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Weibull Distribution for Estimating the Parameters

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1. Introduction

Today, most electrical energy is generated by burning huge fossil fuels and special weather conditions such as acid rain and snow, climate change, urban smog, regional haze, several tornados, etc., have happened around the whole world. It is now clear that the installation of a number of wind turbine generators can effectively reduce environmental pollution, fossil fuel consumption, and the costs of overall electricity generation. Although wind is only an intermittent source of energy, it represents a reliable energy resource from a long-term energy policy viewpoint. Among various renewable energy resources, wind power energy is one of the most popular and promising energy resources in the whole world today. At a specific wind farm, the available electricity generated by a wind power generation system depends on mean wind speed (MWS), standard deviation of wind speed, and the location of installation. Since year-to-year variation on annual MWS is hard to predict, wind speed variations during a year can be well characterized in terms of a probability distribution function (pdf). This paper also addresses the relations among MWS, its standard deviation, and two important parameters of Weibull distribution.
The wind resource varies with of the day and the season of the year and even some extent from year to year. Wind energy has inherent variances and hence it has been expressed by distribution functions. In this paper, we present some methods for estimating Weibull parameters, namely, shape parameter (k) and scale parameter (c). The Weibul distribution is an important distribution especially for reliability and maintainability analysis. The suitable values for both shape parameter and scale parameters of Weibull distribution are important for selecting locations of installing wind turbine generators. The scale parameter of Weibull distribution also important to determine whether a wind farm is good or not. The presented method is the analytical methods and computational experiments on the presented methods are reported.

2. Weibull distribution

The Weibull distribution is characterized by two parameters, one is the shape parameter k (dimensionless) and the other is the scale parameter c (m/s)
The cumulative distribution function is given by

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{1}$$

And the probability function is given by

$$f(v)=\frac{dF(v)}{dv}=\frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{2}$$

The average wind speed can be expressed as

$$\bar{v}=\int_0^\infty vf(v)dv=\int_0^\infty \frac{vk}{c}\left[(\frac{v}{c})^{k-1}\right]\exp\left[-(\frac{v}{c})^{k}\right]dv \tag{3}$$

$$\text{Let}\quad x=(\frac{v}{c})^{k}\quad ,\ x^{\frac{1}{k}}=\frac{v}{c}\quad \text{and}\quad dx=\frac{k}{c}(\frac{v}{c})^{k-1}dv$$

Equation (3) can be simplified as

$$\bar{v}=c\int_0^\infty x^{\frac{1}{k}}\exp(-x)dx \tag{4}$$

By substituting a Gamma Function

$$\Gamma(n)=\int_0^\infty e^{-x}x^{n-1}dx\quad \text{into (4) and let } y=1+\frac{1}{k} \text{ then we have}$$

$$\bar{v}=c\Gamma\left(1+\frac{1}{k}\right) \tag{5}$$

The standard deviation of wind speed v is given by

$$\sigma=\sqrt{\int_0^\infty (v-\bar{v})^2 f(v)dv} \tag{6}$$

i.e.

$$\begin{aligned}\sigma&=\sqrt{\int_0^\infty (v^2-2v\bar{v}+\bar{v}^2)f(v)dv}\\&=\sqrt{\int_0^\infty v^2f(v)dv-2\bar{v}\int_0^\infty vf(v)dv+\bar{v}^2}\\&=\sqrt{\int_0^\infty v^2f(v)dv-2\bar{v}.\bar{v}+\bar{v}^2}\end{aligned} \tag{7}$$

Use

$$\int_0^\infty v^2 f(v)dv = \int_0^\infty v^2 \frac{k}{c}(\frac{v}{c})^{k-1} dv = \int_0^\infty c^2 x^{\frac{2}{k}} \frac{k}{c}(\frac{v}{c})^{k-1} dv = \int_0^\infty c^2 x^{\frac{2}{k}} \exp(-x)dx \tag{8}$$

And put $y = 1 + \frac{2}{k}$, then the following equation can be obtained

$$\int_0^\infty v^2 f(v)dv = c^2 \Gamma(1 + \frac{2}{k}) \tag{9}$$

Hence we get

$$\begin{aligned} \sigma &= \left[c^2 \Gamma(1 + \frac{2}{k}) - c^2 \Gamma^2 (1 + \frac{1}{k}) \right]^{\frac{1}{2}} \\ &= c\sqrt{\Gamma(1 + \frac{2}{k}) - \Gamma^2 (1 + \frac{1}{k})} \end{aligned} \tag{10}$$

