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Simulation of Models and BER Performances of DWT-OFDM versus FFT-OFDM

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1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation system. The transmission channel is divided into a number of subchannel in which each subchannel is assigned a subcarrier. Conventional OFDM systems use IFFT and FFT algorithms at the transmitter and receiver respectively to multiplex the signals and transmit them simultaneously over a number of subcarriers. The system employs guard intervals or cyclic prefixes (CP) so that the delay spread of the channel becomes longer than the channel impulse response (Peled & Ruiz, 1980; Bahai & Saltzberg, 1999; Kalet, 1994; Beek et al.,1999; Bingham, 1990; Nee and Prasad, 2000). The system must make sure that the cyclic prefix is a small fraction of the per carrier symbol duration (Beek et al.,1999; Steendam & Moeneclaey, 1999). The purpose of employing the CP is to minimize inter-symbol interference (ISI). However a CP reduces the power efficiency and data throughput. The CP also has the disadvantage of reducing the spectral containment of the channels (Ahmed, 2000; Dilmirghani & Ghavami, 2007, 2008). Due to these issues, an alternative method is to use the wavelet transform to replace the IFFT and FFT blocks (Ahmed, 2000; Dilmirghani & Ghavami, 2007, 2008; Akansu & Xueming, 1998; Sandberg & Tzannes, 1995). The wavelet transform is referred as Discrete Wavelet Transform OFDM (DWT-OFDM). By using the transform, the spectral containment of the channels is better since they are not using CP (Ahmed, 2000; Dilmirghani & Ghavami, 2007, 2008). The illustration of the superior subchannel containment attributes in wavelet has been described in detailed by (Sandberg & Tzannes, 1995) as compared to Fourier. The wavelet transform also employs Low Pass Filter (LPF) and High Pass Filter (HPF) operating as Quadrature Mirror Filters satisfying perfect reconstruction and orthonormal bases properties. It uses filter coefficients as approximate and detail in LPF and HPF respectively. The approximated coefficients is sometimes referred to as scaling coefficients, whereas, the detailed is referred to wavelet coefficients (Abdullah et al., 2009; Weeks, 2007). In some literatures, these two filters are also called subband coding since the signals are divided into sub-signals of low and high frequencies respectively. The purpose of this chapter is to show the simulation study of using the Matrices Laboratory (MATLAB) on the wavelet based OFDM particularly DWT-

OFDM as alternative substitutions for Fourier based OFDM. MATLAB is preferred for this approach because it offers very powerful matrices calculation with wide range of enriched toolboxes and simulation tools. To the best of the authors' knowledge, there is no study on the descriptive procedures of simulations using MATLAB with regards of flexible transformed models in an OFDM system, especially when dealing with wavelet transform. Therefore, this chapter is divided into three main sections: section 2 will explain conventional FFT-OFDM, section 3 will describe in detail the models for DWT-OFDM, and section 4 will discuss the Bit Error rate (BER) result regarding those two transformed platforms, DWT-OFDM versus FFT-OFDM.

2. Fourier-based OFDM

A typical block diagram of an OFDM system is shown in Figure 1. The inverse and forward blocks can be FFT-based or DWT-based OFDM.

