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On the Dilaton Stabilization by Matter

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1. Introduction

The Dilaton field is a scalar partner of the graviton in the context of superstring theory (1). Then, the background fields in the vacuum state of this theory should involve its component in common with the metric ones in the basic action (2; 3). To the simplest approximation the Dilaton is a massless scalar field showing a special sort of interaction with the matter modes. This type of coupling, determines that a time varying Dilaton induces time-dependent coupling constants. Therefore, to overcome this difficulty this field should remain constant at the current stage of cosmological evolution. In addition, unless it becomes very massive, its existence can imply an observable kind of "Fifth force", being similar to the ones which are currently associated to the observations of the Dark Matter. The constraints posed by current experimental observations determine the lower bound on the mass of the Dilaton to be of the order $m < 10^{-12}$GeV (4) . However, there are attempts to make a time dependent Dilaton consistent with late time cosmology (see (5)).

Therefore, the Dilaton stabilization problem has been the objective of an intense research activity in recent times due to its physical relevance. It can be emphasized that the Dilaton is one of various scalar fields emerging from the formulation of superstring theory in its low-energy limit. Scalar fields describing the sizes and shapes of the extra spatial dimensions associated in this theory are also arising, and are called "moduli fields". The stabilization of such moduli fields has also been the object of recent attention, specially in connection with Type IIB superstring theory. The introduction of fluxes within the compactification spaces has made it possible to stabilize various moduli fields (7). Moreover, gaugino condensation effects (8) has been argued to stabilize the Dilaton field in the framework of heterotic superstring theory (9) and also in string gas cosmology (10).

It can be underlined that, since Dilaton stabilization has special relevance for late time cosmology, there is motivation for finding mechanisms which do not directly rest on the concrete assumptions defining the nature of the extra dimensions. Further motivation to search for alternative Dilaton stabilization mechanisms appears in connection with String Gas Cosmology (SGC). The SGC (11; 12) is a model of early universe cosmology which employs new degrees of freedom and symmetries of string theory, and couples these elements with gravity and Dilaton fields into a classical action model. The Universe is assumed to start as a compact space filled with a gas of strings. Then, because in string theory there is a maximal temperature for a gas of closed strings, the cosmological evolution in SGC starts from a phase of almost constant temperature, called the "Hagedorn phase". The SGC allows to define a non-singular cosmology in which there is no initial Big Bang explosion. Also, it has been identified that the thermal fluctuations in a gas of closed strings in the Hagedorn phase gives

justification to the observed scale-invariant spectrum of cosmological fluctuations in Nature (13; 14), by adding a particular prediction of a slight blue tilt for gravitational waves (15). In this, the consistency of the picture also requires that the Dilaton field be stabilized during the Hagedorn phase. Therefore, in the SGC theory the Dilaton is also required to be fixed at very early times as well as at very late times.
In the present review chapter I will resume the conclusions of two studies previous done in common with various collaborators, in connection with the Dilaton vacuum field. They were presented in Refs. (32; 33). Each of these works assumes different properties for the Dilaton field as described below in the following two subsections:

1.1 a) Small mass Dilaton

In the discussion done in (32), which will be resumed in the section 2 of this chapter, the Dilaton field was assumed to show a small mass. Therefore a static solution of the KG equation for the Dilaton in interaction with gravity and dust matter was searched in that work. The configuration found showed a large region of homogeneity close to a central symmetry point, which becomes increasingly spatially varying at large distances. The existence of this static solution essentially rests on the presence of an interaction of the Dilaton field with pressureless matter. The solution obtained was a generalization of one formerly investigated in Ref. (18; 19) in the absence of matter. The special behavior of the scalar field in such works led to the proposal made in Ref. (18) about considering it as representing the Dilaton of the string theory (20). The idea came from the arising circumstance that when you fix the value of the scalar field (which have dimension of mass) at the central symmetry point to be at the Planck scale, by also requiring an amount of Hubble effect similar to the experimental one, the radius of existence of the solution gets a value $R = 10^{28} cm$ which is near the radius of the Universe. Also very much curious is that the values of KG mass of the scalar field obtained by fixing the above parameters, results to be of the order of $1/R$. That is, a very small value which seems compatible with a very tiny mass acquired by the Dilaton due to boundary conditions or non perturbative effects, which could deviate its mass from its vanishing first approximation.
It should be remarked that the assumption about the isotropic and homogeneous nature of our Universe, that is the Cosmological Principle, is central to modern Cosmology (16). However, recent experimental observations suggest the possibility for the break down of the validity of the principle at large scales (17). Accepting such a breaking will become necessary if the obtained solution result to be realized in Nature. Various static models of the Universe have been considered. Among them are the ones of Einstein, Le Maitre and de'Sitter, respectively. Originally, Einstein (16) examined a Universe filled of uniformly distributed matter but obtained a non-static metric. This result motivated him to introduce in his equations the Cosmological Constant term λ, with the objective of allowing the obtaining of a static solution. In connection with the solution discussed in (32) it follows that the centrally symmetric static scalar field which satisfies the Einstein-Klein-Gordon equations (EKG), curves the space time in a form resembling the one in the de'Sitter space in a large neighborhood of the origin of coordinates (19). The fact that the scalar field is more weakly varying along the radial distances when its value at the origin is lower is an interesting arising property to underline. The associated densities of energy and pressure are positive and negative respectively and weakly varying, approximating the presence of a positive Cosmological Constant. These properties suggested the idea proposed in (18) about considering the Dark Energy (DE) as described by a scalar field in this approximately homogeneous field configuration studied in (19). As mentioned before, this assumption will determine the abandoning of the

Cosmological Principle in favor of what could be imagined as a kind of "Matryoshka" model of the Universe. In this conception, proposed in (18; 19), we could be living inside of a particular configuration in which the Dilaton field has a definite value resulted from the collapse of string matter in fermionic states. Then, the idea comes to the mind about that the Dilaton field could be radiated by the string matter in fermionic states under the extreme conditions of the collapse of fermion matter. The effective realization of this picture in Nature, could lead to the possibility that the astrophysical black-holes (by example the ones which are expected to exist near the centers of the Galaxies) could be no other things that small Universes in which the Dilaton field gets a different value to the external one. This change could be produced again by the collapse of fermion matter in falling to the collapsed configuration, upon the possible radiation of zero angular momentum modes, that is of the Dilaton to furnish the variation of the internal Dilaton field. We find this picture as an interesting one and think that its exploration is worth considering. One point to note, is that the proposed collapsed structures would resemble the so called "gravastars" in Refs. (35; 36). At this point it might be helpful to underline that given that the above recurrent picture is realized in Nature, supports the interest of the ideas argued in Ref. (37), about the connections between the cosmological constant and the quantum behavior of matter in such internal universes.
An important outcome emerged in the examination of the problem, is that the coexistence of the scalar field as described by the EKG equations including also the dust energy momentum tensor does not allow the existence of static solutions, at least in centrally symmetric configurations, in the absence of Dilaton - matter interaction (26). However, when the interaction is allowed a solution appears. The introduction of the coupling does not damage the almost homogenous character of the solution in a relative large region around the origin of the central symmetry, being far away form the limits of the Universe. Another interesting outcome is that the distributions of matter and Dilaton field both show a very close behavior. That is, the scalar field is able to sustain an amount of matter being almost proportional between them.

1.2 a) Large mass stabilized by matter Dilaton

In Ref. (33), which results will be reviewed in section 3 of this chapter, in an alternative way as in Ref. (32), the possibility for the Dilaton to acquire an appreciable mass due to its generic interaction with the matter fields was investigated. In other words, the idea which motivated this study was the universal type of coupling of the Dilaton to the matter fields. This property, could not only lead to an unwanted effect as the mentioned time-dependence of the coupling constants, but it also can give the possibility that quantum effects due to the interaction of the Dilaton with matter, could generate interesting contributions to the Dilaton effective potential. This question was started to be explored in Ref. (28). That work considered the cosmological periods when the additional spatial dimensions of superstring theory were already stabilized and the study was done in the framework of a four-dimensional field theory. The main objective of study was then the interaction of the Dilaton with massive fermions. These masses can be defined by fluxes through internal manifolds. Also, in late time cosmology, the masses could had been generated after supersymmetry breaking. In an alternative early universe cosmology, one may also take into account fermion masses generated by thermal effects. Ref. (28) considered a simple form for the Dilaton gravity action in which a massive Dirac fermion term was included (29). The Einstein frame, was chosen which does not show Dilaton field dependence in the kinetic terms for the fermions. Alternatively, the fermion mass becomes a function of the Dilaton through an universal exponential factor in Dilaton gravity (2; 3). The

chosen action described the low energy effective interaction of Super-Yang-Mills fermions with the Dilaton field in superstring theory (28). The effective potential for the Dilaton field was evaluated up to two loop corrections in the small Dilaton radiative quantum field limit. That leads to a Yukawa like interaction term which allows standard QFT calculations. A fixed value of the cosmological scale factor was assumed. The outcome of the work was, thanks to the appearing of logarithms in the loop calculations, that the Dilaton field appeared in the result in powers multiplied by the exponential factors of the field. This structure, in the one loop approximation clearly indicated the spontaneous generation of vacuum mean value of the Dilaton field.

Motivated by the dynamical generation of the Dilaton result in Ref. (28), in Ref. (33) we addressed the evaluation of next corrections 3-loop terms to the 2-loop evaluation of the effective potential for the Dilaton field. The main issue explored in this work was the possibility of the appearance in the improved potential of stabilizing effects which were in fact absent in the second order correction, and which are suspected to be created by the existence of massive matter upon the mean value of the Dilaton.

The results obtained indicated, for the fermion mass being selected at the GUT or the top quark mass scales, that the mean value of the Dilaton field tends to be stabilized at a high value being close to the Planck mass or the GUT scale, respectively. Therefore, it was suggested that the appearance of mass for matter in the course of the evolution of the Universe can generate a stabilizing action on the vacuum expectation value of the Dilaton field making it unobservable. This effect will tend to stop the time evolution of the mean value, as it is convenient for String Theory consistency.

It should be remarked that in in Ref. (33), in the process of extending the work to include higher loop corrections, we have noticed that in Ref. (28), the kinetic term of the Dilaton Lagrangian was chosen with a negative sign. This selection, although not changing the one loop correction, led to a sign modification of the 2-loop terms, which suggested the existence of minima in the effective action argued in Ref. (28). However, in spite of this non physical adopted assumption, the indication about the dynamical generation in Ref.(28) remained a valid one, because the change in the metric did not affected the one-loop correction, the basic quantity indicating the dynamical generation effect. The results of the work in Ref. (33) and reviewed in this chapter, corrected the evaluation of the two loop term, and indicated that its place in the stabilizing effect over the Dilaton field is played by higher order contributions.

The exposition of section 3 will proceed as follows: In subection 3.1, the notation and basic formulation are given. Subsection 3.2 presents the elements of the one, two and three loops evaluation of the effective potential. Subsection 3.3 discuss the results of the evaluation. In the concluding subsection 3.4 the conclusions of the work are resumed and commented.

2. A cosmological model with a nearly massless Dilaton field

As remarked in the introduction this section 2 will resume the discussion of the work (32) in which the Dilaton field was assumed as a scalar field obeying the Einstein-Klein-Gordon equations in which the mass is assumed to be small. It should be underlined that this previous assumption resulted in radical contrast with the outcome of the later work reported in (33), which will be also reviewed in this chapter. However, the appearance of a large mass suggested by the discussion done in (33), as it will commented at the last section of the chapter devoted to the conclusions, could not result to be excluding some of the most motivating suggestions advanced in Ref. (32).

