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Properties of Macroscopic Quantum Effects and Dynamic Natures of Electrons in Superconductors

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1. Introduction

So-called macroscopic quantum effects(MQE) refer to a quantum phenomenon that occurs on a macroscopic scale. Such effects are obviously different from the microscopic quantum effects at the microscopic scale as described by quantum mechanics. It has been experimentally demonstrated [1-17] that macroscopic quantum effects are the phenomena that have occurred in superconductors. Superconductivity is a physical phenomenon in which the resistance of a material suddenly vanishes when its temperature is lower than a certain value, Tc, which is referred to as the critical temperature of superconducting materials. Modern theories [18-21] tell us that superconductivity arises from the irresistible motion of superconductive electrons. In such a case we want to know "How the macroscopic quantum effect is formed? What are its essences? What are the properties and rules of motion of superconductive electrons in superconductor?" and, as well, the answers to other key questions. Up to now these problems have not been studied systematically. We will study these problems in this chapter.

2. Experimental observation of property of macroscopic quantum effects in superconductor

(1) Superconductivity of material. As is known, superconductors can be pure elements, compounds or alloys. To date, more than 30 single elements, and up to a hundred alloys and compounds, have been found to possess the characteristics [1-17] of superconductors. When $T \le T_c$, any electric current in a superconductor will flow forever without being damped. Such a phenomenon is referred to as perfect conductivity. Moreover, it was observed through experiments that, when a material is in the superconducting state, any magnetic flux in the material would be completely repelled resulting in zero magnetic fields inside the superconducting material, and similarly, a magnetic flux applied by an external magnetic field can also not penetrate into superconducting materials. Such a phenomenon is

called the perfect anti-magnetism or Maissner effect. Meanwhile, there are also other features associated with superconductivity, which are not present here

How can this phenomenon be explained? After more than 40 years' effort, Bardeen, Cooper and Schreiffier proposed the new idea of Cooper pairs of electrons and established the microscopic theory of superconductivity at low temperatures, the BCS theory [18-21],in 1957, on the basis of the mechanism of the electron-phonon interaction proposed by Frohlich [22-23]. According to this theory, electrons with opposite momenta and antiparallel spins form pairs when their attraction, due to the electron and phonon interaction in these materials, overcomes the Coulomb repulsion between them. The so-called Cooper pairs are condensed to a minimum energy state, resulting in quantum states, which are highly ordered and coherent over the long range, and in which there is essentially no energy exchange between the electron pairs and lattice. Thus, the electron pairs are no longer scattered by the lattice but flow freely resulting in superconductivity. The electron pairs in a superconductive state are somewhat similar to a diatomic molecule but are not as tightly bound as a molecule. The size of an electron pair, which gives the coherent length, is approximately 10^{-4} cm. A simple calculation shows that there can be up to 10^6 electron pairs in a sphere of 10^{-4} cm in diameter. There must be mutual overlap and correlation when so many electron pairs are brought together. Therefore, perturbation to any of the electron pairs would certainly affect all others. Thus, various macroscopic quantum effects can be expected in a material with such coherent and long range ordered states. Magnetic flux quantization, vortex structure in the type-II superconductors, and Josephson effect [24-26] in superconductive junctions are only some examples of the phenomena of macroscopic quantum mechanics.

(2) The magnetic flux structures in superconductor. Consider a superconductive ring. Assume that a magnetic field is applied at T >Tc, then the magnetic flux lines ϕ_0 produced by the external field pass through and penetrate into the body of the ring. We now lower the temperature to a value below Tc, and then remove the external magnetic field. The magnetic induction inside the body of circular ring equals zero ($\vec{B} = 0$) because the ring is in the superconductive state and the magnetic field produced by the superconductive current cancels the magnetic field, which was within the ring. However, part of the magnetic fluxes in the hole of the ring remain because the induced current is in the ring vanishes. This residual magnetic flux is referred to as "the frozen magnetic flux". It has been observed experimentally, that the frozen magnetic flux is discrete, or quantized. Using the macroscopic quantum wave function in the theory of superconductivity, it can be shown that the magnetic flux is established by $\Phi' = n\phi_0$ (n=0,1,2,…), where ϕ_0 =hc/2e=2.07×10^{-15} Wb is the flux quantum, representing the flux of one magnetic flux line. This means that the magnetic fluxes passing through the hole of the ring can only be multiples of ϕ_0 [1-12]. In other words, the magnetic field lines are discrete. We ask, "What does this imply?" If the magnetic flux of the applied magnetic field is exactly n, then the magnetic flux through the hole is $n\phi_0$, which is not difficult to understand. However, what is the magnetic flux through the hole if the applied magnetic field is (n+1/4) ϕ_0 ? According to the above, the magnetic flux cannot be $(n+1/4)\phi_0$. In fact, it should be $n\phi_0$. Similarly, if the applied magnetic field is $(n+3/4)\phi_0$ and the magnetic flux passing through the hole is not $(n+3/4)\phi_0$, but rather $(n+1)\phi_0$, therefore the magnetic fluxes passing through the hole of the circular ring are always quantized.

An experiment conducted in 1961 surely proves this to be so, indicating that the magnetic flux does exhibit discrete or quantized characteristics on a macroscopic scale. The above experiment was the first demonstration of the macroscopic quantum effect. Based on quantization of the magnetic flux, we can build a "quantum magnetometer" which can be used to measure weak magnetic fields with a sensitivity of 3×10^{-7} Oersted. A slight modification of this device would allow us to measure electric current as low as 2.5×10^{-9} A.

(3) Quantization of magnetic-flux lines in type-II superconductors. The superconductors discussed above are referred to as type-I superconductors. This type of superconductor exhibits a perfect Maissner effect when the external applied field is higher than a critical magnetic value $\vec{H}_c$. There exists other types of materials such as the NbTi alloy and Nb_3Sn compounds in which the magnetic field partially penetrates inside the material when the external field $\vec{H}$ is greater than the lower critical magnetic field $\vec{H}_{c1}$, but less than the upper critical field $\vec{H}_{c2}$ [1-7]. This kind of superconductor is classified as type-II superconductors and is characterized by a Ginzburg-Landau parameter greater than 1/2. Studies using the Bitter method showed that the penetration of a magnetic field results in some small regions changing from superconductive to normal state. These small regions in normal state are of cylindrical shape and regularly arranged in the superconductor, as shown in Fig.1. Each cylindrical region is called a vortex (or magnetic field line)[1-12]. The vortex lines are similar to the vortex structure formed in a turbulent flow of fluid. Both theoretical analysis and experimental measurements have shown that the magnetic flux associated with one vortex is exactly equal to one magnetic flux quantum ϕ_0, when the applied field $\vec{H} \geq \vec{H}_{c1}$, the magnetic field penetrates into the superconductor in the form of vortex lines, increased one by one. For an ideal type-II superconductor, stable vortices are distributed in triagonal pattern, and the superconducting current and magnetic field distributions are also shown in Fig. 1. For other, non-ideal type-II superconductors, the triagonal pattern of distribution can be also observed in small local regions, even though its overall distribution is disordered. It is evident that the vortex-line structure is quantized and this has been verified by many experiments and can be considered a result of the quantization of magnetic flux. Furthermore, it is possible to determine the energy of each vortex line and the interaction energy between the vortex lines. Parallel magnetic field lines are found to repel each other while anti-parallel magnetic lines attract each other.

(4) The Josephson phenomena in superconductivity junctions [24-26]. As it is known in quantum mechanics, microscopic particles, such as electrons, have a wave property and that can penetrate through a potential barrier. For example, if two pieces of metal are separated by an insulator of width of tens of angstroms, an electron can tunnel through the insulator and travel from one metal to the other. If voltage is applied across the insulator, a tunnel current can be produced. This phenomenon is referred to as a tunneling effect. If two superconductors replace the two pieces of metal in the above experiment, a tunneling current can also occur when the thickness of the dielectric is reduced to about $30\overset{0}{A}$. However, this effect is fundamentally different from the tunneling effect discussed above in quantum mechanics and is referred to as the Josephson effect.

Evidently, this is due to the long-range coherent effect of the superconductive electron pairs. Experimentally, it was demonstrated that such an effect could be produced via many types

of junctions involving a superconductor, such as superconductor-metal-superconductor junctions, superconductor-insulator- superconductor junctions, and superconductor bridges. These junctions can be considered as superconductors with a weak link. On the one hand, they have properties of bulk superconductors, for example, they are capable of carrying certain superconducting currents. On the other hand, these junctions possess unique properties, which a bulk superconductor does not. Some of these properties are summarized in the following.

(A) When a direct current (dc) passing through a superconductive junction is smaller than a critical value Ic, the voltage across the junction does not change with the current. The critical current Ic can range from a few tens of μA to a few tens of mA.

(B) If a constant voltage is applied across the junction and the current passing through the junction is greater than Ic, a high frequency sinusoidal superconducting current occurs in the junction. The frequency is given by υ=2eV/h, in the microwave and far-infrared regions of (5-1000)×10^{9}Hz. The junction radiates a coherent electromagnetic wave with the same frequency. This phenomenon can be explained as follows: The constant voltage applied across the junction produces an alternating Josephson current that, in turn, generates an electromagnetic wave of frequency, υ. The wave propagates along the planes of the junction. When the wave reaches the surface of the junction (the interface between the junction and its surrounding), part of the electromagnetic wave is reflected from the interface and the rest is radiated, resulting in the radiation of the coherent electromagnetic wave. The power of radiation depends on the compatibility between the junction and its surrounding.

(C) When an external magnetic field is applied over the junction, the maximum dc current, Ice , is reduced due to the effect of the magnetic field. Furthermore, I_c changes periodically as the magnetic field increases. The $I_c - H$ curve resembles the distribution of light intensity in the Fraunhofer diffraction experiment , and the latter is shown in Fig. 2. This phenomenon is called quantum diffraction of the superconductivity junction.

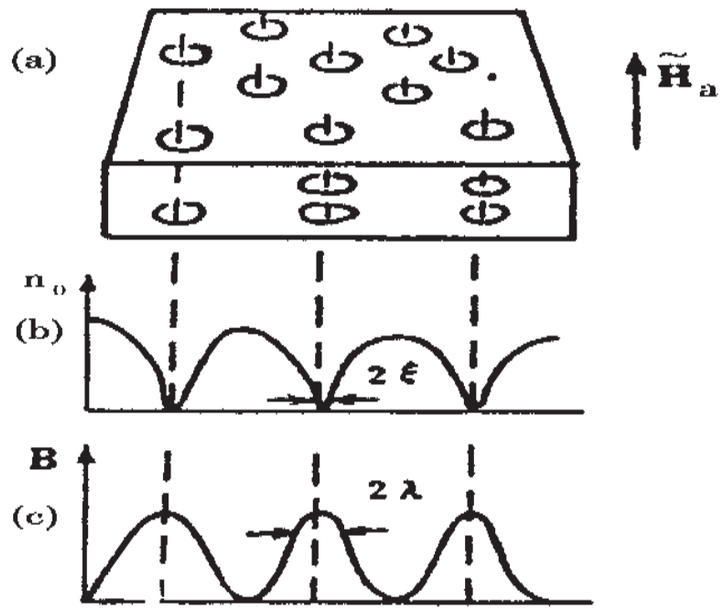


Fig. 1. Current and magnetic field distributionseffect in in a type-II superconductor.

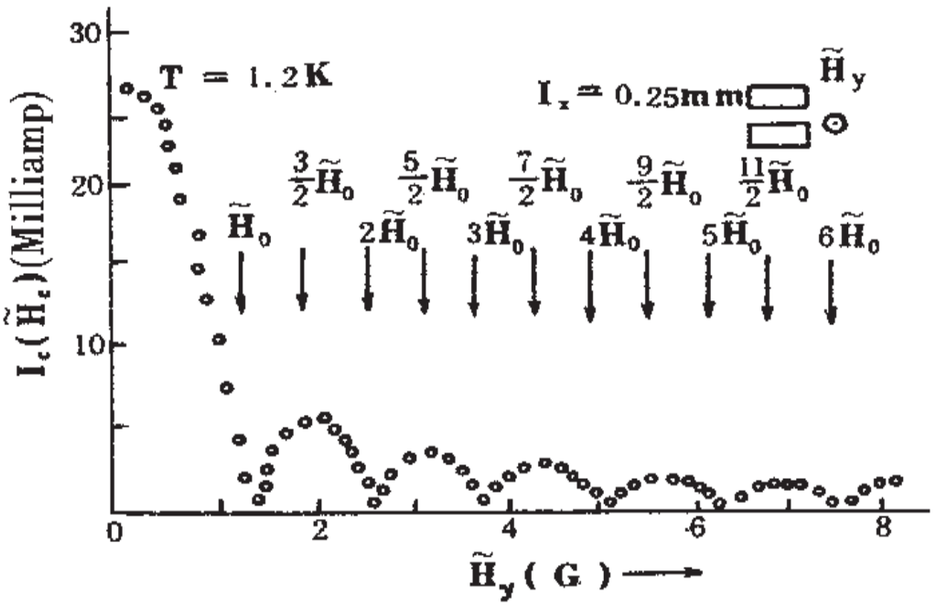


Fig. 2. Quantum diffractionsuperconductor junction

(D) When a junction is exposed to a microwave of frequency, υ, and if the voltage applied across the junction is varied, it can be seen that the dc current passing through the junction increases suddenly at certain discrete values of electric potential. Thus, a series of steps appear on the dc I − V curve, and the voltage at a given step is related to the frequency of the microwave radiation by nυ=2eVn/h(n=0,1,2,3…). More than 500 steps have been observed in experiments.

