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A New Cosmological Model

J.-M. Vigoureux[1], B. Vigoureux[1] and M. Langlois[2]
[1]*Institut UTINAM, UMR CNRS 6213,*
Université de Franche-Comté, Besançon Cedex
[2]*Passavant, 25360*
France

1. Introduction

The constant c was first introduced as the speed of light. However, with the development of physics, it came to be understood as playing a more fundamental role, its significance being not directly that of a usual velocity (even though its dimensions are) and one might thus think of c as being a fundamental constant of the universe (for a discussion on the speed of light, see, for example, (Ellis & Uzan, 2005)). Moreover, the advent of Einsteinian relativity, the fact that c appears in phenomena where there is neither light nor any motion (for example in $E = mc^2$ which shows that c can in principle be measured with a weighing scale and a thermometer (Braunbeck, 1937) or in the relation $(\epsilon_0\mu_0)^{-1/2} = c$ showing that c can be obtained from electro*static* and magneto*static* experiments (Maxwell, 1954)) and its dual-interpretation in terms of "speed" of light and of "speed" of gravitation [1] forces everybody to associate c with the theoretical description of space-time itself rather than that of some of its specific contents. We could not in fact be satisfied by such results and we may think that these different aspects of "c" reflect an underlying structure we do not yet comprehend.
All this invites us to connect c to the geometry of the universe. Noting then that both c and the expansion of the universe *provide a universal relation between space and time* which both have the physical dimension of a velocity, we consider that these two facts cannot be a fortuitous coincidence and that they consequently are two different aspects of a same phenomenon. We thus consider that c must be related to the expansion of the universe and we postulate as a fundamental law of nature (Vigoureux et al., 1988) that

$$c = \alpha\dot{a} = Cst \tag{1}$$

where α is a positive constant and where $a(t)$ is the cosmic scale factor which can be assimilated to the radius of the universe in the case of a spherical geometry (of course, all results also holds when taking $c = 1$). Equation (1) of course means that the scale factor increases at a constant expanding rate. Such a case is usually expected to describe an empty expanding universe (as is for example the Milne universe) or, at the least, an universe in which the density of matter and radiation are so small that they have negligible effect on the flat spacetime geometry. *However, as we shall see, in our model where appears a cosmological constant*

[1] Answering to the question by saying that light and gravitation correspond to zero rest-mass particules does not change the problem.

term, a constant velocity of expansion does not need such an empty universe. [2] Let us also note that eq.(1) verifies the condition $\dot{H} + (1+q)H^2 = 0$ where $q = -\ddot{a}a/\dot{a}^2$ is the deceleration parameter and where H is the Hubble parameter. In our case that equation in fact reduces to $\dot{H} + H^2 = 0$ the solution of which is $H = 1/t$ and consequently $a \sim t$ as expected from eq.(1). Eq.(1) permits to define *c from the knowledge of the geometry of space-time only*, that is from its size and its age. It thus really gives c the statute of a true geometrical fundamental magnitude of the universe, whereas its value 299,792,458 metres per second not only has no geometrical meaning, but also has no meaning at all in the early universe when metres and seconds cannot be defined [3]. On the contrary, it is in fact to be underlined that defining c from the size and the age of the universe has a meaning at all times.

Our aim in this chapter is to show that solving Friedmann's equations with eq.(1), which thus appears as an additional constraint, can explain unnatural features of the standard cosmology without needing any other hypothesis such as those of the inflationary universe or of varying speed of light cosmologies. We thus show that using eq.(1) can solve

- the flatness problem: in our model, the universe dispays the same evolution as a flat universe and *must appear to be flat whatever it may be* (spherical or not);
- the horizon problem: there is no particle horizon;
- the uniformity of the cosmic microwaves background radiation and the small-scale inhomogeneity problem: we show that it is the same tiny part of the early universe that we can observe in any direction around us so that it is quite normal to find the observed background homogeneity. Moreover, it becomes obvious that the universe at time t_{CMB} of the cosmic microwave background radiation can be quite inhomogeneous so that its inhomogeneities can be understood as the seeds of cosmological structures (galaxies and clusters of galaxies).
- We also show that it permits to fit observational data of type Ia supernovae without having to consider an accelerating expansion of the universe: in the standard cosmology, the interpretation of such observations need to use for q a value close to -0.5 for today and a value of 0.5 for very high redshits. On the contrary, our calculations show that all observations can be explained by using $q = 0$ at all times. So, provide we use eq.(1), the linear approach for the cosmological scale factor is well supported by observations;
- Studying then the cosmological term problem which is to understand why ρ_Λ is not only small but also of the same order of magnitude as the present mass density ρ_M of the universe, we finally show how our model also answers that problem.

In each part, we begin by introducing briefly the problem we consider. We then present our results. Some of them have been published (Viennot & Vigoureux, 2009; Vigoureux et al., 1988; 2001; 2003; 2008). However they have not been presented in details. Moreover we also need them for a coherent presentation of our model. We thus present them for clarity and for their subsequent uses in this chapter. In any case, all results are discussed in a detailed way.

In concluding, we first discuss the originality of eq.(1) which has the advantage of giving unity to number of results which, for some of them, have been found by various authors

[2] Usually, such a linear variation of the scale factor leads to at least two special cases. One is an empty universe ($T_{\mu\nu} = 0$) with $k = -1$. The other is a flat universe with the equation of state $p = -\rho c^2/3$. It is consequently concluded that such a variation of the scale factor cannot describe the universe in which we live. However, it would be to conclude too quickly to deduce that any flat-spacetime metric must describe an empty universe: we shall see that in our model, the metric of a spherical universe, *for example*, can be reduced to that of a flat space-time metric.

[3] *For example*, in its 1960 definition, the meter is defined as "the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels 2p10 and 5d5 of the krypton 86 atom." Such a definition has obviously no meaning when atoms did not exist.

from number of different (and sometimes ad hoc) hypotheses. We also open our subject to some of its consequences in other fields of physics. In fact, we consider eq.(1) as *a general law of nature* (Vigoureux et al., 1988) which also concerns other fields of physics such as special relativity, quantum theory or electromagnetism. Some of these ideas will be shortly open in our conclusion.

2. Friedmann equations

We briefly summarize here some well-known results for clarity and for their subsequent uses in this chapter.
Einstein's field equation which relates the geometry of space-time to the energy content of the universe can be written

$$R_{ij} - \frac{1}{2} R \, g_{ij} = 8\pi G \left(T_{ij} - \frac{\Lambda}{8\pi G} g_{ij} \right) \tag{2}$$

As is usual now, the cosmological term Λ has been moved from the left-hand side (curvature side) to the right-hand side of the Einstein equation and has thus been included inside the energy-momentum tensor term. This permits to interpret Λ as a part of the matter content of the universe rather than as a purely geometrical entity.
Taking into account the fact that on very large scale the universe is spatially homogeneous and isotropic to an excellent approximation (which implies that its metric takes the Robertson-Walker form) Einstein's equations reduce to the two Friedmann equations (a dot refers to a derivative with respect to the cosmic time t)

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \tag{3}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3\frac{p}{c^2}) + \frac{\Lambda}{3} \tag{4}$$

where G, ρ and p are the gravitationnal constant, matter-energy density and fluid pressure respectively ; $a(t)$ is the cosmic scale factor characterizing the relative size of the spatial sections as a function of time. As usual, the curvature parameter k takes on values $-1, 0, +1$ for negatively curved, flat, and positive curved spatial sections (open, flat or closed universes) respectively. Note that the cosmological constant Λ will appear in what follows as a time-dependant function.
The energy conservation can be found by differentiation of eq.(3) and by using eq.(4). It can also be found by introducing Λ in the energy-momentum tensor and then using Einstein's field equation. We get

$$\frac{\dot{\Lambda}}{8\pi G} + \dot{\rho} = -3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} \tag{5}$$

3. The solutions of the Friedmann equations

We solve here Friedmann's equations with the additional constraint (1) which expresses a restriction on usual variables characterizing the problem.
Using eq.(1), Friedmann equations (3) and eq.(4) become

$$\frac{\dot{a}^2}{a^2}(1 + k\alpha^2) = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} \tag{6}$$

$$0 = -\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda}{4\pi G} \tag{7}$$

These two above eqs.(6, 7) show that when taking $\Lambda \neq 0$, the linear variation of the scale factor $a(t) = ct/\alpha$ obtained from eq.(1), does not lead to an empty universe. Moreover, the fact that $\ddot{a}(t) = 0$ in the second one could appear inconsistant with observations. It will however be shown that observations which need the condition $\ddot{a}(t) \neq 0$ in the standard model can be explained without it when using eq.(1).