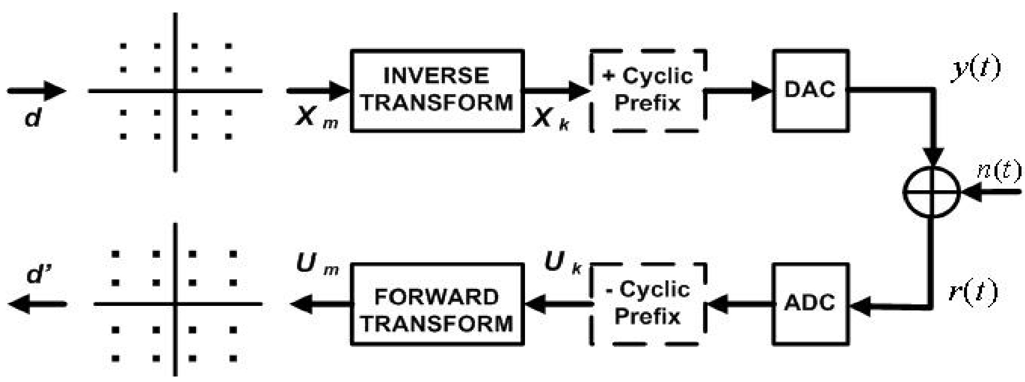

Fig. 1. A Typical model of an OFDM transceiver with inverse and forward transformed blocks which can be substituted as FFT-OFDM or DWT-OFDM.

The system model for FFT-based OFDM will not be discussed in detail as it is well known in the literature. Thus, we merely present a brief description about it. The data d_k is first being processed by a constellation mapping. *M*-ary QAM modulator is used for this work to map the raw binary data to appropriate QAM symbols. These symbols are then input into the IFFT block. This involves taking *N* parallel streams of QAM symbols (*N* being the number of sub-carriers used in the transmission of the data) and performing an IFFT operation on this parallel stream. The output in discrete time domain is as follows:

$$X_k(n) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_m(i) e^{j2\pi \frac{n}{N} i} \tag{1}$$

Where $x_k(n) \mid 0 \le n \le N-1$, is a sequence in the discrete time domain and $X_m(i) \mid 0 \le i \le N-1$ are complex numbers in the discrete frequency domain. The cyclic prefix (CP) is lastly added before transmission to minimize the inter-symbol interference (ISI). At the receiver, the process is reversed to obtain the decoded data. The CP is removed to obtain the data in the discrete time domain and then processed to FFT for data recovery. The output of the FFT in the frequency domain is as follows:

$$U_m(i) = \sum_{i=0}^{N-1} U_k(n) e^{j2\pi \frac{n}{N} i} \tag{2}$$

3. Wavelet-based OFDM

As mentioned in the previous section, the inverse and forward block transforms are flexible and can be substituted with FFT or DWT-OFDM. We have discussed briefly about FFT-OFDM. Thus, this section will describe wavelet based OFDM particularly about DWT-OFDM transceiver. This section is divided into three parts: a description of the DWT-OFDM transmitter and receiver models as well as the Perfect Reconstruction properties' discussion.

3.1 Discrete Wavelet Transform (DWT) transmitter

From Figure 1, it is obvious that the transmitter first uses a 16 QAM digital modulator which maps the serial bits d into the OFDM symbols X_m, within N parallel data stream $X_m(i)$ where $X_m(i)$ $|0 \leq i \leq N-1$. The main task of the transmitter is to perform the discrete wavelet modulation by constructing orthonormal wavelets. Each $X_m(i)$ is first converted to serial representation having a vector *xx* which will next be transposed into *CA* as shown in details as in Figure 2. This means that *CA* not only its imaginary part has inverting signs but also its form is changed to a parallel matrix. Then, the signal is up-sampled and filtered by the LPF coefficients or namely as approximated coefficients. This coefficients are also called scaling coefficients. Since our aim is to have low frequency signals, the modulated signals *xx* perform circular convolution with LPF filter whereas the HPF filter also perform the convolution with zeroes padding signals *CD* respectively. Note that the HPF filter contains detailed coefficients or wavelet coefficients. Different wavelet families have different filter length and values of approximated and detailed coefficients. Both of these filters have to satisfy orthonormal bases in order to operate as wavelet transform. The number of *CA* and *CD* depends on the OFDM subcarriers *N*. Samples of this processing signals *CA* and *CD* that pass through this block model is shown in Figure 4. The above mentioned signals are simulated using MATLAB command $[X_k]$ = *idwt*(*CA;CD;wv*) where *wv* is the type of wavelet family.

