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# Efficient Hedging as Risk-Management Methodology in Equity-Linked Life Insurance

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## 1. Introduction

Using hedging methodologies for pricing is common in financial mathematics: one has to construct a financial strategy that will exactly replicate the cash flows of a contingent claim and, based on the law of one price<sup>1</sup>, the current price of the contingent claim will be equal to the price of the replicating strategy. If the exact replication is not possible, a financial strategy with a payoff “close enough” (in some probabilistic sense) to that of the contingent claim is sought. The presence of budget constraints is one of the examples precluding the exact replication.

There are several approaches used to hedge contingent claims in the most effective way when the exact replication is not possible. The theory of efficient hedging introduced by Fölmer and Leukert (Fölmer & Leukert, 2000) is one of them. The main idea behind it is to find a hedge that will minimize the expected shortfall from replication where the shortfall is weighted by some loss function. In our paper we apply the efficient hedging methodology to equity-linked life insurance contracts to get formulae in terms of the parameters of the initial model of a financial market. As a result risk-management of both types of risks, financial and insurance (mortality), involved in the contracts becomes possible.

Historically, life insurance has been combining two distinct components: an amount of benefit paid and a condition (death or survival of the insured) under which the specified benefit is paid. As opposed to traditional life insurance paying fixed or deterministic benefits, equity-linked life insurance contracts pay stochastic benefits linked to the evolution of a financial market while providing some guarantee (fixed, deterministic or stochastic) which makes their pricing much more complicated. In addition, as opposed to pure financial instruments, the benefits are paid only if certain conditions on death or survival of insureds are met. As a result, the valuation of such contracts represents a challenge to the insurance industry practitioners and academics and alternative valuation techniques are called for. This paper is aimed to make a contribution in this direction.

Equity-linked insurance contracts have been studied since their introduction in 1970's. The first papers using options to replicate their payoffs were written by Brennan and Schwartz (Brennan & Schwartz, 1976, 1979) and Boyle and Schwartz (Boyle & Schwartz, 1977). Since

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<sup>1</sup> The law of one price is a fundamental concept of financial mathematics stating that two assets with identical future cash flows have the same current price in an arbitrage-free market.

then, it has become a conventional practice to reduce such contracts to a call or put option and apply perfect (Bacinello & Ortu, 1993; Aase & Person, 1994) or mean-variance hedging (Möller, 1998, 2001) to calculate their price. All the authors mentioned above had studied equity-linked pure endowment contracts providing a fixed or deterministic guarantee at maturity for a survived insured. The contracts with different kind of guarantees, fixed and stochastic, were priced by Ekern and Persson (Ekern & Persson, 1996) using a fair price valuation technique.

Our paper is extending the great contributions made by these authors in two directions: we study equity-linked life insurance contracts with a stochastic guarantee<sup>2</sup> and we use an imperfect hedging technique (efficient hedging). Further developments may include an introduction of a stochastic model for interest rates and a systematic mortality risk, a combination of deterministic and stochastic guarantees, surrender options and lapses etc.

We consider equity-linked pure endowment contracts. In our setting a financial market consists of a non-risky asset and two risky assets. The first one,  $S_t^1$ , is more risky and profitable and provides possible future gain. The second asset,  $S_t^2$ , is less risky and serves as a stochastic guarantee. Note that we restrict our attention to the case when evolutions of the prices of the two risky assets are generated by the same Wiener process, although the model with two different Wiener processes with some correlation coefficient  $\rho$  between them, as in Margrabe, 1978, could be considered. There are two reasons for our focus. First of all, equity-linked insurance contracts are typically linked to traditional equities such as traded indices and mutual funds which exhibit a very high positive correlation. Therefore, the case when  $\rho = 1$  could be a suitable and convenient approximation. Secondly, although the model with two different Wiener processes seems to be more general, it turns out that the case  $\rho = 1$  demands a special consideration and does not follow from the results for the case when  $\rho < 1$  (see Melnikov & Romaniuk, 2008; Melnikov, 2011 for more detailed information on a model with two different Wiener processes). The case  $\rho = -1$  does not seem to have any practical application although could be reconstructed for the sake of completeness. Note also that our setting with two risky assets generated by the same Wiener process is equivalent to the case of a financial market consisting of one risky asset and a stochastic guarantee being a function of its prices.

We assume that there are no additional expenses such as transaction costs, administrative costs, maintenance expenses etc. The payoff at maturity is equal to  $\max(S_T^1, S_T^2)$ . We reduce it to a call option giving its holder the right to exchange one asset for another at maturity. The formula for the price of such options was given in Margrabe, 1978. Since the benefit is paid on survival of a client, the insurance company should also deal with some mortality risk. As a result, the price of the contract will be less than needed to construct a perfect hedge exactly replicating the payoff at maturity. The insurance company is faced with an initial budget constraint precluding it from using perfect hedging. Therefore, we fix the probability of the shortfall arising from a replication and, with a known price of the contract, control of financial and insurance risks for the given contract becomes possible.

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<sup>2</sup> Although Ekern & Persson, 1996, consider a number of different contracts including those with a stochastic guarantee, the contracts under our consideration differ: we consider two risky assets driven by the same Wiener process or, equivalently, one risky asset and a stochastic guarantee depending on its price evolution. The motivation for our choice follows below.

The layout of the paper is as follows. Section 2 introduces the financial market and explains the main features of the contracts under consideration. In Section 3 we describe efficient hedging methodology and apply it to pricing of these contracts. Further, Section 4 is devoted to a risk-taking insurance company managing a balance between financial and insurance risks. In addition, we consider how the insurance company can take advantage of diversification of a mortality risk by pooling homogeneous clients together and, as a result of more predictable mortality exposure, reducing prices for a single contract in a cohort. Section 5 illustrates our results with a numerical example.

## 2. Description of the model

### 2.1 Financial setting

We consider a financial market consisting of a non-risky asset  $B_t = \exp(rt), t \geq 0, r \geq 0$ , and two risky assets  $S^1$  and  $S^2$  following the Black-Scholes model:

$$dS_t^i = S_t^i (\mu_i dt + \sigma_i dW_t), i = 1, 2, t \leq T. \quad (1)$$

Here  $\mu_i$  and  $\sigma_i$  are a rate of return and a volatility of the asset  $S^i$ ,  $W = (W_t)_{t \leq T}$  is a Wiener process defined on a standard stochastic basis  $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t \leq T}, P)$ ,  $T$  - time to maturity. We assume, for the sake of simplicity, that  $r = 0$ , and, therefore,  $B_t = 1$  for any  $t$ . Also, we demand that  $\mu_1 > \mu_2, \sigma_1 > \sigma_2$ . The last two conditions are necessary since  $S^2$  is assumed to provide a flexible guarantee and, therefore, should be less risky than  $S^1$ . The initial values for both assets are supposed to be equal  $S_0^1 = S_0^2 = S_0$  and are considered as the initial investment in the financial market.