2.1 Linear Least Square Method (LLSM)

Least square method is used to calculate the parameter(s) in a formula when modeling an experiment of a phenomenon and it can give an estimation of the parameters. When using least square method, the sum of the squares of the deviations S which is defined as below, should be minimized.

$$S = \sum_{i=1}^{n} w_i^2 \left[y_i - g(x_i) \right]^2 \tag{11}$$

In the equation, xi is the wind speed, yi is the probability of the wind speed rank, so (xi, yi) mean the data plot, wi is a weight value of the plot and n is a number of the data plot. The estimation technique we shall discuss is known as the Linear Least Square Method (LLSM), which is a computational approach to fitting a mathematical or statistical model to data. It is so commonly applied in engineering and mathematics problem that is often not thought of as an estimation problem. The linear least square method (LLSM) is a special case for the least square method with a formula which consists of some linear functions and it is easy to use. And in the more special case that the formula is line, the linear least square method is much easier. The Weibull distribution function is a non-linear function, which is

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c} \right)^k \right] \tag{12}$$

i.e.

$$\frac{1}{1 - F(v)} = \exp\left[\left(\frac{v}{c} \right)^k \right] \tag{13}$$

i.e.

$$\ln\{\frac{1}{1 - F(v)}\} = \left[\left(\frac{v}{c} \right)^k \right] \tag{14}$$

But the cumulative Weibull distribution function is transformed to a linear function like below:
Again

$$\ln\ln\{\frac{1}{1-F(v)}\} = k\ln v - k\ln c \tag{15}$$

Equation (15) can be written as $Y = bX + a$
where

$$Y = \ln\ln\{\frac{1}{1-F(v)}\},\quad X = \ln v,\ a = -k\ln c,\ b = k$$

By Linear regression formula

$$b = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2} \tag{16}$$

$$a = \frac{\sum_{i=1}^{n} X_i^2 \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i Y_i}{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2} \tag{17}$$

2.2 Maximum Likelihood Estimator(MLE)

The method of maximum likelihood (Harter and Moore (1965a), Harter and Moore (1965b), and Cohen (1965)) is a commonly used procedure because it has very desirable properties.
Let $x_1, x_2, \ldots\ldots\ldots\ldots x_n$ be a random sample of size n drawn from a probability density function $f(x,\theta)$ where θ is an unknown parameter. The likelihood function of this random sample is the joint density of the n random variables and is a function of the unknown parameter. Thus

$$L = \prod_{i=1}^{n} f_{X_i}(x_i, \theta) \tag{18}$$

is the Likelihood function. The Maximum Likelihood Estimator (MLE) of θ, say $\bar{\theta}$, is the value of θ, that maximizes L or, equivalently, the logarithm of L . Often, but not always, the MLE of q is a solution of

$$\frac{dLogL}{d\theta} = 0 \tag{19}$$

Now, we apply the MLE to estimate the Weibull parameters, namely the shape parameter and the scale parameters. Consider the Weibull probability density function (pdf) given in (2), then likelihood function will be

$$L(x_1,x_2,..,x_n,k,c)=\prod_{i=1}^{n}(\frac{k}{c})(\frac{x_i}{c})^{k-1}e^{-\left(\frac{x_i}{c}\right)^k} \tag{20}$$

On taking the logarithms of (20), differentiating with respect to k and *c* in turn and equating to zero, we obtain the estimating equations

$$\frac{\partial \ln L}{\partial k}=\frac{n}{k}+\sum_{i=1}^{n}\ln x_i-\frac{1}{c}\sum_{i=1}^{n}x_i^k\ln x_i=0 \tag{21}$$

$$\frac{\partial \ln L}{\partial c}=\frac{-n}{c}+\frac{1}{c^2}\sum_{i=1}^{n}x_i^k=0 \tag{22}$$

On eliminating *c* between these two above equations and simplifying, we get

$$\frac{\sum_{i=1}^{n}x_i^k\ln x_i}{\sum_{i=1}^{n}x_i^k}-\frac{1}{k}-\frac{1}{n}\sum_{i=1}^{n}\ln x_i=0 \tag{23}$$

which may be solved to get the estimate of k. This can be accomplished by Newton-Raphson method. Which can be written in the form

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \tag{24}$$

Where

$$f(k)=\frac{\sum_{i=1}^{n}x_i^k\ln x_i}{\sum_{i=1}^{n}x_i^k}-\frac{1}{k}-\frac{1}{n}\sum_{i=1}^{n}\ln x_i \tag{25}$$

And

$$f'(k)=\sum_{i=1}^{n}x_i^k(\ln x_i)^2-\frac{1}{k^2}\sum_{i=1}^{n}x_i^k(k\ln x_i-1)-(\frac{1}{n}\sum_{i=1}^{n}\ln x_i)(\sum_{i=1}^{n}x_i^k\ln x_i) \tag{26}$$

Once *k* is determined, c can be estimated using equation (22) as

$$c=\frac{\sum_{i=1}^{n}x_i^k}{n} \tag{27}$$

2.3 Some results

When a location has c=6 the pdf under various values of k are shown in Fig. 1. A higher value of k such as 2.5 or 4 indicates that the variation of Mean Wind speed is small. A lower value of k such as 1.5 or 2 indicates a greater deviation away from Mean Wind speed.