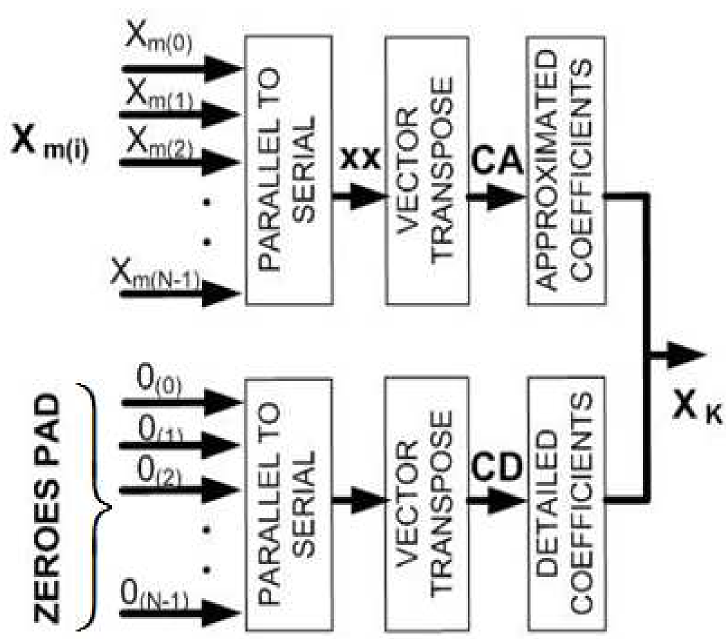

Fig. 2. Discrete Wavelet Transform (DWT)-OFDM transmitter model.

The detailed and approximated coefficients must be orthogonal and normal to each other. By assigning g as LPF filter coefficients and h as HPF filter coefficients, the orthonormal bases can be satisfied via four possible ways (Weeks, 2007): $<g, g^*>= 1$, $<h, h^*>= 1$, $<g, h^*>= 0$ and $<h, g^*>= 0$. The symbol * indicates its conjugate, and the symbol < , > is referring to the dot product. The result which yields to 1 is related to the normal property whereas the result yielding to 0 is for orthogonal property accordingly.

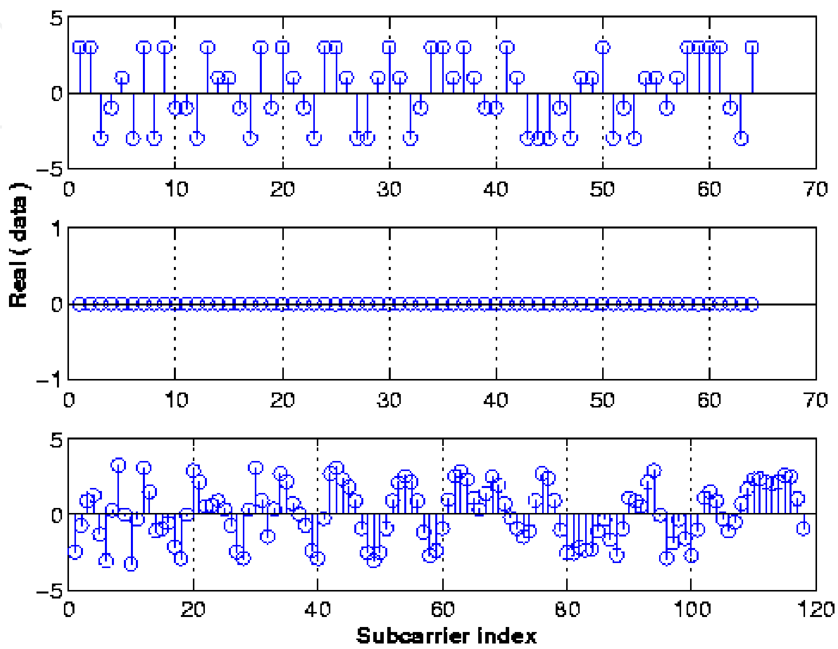

Fig. 3. The processed signals of one symbol DWT-OFDM system using bior5.5 in DWT transmitter. Top: data CA, Middle: data CD, Bottom: data X_k, corresponding to Figure 2.