Josephson first derived these phenomena theoretically and each was experimentally verified subsequently. All these phenomena are, therefore, called Josephson effects [24-26]. In particular, (1) and (3) are referred to as dc Josephson effects while (2) and (4) are referred to as ac Josephson effects. Evidently, Josephson effects are macroscopic quantum effects, which can be explained well by the macroscopic quantum wave function. If we consider a superconducting junction as a weakly linked superconductor, the wave functions of the superconducting electron pairs in the superconductors on both sides of the junction are correlated due to a definite difference in their phase angles. This results in a preferred direction for the drifting of the superconducting electron pairs, and a dc Josephson current is developed in this direction. If a magnetic field is applied in the plane of the junction, the magnetic field produces a gradient of phase difference, which makes the maximum current oscillate along with the magnetic field, and the radiation of the electromagnetic wave occur. If a voltage is applied across the junction, the phase difference will vary with time and results in the Josephson effect. In view of this, the change in the phase difference of the wave functions of superconducting electrons plays an important role in Josephson effect, which will be discussed in more detail in the next section.

The discovery of the Josephson effect opened the door for a wide range of applications of superconductor theory. Properties of superconductors have been explored to produce superconducting quantum interferometer–magnetometer, sensitive ammeter, voltmeter, electromagnetic wave generator, detector, frequency-mixer, and so on.

3. The properties of boson condensation and spontaneous coherence of macroscopic quantum effects

3.1 A nonlinear theoretical model of theoretical description of macroscopic quantum effects

From the above studies we know that the macroscopic quantum effect is obviously different from the microscopic quantum effect, the former having been observed for physical quantities, such as, resistance, magnetic flux, vortex line, and voltage, etc.

In the latter, the physical quantities, depicting microscopic particles, such as energy, momentum, and angular momentum, are quantized. Thus it is reasonable to believe that the fundamental nature and the rules governing these effects are different.

We know that the microscopic quantum effect is described by quantum mechanics. However, the question remains relative to the definition of what are the mechanisms of macroscopic quantum effects? How can these effects be properly described?

What are the states of microscopic particles in the systems occurring related to macroscopic quantum effects? In other words, what are the earth essences and the nature of macroscopic quantum states? These questions apparently need to be addressed.

We know that materials are composed of a great number of microscopic particles, such as atoms, electrons, nuclei, and so on, which exhibit quantum features. We can then infer, or assume, that the macroscopic quantum effect results from the collective motion and excitation of these particles under certain conditions such as, extremely low temperatures, high pressure or high density among others. Under such conditions, a huge number of microscopic particles pair with each other condense in low-energy state, resulting in a highly ordered and long-range coherent. In such a highly ordered state, the collective motion of a large number of particles is the same as the motion of "single particles", and since the latter is quantized, the collective motion of the many particle system gives rise to a macroscopic quantum effect. Thus, the condensation of the particles and their coherent state play an essential role in the macroscopic quantum effect.

What is the concept of condensation? On a macroscopic scale, the process of transforming gas into liquid, as well as that of changing vapor into water, is called condensation. This, however, represents a change in the state of molecular positions, and is referred to as a condensation of positions. The phase transition from a gaseous state to a liquid state is a first order transition in which the volume of the system changes and the latent heat is produced, but the thermodynamic quantities of the systems are continuous and have no singularities. The word condensation, in the context of macroscopic quantum effects has its' special meaning. The condensation concept being discussed here is similar to the phase transition from gas to liquid, in the sense that the pressure depends only on temperature, but not on the volume noted during the process, thus, it is essentially different from the above, first-order phase transition. Therefore, it is fundamentally different from the first-order phase transition such as that from vapor to water. It is not the condensation of particles into a high-density material in normal space. On the contrary, it is the condensation of particles to a single energy state or to a low energy state with a constant or zero momentum. It is thus also called a condensation of momentum. This differs from a first-order phase transition and theoretically it should be classified as a third order phase transition, even though it is really a second order phase transition, because it is related to the discontinuity of the third derivative of a thermodynamic function. Discontinuities can be clearly observed in measured specific heat, magnetic susceptibility of certain systems when condensation

occurs. The phenomenon results from a spontaneous breaking of symmetries of the system due to nonlinear interaction within the system under some special conditions such as, extremely low temperatures and high pressures. Different systems have different critical temperatures of condensation. For example, the condensation temperature of a superconductor is its critical temperature T_c , and from previous discussions[27-32].

From the above discussions on the properties of superconductors, and others we know that, even though the microscopic particles involved can be either Bosons or Fermions, those being actually condensed, are either Bosons or quasi-Bosons, since Fermions are bound as pairs. For this reason, the condensation is referred to as Bose-Einstein condensation[33-36] since Bosons obey the Bose-Einstein statistics. Properties of Bosons are different from those of Fermions as they do not follow the Pauli exclusion principle, and there is no limit to the number of particles occupying the same energy levels. At finite temperatures, Bosons can distribute in many energy states and each state can be occupied by one or more particles, and some states may not be occupied at all. Due to the statistical attractions between Bosons in the phase space (consisting of generalized coordinates and momentum), groups of Bosons tend to occupy one quantum energy state under certain conditions. Then when the temperature of the system falls below a critical value, the majority or all Bosons condense to the same energy level (e.g. the ground state), resulting in a Bose condensation and a series of interesting macroscopic quantum effects. Different macroscopic quantum phenomena are observed because of differences in the fundamental properties of the constituting particles and their interactions in different systems.

In the highly ordered state of the phenomena, the behavior of each condensed particle is closely related to the properties of the systems. In this case, the wave function $\phi = f e^{i\theta}$ or $\phi = \sqrt{\rho} e^{i\theta}$ of the macroscopic state[33-35], is also the wave function of an individual condensed particle. The macroscopic wave function is also called the order parameter of the condensed state. This term was used to describe the superconductive states in the study of these macroscopic quantum effects. The essential features and fundamental properties of macroscopic quantum effect are given by the macroscopic wave function ϕ and it can be further shown that the macroscopic quantum states, such as the superconductive states are coherent and are Bose condensed states formed through second-order phase transitions after the symmetry of the system is broken due to nonlinear interaction in the system.

In the absence of any externally applied field, the Hamiltonian of a given macroscopic quantum system can be represented by the macroscopic wave function ϕ and written as

$$H = \int dx \mathrm{H}' = \int dx [-\frac{1}{2}|\nabla\phi|^2 - \alpha|\phi|^2 + \lambda|\phi|^4] \tag{1}$$

Here H'=H presents the Hamiltonian density function of the system, the unit system in which m=h=c=1 is used here for convenience. If an externally applied electromagnetic field does exist, the Hamiltonian given above should be replaced by

$$\mathrm{H} = \int dx \mathrm{H}' = \int dx \left[-\frac{1}{2}\left|\nabla - \mathrm{i}e^{*}\vec{\mathrm{A}}\phi\right|^2 - \alpha|\phi|^2 + \lambda|\phi|^4 + \frac{\tilde{\vec{\mathrm{H}}}^2}{8\pi} \right] \tag{2}$$

or, equivalently

$$H = \int dx H' = \int dx \left[-\frac{1}{2}\left|(\partial_j - ie^* A_j)\phi\right|^2 - \alpha|\phi|^2 + \lambda|\phi|^4 + \frac{1}{4}F_{ji}.F^{ji} \right]$$

where $F_{ji} = \partial_j A_i - \partial_t A_j$ is the covariant field intensity, $\vec{\tilde{H}}=\nabla\times\vec{A}$ is the magnetic field intensity, e is the charge of an electron, and e*=2e, $\vec{A}$ is the vector potential of the electromagnetic field, α and λ can be said to be some of the interaction constants. The above Hamiltonians in Eqs.(1) and (2) have been used in studying superconductivity by many scientists, including Jacobs de Gennes [37], Saint-Jams [38], Kivshar [39-40], Bullough [41-42], Huepe [43], Sonin [44], Davydov [45], et al., and they can be also derived from the free energy expression of a superconductive system given by Landau et al [46-47]. As a matter of fact, the Lagrangian function of a superconducting system can be obtained from the well-known Ginzberg-Landau (GL) equation [47-54] using the Lagrangian method, and the Hamiltonian function of a system can then be derived using the Lagrangian approach. The results, of course, are the same as Eqs. (1) and (2). Evidently, the Hamiltonian operator corresponding to Eqs. (1) and (2) represents a nonlinear function of the wave function of a particle, and the nonlinear interaction is caused by the electron-phonon interaction and due to the vibration of the lattice in BCS theory in the superconductors. Therefore, it truly exists. Evidently, the Hamiltonians of the systems are exactly different from those in quantum mechanics, and a nonlinear interaction related to the state of the particles is involved in Eqs. (1) –(2). Hence, we can expect that the states of particles depicted by the Hamiltonian also differ from those in quantum mechanics, and the Hamiltonian can describe the features of macroscopic quantum states including superconducting states. These problems are to be treated in the following pages. Evidently, the Hamiltonians in Eqs. (1) and (2) possess the U (1) symmetry. That is, they remain unchanged while undergoing the following transformation:

$$\phi(\vec{r},t) \to \phi'(\vec{r},t) = e^{-iQ_j\theta}\phi(\vec{r},t)$$

where Q_j is the charge of the particle，θ is a phase and, in the case of one dimension, each term in the Hamiltonian in Eq. (1) or Eq. (2) contains the product of the $\phi_j(x,t)$s , then we can obtain:

$$\phi_1'(x,t)\phi_2'(x,t)....\phi_n'(x,t) = e^{-i(Q_1+Q_2+\cdots+Q_n)\theta}\phi_1(x,t)\phi_2(x,t)....\phi_n(x,t)$$

Since charge is invariant under the transformation and neutrality is required for the Hamiltonian, there must be $(Q_1 + Q_2 + \cdots + Q_n) = 0$ in such a case. Furthermore, since θ is independent of x, it is necessary that $\nabla\phi_j \to e^{-i\theta Q_j}\nabla\phi_j$. Thus each term in the Hamiltonian in Eqs. (1) is invariant under the above transformation, or it possesses the U(1) symmetry[16-17].
If we rewrite Eq. (1) as the following

$$H' = -\frac{1}{2}(\nabla\phi)^2 + U_{eff}(\phi), U_{eff}(\phi) = -\alpha\phi^2 + \lambda\phi^4 \tag{3}$$

We can see that the effective potential energy, $U_{eff}(\phi)$, in Eq. (3) has two sets of extrema, $\phi_0 = \pm\sqrt{\alpha/2\lambda}$ and ϕ_0 =0, but the minimum is located at

$$\phi_0 = \pm\sqrt{\alpha/2\lambda} = \langle 0|\phi|0\rangle, \tag{4}$$

rather than at ϕ_0 =0 . This means that the energy at $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$ is lower than that at ϕ_0 =0. Therefore, ϕ_0 =0 corresponds to the normal ground state, while $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$ is the ground state of the macroscopic quantum systems .

In this case the macroscopic quantum state is the stable state of the system. This shows that the Hamiltonian of a normal state differs from that of the macroscopic quantum state, in which the two ground states satisfy $\langle 0|\phi|0\rangle \neq -\langle 0|\phi|0\rangle$ under the transformation, $\phi \to -\phi$. That is, they no longer have the U(1) symmetry. In other words，the symmetry of the ground states has been destroyed. The reason for this is evidently due to the nonlinear term $\lambda\phi^4$ in the Hamiltonian of the system. Therefore, this phenomenon is referred to as a spontaneous breakdown of symmetry. According to Landau's theory of phase transition, the system undergoes a second-order phase transition in such a case, and the normal ground state ϕ_0 ==0 is changed to the macroscopic quantum ground state $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$. Proof will be presented in the following example .

