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Fundamental Problems of the Electrodynamics of Heterogeneous Media with Boundary Conditions Corresponding to the Total-Current Continuity

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1. Introduction

Let us consider the interface S between two media having different electrophysical properties. On each of its side the magnetic-field and magnetic-inductance vectors as well as the electric-field and electric-displacement vectors are finite and continuous; however, at the surface S they can experience a discontinuity of the first kind. Furthermore, at the interface there arise induced surface charges σ and surface currents i (whose vectors lie in the plane tangential to the surface S) under the action of an external electric field.

The existence of a surface charge at the interface S between the two media having different electrophysical properties is clearly demonstrated by the following example. We will consider the traverse of a direct current through a flat capacitor filled with two dielectric materials having relative permittivities ε_1 and ε_2 and electrical conductivities λ_1 and λ_2. A direct-current voltage U is applied to the capacitor plates; the total resistance of the capacitor is R (Fig. 1). It is necessary to calculate the surface electric charge induced by the electric current.

From the electric-charge conservation law follows the constancy of flow in a circuit; therefore, the following equation is fulfilled:

$$\lambda_1 E_{n_1} = \lambda_2 E_{n_2} = U / (RS) \tag{1}$$

where E_{n_1} and E_{n_2} are the normal components of the electric-field vector.

At the interface between the dielectrics, the normal components of the electric-inductance vector change spasmodically under the action of the electric field by a value equal to the value of the induced surface charge σ:

$$\varepsilon_0\varepsilon_1 E_{n_1} - \varepsilon_0\varepsilon_2 E_{n_2} = \sigma \tag{2}$$

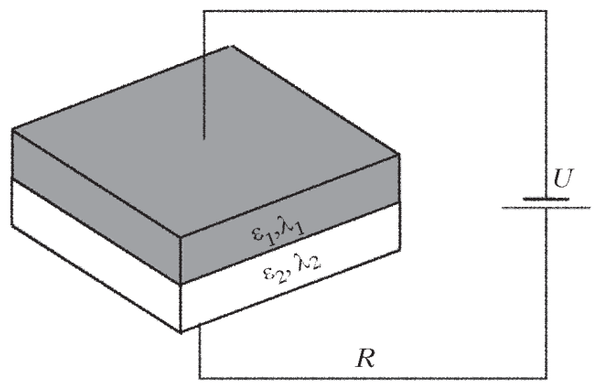


Fig. 1. Dielectric media inside a flat capacitor

Solving the system of Eqs. (see Equations 1 and 2), we obtain the expression for σ

$$\sigma = (U / RS)\varepsilon_0\left[(\varepsilon_1 / \lambda_1) - (\varepsilon_2 / \lambda_2)\right] \tag{3}$$

It follows from (see Equation 3) that the charge σ is determined by the current and the multiplier accounting for the properties of the medium. If

$$(\varepsilon_1 / \lambda_1) - (\varepsilon_2 / \lambda_2) = 0 \tag{4}$$

a surface charge σ is not formed. What is more, recent trends are toward increased use of micromachines and engines made from plastic materials, where the appearance of a surface charge is undesirable. For oiling of elements of such machines, it is best to use an oil with a permittivity ε_{oil} satisfying the relation

$$\varepsilon_1 < \varepsilon_{oil} < \varepsilon_2 \tag{5}$$

This oil makes it possible to decrease the electrization of the moving machine parts made from dielectric materials. In addition to the charge σ, a contact potential difference arises always independently of the current.

An electric field interacting with a material is investigated with the use of the Maxwell equation (1857)

$$\mathbf{j}_{total} = \nabla \times \mathbf{H}, \nabla \cdot \mathbf{D} = \rho \tag{6}$$

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}, \nabla \cdot \mathbf{B} = 0 \tag{7}$$

where $\mathbf{j}_{total} = \lambda\mathbf{E} + \frac{\partial \mathbf{D}}{\partial t}$; $\mathbf{B} = \mu\mu_0\mathbf{H}$; $\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$. In this case, at the interface S the above system of equations is supplemented with the boundary conditions (Monzon, I.; Yonte,T.; Sanchez-Soto, L., 2003; Eremin,Y. & Wriedt,T., 2002)

$$D_{n_1} - D_{n_2} = \sigma \tag{8}$$

$$E_{\tau_1} - E_{\tau_2} = 0 \tag{9}$$

$$B_{n_1} - B_{n_2} = 0 \tag{10}$$

$$\mathbf{H}_{\tau_1} - \mathbf{H}_{\tau_2} = \left[\vec{\mathbf{i}}_\tau \cdot \vec{\mathbf{n}}\right] \tag{11}$$

The indices (subscripts) n and τ denote the normal and tangential components of the vectors to the surface S, and the indices 1 and 2 denote the adjacent media with different electrophysical properties. The index τ denotes any direction tangential to the discontinuity surface. At the same time, a closing relation is absent for the induced surface charge σ, which generates a need for the introduction of an impedance matrix (Wei Hu & Hong Guo, 2002; Danae, D. et al., 2002; Larruquert, J. I., 2001; Koludzija, B. M., 1999; Ehlers, R. A. & Metaxas, A. C., 2003) that is determined experimentally or, in some cases, theoretically from quantum representations (Barta, O.; Pistora, I.; Vesec, I. et al., 2001; Broe, I. & Keller, O., 2002; Keller, 1995; Keller, O., 1995; Keller, O., 1997).

The induced surface charge σ not only characterizes the properties of a surface, but also represents a function of the process, i.e., σ(**E**(∂**E**/∂t, **H**(∂**H**/∂t))); therefore, the surface impedances (Wei Hu & Hong Guo, 2002; Danae, D. et al., 2002; Larruquert, J. I., 2001; Koludzija, B. M., 1999; Ehlers, R. A. & Metaxas, A. C., 2003) are true for the conditions under which they are determined. These impedances cannot be used in experiments conducted under other experimental conditions.

The problem of determination of surface charge and surface current on metal-electrolyte boundaries becomes even more complicated in investigating and modeling nonstationary electrochemical processes, e.g., pulse electrolysis, when lumped parameters L, C, and R cannot be used in principle.

We will show that σ can be calculated using the Maxwell phenomenological macroscopic electromagnetic equations and the electric-charge conservation law accounting for the special features of the interface between the adjacent media.

Separate consideration will be given to ion conductors. In constructing a physicomathematical model, we take into account that $\vec{E}$ and $\vec{H}$ are not independent functions; therefore, the wave equation for $\vec{E}$ or $\vec{H}$ is more preferable than the system of equations (see Equations 6 and 7).

2. Electron conductors. New closing relations on the boundaries of adjacent media

2.1 Generalized wave equation for $\vec{E}$ and conditions on the boundaries in the presence of strong discontinuities of the electromagnetic field

2.1.1 Physicomathematical model

We will formulate a physicomathematical model of propagation of an electromagnetic field in a heterogeneous medium. Let us multiply the left and right sides of the equation for the total current (see Equation 6) by $\mu\mu_0$ and differentiate it with respect to time. Acting by the operator rot on the left and right sides of the first equation of Eq. (see Equation 7) on condition that μ=const we obtain

$$\frac{\partial \mathbf{j}_{total}}{\partial t} = \frac{1}{\mu\mu_0}\nabla^2\mathbf{E} - \frac{1}{\mu\mu_0} grad\left(div\mathbf{E}\right) \tag{12}$$

In Cartesian coordinates, Eq. (see Equation 12) will take the form

$$\frac{\partial \mathbf{j}_{totalx}}{\partial t} = \frac{1}{\mu\mu_0}\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) - \frac{1}{\mu\mu_0}\frac{\partial}{\partial x}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) \tag{13}$$

$$\frac{\partial \mathbf{j}_{totaly}}{\partial t} = \frac{1}{\mu\mu_0}\left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right) - \frac{1}{\mu\mu_0}\frac{\partial}{\partial y}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) \tag{14}$$

$$\frac{\partial \mathbf{j}_{totalz}}{\partial t} = \frac{1}{\mu\mu_0}\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right) - \frac{1}{\mu\mu_0}\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) \tag{15}$$

At the interface, the following relation (Eremin,Y. & Wriedt,T., 2002) is also true:

$$div\mathbf{i}_\tau + I_{qx_1} - I_{qx_2} = -\frac{\partial \sigma}{\partial t} \tag{16}$$

Let us write conditions (see Equations 8–11) in the Cartesian coordinate system:

$$D_{x_1} - D_{x_2} = \sigma \tag{17}$$

$$E_{y_1} - E_{y_2} = 0 \tag{18}$$

$$E_{z_1} - E_{z_2} = 0 \tag{19}$$

$$B_{x_1} - B_{x_2} = 0 \tag{20}$$

$$H_{y_1} - H_{y_2} = i_z \tag{21}$$

$$H_{z_1} - H_{z_2} = i_y \tag{22}$$

where $\mathbf{i}_\tau = i_y\mathbf{j} + i_z\mathbf{k}$ is the surface-current density, and the coordinate x is directed along the normal to the interface. The densities i_y and i_z of the surface currents represent the electric charge carried in unit time by a segment of unit length positioned on the surface drawing the current perpendicularly to its direction.

The order of the system of differential equations (see Equations 13–15) is equal to 18. Therefore, at the interface S, it is necessary to set, by and large, nine boundary conditions. Moreover, the three additional conditions (see Equation 17, 21, and 22) containing (prior to the solution) unknown quantities should be fulfilled at this interface. Consequently, the total number of conjugation conditions at the boundary S should be equal to 12 for a correct solution of the problem.

Differentiating expression (see Equation 17) with respect to time and using relation (see Equation 16), we obtain the following condition for the normal components of the total current at the medium-medium interface:

$$div\mathbf{i}_\tau + \mathbf{j}_{totalx_1} = \mathbf{j}_{totalx_2} \tag{23}$$

that allows one to disregard the surface charge σ. Let us introduce the arbitrary function *f*: $[f]\big|_{x=\xi} = f_1\big|_{x=\xi+0} - f_2\big|_{x=\xi-0}$. In this case, expression (see Equation 23) will take the form

$$\left[div\mathbf{i}_\tau + \mathbf{j}_{totalx_1}\right]\Big|_{x=\xi} = 0 \tag{24}$$

It is assumed that, at the medium-medium interface, E_x is a continuous function of y and z. Then, differentiating Eq. (see Equation 23) with respect to y and z, we obtain

$$\left[\frac{\partial}{\partial y}\mathbf{j}_{totalx_1}\right]\Bigg|_{x=\xi} = -\frac{\partial\left(div\mathbf{i}_\tau\right)}{\partial y} \tag{25}$$

$$\left[\frac{\partial}{\partial z}\mathbf{j}_{totalx_1}\right]\Bigg|_{x=\xi} = -\frac{\partial\left(div\mathbf{i}_\tau\right)}{\partial z} \tag{26}$$

Let us differentiate conditions (see Equations 20–22) for the magnetic induction and the magnetic-field strength with respect to time. On condition that $\mathbf{B}=\mu\mu_0\mathbf{H}$

$$\left[\frac{\partial B_x}{\partial t}\right]\Bigg|_{x=\xi} = 0, \left[\frac{1}{\mu\mu_0}\frac{\partial B_y}{\partial t}\right]\Bigg|_{x=\xi} = \frac{\partial i_z}{\partial t}, \left[\frac{1}{\mu\mu_0}\frac{\partial B_z}{\partial t}\right]\Bigg|_{x=\xi} = \frac{\partial i_y}{\partial t} \tag{27}$$

Using Eq. (see Equation 7) and expressing (see Equation 27) in terms of projections of the electric-field rotor, we obtain

$$\left[rot_x\mathbf{E}\right]\Big|_{x=\xi} = 0 \text{ and } \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right]\Bigg|_{x=\xi} = 0 \tag{28}$$

$$\left[\frac{1}{\mu\mu_0}rot_y\mathbf{E}\right]\Bigg|_{x=\xi} = \frac{\partial i_z}{\partial t} \text{ or } \left[\frac{1}{\mu\mu_0}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\right]\Bigg|_{x=\xi} = \frac{\partial i_z}{\partial t} \tag{29}$$

$$\left[\frac{1}{\mu\mu_0}rot_z\mathbf{E}\right]\Bigg|_{x=\xi} = \frac{\partial i_z}{\partial t} \text{ or } \left[\frac{1}{\mu\mu_0}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right]\Bigg|_{x=\xi} = \frac{\partial i_z}{\partial t} \tag{30}$$

Here, Eq. (see Equation 28) is the normal projection of the electric-field rotor, Eq. (see Equation 29) is the tangential projection of the rotor on y, and Eq. (see Equation 30) is the rotor projection on z.