It can be shown, using the Ito formula, that the model (1) could be presented in the following form:

$$S_t^i = S_0^i \exp \left\{ \left( \mu_i - \frac{\sigma_i^2}{2} \right) t + \sigma_i W_t \right\} \quad (2)$$

Let us define a probability measure  $P^*$  which has the following density with respect to the initial probability measure  $P$ :

$$Z_T = \exp \left\{ -\frac{\mu_1}{\sigma_1} W_T - \frac{1}{2} \left( \frac{\mu_1}{\sigma_1} \right)^2 T \right\}. \quad (3)$$

Both processes,  $S^1$  and  $S^2$ , are martingales with respect to the measure  $P^*$  if the following technical condition is fulfilled:

$$\frac{\mu_1}{\sigma_1} = \frac{\mu_2}{\sigma_2} \quad (4)$$

Therefore, in order to prevent the existence of arbitrage opportunities in the market we suppose that the risky assets we are working with satisfy this technical condition. Further,

according to the Girsanov theorem, the process  $W_t^* = W_t + \frac{\mu_1}{\sigma_1} t = W_t + \frac{\mu_2}{\sigma_2} t$

is a Wiener process with respect to  $P^*$ .

Finally, note the following useful representation of the guarantee  $S_t^2$  by the underlying risky asset  $S_t^1$ :

$$\begin{aligned} S_t^2 &= S_0^2 \exp \left\{ \sigma_2 W_t + \left( \mu_2 - \frac{\sigma_2^2}{2} \right) t \right\} \\ &= S_0 \exp \left\{ \frac{\sigma_2}{\sigma_1} \left( \sigma_1 W_t + \left( \mu_1 - \frac{\sigma_1^2}{2} \right) t \right) - \frac{\sigma_2}{\sigma_1} \left( \mu_1 - \frac{\sigma_1^2}{2} \right) t + \left( \mu_2 - \frac{\sigma_2^2}{2} \right) t \right\} \\ &= \left( S_0 \right)^{1-\sigma_2/\sigma_1} \left( S_t^1 \right)^{\sigma_2/\sigma_1} \exp \left\{ -\frac{\sigma_2}{\sigma_1} \left( \mu_1 - \frac{\sigma_1^2}{2} \right) t + \left( \mu_2 - \frac{\sigma_2^2}{2} \right) t \right\}, \end{aligned}$$

which shows that our setting is equivalent to one with a financial market consisting of a single risky asset and a stochastic guarantee being a function of the price of this asset.

We will call any process  $\pi_t = (\beta_t, \gamma_t^1, \gamma_t^2)$ , adapted to the price evolution  $F_t$ , a *strategy*. Let us define its value as a sum  $X_t^\pi = \beta_t + \gamma_t^1 S_t^1 + \gamma_t^2 S_t^2$ . We shall consider only *self-financing* strategies satisfying the following condition  $dX_t^\pi = \beta_t + \gamma_t^1 dS_t^1 + \gamma_t^2 dS_t^2$ , where all stochastic differentials are well defined. Every  $F_T$ -measurable nonnegative random variable  $H$  is called a *contingent claim*. A self-financing strategy  $\pi$  is a *perfect hedge* for  $H$  if  $X_T^\pi \geq H$  (a.s.). According to the option pricing theory of Black-Scholes-Merton, it does exist, is unique for a given contingent claim, and has an initial value  $X_0^\pi = E^* H$ .

## 2.2 Insurance setting

The insurance risk to which the insurance company is exposed when enters into a pure endowment contract includes two components. The first one is based on survival of a client to maturity as at that time the insurance company would be obliged to pay the benefit to the alive insured. We call it a *mortality risk*. The second component depends on a *mortality frequency risk* for a pooled number of similar contracts. A large enough portfolio of life insurance contracts will result in more predictable mortality risk exposure and a reduced mortality frequency risk. In this section we will work with the mortality risk only dealing with the mortality frequency risk in Section 4.

Following actuarial tradition, we use a random variable  $T(x)$  on a probability space  $(\tilde{\Omega}, \tilde{F}, \tilde{P})$  to denote the remaining lifetime of a person of age  $x$ . Let  ${}_T p_x = \tilde{P}\{T(x) > T\}$  be a survival probability for the next  $T$  years of the same insured. It is reasonable to assume that  $T(x)$  doesn't depend on the evolution of the financial market and, therefore, we consider  $(\Omega, F, P)$  and  $(\tilde{\Omega}, \tilde{F}, \tilde{P})$  as being independent.

We study pure endowment contracts with a flexible stochastic guarantee which make a payment at maturity provided the insured is alive. Due to independency of "financial" and "insurance" parts of the contract we consider the product probability space  $(\Omega \times \tilde{\Omega}, F \times \tilde{F}, P \times \tilde{P})$  and introduce a contingent claim on it with the following payoff at maturity:

$$H(T(x)) = \max\{S_T^1, S_T^2\} \cdot I_{\{T(x) > T\}}. \quad (5)$$

It is obvious that a strategy with the payoff  $H = \max\{S_T^1, S_T^2\}$  at  $T$  is a perfect hedge for the contract under our consideration. Its price is equal to  $E^*H$ .

### 2.3 Optimal pricing and hedging

Let us rewrite the financial component of (5) as follows:

$$H = \max\{S_T^1, S_T^2\} = S_T^2 + (S_T^1 - S_T^2)^+, \quad (6)$$

where  $x^+ = \max(0, x)$ ,  $x \in \mathbb{R}^1$ . Using (2.6) we reduce the pricing of the claim (5) to the pricing of the call option  $(S_T^1 - S_T^2)^+$  provided  $\{T(x) > T\}$ .

According to the well-developed option pricing theory the optimal price is traditionally calculated as an expected present value of cash flows under a risk-neutral probability measure. Note, however, that the “insurance” part of the contract (5) doesn’t need to be risk-adjusted since the mortality risk is essentially unsystematic. It means that the mortality risk can be effectively reduced not by hedging but by diversification or by increasing the number of similar insurance policies.

**Proposition.** The price for the contract (5) is equal to

$${}_T U_x = E^* \times \tilde{E}H(T(x)) = {}_T p_x E^*(S_T^2) + {}_T p_x E^*(S_T^1 - S_T^2)^+, \quad (7)$$

where  $E^* \times \tilde{E}$  is the expectation with respect to  $P^* \times \tilde{P}$ .

We would like to call (7) as the *Brennan-Schwartz price* (Brennan & Schwartz, 1976).