Given the isotropic and stationary character of the solution which was searched in Ref. (32), the structure of the metric was proposed in the standard form

$$ds^2 = v(\rho)dx^{o2} - u(\rho)^{-1}d\rho^2 - \rho^2(sin^2\theta d\varphi^2 + d\theta^2),$$
$$x^o = ct, \quad x^1 = \rho, \tag{1}$$
$$x^2 = \varphi, \;\; x^3 \equiv \theta, \tag{2}$$

from which the components of the Einstein tensor $G_{\mu\nu}$ were computed. Since the metric tensor is diagonal and only depending on ρ, the only non vanishing components of $G_{\mu\nu}$ resulted in

$$G_0^0 = \frac{u'}{\rho} - \frac{1-u}{\rho^2},$$
$$G_1^1 = \frac{u}{v}\frac{v'}{\rho} - \frac{1-u}{\rho^2},$$
$$G_2^2 = G_3^3 = \frac{u}{2v}v'' + \frac{uv'}{4v}(\frac{u'}{u} - \frac{v'}{v}) + \frac{u}{2\rho}(\frac{u'}{u} + \frac{v'}{v}).$$

The components G_2^2 and G_3^3 generated second order equations in the temporal component of the metric, which explicitly did not played an important role thanks to the Bianchi identities (16)

$$G^{\nu}_{\mu;\nu} = 0, \tag{3}$$

which were employed in the discussion. Assumed the satisfaction of the Einstein equations the G_μ^ν tensor was substituted by the energy momentum tensor T_μ^ν. Equation (3) was interpreted as a set of dynamical equations for the variables of the problem e, p and ϕ.

2.1 Matter and Dilaton dark energy

In this subsection let us sketch the way followed in (32) for obtaining two of the necessary equations needed to show the existence of the mentioned static model for the Universe: the Bianchi relations (3) and the static equation for the scalar field coupled to matter.
The action for the scalar field-matter in the given space time was written in the form

$$S_{mat-\phi} = \int L\sqrt{-g}d^4x, \tag{4}$$

where g is the determinant of the metric tensor, and it was considered that the Lagrangian density was taking the form:

$$L = \frac{1}{2}(g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + m^2\phi^2) + j\phi + L_{e,p}. \tag{5}$$

The first and the third terms of the right member of (5) are the Lagrangian densities of the KG scalar field and the dust-like matter respectively, while the second term was an interaction term between both quantities which was assumed to exist. The strength of the interaction was represented by the constant source j. Note that, the existing coupling of the Dilaton to matter fields made this supposition a natural one in our case in which the scalar field was considered as representing the Dilaton.
As it was previously mentioned we assumed for the matter, the perfect fluid expression (16):

$$(T_{e,p})^\nu_\mu = p\,\delta^\nu_\mu + u^\nu u_\mu(p+e), \tag{6}$$

where p was the pressure of the matter. Note that in the work it was assumed a pressureless matter $p = 0$. However, for bookkeeping purposes, it was employed the expression for a general pressure p up to the end when the limit $p = 0$ was fixed.
As usual u^ν denoted the contra-variant components of the 4-velocity of the fluid in the system of reference under consideration. In addition since the search for static configurations was undertaken, the 4-velocity took the simple form $u^\nu = \delta^\nu_0$.
From the Lagrangian L in (5) and the above remarks the energy momentum tensor of the scalar field coupled with the matter got the form

$$T^\nu_\mu = -\frac{\delta^\nu_\mu}{2}(g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + m^2\phi^2 + 2j\,\phi) + g^{\alpha\nu}\phi_{,\alpha}\phi_{,\mu} + p\delta^\nu_\mu + \delta^\nu_0\delta^0_\mu(p+e). \tag{7}$$

From equation (7), the Bianchi relation for $\mu = 1$ in (3) transformed in

$$-\phi j' + p' + \frac{v'}{2v}(p+e) = 0.$$

In case under consideration this is the only one of the four Bianchi relations which became different from zero.
The dynamical equation for the scalar field determining the extremum of the action $S_{mat-\phi}$, resulted in the form

$$\begin{aligned}\frac{\delta S_{mat-\phi}}{\delta\phi} &\equiv \frac{d}{dx^\mu}\frac{\partial L}{\partial\phi_{,\mu}} - \frac{\partial L}{\partial\phi} \\ &\equiv \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}g^{\mu\nu}\phi_{,\nu}) - m^2\phi - j \\ &= 0,\end{aligned} \tag{8}$$

which after introducing the components of the metric tensor was simplified to become

$$u\phi'' + u\phi'(\frac{1}{2}\frac{v'}{v} + \frac{1}{2}\frac{u'}{u} + \frac{2}{\rho}) - m^2\phi - j = 0.$$

Note that if $u = v = 1$, that is, in Minkowski space, relation (9) reduces to the static KG equation for scalar field interacting with an external source j. It might be helpful to notice that natural units

$$[e] = [p] = cm^{-4}, [m] = cm^{-1}, [\phi] = cm^{-1},$$

were employed.

2.2 Einstein equations

The extremum of the action $S_{mat-\phi}$with respect to the metric led to the Einstein equations in the absence of a Cosmological Constant

$$G^\nu_\mu = G\,T^\nu_\mu, \tag{9}$$

where in natural units $G = 8\pi \times l_p^2$ and $l_p = 1.61 \times 10^{-33} cm$ is the Planck length.

From relation (7), the Einstein equations (9) were expressed in the form

$$\frac{u'}{\rho} - \frac{1-u}{\rho^2} = -G[\frac{1}{2}(u\phi_{,\rho}^2 + m^2\phi^2 + 2j\phi) + e], \tag{10}$$

$$\frac{u}{v}\frac{v'}{\rho} - \frac{1-u}{\rho^2} = G[\frac{1}{2}(u\phi_{,\rho}^2 - m^2\phi^2 - 2j\phi) + p\,]. \tag{11}$$

As it was mentioned above, the third Einstein equation was not needed for determining the solution, because its satisfaction was implied by the other equations. This expression only imposed the continuity of the derivative of v with respect to the radial variable since it is a second order differential equation.
It was assumed that j , which gives the form of the interaction term between the dark energy and matter is of the form:

$$j = g\sqrt{e},$$

where g is a coupling constant for the interaction matter-scalar field. In the natural system of units $[g] = cm^{-1}$.
With the aim of working with dimensionless forms of the equations (10) and (11), we defined the new variables and parameters

$$r \equiv m\rho, \quad \Phi \equiv \sqrt{8\pi}l_p\phi,$$
$$J \equiv \frac{\sqrt{8\pi}l_p}{m^2}j, \;\; \epsilon \equiv \frac{8\pi l_p^2}{m^2}e, \quad \gamma \equiv \frac{g}{m}.$$

Also, it was fixed the small mass of the Dilaton field to the value estimated in Ref. (18) for assuring the observed strength of the Hubble effect in the regions near the origin. Interestingly, this value resulted in the very small quantity, $m = 4 \times 10^{-29} cm^{-1}$. This mass is compatible with the zero mass Dilaton in the lowest approximation. In addition the mass was of the order of the inverse of the estimated radius of the Universe, as it was observed in Ref. (18).
Therefore, the set of working equations resulted in the form

$$\frac{u_{,r}}{r} - \frac{1-u}{r^2} = -\frac{1}{2}(u{\Phi_{,r}}^2 + \Phi^2) - J\Phi - \epsilon, \tag{12}$$

$$\frac{u}{v}\frac{v_{,r}}{r} - \frac{1-u}{r^2} = -\frac{1}{2}(-u{\Phi_{,r}}^2 + \Phi^2 + 2J\Phi), \tag{13}$$

$$\epsilon\frac{v_{,r}}{2v} - \Phi J_{,r} = 0, \tag{14}$$

$$u\Phi_{,rr} - \Phi - J = -u\Phi_{,r}(\frac{1}{2}\frac{v_{,r}}{v} + \frac{1}{2}\frac{u_{,r}}{u} + \frac{2}{r}). \tag{15}$$

2.3 The solutions near the center of symmetry

In Ref. (32) it was searched for smooth solutions around the origin. Thus, the continuity of the derivatives v and ϕ, in all places including the origin, was required. Then, after considering

the equations in a neighborhood of the origin, the asymptotic field values were written in the form

$$u = 1 + u_1 r^2 ...,$$
$$v = 1 + v_1 r^2 ...,$$
$$\Phi = \Phi_0 + \Phi_1 r^2 ...,$$
$$\epsilon = \epsilon_0 + \epsilon_1 r^2 ...,$$

where u_1, v_1, Φ_1, ϵ_1 after substitution of the asymptotic solution in the equations were determined in the form

$$u_1 = -\frac{1}{3}(\frac{\Phi_0^2}{2} + J_0\Phi_0 + \epsilon_0), \tag{16}$$

$$v_1 = -\frac{1}{3}(\frac{\Phi_0^2}{2} + J_0\Phi_0 - \frac{\epsilon_0}{2}), \tag{17}$$

$$\Phi_1 = -\frac{1}{6}(\Phi_0 + J_0), \tag{18}$$

$$\epsilon_1 = -\frac{\epsilon_0^{\frac{3}{2}}}{3\gamma\Phi_0}(\frac{\Phi_0^2}{2} + J_0\Phi_0 - \frac{\epsilon_0}{2}), \tag{19}$$

$$J_0 = \gamma\epsilon_0^{\frac{1}{2}}.$$

Note that the spacial dependence of the metric tends to have an homogeneous structure near the center of symmetry. The quantities Φ_0, ϵ_0 and the dimensionless coupling constant γ remained as free parameters. Extensions of this work, could be considered to optimize the parameters, aiming to compare the predictions of the model with redshift vs. stelar magnitude in the supernovae obervations. In the next subsection we resume the study done about the behavior of the solution at all radial distances for given physically motivated values of the parameters.

2.4 The solutions at a arbitrary radial values

The numerical solutions of the equations (12)-(15) were considered, by selecting the parameter values $\gamma = -0.75$, $\Phi_0 = 2.2$ and $\epsilon_0 = 1$. These specific choosing corresponded to a coupling constant $g = 2.9 \times 10^{-29} cm^{-1}$, a value of the scalar field at the origin $\phi_0 = 2.7 \times 10^{32} cm^{-1}$ (that is, laying at the Planck scale) and a matter energy density of $e = 2.3 \times 10^7 cm^{-4}$. The determined numerical solutions of the equations (12)-(15) are illustrated in the figures (1)-(4).

These parameters were a priori selected with the aim of fixing the estimated value of $0.7/0.3$ for the ratio of the Dark Energy to the matter energy content in the Universe (27) and an approximate value of the Hubble effect.
From Fig. (1) the global similarity between the space-time being examined and the de Sitter static solution can be observed. Moreover, due to the chosen value of the Dilaton mass suggested in Ref. (18), the size of the Universe (defined as the radial distance at which the singularity of the structure appears) is of the order of the estimated value $10^{29} cm$. In Fig.(2) the dependence of the temporal metric is shown, it evidenced that the observer near the origin measures a redshift which was imposed to show a value being near to the one currently observed.

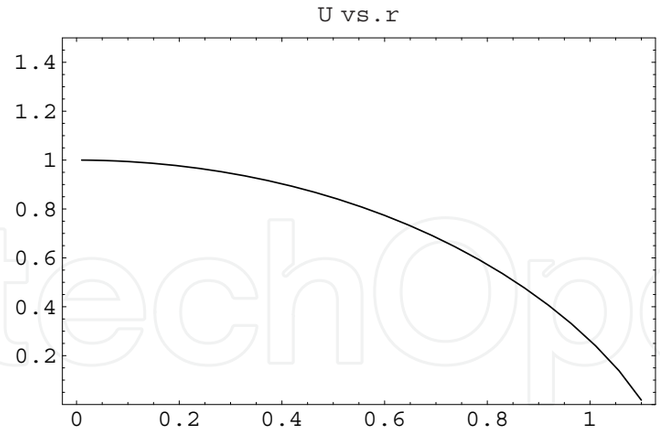

Fig. 1. The radial contraviant component of the metric $g^{11} \equiv u(r)$ behaved basically as in the deSitter Universe having the size $R \equiv 0.25 \times 10^{29}$cm.