Assuming that E_y and E_z are continuous differentiable functions of the coordinates y and z, from conditions (see Equations 18 and 19) we find

$$\left[\frac{\partial E_y}{\partial y}\right]\Bigg|_{x=\xi}=0,\left[\frac{\partial E_y}{\partial z}\right]\Bigg|_{x=\xi}=0$$
$$\left[\frac{\partial E_z}{\partial y}\right]\Bigg|_{x=\xi}=0,\left[\frac{\partial E_z}{\partial z}\right]\Bigg|_{x=\xi}=0 \tag{31}$$

In accordance with the condition that the tangential projections of the electric field on z and y are equal and in accordance with conditions (see Equations 18 and 19), the expressions for the densities of the surface currents i_z and i_y take the form

$$i_z=\bar{\lambda}E_z\Big|_{x=\xi},\ i_y=\bar{\lambda}E_y\Big|_{x=\xi} \tag{32}$$

where

$$\bar{\lambda}=\frac{1}{2}\left(\lambda_1+\lambda_2\right)\Big|_{x=\xi} \tag{33}$$

is the average value of the electrical conductivity at the interface between the adjacent media in accordance with the Dirichlet theorem for a piecewise-smooth, piecewise-differentiable function.

Consequently, formulas (see Equations 31–33) yield

$$\left[div\vec{i_\tau}\right]\Big|_{x=\xi}=0 \tag{34a}$$

Relation (see Equation 34) and hence the equality of the normal components of the total current were obtained (in a different manner) by G.A. Grinberg and V.A. Fok (Grinberg, G.A. & Fok, V.A., 1948). In this work, it has been shown that condition (34a) leads to the equality of the derivatives of the electric field strength along the normal to the surface

$$\left[\frac{\partial E_x}{\partial x}\right]\Bigg|_{x=\xi}=0 \tag{34b}$$

With allowance for the foregoing we have twelve conditions at the interface between the adjacent media that are necessary for solving the complete system of equations (see Equations 13–15):

a. the functions E_y and E_z are determined from Eqs. (see Equations 18 and 19);
b. E_x is determined from condition (see Equation 24);
c. the values of $\partial E_x/\partial y$, $\partial E_x/\partial z$, and $\partial E_x/\partial x$ are determined from relations (see Equations 25 and 26) with the use of the condition of continuity of the total-current normal component at the interface (see Equation 24) and the continuity of the derivative of the total current with respect to the coordinate *x;*
d. the values of $\partial E_y/\partial y$, $\partial E_y/\partial z$, and $\partial E_z/\partial z$ are determined from conditions (see Equations 31 and 32) in consequence of the continuity of the tangential components of the electric field along *y* and *z;*
e. the derivatives $\partial E_y/\partial x$ and $\partial E_z/\partial x$ are determined from conditions (see Equations 29 and 30) as a consequence of the equality of the tangential components of the electric-field rotor along *y* and *z*.

Note that condition (see Equation 23) was used by us in (Grinchik, N. N. & Dostanko, A. P., 2005) in the numerical simulation of the pulsed electrochemical processes in the one-dimensional case. Condition (see Equation 28) for the normal component of the electric-field rotor represents a linear combination of conditions (see Equations 31 and 32); therefore, $\mathrm{rot}_x E = 0$ and there is no need to use it in the subsequent discussion. The specificity of the expression for the general law of electric-charge conservation at the interface is that the components $\partial E_y/\partial y$ and $\partial E_z/\partial z$ are determined from conditions (see Equations 31 and 32) that follow from the equality and continuity of the tangential components E_y and E_z at the boundary S.

Thus, at the interface between the adjacent media the following conditions are fulfilled: the equality of the total-current normal components; the equality of the tangential projections of the electric-field rotor; the electric-charge conservation law; the equality of the electric-field tangential components and their derivatives in the tangential direction; the equality of the derivatives of the total-current normal components in the direction tangential to the interface between the adjacent media, determined with account for the surface currents and without explicit introduction of a surface charge. They are true at each cross section of the sample being investigated.

2.1.2 Features of calculation of the propagation of electromagnetic waves in layered media

The electromagnetic effects arising at the interface between different media under the action of plane electromagnetic waves have a profound impact on the equipment because all real devices are bounded by the surfaces and are inhomogeneous in space. At the same time, the study of the propagation of waves in layered conducting media and, according to (Born, 1970), in thin films is reduced to the calculation of the reflection and transmission coefficients; the function $E(x)$ is not determined in the thickness of a film, i.e., the geometrical-optics approximation is used.

The physicomathematical model proposed allows one to investigate the propagation of an electromagnetic wave in a layered medium without recourse to the assumptions used in (Wei Hu & Hong Guo, 2002; Danae, D. et al., 2002; Larruquert, J. I., 2001; Ehlers, R. A. & Metaxas, A. C., 2003).

Since conditions (see Equations 23-32) are true at each cross section of a layered medium, we will use schemes of through counting without an explicit definition of the interface between the media. In this case, it is proposed to calculate E_x at the interface in the following way.

In accordance with Eq. (see Equation 17), $E_{x1} \neq E_{x2}$, i.e., $E_x(x)$ experiences a discontinuity of the first kind. Let us determine the strength of the electric field at the discontinuity point $x = \xi$ on condition that $E_x(x)$ is a piecewise-smooth, piecewise-differentiable function having finite one-sided derivatives $E'_{x^+}(x)$ and $E'_{x^-}(x)$. At the discontinuity points x_i,

$$E'_{x^+}(x_i) = \lim_{\Delta x_i \to +0} \frac{E(x_i + \Delta x_i) - E(x_i + 0)}{\Delta x_i} \tag{35}$$

$$E'_{x^-}(x_i) = \lim_{\Delta x_i \to -0} \frac{E(x_i + \Delta x_i) - E(x_i - 0)}{\Delta x_i} \tag{36}$$

In this case, in accordance with the Dirichlet theorem (Kudryavtsev, 1970), the Fourier series of the function $E(x)$ at each point x, including the discontinuity point ξ, converges and its sum is equal to

$$E_{x=\xi} = \frac{1}{2}\left[E(\xi - 0) + E(\xi + 0)\right] \tag{37}$$

The Dirichlet condition (see Equation 37) also has a physical meaning. In the case of contact of two solid conductors, e.g., dielectrics or electrolytes in different combinations (metal-electrolyte, dielectric-electrolyte, metal-vacuum, and so on), at the interface between the adjacent media there always arises an electric double layer (EDL) with an unknown (as a rule) structure that, however, substantially influences the electrokinetic effects, the rate of the electrochemical processes, and so on. It is significant that, in reality, the electrophysical characteristics λ, ε, and *E(x)* change uninterruptedly in the electric double layer; therefore, (see Equation 37) is true for the case where the thickness of the electric double layer, i.e., the thickness of the interphase boundary, is much smaller than the characteristic size of a homogeneous medium. In a composite, e.g., in a metal with embedments of dielectric balls, where the concentration of both components is fairly large and their characteristic sizes are small, the interphase boundaries can overlap and condition (see Equation 37) can break down.

If the thickness of the electric double layer is much smaller than the characteristic size *L* of an object, (see Equation 37) also follows from the condition that *E(x)* changes linearly in the EDL region. In reality, the thickness of the electric double layer depends on the kind of contacting materials and can comprise several tens of angstroms (Frumkin, 1987). In accordance with the modern views, the outer coat of the electric double layer consists of two parts, the first of which is formed by the ions immediately attracted to the surface of the metal (a "dense" or a "Helmholtz" layer of thickness *h*), and the second is formed by the ions separated by distances larger than the ion radius from the surface of the layer, and the number of these ions decreases as the distance between them and the interface (the "diffusion layer") increases. The distribution of the potential in the dense and diffusion parts of the electric double layer is exponential in actual practice (Frumkin, 1987), i.e., the condition that *E(x)* changes linearly breaks down; in this case, the sum of the charges of the dense and diffusion parts of the outer coat of the electric double layer is equal to the charge of its inner coat (the metal surface). However, if the thickness of the electric double layer h is much smaller than the characteristic size of an object, the expansion of *E(x)* into a power series is valid and one can restrict oneself to the consideration of a linear approximation. In accordance with the more general Dirichlet theorem (1829), a knowledge of this function in the EDL region is not necessary to substantiate Eq. (see Equation 37). Nonetheless, the above-indicated physical features of the electric double layer lend support to the validity of condition (see Equation 37).

The condition at interfaces, analogous to Eq. (see Equation 37), has been obtain earlier (Tikhonov, A. N. & Samarskii, A. A., 1977) for the potential field (where rot **E** = 0) on the basis of introduction of the surface potential, the use of the Green formula, and the consideration of the discontinuity of the potential of the double layer. In (Tikhonov, A. N. & Samarskii, A. A., 1977), it is also noted that the consideration of the thickness of the double layer and the change in its potential at $h/L \ll 1$ makes no sense in general; therefore, it is advantageous to consider, instead of the volume potential, the surface potential of any density. Condition (see Equation 37) can be obtained, as was shown in (Kudryavtsev, 1970), from the more general Dirichlet theorem for a nonpotential vorticity field (Tikhonov, A. N. & Samarskii, A. A., 1977).

Thus, the foregoing and the validity of conditions (see Equations 17-19 and 25-.32) at each cross section of a layered medium show that, for numerical solution of the problem being considered it is advantageous to use schemes of through counting and make the

discretization of the medium in such a way that the boundaries of the layers have common points.

The medium was divided into finite elements so that the nodes of a finite-element grid, lying on the separation surface between the media with different electrophysical properties, were shared by these media at a time. In this case, the total currents or the current flows at the interface should be equal if the Dirichlet condition (see Equation 37) is fulfilled.