These equations can be solved in the most general case by using the equation of state parameter w of a perfect fluid:

$$p(t) = w\,\rho(t)\,c^2 \tag{8}$$

with w a constant ($w = \frac{1}{3}$ for the radiation dominated epoch and $w = 0$ in the case of an universe dominated by cold matter). Solving eq.(6,7) with (8) we obtain

$$\rho(t) = \frac{\left(1 + k\alpha^2\right)c^2}{4\pi G(1+w)\alpha^2}\,\frac{1}{a(t)^2} \tag{9}$$

showing that the cosmic mass density varies with the reciprocal of the squared cosmic scale, and

$$\Lambda(t) = (1+3w)4\pi G\,\rho(t) = \frac{(1+3w)\left(1+k\alpha^2\right)c^2}{(1+w)\alpha^2}\frac{1}{a(t)^2} \tag{10}$$

Such a variation of $\rho(t)$ and of $\Lambda(t)$ with $a(t)^{-2}$ will be discussed at the end of this part. It comes from the presence of the term $\dot{\Lambda}$ in eq.(5). This can be seen by introducing eq.(10) into the left-hand side of eq.(5) which becomes

$$\frac{\dot{\Lambda}}{8\pi G} + \dot{\rho} = \frac{(1+3w)}{2}\dot{\rho} + \dot{\rho} = \frac{3}{2}(1+w)\dot{\rho} \tag{11}$$

so that the energy conservation becomes

$$\dot{\rho} = -2\rho\frac{\dot{a}}{a} \tag{12}$$

where the multiplicating factor 2 appears instead of 3.

Eq.(9) also gives (for a spherical universe):

$$M = \frac{4\pi}{3}a^3\rho = \frac{(1+k\alpha^2)c^2}{3G(1+w)\alpha^2}a(t) \overset{k=1,\,w=0}{=} \frac{c^2(1+\alpha^2)}{3G\,\alpha^2}a(t) \tag{13}$$

showing that the total mass of the universe scales with its cosmic radius (that unexpected result is discussed at the end of that part). Using that last equation, we note that

$$\frac{GM}{Rc^2} = \frac{(1+k\alpha^2)}{3(1+w)\alpha^2} \overset{k=1,\,w=0}{=} \frac{(1+\alpha^2)}{3\alpha^2} \tag{14}$$

which is a general expression of Mach's principle (Assis, 1994; Brans & Dicke, 1961) showing that our model can fulfil the principle of equivalence of rotation (Fahr & Heyl, 2006).

It is often useful to introduce the critical density ρ_c:

$$\rho_c = \frac{3H^2}{8\pi G} \overset{\text{eq.(1)}}{=} \frac{3c^2}{8\pi G\alpha^2 a^2} \tag{15}$$

and the density parameter Ω (we take the effects of a cosmological constant into account by including the vacuum energy density $\rho_\Lambda = \Lambda/8\pi G$ into the total density). We thus find, whatever may be the value of w

$$\Omega = \frac{\rho_{total}}{\rho_c} = \frac{\rho + \rho_\Lambda}{\rho_c} = (1 + k\alpha^2) \tag{16}$$

We thus find that the density ρ of the universe may be, as expected on the basis of number of recent observations, of the same order of the critical density ρ_c.
The expressions for Ω and Ω_Λ are

$$\Omega = \frac{\rho}{\rho_c} = \frac{(1 + k\alpha^2)}{3} \frac{2}{1 + w} \tag{17}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{(1 + k\alpha^2)}{3} \frac{1 + 3w}{1 + w} \tag{18}$$

Solving the three above results for Ω_Λ and Ω we obtain in the case of an universe dominated by cold matter ($w = 0$) and vacuum energy

$$\Omega = \frac{2}{3}(1 + k\alpha^2) \qquad \Omega_\Lambda = \frac{1}{3}(1 + k\alpha^2) \tag{19}$$

so that we get $(\Omega, \Omega_\Lambda) = (0.66(1 + k\alpha^2), 0.33(1 + k\alpha^2))$. This result gives $\Omega/\Omega_\Lambda = 2$ instead of the value $\Omega/\Omega_\Lambda = 1/2$ usually obtained from recent observations. However, it is to be emphasized, firstly, that this latter *numerical* result has not be obtained from direct measurements but from interpretations using explicitely the standard model, and secondly that it comes from explaining recent observations of type Ia supernovae in terms of an accelerating expansion of the universe which will appear as unnecessary in our model. It is worth recalling (an example will be given in the next part) that the same observations can lead to different numerical results when interpreted with different theories.
Discussion : the above results call two remarks:
- The first one concerns the variation of Λ with respect to time and, more precisely, its $a(t)^{-2}$ variation in eq. (10). In this connection, let us note that cosmologies with a time variable cosmological "constant" have been extensively discussed in the litterature (Dolgov, 1983; Ford, 1985; Ratra & Peebles, 1988) and that it has been shown that they not only lead to no conflict with existing observations (Riess et al., 2004) but also that they are suggested by recent observations (Axenides & Perivolaropoulos, 2002; Baryshev et al., 2001; Chernin et al., 2000; Overduin & Cooperstock, 1998) for example to solve the so-called coincidence problem. More precisely, the $a(t)^{-2}$ variation of Λ has been shown to be in conformity with quantum gravity by Chen and Wu (Chen & Wu, 1990) and consistent with the result of Özer (Özer & Taha, 1987) and other authors (Khadekar & Butey, 2009; Mukhopadhyay et al., 2011; Ray et al., 2011) who obtained it in different contexts (S. Ray, for example, consider $\Lambda \sim H^2$ leading thus, in our case (i.e. when using eq.(1)), to $\Lambda \sim a^{-2}$).
- The second remark deals with the variation of masses with $a(t)$. That result could appear surprising, but, as explained in (Fahr & Heyl, 2007), it has yet been emphasized as possibly true from completely different reasonings by many physicists (Dirac, 1937; Einstein, 1917; Fahr & Heyl, 2006; Fahr & Zoennchen, 2006; Hoyle, 1990; 1992; Whitrow, 1946). It moreover appears, on one hand, that a scaling of masses with the cosmic scale factor is the most natural scale required to make the theory of general relativity conformally scale-invariant (H. Weyl's

requirement) and, on the other hand, that it expresses a necessary condition to extend the equivalence principle with respect to rotating reference systems to the whole universe (Mach's principle). We do not discuss here the possible explanations for such a variation of masses with the scale factor. They are discussed in (Fahr & Heyl, 2007). It still remains that here is an important point to explore more deeply.