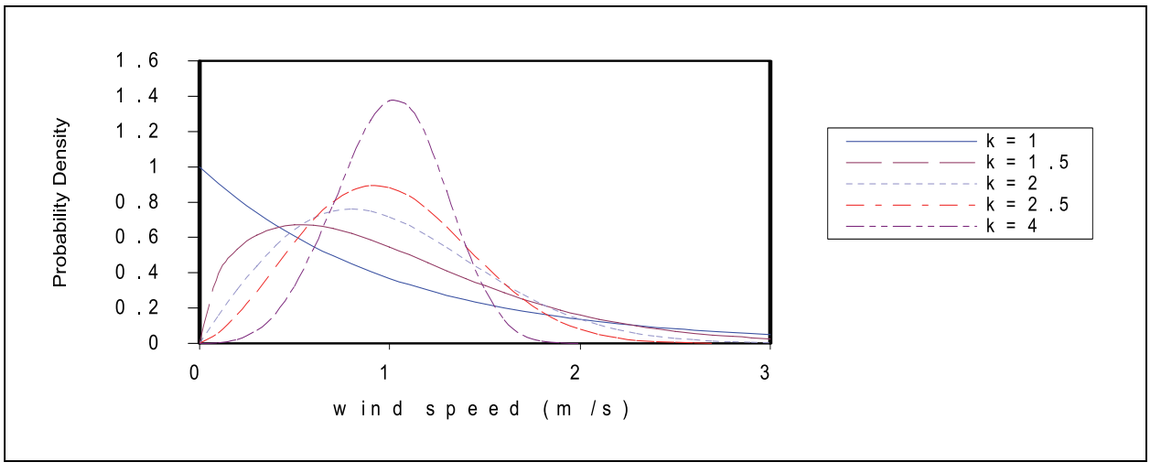


Fig. 1. Weibull Distribution Density versus wind speed under a constant value of c and different values of k

When a location has k=3 the pdf under various valus of c are shown in Fig.2. A higher value of c such as 12 indicates a greater deviation away from Mean Wind speed.

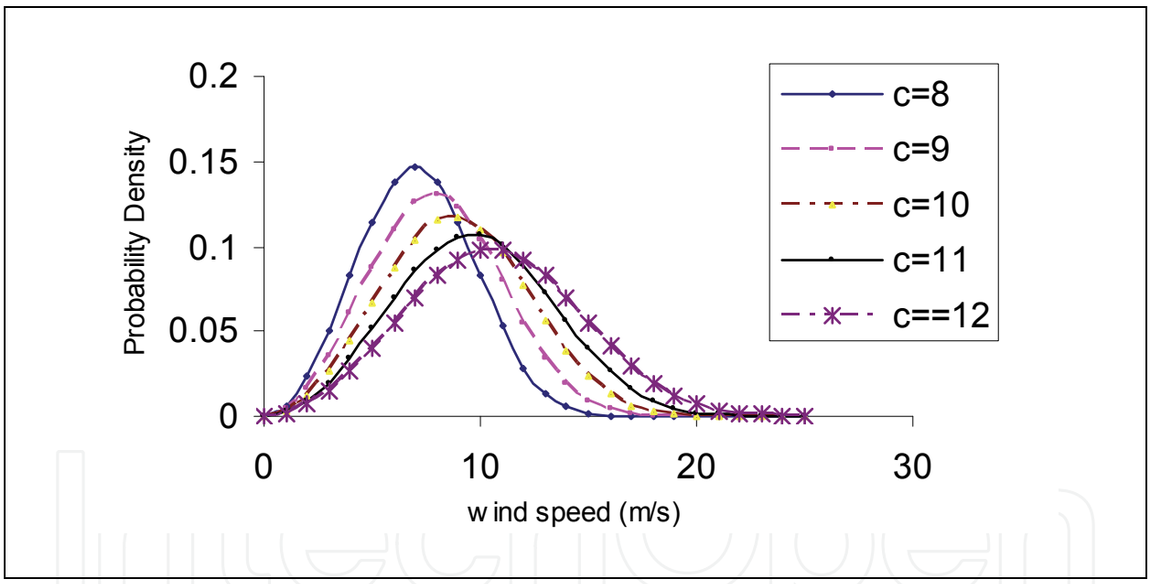


Fig. 2. Weibull Distribution Density versus wind speed under a constant value of k=3 and different values of c

Fig. 3 represents the characteristic curve of $\Gamma\left(1+\frac{1}{k}\right)$. versus shape parameter k. The values of $\Gamma\left(1+\frac{1}{k}\right)$. varies around .889 when k is between 1.9 to 2.6.