2.1.3 Results of numerical simulation of the propagation of electromagnetic waves in layered media

Let us analyze the propagation of an electromagnetic wave through a layered medium that consists of several layers with different electrophysical properties in the case where an electromagnetic-radiation source is positioned on the upper plane of the medium. It is assumed that the normal component of the electric-field vector $E_x = 0$ and its tangential component $E_y = a \sin(\omega t)$, where a is the electromagnetic-wave amplitude (Fig. 2).

In this example, for the purpose of correct specification of the conditions at the lower boundary of the medium, an additional layer is introduced downstream of layer 6; this layer has a larger conductivity and, therefore, the electromagnetic wave is damped out rapidly in it. In this case, the condition $E_y = E_z = 0$ can be set at the lower boundary of the medium. The above manipulations were made to limit the size of the medium being considered because, in the general case, the electromagnetic wave is attenuated completely at an infinite distance from the electromagnetic-radiation source.

Numerical calculations of the propagation of an electromagnetic wave in the layered medium with electrophysical parameters $\varepsilon_1 = \varepsilon_2 = 1$, $\lambda_1 = 100$, $\lambda_2 = 1000$, and $\mu_1 = \mu_2 = 1$ were carried out. Two values of the cyclic frequency $\omega = 2\pi/T$ were used: in the first case, the electromagnetic-wave frequency was assumed to be equal to $\omega = 10^{14}$ Hz (infrared radiation), and, in the second case, the cyclic frequency was taken to be $\omega = 10^9$ Hz (radiofrequency radiation).

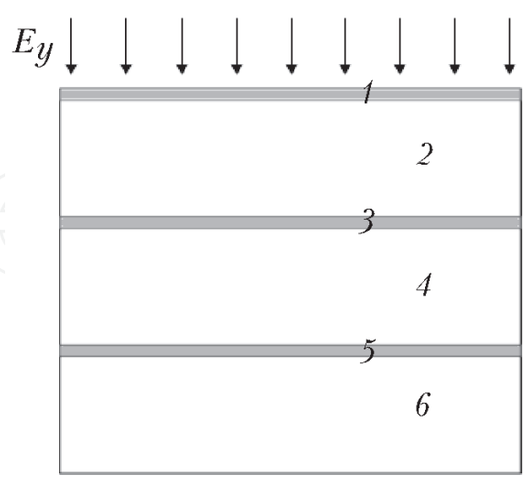


Fig. 2. Scheme of a layered medium: layers 1, 3, and 5 are characterized by the electrophysical parameters ε_1, λ_1, and μ_1, and layers 2, 4, and 6 — by ε_2, λ_2, μ_2.

As a result of the numerical solution of the system of equations (see Equations 13–15) with the use of conditions S (see Equations 24-34) at the interfaces, we obtained the time

dependences of the electric-field strength at different distances from the surface of the layered medium (Fig. 3).

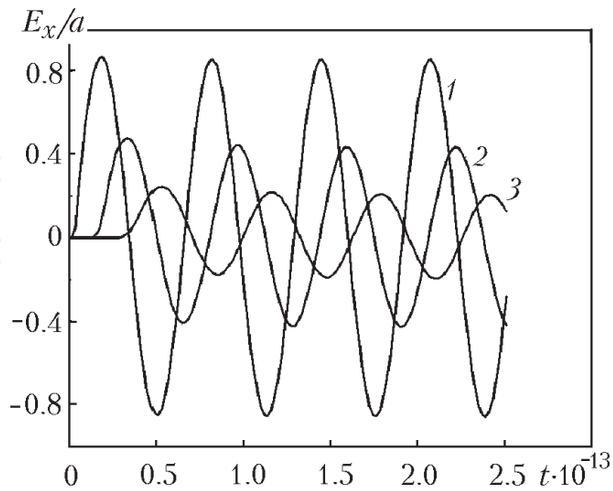


Fig. 3. Time change in the tangential component of the electric-field strength at a distance of 1 μm (1), 5 μm (2), and 10 μm (3) from the surface of the medium at $\lambda_1 = 100$, $\lambda_2 = 1000$, $\varepsilon_1 = \varepsilon_2 = 1$, $\mu_1 = \mu_2 = 1$, and $\omega = 10^{14}$ Hz. *t*, sec.

The results of our simulation (Fig. 4) have shown that a high-frequency electromagnetic wave propagating in a layered medium is damped out rapidly, whereas a low-frequency electromagnetic wave penetrates into such a medium to a greater depth. The model developed was also used for calculating the propagation of a modulated signal of frequency 20 kHz in a layered medium. As a result of our simulation (Fig. 5), we obtained changes in the electric-field strength at different depths of the layered medium, which points to the fact that the model proposed can be used to advantage for calculating the propagation of polyharmonic waves in layered media; such a calculation cannot be performed on the basis of the Helmholtz equation.

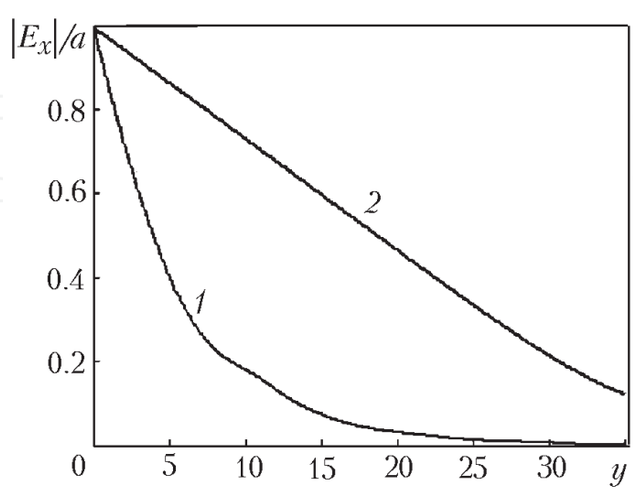


Fig. 4. Distribution of the amplitude of the electric-field-strength at the cross section of the layered medium: $\omega = 10^{14}$ (1) and 10^9 Hz (2). *y*, μm.

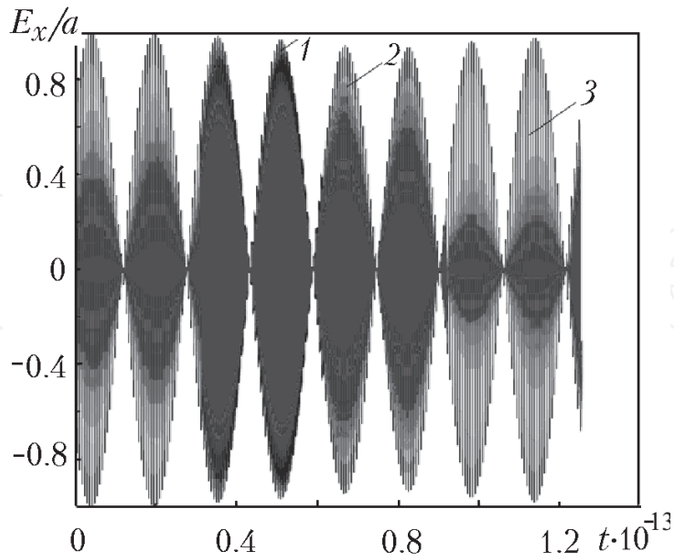
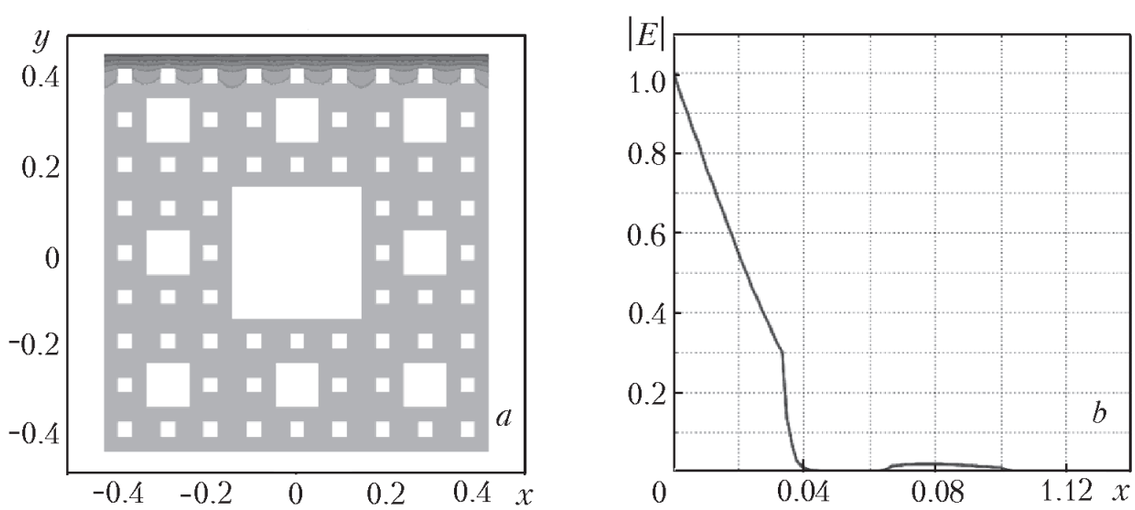


Fig. 5. Time change in the electric-field strength at a distance of 1 (1), 5 (2), and 10 μm (3) from the surface of the medium. *t*, sec.

The physicomathematical model developed can also be used to advantage for simulation of the propagation of electromagnetic waves in media with complex geometric parameters and large discontinuities of the electromagnetic field (Fig. 6).



(a) Distribution of the amplitude of the electric-field strength in the two-dimensional medium

(b) Distribution of the amplitude of the electric-field strength in depth

Fig. 6. Distribution of the amplitude of the electric-field strength in the two-dimensional medium and in depth at $\varepsilon_1 = 15$, $\varepsilon_2 = 20$, $\lambda_1 = 10^{-6}$, $\lambda_2 = 10$, $\mu_1 = \mu_2 = 1$, and $\omega = 10^9$ Hz (the dark background denotes medium 1, and the light background – medium 2). *x*, *y*, mm; *E*, V/m.

Figure 6a shows the cross-sectional view of a cellular structure representing a set of parallelepipeds with different cross sections in the form of squares. The parameters of the materials in the large parallelepiped are denoted by index 1, and the parameters of the materials in the small parallelepipeds (the squares in the figure) are denoted by index 2.

An electromagnetic wave propagates in the parallelepipeds (channels) in the transverse direction. It is seen from Fig. 6b that, in the cellular structure there are "silence regions," where the amplitude of the electromagnetic-wave strength is close to zero, as well as inner regions where the signal has a marked value downstream of the "silence" zone formed as a result of the interference.

2.1.4 Results of numerical simulation of the scattering of electromagnetic waves in angular structures

It is radiolocation and radio-communication problems that are among the main challenges in the set of problems solved using radio-engineering devices.

Knowledge of the space-time characteristics of diffraction fields of electromagnetic waves scattered by an object of location into the environment is necessary for solving successfully any radiolocation problem. Irradiated object have a very intricate architecture and geometric shape of the surface consisting of smooth portions and numerous wedge-shaped for formations of different type-angular joints of smooth portions, surface fractures, sharp edges, etc. – with rounded radii much smaller than the probing-signal wavelength. Therefore, solution of radiolocation problems requires that the methods of calculation of the diffraction fields of electromagnetic waves excited and scattered by different surface portions of the objects, in particular, by wedge-shaped formations, be known, since the latter are among the main sources of scattered waves.