4. The flatness problem

The observable universe is close to a flat Friedmann universe in which the energy density ρ_M takes the critical value ρ_c ($\Omega_0 \sim 1$) and the homogeneous spatial surfaces are euclidean. That result is all the more surprising that the flat Friedmann model is unstable. In fact, small deviations from $\Omega = 1$ must quickly grow as time increases. The observation of $\Omega_0 \sim 1$ now therefore requires extreme fine-tuning of the cosmological initial conditions at the beginning of the universe. The question has thus been asked to know how Ω could have been so highly fine-tuned in the past.
A solution of this problem has been proposed in the context of inflationary scenarios. In these scenarios, k has not to vanish and ρ may not start out close to ρ_c, but there is an early period of rapid growth of the universe in which Ω rapidly approachs unity. In few words, the flatness problem is thus resolved from the fact that when a geometry is scale up by a great factor then it appears locally flat.
In this part, we show that when using eq.(1), the universe dispays the same evolution as a flat universe and *must appear to be flat whatever it may be* (spherical or not). Within our model, it is consequently not surprising to find it to be flat:
In the conventional cosmology, the Friedmann eq.(3) gives in the case of a flat universe $(k = 0)$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} \tag{20}$$

That equation is to be compared with eq.(6) we have obtained by using eq.(1) in eq.(3):

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\frac{\rho}{1+k\alpha^2} + \frac{\Lambda}{3(1+k\alpha^2)}. \tag{21}$$

That eq.(21) *which describes both flat, closed or open universes following the value of k* is quite similar to eq.(20) which characterizes *a flat universe* in the standard cosmology. A comparison between these two equations thus shows that if eq.(1) is valid, the universe *must appear to be flat* whatever may be its geometric form (*whatever may be the value of k*) but with more or less matter than expected in the standard model following the value -1 or $+1$ of k since the density $\rho/(1 + k\alpha^2)$ appears in eq.(21) instead of ρ in eq.(20).
These unexpected results can easily be verified: let us consider a flat universe with energy matter density ρ' and cosmological constant Λ'. Whatever may be ρ' and Λ', we may write their values $\rho' = \rho/(1 + k\alpha^2)$ and $\Lambda' = \Lambda/(1 + k\alpha^2)$ with $k = \pm 1$. Using then the Robertson-Walker metric of a *flat universe*

$$ds^2 = -c^2dt^2 + a(t)^2\left(dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right) \tag{22}$$

and using $\rho/(1 + k\alpha^2)$ and $\Lambda/(1 + k\alpha^2)$ instead of ρ' and of Λ' respectively in the energy-momentum tensor of the Einstein equation, directly lead to eq.(21) which, using eq.(1), becomes:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \tag{23}$$

Although it has been obtained from equations describing a flat universe in usual cosmologies, we thus find the Friedmann equation which corresponds (when $k = \pm 1$) to *non flat universes*

5. The horizon and the smoothness problems

The horizon and the smoothness problems were identified in the 1970s. They point out that different regions of the universe which cannot have "contacted" each other due to the great distances between them, have nevertheless the same temperature and the same density to a high degree of accuracy (one part in one hundred thousand). Given the fact that the exchange of information or energy cannot take place at velocities greater than that of light such a result, which underlines the uncanny homogeneity of the universe across apparently causally disconnected regions, should not be possible. In the standard cosmology the problem is consequently to understand how the universe can be so smooth at large angular sizes, if different parts of it were never in contact or in communication [4]. That problem may have been answered by inflationary theory or by variable speed of light theory.

- Inflation provides the following explanation: before the inflationary area, the part of the universe that we can observe would have occupied a very tiny space and there would have been plenty of time for everything in this space to be homogeneized. However, it gives no clear explanations of why the universe would have then exponentially grown.

- The idea of varying speed of light cosmologies, as originally proposed by Moffat (Moffat, 1993) is that a higher propagation velocity for light in the cosmological past would have increased the propagation of causality so that all or most of the universe could thus have been causally connected.

In this part, we first show that, using eq.(1), the space-time of any observer is closed on itself so that there is no horizon problem. We then show that it is the same tiny part of the early universe that we see in every directions around us, so that it is quite logic to find the observed uniformity in terms of temperature and density of the cosmological microwaves background (CMB).

In the standard isotropic and homogeneous model of the universe, the Robertson-Waker metric may be written

$$ds^2 = -c^2dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right) = -c^2dt^2 + a(t)^2 dl^2 \tag{24}$$

where t is the co-moving proper time and where $d\Omega^2 = d\theta^2 + sin^2\theta \, d\phi^2$ is the metric on a two-sphere. More generally, that equation can also be written

$$ds^2 = -c^2dt^2 + a(t)^2 \left(d\chi^2 + \sigma^2(\chi) \, d\Omega^2 \right) \tag{25}$$

where χ is the standard radial coordinate. In that equation, the three possible elementary topologies are defined by $\sigma(\chi) = \chi$ for a flat universe, $\sigma(\chi) = \sin\chi$ for a closed universe and $\sigma(\chi) = \sinh\chi$ for an open universe. Using the line element (25) the coordinate of the particle horizon is obtained by writing that the light we detect now at $t = t_0$ must have been emitted at

[4] In conventional cosmologies, the horizon at time of last scattering ($z \sim 1100 - 1500$) now substends an angle of order 1.5 degree. Therefore no physical influence could have smoothed out initial inhomogeneities and brought points at a redshif $z = 1100 - 1500$ that are separated by more than a few degrees to the same temperature and density.

the beginning of the universe ($t = 0$). Noting that the path of light is given by setting $ds^2 = 0$ and taking light rays travelling in the radial direction, eq.(25) gives for the particle horizon

$$ds^2 = -c^2dt^2 + a(t)^2d\chi^2 = 0 \overset{\text{eq.(1)}}{\Longrightarrow} \chi_H = \pm \int_0^{t_0} \frac{cdt}{a(t)} \overset{\text{eq.(1)}}{=} \pm\alpha \int_0^{t_0} \frac{dt}{t} = \pm\infty \tag{26}$$

The integral does not converge and it can easily be shown that there is no particle horizon whatever may be the geometry of the universe ($k = -1, 0, +1$). Our model is thus horizon-free and allows the interactions to eventually homogenize the whole universe. Moreover, in the case of a spherical universe, it implies that the our "antipodes" can be seen by us now.
Our model could thus explain the observed uniformity in terms of temperature and density of the cosmological microwaves background radiation (CMB) without needing an inflationary expansion or a varying speed of light hypothesis. However, although it has no particle horizon so that all space points could have undergone physical interactions with each others, it shows that *the observed homogeneity does not come from such causal interactions, but from the fact that it is the same "tiny part" of the primitive universe that we see in any direction around us*:
Let us consider the case of *a spherical universe* ($k = +1$). Because of the symmetry, the rays that correspond to photons' world lines can be chosen so that $d\phi = d\theta = 0$. Solving then eq.(25) for light ($ds^2 = 0$) with these conditions and using eq.(1) give the radial coordinate χ as a function of time

$$ds^2 = -c^2dt^2 + a(t)^2d\chi^2 = 0 \overset{\text{eq.(1)}}{\Longrightarrow} \chi(t) = -\alpha \ln \frac{a(t)}{a(t_0)} = -\alpha \ln \frac{t}{t_0} \tag{27}$$

where we take for the initial condition $\chi = 0$ at the present value $t = t_0$ of the cosmic time and where χ increases toward the past. Eq.(27) shows that when using eq.(1), the space-time of any observer is closed on itself at early times defined by $\chi(t) = n\pi$. The first of these (our spatio-temporal antipode which is defined as the point where the radial coordinate $\chi(t)$ takes the value π), is denoted A on fig. 1.
- Since it can be seen identically in any direction around us, it can reasonably be identified to the source of the cosmic microwaves background radiation (CMB).
- Since it is then *the same "tiny part"* of the early universe that we can observe *in any directions around us* (the cosmic microwave background radiation arriving at the earth from all directions in the sky *does come* from the *same* tiny part of the early universe), *it is not surprising to observe a very high uniformity in terms of temperature and of density of the CMB.* [5]
- Neither inflation nor other hypotheses are consequently required to explain the high isotropy of the *CMB*.
All these results are shown on fig. 1, on which the logarithmic spiral (eq.27) corresponds to our past light cone (present observers are at point O). The point A represents our spatiotemporal antipode and thus corresponds to that "tiny part" of the universe that we observe in any directions around us (the "source" of the CMB).

[5] To give a simple example, consider we are on the north Pole of the Earth and that light must propagate by following Earth's surface. Looking at the farthest point of us, we would see the same point of the south Pole all around us and our background would then appear surprisingly homogeneous.