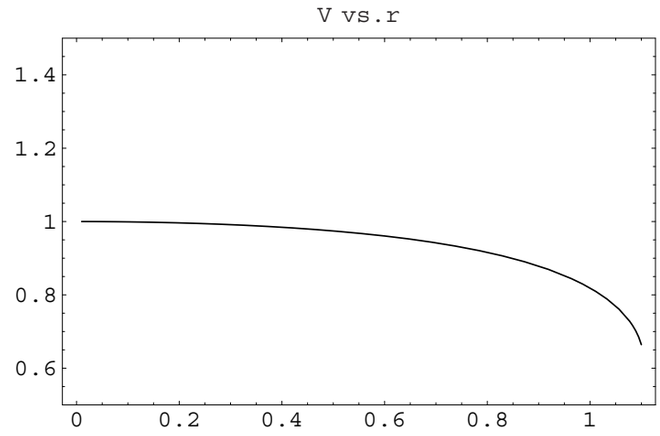

Fig. 2. Temporal component of the metric $g_{00} \equiv$v(r). Its decreasing behavior shows the redshift of the light arriving to the central zone regions from the outside regions. The radius of the singularity at the far away regions is $R \equiv 0.25 \times 10^{29}$cm.

Figures (3) and (4) illustrate the obtained distribution of energy and scalar field respectively. Note the similarity between both quantities. That is, the presence of the Dilaton-Matter coupling not only allowed the existence of the static solution, but in addition it also produced a configuration in which the proportion of matter and dark energy became approximately constant over large regions of the space time.

3. Large mass Dilaton stabilization by matter

As it had been mentioned in the Introduction, this section will review the results of the work presented in Ref. (33) . In this study it was investigated the possibility that the Dilaton could be stabilized at large values and masses as a direct consequence its universal type of interaction with matter. The review will be ordered as follows: In subsection 3.1, the notation and basic formulation employed in Ref. (33) will be given. Subsection 3.2 will review the

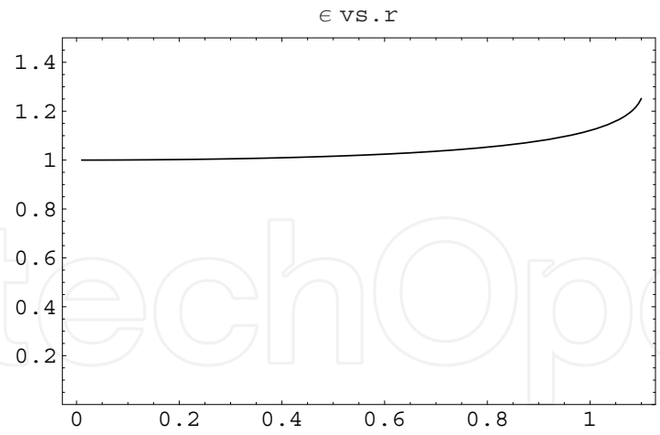

Fig. 3. The matter distribution $e(r)$ resulted as slowly varying with the radial distance. The coupling between the scalar field and the matter $J\,\Phi$ was central in allowing the existence of the static solution, in which also the matter to Dark energy content ratio resulted a slowly varying. The radial singularity defining the end of the space time at $R = 0.25 \times 10^{29}$cm.

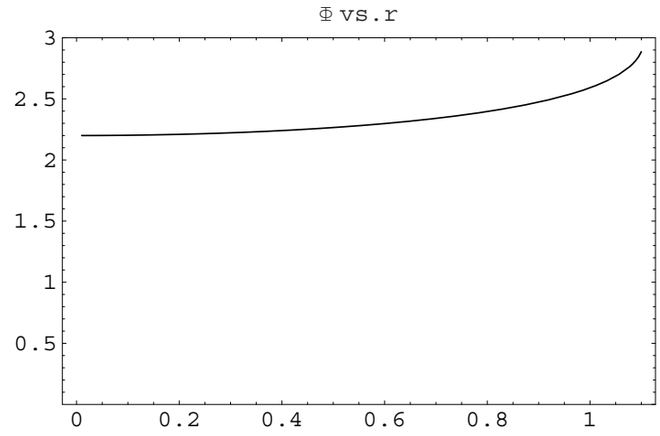

Fig. 4. The scalar field slowly varied with the radial component and behaved very closely with the matter density $e(r)$; The radial singularity defining the end of the space time is at $R = 0.25 \times 10^{29}$cm. There is no static metric with Dilaton and matter in coexistence without interaction.

elements of the one, two and three loops evaluation of the effective potential. The subsection 3.3 discuss the results of the evaluations done.

3.1 The Dilaton action and generating functional

In Ref. (33) it was considered a model of the Dilaton field interacting with fermion matter in the form

$$S = \int d^4x\sqrt{-g(x)}\Big(\frac{1}{2\kappa^2}g^{\mu\nu}(x)\partial_\mu\phi^r(x)\partial_\nu\phi^r(x) + \overline{\Psi}(x)\Big(i\frac{g^{\mu\nu}\gamma_\mu\overleftrightarrow{\partial}_\nu}{2} - m\Big)\Psi(x)$$
$$-\overline{\Psi}(x)g_Y^*\,\phi^r(x)\Psi(x) + j(x)\phi^r(x) + \overline{\Psi}(x)\eta(x) \;+\overline{\eta}(x)\Psi(x)\Big), \tag{20}$$

$$m = \exp(\alpha^* \phi) m_f, \tag{21}$$

$$g_Y^* = \alpha^* m, \tag{22}$$

$$\alpha^* = -\frac{3}{4}, \tag{23}$$

$$x^\mu = (x^0, x^1, x^2, x^3), \quad \overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}, \quad \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}(x), \tag{24}$$

$$g_{\mu\nu}(x) = \begin{pmatrix} 1\,0 & 0 & 0 \\ 0\,-1 & 0 & 0 \\ 0\,0 & -1 & 0 \\ 0\,0 & 0 & -1 \end{pmatrix}, \quad \sqrt{-g(x)} = 1. \tag{25}$$

That is, we considered the Dilaton field interacting with a massive fermion in the Einstein frame, in which the metric $g_{\mu\nu}$ was approximated by the Minkowski one in order to simplify the evaluations. The gravitational constant was explicitly introduced, and natural units were employed for the distances and mass. The vacuum value of the Dilaton field is named as ϕ and its radiative part is called ϕ^r. Note that is was assumed that the radiative part is small in order to retain only the first term in the expansion of the exponential. This was the Yukawa approximation which was employed in Ref. (33). All the results are functions of the vacuum field ϕ.

The parameter defining the Dilaton field dependent exponential, the Planck length $\kappa = l_P$ and mass m_P were defined by the expressions

$$\kappa^2 = \frac{8\pi G\hbar}{c^3}, \tag{26}$$

$$\kappa = l_P = \frac{1}{m_P} = 8.10009 \times 10^{-33} \text{ cm}, \tag{27}$$

$$G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}, \tag{28}$$

$$\hbar = 1.05457 \times 10^{-27} \text{ cm}^2 \text{ g s}^{-1}, \tag{29}$$

$$c = 2.9979245800 \times 10^{10} \text{ cm s}^{-1}. \tag{30}$$

In the above formula for the action, the coordinates and times are measured in cm, the masses m in the natural unit cm^{-1} and the Dilaton field is dimensionless.

Starting from the classical action, the work considered corrections up to 3-loops for the effective action, assuming a homogenous and time independent value of the Dilaton mean field ϕ as

$$\frac{\Gamma[\phi]}{V^{(4)}} = -V^{eff}(\phi), \tag{31}$$

where $V^{(4)}$ is the four dimensional volume. In order to eliminate the explicit appearance of the gravitational constant from the diagram technique for evaluating the effective action, its appearance was eliminated from the equations by redefining the Dilaton field value, the α^* constant and the coupling as

$$\varphi = \phi/\kappa, \tag{32}$$

$$\alpha = \alpha^* \kappa = -\frac{3}{4}\kappa, \tag{33}$$

$$g_Y = g_Y^* \kappa. \tag{34}$$

After these changes, the above written classical action S, to be used for generating the Feynman expansion, was expressed as follows

$$S\,[\overline{\Psi},\Psi,\varphi^r,\varphi] = \int d^4x\,(\frac{1}{2}g^{\mu\nu}(x)\partial_\mu\varphi^r(x)\partial_\nu\varphi^r(x) + \overline{\Psi}(x)(i\frac{g^{\mu\nu}\gamma_\mu\overleftrightarrow{\partial}_\nu}{2} - m)\Psi(x)$$
$$-\overline{\Psi}(x)g_Y\,\varphi^r(x)\Psi(x) + j(x)(\,\varphi+\varphi^r(x)) + \overline{\Psi}(x)\,\eta(x)\;+\overline{\eta}(x)\,\Psi(x)). \quad (35)$$

The expansion was considered in $d = 4-2\epsilon$ dimensions for implementing dimensional regularization scheme. Accordingly, the coupling constant g_Y was modified by the introduction of the regularization scale parameter μ as follows

$$g_Y^2 = \mu^{2\epsilon}(g_Y^0)^2,$$

where g_Y^0 is the usual coupling constant in four dimensions.

3.1.1 The generating functional and the effective action

In this subsection, we will sketch the main expressions defining the perturbative calculation which was considered in Ref. (33). The generating functional of the Green functions Z , its connected part W and the mean field values were defined by the formulae

$$Z[\overline{\eta},\eta,j] = \int \mathcal{D}\overline{\Psi}\mathcal{D}\Psi\mathcal{D}\varphi^r\exp(i\,S\,[\overline{\Psi},\Psi,\varphi^r,\varphi]), \quad (36)$$

$$W[\overline{\eta},\eta,j] = \frac{1}{i}\log Z[\overline{\eta},\eta,j], \quad (37)$$

$$\frac{\delta\,W}{i\,\delta j(x)} = \varphi + \langle\varphi^r(x)\rangle, \quad (38)$$

$$\frac{\delta\,W}{i\,\delta\overline{\eta}(x)} = \langle\Psi(x)\rangle, \quad (39)$$

$$\frac{\delta\,W}{-i\,\delta\eta(x)} = \langle\overline{\Psi}(x)\rangle. \quad (40)$$

Note that the mean Dilaton field φ was considered as homogeneous and the mean value of the radiative part $\langle\varphi^r(x)\rangle$ was assumed to vanish when the sources are zero. The effective action was defined as the Legendre transform of Z depending on the mean field values as:

$$\Gamma[\langle\Psi\rangle,\langle\overline{\Psi}\rangle,\varphi+\langle\varphi^r\rangle] = \frac{1}{i}\log Z[\overline{\eta},\eta,j] - \int dx[j(x)(\,\varphi+\langle\varphi^r(x)\rangle) +$$
$$\langle\overline{\Psi}(x)\rangle\,\eta(x)\;+\overline{\eta}(x)\,\langle\Psi(x)\rangle\,], \quad (41)$$

$$\frac{\delta\,\Gamma}{\delta\langle\varphi^r(x)\rangle} = -j(x), \quad (42)$$

$$\frac{\delta\,\Gamma}{\delta\langle\overline{\Psi}(x)\rangle} = -\eta(x), \quad (43)$$

$$\frac{\delta\,\Gamma}{\delta\langle\Psi(x)\rangle} = \overline{\eta}(x). \quad (44)$$

The expression for Z, after writing the Yukawa vertex part of the Lagrangian in terms of the functional derivatives over the sources and integrating the gaussian functional integral that

remains, led to the Wick expansion formula:

$$Z[\overline{\eta},\eta,j] = \exp\,[i\int dx\; g_Y \frac{\delta}{i\delta j(x)}\frac{\delta}{-i\delta\eta(x)}\frac{\delta}{i\delta\overline{\eta}(x)}]\times$$
$$\exp\left[\int dx\,dy\;(\overline{\eta}(x)S(x-y)\eta(y)+\frac{1}{2}j(x)D(x-y)j(y))\right], \tag{45}$$

$$S(x-y) = \int \frac{dp^d}{(2\pi)^d}\frac{\exp(-i\;p.(x-y))}{m-\gamma^\mu p_\mu}, \tag{46}$$

$$D(x-y) = \int \frac{dk^d}{(2\pi)^d}\frac{\exp(-i\;k.(x-y))}{-(k^2-i\epsilon)}, \tag{47}$$

in which S and D are the fermion and Dilaton free propagators, respectively. The notation for fermions and scalar field related quantities, and the definition of the Feynman rules for the generation of the analytic expressions for the various contributions, were exactly the ones described in Ref. (30), for the cases of scalar and fermion fields. Specifically, for the momentum space rules, the propagators and the only existing vertex are graphically illustrated in figure 5.