Both filters are also assumed to have perfect reconstruction property. The input and output of the two filters are expected to be the same. A further discussion can be found in section 3.3.

3.2 Discrete Wavelet Transform (DWT) receiver

The DWT receiver is the reverse process which is simulated using the MATLAB command [*ca; cd*] = *dwt*(U_k;*wv*). The receiver system model that processes the data *ca*, *cd* and U_k is shown in Figure 4. The parameter *wv* is to indicate the wavelet family that is used in this simulation. U_k is the front-end receiver data. This data is decomposed into two filters, high and low pass filters corresponding to detailed and approximated coefficients accordingly. The *ca* signal which is the output of the approximated coefficients or low pass filter will finally be processed to the QAM demodulator for data recovery. To perform that operation, data is first transposed before converting into parallel representation. The output $U_{m(i)}$ is passed to QAM demodulator. The index i depends on the number of OFDM subcarriers. The data *cd* is explained next. Due to the effect of CD data generated in the transmitter, U_k has some zeroes elements which is decomposed as the detailed coefficients. The signal output of these coefficients is *cd*. Comparing to *ca*, the *cd* signal is discarded because it does not contain any useful information instead. Samples of this processing signals that pass through the DWT-OFDM receiver model is shown in Figure 5.

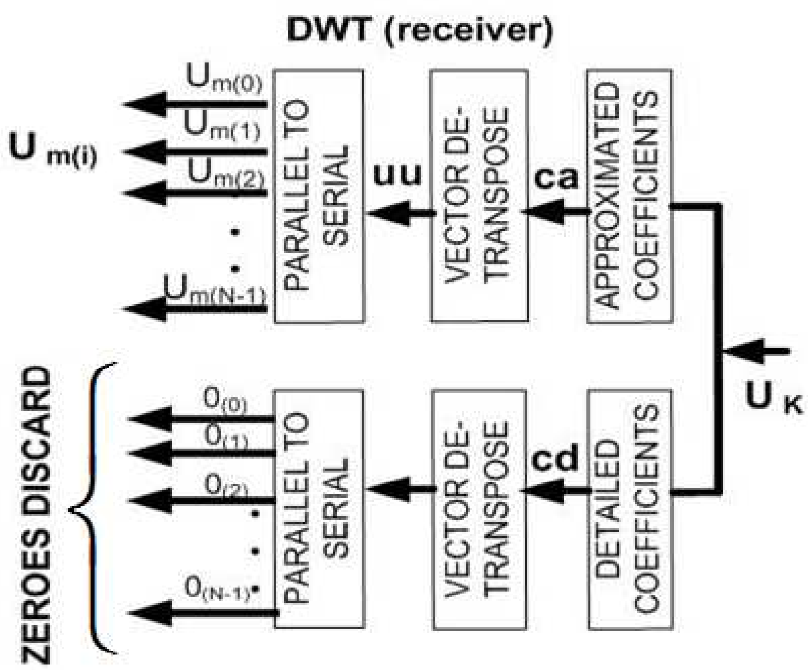

Fig. 4. Discrete Wavelet Transform (DWT)-OFDM receiver model.