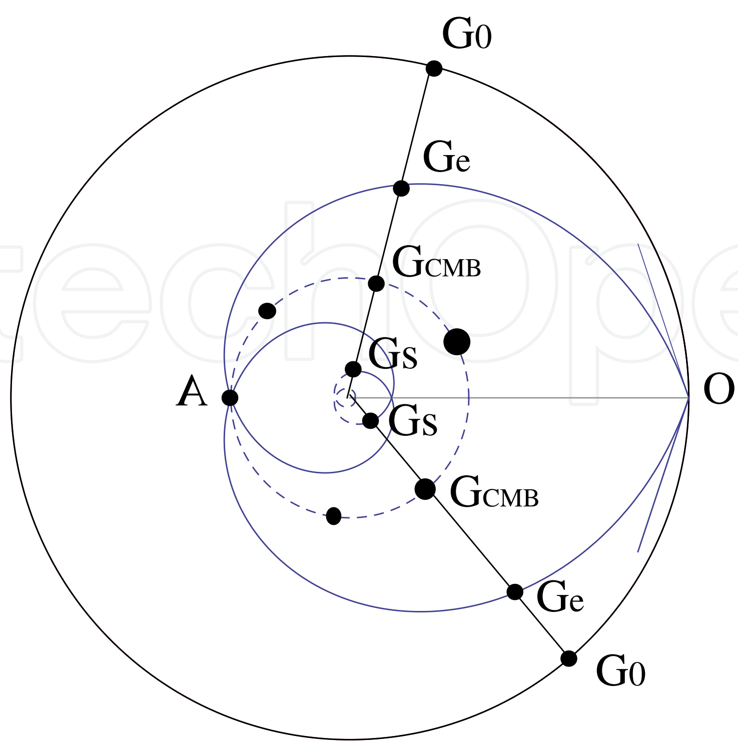

Fig. 1. We consider a spherical universe. The circle of radius $R(t_0)$ represents the universe at time t_0. The logarithmic spiral corresponds to the past light cone of the observer O, that is, to trajectories of all the light rays that he/she receives at $t = t_0$. The point A can be seen in any direction around O. It can thus be identified to the "source" of the CMB. The dashed circle corresponds to the universe at time t_{CMB} when the CMB was formed. A represents only a very tiny part of the universe at that time so that, *at that time*, the seeds of galaxies we observe now (points G_e) were not at A, but here and there on that dashed circle. They are *symbolically* illustrated by grey circles on the dashed circle. Note that they have not the same size. In fact at t_{CMB} the universe did not need to be homogeneous (and was certainly not) so that the seeds of these galaxies at that time could be quite different the ones from the others. The two radius are the world lines of two galaxies: G_{CMB} are galaxies (or their seeds) at time t_{CMB}; G_e gives their positions at the time t_e they emitted the light we receive now at t_0; G_0 are their current (and unknown) positions now. Because of the spiral form of the light cone, it could *theoretically* be possible (if universe was not opaque before t_{CMB}) to see *behind galaxies* G_e we see, their earlier seeds (the points G_s near the big-bang, on the galaxy world lines and on the light cone of O). However, no *radiation* coming from "before the last scattering surface" can be "visible" now *by definition*. May be other "isotropic points" $\chi = n\pi$ with $n > 1$ could be the "source" of isotropic cosmic particles backgrounds.

To be clear, and to show that we do see the same "tiny part A" of the universe in every direction we look, let us note that the spatial volume enclosed between the coordinate hyperspheres of radius $\chi_0 - \Delta\chi_0$ and χ_0 is

$$\Delta V_{\chi_0} = \int_{\chi_0 - \Delta\chi_0}^{\chi_0} \int_0^{\pi} \int_0^{2\pi} (a_0 e^{-\frac{\chi}{\alpha}})^3 \sin^2\chi \, \sin\theta \, d\chi \, d\theta \, d\phi \tag{28}$$

Making the change of variable $\chi \to -\alpha \ln \frac{t}{t_0}$ (eq.(27)), that expression becomes

$$\Delta V_t = \int_{t-\Delta t}^{t} \int_0^{\pi} \int_0^{2\pi} \frac{c^3 t^2}{\alpha^2} \sin^2(-\alpha \ln \frac{t}{t_0}) \sin\theta \, dt \, d\theta \, d\phi \tag{29}$$

It expresses the value of the spatial volume of the observed universe corresponding to past times between $t - \Delta t$ and t. Its expression being not simple, we only present its variation with respect to t on fig. 2. That figure shows that the farther back we look in the past, the smaller ΔV is, or, in other words, that the volume of the universe we progressively add to our observed universe when looking farther and farther tends toward 0 when t tends toward t_{CMB}.

Integrating eq.(29) over the past history of the universe, from t_{CMB} up to the present, we find the apparent volume V_{app} of the universe (the volume which is seen). Taking then $\alpha = 1$ or 0.3 (see at the end of that paragraph) this volume is only few percents of the universe at the present time t_0.

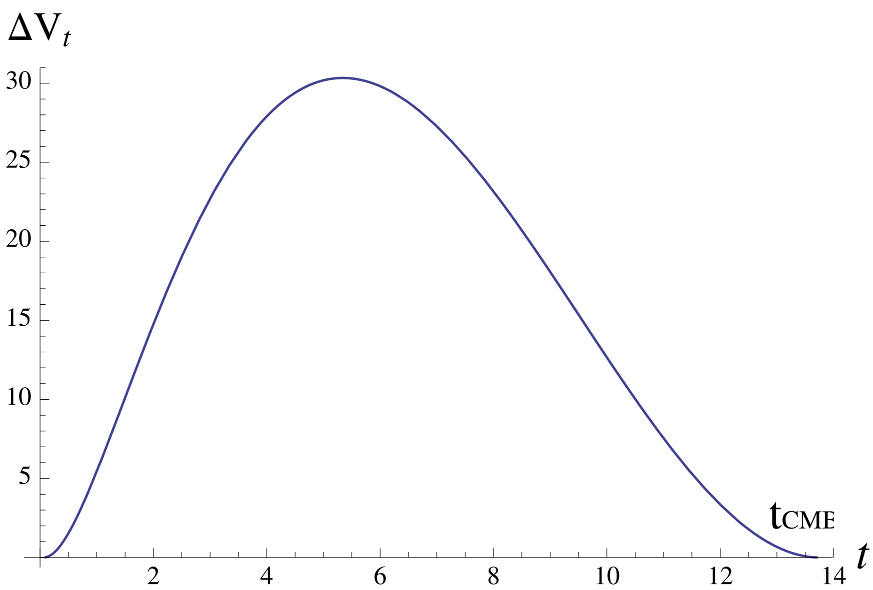

Fig. 2. ΔV_t (arbitrary units) versus time t (in billion years) : ΔV_t represents the volume of the universe we can observe now corresponding to past cosmic times between $t - \Delta t$ and t. We see that when time tends toward t_{CMB} (at about 13.7 Gy) that volume tends toward 0. In other words, as we look back in time, the spatial part of the observed volume of the universe that corresponds to times between $t - \Delta t$ and t, spreads out, then reaches a maximum and then starts to decrease to be all the more small that we approach t_{CMB}. That figure has been drawn by taking $c = 1$, $\alpha = 0.35$ and $\Delta t = 100$ million years. Time increases from $t_0 = 0$ (present time) to 14 billion years in the past.

It can be added that the identification of A with the source of the CMB *could* permit to calculate the value of α in eq.(1). Using both the right-hand part of eq.(27) with $\chi = \pi$ and taking the usual value given by nucleosynthesis for t_{CMB} (the radiation was created when atoms formed at around 360 000 years after the big-bang) thus would give $\alpha \sim 0.3$. With such a value, the theoretical value of Ω (16) would be $\Omega \sim 1.1$.