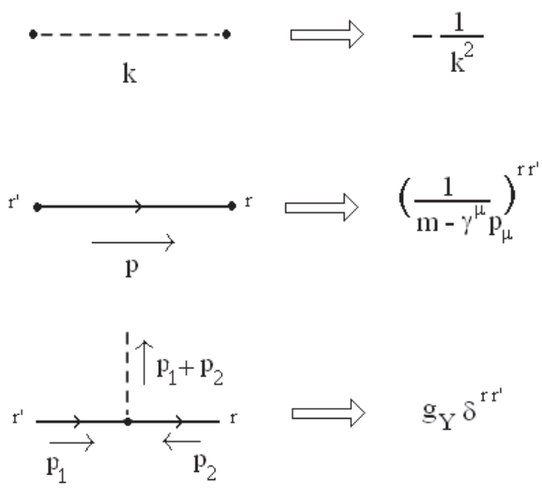

Fig. 5. The figure illustrates the Feynman rules for the particular Yukawa model approximation adopted for the Dilaton action in Ref. (33)

3.2 Effective potential evaluation

Let us resume in this section the evaluations of the effective potential for the Dilaton field done in Ref. (33). They followed after employing the perturbative expansion described in the past section. The diagrams which were considered are depicted in Fig. 6. They included up to three loops corrections. The contributions were exactly evaluated for the one and two loops terms. In addition, the three loop term D_{32} also was analytically calculated in terms of Master integrals. However, the three loop diagrams D_{31} and D_{33} were determined only in their leading terms of order $\log\left(\frac{m}{\mu}\right)^3$. We expect to be able in evaluating the non leading

corrections (lower powers of $log\left(\frac{m}{\mu}\right)$) in extending the work done in Ref.(33). The results for each diagram are reviewed in various subsections below.

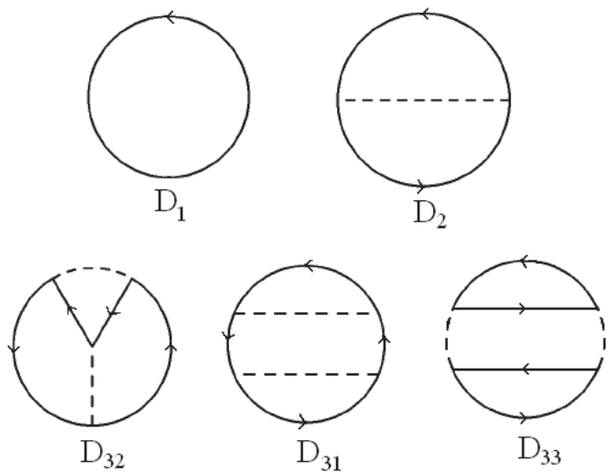

Fig. 6. The one, two and three loops Feynman diagrams considered in Ref. (33) . The one and two loop corrections D_1 and D_2 were exactly calculated. In the case of the three loops terms, the D_{32} was completely evaluated in terms of the listed Master integrals in Ref. (31). The D_{31} and D_{33} were determined only in their leading logarithm correction.

3.2.1 One loop term D_1

The analytic expression for the one loop diagram D_1 and its derivative over m^2 had the forms

$$\Gamma^{(1)} = V^{(d)} \int \frac{dp^d}{(2\pi)^d i} Tr \log(m^2 - p^2), \tag{48}$$

$$\frac{d}{d\, m^2}\Gamma^{(1)} = 4V^{(d)} \int \frac{dp^d}{(2\pi)^d i} \frac{1}{m^2 - p^2}. \tag{49}$$

The result for the momentum integral entering in the derivate of $\Gamma^{(1)}$ over m^2, after divided by $\mu^{2\epsilon}\ V^{(d)}$ (in order to define a 4-dimensional energy density) and integrated over m^2, allowed to write for the one loop effective action density the expression (See Ref. (31))

$$\gamma_1(m,\epsilon,\mu) \equiv \frac{\Gamma^{(1)}}{\mu^{2\epsilon} V^{(d)}} = m^4 (\frac{m}{\mu})^{-2\epsilon} \frac{8\pi^{2-\epsilon}}{(2\pi)^{4-2\epsilon}} \Gamma\left(-1+\epsilon\right). \tag{50}$$

After employing the minimal substraction (MS) scheme, that is, getting the finite part by eliminating the pure pole part in ϵ the Laurent expansion of $\gamma(m,\epsilon)$ and taking the limit $\epsilon \to 0$, the one loop contribution to the effective action density as a function of m and μ becomes written in the form

$$\gamma_1(m,\mu) = 0.0506606 m^4 \left(2.\log\left(\frac{m}{\mu}\right) - 2.95381\right). \tag{51}$$

Note that the negative of this term, which defines the one loop effective potential led to a the dynamical generation of the Dilaton field for positive values of $\alpha^* \phi$ as follows from $log(m) = log(m_f) + \alpha^* \phi$. This was the effect which motivated the study started in Ref. (28).

3.2.2 Two loop term D_2

For the two loop contribution D_2 the analytic expression was

$$\begin{aligned}\gamma_2(m,\epsilon,\mu) &\equiv \frac{\Gamma^{(2)}}{\mu^{2\epsilon}V^{(d)}} = \frac{1}{2}(g_Y^0)^2 \int \frac{dp_1^d}{(2\pi)^d i}\frac{dp_2^d}{(2\pi)^d i}\frac{4(m^2+p_1.p_2)}{(m^2-p_1^2)(m^2-p_2^2)(p_1-p_2)^2} \\ &= \frac{1}{2}(g_Y^0)^2 m^{2d-4}\int \frac{dq_1^d}{(2\pi)^d i}\frac{dq_2^d}{(2\pi)^d i}\frac{4(1+q_1.q_2)}{(1-q_1^2)(1-q_2^2)(q_1-q_2)^2} \\ &= 2(g_Y^0)^2 m^4 m^{-4\epsilon}(2\int \frac{dq_1^d}{(2\pi)^d i}\frac{dq_2^d}{(2\pi)^d i}\frac{1}{(1-q_1^2)(1-q_2^2)(q_1-q_2)^2} - \\ &\quad -\frac{1}{2}(\int \frac{dq_1^d}{(2\pi)^d i}\frac{1}{(1-q_1^2)})^2), \end{aligned} \tag{52}$$

where the identity $q_1.q_2 = \frac{1}{2}(q_1^2 - 1 + {q_2}^2 - 1) + 1 - \frac{1}{2}(q_1-q_2)^2$ was employed. The two momentum integrals appearing in the last line are the simplest Master integrals for scalar fields as listed in Ref. (31). The results for them in that reference are:

$$\int \frac{dq_1^d}{(2\pi)^d i}\frac{dq_2^d}{(2\pi)^d i}\frac{1}{(1-q_1^2)(1-q_2^2)(q_1-q_2)^2} = \frac{(d-2)(\pi)^d\Gamma\left(1-\frac{d}{2}\right)^2}{2(d-3)(2\pi)^{2d}}, \tag{53}$$

$$\int \frac{dq_1^d}{(2\pi)^d i}\frac{1}{(1-q_1^2)} = \frac{(\pi)^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)}{(2\pi)^d}. \tag{54}$$

They allowed to write for the regularized two loop effective action density the expression

$$\gamma_2(m,\epsilon,\mu) = -m^4(\frac{m}{\mu})^{-4\epsilon}\frac{2(g_Y^0)^2(\pi)^d}{(2\pi)^{2d}}(-\frac{d-2}{d-3}+\frac{1}{2})\Gamma\left(1-\frac{d}{2}\right)^2. \tag{55}$$

Expanding in Laurent series in ϵ and disregarding the pole part in the limit $\epsilon \to 0$, led in Ref. (33) to the two loop perturbative contribution to the effective action

$$\gamma_2(m,\mu) = \ 0.0000200507(g_Y^0)^2 m^4\ (48.\log^2\left(\frac{m}{\mu}\right) - 173.783\log\frac{m}{\mu}\ + 183.83\ ). \tag{56}$$

As it was noted in the Introduction, in Ref. (28) it was employed an inappropriate negative kinetic term for the Dilaton field. This change, although not affecting the one fermion loop contribution, which is not altered by the sign of the boson propagator, drastically modified the sign of the two loop term which linearly depends on the Dilaton propagator. In the previous evaluation, the two loop terms determined the existence of minima for the Dilaton potential. Therefore, the consequence of the change in sign fixed by the consideration in Ref. (33) of the correct positive kinetic energy term, should be further investigated in connection with the existence of stabilizing minima for the scalar field. This circumstance determined the motivation for the new three loop corrections considered in Ref. (33) and reviewed in this chapter.

3.2.3 Three loops terms

Let us resume the evaluation of the three loop terms in Ref. (33).