For another topical problem, i.e., radio communication effected between objects, the most difficult are the issues of designing of antennas arranged on an object, since their operating efficiency is closely related to the geometric and radiophysical properties of its surface.

The issues of diffraction of an electromagnetic wave in wedge-shaped regions are the focus of numerous of the problems for a perfectly conducting and impedance wedge for monochromatic waves is representation of the diffraction field in an angular region in the form of a Sommerfeld integral (Kryachko, A.F. et al., 2009).

Substitution of Sommerfeld integrals into the system of boundary conditions gives a system of recurrence functional equations for unknown analytical integrands. The system's coefficients are Fresnel coefficients defining the reflection of plane media or their refraction into the opposite medium. From the system of functional equations, one determines, in a recurrence manner, sequences of integrand poles and residues in these poles.

The edge diffraction field in both media is determined using a pair of Fredholm-type singular integral equations of the second kind which are obtained from the above-indicated systems of functional equations with subsequent computation of Sommerfeld integrals by the saddle-point approximation. The branching points of the integrands condition the presence of creeping waves excited by the edge of the dielectric wedge.

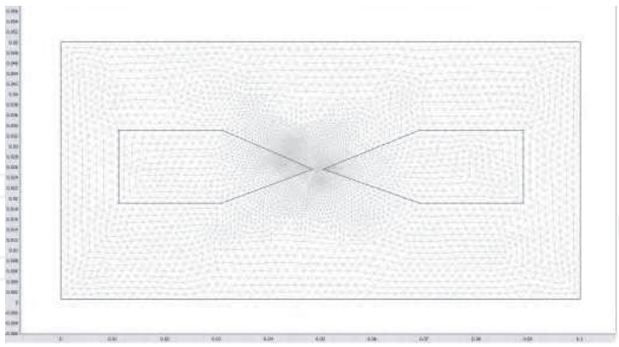
The proposed method is only true of monochromatic waves and of the approximate Leontovich boundary conditions, when the field of the electromagnetic wave slowly varies from point to point on a wavelength scale (Leontovich, 1948).
We note that the existing approximate Leontovich conditions have a number of other constants and should be used with caution (Leontovich, 1948).
In actual fact, the proposed calculation method does not work in the presence of, e.g., two wedges, when the sharp angles are pointed at each other, i.e., an optical knife, or in diffraction of the electromagnetic wave on a system of parallel lobes, when the gap between the lobes is in the region of microns, and the electromagnetic field is strongly "cut" throughout the space with a step much than the wavelength.

A) Optical Knife

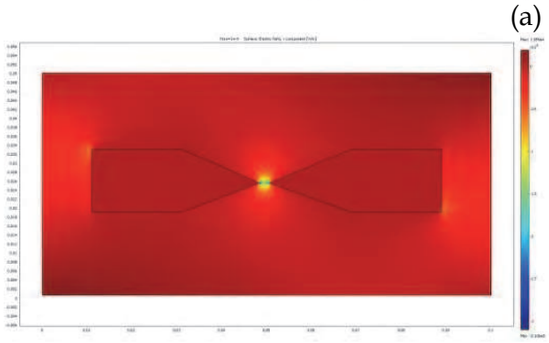
Figure 7 shows the field of an electromagnetic wave in its diffraction on the optical knife. The parameters of the wave at entry and at exit are $E_x=10^4\sin(10^{10}t)$, $E_y=10^4\cos(10^{10}t)$.
The electrophysical characteristics are as follows: the wedge is manufactured from aluminum: $\varepsilon=1$; $\mu=1$; $\sigma=3.774\cdot10^7$ S/m; the ambient medium is air.
The dimensions of the computational domain are 0.1×0.05 m. The calculation time 10^{-9} sec, and the time step is 10^{-11} sec.
Numerical solution of the system of equations (13)-(15) yields the dependences of the distribution Ex (x,y) и Ey (x,y) on the optical knife. The calculations results are in good agreement with the existing experimental data and experiments specially conducted at the Department of the Physics and Chemistry of Nonequilibrium Media of the A.V. Luikov Heat and Mass Transfer Institute of the National Academy of Sciences by A.I. Bereznyak.
The experiments were carried out with an optical-range laser and were tentative in character but the obtained experimental photographs of diffraction fields and the calculated results turned out to be in good qualitative agreement. The authors express their thanks to A.I. Bereznyak for the conducting of the experiments.

B) Diffraction Grating

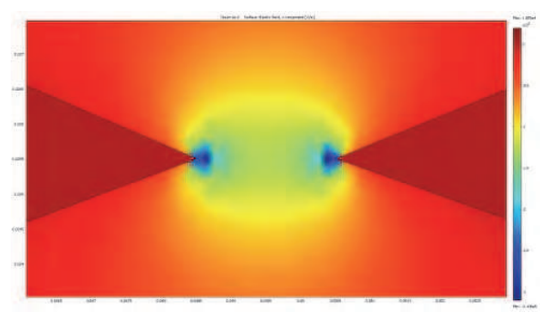
The parameters of the wave and the interfacial conditions are the same, as those for the case "optical knife". The electrophysical characteristics are as follows: 2D lobes, $\varepsilon=12$; and $\sigma=100$ S/m; the ambient medium is air; the characteristics of the prism and the square are identical to those of the lobes.
Figure 8 corresponds to a calculation time of 10^{-10} sec; the time step is 10^{-12} sec. Figure 9 corresponds to a calculation time of 10^{-9} sec; the time step is 10^{-11} sec.
It is seen from the modeling results that the proposed "comb" cab be used as a filter of a high-frequency signal. Furthermore, we carried out numerical calculations of a modulated signal at a frequency of 20 kHz. The results of the modulated-signal calculations are not given. To analyze the difference scheme for stability was analyzed by the initial data. When the time and space steps are large there appear oscillations of the grid solution and of its "derivatives" ("ripple") which strongly decrease the accuracy of the scheme. Undoubtedly, this issue calls for separate consideration. The proposed algorithm of solution of Maxwell equations allows circuitry-engineering modeling of high-frequency radio-engineering devices and investigation of the propagation of electromagnetic waves in media of intricate geometry in the presence of strong discontinuities of electromagnetic field.
The result of Para 2.1 were published in part (Grinchik, N.N et al., 2009).