The insurance company acts as a hedger of  $H$  in the financial market. It follows from (7) that the initial price of  $H$  is strictly less than that of the perfect hedge since a survival probability is always less than one or

$${}_T U_x < E^*(S_T^2 + (S_T^1 - S_T^2)^+) = E^*H.$$

Therefore, perfect hedging of  $H$  with an initial value of the hedge restricted by the Black-Scholes-Merton price  $E^*H$  is not possible and alternative hedging methods should be used. We will look for a strategy  $\pi^*$  with some initial budget constraint such that its value  $X_T^{\pi^*}$  at maturity is close to  $H$  in some probabilistic sense.

## 3. Efficient hedging

### 3.1 Methodology

The main idea behind efficient hedging methodology is the following: we would like to construct a strategy  $\pi$ , with the initial value

$$X_0^\pi \leq X_0 < E^*H, \quad (8)$$

that will minimize the expected shortfall from the replication of the payoff  $H$ . The shortfall is weighted by some loss function  $l: \mathbb{R}_+ \rightarrow \mathbb{R}_+ = [0, \infty)$ . We will consider a power loss function  $l(x) = \text{const} \cdot x^p$ ,  $p > 0, x \geq 0$  (Fölmer & Leukert, 2000). Since at maturity of the contract  $X_T^\pi$



should be close to  $H$  in some probabilistic sense we will consider  $El\left(\left(H - X_T^\pi\right)^+\right)$  as a measure of closeness between  $X_T^\pi$  and  $H$ .

**Definition.** Let us define a strategy  $\pi^*$  for which the following condition is fulfilled:

$$El\left(\left(H - X_T^{\pi^*}\right)^+\right) = \inf_{\pi} El\left(\left(H - X_T^\pi\right)^+\right), \quad (9)$$

where infimum is taken over all self-financing strategies with positive values satisfying the budget restriction (8). The strategy  $\pi^*$  is called the *efficient hedge*.

Once the efficient hedge is constructed we will set the price of the equity-linked contract (5) being equal to its initial value  $X_0^{\pi^*}$  and make conclusions about the appropriate balance between financial and insurance risk exposure.

Although interested readers are recommended to get familiar with the paper on efficient hedging by Fölmer & Leukert, 2000, for the sake of completeness we formulate the results from it that are used in our paper in the following lemma.

**Lemma 1.** Consider a contingent claim with the payoff (6) at maturity with the shortfall from its replication weighted by a power loss function

$$l(x) = \text{const} \cdot x^p, p > 0, x \geq 0. \quad (10)$$

Then the efficient hedge  $\pi^*$  satisfying (9) exists and coincides with a perfect hedge for a modified contingent claim  $H_p$  having the following structure:

$$\begin{aligned} H_p &= H - a_p Z_T^{1/(p-1)} \wedge H && \text{for } p > 1, \text{ const} = 1/p, \\ H_p &= H \cdot I_{\{Z_T^{-1} > a_p H^{1-p}\}} && \text{for } 0 < p < 1, \text{ const} = 1, \\ H_p &= H \cdot I_{\{Z_T^{-1} > a_p\}} && \text{for } p = 1, \text{ const} = 1, \end{aligned} \quad (11)$$

where a constant  $a_p$  is defined from the condition on its initial value  $E^* H_p = X_0$ .

In other words, we reduce a construction of an efficient hedge for the claim  $H$  from (9) to an easier-to-do construction of a perfect hedge for the modified claim (11). In the next section we will apply efficient hedging to equity-linked life insurance contracts.

### 3.2 Application to equity-linked life insurance contracts

Here we consider a single equity-linked life insurance contract with the payoff (5). Since (6) is true, we will pay our attention to the term  $\left(S_T^1 - S_T^2\right)^+ \cdot I_{\{T(x) > T\}}$  associated with a call option. Note the following equality that comes from the definition of perfect and efficient hedging and Lemma 1:

$$X_0 = {}_T p_x E^* \left(S_T^1 - S_T^2\right)^+ = E^* \left(S_T^1 - S_T^2\right)_{p, p > 0}^+ \quad (12)$$

where  $\left(S_T^1 - S_T^2\right)_p^+$  is defined by (11). Using (12) we can separate insurance and financial components of the contract:

$${}_T p_x = \frac{E^* \left( S_T^1 - S_T^2 \right)_p^+}{E^* \left( S_T^1 - S_T^2 \right)^+}. \quad (13)$$

The left-hand side of (13) is equal to the survival probability of the insured, which is a mortality risk for the insurer, while the right-hand side is related to a pure financial risk as it is connected to the evolution of the financial market. So, the equation (13) can be viewed as a *key balance equation* combining the risks associated with the contract (5).

We use efficient hedging methodology presented in Lemma 1 for a further development of the numerator of the right-hand side of (13) and the Margrabe formula (Margrabe, 1978) for its denominator.

**Step 1.** Let us first work with the denominator of the right-hand side of (13). We get

$$E^* \left( S_T^1 - S_T^2 \right)^+ = S_0 \left\{ \Phi \left( b_+(1,1,T) \right) - \Phi \left( b_-(1,1,T) \right) \right\}, \quad (14)$$

$$\text{where } b_{\pm}(1,1,T) = \frac{\ln 1 \pm (\sigma_1 - \sigma_2)^2 \frac{T}{2}}{(\sigma_1 - \sigma_2) \sqrt{T}}, \quad \Phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^x \exp(-y^2/2) dy.$$

The proof of (14) is given in Appendix. Note that (14) is a variant of the Margrabe formula (Margrabe, 1978) for the case  $S_0^1 = S_0^2 = S_0$ . It shows the price of the option that gives its holder the right to exchange one risky asset for another at maturity of the contract.

**Step 2.** To calculate the numerator of the right-hand side of (13), we want to represent it in terms of  $Y_T = S_T^1 / S_T^2$ . Let us rewrite  $W_T$  with the help a free parameter  $\gamma$  in the form

$$\begin{aligned} W_T &= (1 + \gamma)W_T - \gamma W_T \\ &= \frac{1 + \gamma}{\sigma_1} \left( \sigma_1 W_T + \left( \mu_1 - \frac{\sigma_1^2}{2} \right) T \right) - \frac{\gamma}{\sigma_2} \left( \sigma_2 W_T + \left( \mu_2 - \frac{\sigma_2^2}{2} \right) T \right) \\ &\quad - \frac{1 + \gamma}{\sigma_1} \left( \mu_1 - \frac{\sigma_1^2}{2} \right) T + \frac{\gamma}{\sigma_2} \left( \mu_2 - \frac{\sigma_2^2}{2} \right) T. \end{aligned} \quad (15)$$

Using (3) and (15), we obtain the next representation of the density  $Z_T$ :

$$Z_T = G \cdot \left( S_T^1 \right)^{-\frac{(1+\gamma)\mu_1}{\sigma_1^2}} \left( S_T^2 \right)^{\frac{\gamma\mu_1}{\sigma_1\sigma_2}} \quad (16)$$

where

$$\begin{aligned} G &= \left( S_0^1 \right)^{\frac{(1+\gamma)\mu_1}{\sigma_1^2}} \left( S_0^2 \right)^{-\frac{\gamma\mu_1}{\sigma_1\sigma_2}} \\ &\quad \times \exp \left( \frac{(1+\gamma)\mu_1}{\sigma_1^2} \left( \mu_1 - \frac{\sigma_1^2}{2} \right) T - \frac{\gamma\mu_1}{\sigma_1\sigma_2} \left( \mu_2 - \frac{\sigma_2^2}{2} \right) T - \frac{1}{2} \left( \frac{\mu_1}{\sigma_1} \right)^2 T \right). \end{aligned}$$



Now we consider three cases according to (11) and choose appropriate values of the parameter  $\gamma$  for each case (see Appendix for more details). The results are given in the following theorem.