3.2.4 Diagram D_{32}

The D_{32} term is the only of the 3-loops diagrams which is not composed of two fermion or boson self energy insertions connected in series. For the D_{31} and D_{33} cases we had difficulties in reducing their contributions to a linear combination of tabulated Master integrals. This obstacle only allowed us to calculate their leading term in the expansion in $\log(\frac{m}{\mu})$. However, for D_{32} it was possible to express it as a sum over the Master integrals given in Ref. (31). The analytic expression of the diagram was

$$\begin{aligned}\Gamma^{(32)} &= -V^{(d)}\frac{1}{4}(g_Y)^4\int\frac{dp_1^d}{(2\pi)^d i}\frac{dp_2^d}{(2\pi)^d i}\frac{dp_3^d}{(2\pi)^d i}\times \\ &\frac{\mathrm{Tr}\left[(m+p_2^\mu\gamma_\mu)(m+(p_2^\mu+p_3^\mu-p_1^\mu)\gamma_\mu)(m+p_3^\mu\gamma_\mu)(m+p_1^\mu\gamma_\mu)\right]}{(m^2-p_1^2)(m^2-p_2^2)(m^2-p_3^2)(m^2-(p_2+p_3-p_1)^2)(p_1-p_3)^2(p_1-p_2)^2} \\ &= -V^{(d)}\frac{1}{4}(g_Y)^4\int\frac{dp_1^d}{(2\pi)^d i}\frac{dp_2^d}{(2\pi)^d i}\frac{dp_3^d}{(2\pi)^d i}\times \qquad (57)\\ &\frac{m^4+c_1(p_1,p_2,p_3)m^2+c_2(p_1,p_2,p_3)}{(m^2-p_1^2)(m^2-p_2^2)(m^2-p_3^2)(m^2-(p_2+p_3-p_1)^2)(p_1-p_3)^2(p_1-p_2)^2},\end{aligned}$$

$$c_1(p_1,p_2,p_3) = 3p_2.p_{3+}p_1.p_2+p_1.p_3+p_2^2+p_3^2-p_1^2 \qquad (58)$$

$$c_2(p_1,p_2,p_3) = p_1^2\ p_2.p_3+p_2^2\ p_1.p_3+p_3^2\ p_1.p_2-2\ p_1.p_2\ p_1.p_3. \qquad (59)$$

After defining

$$\begin{aligned}z_1 &= p_1^2-m^2,\\ z_2 &= p_2^2-m^2,\\ z_3 &= p_3^2-m^2,\\ z_4 &= (p_1-p_2)^2,\\ z_5 &= (p_1-p_3)^2,\\ z_6 &= (p_2-p_1+p_3)^2-m^2, \qquad (60)\end{aligned}$$

and employing various vectorial identities expressing the squares of the differences between any two momenta in terms of the scalar product between them and the squares of the considered momenta, the integral defining Γ^{32} was written as follows

$$\begin{aligned}\Gamma^{32} &= -V^{(d)}\frac{1}{4}(g_Y)^4\int\frac{dp_1^d}{(2\pi)^d i}\frac{dp_2^d}{(2\pi)^d i}\frac{dp_3^d}{(2\pi)^d i}\times \\ &\frac{m^4+c_1(z)\ m^2+c_2(z)}{z_1\ z_2\ z_3\ z_4\ z_5\ z_6},\\ z &= (z_1,z_2,z_3,\ z_4,\ z_5,z_6), \qquad (61)\\ c_1(z) &= \frac{3}{2}(z_1+\ z_2+\ z_3+z_6)-2(\ z_4\ +z_5)\ +6m^2, \qquad (62)\\ c_2(z) &= \frac{1}{2}(z_1z_6+z_2z_3-z_4z_5+m^2(z_1+z_2+z_3+z_6)+2m^4). \qquad (63)\end{aligned}$$

Therefore, there exist one or two z factors in the denominator that can be canceled by the terms of the quadratic polynomial in these quantities. This property allowed the integral to be decomposed in a linear combination of the Master integrals listed in Ref. (31). The result for the action density

$$\gamma_{32}(m,\mu,\epsilon) = \frac{\Gamma^{(32)}}{\mu^{2\epsilon}V^{(d)}} \tag{64}$$

was expressed in terms of only five of them as follows

$$\gamma_{32}(m,\mu,\epsilon) = -(g_Y^0)^4\, m^4 \left(\frac{m}{\mu}\right)^{-6\epsilon} \left(8I_1(\epsilon) + 8I_2(\epsilon) - 4I_3(\epsilon) + I_5(\epsilon) - \frac{I_7(\epsilon)}{2}\right),$$

where the functions $I_1(\epsilon)$, $I_2(\epsilon)$, $I_3(\epsilon)$, $I_5(\epsilon)$ and $I_7(\epsilon)$ resulted to be given by

$$I_1(\epsilon) = \frac{2^{-3(4-2\epsilon)-9}\pi^{-\frac{3}{2}(4-2\epsilon)}(5(4-2\epsilon)-18)M_1(\epsilon)^3}{1-2\epsilon} + \frac{2^{-3(4-2\epsilon)-6}\pi^{-3(4-2\epsilon)}(3(4-2\epsilon)-10)(3(4-2\epsilon)-8)\left(M_5(\epsilon) - \frac{8\epsilon}{2(4-2\epsilon)-7}M_4(\epsilon)\right)}{\epsilon^2}, \tag{65}$$

$$I_2(\epsilon) = -\frac{2^{-3(4-2\epsilon)-2}\pi^{-3(4-2\epsilon)}}{1-2\epsilon}\left(\frac{M_1(\epsilon)^3(2-2\epsilon)^2}{1-2\epsilon} + (3(4-2\epsilon)-8)M_4(\epsilon)\right), \tag{66}$$

$$I_3(\epsilon) = -\frac{2^{-3(4-2\epsilon)-3}\pi^{-3(4-2\epsilon)}}{\epsilon}\left(\frac{2(2-2\epsilon)^2M_1(\epsilon)^3}{1-2\epsilon} + (3(4-2\epsilon)-8)M_5(\epsilon)\right), \tag{67}$$

$$I_5(\epsilon) = (2\pi)^{-3(4-2\epsilon)}M_4(\epsilon), \tag{68}$$

$$I_7(\epsilon) = (2\pi)^{-3(4-2\epsilon)}M_5(\epsilon),$$

in terms of the Master integrals (See Ref. (31)):

$$M_1(\epsilon) = \pi^{\frac{1}{2}(4-2\epsilon)}\Gamma\left(\frac{1}{2}(2\epsilon-4)+1\right), \tag{69}$$

$$M_2(\epsilon) = -\frac{(2-2\epsilon)\,M_1(\epsilon)^2}{2(1-2\epsilon)}, \tag{70}$$

$$M_3(\epsilon) = 2^{\frac{1}{2}(2\epsilon-4)}\Gamma\left(\frac{1}{2}(4-2\epsilon)\right)\Gamma\left(\frac{1}{2}(2\epsilon-1)\right)M_1(\epsilon)^2, \tag{71}$$

$$M_4(\epsilon) = \frac{2^{1-2\epsilon}\Gamma\left(\frac{1}{2}(8-3(4-2\epsilon))\right)\Gamma\left(\frac{1}{2}(2\epsilon-1)\right)}{\Gamma\left(\frac{1}{2}(7-2(4-2\epsilon))\right)\Gamma\left(\frac{1}{2}(2\epsilon-2)\right)}M_1(\epsilon)^3, \tag{72}$$

$$M_5(\epsilon) = (-2 - \frac{5}{3}\epsilon - \frac{1}{2}\epsilon^2 + \frac{103}{12}\epsilon^3 + \frac{7}{24}(163-128\zeta(3))\epsilon^4 + (\frac{9055}{48} + \frac{136\pi^4}{45} + \frac{1}{3}(\pi^2-\log(2)^2)(32\log(2)^2) - 168\zeta(3) - 256\mathrm{Li}_4(\frac{1}{2}))\epsilon^5)\,M_1(\epsilon)^3, \tag{73}$$

where the special functions $\mathrm{Li}_n(\frac{1}{2})$ and $\zeta(n)$ are defined as

$$\mathrm{Li}_n(x) = \sum_{k=1}^{\infty} \frac{1}{2^k k^n}, \tag{74}$$

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}. \tag{75}$$

Finally, the application of the before described MS procedure led to the following formula for the contribution to the vacuum effective action density of the diagram D_{32}

$$\begin{aligned}\gamma_{32}(m,\mu) = \; & (g_Y^0)^4 m^4 \, (0.0000329114 \log^5\left(\frac{m}{\mu}\right) - 0.000105904 \log^4\left(\frac{m}{\mu}\right) + \\ & 0.0000165851 \log^3\left(\frac{m}{\mu}\right) + 0.000441159 \log^2\left(\frac{m}{\mu}\right) \\ & -0.00074347 \log\left(\frac{m}{\mu}\right) + 0.000388237).\end{aligned} \tag{76}$$

It can be noted that this term has a high quintic power of $\log^5(\frac{m}{\mu})$ which was determined by the also high pole of the ϵ expansion present in the function I_1. This represents the highest power of the $\log\left(\frac{m}{\mu}\right)$ expansion appearing in the results. The next higher power, the fourth one, also is arising in this term.

3.2.5 Diagram $\mathbf{D}_{31}$

We were not able to exactly evaluate this contribution (and also the one associated to D_{33}) in terms of Master integrals. Therefore, for both of these terms we limited ourself to evaluate their leading terms in the expansion in powers of $\log\left(\frac{m}{\mu}\right)$. For this purpose, use was made of the circumstance that (at variance with D_{32}, but in coincidence with the case of D_{33}) this term corresponds to a loop formed by two one loop self-energy insertions. Since these self-energy terms are explicitly calculable in terms of hypergeometric functions, both terms were expressed as single momentum integral in d dimensions. The diagram had the original analytic expression

$$\begin{aligned}\Gamma^{31} &= -V^{(d)} \frac{1}{2} (g_Y)^4 \int \frac{dp_1^d}{(2\pi)^d i} \frac{dp_2^d}{(2\pi)^d i} \frac{dp_3^d}{(2\pi)^d i} \times \\ &\quad \frac{Tr\left[(m + p_2^\mu \gamma_\mu)(m + p_1^\mu \gamma_\mu)(m + p_3^\mu \gamma_\mu)(m + p_1^\mu \gamma_\mu)\right]}{(m^2 - p_1^2)^2 (m^2 - p_2^2)(m^2 - p_3^2)(p_1 - p_3)^2 (p_1 - p_2)^2} \\ &= -V^{(d)} \frac{1}{2} (g_Y)^4 \int \frac{dp_1^d}{(2\pi)^d i} \frac{dp_2^d}{(2\pi)^d i} \frac{dp_3^d}{(2\pi)^d i} \times \\ &\quad \frac{m^4 + d_1(p_1,p_2,p_3) m^2 + d_2(p_1,p_2,p_3)}{(m^2 - p_1^2)^2 (m^2 - p_2^2)(m^2 - p_3^2)(p_1 - p_3)^2 (p_1 - p_2)^2},\end{aligned} \tag{77}$$

$$d_1(p_1,p_2,p_3) = p_1^2 + 2p_1.p_2 + 2p_1.p_3 + p_2.p_3 \quad , \tag{78}$$

$$d_2(p_1,p_2,p_3) = 2\, p_1.p_2\; p_1.p_3 - \; p_1^2\; p_2.p_3. \tag{79}$$

Then, it was defined the fermion self-energy integral and its related vector as follows

$$\begin{aligned} s_{31}(p^2) &= \int \frac{dp_1^d}{(2\pi)^d i} \frac{1}{(m^2-p_1^2)(p_1-p)^2} \\ &= -\frac{\pi^{\frac{d}{2}}}{(2\pi)^d}\Gamma(\epsilon)\int_0^1 dx\, x^{-\epsilon}(m^2-p^2(1-x)-i\delta)^{-\epsilon} \\ &= \frac{\pi^{\frac{d}{2}}}{(2\pi)^d}\Gamma(\epsilon)(m^2)^{-\epsilon}\frac{{}_2F_1(1-\epsilon,\epsilon,2-\epsilon,-\frac{(\frac{p}{m})^2}{1-(\frac{p}{m})^2})}{\epsilon-1}, \end{aligned} \tag{80}$$

$$\begin{aligned} v_\mu(p^2) &= \int \frac{dp_1^d}{(2\pi)^d i} \frac{p_{1\mu}}{(m^2-p_1^2)(p_1-p)^2} \\ &= a(p^2)\ p_\mu, \end{aligned} \tag{81}$$

$$a(p^2) = \frac{p^2+m^2}{2p^2}s_{31}(p^2) - \frac{L(m,\epsilon)}{2p^2}, \tag{82}$$

$$L(m,\epsilon) = \int \frac{dp_1^d}{(2\pi)^d i}\frac{1}{(m^2-p_1^2)} = \frac{\pi^{\frac{d}{2}}m^{d-2}}{(2\pi)^d}\Gamma(1-\frac{d}{2}). \tag{83}$$