6. The cosmic microwaves background radiation and the small scale homogeneity

A related comment concerns the problem of the small-scale inhomogeneities needed to produce astronomical structures that are now observed. Cosmologists are usually searching in fluctuations of the CMB the density fluctuations that led to galaxies clusters and giant voids. In this context, the uniformity of the CMB leads to another problem of the standard cosmology: if the universe was so smooth, then how did anything form ? There must have been some bumps in the early universe that could grow to create the structures (galaxies and clusters of galaxies) we see locally. This problem no more exists when using eq.(1).

In our model, the small fluctuations that we observe now in the CMB *are not those which gave birth to the structures of the universe we can observe*. In fact, as shown on fig. 1, the galaxies which emitted the light we receive at $t = t_0$ were not at A at time $t_A = t_{CMB}$ (and consequently *their seeds were not in the CMB we observe*) but on the circle of radius $ct_A = ct_{CMB}$ which represents the universe at time t_{CMB}. In that light, the uniformity of the CMB not only is obvious (since it is the same tiny part of the universe that we see in any direction we look), but also it does not pose any problem to understand the cosmic structures we observe now. In fact nothing imposes that inhomogeneities of the universe at that time (that is on the dashed circle in fig. 1) be so small as thoses observed in its very tiny part A (that is to say in the CMB). We cannot know others regions (other than A) of the circle of radius $ct_A = ct_{CMB}$ and they, in fact, may be have overdense parts. Of course, it remains that studying the small inhomogeneities of the microwaves background may be useful to understand the past history of the universe.

We can note that it could *theoretically* be possible (if the universe was not opaque before t_{CMB}) to observe the seeds G_s (G_s for G_{seed}) which gave birth to galaxies and cosmic structures. The two images would then be observed the one behind the other (see fig. 1: behind galaxy G_e, and beyond the point A, the black points G_s are simultaneously on the world line of G_e and in our light cone).

We can also note that others points defined by $\chi = n\pi$ with $n > 1$ (n integer) are also "isotropic points" which could be "seen" as a homogeneous background in all directions around us (as does the CMB). Of course, they cannot correspond to light sources since the universe was *by definition* opaque before the "last scattering time". However they may correspond to sources of isotropic cosmic particles backgrounds.

Remark: These above results can be illustrated by mapping the 3-spatial sphere onto a 3-dimensional hyperplane by a 3-dimensional stereographic projection. Restricting ourselves to the spherical case (k=1) and using $\sigma(\chi) = \sin\chi$ in eq.(25) gives

$$ds^2 = -c^2dt^2 + a(t)^2(d\chi^2 + \sin^2\chi\, d\Omega^2) \tag{30}$$

Making then the change of variable $\mathcal{R} = 2\tan(\chi/2)$ we get the metric on the 3-hyperplane

$$ds^2 = -c^2dt^2 + \frac{a(t)^2}{(1+\frac{\mathcal{R}^2}{4})^2}(d\mathcal{R}^2 + \mathcal{R}^2 d\Omega^2) \tag{31}$$

Using it, it is straigthforward to show that all points at infinity are the image of the same antipodal point on S^3 so that we can understand that it is really the same point we see in all directions around us when looking at the CMB. Such a stereographic projection sends meridians of the 3-sphere (light world lines that do pass through the place of the observer) to straight lines on the hyperplane making their way toward the observer. Apart from a change of scale when looking increasingly far, the 3-hyperplane consequently corresponds more closely to the universe which is seen by each of us.

Also note that, whatever may be the value of k, using eq.(1) transforms the above metric (25) into a flat spacetime metric admitting Minkowski coordinates: writing $a = ct/\alpha$ from eq.(1) and $t = t_0 e^{u/\alpha}$ as suggested by eq.(27) gives $dt/t = du/\alpha$ so that eq.(25) gives

$$ds^2 = \frac{c^2\, t_0^2}{\alpha^2}\, e^{2u/\alpha}(-du^2 + d\chi^2 + \sigma^2(\chi)\, d\Omega^2) = a(t)^2(-du^2 + d\chi^2 + \sigma^2(\chi)\, d\Omega^2) \qquad (32)$$

where the term $a(t)^2 = c^2 t^2/\alpha^2$ represents the factor by which the scale changes in different locations. Using the conformal time u ($u = \int \frac{cdt}{a} = \alpha \int \frac{dt}{t}$) has thus the advantage of leading to a "conformally flat" metric.

7. Apparent luminosity and observation of type Ia supernovae

Following pioneering works related in (Norgaard-Nielsen et al., 1989), recent observations of type Ia supernovae (Perlmutter et al., 1999; Riess et al., 1998; 2004; Schwarzschild, 2004; Tonry et al., 2003; Wang et al., 2003) have provided a robust extension of the Hubble diagram to $1 < z < 1.8$. These results have shown that observations *cannot be fitted by using the usual distance modulus expression with* $\Lambda = 0$ *both for* $z < 1$ *and for* $z > 1$. To fit new data points at redshift 1.755 the standard model thus needs to consider that the expansion of the universe is accelerating, an effect that is generally attributed to the existence of an hypothetic "dark energy".

In that part, we show that eq.(1) leads to another expression for the distance-moduli which can fit all the data without needing for an acceleration of the expansion (fig. 3).

Distances are measured in terms of the "distance modulus" $\mu = m - M$ where m is the apparent magnitude of the source and M its absolute magnitude. The standard expression for the distance-moduli with respect to z can be found in (Tonry et al., 2003; Weinberg, 1972). Our aim here is to calculate μ in our model:

let an object be at cosmic radial coordinate χ and consider that the light that it emitted at cosmic time t_e is just reaching us at time t_0. The luminosity distance d_L of the object can be expressed as (Weinberg, 1972)

$$d_L = (\frac{a^2(t_o)}{a(t_e)})\chi = a(t_o)(1+z)\chi \qquad (33)$$

Using eq.(1) and noting H_0 the Hubble constant at the present time, that expression becomes

$$d_L = (\frac{c}{\alpha H_0})(1+z)\chi. \qquad (34)$$

χ can be obtained from calculations similar to that of eq.(26):

$$\chi = \int_{t_e}^{t_o} c\frac{dt}{a(t)} \overset{\text{eq.(1)}}{=} \alpha \ln(\frac{a(t_o)}{a(t_e)}) = \alpha \ln(1+z) \qquad (35)$$

so that

$$d_L = \frac{c}{H_0}(1+z)\ln(1+z) \qquad (36)$$

Expressing the distance modulus μ in terms of d_L then gives

$$\mu = 25 + 5\log d_L = 25 + 5\log(\frac{c}{H(t_0)}) + 5\log(1+z) + 5\log\ln(1+z) \qquad (37)$$

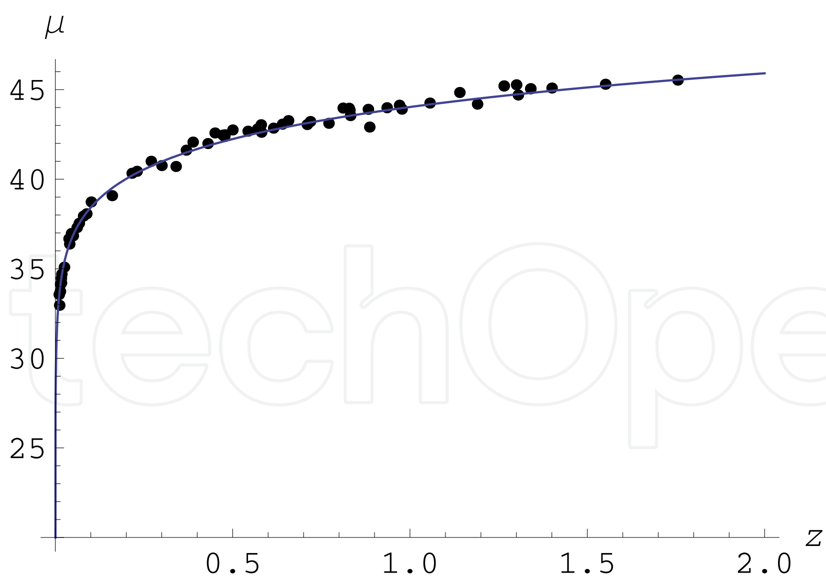

Fig. 3. Distance modulus μ vs redshift z in our model. The data points are taken from table 5 of the High-z Supernova Search Team (Riess et al., 2004). Whereas conventional cosmologies fail to fit all experimental data *both for* $z < 1$ *and for* $z > 1$, this is possible when using eq.(1). The full line, which represents predictions of the present model (eq.37), has been drawn by using $H_0 = 68\ km.s^{-1}.Mpc^{-1}$ (note a typewriting error in (Vigoureux et al., 2008) where we wrote $H_0 = 58\ km.s^{-1}.Mpc^{-1}$).