In order to make the expectation value in a new ground state zero in the macroscopic quantum state, the following transformation [16-17] is done:

$$\phi' = \phi + \phi_0 \tag{5}$$

so that

$$\langle 0|\phi'|0\rangle = 0 \tag{6}$$

After this transformation, the Hamiltonian density of the system becomes

$$H'(\phi+\phi_0) = \frac{1}{2}|\nabla\phi|^2 + (6\lambda\phi^2 - \alpha)\phi^2 + 4\lambda\phi_0\phi^3 + (4\lambda\phi_0^3 - 2\alpha\phi_0)\phi + \lambda\phi^4 - \alpha\phi_0^2 + \lambda\phi_0^4 \tag{7}$$

Inserting Eq. (4) into Eq. (7), we have $\left\langle \phi_0 \left| 4\lambda\phi_0^2 - 2\alpha \right| \phi_0 \right\rangle = 0$.

Consider now the expectation value of the variation $\delta H'/\delta\phi$ in the ground state, i.e. $\langle 0|\frac{\delta H'}{\delta\phi}|0\rangle = 0$, then from Eq. (1), we get

$$\langle 0|\frac{\delta H'}{\delta\phi}|0\rangle = \left\langle 0 \left| -\nabla^2\phi + 2\alpha\phi - 4\lambda\phi^3 \right| 0 \right\rangle = 0 \tag{8}$$

After the transformation Eq. (6), it becomes

$$\nabla^2\phi_0 + (4\lambda\phi_0^2 - 2\alpha)\phi_0 + 12\lambda\phi_0\left\langle 0\left|\phi^2\right|0\right\rangle + 4\lambda\left\langle 0\left|\phi^3\right|0\right\rangle - (2\alpha - 12\lambda\phi_0^2)\langle 0|\phi|0\rangle = 0 \tag{9}$$

where the terms $\langle 0|\phi^3|0\rangle$ and $\langle 0|\phi|0\rangle$ are both zero, but the fluctuation $12\lambda\phi_0\langle 0|\phi^2|0\rangle$ of the ground state is not zero. However, for a homogeneous system, at T=0K, the term $\langle 0|\phi^2|0\rangle$ is very small and can be neglected.

Then Eq. (9) can be written as

$$-\nabla^2\phi_0 - (4\lambda\phi_0^2 - 2\alpha)\phi_0 = 0 \tag{10}$$

Obviously, two sets of solutions, $\phi_0 = 0$, and $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$, can be obtained from the above equation, but we can demonstrate that the former is unstable, and that the latter is stable.

If the displacement is very small, i.e. $\phi_0 \to \phi_0 + \delta\phi_0 = \phi_0^{'}$, then the equation satisfied by the fluctuation $\delta\phi_0$ is relative to the normal ground state $\phi_0 = 0$ and is

$$\nabla^2\delta\phi_0 - 2\alpha\delta\phi_0 = 0 \tag{11}$$

Its' solution attenuates exponentially indicating that the ground state, $\phi_0 = 0$ is unstable. On the other hand, the equation satisfied by the fluctuation $\delta\phi_0$, relative to the ground state $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$ is $\nabla^2\delta\phi_0 + 2\alpha\delta\phi_0 = 0$. Its' solution, $\delta\phi_0$, is an oscillatory function and thus the macroscopic quantum state ground state $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$ is stable. Further calculations show that the energy of the macroscopic quantum state ground state is lower than that of the normal state by $\varepsilon_0 = -\alpha^2 / 4\lambda < 0$ .Therefore, the ground state of the normal phase and that of the macroscopic quantum phase are separated by an energy gap of $\alpha^2 / (4\lambda)$ so then, at T=0K, all particles can condense to the ground state of the macroscopic quantum phase rather than filling the ground state of the normal phase. Based on this energy gap, we can conclude that the specific heat of the macroscopic quantum systems has an exponential dependence on the temperature, and the critical temperature is given by: $T_c = 1.14\omega_p \exp[-1/(3\lambda/\alpha)N(0)]$ [16-17]. This is a feature of the second-order phase transition. The results are in agreement with those of the BCS theory of superconductivity.

Therefore, the transition from the state $\phi_0 = 0$ to the state $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$ and the corresponding condensation of particles are second-order phase transitions. This is obviously the results of a spontaneous breakdown of symmetry due to the nonlinear interaction, $\lambda\phi^4$.

In the presence of an electromagnetic field with a vector potential $\vec{A}$, the Hamiltonian of the systems is given by Eq. (2). It still possesses the U (1) symmetry.Since the existence of the nonlinear terms in Eq. (2) has been demonstrated, a spontaneous breakdown of symmetry can be expected. Now consider the following transformation:

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + i\phi_2(x)] \to \frac{1}{\sqrt{2}}[\phi_1(x) + \phi_0 + i\phi_2(x)] \tag{12}$$

Since $\langle 0|\phi_i|0\rangle = 0$ under this transformation, then the equation (2) becomes

$$\begin{aligned} H' = {} & \frac{1}{4}(\partial_i A_j - \partial_j A_i)^2 - \frac{1}{2}(\nabla\phi_2)^2 - \frac{1}{2}(\nabla\phi_1)^2 + \frac{(e^*)^2}{2}[(\phi_1 + \phi_0)^2 + \phi_2^2]A_i^2 - e^*\phi_0 A_i\nabla\phi_2 + \\ & e^*(\phi_2\nabla\phi_1 - \phi_1\nabla\phi_2)A_i - \frac{1}{2}(-12\lambda\phi_0^2 + 2\alpha)\phi_1^2 - \frac{1}{2}(12\lambda\phi_0^2 + 2\alpha)\phi_2^2 + 4\lambda\phi_0\phi_1(\phi_1^2 + \phi_2^2) + \\ & 4\lambda(\phi_1^2 + \phi_2^2)^2 - \phi_0(4\lambda\phi_0^2 + 2\alpha)\phi_1 - \alpha\phi_0^2 + \lambda\phi_0^2 \end{aligned} \tag{13}$$

We can see that the effective interaction energy of ϕ_0 is still given by:

$$U_{\text{eff}}(\phi_0) = -\alpha\phi_0^2 + \lambda\phi_0^4 \tag{14}$$

and is in agreement with that given in Eq. (4). Therefore, using the same argument, we can conclude that the spontaneous symmetry breakdown and the second-order phase transition also occur in the system. The system is changed from the ground state of the normal phase, $\phi_0 = 0$ to the ground state $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$ of the condensed phase in such a case. The above result can also be used to explain the Meissner effect and to determine its critical temperature in the superconductor. Thus, we can conclude that, regardless of the existence of any external field macroscopic quantum states, such as the superconducting state, are formed through a second-order phase transition following a spontaneous symmetry breakdown due to nonlinear interaction in the systems.

3.2 The features of the coherent state of macroscopic quantum effects

Proof that the macroscopic quantum state described by Eqs. (1) - (2) is a coherent state, using either the second quantization theory or the solid state quantum field theory is presented in the following paragraphs and pages.

As discussed above, when $\delta H' / \delta\phi = 0$ from Eq. (1), we have

$$\nabla^2\phi - 2\alpha\phi + 4\lambda|\phi|^2\phi = 0 \tag{15}$$

It is a time-independent nonlinear Schrödinger equation (NLSE), which is similar to the GL equation. Expanding ϕ in terms of the creation and annihilation operators, b_p^+ and b_p

$$\phi = \frac{1}{\sqrt{\bar{V}}}\sum_p \frac{1}{\sqrt{2\varepsilon_p}}(b_p e^{-ipx} + b_p^+ e^{ip.x}) \tag{16}$$

where $\bar{V}$ is the volume of the system. After a spontaneous breakdown of symmetry, ϕ_0, the ground-state of ϕ, for the system is no longer zero, but $\phi_0 = \pm\sqrt{\alpha / 2\lambda}$. The operation of the annihilation operator on $|\phi_0\rangle$ no longer gives zero, i.e.

$$b_p|\phi_0\rangle \neq 0 \tag{17}$$

A new field ϕ' can then be defined according to the transformation Eq. (5), where ϕ_0 is a scalar field and satisfies Eq. (10) in such a case. Evidently, ϕ_0 can also be expanded into

$$\phi_0 = -\frac{1}{\sqrt{\bar{V}}}\sum_p \frac{1}{\sqrt{2\varepsilon_p}}(\zeta_p e^{-ip.x} + \zeta_p^+ e^{ip.x}) \tag{18}$$

The transformation between the fields ϕ and ϕ' is obviously a unitary transformation, that is

$$\phi' = U\phi U^{-1} = e^{-s}\phi e^{s} = \phi + \phi_0 \tag{19}$$

where

$$S = i\int dx'[\phi'(x',t)\phi_0(x',t) - \phi_0(x',t)\phi(x',t)] \tag{20}$$

ϕ and ϕ' satisfy the following commutation relation

$$[\phi'(x',t),\phi(x,t)] = i\delta(x'-x) \tag{21}$$

From Eq. (6) we now have $\langle 0|\phi'|0\rangle = \phi_0' = 0$. The ground state $|\phi_0'\rangle$ of the field ϕ' thus satisfies

$$b_p|\phi_0'\rangle = 0 \tag{22}$$

From Eq. (6), we can obtain the following relationship between the annihilation operator a_p of the new field ϕ' and the annihilation operator b_p of the ϕ field

$$a_p = e^{-S}b_p e^{S} = b_p + \zeta_p \tag{23}$$

where

$$\zeta_p = \frac{1}{(2\pi)^{3/2}}\int \frac{dx}{\sqrt{\varepsilon_p}}[\phi_0(x,t)e^{ip.x} + i\phi_0^*(x,t)e^{-ip.x}] \tag{24}$$

Therefore, the new ground state $|\phi_0'\rangle$ and the old ground state $|\phi_0\rangle$ are related through $|\phi_0'\rangle = e^{S}|\phi_0\rangle$.

Thus we have

$$a_p|\phi_0'\rangle = (b_p + \zeta_p)|\phi_0'\rangle = \zeta_p|\phi_0'\rangle \tag{25}$$

According to the definition of the coherent state, equation (25) we see that the new ground state $|\phi_0'\rangle$ is a coherent state. Because such a coherent state is formed after the spontaneous breakdown of symmetry of the systems, thus, it is referred to as a spontaneous coherent state. But when $\phi_0 = 0$, the new ground state is the same as the old state, which is not a coherent state.The same conclusion can be directly derived from the BCS theory [18-21]. In the BCS theory, the wave function of the ground state of a superconductor is written as

$$|\phi_0'\rangle = \prod_k(\mu_k + \upsilon_k\hat{a}_k^+a_{-k}^+)|\phi_0\rangle = \prod_k(\mu_k + \upsilon_k\hat{b}_{k-k}^+)|\phi_0\rangle \sim \eta'\exp(\sum_k\frac{\upsilon_k}{\mu_k}\hat{b}_{k-k}^+)|\phi_0\rangle \tag{26}$$

where $\hat{b}_{k-k}^+ = \hat{a}_k^+\hat{a}_{-k}^+$. This equation shows that the superconducting ground state is a coherent state. Hence, we can conclude that the spontaneous coherent state in superconductors is formed after the spontaneous breakdown of symmetry.

By reconstructing a quasiparticle-operator-free new formulation of the Bogoliubov-Valatin transformation parameter dependence [55], W. S. Lin et al [56] demonstrated that the BCS state is not only a coherent state of single-Cooper-pairs, but also the squeezed state of the double-Cooper- pairs, and reconfirmed thus the coherent feature of BCS superconductive state.