In the above expressions, the Feynman parametric integral was explicitly evaluated by employing the algebraic calculation program Mathematica. After again defining the action density contribution as

$$\gamma_{31}(m,\mu,\epsilon) = \frac{\Gamma^{(31)}}{\mu^{2\epsilon}V^{(d)}}, \tag{84}$$

performing the Wick rotation in the momenta and extracting the d-dimensional solid angle arising form the angular integrals, this quantity is expressed as the integral

$$\gamma_{31}(m,\mu,\epsilon) = -\frac{2(g_Y^0)^4\left(\frac{m}{\mu}\right)^{-6\,\epsilon}c(m,\epsilon)}{\epsilon^2}\int_0^\infty \frac{p^{3-2\epsilon}}{(p^2+1)^2}\,f(p,\epsilon)dp, \tag{85}$$

$$f(p,\epsilon) = \epsilon^2 f_1(p,\epsilon)\Gamma(\epsilon)^2 + f_2(p,\epsilon)\,\epsilon\,\Gamma(\epsilon) + f_3(p,\epsilon), \tag{86}$$

$$f_1(p,\epsilon) = (1-p^2)(3-\frac{(1-p^2)^2}{4p^2})(s_{31}^*(p^2,\epsilon))^2, \tag{87}$$

$$f_2(p,\epsilon) = (2-\frac{(1-p^2)^2}{2p^2})s_{31}^*(p^2,\epsilon)L^*(\epsilon), \tag{88}$$

$$f_3(p,\epsilon) = -\frac{\left(1-p^2\right)^2(L^*(\epsilon))^2}{4p^2}, \tag{89}$$

$$s_{31}^*(p,\epsilon) = -\frac{2^{2\epsilon-4}\pi^{\frac{1}{2}(4-2\epsilon)+2\epsilon-4}\,{}_2F_1\left(1-\epsilon,\epsilon;2-\epsilon;\frac{p^2}{p^2+1}\right)}{\epsilon-1}, \tag{90}$$

$$c(m,\epsilon) = \frac{2^{2\epsilon-3}m^4\pi^{\frac{1}{2}(4-2\epsilon)+2\epsilon-4}}{\Gamma\left(\frac{1}{2}(4-2\epsilon)\right)}, \tag{91}$$

$$L^*(\epsilon) = \epsilon L(1,\epsilon). \tag{92}$$

As it was mentioned before, we were not able yet to find an epsilon expansion (rigorous or sufficiently approximated numerical one) allowing to exactly evaluate this integral after removing the regularization. Therefore, in order to determine an approximation for γ_{31} we have made use of an assumption suggested by an exploration done about the asymptotic power expansion at infinity of the integrand as a function of the momentum integration variable p. It followed that all the terms of the expansion after integrated, show a single pole structure in their Laurent expansion in ϵ. Then, it suggests that the full divergence of the integral at $d = 4$ is defined by a single pole in ϵ. Assuming this property, the extraction of the leading correction in $\log(\frac{m}{\mu})$ should be defined by the maximal power of $\log(\frac{m}{\mu})$ appearing in the coefficient of the zero order term in the expansion of the modified integral $\gamma_{31}(m,\mu,\epsilon)$

$$\gamma^*_{31}(m,\mu,\epsilon) = -\frac{2(g^0_Y)^4\left(\frac{m}{\mu}\right)^{-6\,\epsilon} c(m,0)}{\epsilon^2}\int_0^\infty \frac{p^{3-2\epsilon}}{\left(p^2+1\right)^2} f(p,0)dp. \tag{93}$$

Note that any other power of ϵ in the expansions of $c(m,\epsilon)$ and $f(p,\epsilon)$ will reduce the maximal order of the negative powers of epsilon in the full expansion of $\gamma_{31}(m,\mu,\epsilon)$, which determines the leading correction in the expansion. For $f(p,0)$ it followed

$$f(p,0) = \frac{p^4}{1024\pi^4} - \frac{17p^2}{1024\pi^4} + \frac{7}{256\pi^4} - \frac{1}{256\pi^4 p^2}. \tag{94}$$

Then, the use of the formula

$$\int_0^\infty \frac{p^{3-2\epsilon+m}}{\left(p^2+1\right)^2}dp = -\frac{\pi}{4}(m-2\epsilon+2)\csc(\frac{\pi}{2}(m-2\epsilon)), \tag{95}$$

which shows the $\frac{1}{\epsilon}$ singularity, allowed to write for γ_{31} the leading logarithm correction to its finite part

$$\gamma_{31}(m,\mu) = -0.0000228551(g^0_Y)^4\, m^4 \log^3\left(\frac{m}{\mu}\right). \tag{96}$$

3.2.6 Diagram D_{33}

As it was remarked before, this term was treated in a similar way as it was D_{31}. Now, the corresponding self-energy insertions were boson ones. Again, the two self-energy loops were explicitly calculable in terms of hypergeometric functions. The starting analytic expression of the diagram was

$$\begin{aligned}
\Gamma^{33} &= V^{(d)}\frac{1}{4}(g_Y)^4\int\frac{dp^d}{(2\pi)^d i}\frac{dp_1^d}{(2\pi)^d i}\frac{dp_2^d}{(2\pi)^d i}\times\frac{1}{(p^2)^2}\\
&\quad\frac{\mathrm{Tr}[(m+p_1^\mu\gamma_\mu)(m+(p+p_1)^\nu\gamma_\nu)]\mathrm{Tr}[(m+p_2^\mu\gamma_\mu)(m+(p_2+p)^\nu\gamma_\nu)]}{(m^2-p_1^2)^2(m^2-(p_1+p)^2)(m^2-p_2^2)^2(m^2-(p_2+p)^2)},\\
&= V^{(d)}4(g_Y)^4\int\frac{dp^d}{(2\pi)^d i}\frac{dp_1^d}{(2\pi)^d i}\frac{dp_2^d}{(2\pi)^d i}\times\frac{1}{(p^2)^2}\times\\
&\quad\frac{(m^2+p_1.(p_1+p))(m^2+p_2.(p_2+p))}{(m^2-p_1^2)^2(m^2-(p_1+p)^2)(m^2-p_2^2)^2(m^2-(p_2+p)^2)},
\end{aligned} \tag{97}$$

where the fermion traces were evaluated for writing the second form of the integral. The last expression evidenced the decomposition in two serial self-energy terms.
After rotating to Euclidean space the momenta variables of the integration regions and the external momentum, the fermion selfenergy integral and its related vector integral were written as follows (See Ref. (30))

$$\begin{aligned}
s_{33}(q^2,\epsilon) &= \int \frac{dq_1^d}{(2\pi)^d i}\frac{1}{(m^2+q_1^2)(m^2+(q+q_1)^2} \\
&= \frac{(m)^{-2\epsilon}}{(4\pi)^{\frac{d}{2}}}\Gamma(\epsilon)\int_0^1 dx\,(1+(\frac{q}{m})^2x(1-x))^{-\epsilon} \\
&= \frac{(m)^{-2\epsilon}}{(4\pi)^{\frac{d}{2}}}\Gamma(\epsilon)\mathcal{F}(\frac{q^2}{m^2}), \\
\mathcal{F}(q^2) &= \int_0^1 dx\,(1+q^2x(1-x))^{-\epsilon} \\
&= -\frac{2^{-\epsilon-1}(q+\sqrt{q^2+1})(1-\frac{q}{\sqrt{q^2+1}})^{\epsilon}F_1(1-\epsilon,\epsilon,2-\epsilon,\frac{1}{2}(\frac{q}{\sqrt{q^2+1}}+1))}{q(\epsilon-1)} - \\
&\quad \frac{2^{-\epsilon-1}(q-\sqrt{q^2+1})(1+\frac{q}{\sqrt{q^2+1}})^{\epsilon}F_1(1-\epsilon,\epsilon,2-\epsilon,\frac{1}{2}(-\frac{q}{\sqrt{q^2+1}}+1))}{q(\epsilon-1)}, \\
v_{33\mu}(p^2) &= \int \frac{dp_1^d}{(2\pi)^d i}\frac{p_{1\mu}}{(m^2-p_1^2)(m^2-(p+p_1)^2} \\
&= a(p^2)\;\; p_\mu, \\
a(p^2) &= -\frac{1}{2}s_{33}(p^2,\epsilon).
\end{aligned}$$

Again the result for parametric Feynman integral was analytically evaluated thanks to the use of the algebraic calculation program Mathematica. Thus, after extracting the Euclidean angular integrals and performing some transformations, the effective action density contribution

$$\gamma_{33}(m,\mu,\epsilon) = \frac{\Gamma^{(33)}}{\mu^{2\epsilon}V^{(d)}} \tag{98}$$

was expressed as single momentum integral in the range (0,∞) as follows

$$\gamma_{33}(m,\mu,\epsilon) = \frac{4c(m,\epsilon)(g_Y^0)^4\left(\frac{m}{\mu}\right)^{-6\epsilon}}{\epsilon^2}\int_0^\infty dp\,\frac{p^{3-2\epsilon}}{\left(p^2+r^2\right)^2}g(p,\epsilon), \tag{99}$$

$$g(p,\epsilon) = \epsilon^2 g_1(p,\epsilon)\Gamma(\epsilon)^2 + g_2(p,\epsilon)\,\epsilon\,\Gamma(\epsilon) + g_3(p,\epsilon), \tag{100}$$

$$g_1(m,\epsilon) = \left(\frac{p^2}{2}+2\right)^2 s_{33}^*(p,\epsilon)^2, \tag{101}$$

$$g_2(m,\epsilon) = -2\left(\frac{p^2}{2}+2\right)L(\epsilon)\,s_{33}^*(p,\epsilon), \tag{102}$$

$$g_3(m,\epsilon) = L(\epsilon)^2, \tag{103}$$

$$s^*_{33}(p,\epsilon) = \frac{s_{33}(p,\epsilon)}{\Gamma(\epsilon)}, \tag{104}$$

$$c(m,\epsilon) = \frac{2^{2\epsilon-3} m^4 \pi^{\frac{1}{2}(4-2\epsilon)+2\epsilon-4}}{\Gamma\left(\frac{1}{2}(4-2\epsilon)\right)}, \tag{105}$$

$$L(\epsilon) = \frac{\pi^{2-\epsilon}}{(2\pi)^{4-2\epsilon}}\Gamma(-1+\epsilon). \tag{106}$$

Finally, by employing a similar procedure for extracting the leading logarithmic correction in $\log\left(\frac{m}{\mu}\right)$ for D_{31}, the analogous contribution for D_{33} followed in the form

$$\gamma_{33}(m,\mu) = -0.000329114\ (g_Y^0)^4\ m^4 \log^3\left(\frac{m}{\mu}\right).$$