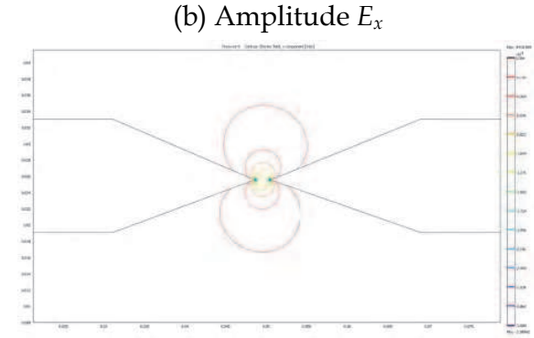
(a) Mesh



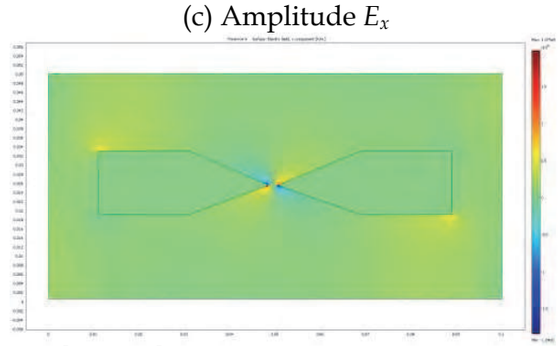
(b) Amplitude E_x



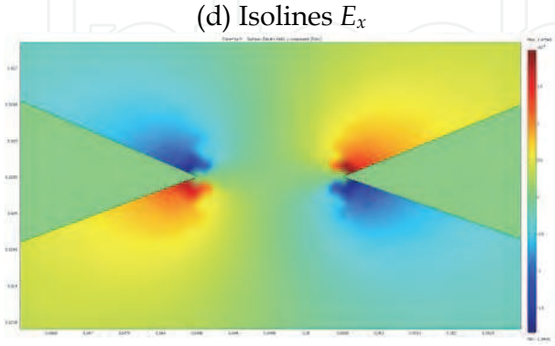
(c) Amplitude E_x



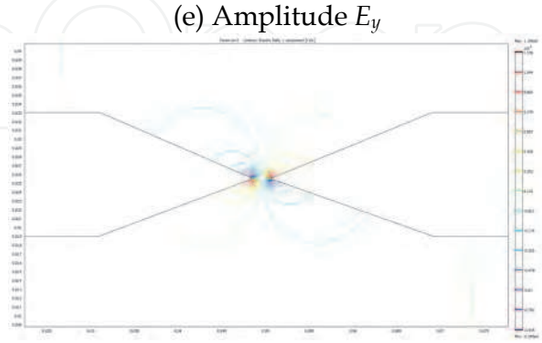
(d) Isolines E_x



(e) Amplitude E_y

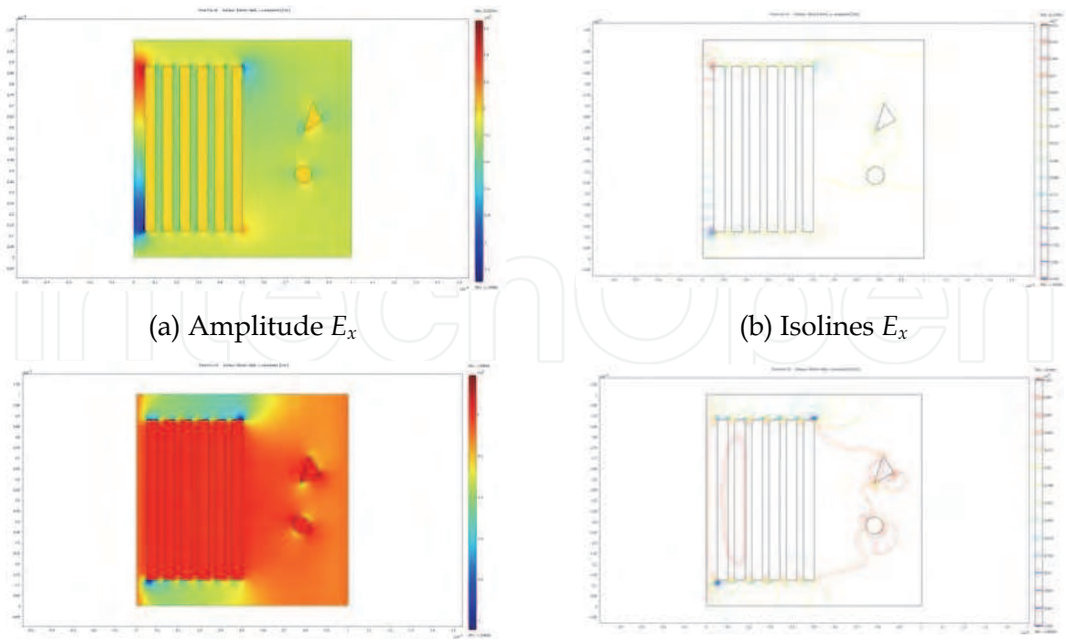


(f) Amplitude E_y



(g) Isolines E_y

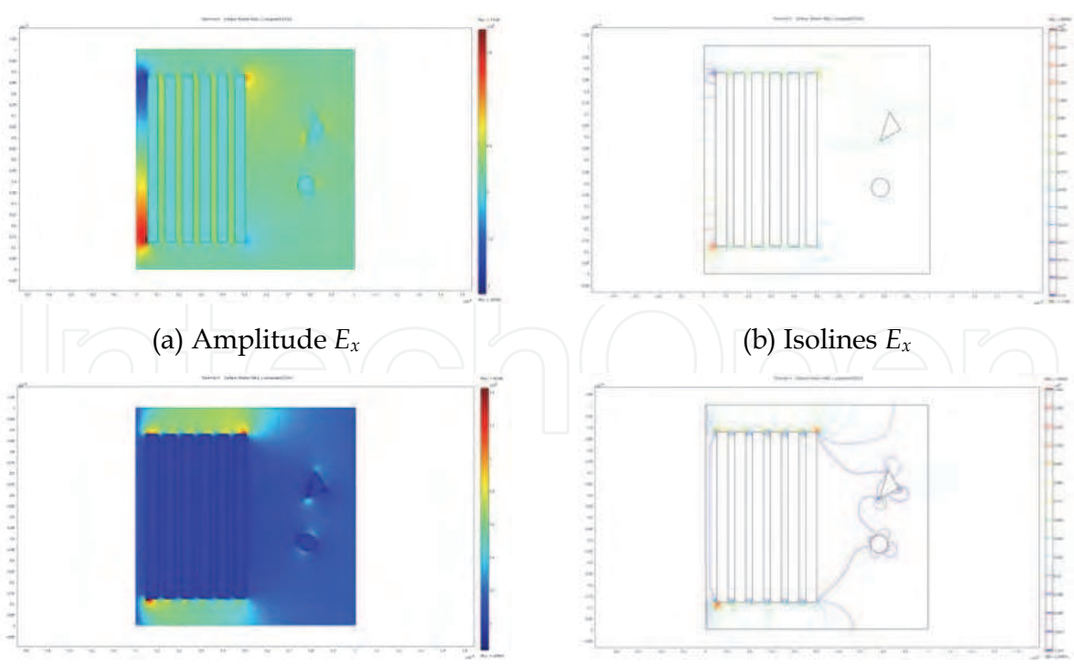
Fig. 7. Mesh, amplitude E_x and isolines, amplitude E_y and isolines of the electromagnetic field strength



(a) Amplitude E_x (b) Isolines E_x

(c) Amplitude E_y (d) Isolines E_y

Fig. 8. Amplitude E_x and isolines, amplitude E_y and isolines of the electromagnetic field strength



(a) Amplitude E_x (b) Isolines E_x

(c) Amplitude E_y (d) Isolines E_y

Fig. 9. Amplitude E_x and isolines, amplitude E_y and isolines of the electromagnetic field strength

2.1.5 Conclusions

We were the first to construct a consistent physicomathematical model of propagation of electromagnetic waves in layered media without recourse to the matrices of the induced-surface-charge impedances. This model is based on the Maxwell equations, the electric-charge conservation law, the total-current continuity, and the Dirichlet theorem. Our numerical investigations have shown that the physical and mathematical model proposed can be used to advantage for simulation of the propagation of a high-frequency electromagnetic wave in a medium consisting of layers having different electrophysical properties.

2.2 Wave equation for $\vec{H}$ and conditions on the boundaries in the presence of strong discontinuities of the electromagnetic field. numerical modeling of electrodynamic processes in the surface layer

2.2.1 Introduction

During the interaction of an external magnetic field and magnetic abrasive particles, the particles are magnetized, and magnetic dipoles with the moment oriented predominantly along the field are formed. "Chains" along the force lines of the field (Shul'man, Z. P. & Kordonskii ,V. I., 1982; Khomich, 2006) appear that periodically act on the processable surface with a frequency $\omega=l/v$. A fixed elemental area of the material periodically experiences the effect of the magnetic field of one direction. Actually, the frequency and duration of the pulse will be still higher because of the rotation of the magnetic abrasive particle due to the presence of the moment of forces on contact and of the friction of the particle against the processable part. In what follows, we will not take into account the effect of rotation.

We assume that the particle velocity on the polisher is v. If the particle radius is r, then the angular frequency is $\omega=2\pi v/r$, and precisely this frequency determines the frequency of the effect of the variable magnetic field component due to the fact that for a ferromagnetic $\mu>1$. The magnetic permeability μ of ferromagnetics, which are usually used in magnetic abrasive polishing, is measured by thousands of units in weak fields. However, in polishing, the constant external magnetic field is strong and amounts to 10^5-10^6 A/m, and in this case the value of μ for compounds of iron and nickel and for Heusler alloy decreases substantially.

Because of the presence of a strong external magnetic field H_0 the "small" absolute value of μ of an abrasive particle leads to a periodic "increase" and "decrease" in the normal component of the magnetic induction near the processable surface. In the present work we used neodymium magnets (neodymium-iron-boron) with H_0>485,000 A/m. The magnetic permeability of a magnetic abrasive particle based on carbonyl iron was assumed in this case to be equal to $\mu_1=100$.

Due to the continuity of the normal magnetic induction component $B_{n1}=B_{n2}$, where $B_{n1}=\mu_1\mu_0H_1$; $B_{n2}=\mu_2\mu_0H_2$. For example, in glasses ($\mu_2=1$; therefore at the boundary of contact of the glass with the magnetic abrasive particle an additional variable magnetic field of strength $H_1 > H_0$ appears.

In (Levin, M. N. et al., 2003; Orlov, A. M. et al., 2001; Makara, V. A. et al., 2001; Rakomsin, 2000), magnetic field-induced effects in silicon are considered: a nonmonotonic change in the crystal lattice parameters in the surface layer of silicon, the gettering of defects on the surface, the change in the sorption properties of the silicon surface, and the change in the mobility of the edge dislocations and in the microhardness of silicon.

In (Golovin, Yu. I. et al.; Makara ,V. A. et al., 2008; Orlov, A. M. et al., 2003), the influence of an electromagnetic field on the domain boundaries, plasticity, strengthening, and on the reduction of metals and alloys was established.

In view of the foregoing, it is of interest to find the relationship between the discrete-impulse actions of a magnetic field of one direction on the surface layer of the processable material that contains domains. According to (Shul'man, Z. P. & Kordonskii ,V. I., 1982), the size of domains is as follows: 0.05 μm in iron, 1.5 μm in barium ferrite; 8 μm in the MnBi compound, and 0.5-1 μm in the acicular gamma ferric oxide. According to (Akulov, 1961), the size of a domain may reach 10^{-6} cm^3 (obtained by the method of magnetic metallography).

As a rule, an abrasive exhibits a distinct shape anisotropy, whereas the frequency of the effect is determined by the concentration of abrasive particles in a hydrophobic solution and by the velocity of its motion. We assume that on the surface of a processable crystal the magnetic field strength $H(t)=H_1 \sin^4(\omega t) + H_0$.

It is required to find the value of the magnetic field strength in the surface layer that has the characteristics λ_1, ε_1, and μ_1 and contains domains with electrophysical properties λ_2, ε_2, and μ_2. The domains may have the form of a triangular prism, a bar, a cylinder, etc.

2.2.2 Physicomathematical model. wave equation for $\vec{H}$

We will formulate a physicomathematical model of propagation of electromagnetic waves in a heterogeneous medium. The media in contact are considered homogeneous. We operate with the operator rot on the left- and right-hand sides of the first equation for the total current (see Equation 6) and multiply by $\mu_0\mu$; then we differentiate the second equation in Eq. (see Equation 7) with respect to time. Taking into consideration the solenoidality of the magnetic field (see Equation 7) and the rule of repeated application of the operator ∇ to the vector **H**, we obtain

$$\mu_0\varepsilon\varepsilon_0\frac{\partial^2\mathbf{H}}{\partial t^2}+\lambda\mu_0\frac{\partial\mathbf{H}}{\partial t}=\frac{1}{\mu}\nabla^2\mathbf{H} \tag{38}$$

In the Cartesian coordinates Eq. (see Equation 38) will have the form

$$\begin{aligned}
\varepsilon\varepsilon_0\frac{\partial^2 H_x}{\partial t^2}+\lambda\mu_0\frac{\partial H_x}{\partial t}&=\frac{1}{\mu}\left(\frac{\partial^2 H_x}{\partial x^2}+\frac{\partial^2 H_x}{\partial y^2}+\frac{\partial^2 H_x}{\partial z^2}\right)\\
\varepsilon\varepsilon_0\frac{\partial^2 H_y}{\partial t^2}+\lambda\mu_0\frac{\partial H_y}{\partial t}&=\frac{1}{\mu}\left(\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial y^2}+\frac{\partial^2 H_y}{\partial z^2}\right)\\
\varepsilon\varepsilon_0\frac{\partial^2 H_z}{\partial t^2}+\lambda\mu_0\frac{\partial H_z}{\partial t}&=\frac{1}{\mu}\left(\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+\frac{\partial^2 H_z}{\partial z^2}\right)
\end{aligned} \tag{39}$$

One fundamental electromagnetic field equation is the equation $div\vec{B}=0$. The use of the Dirichlet theorem for approximation of the value of the magnetic field strength on the boundaries between adjacent media analogously to that of the electric field strength does not necessarily guarantees the observance of the condition of solenoidality of the magnetic field; furthermore, the magnetic properties of heterogeneous media were assumed constant

in deriving generalized wave equations. The experience of numerical calculations has shown that when it is necessary to model nonstationary magnetic phenomena it is better in many cases to use a generalized wave equation for $\vec{E}$, accordingly expressing $\vec{H}(t,\vec{r})$ by $\vec{E}(t,\vec{r})$ and, if need be, to perform backward recalculation to $\vec{H}(t,\vec{r})$. This approach is difficult to apply to modeling of heterogeneous media with different magnetic properties, when the magnetic permeability μ is dependent on coordinates.

In media with a weak heterogeneity where μ (x, y, z) is a piecewise continuous quantity, the application of the proposed method of through counting is quite justified. Indeed, the system of equations (see Equations 13-15, 39) yields that the function's discontinuity on the boundaries between adjacent media is determined by the complexes which will be called the generalized permeability $\varepsilon^* \equiv \varepsilon\mu\varepsilon_0\mu_0$ and the generalized conductivity $\lambda^* \equiv \lambda\mu\mu_0$. Using the Direchlet theorem for ε^* and λ^*, we obtain their values on the boundaries between adjacent media and the values for the electric field strength at the discontinuity point (see Equation 37); here, we note that the value of the electric field strength is obtained without solving Maxwell equations. In fact, at the discontinuity point, we use linear interpolation of the function to obtain the values of ε^*, λ^*, and $E_{x=\xi} = \frac{1}{2}\left[E(\xi-0)+E(\xi+0)\right]$. Consequently, for piecewise continuous quantity μ (x, y, z), the application of the proposed method of through counting is justified. We note that the equality of the derivatives of the electric field strength along the normal to the surface at the discontinuity point according to Eq. (see Equation 34b) holds. When the wave equation for $\vec{H}$ is used for media with different magnetic permeabilities the condition of equality of the derivatives fails, i.e.,

$$\left.\frac{\partial H_x}{\partial x}\right|_{x=\xi-0} \neq \left.\frac{\partial H_x}{\partial x}\right|_{x=\xi+0}$$

which is a consequence of Eq. (see Equation 10); therefore, the use of through-counting schemes for the wave equation for $\vec{H}$ is difficult.