3.3 The Boson condensed features of macroscopic quantum effects

We will now employ the method used by Bogoliubov in the study of superfluid liquid helium 4He to prove that the above state is indeed a Bose condensed state. To do that, we rewrite Eq. (16) in the following form [12-17]

$$\phi(x)=\frac{1}{\sqrt{\bar{V}}}\sum_{\mathrm{p}}q_{\mathrm{p}}e^{\mathrm{ipx}},q_{\mathrm{p}}=\frac{1}{\sqrt{2\varepsilon_{\mathrm{p}}}}\left(b_{\mathrm{p}}+b_{\text{-p}}^{+}\right) \tag{27}$$

Since the field ϕ describes a Boson, such as the Cooper electron pair in a superconductor and the Bose condensation can occur in the system, we will apply the following traditional method in quantum field theory, and consider the following transformation:

$$b_{\mathrm{p}}=\sqrt{N_0}\delta(p)+\gamma_{\mathrm{p}},b_{\mathrm{p}}^{+}=\sqrt{N_0}\delta(p)+\beta_{\mathrm{p}} \tag{28}$$

where N_0 is the number of Bosons in the system and $\delta(p)=\begin{cases}0\text{ ,if p}\neq 0\\ 1\text{ , if p=0}\end{cases}$. Substituting Eqs. (27) and (28) into Eq. (1), we can arrive at the Hamiltonian operator of the system as follows

$$\begin{aligned}\widehat{H}=&\left(\frac{4\lambda N_0^2}{\varepsilon_0^2\bar{V}}-\frac{\alpha}{\varepsilon_0}\right)\sqrt{N_0}\left(\gamma_0+\gamma_0^{+}+\beta_0+\beta_0^{+}\right)+\sum_{\mathrm{p}}\left(\frac{4\lambda}{\varepsilon_0\varepsilon_{\mathrm{P}}}\frac{N_0}{\bar{V}}-\varepsilon_{\mathrm{p}}\right)\left(\gamma_{\mathrm{p}}^{+}\beta_{\text{-p}}^{+}+\gamma_{\mathrm{p}}\beta_{\text{-p}}\right)+\frac{4\lambda N_0^2}{\varepsilon_0^2\bar{V}}-\\ &\frac{2N_0\alpha}{\varepsilon_0}+\frac{\lambda N_0}{\varepsilon_0\bar{V}}\sum_{\mathrm{p}}\frac{1}{\varepsilon_{\mathrm{p}}}\begin{pmatrix}\beta_{\mathrm{p}}^{+}\beta_{\text{-p}}^{+}+\beta_{\mathrm{p}}\beta_{\text{-p}}+\gamma_{\mathrm{p}}^{+}\gamma_{\text{-p}}^{+}+\\ \gamma_{\mathrm{p}}\gamma_{\text{-p}}+2\gamma_{\mathrm{p}}^{+}\beta_{\mathrm{p}}^{+}+2\beta_{\mathrm{p}}^{+}\gamma_{\mathrm{p}}\end{pmatrix}+\\ &\sum_{\mathrm{p}}\left(\varepsilon_{\mathrm{p}}-\frac{\alpha}{2\varepsilon_0}+\frac{4\lambda N_0}{\varepsilon_0\varepsilon_p\bar{V}}\right)\left(\gamma_{\mathrm{p}}^{+}\gamma_{\mathrm{p}}+\beta_{\mathrm{p}}^{+}\beta_{\mathrm{p}}\right)+\sum_{P}\frac{4\lambda}{\varepsilon_0\varepsilon_{\mathrm{P}}}\frac{N_0}{\bar{V}}+O\left(\frac{\sqrt{N_0}}{\bar{V}}\right)+O\left(\frac{N_0}{\bar{V}^2}\right)\end{aligned} \tag{29}$$

Because the condensed density $N_0/\bar{V}$ must be finite, it is possible that the higher order terms $0\left(\sqrt{N_0}/\bar{V}\right)$ and $0\left(N_0/\bar{V}^2\right)$ may be neglected. Next we perform the following canonical transformation

$$\gamma_{\mathrm{p}}=u_{\mathrm{p}}^{*}c_{\mathrm{p}}+\upsilon_{\mathrm{p}}c_{\text{-p}}^{+},.\beta_{\mathrm{p}}=u_{\mathrm{p}}^{*}d_{\mathrm{p}}+\upsilon_{\mathrm{p}}d_{\text{-p}}^{+} \tag{30}$$

where v_p and u_p are real and satisfy $\left(u_{\mathrm{p}}^2-\upsilon_{\mathrm{p}}^2\right)=1$. This introduces another transformation

$$\varsigma_{\mathrm{p}}=\frac{1}{\sqrt{2}}\left(u_{\mathrm{p}}\gamma_{\mathrm{p}}^{+}-\upsilon_{\mathrm{p}}\gamma_{\text{-p}}+u_{\mathrm{p}}\beta_{\mathrm{p}}^{+}-\upsilon_{\mathrm{p}}\beta_{\text{-p}}\right),\eta_{\mathrm{p}}^{+}=\frac{1}{\sqrt{2}}\left(u_{\mathrm{p}}\gamma_{\mathrm{p}}^{+}-\upsilon_{\mathrm{p}}\gamma_{\text{-p}}-u_{\mathrm{p}}\beta_{\mathrm{p}}^{+}+\upsilon_{\mathrm{p}}\beta_{\text{-p}}\right) \tag{31}$$

the following relations can be obtained

$$\left[\varsigma_{\mathrm{p}},H\right]=g_{\mathrm{p}}\varsigma_{\mathrm{p}}+M_{\mathrm{p}}\varsigma_{\text{-p}}^{+},\qquad\left[\eta_{\mathrm{p}},H\right]=g_{\mathrm{p}}'\eta_{\mathrm{p}}+M_{\mathrm{p}}'\eta_{\text{-p}}^{+} \tag{32}$$

where

$$\begin{cases}g_{\mathrm{p}}=G_{\mathrm{p}}\left(u_{\mathrm{p}}^2+\upsilon_{\mathrm{p}}^2\right)+F_{\mathrm{p}}2u_{\mathrm{p}}\upsilon_{\mathrm{p}},\ \mathrm{M}_{\mathrm{p}}=F_{\mathrm{p}}\left(u_{\mathrm{p}}^2+\upsilon_{\mathrm{p}}^2\right)+G_{\mathrm{p}}2u_{\mathrm{p}}\upsilon_{\mathrm{p;}}\\ g_{\mathrm{p}}'=G_p'\left(u_{\mathrm{p}}^2+\upsilon_{\mathrm{p}}^2\right)+F_{\mathrm{p}}'2u_{\mathrm{p}}\upsilon_{\mathrm{p}},\ \mathrm{M}_{\mathrm{p}}'=F_{\mathrm{p}}'\left(u_{\mathrm{p}}^2+\upsilon_{\mathrm{p}}^2\right)+G_{\mathrm{p}}'2u_{\mathrm{p}}\upsilon_{\mathrm{p}}\end{cases} \tag{33}$$

while

$$G_{\mathrm{p}} = \varepsilon_{\mathrm{p}} - \frac{\alpha}{2\varepsilon_{\mathrm{p}}} + 6\xi'_{\mathrm{p}},\ \mathrm{F}_{\mathrm{p}} = -\frac{\alpha}{2\varepsilon_{\mathrm{p}}} + 6\xi'_{\mathrm{p}},\ \ G'_{\mathrm{p}} = \varepsilon_{\mathrm{p}} - \frac{\alpha}{2\varepsilon_{\mathrm{p}}} + 2\xi'_{\mathrm{p}},\ \mathrm{F}'_{p} = \frac{\alpha}{2\varepsilon_{\mathrm{p}}} - 2\xi'_{\mathrm{p}} \tag{34}$$

where $\xi'_{p} = \frac{\lambda \mathrm{N}_0}{\varepsilon_0 \varepsilon_{\mathrm{p}} \overline{\mathrm{V}}}$.

We will now study two cases to illustrate the concepts.

(A) Let $M'_{\mathrm{p}} = 0$, then it can be seen from Eq. (32) that η^{+}_{p} is the creation operator of elementary excitation and its energy is given by

$$g'_{\mathrm{p}} = \sqrt{\varepsilon^2_{\mathrm{p}} + 4\varepsilon_{\mathrm{p}}\xi'_{\mathrm{p}} - 2\alpha} \tag{35}$$

Using this concept, we can obtain the following form from Eqs. (32) and (34)

$$\left(u'_{\mathrm{p}}\right)^2 = \frac{1}{2}\left(1 + \frac{G'_{\mathrm{p}}}{g'_{\mathrm{p}}}\right) \quad \text{and} \quad \left(\upsilon'_{\mathrm{p}}\right)^2 = \frac{1}{2}\left(-1 + \frac{G'_{\mathrm{p}}}{g'_{\mathrm{p}}}\right) \tag{36}$$

From Eq. (32), we know that ξ^{+}_{p} is not a creation operator of the elementary excitation. Thus, another transformation must be made

$$B_{\mathrm{p}} = \chi_{\mathrm{p}}\varsigma'_{\mathrm{p}} + \mu_{\mathrm{p}}\varsigma'^{+}_{\mathrm{p}},\ \left|\chi_{\mathrm{p}}\right|^2 - \left|\mu_{\mathrm{p}}\right|^2 = 1 \tag{37}$$

We can then prove that

$$[B_p, \widehat{H}] = E_{\mathrm{p}} B_p \tag{38}$$

where $\mathrm{E}_{\mathrm{p}} = \sqrt{12\varepsilon_{\mathrm{p}}\xi'_{\mathrm{p}} + \varepsilon^2_{p} - 2\alpha}$

Now, inserting Eqs. (30), (37)-(38) and $\mathrm{M}'_{\mathrm{p}} = 0$ into Eq. (29), and after some reorganization, we have

$$\widehat{H} = U + E_0 + \sum_{\mathrm{p}>0}\left[E_{\mathrm{p}}\left(B^{+}_{\mathrm{p}}B_{\mathrm{p}} + B^{+}_{\text{-p}}B_{\text{-p}}\right) + g'_{\mathrm{p}}\left(\eta^{+}_{\mathrm{p}}\eta_{\mathrm{p}} + \eta^{+}_{\text{-p}}\eta_{\text{-p}}\right)\right] \tag{39}$$

where

$$U = \frac{\lambda N_0^2}{\varepsilon_0^2 \overline{V}} - \frac{2\alpha N_0}{\varepsilon_0} + \sum_{\mathrm{p}} 4\xi'_{\mathrm{p}} + \sum_{\mathrm{p}}\left(\varepsilon_{\mathrm{p}} + \frac{\alpha}{2\varepsilon_{\mathrm{p}}} + 4\xi'\right)4\upsilon'^2_{\mathrm{p}} + \sum_{\mathrm{p}>0} 4\eta \cdot 2u'_{\mathrm{p}}\upsilon'_{\mathrm{p}}$$

$$E_0 = -2\sum_{\mathrm{p}>0} E_{\mathrm{p}}\left|\mu_{\mathrm{p}}\right|^2 = -\sum_{\mathrm{p}>0}\left(g'_{\mathrm{p}} - E_{\mathrm{p}}\right) \tag{40}$$

Both U and E_0 are now independent of the creation and annihilation operators of the Bosons. $U + E_0$ gives the energy of the ground state. N_0 can be determined from the condition, $\frac{\delta\left(U + E_0\right)}{\delta N_0} = 0$, which gives

$$\frac{N_0}{\bar{V}} = \frac{\alpha\varepsilon_0}{4\lambda} = \frac{1}{2}\varepsilon_0\phi_0^2 \tag{41}$$

This is the condensed density of the ground state ϕ_0. From Eqs. (36), (37) and (40), thus we can arrive at:

$$g'_{\mathrm{p}} = \sqrt{\varepsilon_{\mathrm{p}}^2 - \alpha}, \quad \mathrm{E_p} = \sqrt{\varepsilon_{\mathrm{p}}^2 - \alpha} \tag{42}$$

These correspond to the energy spectra of η_p^+ and B_p^+,respectively, and they are similar to the energy spectra of the Cooper pair and phonon in the BCS theory. Substituting Eq. (42) into Eq. (36), thus we now have:

$$\mathrm{u}_{\mathrm{p}}'^2 = \frac{1}{2}\left(1+\frac{2\varepsilon_{\mathrm{p}}^2\text{-}\alpha}{2\sqrt{\varepsilon_{\mathrm{p}}^2\text{-}\alpha\mathrm{e_p}}}\right), \quad \upsilon_{\mathrm{p}}'^2 = \frac{1}{2}\left(\text{-}1+\frac{2\varepsilon_{\mathrm{p}}^2 - \alpha}{2\sqrt{\varepsilon_{\mathrm{p}}^2 - \alpha\varepsilon_{\mathrm{p}}}}\right) \tag{43}$$