3.3 Discussion of the results

This subsection resume the results obtained in Ref. (33) for the effective action density. The total effective potential value $v(m,\mu)$, resulted to be given by the sum of all the evaluated terms after changing their sign. The total potential and its various contributions were written as follows

$$v(m,\mu) = v_1(m,\mu) + v_2(m,\mu) + v_{31}(m,\mu) + v_{33}(m,\mu) + v_{32}(m,\mu), \tag{107}$$

$$\frac{v_1(m,\mu)}{m^4} = -\frac{\gamma_1(m,\mu)}{m^4}$$
$$= -0.0506606\left(2.\log\left(\frac{m}{\mu}\right) - 2.95381\right), \tag{108}$$

$$\frac{v_2(m,\mu)}{m^4} = -\frac{\gamma_2(m,\mu)}{m^4}$$
$$= -0.0000200507(g_Y^0)^2(183.83 - 173.783\log\left(\frac{m}{\mu}\right) + 48.\log^2\left(\frac{m}{\mu}\right)), \tag{109}$$

$$\frac{v_{31}(m,\mu)}{m^4} = -\frac{\gamma_{31}(m,\mu)}{m^4} = 0.0000228551(g_Y^0)^4 m^4 \log^3\left(\frac{m}{\mu}\right), \tag{110}$$

$$\frac{v_{33}(m,\mu)}{m^4} = -\frac{\gamma_{33}(m,\mu)}{m^4} = 0.000329114(g_Y^0)^4 m^4 \log^3\left(\frac{m}{\mu}\right), \tag{111}$$

$$\frac{v_{32}(m,\mu)}{m^4} = -\frac{\gamma_{32}(m,\mu)}{m^4}$$
$$= -(g_Y^0)^4 m^4\ 10^{-3}(0.0329114\log^5\left(\frac{m}{\mu}\right) - 0.105904\log^4\left(\frac{m}{\mu}\right)$$
$$+ 0.0165851\log^3\left(\frac{m}{\mu}\right) + 0.441159\log^2\left(\frac{m}{\mu}\right) -$$
$$0.74347\log\left(\frac{m}{\mu}\right) + 0.388237). \tag{112}$$

The renormalization point for μ was chosen at the same value of the fermion mass m_f, under consideration, that is $Log(\frac{m_f}{\mu}) - > 0$. Also, it was defined a new scaled scalar field Φ and interaction parameter g by mean of

$$\Phi = \alpha\varphi, \tag{113}$$

$$g_Y^0 = \alpha m = g\exp(\Phi), \tag{114}$$

$$g = \alpha m_f. \tag{115}$$

Then, the evaluated total contribution to the effective potential for the Dilaton $v(m,\mu)$ was expressed as a function $v(\Phi, g)$ as follows

$$\begin{aligned}\frac{v(\Phi,g)}{m_f^4} \equiv \frac{v(m,\mu)}{m_f^4} = & -0.0000329114e^{8\Phi}g^4\Phi^5 + 0.000105904e^{8\Phi}g^4\Phi^4 \\ & +0.000289673e^{8\Phi}g^4\Phi^3 + e^{4\Phi}\left(-0.000441159e^{4\Phi}g^4 - 0.000962436e^{2\Phi}g^2\right)\Phi^2 + \\ & e^{4\Phi}\left(0.00074347e^{4\Phi}g^4 + 0.00348448e^{2\Phi}g^2 - 0.101321\right)\Phi + \\ & e^{4\Phi}\left(-0.000388237e^{4\Phi}g^4 - 0.00368594e^{2\Phi}g^2 + 0.149642\right).\end{aligned} \tag{116}$$

New functions $u_{5,}$ u_4 and u_3 representing approximations of the potential were defined in the form

$$\frac{u_5(\Phi,g)}{m_f^4} = \frac{v(\Phi,g)}{m_f^4}, \tag{117}$$

$$\begin{aligned}\frac{u_4(\Phi,g)}{m_f^4} = & \ 0.000105904e^{8\Phi}g^4\Phi^4 + 0.000289673e^{8\Phi}g^4\Phi^3 + \\ & e^{4\Phi}\left(-0.000441159e^{4\Phi}g^4 - 0.000962436e^{2\Phi}g^2\right)\Phi^2 + \\ & e^{4\Phi}\left(0.00074347e^{4\Phi}g^4 + 0.00348448e^{2\Phi}g^2 - 0.101321\right)\Phi + \\ & e^{4\Phi}\left(-0.000388237e^{4\Phi}g^4 - 0.00368594e^{2\Phi}g^2 + 0.149642\right),\end{aligned} \tag{118}$$

$$\begin{aligned}\frac{u_3(\Phi,g)}{m_f^4} = & \ +0.000289673e^{8\Phi}g^4\Phi^3 + \\ & e^{4\Phi}\left(-0.000441159e^{4\Phi}g^4 - 0.000962436e^{2\Phi}g^2\right)\Phi^2 + \\ & e^{4\Phi}\left(0.00074347e^{4\Phi}g^4 + 0.00348448e^{2\Phi}g^2 - 0.101321\right)\Phi + \\ & e^{4\Phi}\left(-0.000388237e^{4\Phi}g^4 - 0.00368594e^{2\Phi}g^2 + 0.149642\right).\end{aligned} \tag{119}$$

Note that u_5 coincided v and is of order five in the powers of Φ. The function u_4, u_3 were defined as retaining only all the terms up to order Φ^4 and Φ^3 respectively of the original function u_5 . Therefore, these functions basically correspond to the expansion of order five, four and three in powers of $\log\left(\frac{m}{\mu}\right)$. They were defined in order to study the influence of increasing the order of the perturbative expansion in powers of $\log\left(\frac{m}{\mu}\right)$.

To evidence the dependence on Φ and g of the three functions (after divided by the common factor m_f^4), they were plotted in figure 7. The range of values of $g = m_f \alpha$ was chosen $(0, 1)$ as suggested by the fact that α is of the order of the Planck length and thus the physical values of the considered fermion mass are expected to determine g to be smaller than one. The plot of u_5 showed that there is a threshold value of g, below which the potential exhibits minima tending to stabilize the vacuum mean value of the Dilaton field. This behavior was also shown by the approximated potentials u_4 and u_3, a fact that indicated that after disregarding the higher quintic and quartic terms in the expansion in $log(\frac{m}{\mu})$, the existence of Dilaton stabilizing minima is not affected.
When considering the full evaluated potential curve u_5, illustrated at the top plot of figure 7, it can be observed that after lowering the g value below a critical threshold, the minimum as a function of Φ stops to exist at a critical value g_{min}. However, in the case of u_4 and u_3, the minimum exists for arbitrary values of $g- > 0$. That is, when the potential approximation is bounded from below, the potential shows stabilizing minima at any small value of g close to zero. The field value at the minima grow when the coupling tends to vanish. It can be noted, that the non bounded from below character of the approximated potential calculated, is determined by the fact that the quintic power of Φ correction turned to be negative. However, the physical system under consideration is one in which the total effective potential can be expected to show an exact bounded from below character. Thus, the next corrections are expected to exhibit a bounded from below behavior. In accordance with this expectation, in studying the g dependence at small values, we employed the approximated potential function u_4, assuming that it represents a reasonably good approximation of the exact potential.

3.4 Dilaton field and mass for m_f at the GUT scale

In Ref. (33) it was firstly considered that the highest fermion mass m_f is given by the GUT mass scale

$$\begin{aligned} m_f &= m_{GUT} = 5.06773 \times 10^{29} \text{cm}^{-1} \\ &\equiv 10^{16} \text{ GeV}, \end{aligned} \tag{120}$$

which produced for the coupling g the value

$$g = m_f \, \alpha = -\frac{3}{4} \kappa \, m_{GUT} = -0.0030789542773.$$

The potential u_4 as a function of the field Φ for this particular value of g is shown in figure 8. The minimum of the curve determined an estimate for the vacuum value of the Dilaton field given by

$$\Phi_{vac} = 5.8576156 \ = \alpha \, \varphi_{vac}, \tag{121}$$

$$\varphi_{vac} = -\frac{4}{3} 5.8576156 \frac{1}{\kappa}. \tag{122}$$

This result indicated that the vacuum mean value of the Dilaton field, after assuming that the fermion mass is in the GUT scale, became stabilized in the scale of the Planck mass.
For the mass of the field excitation it followed that its value was determined by the second derivative of the potential curve taken at the minimum, which is given by

$$\frac{d^2}{d\Phi^2} u_4(\Phi, -0.0030789542773) \Big|_{\Phi = \Phi_{vac}^{(m_{GUT})}} = 1.28179 \times 10^{11} m_f^4. \tag{123}$$

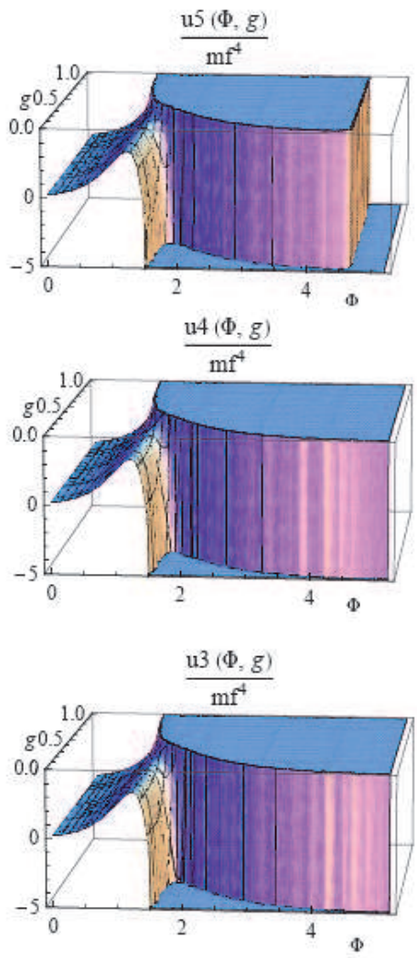

Fig. 7. The three figures show, from top to bottom, the potentials u_5, u_4 and u_3 dependence on the field Φ and the coupling g, respectively. The potential scale is chosen for a high magnification range (the minima of the surface at fixed g values are very far below the plotted range) in order to evidence the presence of a threshold for the appearing of the minima when the value of g decreases below $g = 1$. Note that for Φ smaller than some units and not to small values of g, the three plotted graphs are similar, indicating that the elimination of the highest fifth, and also the next to highest fourth, powers of the field (or, of the logarithm in the original expansion) in defining u_4 and u_3 respectively, are not affecting the results in the mentioned region. The circumstance that the exact evaluated contribution has a negative leading term of order five (which makes the result unbounded from below) explains that for the plot of u_5 the minima disappear for sufficiently small values of g. However, the fact that exact potential should be expected to be bounded from below, we consider that supports our assumption about employing the bounded from below approximations of the potential u_4 (or u_3) in evaluating the Dilaton properties at the small values of g defined by the GUT and m_{top} mass scales.

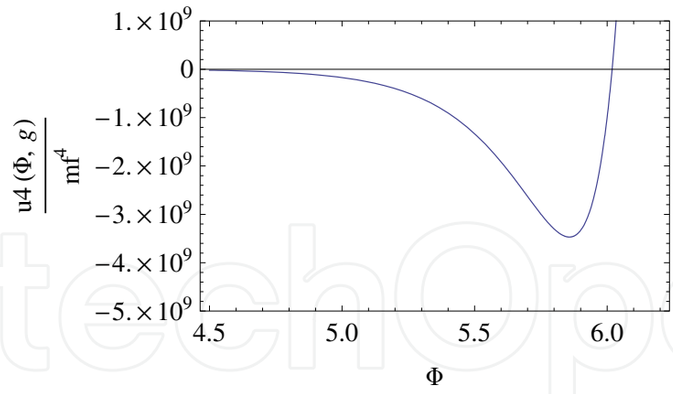

Fig. 8. The effective potential u_4 defined by Eq. 118 as a function of the Dilaton field Φ. The fermion mass was fixed to correspond to the GUT mass m_{GUT} and the renormalization scale μ was chosen to coincide with this mass. The minimum of the potential was near the value $\Phi = 5.7$, which indicates that the field is bound to a high value near the Planck scale.