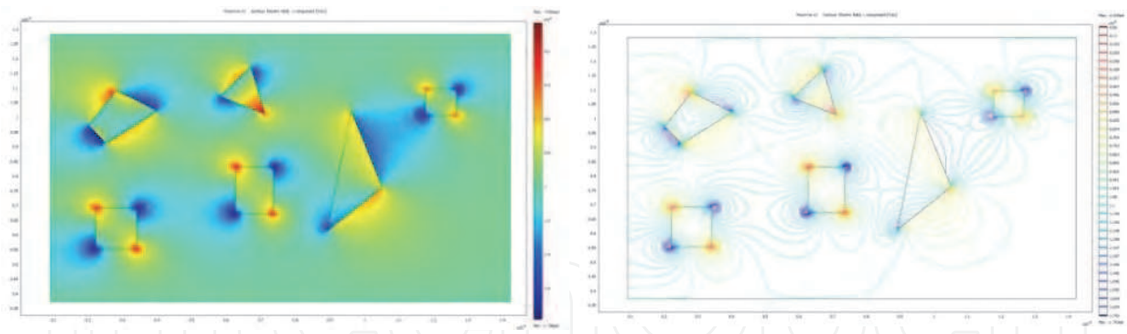
The generalized wave equation for $\vec{E}$ contains the term $grad\ div\vec{E}$ which directly allows for the influence of induced surface charges on the propagation of waves. We note that the proposed method of calculation can be used on condition that there are no built-in space charges and extraneous electromotive forces (Grinberg, G.A. & Fok, V.A., 1948).

By virtue of what has been stated above, for modeling of the propagation of electromagnetic waves in glasses having roughness and defects, we used system (see Equations 13-15) with boundary conditions (see Equations 24-34)

2.2.3 Results of numerical simulation

The physicomathematical model developed can also efficiently be used in modeling the propagation of electromagnetic waves in media with complex geometries and strong electromagnetic field discontinuities.

The transverse cut of a cellular structure represents a set of parallelepipeds and triangular prisms of various cross sections, as depicted in Fig. 10a and 11a. An electromagnetic wave propagates across the direction of parallelepipeds and triangular prisms (channels) along the coordinate *x*.

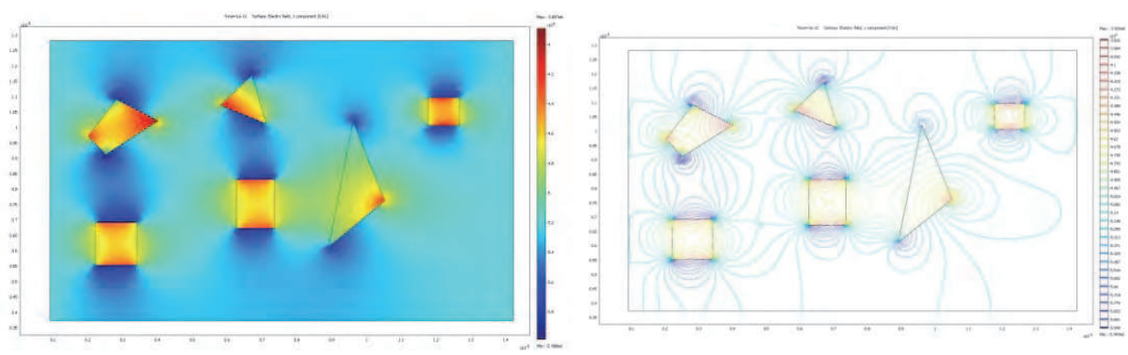


(a) Amplitude H_x (b) Isolines

Fig. 10. Amplitude H_x and isolines of the magnetic field strength

The size of the investigated two-dimensional object is $14\times20\cdot10^{-6}$ m, and the sizes of the domains are 2–4 μm. The frequency of the influence of the magnetic field is $\omega= 2\pi\cdot10^{6}$, and the strength of the field is

$$H_x = 21\cdot10^5 \sin^4\left(2\pi\cdot10^6 t\right) A/m \tag{40}$$



(a) Amplitude H_y (b) Isolines

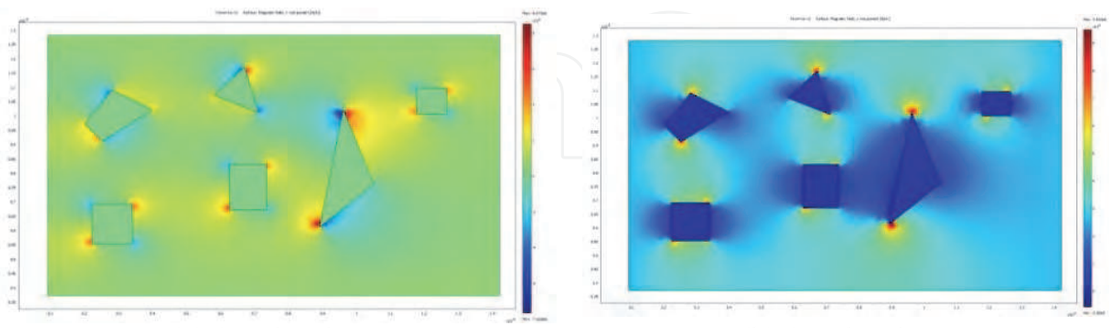
Fig. 11. Amplitude H_y and isolines of the magnetic field strength

The electrophysical properties are as follows: of the large parallelepiped, $\mu= 1$, $\varepsilon= 8$, $\sigma = 10^{-9}$ Ω·m; of domains, $\mu= 1$, $\varepsilon= 6$, $\sigma = 10^{-8}$ Ω·m. They correspond to the electrophysical properties of glasses.

It was assumed that in a layer of thickness 15-20 μm an electromagnetic wave propagates without attenuation; therefore, on all the faces of the large parallelepiped the fulfillment of condition (see Equation 40) was considered valid. On the faces of the parallelepiped that are parallel to the OX axis condition (see Equation 40) corresponded to the "transverse" tangential component of the wave; on the faces parallel to OY condition (see Equation 40) corresponded to the normal component of the field.

The calculations were carried out with a time step of 10^{-13} sec up to a time instant of 10^{-10} sec. Figures 10a and 11a present the amplitude values of the magnetic field strength along H_x and H_y with a comparison scale, whereas Figs. 11b and 11b present the corresponding isolines. An analysis of these figures shows that at the places of discontinuity, on the wedges, force lines of the electromagnetic field concentrate. According to (Akulov, N. S.,

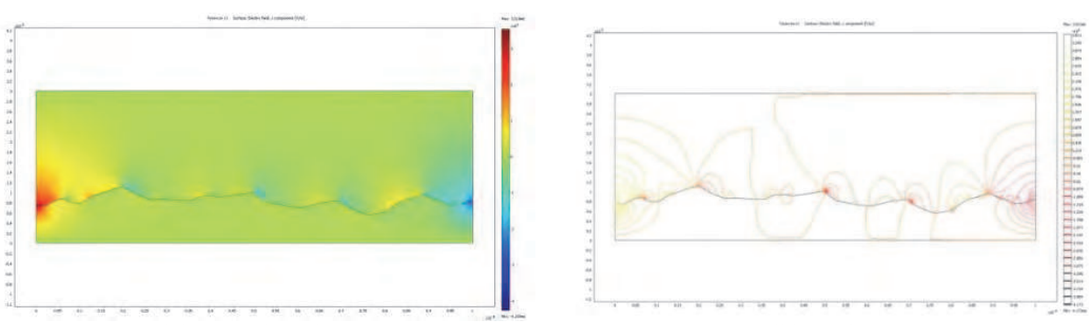
1939), precisely wedges are often the sources and sinks of the vacancies that determine, for example, the hardness and plasticity of a solid body.
Also, we modeled the propagation of waves in media, when domains possess magnetic properties. We assumed, in the calculations, that μ=100; the remaining parameters correspond to the previous example of solution (Fig. 12)



(a) Amplitude H_x (b) Amplitude H_y

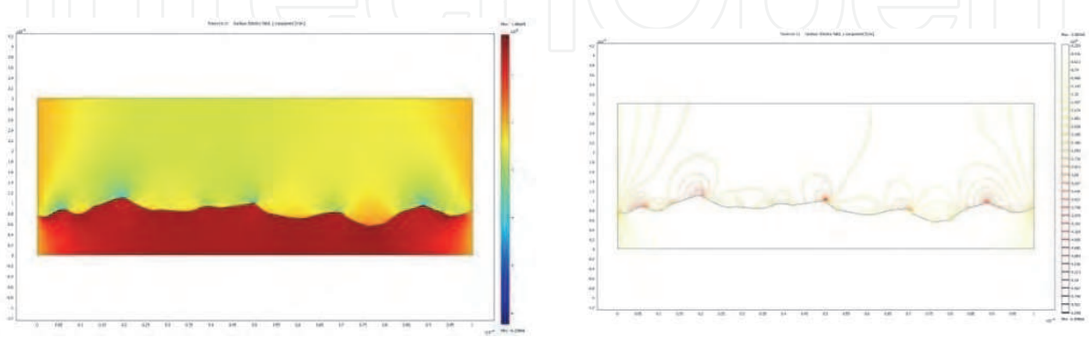
Fig. 12. Amplitude H_x and H_y of the magnetic field strength

Of interest is the interaction of the electromagnetic wave with the rough surface shown in Fig. 13 and 14. As in the previous examples, we observe the concentration of electromagnetic energy on angular structures.



(a) Amplitude H_x (b) Isolines

Fig. 13. Amplitude H_x and isolines of the magnetic field strength



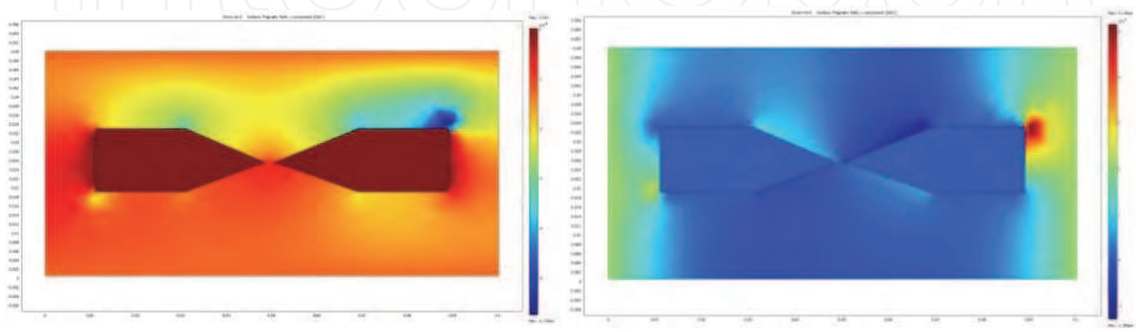
(a) Amplitude H_x (b) Isolines

Fig. 14. Amplitude H_y and isolines of the magnetic field strength

From Fig. 13 and 14, it is seen that electromagnetic heating of tapered structures may occur in addition to mechanical heating in magnetic abrasive machining.