**Theorem 1.** Consider an insurance company measuring its shortfalls with a power loss function (10) with some parameter  $p > 0$ . For an equity-linked life insurance contract with the payoff (5) issued by the insurance company, it is possible to balance a survival probability of an insured and a financial risk associated with the contract.

**Case 1:**  $p > 1$

For  $p > 1$  we get

$${}_T p_x = \frac{\Phi(b_+(1, C, T)) - \Phi(b_-(1, C, T))}{\Phi(b_+(1, 1, T)) - \Phi(b_-(1, 1, T))} + \frac{(C-1)^+}{C^{\alpha_p}} \exp \left\{ \alpha_p (1 - \alpha_p) \frac{(\sigma_1 - \sigma_2)^2}{2} T \right\} \times \frac{\Phi(b_-(1, C, T) + \alpha_p (\sigma_1 - \sigma_2) \sqrt{T})}{\Phi(b_+(1, 1, T)) - \Phi(b_-(1, 1, T))}, \quad (17)$$

where  $C$  is found from  $a_p G^{1/(p-1)} C^{\alpha_p} = C - 1$  and  $\alpha_p = -\frac{\mu_1}{\sigma_1(\sigma_1 - \sigma_2)(p-1)} - \frac{\sigma_2}{\sigma_1 - \sigma_2}$ .

**Case 2:**  $0 < p < 1$

Denote  $\alpha_p = \frac{\sigma_1 \sigma_2 (1-p) - \mu_1}{\sigma_1(\sigma_1 - \sigma_2)}$ .

2.1. If  $-\alpha_p \leq 1-p$  (or  $\frac{\mu_1}{\sigma_1^2} \leq 1-p$ ) then

$${}_T p_x = 1 - \frac{\Phi(b_+(1, C, T)) - \Phi(b_-(1, C, T))}{\Phi(b_+(1, 1, T)) - \Phi(b_-(1, 1, T))}, \quad (18)$$

where  $C$  is found from

$$C^{-\alpha_p} = a_p \cdot G \cdot ((C-1)^+)^{1-p}. \quad (19)$$

2.2. If  $-\alpha_p > 1-p$  (or  $\frac{\mu_1}{\sigma_1^2} > 1-p$ ) then

2.2.1. If (19) has no solution then  ${}_T p_x = 1$ .

2.2.2. If (19) has one solution  $C$ , then  ${}_T p_x$  is defined by (18).

2.2.3. If (19) has two solutions  $C_1 < C_2$  then

$${}_T p_x = 1 - \frac{\Phi(b_+(1, C_1, T)) - \Phi(b_-(1, C_1, T))}{\Phi(b_+(1, 1, T)) - \Phi(b_-(1, 1, T))} + \frac{\Phi(b_+(1, C_2, T)) - \Phi(b_-(1, C_2, T))}{\Phi(b_+(1, 1, T)) - \Phi(b_-(1, 1, T))}. \quad (20)$$

**Case 3:**  $p = 1$

For  $p = 1$  we have

$${}_T p_x = 1 - \frac{\Phi(b_+(1, C, T)) - \Phi(b_-(1, C, T))}{\Phi(b_+(1, 1, T)) - \Phi(b_-(1, 1, T))}, \quad (21)$$

where  $C = (Ga_p)^{\frac{\sigma_1(\sigma_1 - \sigma_2)}{\mu_1}}$  and  $\alpha_p = -\frac{\mu_1}{\sigma_1(\sigma_1 - \sigma_2)}$ .

The proof of (17), (18), (20), and (21) is given in Appendix.

**Remark 1.** One can consider another approach to find  $C$  (or  $C_1$  and  $C_2$ ) for (18), (20) and (21). Let us fix a probability of the set  $\{Y_T \leq C\}$  (or  $\{Y_T \leq C_1\} \cup \{Y_T > C_2\}$ ):

$$P(Y_T \leq C) = 1 - \varepsilon, \varepsilon > 0, \quad (22)$$

$$P(\{Y_T \leq C_1\} \cup \{Y_T > C_2\}) = 1 - \varepsilon, \varepsilon > 0$$

and calculate  $C$  (or  $C_1$  and  $C_2$ ) using log-normality of  $Y_T$ . Note that a set for which (22) is true coincides with  $\{X_T^\pi \geq H\}$ . The latter set has a nice financial interpretation: fixing its probability at  $1 - \varepsilon$ , we specify the level of a financial risk that the company is ready to take or, in other words, the probability  $\varepsilon$  that it will not be able to hedge the claim (6) perfectly. We will explore this remark further in the next section.

#### 4. Risk-management for risk-taking insurer

The loss function with  $p > 1$  corresponds to a company avoiding risk with risk aversion increasing as  $p$  grows. The case  $0 < p < 1$  is appropriate for companies that are inclined to take some risk. In this section we show how a risk-taking insurance company could use efficient hedging for management of its financial and insurance risks. For illustrative purposes we consider the extreme case when  $p \rightarrow 0$ . While the effect of a power  $p$  close to zero on efficient hedging was pointed out by Föllmer and Leukert (Föllmer & Leukert, 2000), we give it a different interpretation and implementation which are better suited for the purposes of our analysis. In addition, we restrict our attention to a particular case for which the equation (19) has only one solution: that is Case 2.1. This is done for illustrative purposes only since the calculation of constants  $C$ ,  $C_1$  and  $C_2$  for other cases may involve the use of extensive numerical techniques and lead us well beyond our research purposes.