Fig.4 represents the characteristic curve of $\frac{c}{v}=$ versus shape parameter k .Normally the wind speed data collected at a specified location are used to calculate Mean Wind speed. A good

estimate for parameter c can be obtained from Fig.4 as $c = 1.128\bar{v}$ where k ranges from 1.6 to 4. If the parameter k is less than unity , the ratio $\frac{c}{v}$ decrease rapidly. Hence c is directly proportional to Mean Wind speed for $1.6 \leq k \leq 4$ and Mean Wind speed is mainly affected by c. The most good wind farms have k in this specified range and estimation of c in terms of $\bar{v}$ may have wide applications.

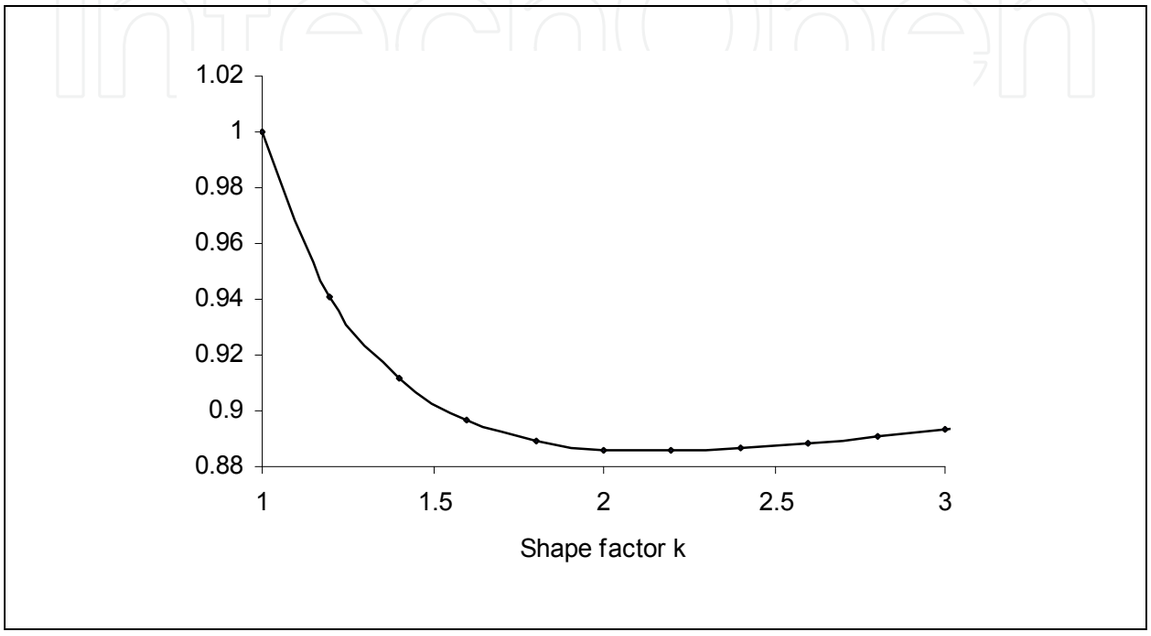


Fig. 3. Characteristic curve of $\Gamma(1+1/k)$ versus Shape parameter k

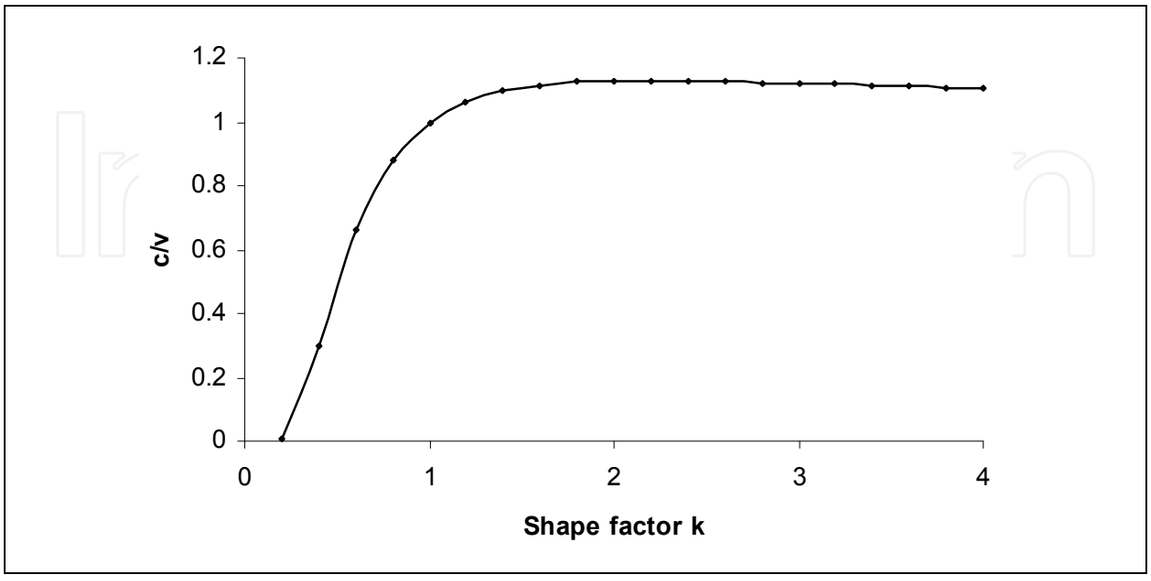


Fig. 4. Characteristic curve of c/ $\tilde{v}$ versus shape parameter k

March, 2009	Wind Speed (m/s)	March, 2009	Wind Speed (m/s)
1	0.56	17	0.28
2	0.28	18	0.83
3	0.56	19	1.39
4	0.56	20	1.11
5	1.11	21	1.11
6	0.83	22	0.83
7	1.11	23	0.56
8	1.94	24	0.83
9	1.11	25	1.67
10	0.83	26	1.94
11	1.11	27	1.39
12	1.39	28	0.83
13	0.28	29	2.22
14	0.56	30	1.67
15	0.28	31	2.22
16	0.28		

Example: Consider the following example where x_i represents the Average Monthly Wind Speed (m/s) at kolkata (from 1st March, 2009 to 31st March, 2009)

Also let $F(x_i) = \frac{i}{n+1}$ and using equations (16) and (17) we get k= 1.013658 and c=29.9931

But if we apply maximum Likelihood Method we get k = 1.912128 and c=1.335916. There is a huge difference in value of c by the above two methods. This is due to the mean rank of $F(x_i)$ and k value is tends to unity.