As we have mentioned above, for investigation of the propagation of electromagnetic waves in nonmagnetic materials, it is more expedient to use the generalized equation for $\vec{E}$. For the purpose of illustration we give an example of numerical calculation of an optical knife with the wave equation for $\vec{H}$ (Fig. 15).

From Fig. 15, it is seen that the actual problem of diffraction on the optical knife remains to be solved, i.e., there is no "glow" on the optical-knife section, which is inconsistent with experimental data.



(a) Amplitude H_x (b) Amplitude H_y

Fig. 15. Amplitude H_x and H_y of the magnetic field strength

As is known (Bazarov, 1991), in thermodynamically equilibrium systems the temperature T and the electrical φ and chemical μ_c potentials are constant along the entire system:

$$grad\ T = 0,\ grad\ \varphi = 0,\ grad\ \mu_c = 0 \tag{41}$$

If these conditions are not fulfilled (grad $T \neq 0$, grad $\varphi \neq 0$, grad $\mu_c \neq 0$), irreversible processes of the transfer of mass, energy, electrical charge, etc. appear in the system.

The chemical potential of the j-th component is determined, for example, as a change of the free energy with a change in the number of moles:

$$\mu_{cj} = \left(\partial F / \partial n_j\right)_{T,V} \tag{42}$$

Where

$$dF = -SdT - PdV + HdB \tag{43}$$

The last term in Eq. (see Equation 43) takes into account the change in the free energy of a dielectric due to the change in the magnetic induction. The free energy of a unit volume of the dielectric in the magnetic field in this case has the form

$$F(T,D) = F_0 + \mu\mu_0 \frac{H^2}{2} \tag{44}$$

We assume that changes in the temperature and volume of the dielectric are small. Then the mass flux is determined by a quantity proportional to the gradient of the chemical potential or, according to Eq. (see Equation 43), we obtain

$$q_i = -D_{\mu_c} grad\left(HdB\right) = -D_{\mu_c} grad\, W \tag{45}$$

where $W = \mu\mu_0 \frac{H^2}{2}$ is the density of the magnetic field in the unit volume of the dielectric.
In magnetic abrasive polishing on the sharp protrusions of domains the gradients of magnetic energy are great, which can lead to the origination of vacancy flows.
An analysis of the results shows that the nonstationary component of the full electromagnetic energy is also concentrated in the region of fractures and wedges, i.e., at the sharp angles of domains, which may lead to the improvement of the structure of the sublayer of the treated surface due to the "micromagnetoplastic" effect. Maximum values of the nonstationary part of the total electromagnetic energy W_{max} in the sublayer correspond to a maximum value of the function sin $(2\pi \cdot 10^6\, t)$ and occur for the time instants $t = (n/4)10^{-6}$ sec, where n is the integer, with the value of W_{max} for a neodymium magnet and a magnetoabrasive particle on the basis of carboxyl iron amounting to a value of the order of (see Equations 5-6)$\cdot 10^6$ J/m^3 . Having multiplied W_{max} by the volume of a domain, vacancy, or atom, we may approximately obtain the corresponding energy. The density of the electromagnetic energy in all of the cases is much smaller than the bonding energy of atoms, 10^{-18}-10^{-19} J. However, a periodic change in the magnetic field in one direction leads to a ponderomotive force that may influence the motion of various defects and dislocations to create a stable and equilibrium structure of atoms and molecules in magnetic abrasive polishing and, in the long run, in obtaining a surface with improved characteristics due to the "micromagnetoplastic" effect. The result of Para 2.2 were published in part (Grinchik, N.N. et al., 2010).

3. Interaction of nonstationary electric and thermal fields with allowance for relaxation processes

We investigate electric and thermal fields created by macroscopic charges and currents in continuous media. Of practical interest is modeling of local heat releases in media on exposure to a high-frequency electromagnetic field. We should take into account the influence of the energy absorption on the propagation of an electromagnetic wave, since the transfer processes are interrelated.
In an oscillatory circuit with continuously distributed parameters, the energy dissipation is linked (Kolesnikov, 2001) to the dielectric loss due to the dependence of the relative permittivity $\varepsilon(\omega)$ on frequency. In the general case ε is also complex, and the relationship between the electric displacement and electric field vectors has the form $D=\varepsilon(\omega)E$, where $\varepsilon(\omega)= \varepsilon'(\omega) - i\varepsilon''(\omega)$; here, ε' and ε'' are determined experimentally. As of now, the problems of dielectric heating of a continuous medium are reduced in many cases to consideration of an equivalent circuit based on lumped parameters, such as capacitance, inductance, loss angle, and relative-dielectric-loss factor (Skanavi, 1949; Perre P.; Turner I. W., 1996), that are established experimentally.
With this approach, there arise substantial difficulties in determining the temperature field of equivalent circuits. Also, we have polarization and the occurrence of an electric double layer of a prescribed electric moment in contact of media with different properties. Equivalent circuits in lamellar media additionally involve empirical lumped parameters: surface capacitance and surface resistance (Jaeger, 1977). The total current can always be separated into a dissipative, or conduction current which is in phase with the applied voltage and a displacement current shifted in time relative to the voltage. The exact physical

meaning of these components of the current is largely dependent on selection of an equivalent electric circuit. A unique equivalent circuit – series or parallel connection of the capacitor, the resistor, and the inductor – does not exist; it is determined by a more or less adequate agreement with experimental data.

In the case of electrolytic capacitors, the role of one plate is played by the electric double layer with a specific resistance much higher than the resistance of metallic plates. Therefore, decrease in the capacitance with frequency is observed, for such capacitors, even in the acoustic-frequency range (Jaeger, 1977). Circuits equivalent to an electrolytic capacitor are very bulky: up to 12 R, L, and C elements can be counted in them; therefore, it is difficult to obtain a true value of, e.g., the electrolyte capacitance. In (Jaeger, 1977) experimental methods of measurement of the dielectric properties of electrolyte solutions at different frequencies are given and ε' and ε" are determined. The frequency dependence of dispersion and absorption are essentially different consequences of one phenomenon: "dielectric-polarization inertia" (Jaeger, 1977). In actual fact, the dependence ε(ω) is attributable to the presence of the resistance of the electric double layer and to the electrochemical cell in the electrolytic capacitor being a system with continuously distributed parameters, in which the signal velocity is a finite quantity.

Actually, ε' and ε" are certain integral characteristics of a material at a prescribed constant temperature, which are determined by the geometry of the sample and the properties of the electric double layer. It is common knowledge that in the case of a field arbitrarily dependent on time any reliable calculation of the absorbed energy in terms of ε(ω) turns out to be impossible (Landau, L.D. & Lifshits, E.M., 1982). This can only be done for a specific dependence of the field *E* on time. For a quasimonochromatic field, we have (Landau, L.D. & Lifshits, E.M., 1982)

$$\mathbf{E}(t)=\frac{1}{2}\left[\mathbf{E}_0(t)e^{-i\omega t}+\mathbf{E}_0^*(t)e^{i\omega t}\right] \tag{46}$$

$$\mathbf{H}(t)=\frac{1}{2}\left[\mathbf{H}_0(t)e^{-i\omega t}+\mathbf{H}_0^*(t)e^{i\omega t}\right] \tag{47}$$

The values of $\mathbf{E}_0$(t) and $\mathbf{H}_0$(t), according to (Barash, Yu. & Ginzburg, V.L., 1976), must very slowly vary over the period T = 2π/ω. Then, for absorbed energy, on averaging over the frequency ω, we obtain the expression (Barash, Yu. & Ginzburg, V.L., 1976)

$$\begin{aligned}\frac{\partial\mathbf{D}(t)}{\partial t}E(t)=&\frac{1}{4}\frac{d(\omega\varepsilon'(\omega))}{d(\omega)}\frac{\partial}{\partial t}\left[\mathbf{E}_0(t)\mathbf{E}_0^*(t)\right]+\frac{\omega\varepsilon''}{2}\mathbf{E}_0(t)\mathbf{E}_0^*(t)+\\&i\frac{d(\omega\varepsilon''(\omega))}{4d\omega}\left(\frac{\partial\mathbf{E}_0(t)}{\partial t}\mathbf{E}_0^*(t)-\frac{\mathbf{E}_0^*(t)}{\partial t}\mathbf{E}_0(t)\right)\end{aligned} \tag{48}$$

where the derivatives with respect to frequency are taken at the carrier frequency ω. We note that for an arbitrary function **E**(t), it is difficult to represent it in the form

$$E(t)=a(t)\cos\varphi(t) \tag{49}$$

since we cannot unambiguously indicate the amplitude *a*(*t*) and the phase φ (*t*). The manner in which *E*(*t*) is decomposed into factors *a* and cosφ is not clear. Even greater difficulties

appear in the case of going to the complex representation $W(t)=U(t)+iV(t)$ when the real oscillation $E(t)$ is supplemented with the imaginary part $V(t)$. The arising problems have been considered in (Vakman, D.E. & Vanshtein, L.A., 1977) in detail. In the indicated work, it has been emphasized that certain methods using a complex representation and claiming higher-than-average accuracy become trivial without an unambiguous determination of the amplitude, phase, and frequency.

Summing up the aforesaid, we can state that calculation of the dielectric loss is mainly empirical in character. Construction of the equivalent circuit and allowance for the influence of the electric double layer and for the dependence of electrophysical properties on the field's frequency are only true of the conditions under which they have been modeled; therefore, these are fundamental difficulties in modeling the propagation and absorption of electromagnetic energy.

As we believe, the release of heat in media on exposure to nonstationary electric fields can be calculated on the basis of allowance for the interaction of electromagnetic and thermal fields as a system with continuously distributed parameters from the field equation and the energy equation which take account of the distinctive features of the boundary between adjacent media. When the electric field interacting with a material medium is considered we use Maxwell equations (see Equations 6–7). We assume that space charges are absent from the continuous medium at the initial instant of time and they do not appear throughout the process. The energy equation will be represented in the form

$$\rho C_p \frac{dT}{dt} = div\left[k(T)\,grad(T)\right] + Q \tag{50}$$

where Q is the dissipation of electromagnetic energy.

According to (Choo, 1962), the electromagnetic energy converted to heat is determined by the expression

$$Q = \rho\left[\mathbf{E}\frac{d}{dt}\left(\frac{\mathbf{D}}{\rho}\right) + H\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right)\right] + \mathbf{J}_q\mathrm{E} \tag{51}$$

In deriving this formula, we used the nonrelativistic approximation of Minkowski's theory. If ε, μ, and ρ = const, there is no heat release; therefore, the intrinsic dielectric loss is linked to the introduction of ε'(ω) and ε"(ω). The quantity Q is affected by the change in the density of the substance ρ(T).