As was mentioned above, the characteristic equation (19) with  $p \leq 1 + \alpha_p$  (or, equivalently,

$p \leq 1 - \frac{\mu_1}{\sigma_1^2}$ ) admits only one solution  $C$  which is further used for determination of a

modified claim (11) as follows

$$H_p = H \cdot I_{\{Y_T \leq C\}} \quad (23)$$

where  $H = (S_T^1 - S_T^2)^+$ ,  $Y_T = S_T^1/S_T^2$ , and  $0 < p < 1$ . Denote an efficient hedge for  $H$  and its initial value as  $\pi^*$  and  $x = X_0$  respectively. It follows from Lemma 1 that  $\pi^*$  is a perfect hedge for  $H_p = (S_T^1 - S_T^2)^+_p$ . Since the inequality  $((a-b)^+)^p \leq a^p$  is true for any positive  $a$  and  $b$ , we have

$$\begin{aligned} E\left((H - X_T^{\pi^*}(x))^+\right)^p &= E\left[\left(H_p - X_T^{\pi^*}(x)\right)^+ \cdot I_{\{Y_T \leq C\}} + \left(H - X_T^{\pi^*}(x)\right)^+ \cdot I_{\{Y_T > C\}}\right]^p \\ &= E\left[\left(H - X_T^{\pi^*}(x)\right)^+ \cdot I_{\{Y_T > C\}}\right]^p \\ &= E\left[\left(H - X_T^{\pi^*}(x)\right)^+\right]^p \cdot I_{\{Y_T > C\}} \leq EH^p \cdot I_{\{Y_T > C\}}. \end{aligned} \quad (24)$$

Taking the limit in (24) as  $p \rightarrow 0$  and applying the classical dominated convergence theorem, we obtain

$$EH^p \cdot I_{\{Y_T > C\}} \xrightarrow{p \rightarrow 0} EI_{\{Y_T > C\}} = P(Y_T > C) \quad (25)$$

Therefore, we can fix a probability  $P(Y_T > C) = \varepsilon$  which quantifies a financial risk and is equivalent to the probability of failing to hedge  $H$  at maturity.

Note that the same hedge  $\pi^*$  will also be an efficient hedge for the claim  $\delta \cdot H$  where  $\delta$  is some positive constant but its initial value will be  $\delta \cdot x$  instead of  $x$ . We will use this simple observation for pricing cumulative claims below when we consider the insurance company taking advantage of diversification of a mortality risk and further reducing the price of the contract.

Here, we pool together the homogeneous clients of the same age, life expectancy and investment preferences and consider a cumulative claim  $l_{x+T} \cdot H$ , where  $l_{x+T}$  is the number of insureds alive at time  $T$  from the group of size  $l_x$ . Let us measure a mortality risk of the pool of the equity-linked life insurance contracts for this group with the help of a parameter  $\alpha \in (0, 1)$  such that

$$\tilde{P}(l_{x+T} \leq n_\alpha) = 1 - \alpha, \quad (26)$$

where  $n_\alpha$  is some constant. In other words,  $\alpha$  equals the probability that the number of clients alive at maturity will be greater than expected based on the life expectancy of homogeneous clients. Since it follows a frequency distribution, this probability could be calculated with the help of a binomial distribution with parameters  ${}_T p_x$  and  $l_x$  where  ${}_T p_x$  is found by fixing the level of the financial risk  $\varepsilon$  and applying the formulae from Theorem 1.

We can rewrite (26) as follows

$$\tilde{P}\left(\frac{l_{x+T}}{l_x} \leq \frac{n_\alpha}{l_x}\right) = \tilde{P}\left(\frac{l_{x+T}}{l_x} \leq \delta\right) = 1 - \alpha,$$

where  $\delta = n_\alpha/l_x$ . Due to the independence of insurance and financial risks, we have

$$\begin{aligned}
 P \times \tilde{P}(l_x X_T^\pi(\delta x) \geq l_{x+T} H) &\geq P \times \tilde{P}(X_T^\pi(\delta x) \geq \frac{l_{x+T}}{l_x} H) \\
 &\geq P(X_T^\pi(\delta x) \geq \delta H) \cdot \tilde{P}(n_\alpha \geq l_{x+T}) \geq (1 - \varepsilon)(1 - \alpha) \geq 1 - (\varepsilon + \alpha).
 \end{aligned}
 \tag{27}$$

So, using the strategy  $\pi^*$  the insurance company is able to hedge the cumulative claim  $l_{x+T} \cdot H$  with the probability at least  $1 - (\varepsilon + \alpha)$  which combines both financial and insurance risks. The price of a single contract will be further reduced to  $\frac{n_\alpha}{l_x} T p_x E^* H$ .

## 5. Numerical example

Using the same reasons as in the previous section, we restrict our attention to the case when  $p \rightarrow 0$  and the equation (19) has only one solution as is in Case 2.1. Consider the following parameters for the risky assets:

$$\mu_1 = 5\%, \quad \sigma_1 = 23\%,$$

$$\mu_2 = 4\%, \quad \sigma_2 = 19\%.$$

The condition (4) is approximately fulfilled to preclude the existence of arbitrage opportunities. Also, since  $1 - \mu_1 / \sigma_1^2 \cong 0.05$ ,  $p$  should be very small, or  $p \leq 0.05$ , and we are able to use (25) instead of (19) and exploit (18) from Theorem 1. For survival probabilities we use the Uninsured Pensioner Mortality Table UP94 (Shulman & Kelley, 1999) which is based on best estimate assumptions for mortality. Further, we assume that a single equity-linked life insurance contract has the initial value  $S_0 = 100$ . We consider contracts with the maturity terms  $T = 5, 10, 15, 20, 25$  years. The number of homogeneous insureds in a cohort is  $l_x = 100$ .

Figure 1 represents the offsetting relationships between financial and insurance risks. Note that financial and insurance risks do offset each other. As perfect hedging is impossible, the insurer will be exposed to a financial risk expressed as a probability that it will be unable to hedge the claim (6) with the probability one. At the same time, the insurance company faces a mortality risk or a probability that the insured will be alive at maturity and the payment (6) will be due at that time. Combining both risks together we conclude that if the financial risk is big, the insurance company may prefer to be exposed to a smaller mortality risk. By contrast, if the claim (6) could be hedged with greater probability the insurance company may wish to increase its mortality risk exposure. Therefore, there is an offset between financial and mortality risks the insurer can play with: by fixing one of the risks, the appropriate level of another risk could be calculated.

For Figure 1 we obtained survival probabilities using (18) for different levels of a financial risk  $\varepsilon$  and found the corresponding ages for clientele using the specified mortality table. Note that whenever the risk that the insurance company will fail to hedge successfully increases, the recommended ages of the clients rise as well. As a result, the company diminishes the insurance component of risk by attracting older and, therefore, safer clientele to compensate for the increasing financial risk. Also observe that with longer contract

maturities, the company can widen its audience to younger clients because a mortality risk, which is a survival probability in our case, is decreasing over time.

Different combinations of a financial risk  $\varepsilon$  and an insurance risk  $\alpha$  give us the range of prices for the equity-linked contracts. The results for the contracts are shown in Figure 2.