where c is in km.s^{-1} and H in km.s^{-1}.Mpc^{-1}.
The variation of μ with respect to z is shown on fig 3 . Fig. 3 has been obtained by using the value $H(t_0) = 68\,km/sec/Mpc$ which agrees well with usual determinations of the Hubble constant ($H(t_0) = 73 \pm 4km/sec/Mpc$). It shows that eq.(37) can permit to fit all experimental values in the whole range $z < 1$ and for $z > 1$ without any other hypothesis. The use of eq.(1) thus succeeds in explaining all the data without having to consider an acceleration of the expansion of the universe. To be clear, whereas in the standard model observations of type Ia supernovae lead to give the deceleration parameter q a value close to -0.5 for today and close to $+0.5$ for very high redshifts, we are able to explain all these observations by taking $q = 0$ at all times, as required by eq.(1).
Noting that different fitting of experimental points gives 63 < H < 70 at the present time and that eq.(1) leads to a scale factor proportional to time (and thus to $H(t) = 1/t$) the age of the universe in our model is about 14 billion years.
Remark: the above calculation uses the usual relation $a(t_0)/a(t_e) = 1 + z$ where $a(t_e)$ is the scale factor at the time of emission and where $a(t_0)$ is the scale factor at the time of observation. The redshift z undergone by radiation from a comoving object as it travels to us today is thus related to the scale factor at which it was emitted. It can easily be shown that this relation is still valid in our model and that it may consequently be used in calculations leading to eq.(37): using the Robertson-Walker metric (25), consider ligth reaching us (at $\chi = 0$) at the present time t_0 and emitted by a galaxy at a distant position $\chi = \chi_e$ and at a time t_e. Two crests arriving at t_0 and $t_0 + \Delta t_0$ were emitted at $t_e + \Delta t_e$. Since light has travelled radially inwards along a null geodesic, we get

$$c\int_{t_e}^{t_0} \frac{dt}{a(t)} \; = -\int_0^{\chi_e} d\chi = c\int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{dt}{a(t)} \tag{38}$$

Over the period of one cycle of a light wave, the scale factor is essentially a constant. This yields $\Delta t_e / a(t_e) = \Delta t_0 / a(t_0)$. Now, the observed and emitted wavelength λ_0 and λ_e are related to Δt_0 and Δt_e by $\lambda_{e,0} = c\Delta t_{e,0}$ so that the cosmological redshift $z = (\lambda_0 - \lambda_e)/\lambda_e = a_0/a_e - 1$ takes it usual expression and its use is consequently valid in the above calculation.

8. The cosmological constant and the cosmic coincidence problem

In the standard model, the cosmological constant has been introduced to account for anomalies observed in cosmological data and especially for explaining supernovae observations (Carroll, 2001). That introduction rises a new cosmological problem which is to explain the so-called *cosmic coincidence problem*, that is to understand why ρ_Λ (the dark energy density) is not only small but also, as current type Ia supernovae observations indicate, of the same order of magnitude as the present mass density ρ_M of the universe.

In fact, in usual models, the mass density ρ_M changes with time whereas the vacuum energy is constant. These two energy densities have thus evolved differently throughout the history of the universe and it is consequently very hard to explain why ρ_M and ρ_Λ would coincide today. Such a coincidence would require that the early universe had been very fine-tuned (Henttunen et al., 2006) but the underlaying models of particle physics cannot provide a natural explanation to the necessity of a so carefully fine-tuning.

That problem can be solved, arguably at least, by the anthropic principle argument. There are however other potential solutions based on physical arguments alone.

The most common is to consider that ρ_Λ really is not a constant. Peebles and Ratra, for example, (Peebles & Ratra, 1988; Ratra & Peebles, 1988) have thus considered a model in which the vacuum energy depends on a scalar field that changes as the universe expands. The vacuum is then treated as a form of matter and the cosmological constant thus turns out to be a measure of the energy density of the vacuum.

The quintessence model has then been proposed. It consists in a slowly varying energy component with a negative equation of state. That "dark energy" associated with the scalar field slowly evolves down its potential according to an attractor-like solution of the equation of motion, *regardless of the initial conditions* and can thus resolve the coincidence problem.

However the proposed solutions cannot satisfy *exactly* the *necessary* conditions $p_\Lambda = -\rho_\Lambda c^2$ and $\rho_\Lambda(t) \sim a(t)^{-n}$ with $n \neq 0$. They consequently cannot exactly generate the cosmological constant.

In the above part, we have shown that we do not need introducing a cosmological constant in order to explain type Ia supernovae observations. As explained just under eq.(7), we however need it to satisfy the Friedmann equations when adding them the additionnal constraint (1). Our aim in that part is to show that in the model we propose, we find not only that vacuum can *exactly* verify the condition $p_\Lambda = -\rho_\Lambda c^2$ but also that ρ_Λ and ρ_M have the same order of magnitude *at all times*.

To explain the origin of the cosmological constant, let us consider a quintessence fluid the density and the pressure of which (denoted ρ_Λ and p_Λ) being thus to be included in the Friedmann's equations. Assuming, as is usual, that the equation of state of that fluid has the form

$$p_\Lambda = \gamma \rho_\Lambda c^2 \tag{39}$$

where the constant γ, which has to be determined, must be negative to get an anti-gravity. The cosmological constant can thus be written

$$\Lambda = 8\pi G \rho_\Lambda \tag{40}$$

in eq.(3), and

$$\Lambda = -4\pi G \left(\rho_\Lambda + 3\frac{p_\Lambda}{c^2}\right) = -4\pi G \rho_\Lambda (1+3\gamma) \tag{41}$$

in eq.(4). Of course, these two expressions for Λ must be equal so that the two Friedmann equations are coherent (and consequently the quintessence fluid can generate *exactly* the cosmological constant Λ) *if and only if*

$$8\pi G \rho_\Lambda = -4\pi G \rho_\Lambda (1+3\gamma) \quad \Rightarrow \quad \gamma = -1 \tag{42}$$

or, by inserting this result inside eq.(39), *if and only if*

$$p_\Lambda = -\rho_\Lambda c^2 \tag{43}$$

So, the value $\gamma = -1$ is that which must be found. Apart from that "coherence reason", two other reasons can be considered in support of it: first, observations of supernovae indicate that $\gamma = -1.02^{+0.13}_{-0.19}$ (Riess et al., 2004); second, the value $\gamma = -1$ is a necessary and sufficient condition for the energy-momentum tensor of the vacuum to be Lorentz invariant [6] (see for example (Jordan, 2005)).
In that part we first show that the standard model cannot satisfy *exactly* that value and consequently that it cannot exactly generate the cosmological constant. We then show that the present model can generate it:
Let us first consider the conventional model ($\dot{a} \neq Cst$). Introducing eq.(40) and eq.(41) into Friedmann equations (3) and eq.(4) respectively gives

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G(\rho+\rho_\Lambda)}{3} - \frac{kc^2}{a^2} \tag{44}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho(1+3w) + \rho_\Lambda(1+3\gamma)\right) \tag{45}$$