(B) In the case of Mp=0, a similar approach can be used to arrive at the energy spectrum corresponding to ξ_p^+ as $\mathrm{E_p} = \sqrt{\varepsilon_{\mathrm{p}}^2 + \alpha}$, while that corresponding to $\mathrm{A_p^+} = \chi_{\mathrm{p}}\eta_{\mathrm{p}}^+ + \mu_{\mathrm{p}}\eta_{\text{-p}}$ is $g'_{\mathrm{p}} = \sqrt{\varepsilon_{\mathrm{p}}^2 + \alpha}$, where

$$\mathrm{u}_{\mathrm{p}}^2 = \frac{1}{2}\left(1+\frac{2\varepsilon_{\mathrm{p}}^2 + \alpha}{2\varepsilon_{\mathrm{p}}\sqrt{\varepsilon_{\mathrm{p}}^2 + \alpha}}\right), \quad \upsilon_{\mathrm{p}}^2 = \frac{1}{2}\left(\text{-}1+\frac{2\varepsilon_{\mathrm{p}}^2 + \alpha}{2\varepsilon_{\mathrm{p}}\sqrt{\varepsilon_{\mathrm{p}}^2 + \alpha}}\right) \tag{44}$$

Based on experiments in quantum statistical physics, we know that the occupation number of the level with an energy of ε_p, for a system in thermal equilibrium at temperature $\mathrm{T}(\neq 0)$ is shown as:

$$\mathrm{N_p} = \left\langle \mathrm{b_p^+ b_p} \right\rangle = \frac{1}{\mathrm{e}^{\varepsilon_{\mathrm{P}}/\mathrm{K_B T}} - 1} \tag{45}$$

where $\langle\cdots\rangle$ denotes Gibbs average, defined as $\langle\cdots\rangle = \dfrac{\mathrm{SP}\left[\mathrm{e}^{-\mathrm{H}/\mathrm{K_B T}}\cdots\right]}{\mathrm{SP}\left[\mathrm{e}^{-\mathrm{H}/\mathrm{K_B T}}\right]}$, here SP denotes the trace in a Gibbs statistical description. At low temperatures, or $\mathrm{T} \to 0$ K, the majority of the Bosons or Cooper pairs in a superconductor condense to the ground state with $\mathrm{p} = 0$. Therefore $\left\langle \mathrm{b_0^+ b_0} \right\rangle \approx \mathrm{N_0}$, where $\mathrm{N_0}$ is the total number of Bosons or Cooper pairs in the system and $\mathrm{N_0} >> 1$, i.e. $\left\langle \mathrm{b^+ b} \right\rangle = 1 << \left\langle \mathrm{b_0^+ b_0} \right\rangle$.

As can be seen from Eqs. (27) and (28), the number of particles is extremely large when they lie in condensed state, that is to say:

$$\phi_0 = \phi_{\mathrm{P}=0} = \frac{1}{\sqrt{2\varepsilon_0 \bar{\mathrm{V}}}}\left(\mathrm{b}_0 + \mathrm{b}_0^+\right) \tag{46}$$

Because $\gamma_0 \left| \phi_0 \right\rangle = 0$ and $\beta_0 \left| \phi_0 \right\rangle = 0$, b_0 and b_0^+ can be taken to be $\sqrt{N_0}$. The average value of $\phi^*\phi$ in the ground state then becomes

$$\left\langle \phi_0 \left| \phi^* \phi \right| \phi_0 \right\rangle = \left\langle \phi^* \phi \right\rangle_0 = \frac{1}{2\varepsilon_0 \bar{V}} \cdot 4N_0 = \frac{2N_0}{\varepsilon_0 \bar{V}} \tag{47}$$

Substituting Eq. (41) into Eq. (47), we can see that:

$$\left\langle \phi^* \phi \right\rangle_0 = \frac{\alpha}{2\lambda} \quad \text{or} \quad \left\langle \phi^* \right\rangle_0 = \pm\sqrt{\frac{\alpha}{2\lambda}}$$

which is the ground state of the condensed phase, or the superconducting phase, that we have known. Thus, the density of states, $N_0/\bar{V}$, of the condensed phase or the superconducting phase formed after the Bose condensation coincides with the average value of the Boson's (or Copper pair's) field in the ground state. We can then conclude from the above investigation shown in Eqs. (1) - (2) that the macroscopic quantum state or the superconducting ground state formed after the spontaneous symmetry breakdown is indeed a Bose-Einstein condensed state. This clearly shows the essences of the nonlinear properties of the result of macroscopic quantum effects.

In the last few decades, the Bose-Einstein condensation has been observed in a series of remarkable experiments using weakly interacting atomic gases, such as vapors of rubidium, sodium lithium, or hydrogen. Its' formation and properties have been extensively studied. These studies show that the Bose-Einstein condensation is a nonlinear phenomenon, analogous to nonlinear optics, and that the state is coherent, and can be described by the following NLSE or the Gross-Pitaerskii equation [57-59]:

$$\mathrm{i}\frac{\partial \phi}{\partial \mathrm{t}'} = -\frac{\partial^2 \phi}{\partial \mathrm{x}'^2} - \lambda|\phi|^3 + \mathrm{V}(\mathrm{x})\phi \tag{48}$$

where $\mathrm{t}'=\mathrm{t}/\hbar$, $\mathrm{x}'=\mathrm{x}\sqrt{2\mathrm{m}}/\hbar$. This equation was used to discuss the realization of the Bose-Einstein condensation in the $\mathrm{d}+1$ dimensions $(\mathrm{d}=1,2,3)$ by H. K. Bullough et.al [41-42]. Too, Elyutin et al [60-61]. gave the corresponding Hamiltonian density of a condensate system as follows:

$$H' = \left|\frac{\partial \phi}{\partial x'}\right|^2 + V(x')|\phi|^2 - \frac{1}{2}\lambda|\phi|^4 \tag{49}$$

where H'=H, the nonlinear parameters of λ are defined as $\lambda = -2Naa_1 / a_0^2$, with N being the number of particles trapped in the condensed state, a is the ground state scattering length, a_0 and a_1 are the transverse (y, z) and the longitudinal (x) condensate sizes (without self-interaction) respectively, (Integrations over y and z have been carried out in obtaining the above equation). λ is positive for condensation with self-attraction (negative scattering length).The coherent regime was observed in Bose-Einstein condensation in lithium. The specific form of the trapping potential V (x') depends on the details of the experimental setup. Work on Bose-Einstein condensation based on the above model Hamiltonian were carried out and are reported by C. F. Barenghi et al [31].

It is not surprising to see that Eq. (48) is exactly the same as Eq. (15), corresponding to the Hamiltonian density in Eq. (49) and, where used in this study is naturally the same as Eq. (1). This prediction confirms the correctness of the above theory for Bose-Einstein

condensation. As a matter of fact, immediately after the first experimental observation of this condensation phenomenon, it was realized that the coherent dynamics of the condensed macroscopic wave function could lead to the formation of nonlinear solitary waves. For example, self-localized bright, dark and vortex solitons, formed by increased (bright) or decreased (dark or vortex) probability density respectively, were experimentally observed, particularly for the vortex solution which has the same form as the vortex lines found in type II-superconductors and superfluids. These experimental results were in concordance with the results of the above theory. In the following sections of this text we will study the soliton motions of quasiparticles in macroscopic quantum systems, superconductors. We will see that the dynamic equations in macroscopic quantum systems do have such soliton solutions.

3.4 Differences of macroscopic quantum effects from the microscopic quantum effects

From the above discussion we may clearly understand the nature and characteristics of macroscopic quantum systems. It would be interesting to compare the macroscopic quantum effects and microscopic quantum effects. Here we give a summary of the main differences between them.

1. Concerning the origins of these quantum effects; the microscopic quantum effect is produced when microscopic particles, which have only a wave feature are confined in a finite space, or are constituted as matter, while the macroscopic quantum effect is due to the collective motion of the microscopic particles in systems with nonlinear interaction. It occurs through second-order phase transition following the spontaneous breakdown of symmetry of the systems.
2. From the point-of-view of their characteristics, the microscopic quantum effect is characterized by quantization of physical quantities, such as energy, momentum, angular momentum, etc. wherein the microscopic particles remain constant. On the other hand, the macroscopic quantum effect is represented by discontinuities in macroscopic quantities, such as, the resistance, magnetic flux, vortex lines, voltage, etc. The macroscopic quantum effects can be directly observed in experiments on the macroscopic scale, while the microscopic quantum effects can only be inferred from other effects related to them.
3. The macroscopic quantum state is a condensed and coherent state, but the microscopic quantum effect occurs in determinant quantization conditions, which are different for the Bosons and Fermions. But, so far, only the Bosons or combinations of Fermions are found in macroscopic quantum effects.
4. The microscopic quantum effect is a linear effect, in which the microscopic particles and are in an expanded state, their motions being described by linear differential equations such as the Schrödinger equation, the Dirac equation, and the Klein-Gordon equations.

On the other hand, the macroscopic quantum effect is caused by the nonlinear interactions, and the motions of the particles are described by nonlinear partial differential equations such as the nonlinear Schrödinger equation (17).

Thus, we can conclude that the macroscopic quantum effects are, in essence, a nonlinear quantum phenomenon. Because its' fundamental nature and characteristics are different from those of the microscopic quantum effects, it may be said that the effects should be depicted by a new nonlinear quantum theory, instead of quantum mechanics.

4. The nonlinear dynamic natures of electrons in superconductors

4.1 The dynamic equations of electrons in superconductors

It is quite clear from the above section that the superconductivity of material is a kind of nonlinear quantum effect formed after the breakdown of the symmetry of the system due to the electron-phonon interaction, which is a nonlinear interaction.

In this section we discuss the properties of motion of superconductive electrons in superconductors and the relation of the solutions of dynamic equations in relation to the above macroscopic quantum effects on it. The study presented shows that the superconductive electrons move in the form of a soliton, which can result in a series of macroscopic quantum effects in the superconductors. Therefore, the properties and motions of the quasiparticles are important for understanding the essences and rule of superconductivity and macroscopic quantum effects.

As it is known, in the superconductor the states of the electrons are often represented by a macroscopic wave function,

$$\phi(\vec{r},t) = f(\vec{r},t)\phi_0 e^{i\theta(\vec{r},t)}, \quad \text{or} \quad \phi = \sqrt{\rho}e^{i\theta},$$

as mentioned above, where $\phi_0^2 = \alpha / 2\lambda$. Landau et al [45,46] used the wave function to give the free energy density function, f, of a superconducting system, which is represented by

$$f_s = f_n - \frac{\hbar^2}{2m}|\nabla\phi|^2 - \alpha|\phi|^2 + \lambda|\phi|^4 \tag{50}$$

in the absence of any external field. If the system is subjected to an electromagnetic field specified by a vector potential $\vec{A}$, the free energy density of the system is of the form:

$$f_s = f_n - \frac{\hbar^2}{2m}\left|(\nabla - \frac{ie^*}{c\hbar}\vec{A})\phi\right|^2 - \alpha|\phi|^2 + \lambda|\phi|^4 + \frac{1}{8\pi}\vec{\tilde{H}}^2 \tag{51}$$

where e*=2e, $\vec{\tilde{H}}=\nabla\times\vec{A}$, α and λ are some interactional constants related to the features of superconductor, m is the mass of electron, e* is the charge of superconductive electron, c is the velocity of light, h is Planck constant, $\hbar = h/2\pi$, fn is the free energy of normal state. The free energy of the system is $F_s = \int f_s d^3x$. In terms of the conventional field, $F_{jl} = \partial_j A_l - \partial_l A^j$, (j, l=1, 2, 3), the term $\tilde{H}^2/8\pi$ can be written as $F_{jl}F^{jl}/4$. Equations (50) - (51) show the nonlinear features of the free energy of the systems because it is the nonlinear function of the wave function of the particles, $\phi(\vec{r},t)$. Thus we can predict that the superconductive electrons have many new properties relative to the normal electrons. From $\delta F_s / \delta\phi = 0$ we get

$$\frac{\hbar^2}{2m}\nabla^2\phi - \alpha\phi + 2\lambda\phi^3 = 0 \tag{52}$$

and

$$\frac{\hbar^2}{2m}(\nabla - \frac{ie^*}{c\hbar}\vec{A})^2\phi - \alpha\phi + 2\lambda\phi^3 = 0 \tag{53}$$

in the absence and presence of an external fields respectively, and

$$\vec{J} = +\frac{e^*\hbar}{2mi}(\phi^*\nabla\phi - \phi\nabla\phi^*) - \frac{e^*}{mc}|\phi|^2\vec{A} \tag{54}$$

Equations (52) - (54) are just well-known the Ginzburg-Landau (GL) equation [48-54] in a steady state, and only a time-independent Schrödinger equation. Here, Eq. (52) is the GL equation in the absence of external fields. It is the same as Eq. (15), which was obtained from Eq. (1). Equation (54) can also be obtained from Eq. (2). Therefore, Eqs. (1)-(2) are the Hamiltonians corresponding to the free energy in Eqs. (50)- (51).
From equations (52) - (53) we clearly see that superconductors are nonlinear systems. Ginzburg-Landau equations are the fundamental equations of the superconductors describing the motion of the superconductive electrons, in which there is the nonlinear term of $2\lambda\phi^3$. However, the equations contain two unknown functions ϕ and $\vec{A}$ which make them extremely difficult to resolve.