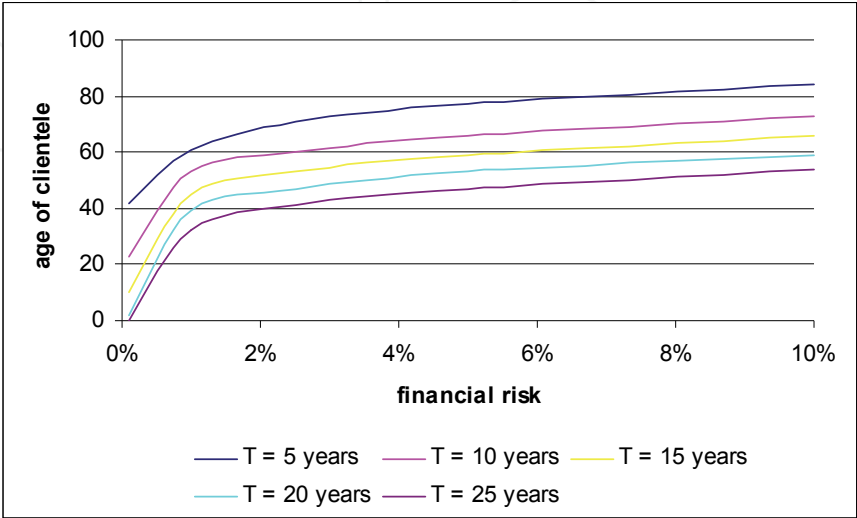


Fig. 1. Offsetting financial and mortality risks

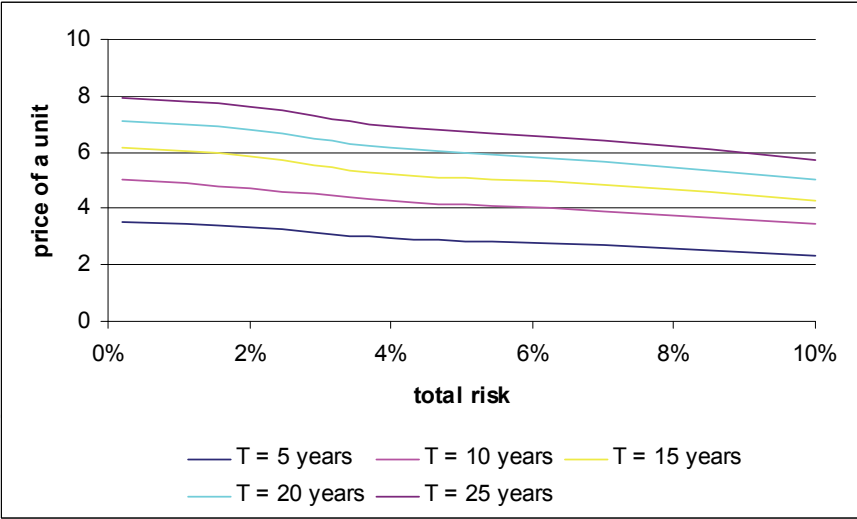


Fig. 2. Prices of \$100 invested in equity-linked life insurance contracts

The next step is to construct a grid that enables the insurance company to identify the acceptable level of the financial risk for insureds of any age. We restrict our attention to a group of clients of ages 30, 40, 50, and 60 years. The results are presented in Table 1. The financial risk found reflects the probability of failure to hedge the payoff that will be offset by the mortality risk of the clients of a certain age.

Age of clients	$T = 5$	$T = 10$	$T = 15$	$T = 20$	$T = 25$
30	0.05%	0.13%	0.25%	0.45%	0.8%
40	0.1%	0.25%	0.55%	1.2%	2.3%
50	0.2%	0.7%	1.8%	3.7%	7%
60	0.8%	2.5%	5.5%	10.5%	18.5%

Table 1. Acceptable Financial Risk Offsetting Mortality Risk of Individual Client

Age of clients	$T = 5$	$T = 10$	$T = 15$	$T = 20$	$T = 25$
30	3.45	4.86	5.87	6.66	7.22
40	3.45	4.79	5.69	6.25	6.45
50	3.39	4.56	5.11	5.10	4.53
60	3.17	3.84	3.76	2.99	1.70
Margrabe price	3.57	5.04	6.17	7.13	7.97

Table 2. Prices of contracts with cumulative mortality risk  $\alpha = 2.5\%$

Prices of the contracts for the same group of clients are given in Table 2. Note that the price of a contract is a function of financial and insurance risks associated with this contract. The level of the insurance risk is chosen to be  $\alpha = 2.5\%$ . In the last row, the Margrabe prices are compared with reduced prices of equity-linked contracts. The reduction in prices was possible for two reasons: we took into account the mortality risk of an individual client (the probability that the client would not survive to maturity and, therefore, no payment at maturity would be made) and the possibility to diversify the cumulative mortality risk by pooling homogeneous clients together.

6. Appendix

6.1 Proof of (14)

Let  $Y_T = S_T^1/S_T^2$ . Then we have

$$\begin{aligned} E^* \left( S_T^1 - S_T^2 \right)^+ &= E^* \left( S_T^1 - S_T^2 \right) \cdot I_{\{Y_T > 1\}} \\ &= E^* S_T^1 - E^* S_T^2 - E^* \left( S_T^1 - S_T^2 \right) \cdot I_{\{Y_T \leq 1\}}. \end{aligned}$$

(28)

Since  $S^1, S^2$  are martingales with respect to  $P^*$ , we have  $E^* S_T^i = S_0^i = S_0$ ,  $i=1,2$ . For the last term in (28), we get

$$E^* S_T^i \cdot I_{\{Y_T \leq 1\}} = E^* \exp \{ -\eta_i \} \cdot I_{\{ \xi \leq \ln 1 \}}$$

(29)

where  $\eta_i = -\ln S_T^i$ ,  $\xi = \ln Y_T$  are Gaussian random variables. Using properties of normal random variables (Melnikov, 2011) we find that



$$E^* \exp\{-\eta_i\} \cdot I_{\{\xi \leq \ln 1\}} = \exp\left\{\frac{\sigma_{\eta_i}^2}{2} - \mu_{\eta_i}\right\} \Phi\left(\frac{\ln 1 - (\mu_{\xi} - \text{cov}(\xi, \eta_i))}{\sigma_{\xi}}\right) \quad (30)$$

where  $\mu_{\eta_i} = E^* \eta_i, \sigma_{\eta_i}^2 = \text{var}(\eta_i), \mu_{\xi} = E^* \xi, \sigma_{\xi}^2 = \text{var} \xi, \Phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^x \exp(-y^2/2) dy$ .

Using (29), (30), we arrive at (14).