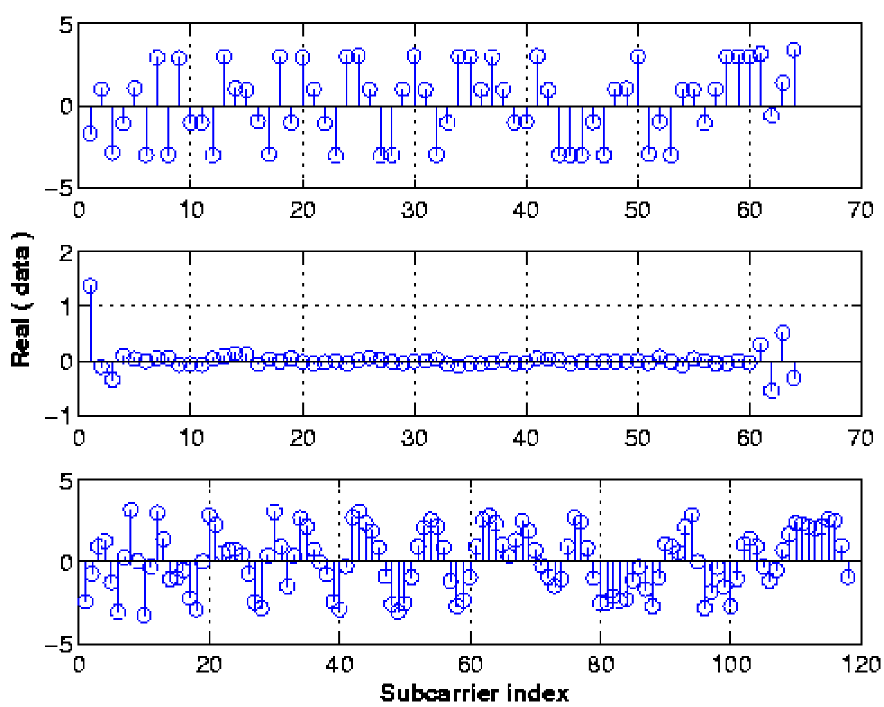

Fig. 5. The processed signals of one symbol DWT-OFDM system using bior5.5 in DWT receiver. Top: data ca. Middle: data cd. Bottom: data U_k, corresponding to Figure 4.

3.3 Perfect reconstruction

A block diagram of perfect reconstruction (PR) system operation is illustrated in Figure 6. The PR property is performed by a two-channel filter bank which is represented by the LPF and HPF. The first level of analysis filter in the receiver part can be folded and the decimator and the expander are cancelled out by each other.

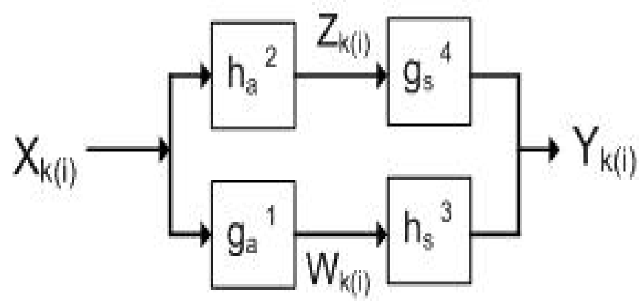

Fig. 6. A simple and modified model of two-channel filter bank illustrating a perfect reconstruction property with the superscript number is referring to the steps.

To satisfy a perfect reconstruction operation, the output $Y_k(i)$ is expected to be the same as $X_k(i)$. With the exception of a time delay, the input can be considered as $Y_k(i) = X_k(i\text{-}n)$ where n can be substituted as 1 to describe this simple task. The steps to perform the mathematical operation of PR can be summarized as follows (Weeks, 2007):

1. Selecting the filter coefficients for g_a, i.e., a and b. Thus, $g_a = \{a; b\}$.
2. h_a is a reversed version of g_a with every other value negated. Thus, $h_a = \{b; -a\}$. If the system has 4 filter coefficients with $g_a = \{a; b; c; d\}$, then $h_a = \{d; -c; b; -a\}$.
3. h_s is the reversed version of g_a, thus $h_s = \{b; a\}$.
4. g_s is also a reversed version of h_a, therefore $g_s = \{-a; b\}$.