4.2 The dynamic properties of electrons in steady superconductors

We first study the properties of motion of superconductive electrons in the case of no external field. Then, we consider only a one-dimensional pure superconductor [62-63], where

$$\phi = \phi_0\varphi(x,t),\ \xi'^2(T) = \hbar^2 / 2m|\alpha|,\ x' = x / \xi'(T) \tag{55}$$

and where $\xi'(T)$ is the coherent length of the superconductor, which depends on temperature. For a uniform superconductor, $\xi'(T) = 0.94\xi_0[T_c/(T_c - T)]^2$, where T_c is the critical temperature and ξ_0 is the coherent length of superconductive electrons at T=0. In boundary conditions of $\varphi(x'=0)=1$, and $\varphi(x' \to \pm\infty) = 0$, from Eqs. (52) and (54) we find easily its solution as:

$$\varphi = \pm\sqrt{2}\,\mathrm{sec}h\left[\frac{x - x_0}{\xi'(T)}\right]$$

or

$$\phi = \pm\sqrt{\frac{\alpha}{\lambda}}\,\mathrm{sec}h[\frac{x - x_0}{\xi'(T)}] = \pm\sqrt{\frac{\alpha}{\lambda}}\,\mathrm{sec}h[\frac{\sqrt{2m\alpha}}{\hbar}(x - x_0)] \tag{56}$$

This is a well-known wave packet-type soliton solution. It can be used to represent the bright soliton occurred in the Bose-Einstein condensate found by Perez-Garcia et. al. [64]. If the signs of α and λ in Eq. (52) are reversed, we then get a kink-soliton solution under the boundary conditions of $\varphi(x'=0)=0$, $\varphi(x' \to \pm\infty) = \pm 1$,

$$\phi = \pm(\alpha / 2\lambda)^{1/2}\tanh\{[m\alpha(x - x_0 / \hbar^2]^{1/2}\} \tag{57}$$

The energy of the soliton, (56), is given by

$$E_{so1}=\int_{-\infty}^{\infty}\left[\frac{\hbar^2}{2m}(\frac{d\phi}{dx})^2-\alpha\phi^2-\lambda\phi^4\right]dx=\frac{4\hbar\alpha^{3/2}}{3\lambda\sqrt{2m}} \tag{58}$$

We assume here that the lattice constant, r_0=1. The above soliton energy can be compared with the ground state energy of the superconducting state, Eground= $-\alpha^2/4\lambda$. Their difference is $E_{so1}-E_{\text{ground}}=\alpha^{3/2}\left(\sqrt{\alpha}+\frac{16\hbar}{3\sqrt{2m}}\right)/2\lambda>0$. This indicates clearly that the soliton is not in the ground state, but in an excited state of the system, therefore, the soliton is a quasiparticle.

From the above discussion, we can see that, in the absence of external fields, the superconductive electrons move in the form of solitons in a uniform system. These solitons are formed by a nonlinear interaction among the superconductive electrons which suppresses the dispersive behavior of electrons. A soliton can carry a certain amount of energy while moving in superconductors. It can be demonstrated that these soliton states are very stable.

4.3 The features of motion of superconductive electrons in an electromagnetic field and its relation to macroscopic quantum effects

We now consider the motion of superconductive electrons in the presence of an electromagnetic field $\vec{A}$; its equation of motion is denoted by Eqs. (53)-(54)· Assuming now that the field $\vec{A}$ satisfies the London gauge $\nabla\cdot\vec{A}=0$ [65], and that the substitution of $\phi(\vec{r},t)=\varphi(\vec{r},t)\phi_0 e^{i\theta(\vec{r})}$ into Eqs. (53) and (54) yields [66-67]:

$$J=\frac{e^*\phi_0^2}{m}(\hbar\nabla\theta-\frac{e^*}{c}\vec{A})\varphi^2 \tag{59}$$

and

$$\nabla^2\varphi-[(\nabla\theta-\frac{e^*}{\hbar c}\vec{\mathbf{A}})^2\varphi]-\frac{2m}{\hbar^2}(\alpha-2\lambda\phi_0^2\varphi^2)\varphi=0 \tag{60}$$

For bulk superconductors, J is a constant (permanent current) for a certain value of $\vec{A}$, and it thus can be taken as a parameter. Let $B^2=m^2J^2/\hbar^2(e^*)^2\phi_0^4$, $b=2m\alpha/\hbar^2=\xi^{-2}$, from Eqs. (59) and (60), we can obtain [66-67]:

$$(\hbar\nabla\theta-\frac{e^*}{c}\vec{\mathbf{A}})=\frac{Jm}{e^*\phi_0^2\varphi^2} \tag{61}$$

$$\frac{d^2\varphi}{dx^2}=-\frac{d}{d\varphi}U_{\text{eff}}(\varphi),U_{\text{eff}}(\varphi)=\frac{B^2}{2\varphi^2}-\frac{1}{2}b\varphi^2+\frac{1}{4}b\varphi^4 \tag{62}$$

where Ueff is the effective potential of the superconductive electron in this case and it is schematically shown in Fig. 2. Comparing this case with that in the absence of external fields, we found that the equations have the same form and the electromagnetic field changes only the effective potential of the superconductive electron. When $\vec{A}=0$, the

effective potential well is characterized by double wells. In the presence of an electromagnetic field, there are still two minima in the effective potential, corresponding to the two ground states of the superconductor in this condition. This shows that the spontaneous breakdown of symmetry still occurs in the superconductor, thus the superconductive electrons also move in the form of solitons. To obtain the soliton solution, we integrate Eq. (62) and can get:

$$x = \int_{\varphi_1}^{\varphi} \frac{d\varphi}{\sqrt{2[E - U_{\text{eff}}(\varphi)}} \tag{63}$$

Where E is a constant of integration which is equivalent to the energy, the lower limit of the integral, φ_1, is determined by the value of φ at x=0, i.e. $E = U_{\text{eff}}(\varphi_0) = U_{\text{eff}}(\varphi_1)$. Introduce the following dimensionless quantities $\varphi^2 = u$, $E = \frac{b}{2}\varepsilon$, $2\tilde{d} = \frac{4J^2 m\lambda}{(e^*)^2\alpha^2}$, and equation (63) can be written as the following upon performing the transformation u→–u,

$$-\sqrt{2b}x = \int_{u_1}^{u} \frac{du}{\sqrt{u^3 - 2u^2 - 3\varepsilon u - 2\tilde{d}^2}} \tag{64}$$

It can be seen from Fig. 3 that the denominator in the integrand in Eq. (64) approaches zero linearly when $u=u_1=\varphi_1^2$, but approaches zero gradually when $u=u_2=\varphi_0^2$. Thus we give [66-67]

$$u(x) = \varphi^2(x) = u_0 - g\sec h^2\left(\sqrt{\frac{1}{2}gb}x\right) = u_1 + g\tan h^2\left(\sqrt{\frac{1}{2}gb}x\right) \tag{65}$$

where $g= u_0–u_1$ and satisfies

$$(2+g)^2(1-g) = 27\tilde{d}^2,\ 2u_0 + u_1 = 2,\ u_0^2 + 2u_0u_1 = -2\varepsilon,\ u_1u_0^2 = 2\tilde{d}^2 \tag{66}$$

It can be seen from Eq. (65) that for a large part of sample, u_1 is very small and may be neglected; the solution u is very close to u_0. We then get from Eq. (65) that

$$\varphi(x) = \varphi_0 \tan h\left(\sqrt{\frac{1}{2}gb}x\right) \tag{67}$$

Substituting the above into Eq. (61), the electromagnetic field $\vec{A}$ in the superconductors can be obtained

$$\vec{A} = -\frac{\vec{J}mc}{(e^*)^2\phi_0^2}\frac{1}{\varphi^2} - \frac{\hbar c}{e^*}\nabla\theta \quad = \frac{\vec{J}mc}{(e^*)^2\phi_0^2\varphi_0^2}\cot h\left(\sqrt{\frac{1}{2}gb}x\right) - \frac{\hbar c}{e^*}\nabla\theta$$

For a large portion of the superconductor, the phase change is very small. Using $\vec{H} = \nabla \times \vec{A}$ the magnetic field can be determined and is given by [66-67]

$$\vec{H} = \frac{\vec{J}mc\sqrt{2gb}}{(e^*)^2\phi_0^2\varphi_0^2}\left[\cot h^3\left(\sqrt{\frac{1}{2}gb}x\right) + \cot h\left(\sqrt{\frac{1}{2}gb}x\right)\right] \tag{68}$$

Equations (67) and (68) are analytical solutions of the GL equation.(63) and (64) in the one-dimensional case, which are shown in Fig. 3. Equation (67) or (65) shows that the superconductive electron in the presence of an electromagnetic field is still a soliton. However, its amplitude, phase and shape are changed, when compared with those in a uniform superconductor and in the absence of external fields, Eq. (66). The soliton here is obviously influenced by the electromagnetic field, as reflected by the change in the form of solitary wave. This is why a permanent superconducting current can be established by the motion of superconductive electrons along certain direction in such a superconductor, because solitons have the ability to maintain their shape and velocity while in motion.
It is clear from Fig.4 that $\tilde{H}(\mathrm{x})$ is larger where $\phi(\mathrm{x})$ is small, and vice versa. When $x \to 0$, $\tilde{H}(x)$ reaches a maximum, while ϕ approaches to zero. On the other hand, when $x \to \infty$, ϕ becomes very large, while $\tilde{H}(x)$ approaches to zero. This shows that the system is still in superconductive state.These are exactly the well-known behaviors of vortex lines-magnetic flux lines in type-II superconductors [66-67]. Thus we explained clearly the macroscopic quantum effect in type-II superconductors using GL equation of motion of superconductive electron under action of an electromagnetic-field.

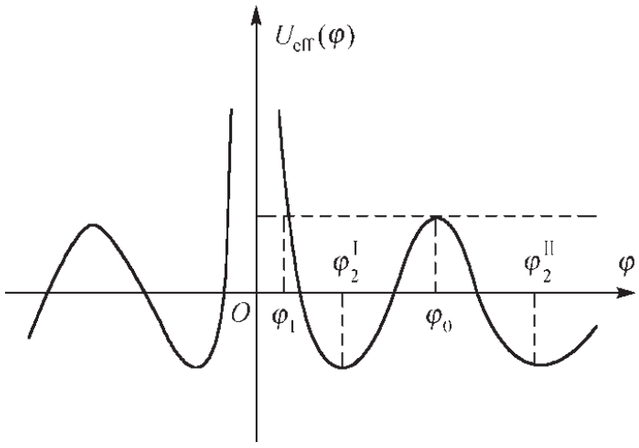


Fig. 3. The effective potential energy in Eq. (67).

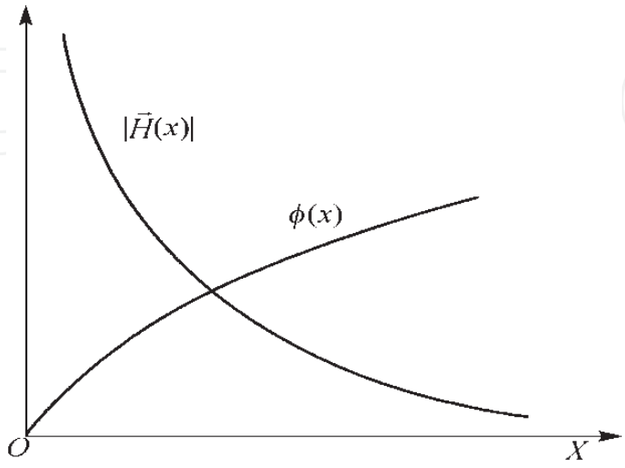


Fig. 4. Changes of $\phi(\mathrm{x})$ and $\left|\tilde{H}(\mathrm{x})\right|$ with x in Eqs. (67)-(68)

Recently, Garadoc-Daries et al. [68], Matthews et al. [69] and Madison et al.[70] observed vertex solitons in the Boson-Einstein condensates. Tonomure [71] observed experimentally magnetic vortexes in superconductors. These vortex lines in the type-II-superconductors are quantized. The macroscopic quantum effects are well described by the nonlinear theory discussed above, demonstrating the correctness of the theory.