## 6.2 Proof of (17)

According to (16), we have

$$\begin{aligned} Z_T^{1/(p-1)} \cdot (S_T^2)^{-1} &= G^{1/(p-1)} \cdot (S_T^1)^{-\frac{(1+\gamma)\mu_1}{\sigma_1^2(p-1)}} (S_T^2)^{-1+\frac{\mu_1}{\sigma_1\sigma_2(p-1)}} \\ &= G^{1/(p-1)} \cdot Y_T^{\alpha_p} \end{aligned} \quad (31)$$

with

$$\alpha_p = -\frac{(1+\gamma)\mu_1}{\sigma_1^2(p-1)} = 1 - \frac{\mu_1}{\sigma_1\sigma_2(p-1)}. \quad (32)$$

Equation (32) has the unique solution

$$\gamma = \gamma_p = \frac{\sigma_1^2\sigma_2(p-1) + \mu_1\sigma_2}{\mu_1(\sigma_1 - \sigma_2)}. \quad (33)$$

It follows from (33) that  $\gamma_p > 0$  and, therefore, from (32) we conclude that  $\alpha_p < 0$  and the equation

$$a_p G^{1/(p-1)} y^{\alpha_p} = (y-1)^+, \quad y \geq 1 \quad (34)$$

has the unique solution  $C = C(p) \geq 1$ . Using (31)-(34), we represent  $(S_T^1 - S_T^2)_p^+$  as follows

$$\begin{aligned} (S_T^1 - S_T^2)_p^+ &= S_T^2 (Y_T - 1)^+ - \left(a_p G^{1/(p-1)} Y_T^{\alpha_p} S_T^2\right) \wedge S_T^2 (Y_T - 1)^+ \\ &= S_T^2 \left((Y_T - 1)^+ - \left(a_p G^{1/(p-1)} Y_T^{\alpha_p}\right) \wedge (Y_T - 1)^+\right) \\ &= S_T^2 \left((Y_T - 1)^+ - (Y_T - 1)^+ I_{\{Y_T \leq C(p)\}} - a_p G^{1/(p-1)} Y_T^{\alpha_p} I_{\{Y_T > C(p)\}}\right). \end{aligned}$$

Taking into account that  $I_{\{Y_T > C(p)\}} = 1 - I_{\{Y_T \leq C(p)\}}$ , we get

$$\begin{aligned} E^* (S_T^1 - S_T^2)_p^+ &= E^* (S_T^1 - S_T^2)^+ - E^* (S_T^1 - S_T^2)^+ I_{\{Y_T \leq C(p)\}} \\ &\quad - a_p G^{1/(p-1)} \left(E^* S_T^2 Y_T^{\alpha_p} - E^* S_T^2 Y_T^{\alpha_p} I_{\{Y_T \leq C(p)\}}\right). \end{aligned} \quad (35)$$

Since  $C(p) \geq 1$ , we have

$$\begin{aligned} E^* \left( S_T^1 - S_T^2 \right)^+ - E^* \left( S_T^1 - S_T^2 \right)^+ I_{\{Y_T \leq C(p)\}} &= E^* \left( S_T^1 - S_T^2 \right)^+ I_{\{Y_T > C(p)\}} \\ &= E^* \left( S_T^1 - S_T^2 \right) - E^* \left( S_T^1 - S_T^2 \right) I_{\{Y_T \leq C(p)\}}. \end{aligned} \quad (36)$$

Using (36), we can calculate the difference between the first two terms in (35) reproducing exactly the same procedure as in (28)-(30) and arrive at

$$\begin{aligned} E^* \left( S_T^1 - S_T^2 \right)^+ - E^* \left( S_T^1 - S_T^2 \right)^+ I_{\{Y_T \leq C(p)\}} \\ = S_0 \left\{ \Phi \left( b_+(1, C, T) \right) - \Phi \left( b_-(1, C, T) \right) \right\}. \end{aligned} \quad (37)$$

To calculate the other two terms in (35), we represent the product  $S_T^2 Y_T^{\alpha_p}$  as follows

$$\begin{aligned} S_T^2 Y_T^{\alpha_p} &= S_0 \times \exp \left\{ \left( \sigma_1 \alpha_p + \sigma_2 (1 - \alpha_p) \right) W_T^* - \frac{1}{2} \left( \sigma_1^2 \alpha_p + \sigma_2^2 (1 - \alpha_p) \right) T \right\} \\ &= S_0 \times \exp \left\{ \left( \sigma_1 \alpha_p + \sigma_2 (1 - \alpha_p) \right) W_T^* - \frac{1}{2} \left( \sigma_1 \alpha_p + \sigma_2 (1 - \alpha_p) \right)^2 T \right. \\ &\quad \left. + \frac{1}{2} \left( \sigma_1 \alpha_p + \sigma_2 (1 - \alpha_p) \right)^2 T - \frac{1}{2} \left( \sigma_1^2 \alpha_p + \sigma_2^2 (1 - \alpha_p) \right) T \right\} \\ &= S_0 \times \exp \left\{ \left( \sigma_1 \alpha_p + \sigma_2 (1 - \alpha_p) \right) W_T^* - \frac{1}{2} \left( \sigma_1 \alpha_p + \sigma_2 (1 - \alpha_p) \right)^2 T \right. \\ &\quad \left. - \alpha_p (1 - \alpha_p) (\sigma_1 - \sigma_2)^2 \frac{T}{2} \right\}. \end{aligned} \quad (38)$$

Taking an expected value of (38) with respect to  $P^*$ , we find that

$$E^* S_T^2 Y_T^{\alpha_p} = S_0 \exp \left\{ -\alpha_p (1 - \alpha_p) (\sigma_1 - \sigma_2)^2 \frac{T}{2} \right\}. \quad (39)$$

Using (38), (39) and following the same steps as in (28)-(30), we obtain

$$\begin{aligned} -a_p G^{1/(p-1)} \left( E^* S_T^2 Y_T^{\alpha_p} - E^* S_T^2 Y_T^{\alpha_p} I_{\{Y_T \leq C(p)\}} \right) \\ = -a_p G^{1/(p-1)} \left( S_0 \right) \exp \left\{ -\alpha_p (1 - \alpha_p) (\sigma_1 - \sigma_2)^2 \frac{T}{2} \right\} \\ \times \Phi \left( b_-(1, C, T) + a_p (\sigma_1 - \sigma_2) \sqrt{T} \right). \end{aligned} \quad (40)$$

Combining (13), (14), (35), (37), and (40), we arrive at (17). <sup>†</sup>

### 6.3 Proof of (18)

Taking into account the structure of  $\left( S_T^1 - S_T^2 \right)_p^+$  in (11) we represent the product  $Z_T \left( S_T^2 \right)^{1-p}$  with the help of a free parameter  $\gamma$  (see (15), (16), (31)-(34)) and get

$$Z_T \left( S_T^2 \right)^{1-p} = G \left( S_T^1 \right)^{-\frac{(1+\gamma)\mu_1}{\sigma_1^2}} \left( S_T^2 \right)^{\frac{\gamma\mu_1}{\sigma_1\sigma_2}} \left( S_T^2 \right)^{1-p} = G Y_T^{\alpha_p}, \quad (41)$$

where  $\alpha_p = -\frac{(1+\gamma)\mu_1}{\sigma_1^2} = -\frac{\gamma\mu_1}{\sigma_1\sigma_2} - (1-p)$  and, hence,

$$\begin{aligned} \gamma = \gamma_p &= \frac{\sigma_2(\mu_1 - (1-p)\sigma_1^2)}{\mu_1(\sigma_1 - \sigma_2)}, \\ -\alpha_p &= \frac{\mu_1}{\sigma_1^2} \left( 1 + \frac{\sigma_2(\mu_1 - (1-p)\sigma_1^2)}{\mu_1(\sigma_1 - \sigma_2)} \right) = \frac{\mu_1}{\sigma_1^2} + \frac{\sigma_2}{(\sigma_1 - \sigma_2)} \left( \frac{\mu_1}{\sigma_1^2} - (1-p) \right). \end{aligned} \quad (42)$$