The above steps can be rewritten as follows:

$$g_a = \{a,b\},\ h_a = \{b,-a\},\ h_s = \{b,a\},\ g_s = \{-a,b\} \tag{3}$$

Considering that the input with delay are applied to h_a and g_a in Figure 4, then the output of these filters are

$$Z_k(i) = b(X_k(i) - a(X_k(i\text{-}1)) \tag{4}$$

$$W_k(i) = a(X_k(i) + b(X_k(i\text{-}1)) \tag{5}$$

Considering also that $Z_k(i)$ and $W_k(i)$ are delayed by 1, then i can be replaced by (i-1) as follows

$$Z_k(i\text{-}1) = a(X_k(i\text{-}1) + b(X_k(i\text{-}2)) \tag{6}$$

$$W_k(i\text{-}1) = b(X_k(i\text{-}1) - a(X_k(i\text{-}2)) \tag{7}$$

The output $Y_k(i)$ can be written as

$$Y_k(i) = g_s Z_k(i) + h_s W_k(i) \tag{8}$$

or,

$$Y_k(i) = -aZ_k(i) + bZ_k(i\text{-}1) + bW_k(i) + aW_k(i\text{-}1) \tag{9}$$

Substituting equations (5), (6), (7) and (8) into (9) yields to

$$Y_k(i) = 2(a2 + b2)X_k(i\text{-}1) \tag{10}$$

The output $Y_k(i)$ is the same as the input $X_k(i)$ except that it is delayed by 1 if we substitute the coefficient factor $2(a2 + b2)$ by 1. The PR condition is satisfied.

4. Simulation results

Simulation variables and their matrix values are shown in Table I. The number of samples for the subcarriers N is 64, and the number of samples for the symbols ns is 1000. Data is similar between FFT and DWT OFDM in all parameters except the multiplexed one. For DWT-OFDM, it is required the transmitted signal to have double the data of FFT-OFDM. This is due to the fact that the DWT transmitter has zeroes padding component. An element value in the table that has a multiplier is referred to its matrix representation of row and column. If the element has 64 x 1000, it means that it has 64 numbers of rows and 1000 numbers of columns.

	Variables and Parameters	FFT-OFDM	DWT-OFDM
Minimum requirement	Subcarriers	64	64
	OFDM symbols	1000	1000
Transmitter	input binary generated	64 x 1000	64 x 1000
	parallel transmitted data	64 x 1000	64 x 1000
	serial transmitted data	1 x 64000	1 x 64000
	multiplexed data transmitted	64000 x 1	128000 x 1
Receiver	multiplexed data received	64000 x 1	128000 x 1
	serial received data	1 x 64000	1 x 64000
	parallel received data	64 x 1000	64 x 1000
	output binary recovered	64 x 1000	64 x 1000

Table 1. Simulation variables and their matrix values.

The curves in Figure 8 could have been better if we used more number of samples for the symbols. However, this yields longer time of running the simulations. Other variables are listed according to their use as in Figures 1, 2 and 3. Figure 7 shows the OFDM symbols in time domain for the two transformed platforms FFT and DWT. Some of the simulation parameters related to this figure are: the OFDM symbol period T_o = 9 ms, the total simulation time $t = 10 \times T_o$ = 90 ms, the sampling frequency f_s = 71.11 kHz, the carriers spacing ΔN = 1.11 kHz and the bandwidth $B = \Delta N \times 64$ = 71.11 kHz. Thus, the simulation satisfied the Nyquist criterion where $f_s < 2B$. Both platforms used the same parameters. It is interesting to observe that the DWT-OFDM symbol is less in term of the mean of amplitude vectors as compared to FFT-OFDM. The mean of FFT is 1.4270, whereas, the mean of DWT is -9.667E-04. This is due to the fact that zero - padding was performed in the DWT