We now proceed to determine the energy of the soliton given by (67). From the earlier discussion, the energy of the soliton is given by:

$$E=\int_{-\infty}^{+\infty}\left[\frac{1}{2}(\frac{d\varphi}{dx})^2+\frac{b}{2}\varphi^2-\frac{\mathrm{b}}{4}\varphi^4-\frac{B^2}{2\varphi^2}\right]dx\approx\varphi_0{}^2\left[\frac{2b\varphi_0^2}{3}-1+\frac{b}{2}(1-\frac{\varphi_0^2}{2})\right]-\frac{B^2}{2\varphi_0^2}$$

which depends on the interaction between superconductive electrons and electromagnetic field.

From the above discussion, we understand that for a bulk superconductor, the superconductive electrons behave as solitons, regardless of the presence of external fields. Thus, the superconductive electrons are a special type of soliton. Obviously, the solitons are formed due to the fact that the nonlinear interaction $\lambda|\phi|^2\phi$ suppresses the dispersive effect of the kinetic energy in Eqs. (52) and (53). They move in the form of solitary wave in the superconducting state. In the presence of external electromagnetic fields, we demonstrate theoretically that a permanent superconductive current is established and that the vortex lines or magnetic flux lines also occur in type-II superconductors.

5. The dynamic properties of electrons in superconductive junctions and its relation to macroscopic quantum effects

5.1 The features of motion of electron in S-N junction and proximity effect

The superconductive junction consists of a superconductor (S) which contacts with a normal conductor (N), in which the latter can be superconductive. This phenomenon refers to a proximity effect. This is obviously the result of long- range coherent property of superconductive electrons. It can be regarded as the penetration of electron pairs from the superconductor into the normal conductor or a result of diffraction and transmission of superconductive electron wave. In this phenomenon superconductive electrons can occur in the normal conductor, but their amplitudes are much small compare to that in the superconductive region, thus the nonlinear term $\lambda|\phi|^2\phi$ in GL equations (53)-(54) can be neglected. Because of these, GL equations in the normal and superconductive regions have different forms. On the S side of the S-N junction, the GL equation is [72]

$$\frac{\hbar^2}{2\mathrm{m}}(\nabla-\frac{\mathrm{ie}^*}{\mathrm{ch}}\vec{\mathrm{A}})\phi-\alpha\phi+2\lambda\phi^3=0 \tag{69}$$

while that on the N side of the junction is

$$\frac{\hbar^2}{2\mathrm{m}}(\nabla-\frac{\mathrm{ie}^*}{\mathrm{ch}}\vec{\mathrm{A}})\phi-\alpha'\phi=0 \tag{70}$$

Thus, the expression for $\vec{J}$ remains the same on both sides.

$$\vec{J} = \frac{e^{*}\hbar}{2mi}(\phi^{*}\nabla\phi - \phi\nabla\phi^{*}) - \frac{(e^{*})^{2}}{mc}|\phi|^{2}\vec{A} \tag{71}$$

In the S region, we have obtained solution of (69) in the previous section, and it is given by (65) or (67) and (68). In the N region, from Eqs. (70)- (71) we can easily obtain

$$\begin{cases} \varphi^{2} = \frac{1}{2}\sqrt{(\varepsilon')^{2} - 4\tilde{d}^{2}}\,\sin(2\sqrt{b'}x) + \frac{\varepsilon'}{2} \\ \phi_{N}^{2} = \varphi^{2}\phi_{0}^{2}e^{-2i\theta} = \frac{1}{2}\sqrt{(\varepsilon')^{2} - 4\tilde{d}^{2}}\,\sin(2\sqrt{b'}x)e^{-i2\theta} + \frac{\varepsilon'}{2}\phi_{0}^{2}e^{-i2\theta} \end{cases} \tag{72}$$

where $b' = \frac{2m\alpha'}{\hbar^{2}} = \frac{1}{\xi'^{2}}$, $2\tilde{d}^{2} = \frac{4\bar{J}^{2}m\lambda}{(e^{*})^{2}\alpha'^{2}}$, $E' = \frac{b'}{2}\varepsilon'$. .

here ε' is an integral constant. A graph of ϕ vs. x in both the S and the N regions, as shown in Fig.5, coincides with that obtained by Blackbunu [73]. The solution given in Eq. (72) is the analytical solution in this case. On the other hand, Blackbunu's result was obtained by expressing the solution in terms of elliptic integrals and then integrating numerically. From this, we see that the proximity effect is caused by diffraction or transmission of the superconductive electrons

5.2 The Josephson effect in S-I-S and S-N-S as well as S-I-N-S junctions

A superconductor-normal conductor -superconductor junction (S-N-S) or a superconductor-insulator-superconductor junction (S-I-S) consists of a normal conductor or an insulator sandwiched between two superconductors as is schematically shown in Fig.6a · The thickness of the normal conductor or the insulator layer is assumed to be L and we choose the z coordinate such that the normal conductor or the insulator layer is located at $-L/2 \le x \le L/2$. The features of S-I-S junctions were studied by Jacobson et al.[74]. We will treat this problem using the above idea and method [75-76].

The electrons in the superconducting regions ($|x| \ge L/2$) are depicted by GL equation (69). Its' solution was given earlier in Eq.(67). After eliminating u_1 from Eq.(66), we have [73-74]

$J = \frac{1}{2}e^{*}\alpha u_{0}\sqrt{\frac{\alpha}{m\lambda}(1-)u_{0}}$.

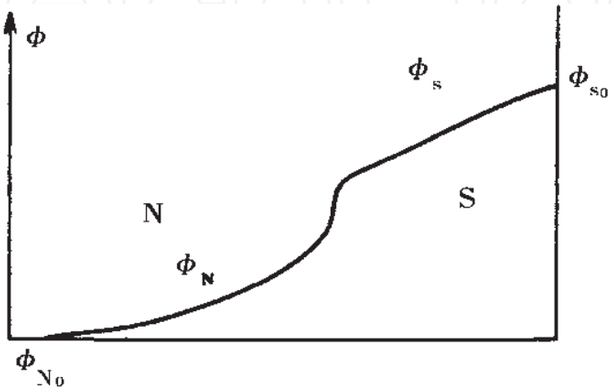


Fig. 5. Proximity effect in S-N junction

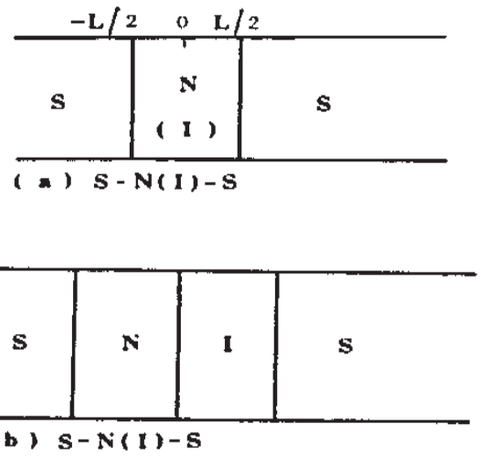


Fig. 6. Superconductive junction of S-N(I)-S and S-N-I-S

The electrons in the superconducting regions ($|x| \geq L/2$) are depicted by GL equation (69). Its' solution was given earlier. Setting $dJ/du_0 = 0$, we get the maximum current $J_c = \frac{e^* \alpha}{3}\sqrt{\frac{\alpha}{3m\lambda}}$. This is the critical current of a perfect superconductor, corresponding to the three-fold degenerate solution of Eq.(66), i.e.,u_1=u_0.

From Eq.(71), we have $\vec{A} = -\frac{m\vec{J}c}{(e^*)^2\phi_0^2\varphi^2} + \frac{hc}{e^*}\nabla\theta$. Using the London gauge, $\nabla.\vec{A} = 0$, we can get[75-76] $\frac{d^2\theta}{dx^2} = \frac{mJ}{e^*\phi_0^2\hbar}\frac{d}{dx}(\frac{1}{\varphi^2})$. Integrating the above equation twice , we get the change of the phase to be

$$\Delta\theta = \frac{mJ}{e^*\phi_0^2\hbar}\int(\frac{1}{\varphi^2} - \frac{1}{\varphi_\infty^2})dx \tag{73}$$

where $\varphi^2 = u$, and $\varphi_\infty^2 = u_0$. Here we have used the following de Gennes boundary conditions in obtaining Eq. (73)

$$\left.\frac{d\phi}{dx}\right|_{|x|\to\infty} = 0, \left.\frac{d\theta}{dx}\right|_{|x|\to\infty} = 0, \ \phi(|x| \to \infty) = \phi_\infty \tag{74}$$

If we substitute Eqs.(64) - (67) into Eq.(73), the phase shift of wave function from an arbitrary point x to infinite can be obtained directly from the above integral, and takes the form of:

$$\Delta\theta_L(x \to \infty) = -\tan^{-1}\sqrt{\frac{u_1}{u_0 - u_1}} + \tan^{-1}\sqrt{\frac{u_1}{u - u_1}} \tag{75}$$

For the S-N-S or S-I-S junction, the superconducting regions are located at $|x| \geq L/2$ and the phase shift in the S region is thus

$$\Delta\theta_s=2\Delta\theta_L(\frac{L}{2}\to\infty)\approx 2\tan^{-1}\sqrt{\frac{u_1}{u_s-u_1}} \tag{76}$$

According to the results in (70) - (71) and the above similar method, the change of the phase in the I or N region of the S-N-S or S-I-S junction may be expressed as [75-76]

$$\Delta\theta_N=-2\tan^{-1}[\frac{2e^*h'}{\dot{J}}\sqrt{\frac{\alpha^2}{8m\lambda}}\tan(\frac{\sqrt{b'}L}{2})]+\frac{m\dot{J}L}{2e^*h'\mu_0} \tag{77}$$

where $h'=\sqrt{\frac{8m\lambda}{\alpha^2}}\frac{\dot{J}}{2e^*}\frac{\tan(\Delta\theta_N/2)}{\tan(\sqrt{b'}L/2)}$, $\frac{m\dot{J}L}{2e^*h'\mu_0}$ is an additional term to satisfy the boundary conditions (74),and may be neglected in the case being studied.

Near the critical temperature (T<Tc), the current passing through a weakly linked superconductive junction is very small ($\dot{J}<<1$), we then have $\mu_1'=\frac{4\dot{J}^2m\lambda}{(e^*)^2\alpha^2}=2\overline{A}^2$, and g′=1. Since $\eta\varphi^2$ and $d\varphi^2/dx$ are continuous at the boundary x=L/2, we have

$$\frac{d\mu_s}{dx}\Big|_{x=L/2}=\frac{d\mu_N}{dx}\Big|_{x=L/2},\quad \eta_s\mu_s\Big|_{x=L/2}=\eta_N\mu_N\Big|_{x=L/2},$$

where η_s and η_N are the constants related to features of superconductive and normal phases in the junction, respectively. These give [75-76]

$$2\sqrt{b'}\overline{A}\sin(2\Delta\theta_N)=\varepsilon_1[1-\cos(2\Delta\theta_s)]\sin(\sqrt{b'}L),$$

$$\cos(\sqrt{b'}L)\sin(2\Delta\theta_s)=\varepsilon\sin(2\Delta\theta_N)+\sin(2\Delta\theta_s+\Delta\theta_N)$$

where $\varepsilon_1=\eta_N/\eta_S$. From the two equations, we can get

$$\sin(\Delta\theta_s+\Delta\theta_N)=\frac{2\sqrt{2m\lambda}\dot{J}}{e^*\alpha}\sqrt{b'}\sin(\sqrt{b'}L).$$

Thus

$$\dot{J}=\dot{J}_{max}\sin(\Delta\theta_s+\Delta\theta_N)=\dot{J}_{max}\sin(\Delta\theta) \tag{78}$$

where

$$\dot{J}_{max}=\frac{e^*\alpha_s}{2\sqrt{2m\lambda b'}}.\frac{1}{\sin(\sqrt{b'}L)},\Delta\theta=\Delta\theta_s+\Delta\theta_N \tag{79}$$

Equation (78) is the well-known example of the Josephson current. From Section I we know that the Josephson effect is a macroscopic quantum effect. We have seen now that this effect can be explained based on the nonlinear quantum theory. This again shows that the macroscopic quantum effect is just a nonlinear quantum phenomenon.

From Eq. (79) we can see that the Josephson critical current is inversely proportional to sin ($\sqrt{b'}L$), which means that the current increases suddenly whenever $\sqrt{b'}L$ approaches to $n\pi$,

suggesting some resonant phenomena occurs in the system · This has not been observed before. Moreover $\dot{J}_{max}$ is proportional to $e^*\alpha_s / 2\sqrt{2m\lambda b'} = (e^*\hbar\alpha_S / 4m\sqrt{\lambda\alpha_N})$, which is related to $(T\text{-}T_c)^2$ ·

Finally, it is worthwhile to mention that no explicit assumption was made in the above on whether the junction is a potential well (α <0) or a potential barrier (α >0). The results are thus valid and the Josephson effect in Eq. (2.78), occurs for both potential wells and for potential barriers.