3. Conclusions

In this paper, we have presented two analytical methods for estimating the Weibull distribution parameters. The above results will help the scientists and the technocrats to select the location for Wind Turbine Generators.

4. Appendix

Those who are not familiar with the units or who have data given in units of other systems (For example wind speed in kmph), here is a short list with the conversion factors for the units that are most relevant for design of Wind Turbine Generators

Length	1m = 3.28 ft
Area	$1m^2=10.76\ ft^2$
Volume	$1m^3 = 35.31\ ft^3=264.2$ gallons
Speed	1m/s=2.237mph 1knot=.5144 m/s=1.15 mph
Mass	1 kg=2.20 5lb
Force	1N=0.225 lbf=0.102 kgf
Torque	1Nm=.738 ft lbf
Energy	$1J= 0.239$ Calories= $0.27777*10^{-6}$ kWh= 1 Nm
Power	1 W=1Watt=1 J/s=0.738 ft lbf/s= 1Nm/s 1 hp = 0.7457 kW 1 pk = 0.7355 Kw

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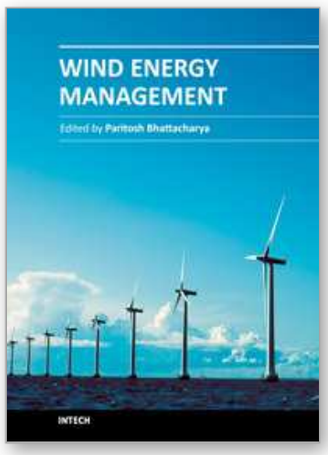
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The book "Wind Energy Management" is a required part of pursuing research work in the field of Renewable Energy at most universities. It provides in-depth knowledge to the subject for the beginners and stimulates further interest in the topic. The salient features of this book include: - Strong coverage of key topics - User friendly and accessible presentation to make learning interesting as much as possible - Its approach is explanatory and language is lucid and communicable - Recent research papers are incorporated

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