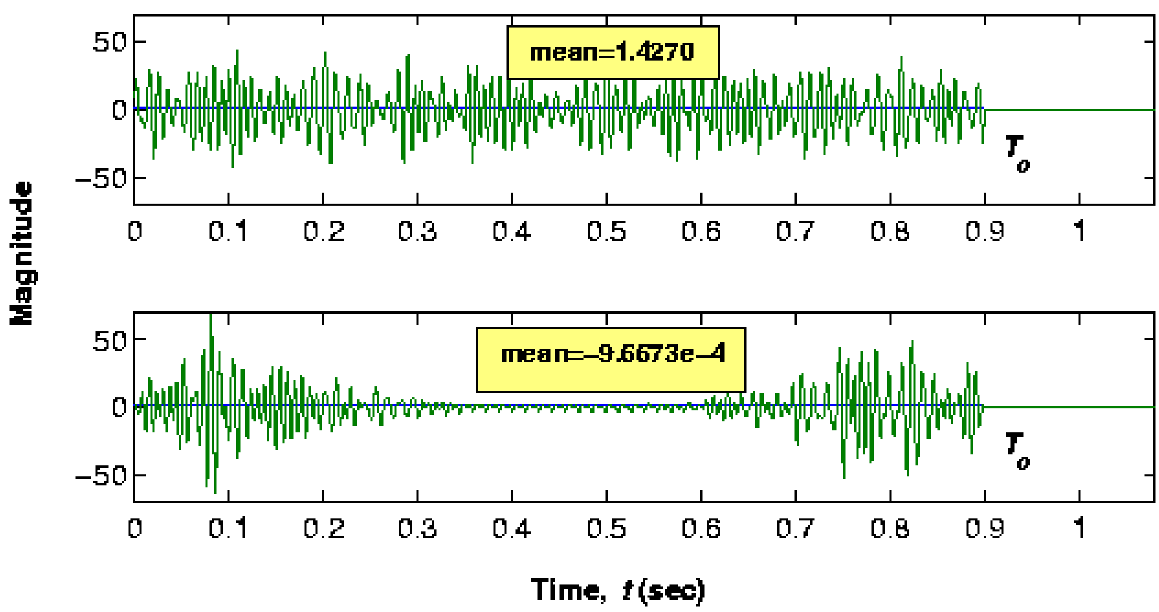

Fig. 7. An OFDM symbol in time domain: FFT-OFDM (Top), DWT-OFDM (Bottom).