Derivating eq.(44) and inserting eq.(45) into the result then leads to

$$-3\frac{\dot{a}}{a}\left(\rho(1+w) + \rho_\Lambda(1+\gamma)\right) = \dot{\rho} + \dot{\rho}_\Lambda \tag{46}$$

the solution of which for ρ_Λ is

$$\rho_\Lambda \propto \frac{1}{a^{3(\gamma+1)}} \tag{47}$$

Introducing the coherence condition $\gamma = -1$ (eq.42) into eq.(47) then leads to $\rho_\Lambda = Cst$, and to $\Lambda = Cst$. These results make Λ to be a pure constant but in that case the quintessence fluid does not dilute when the universe expands. The key problem then remains to explain the cosmic coincidence: if ρ_Λ is constant whereas ρ_M varies, why these two quantities should be comparable today ? This shows, that, in the usual cosmology
- if the cosmic fluid can generate *exactly* the cosmological constant ($\gamma = -1$ *exactly*), then $\rho_\Lambda = Cst$ and consequently the standard model cannot explain the cosmic coincidence, and
- if the standard model want to explain the cosmic coincidence (ρ_Λ does vary with respect

[6] The vacuum must be Lorentz invariant or one would have a preferred frame. The stress-energy tensor of the vacuum is diagonal and this tensor must be invariant. The only Lorentz invariant nonzero rank tensor is the metric $\mathrm{diag}(-1,1,1,1)$ in a local inertial frame so if the vacuum energy density is non-zero the pressure has to be $-\rho c^2$.

to $a(t)$), then, eq.(47) shows that ($\gamma \neq -1$) and consequently it cannot *exactly* generate the cosmological constant.
In other words, in usual theories, the condition $\gamma = -1$ is not compatible with the other condition $\rho_\Lambda \sim a(t)^{-n}$ ($n \neq 0$) and thus provides no answer to the fine-tuning problem.
Contrarily to what is found with these theories, the two conditions $\gamma = -1$ (or $p_\Lambda = -\rho_\Lambda c^2$) and $\rho_\Lambda \propto R^{-n}$ with $n \neq 0$ can be *simultaneously* fulfilled when using eq.(1):
When using eq.(1) and eqs.(40, 41), the two Friedmann's equations (6) and (7) can be written:

$$\left(\frac{c}{\alpha a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_M + \rho_\Lambda}{1 + k\alpha^2} \tag{48}$$

$$0 = \rho_M(1 + 3w) + \rho_\Lambda(1 + 3\gamma) \tag{49}$$

It is obvious that these two equations do have solutions even when $\gamma = -1$. They are

$$\rho_M = \frac{c^2}{8\pi G}\frac{(1 + k\alpha^2)}{\alpha^2}\frac{2}{1 + w}\frac{1}{a^2} \tag{50}$$

$$\rho_\Lambda = \frac{c^2}{8\pi G}\frac{(1 + k\alpha^2)}{\alpha^2}\frac{1 + 3w}{1 + w}\frac{1}{a^2} \tag{51}$$

As discussed in section 3, such a variation of ρ_Λ and of the cosmological "constant" term as a^{-2} has been shown to lead to no conflict with existing observations (Riess et al., 2004) and to be in conformity with quantum cosmology (Chen & Wu, 1990).
We thus have

$$\rho \propto \frac{1}{a^2} \qquad \rho_\Lambda \propto \frac{1}{a^2} \qquad \Lambda \propto \frac{1}{a^2} \tag{52}$$

whatever may be the equation of state of the cosmic fluid. Contrarily to what is obtained in the standard cosmology, the present model thus do fulfil the two conditions $\gamma = -1$ and $\rho_\Lambda \propto a^{-n}$ (with $n \neq 0$) simultaneously. It can consequently explain the origin of the cosmological constant with a quintessence fluid which dilutes when the universe expands. It can also solve the problem of the "cosmic coincidence": in this model, the "cosmological constant" in fact varies in the same way as ρ_M and has always been comparable to it. Since the two fluids dilute in the same way and evolve together, it is not suprising to find that they can coincide now.
Moreover, the above equations (50, 51) also show
- that the two energy densities ρ_M and ρ_Λ are exactly equal when $w = \frac{1}{3}$ that is to say in the radiation dominated epoch.
- that $\rho_M = 2\rho_\Lambda$ in the matter epoch.
Let us also recall that eq.(1) can also explain why the mass density of the cosmic fluid is so near the critical density ρ_c: using eq.(15) in fact gives

$$\rho_\Lambda = \rho_c\frac{(1 + k\alpha^2)}{3}\frac{1 + 3w}{1 + w} \overset{w=0}{=} \rho_c\frac{(1 + k\alpha^2)}{3} \tag{53}$$

$$\rho_M = \rho_c\frac{(1 + k\alpha^2)}{3}\frac{2}{1 + w} \overset{w=0}{=} \rho_c\frac{2(1 + k\alpha^2)}{3} \tag{54}$$

so that

$$\rho_\Lambda \sim \rho_c \qquad \rho_{total} = \rho_c(1 + k\alpha^2) \tag{55}$$

9. Conclusions

We advocate the possibility that the universal relations existing between space and time in the so-called "speed of light" and in the expansion of the universe are two aspects of a same phenomenon:
Introducing eq.(1) as an additionnal constraint to solve the Friedmann equations leads to interesting ways to explain number of unanswered problems of the standard cosmology without needing usual hypotheses as, for example, the present accelerating expansion of the universe or the inflation scenario which assumes that the universe went through an early period of exponential growth without worrying about how this came about.
We have shown how eq.(1) can solve the flatness and the horizon problems, the problem of the observed uniformity in term of temperature and density of the cosmological background radiation, the small-scale inhomogeneity problem (with the one of the seeds of galaxies and of cosmic structures) and the cosmic coincidence problem. Reconsidering the Hubble diagram of distance moduli and redshifts as obtained from recent observations of type Ia supernovae, we have also shown that all the new data can be understood without needing an accelerating universe.
Whereas a cosmological constant is useless in the present model to explain such observations, we however need it for coherence in Friedmann's equations. Concerning that point, one appealing feature of our results is that eq.(1) permits to accommodate simultaneously the equation of state $p_\Lambda = -\rho_\Lambda c^2$ of the quintessence fluid which generates the cosmological constant Λ (so that it can *perfectly generate* the cosmological constant), with a varying density $\rho_\Lambda \propto a^{-n}$ (with $n = 2$ in our case) which appears to be a necessary condition to avoid the cosmic coincidence problem.
The present model also explains why ρ, ρ_c and ρ_Λ are comparable today. At this point, let us recall (Vigoureux et al., 2008) that, with eq.(1), a spherical universe (for example) displays the same evolution as a flat universe in the standard model (section 4).
One of our results may however appear unnatural: the total mass M of the universe would scale with $a(t)$. Although such a variation has been shown to be the most natural one to extend the equivalence principle with respect to rotating reference frame to the whole universe (Mach's principle); although it appears to be the most natural scale to fulfil the Weyl's requirement of conformally scale invariance; although it has also been emphasized as possibly true by physicists as Dirac, Einstein or Hoyle as discussed in (Fahr & Heyl, 2007), it however remains to be carefully studied.
Eq.(1) may provide an alternative way to solve the standard cosmological problems and our results appear compatible with astronomical observations. It leads however to some numerical values which may seem contradict with some of these (for example, concerning the proportion of ordinary matter and of black matter, we find $(\Omega_M, \Omega_\Lambda) = (0.66, 0.33)$ when usual experiments would rather give $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$). However, one has to be careful before concluding such a question: as liked to recall Einstein, theory and observations are interdependent and there are no observation which can be directly interpretable without referring to a given theory. To be able to construct a picture of the world, we must interpret the observational data *within a given theory* and we may occasionally forget that we use theories all the time while we may think of us as giving observational results independently of any theory. Because of this, our results cannot be too quickly compared with numerical values *deduced from* the standard big-bang cosmology. An example of this is given by looking at eqs.(20, 21) showing that a flat universe corresponding to a given value of the energy density of matter ρ in usual cosmology, may correspond to a spherical universe with another density $\rho/(1 + k\alpha^2)$

in our model. Another example is given by the interpretations of observations of type Ia supernovae: the values $q \sim -0.5$ for today and $q \sim +0.5$ obtained in the standard model for very high redshifts are not independant of any theory. On the contrary, they correspond to the values that must be used *inside the usual theory* to explain observations. As explained above, *in our model*, these same observations lead to the quite different value $q = 0$ at all times.