A characteristic feature of high frequencies is the lag of the polarization field behind the charge in the electric field in time. Therefore, the electric-polarization vector is expediently determined by solution of the equation $\mathbf{P}(t+\tau_e)=(\varepsilon-1)\varepsilon_0\mathbf{E}(t)$ with allowance for the time of electric relaxation of dipoles τ_e. Restricting ourselves to the first term of the expansion $\mathbf{P}(t+\tau_e)$ in a Taylor series, from this equation, we obtain

$$\mathbf{P}(t) + \tau_e\frac{d\mathbf{P}(t)}{dt} = (\varepsilon - 1)\varepsilon_0\mathbf{E}(t) \tag{52}$$

The solution (see Equation 52), on condition that **P**=0 at the initial instant of time, will take the form

$$\mathbf{P} = \frac{(\varepsilon - 1)\varepsilon_0}{\tau_e} \int_{t_0}^{t} \mathbf{E}(\tau) e^{-(t-\tau)/\tau_e} d\tau \tag{53}$$

It is noteworthy that Eq. (see Equation 52) is based on the classical Debay model. According to this model, particles of a substance possess a constant electric dipole moment. The indicated polarization mechanism involves partial arrangement of dipoles along the electric field, which is opposed by the process of disorientation of dipoles because of thermal collisions. The restoring "force", in accordance with Eq. (see Equation 52), does not lead to oscillations of electric polarization. It acts as if constant electric dipoles possessed strong damping.

Molecules of many liquids and solids possess the Debay relaxation polarizability. Initially polarization aggregates of Debay oscillators turn back to the equilibrium state $\mathbf{P}(t)=\mathbf{P}(0)\exp(-t/\tau_e)$.

A dielectric is characterized, as a rule, by a large set of relaxation times with a characteristic distribution function, since the potential barrier limiting the motion of weakly coupled ions may have different values (Skanavi, 1949); therefore, the mean relaxation time of the ensemble of interacting dipoles should be meant by τ_e in Eq. (see Equation 52).

To eliminate the influence of initial conditions and transient processes we set $t_0 = -\infty$, $\mathbf{E}(\infty)=0$, $\mathbf{H}(\infty)=0$, as it is usually done. If the boundary regime acts for a fairly long time, the influence of initial data becomes weaker with time owing to the friction inherent in every real physical system. Thus, we naturally arrive at the problem without the initial conditions:

$$\mathbf{P} = \frac{(\varepsilon - 1)\varepsilon_0}{\tau_e} \int_{-\infty}^{t} \mathbf{E}(\tau) e^{-(t-\tau)/\tau_e} d\tau \tag{54}$$

Let us consider the case of the harmonic field $\mathbf{E} = \mathbf{E}_0 \sin\omega t$; then, using Eq. (see Equation 54) we have, for the electric induction vector

$$\begin{aligned} \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \frac{(\varepsilon - 1)\varepsilon_0}{\tau_e} \int_{-\infty}^{t} \mathbf{E}(\tau) e^{-(t-\tau)/\tau_e} d\tau + \varepsilon_0 \mathbf{E}_0 \sin\omega t = \\ \frac{\mathbf{E}_0 (\varepsilon - 1)\varepsilon_0}{1 + \omega^2 \tau_e^2} (\sin\omega t - \omega\tau_e \cos\omega t) + \varepsilon_0 \mathbf{E}_0 \sin\omega t \end{aligned} \tag{55}$$

The electric induction vector is essentially the sum of two absolutely different physical quantities: the field strength and the polarization of a unit volume of the medium.

If the change in the density of the substance is small, we obtain, from formula (see Equation 51), for the local instantaneous heat release

$$\mathbf{Q} = \mathbf{E}\frac{d\mathbf{D}}{dt} = \frac{E_0^2 (\varepsilon - 1)\varepsilon_0}{1 + \omega^2 \tau_e^2} \left(\omega \sin\omega t \cos\omega t + \omega^2 \tau_e \sin^2 \omega t \right) \tag{56}$$

when we write the mean value of Q over the total period T:

$$\mathbf{Q} = \frac{1}{2} \frac{\mathbf{E}_0^2 (\varepsilon - 1)\varepsilon_0}{1 + \omega^2 \tau_e^2} \omega^2 \tau_e + \lambda \mathbf{E}^2 / 2 \tag{57}$$

For high frequencies (ω→∞), heat release ceases to be dependent on frequency, which is consistent with formula (see Equation 57) and experiment (Skanavi, 1949).
When the relaxation equation for the electric field is used we must also take account of the delay of the magnetic field, when the magnetic polarization lags behind the change in the strength of the external magnetic field:

$$\mathbf{I}(t) + \tau_i \frac{d\mathbf{I}(t)}{dt} = \mu\mu_0 \mathbf{H}(t) \tag{58}$$

Formula (see Equation 57) is well known in the literature; it has been obtained by us without introducing complex parameters. In the case of "strong" heating of a material where the electrophysical properties of the material are dependent on temperature expression (see Equation 52) will have a more complicated form and the expression for Q can only be computed by numerical methods. Furthermore, in the presence of strong field discontinuities, we cannot in principle obtain the expression for Q because of the absence of closing relations for the induced surface charge and the surface current on the boundaries of adjacent media; therefore, the issue of energy relations in macroscopic electrodynamics is difficult, particularly, with allowance for absorption.
Energy relations in a dispersive medium have repeatedly been considered; nonetheless, in the presence of absorption, the issue seems not clearly understood (or at least not sufficiently known), particularly in the determination of the expression of released heat on the boundaries of adjacent media.
Indeed, it is known from the thermodynamics of dielectrics that the differential of the free energy F has the form

$$dF = -SdT - pdV + \mathbf{E}d\mathbf{D} \tag{59}$$

If the relative permittivity and the temperature and volume of the dielectric are constant quantities, from Eq. (see Equation 59) we have

$$F(T, \mathbf{D}) = F_0 + D^2/2 \tag{60}$$

where F_0 is the free energy of the dielectric in the absence of the field.
The change of the internal energy of the dielectric during its polarization at constant temperature and volume can be found from the Gibbs-Helmholtz equation, in which the external parameter **D** is the electric displacement. Disregarding F_0 which is independent of the field strength, we can obtain

$$U(T, \mathbf{D}) = F(T, \mathbf{D}) - T\left(dF/dT\right)_D \tag{61}$$

If the relative dielectric constant is dependent on temperature ($\varepsilon(T)$), we obtain

$$U(T, \mathbf{D}) = \varepsilon_0 E^2/2\left(\varepsilon + T\left(d\varepsilon/dT\right)_D\right) \tag{62}$$

Expression (see Equation 62) determines the change in the internal energy of the dielectric in its isothermal polarization but with allowance for the energy transfer to a thermostat, if the polarization causes the dielectric temperature to change. A more detailed substantiation of Eq. (see Equation 62) will be given in the book. In the works on microwave heating, that we know, expression (see Equation 62) is not used.

A characteristic feature of high frequencies is that the polarization field lags behind the change in the external field in time; therefore, the polarization vector is expediently determined by solution of the equation

$$\mathbf{P}(t+\tau_e)=\left(\varepsilon-1+T\left(d\varepsilon/dT\right)_D\right)\varepsilon_0\mathbf{E}(t) \tag{63}$$

With allowance for the relaxation time, i.e., restricting ourselves to the first term of the expansion $\mathbf{P}(t+\tau_e)$ in a Taylor series, we obtain

$$\mathbf{P}(t)+\tau_e\, d\mathbf{P}(t)/dT=\left(\varepsilon-1+T\left(d\varepsilon/dT\right)_D\right)\varepsilon_0\mathbf{E}(t) \tag{64}$$

In the existing works on microwave heating with the use of complex parameters, they disregard the dependence ε"(T). In (Antonets, I.V.; Kotov, L.N.; Shavrov, V.G. & Shcheglov, V.I., 2009), consideration has been given to the incidence of a one-dimensional wave from a medium with arbitrary complex parameters on one or two boundaries of media whose parameters are also arbitrary. The amplitudes of waves reflected from and transmitted by each boundary have been found. The refection, transmission, and absorption coefficients have been obtained from the wave amplitudes. The well-known proposition that a traditional selection of determinations of the reflection, transmission, and absorption coefficients from energies (reflectivity, transmissivity, and absorptivity) in the case of complex parameters of media comes into conflict with the law of conservation of energy has been confirmed and exemplified. The necessity of allowing for ε"(T) still further complicates the problem of computation of the dissipation of electromagnetic energy in propagation of waves through the boundaries of media with complex parameters.

The proposed method of computation of local heat release is free of the indicated drawbacks and makes it possible, for the first time, to construct a consistent model of propagation of nonmonochromatic waves in a heterogeneous medium with allowance for frequency dispersion without introducing complex parameters.

In closing, we note that a monochromatic wave is infinite in space and time, has infinitesimal energy absorption in a material medium, and transfers infinitesimal energy, which is the idealization of real processes. However with these stringent constraints, too, the problem of propagation of waves through the boundary is open and far from being resolved even when the complex parameters of the medium are introduced and used. In reality, the boundary between adjacent media is not infinitely thin and has finite dimensions of the electric double layers; therefore, approaches based on through-counting schemes for a hyperbolic equation without explicit separation of the boundary between adjacent media are promising.

6. Conclusion

The consistent physicomathematical model of propagation of an electromagnetic wave in a heterogeneous medium has been constructed using the generalized wave equation and the Dirichlet theorem. Twelve conditions at the interfaces of adjacent media were obtained and justified without using a surface charge and surface current in explicit form. The conditions are fulfilled automatically in each section of the heterogeneous medium and are conjugate, which made it possible to use through-counting schemes for calculations. For the first time

the effect of concentration of "medium-frequency" waves with a length of the order of hundreds of meters at the fractures and wedges of domains of size 1-3 μm has been established. Numerical calculations of the total electromagnetic energy on the wedges of domains were obtained. It is shown that the energy density in the region of wedges is maximum and in some cases may exert an influence on the motion, sinks, and the source of dislocations and vacancies and, in the final run, improve the near-surface layer of glass due to the "micromagnetoplastic" effect.

The results of these calculations are of special importance for medicine, in particular, when microwaves are used in the therapy of various diseases. For a small, on the average, permissible level of electromagnetic irradiation, the concentration of electromagnetic energy in internal angular structures of a human body (cells, membranes, neurons, interlacements of vessels, etc) is possible.

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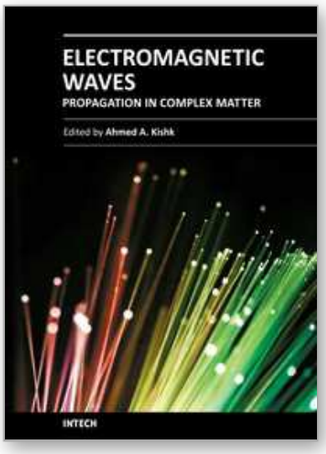

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This volume is based on the contributions of several authors in electromagnetic waves propagations. Several issues are considered. The contents of most of the chapters are highlighting non classic presentation of wave propagation and interaction with matters. This volume bridges the gap between physics and engineering in these issues. Each chapter keeps the author notation that the reader should be aware of as he reads from chapter to the other.

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