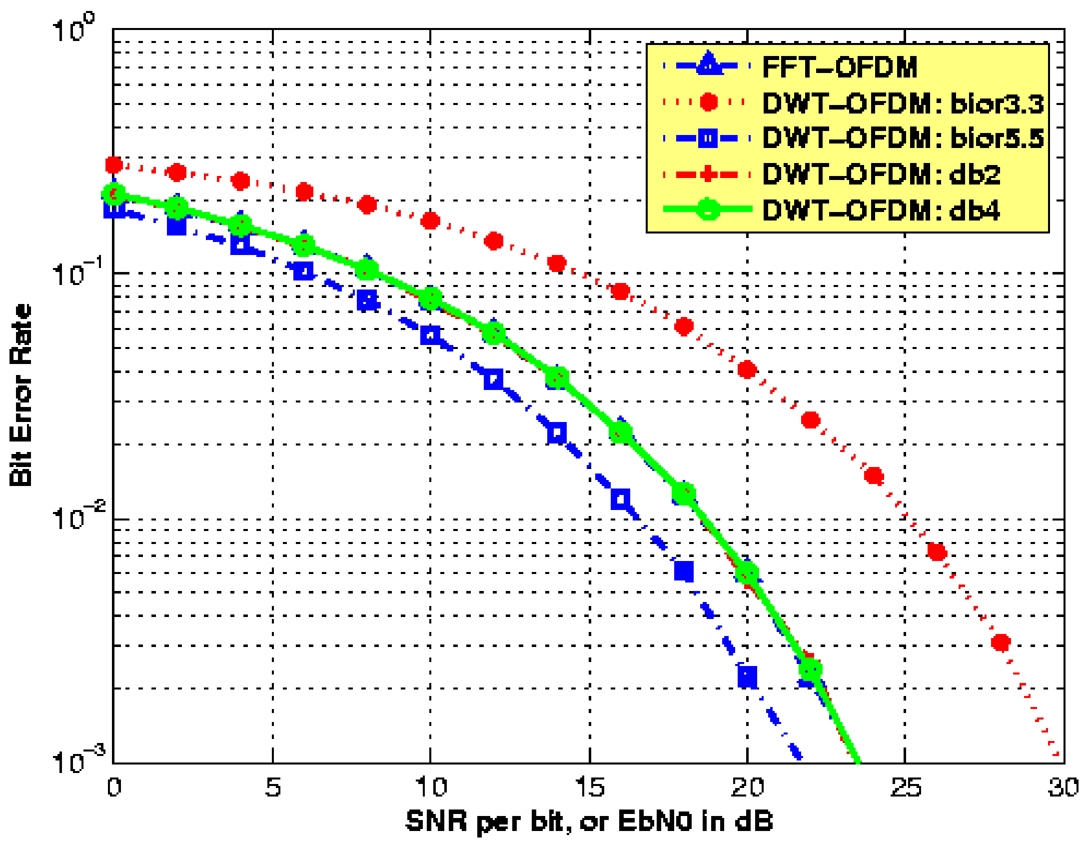

Fig. 8. BER performance for DWT-OFDM.

(transmitter) system model. As a result, most samples in the middle of DWT-OFDM symbol is almost zeroes. The DWT-OFDM performance can be observed from Figure 8. The wavelet families Biorthogonal and Daubechies are compared with FFT-OFDM. It is shown that bior5.5 is superior among all others. It outperforms FFT and Daubechies by about 2 dB and bior3.3 by 8 dB at 0.001 BER.

5. Conclusions

Simulation approaches using MATLAB for wavelet based OFDM, particularly in DWT-OFDM as alternative substitutions for Fourier based OFDM are presented. Conventional OFDM systems use IFFT and FFT algorithms at the transmitter and receiver respectively to multiplex the signals and transmit them simultaneously over a number of subcarriers. The system employs guard intervals or cyclic prefixes so that the delay spread of the channel becomes longer than the channel impulse response. The system must make sure that the cyclic prefix is a small fraction of the per carrier symbol duration. The purpose of employing the CP is to minimize inter-symbol interference (ISI). However a CP reduces the power efficiency and data throughput. The CP also has the disadvantage of reducing the spectral containment of the channels. Due to these issues, an alternative method is to use the wavelet transform to replace the IFFT and FFT blocks. The wavelet transform is referred as Discrete Wavelet Transform OFDM (DWT-OFDM). By using the transform, the spectral containment of the channels is better since they are not using CP. The wavelet based OFDM (DWT-OFDM) is assumed to have ortho-normal bases properties and satisfy the perfect reconstruction property. We use different wavelet families particularly, Biorthogonal and Daubechies and compare with conventional FFT-OFDM system. BER performances of both OFDM systems are also obtained. It is found that the DWT-OFDM platform is superior as compared to others as it has less error rate, especially using bior5.5 wavelet family.

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The discrete wavelet transform (DWT) algorithms have a firm position in processing of signals in several areas of research and industry. As DWT provides both octave-scale frequency and spatial timing of the analyzed signal, it is constantly used to solve and treat more and more advanced problems. The present book: Discrete Wavelet Transforms: Algorithms and Applications reviews the recent progress in discrete wavelet transform algorithms and applications. The book covers a wide range of methods (e.g. lifting, shift invariance, multi-scale analysis) for constructing DWTs. The book chapters are organized into four major parts. Part I describes the progress in hardware implementations of the DWT algorithms. Applications include multitone modulation for ADSL and equalization techniques, a scalable architecture for FPGA-implementation, lifting based algorithm for VLSI implementation, comparison between DWT and FFT based OFDM and modified SPIHT codec. Part II addresses image processing algorithms such as multiresolution approach for edge detection, low bit rate image compression, low complexity implementation of CQF wavelets and compression of multi-component images. Part III focuses watermaking DWT algorithms. Finally, Part IV describes shift invariant DWTs, DC lossless property, DWT based analysis and estimation of colored noise and an application of the wavelet Galerkin method. The chapters of the present book consist of both tutorial and highly advanced material. Therefore, the book is intended to be a reference text for graduate students and researchers to obtain state-of-the-art knowledge on specific applications.

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