Consider the following characteristic equation:

$$y^{-\alpha_p} = a_p G \left( (y-1)^+ \right)^{1-p}, \quad y \geq 0. \quad (43)$$

1. If  $-\alpha_p > 1-p$ , then according to (42)

$$\frac{\mu_1}{\sigma_1^2} + \frac{\sigma_2}{(\sigma_1 - \sigma_2)} \left( \frac{\mu_1}{\sigma_1^2} - (1-p) \right) > 1-p \quad \text{or} \quad \frac{\mu_1}{\sigma_1^2} > 1-p > 0. \quad (44)$$

In this case the equation (43) has zero, one, or two solutions. All these situations can be considered in a similar way as Case 1.

1.1. If (43) has no solution then  $I_{\{Z_T^{-1} > a_p H^{1-p}\}} \equiv 1$ ,  $H_p = H$  and, therefore,  ${}_T p_x = 1$ .

1.2. If (43) has one solution  $C = C(p)$  then  $\left( S_T^1 - S_T^2 \right)_p^+ = \left( S_T^1 - S_T^2 \right)^+ I_{\{Y_T \leq C(p)\}}$  and, according to (13), we arrive at (18).

1.3. If there are two solutions  $C_1(p) < C_2(p)$  to (43) then the structure of a modified claim is

$$\left( S_T^1 - S_T^2 \right)_p^+ = \left( S_T^1 - S_T^2 \right)^+ I_{\{Y_T \leq C_1(p)\}} + \left( S_T^1 - S_T^2 \right)^+ I_{\{Y_T > C_2(p)\}}$$
 and we arrive at (20).

2. If  $-\alpha_p \leq 1-p$ , then  $\frac{\mu_1}{\sigma_1^2} \leq 1-p < 1$  and, therefore, the equation (43) has only one solution  $C = C(p)$ . This is equivalent to 1.2 and, reproducing the same reasons, we arrive at (18).

## 6.4 Proof of (21)

According to (16), we represent the density  $Z_T$  as follows

$$Z_T = G \left( S_T^1 \right)^{-\frac{(1+\gamma)\mu_1}{\sigma_1^2}} \left( S_T^2 \right)^{\frac{\gamma\mu_1}{\sigma_1\sigma_2}} = G Y_T^{\alpha_p} \quad (45)$$

where  $\alpha_p = -\frac{(1+\gamma)\mu_1}{\sigma_1^2} = -\frac{\gamma\mu_1}{\sigma_1\sigma_2}$  and, therefore,

$$\begin{aligned}\gamma &= \gamma_p = \frac{\sigma_1 \sigma_2}{\sigma_1 (\sigma_1 - \sigma_2)}, \\ -\alpha_p &= \frac{\mu_1}{\sigma_1 \sigma_2} \gamma_p = \frac{\mu_1}{\sigma_1 (\sigma_1 - \sigma_2)}, \quad \sigma_1 > \sigma_2.\end{aligned}\quad (46)$$

From (16) and (46) we find that

$$\left(S_T^1 - S_T^2\right)^+ I_{\left\{Y_T^{-\alpha_p} > Ga_p\right\}} = \left(S_T^1 - S_T^2\right)^+ I_{\left\{Y_T^{\frac{\mu_1}{\sigma_1(\sigma_1 - \sigma_2)}} > Ga_p\right\}} = \left(S_T^1 - S_T^2\right)^+ I_{\{Y_T > C\}} \quad (47)$$

where

$$C = \left(Ga_p\right)^{\frac{\sigma_1(\sigma_1 - \sigma_2)}{\mu_1}}. \quad (6.20)$$

Using (13), (14), (36), (37), and (47), we arrive at (21).

## 7. Conclusion

As financial markets become more and more complicated over time new techniques emerge to help dealing with new types of uncertainties, either not present or not recognized before, or to refine measurements of already existing risks. The insurance industry being a part of the bigger and more dynamic financial industry could benefit from new developments in financial instruments and techniques. These may include introduction of new types of insurance contracts linked to specific sectors of the financial market which were not possible or not thought of before, new ways of hedging already existing types of insurance contracts with the help of financial instruments, more refined measurement of financial or insurance risks existing or emerging in the insurance industry that will improve their management through better hedging or diversification and thus allow insurance companies to take more risk. In any way, the insurance industry should stay attune to new developments in the financial industry. Stochastic interest rate models, jump-diffusion models for risky assets, a financial market with  $N$  ( $N > 2$ ) correlated risky assets, modeling of transaction costs are few examples of the developments in the financial mathematics which could be incorporated in the financial setting of the model for equity-linked life insurance under our consideration. Some actuarial modeling including lapses, surrender options, the ability of the insured to switch between different benefit options, mortality risk models will also be able to enrich the insurance setting of the model. Methods of hedging/risk management other than efficient hedging could be used as well.

A balanced combination of two approaches to risk-management: risk diversification (pooling homogenous mortality risks, a combination of maturity benefits providing both a guarantee and a potential gain for the insured) and risk hedging (as for hedging maturity benefits with the help of financial market instruments) are going to remain the main focus for combining financial and insurance risk. A third risk management method – risk insurance (reinsurance, insurance of intermediate consumption outflows, insurance of extreme events in the financial market) – could be added for benefits of both the insurance company and the insured.

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## **Risk Management Trends**

Edited by Prof. Giancarlo Nota

ISBN 978-953-307-314-9

Hard cover, 266 pages

**Publisher** InTech

**Published online** 28, July, 2011

**Published in print edition** July, 2011

In many human activities risk is unavoidable. It pervades our life and can have a negative impact on individual, business and social levels. Luckily, risk can be assessed and we can cope with it through appropriate management methodologies. The book Risk Management Trends offers to both, researchers and practitioners, new ideas, experiences and research that can be used either to better understand risks in a rapidly changing world or to implement a risk management program in many fields. With contributions from researchers and practitioners, Risk Management Trends will empower the reader with the state of the art knowledge necessary to understand and manage risks in the fields of enterprise management, medicine, insurance and safety.

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