Eq.(1) can thus solve usual problems of cosmology. An important remark about this is that these latter have been solved by using one single hypothesis. It is in fact to be emphasized that *all our results have been obtained from the only hypothesis that the speed of light is related to the expansion of the universe*. An important feature of eq.(1) is thus *its unifying power*. Eq.(1) gives unity to number of results which, for some of them, have yet been obtained by other authors by introducing many quite different, and sometimes *ad hoc*, hypotheses.

In order to illustrate the importance of such an unifying power of our proposition, let us present a brief outline of some of the wide variety of hypotheses which have yet been used to solve one or the other problem:

The variation of ρ and Λ as a^{-2} in our equations (9, 10), has yet been obtained from some very general arguments in line with quantum cosmology and with dimensional considerations (Chen & Wu, 1990) or by postulating the invariance of equations under a change of scale (Canuto et al., 1977). It has also been directly postulated to explore its consequences as did, for example, Berman (Berman, 1991) who made the hypothesis that $\Lambda(t) = Bt^{-2}$ and $\rho(t) = At^{-2}$ (leading then to some of our results). Others (Lima & Carvalho, 1994; Mukhopadhyay et al., 2011) consider the phenomenological assumption $\Lambda \sim H^2$ (Overduin & Cooperstock, 1998). Fahr and Heyl (Fahr & Heyl, 2007) also make the assumption that the total mass density of the universe (matter and vacuum) scales with a^{-2} and find the relation $c = \dot{a}(t)$ in the particular case $k = 0$. They then show that such a scaling abolishes the horizon problem and that the cosmic vacuum energy density can then be reconcilied with its theoretical expected value. Others postulated the Mach's principle or, as did Özer (Özer & Taha, 1987), make the assumption that the equality $\rho_M = \rho_c$ is a time-independant feature of the universe from which they deduce $\Lambda \sim a(t)^{-2}$. Similarly it has also been postulated the ratio ρ_Λ/ρ_M to be constant in time (Freese et al., 1987)... In a similar way Bacinich and Kriz (Bacinich & Kriz, 1995) found the same logarithmic spiral form of the light cone from the quite different consideration of a local conservation of the CMB flux...

Eq.(1) may not only unify different results which can have been proposed from number of different hypotheses, but it may also illustrate and unify different questions about light (see the introduction). It may thus interest other fields of physics such as special relativity, quantum theory or electromagnetism.

In its light

- the energy $E = m_0c^2$ of a given rest mass m_0 can be seen as originating from the expansion of the universe: it would in fact correspond to a form of "comoving kinetic energy" of any comoving object carried away by the expansion of the universe ($E = m_0c^2 = \alpha^2 m_0 \dot{a}^2$);
- by connecting the light phenomenon (and more generally electromagnetic radiations) to the expansion of the universe, eq.(1) also *illustrates* the assumption that the speed of electromagnetic radiations is indifferent to both its emitter and its absorber and that it can be neither compounded with that of an object nor transformed away by the choice of a suitable reference frame. This independance of place (homogeneity), direction (isotropy), source and detector motions can be understood when connecting c to the expansion of the universe. It can thus be *illustrated* by imagining an insect moving on an expanding balloon: the velocity of the insect is obviously independant of that of the ballon expansion and it is not because the

insect would go faster or slower that the balloon would expand differently.
In these views, an essential feature of eq.(1) is perhaps to suggest a cosmic interpretation of light phenomena which would thus *essentially* appear essentially as a consequence of the expansion of the universe rather than as a propagating phenomenon.
The expansion of the universe in fact induces two kinds of change in the universe : a growth of its radius (cdt) and a growth of its circumference ($dx = ad\chi$), the second being a consequence of the first. Both are equivalent so that $cdt = ad\chi = dx$. That equivalence makes the expansion to appear in space although it is *essentially* a time phenomenon. In the same way, light appears to propagate into space although its 4-velocity (c, 0, 0, 0) clearly expresses *its temporal nature*. In other words, light, and electromagnetic phenomena, are carried by the time axis (the radius of the universe) but, because of the expansion, they *appear* to propagate *into* space (and so they appear "diagonally" in space-time diagrams). To be clear, consider a comoving point A in the expanding universe. Because of the expansion, although it has no dynamical motion, its relation to us *in our ligth cone* is expressed by the time extension of the distance $D = \int a(t)d\chi = \int cdt = c\Delta t$ so that its instant relation to us *appears* to propagate at velocity $dD/dt = c$ whatever may be its comoving coordinates.
This may perhaps throw light on current and fondamental problems that are the time symmetry of Maxwell's equations, the emission theory or other problems in quantum theory where considering light as a *propagating* phenomenon often leads to paradoxes.
The complete time symmetry of Maxwell's equations (whereas the observed electromagnetic phenomena are *asymmetric* with respect to time) in fact tells us that electromagnetic interaction proceeds not only forward in time (from the emitter to the detector), but also backwards in time (from the detector to the emitter). In practice, retarded fields are selected because they appear to correspond to reality, whereas advanced fields are discarded on the grounds that they are contrary to experiments. However, it seems we need it on a theoretical ground: purely retarded solutions of Maxwell's equations embodies an electrodynamical arrow of time not recognized by the equations themselves. That question has been asked for a long time: it is generally assumed that a radiating body emits light in every direction, quite regardless of whether there are near or distant objects which may ultimately absorb that light (in other words, it radiates "into space"). However, Tetrode, yet in 1922, (Tetrode, 1922) made the assumption that an atom never emits light except to another atom so that the emitter and the absorber both act in the emission process, the first one to emit light and the other one "to tell" the emitter that it is ready to absorb. He thus proposed to eliminate the idea of a mere emission of light and substituted the idea of a process of exchange of energy between two definite atoms or molecules. Such propositions were reconsidered by G. N. Lewis in 1926, and then, in 1927, by Bridgman *who held that it is wrong to speak of light as something travelling*. Their paper gave birth to the Wheeler-Feynman absorber theory of radiation (Wheeler & Feynman, 1945) in which there is no radiation proper (see also (Hoyle & Narlikar, 1995)). They thus anticipated a quantitative theory of electrodynamics using both retarded and advanced potential the interest of which is perhaps to try to give both to the emitter and the absorber the same importance.
Such a use of advanced waves may be somewhat provoking. In fact, it is. However it seems possible to consider such a dual interaction between an emitter and a detector as the translation in the langage of classical waves physics of what may reallycorrespond to an elongation (a dilation) phenomenon (as in the stretching of an elastic band where the "interaction" between the two ends cannot be accredited to one or to the other end). As written above, because of the expansion of the universe, the relation of two comoving objects (the two

ends of the elastic band in our example) in fact appears to us *as if* a signal was propagating between them at velocity c. Such a description would suppress the provoking acausal action from an absorber to an emitter.
As expected by all the above authors, our aim is thus to note that connecting the light phenomenon to the expansion of the universe may perhaps permit to consider light as an effect of the stretching of the spacetime rather than as a propagating phenomenon. May be eq.(1), could thus also open a way to reconsider the origin of electromagnetism.

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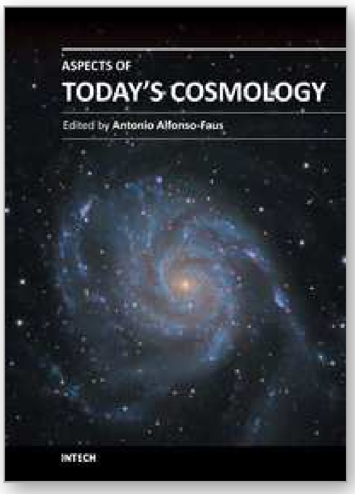

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