, which guarantee asymptotic convergence of the closed loop system.

As future work the proposed controller will be implemented at a real robot system, it is expected that the experimental results confirm the simulated ones, besides that it is well know that an integral action renders robustness against signal noise, by filtering it.

5. Acknowledgment

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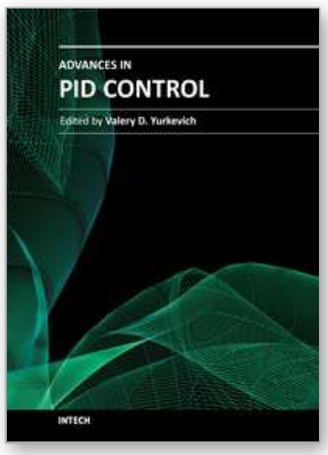

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Since the foundation and up to the current state-of-the-art in control engineering, the problems of PID control steadily attract great attention of numerous researchers and remain inexhaustible source of new ideas for process of control system design and industrial applications. PID control effectiveness is usually caused by the nature of dynamical processes, conditioned that the majority of the industrial dynamical processes are well described by simple dynamic model of the first or second order. The efficacy of PID controllers vastly falls in case of complicated dynamics, nonlinearities, and varying parameters of the plant. This gives a pulse to further researches in the field of PID control. Consequently, the problems of advanced PID control system design methodologies, rules of adaptive PID control, self-tuning procedures, and particularly robustness and transient performance for nonlinear systems, still remain as the areas of the lively interests for many scientists and researchers at the present time. The recent research results presented in this book provide new ideas for improved performance of PID control applications.

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