In order to estimate the Dilaton mass it was considered the linearized equation of motion for the mean field

$$(\frac{1}{\alpha^2}\partial^2 + \frac{d^2}{d\Phi^2}u_4(\Phi, -0.00307895)\Big|_{\Phi=\Phi_{vac}^{(m_{GUT})}})\,\Phi = 0, \tag{124}$$

in which the factor $\frac{1}{\alpha^2}$ multiplying the D'Alembertian appeared due to the previously done change of field variables $\Phi = \alpha\,\varphi$.
The above wave equation led to the dispersion relation for the Dilaton modes

$$(-\frac{1}{\alpha^2}p^2 + \frac{d^2}{d\Phi^2}u_4(\Phi, -0.00307895)\Big|_{\Phi=\Phi_{vac}^{(m_{GUT})}}) = 0, \tag{125}$$

which for the case of the particle at rest $p = (m_D^{(m_{GUT})}, 0, 0, 0)$ determined for the Dilaton the mass estimate

$$\begin{aligned} m_D^{(m_{GUT})} &= \sqrt{\frac{d^2}{d\Phi^2}u_4(\Phi, -0.00307895)\Big|_{\Phi=\Phi_{vac}^{(m_{GUT})}} m_{GUT}^2 \mid \alpha \mid} \\ &= \quad 5.58626 \times 10^{32} \qquad \mathrm{cm}^{-1}. \end{aligned} \tag{126}$$

Therefore, the predicted order of the mass for the Dilaton also became an extremely high value which makes this field mode undetectable in a direct way.

3.5 Dilaton mean value and mass for m_f at the top quark mass scale

It was also of interest to take as m_f the highest currently known fermion mass: that is, the top quark one

$$m_{top} = 172.0 \pm 0.9 \quad \mathrm{GeV} = 8.7164 \times 10^{15} \quad \mathrm{cm}^{-1}. \tag{127}$$

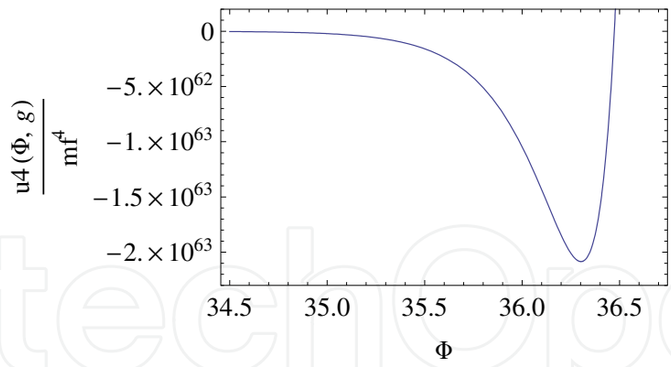

Fig. 9. The effective potential u_4 plotted as a function of the Dilaton field Φ. In this case the coupling was defined by a fermion mass correspond to the top quark one m_{top} and the renormalization scale μ was also chosen to coincide with this value. The minimum of the potential is now near the value $\Phi = 36.7765$, which indicated that the field is again staying at a high value.

Then, the coupling g in this case got the small value

$$g = m_f\,\alpha = -\frac{3}{4}\kappa\, m_{top} = -5.32659 \times 10^{-17}. \tag{128}$$

Figure 9 shows the dependence of the potential u_4 as a function of the field Φ at the above value of the coupling g. The minimum of the curve in this case gave for the mean Dilaton field at the vacuum

$$\Phi_{vac}^{(m_{top})} = 36.3020096 = \alpha\ \varphi_{vac}^{(m_{top})}, \tag{129}$$

$$\varphi_{vac}^{(m_{top})} = -\frac{4}{3}36.3020096\frac{1}{\kappa}. \tag{130}$$

This result predicts that, assuming that the maximal fermion mass in Nature is given by the top quark one, which means a lower bound for the physical masses, the vacuum field of the Dilaton, again becames stabilized in a scale, which although not being so high, is yet close to the Planck mass.

In this case the dispersion relation for the Dilaton modes resulted in the form

$$(-\frac{1}{\alpha^2}p^2 + \frac{d^2}{d\Phi^2}u_4(\Phi, -5.32659 \times 10^{-17})\Big|_{\Phi=\Phi_{vac}^{(m_{top})}}) = 0. \tag{131}$$

But, after evaluating for the second derivative of the potential at the minimum to be

$$\frac{d^2}{d\Phi^2}u_4(\Phi, -5.32659 \times 10^{-17})\Big|_{\Phi=\Phi_{vac}^{(m_{top})}} = 6.86404 \times 10^{64}, \tag{132}$$

and fixing again the rest frame momentum $p = (m_D^{(m_{top})}, 0, 0, 0)$ estimated for the Dilaton mass the value

$$m_D^{(m_{top})} = \sqrt{\frac{d^2}{d\Phi^2} u_4(\Phi, -5.32659 \times 10^{-17})\Big|_{\Phi=\Phi_{vac}^{(m_{top})}}} \times m_{top}^2 \mid \alpha \mid$$

$$= 7.07209 \times 10^{29} \quad \text{cm}^{-1}. \tag{133}$$

Henceforth, also in this case the predicted mass for the Dilaton turned to be a high value being now close to the GUT scale. Thus, it can be expected that for maximal fermion masses in Nature ranging between the lower bound m_{top} and the GUT scale one, the Dilaton gets stabilized at a large field value as required by string phenomenology. In addition the resulting values of its mass, for the same range of m_f, became also out of the current observability range of particle detectors.

4. Conclusions

We had reviewed and commented some issues linked with the possible roles of the Dilaton in Cosmology and its stabilization properties under the existence of massive fermion matter, which were advanced in Refs. (32; 33).

In the work (33), the fermion field mass values were considered in two cases: the top quark mass representing the lower bound of all existing but yet unknown fermion masses in Nature, and the energy scale of the grand unification theories of order 10^{16} GeV. In both situations, the results indicated that the Dilaton mean field becomes stabilized at the very high values required by its role in allowing gravity to have its observed properties. Then, the same existence of matter seems to be a possible source of the dynamical fixation of the Dilaton field at the high values, required by String Theory to imply the observable Einstein theory of gravity. Furthermore, the evaluations pointed out that the Dilaton field also resulted to be strongly bound around its mean value, by showing a large mass being close to the GUT or Planck scales. Therefore, a possible explanation for the lack of observable consequences of the Dilaton scalar field in Nature was underlined. The discussion included contributions to the effective potential up to 3-loops. They allowed to consider the influence of the inclusion of different leading perturbative correction on the main conclusions. After, disregarding in the evaluated potential: a) the highest order term (quintic) in the expansion in powers of $Log(\frac{m}{\mu})$ (which determined the unbounded from below structure of the potential at large Φ values) or b) the two highest orders (the quintic and the quartic ones), the obtained modified potentials were both bounded from below at high field values. This procedure allowed that minima as functions of Φ exist for arbitrarily small values of the coupling g, which allowed to evaluate at the small coupling values fixed by the GUT and top quark masses. The fact that the Yukawa theory under consideration should exhibit a bounded from below potential, then supported the adopted procedure for estimating the vacuum mean values and mass of the Dilaton field. However, further higher loop evaluations are convenient to define more precise estimated values of the Dilaton vacuum field and mass and also for checking that they do not affect the picture. Moreover, it will be also helpful to perform an evaluation of the vacuum mean value of the square of the radiation Dilaton field (basically defined by a Dilaton propagator tadpole diagram). A result of $< 0|((\Phi^r(x))^2|0 >$ being much smaller than 1 will check a main assumption adopted in this work: that the QFT with exponential interaction associated to the

Dilaton field could be well approximated by the here employed Yukawa QFT description for each value of the mean field Φ.
On another hand the results of the work (32) presented a static solution of the EKG equations in which the Dilaton was represented by a scalar field showing a small mass of the order of the inverse of the estimated radius of the Universe. An interaction of the Dilaton with matter was also included. It arose that the existence of the interaction was central in allowing the arising of the static solution. The model parameters were able to be fixed for determining a relation matter-Dark Energy ratio close to the one observed, and which slowly changes with the increase of the radial distance. However, the assumed small mass of the Dilaton rises doubts about the validity of this picture for the Dilaton. Consequently, these doubts also translate to the possible feasibility of the picture speculated in the introduction in which the Universe could show a kind a "Matryoshka" structure, in which our Universe could result to be the interior of a kind of "gravastar" (See Refs. (35; 36)). However, the fixation of the Dilaton field to a high and rigid value induced by the validity of the results of Ref. (33) could perhaps not to exclude the realization of the mentioned picture. We would like to underline here an idea, which is already being discussed in the literature, and that in our view could support the considered picture, by also furnishing a concrete explanation for the origin and smallness of the Cosmological Constant. The first reference about this point of view we received from the work reported in Ref. (34). In this article it was pointed out that the one loop quantization of pure gravity determines deSitter spacetime as a natural solution. More importantly, it was also underlined that the effect is a consequence of a very much natural effect: the condensation of the massless and also gravitationally attracting between themselves gravitons. In other words that work proposed as the source of the Cosmological Constant (that is, the validity in Nature of the deSitter space time) the expected to occur condensation of gravitons once the gravity is assumed to be quantized. The smallness of the CC could be associated to the weakness of the attraction between gravitons. It should be also pointed out that the technical difficulty linked with the non renormalizability of pure gravity, should not be considered as a serious obstacle to the possibility of the relevance in Nature of this effect. This seems to be so, whenever we accept the relevance of string theory in Physics, because a quantized gravity should be described by string theory in an expected to be finite way. If such is the case, the just mentioned graviton condensation effect should be expected in the finite calculational framework of string theory for the quantum gravitational effects. Finally, the connection of this picture with our discussion comes form the possibility that the graviton condensation effect could allow a possible realization of the "Matryoshka" model of the Universe. In it, the collapse of usual matter could occur between regions showing a difference in the Cosmological Constant values. Such configurations can be imagined as being closely resembling the so called "gravastars" discussed in Refs. (35; 36). In ending, I would like to remark about the fact that the observed CC is considered as a surprisingly small quantity as compared with the energy density of the vacuum modes for the fields associated to the observed particles in Nature. However, it can be noted that in the framework of QFT in Minkowski spacetime, such vacuum densities are automatically canceled by the normal ordering rules in the field quantization procedure. Therefore, one could suspect that such vacuum contributions can be also efficiently canceled by consistent quantization procedures in curved spacetimes. The validity of this expectation could perhaps enforce the vanishing of the CC when QFT is considered in Minkowski space time and the gravitons are assumed as pure classical modes. However, just the gravitons are assumed to be quantum waves, their massless character in addition with the natural gravitational attraction existing among them,

strongly suggest the appearance of the graviton condensate already underlined in Ref. (34), as being equivalent to the instability of the Minkowski spacetime to become a deSitter one. This last point also rises the idea about that the one loop effective action for gravity which should be generated by the graviton condensation effect, could also play a role in explaining the large scale effects currently attributed the existence of the Dark Matter. We expect to be able of considering some of these questions in further extensions of the work.

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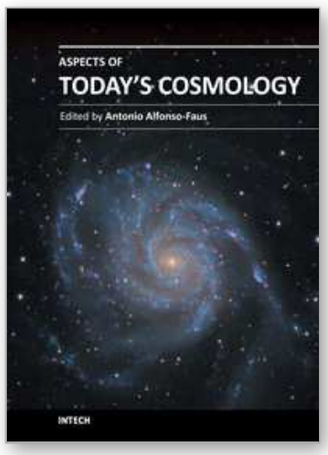

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This book presents some aspects of the cosmological scientific odyssey that started last century. The chapters vary with different particular works, giving a versatile picture. It is the result of the work of many scientists in the field of cosmology, in accordance with their expertise and particular interests. Is a collection of different research papers produced by important scientists in the field of cosmology. A sample of the great deal of efforts made by the scientific community, trying to understand our universe. And it has many challenging subjects, like the possible doomsday to be confirmed by the next decade of experimentation. May be we are now half way in the life of the universe. Many more challenging subjects are not present here: they will be the result of further future work. Among them, we have the possibility of cyclic universes, and the evidence for the existence of a previous universe.

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