We now study Josephson effect in the superconductor -normal conductor-insulator-superconductor junction (SNIS) is shown schematically in Fig. 6b. It can be regarded as a multilayer junction consists of the S-N-S and S-I-S junctions. If appropriate thicknesses for the N and I layers are used (approximately 20 °A– 30 °A), the Josephson effect similar to that discussed above can occur in the SNIS junction. Since the derivations are similar to that in the previous sections, we will skip much of the details and give the results in the following. The Josephson current in the SNIS junction is still given by

$$\dot{J}=\dot{J}_{max}\sin(\Delta\theta)$$

but, where $\Delta\theta = \Delta\theta_{s1} + \Delta\theta_N + \Delta\theta_I + \Delta\theta_{s2}$ and

$$\dot{J}_{max} = \frac{1}{\sqrt{b'_N}}\{\frac{\varepsilon_1\sinh(\sqrt{b'_N}L)}{2[\cosh(\sqrt{b'_N}L)-\cos(2\Delta\theta_N)]}\}\times \frac{1}{\sqrt{[1+\cos(2\Delta\theta_N)][1+\cos(2\Delta\theta_I)]}-\sqrt{[1-\cos(2\Delta\theta_N)][1-\cos(2\Delta\theta_I)]}}-$$

$$\frac{1}{\sqrt{b'_N}}\{\frac{\varepsilon_1\sqrt{[1-\cos^2(2\Delta\theta_N)]}\sinh(\sqrt{b'_N}L)}{2[\cosh(\sqrt{b'_N}L)-\cos(2\Delta\theta_N)]^2-1+\cos^2(2\Delta\theta_N)}\}\times \frac{1}{\sqrt{[1-\cos(2\Delta\theta_N)][1-\cos(2\Delta\theta_I)]}+\sqrt{[1+\cos(2\Delta\theta_N)][1+\cos(2\Delta\theta_I)]}}$$

It can be shown that the temperature dependence of J_{max} is $J_{max} \propto (T_c - T_0)^2$,which is quite similar to the results obtained by Blackburm et al[73] for the SNIS junction and those by Romagnan et al[7] using the Pb-PbO-Sn-Pb junction. Here, we obtained the same results using a complete different approach. This indicates again that we can obtain some results, which agree with the experimental data.

6. The nonlinear dynamic-features of time- dependence of electrons in superconductor

6.1 The soliton solution of motion of the superconductive electron

We studied only the properties of motion of superconductive electrons in steady states in superconductors in section 2.3.2, and which are described by the time-independent GL equation. In such a case, the superconductive electrons move as solitons. We ask, "What are the features of a time-dependent motion in non-equilibrium states of a superconductor?" Naturally, this motion should be described by the time-dependent Ginzburg-Landau (TDGL) equation [48-54,77] in this case. Unfortunately, there are many different forms of the

TDGL equation under different conditions. The one given in the following is commonly used when an electromagnetic field $\vec{A}$ is involved

$$\Gamma\left[\hbar\frac{\partial}{\partial t}-2ie\mu(r)\right]\phi=\frac{-1}{2m}\left(\hbar\nabla-\frac{2ie}{c}\vec{A}\right)^2+\alpha\phi-\lambda|\phi|^2\phi \tag{80}$$

and

$$\vec{J}=\sigma\left[-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}-\nabla\mu(r)\right]+\frac{ie\hbar}{m}(\phi^*\nabla\phi-\phi\nabla\phi^*)-\frac{4e^2}{mc}\vec{A}|\phi|^2 \tag{81}$$

here $i=\sqrt{-1}$, $\nabla\times\nabla\times\vec{A}=\frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}-\nabla\mu\right)+\frac{4\pi\vec{J}}{c}$ and σ is the conductivity in the normal state, Γ is an arbitrary constant, and μ is the chemical potential of the system. In practice, Eq. (80) is simply a time-dependent Schrödinger equation with a damping effect.
In certain situations, the following forms of the TDGL equation are also used.

$$i\hbar\frac{\partial\phi}{\partial t}=-\frac{\hbar^2}{2m}\left(\nabla-\frac{2ie}{\hbar c}\vec{A}\right)^2\phi+\alpha\phi-\lambda|\phi|^2\phi \tag{82}$$

or

$$i\left(\hbar\frac{\partial}{\partial t}-i2e\mu\right)\phi=\frac{1}{\Gamma}(\alpha-\lambda|\phi|^2)\phi+\frac{\xi'^2}{\Gamma}\left(\nabla-\frac{2ie}{\hbar c}\vec{A}\right)^2\phi \tag{83}$$

here $\xi'=\hbar/\sqrt{2m}$, and equation (82) is a nonlinear Schrödinger equation under an electromagnetic field having soliton solutions. However, these solutions are very difficult to find, and no analytic solutions have been obtained. An approximate solution was obtained by Kusayanage et al [78] by neglecting the ϕ^3 term in Eq. (80) or Eq. (82), in the case of $\vec{A}=(0,\tilde{H}x,0)$, $\mu=-K\tilde{E}x$, $\vec{H}$=(0, 0, $\tilde{H}$) and $\vec{E}$=($\tilde{E}$,0,0), where $\vec{H}$ is the magnetic field, while $\vec{E}$ is the electric field .We will solve the TDGL equation in the case of weak fields in the following.
TDGL equation (83) can be written in the following form when $\vec{A}$ is very small[80-81]

$$i\hbar\frac{\partial\phi}{\partial t}+\frac{\hbar^2}{2m\Gamma}\nabla^2\phi+\frac{\lambda}{\Gamma}|\phi|^2\phi=\left(\frac{\alpha}{\Gamma}\text{-}2e\mu\right)\phi \tag{84}$$

Where α and Γ are material dependent parameters, λ is the nonlinear coefficient, m is the mass of the superconductive electron. Equation (84) is actually a nonlinear Schrödinger equation in a potential field $\alpha/\Gamma-2e\mu$. Cai, Bhattacharjee et al [79], and Davydov [45] used it in their studies of superconductivity. However, this equation is also difficult to solve · In the following, Pang solves the equation only in the one-dimensional case.
For convenience, let $t'=t/\hbar$, $x'=x\sqrt{2m\Gamma}/\hbar$, then Eq. (84) becomes

$$i\frac{\partial\phi}{\partial t'}+\frac{\partial^2\phi}{\partial x'^2}+\frac{\lambda}{\Gamma}|\phi|^2\phi=\left[\frac{\alpha}{\Gamma}\text{-}2e\mu(x')\right]\phi \tag{85}$$

If we let $\frac{\alpha}{\Gamma} - 2e\mu = 0$, then Eq. (85) is the usual nonlinear Schrödinger equation whose solution is of the form [80-81]

$$\phi_s^0 = \varphi_0(x',t')e^{i\theta_0(x',t')}, \tag{86}$$

$$\varphi_0(x',t') = \sqrt{\frac{\Gamma(\upsilon_e^2 - 2\upsilon_c\upsilon_e)}{2\lambda}} \times \sec h\left[\sqrt{\frac{(\upsilon_e^2 - 2\upsilon_c\upsilon_e)}{4}}(x' - \upsilon_e t')\right] \tag{87}$$

here $\theta_0(x',t') = \frac{1}{2}\upsilon_e(x' - \upsilon_c t')$. In the case of $\frac{\alpha}{\Gamma}$-$2e\mu \neq 0$, let $\mu = -K\tilde{E}x'$, where K is a constant, and assume that the solution is of the form [80-81]

$$\phi = \varphi'(x',t')e^{i\theta(x',t')} \tag{88}$$

Substituting Eq. (88) into Eq. (86), we get:

$$-\varphi'\frac{\partial\theta}{\partial t'} - \varphi'\left(\frac{\partial\theta}{\partial t'}\right)^2 + \frac{\partial^2\varphi'}{\partial(x')^2} + \frac{\lambda}{\Gamma}(\varphi')^3 = \left(2Ke\tilde{E}x' + \frac{\alpha}{\Gamma}\right)\varphi' \tag{89}$$

$$\frac{\partial\varphi'}{\partial t'} + 2\frac{\partial\varphi'}{\partial x'}\frac{\partial\theta}{\partial x'} + \varphi'\frac{\partial^2\theta}{\partial(x')^2} = 0 \tag{90}$$

Now let $\varphi'(x',t') = \varphi(\xi)$, $\xi = x' - u(t')$, $u(t') = -2\tilde{E}Ke(t')^2 + \upsilon t' + d$, where $u(t')$ describes the accelerated motion of $\varphi'(x',t')$. The boundary condition at $\xi' \to \infty$ requires $\varphi(\xi)$ to approach zero rapidly. When $2\,\partial\theta/\partial\xi - \dot{u} \neq 0$, equation (90) can be written as: $\varphi^2 = \frac{g(t')}{(\partial\theta/\partial\xi - \dot{u}/2)}$, or

$$\frac{\partial\theta}{\partial x'} = \frac{g(t')}{\varphi^2} + \frac{\dot{u}}{2} \tag{91}$$

where $\dot{u} = du/dt'$. Integration of (91) yields:

$$\theta(x',t') = g(t')\int_0^{x'}\frac{dx''}{\varphi^2} + \frac{\dot{u}}{2}x' + h(t') \tag{92}$$

and where $h(t')$ is an undetermined constant of integration. From Eq. (92) we can get:

$$\frac{\partial\theta}{\partial t'} = \dot{g}(t')\int_0^{x'}\frac{dx''}{\varphi^2} - \frac{g\dot{u}}{\varphi^2} + \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0} + \frac{\ddot{u}}{2}x' + \dot{h}(t') \tag{93}$$

Substituting Eqs. (92) and (93) into Eq. (89), we have:

$$\frac{\partial^2\varphi}{\partial(x')^2} = \left[\left(2K\tilde{E}ex' + \frac{\alpha}{\Gamma}\right) + \frac{\ddot{u}}{2}x' + \dot{h}(t') + \frac{\dot{u}^2}{4} + \dot{g}\int_0^{x'}\frac{dx''}{\varphi^2} + \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0}\right]\varphi - \frac{\lambda}{\Gamma}\varphi^3 + \frac{g^2}{\varphi^3} \tag{94}$$

Since $\frac{\partial^2\varphi}{\partial(x')^2}=\frac{d^2\varphi}{d\xi^2}$, which is a function of ξ only, the right-hand side of Eq. (94) is also a function of ξ only, so it is necessary that $g(t')=g_0=\text{constant}$, and $(2K\tilde{E}ex'+\frac{\alpha}{\Gamma})+\frac{\ddot{u}}{2}x'+\dot{h}(t')+\frac{\dot{u}^2}{4}+\frac{g\dot{u}}{f^2}\Big|_{x'=0}=\overline{V}(\xi)$. Next, we assume that $V_0(\xi)=\overline{V}(\xi)-\beta$, where β is real and arbitrary, then

$$2K\tilde{E}ex'+\frac{\alpha}{\Gamma}=V_0(\xi)-\frac{\ddot{u}}{2}x'+\left[\beta-\frac{g\dot{u}}{\varphi^2}\Big|_{x'=0}-\dot{h}(t')-\frac{\dot{u}^2}{4}\right] \tag{95}$$

Clearly in the case discussed, $V_0(\xi)=0$, and the function in the brackets in Eq. (95) is a function of t'. Substituting Eq. (95) into Eq. (94), we can get [80-81]:

$$\frac{\partial^2\tilde{\varphi}}{\partial\xi^2}=\beta\tilde{\varphi}-\frac{\lambda}{\Gamma}\tilde{\varphi}^3+g_0^2/\tilde{\varphi}^3 \tag{96}$$

This shows that $\tilde{\varphi}$ is the solution of Eq. (96) when β and g are constant. For large $|\xi|$, we may assume that $|\tilde{\varphi}|\le\beta/|\xi|^{1+\Delta}$, when Δ is a small constant. To ensure that $\tilde{\varphi}$ and $d^2\tilde{\varphi}/d\xi^2$ approach zero when $|\xi|\to\infty$, only the solution corresponding to g_0=0 in Eq. (96) is kept, and it can be shown that this soliton solution is stable in such a case. Therefore, we choose g_0=0 and obtain the following from Eq. (91):

$$\partial\theta/\partial x'=\dot{u}/2 \tag{97}$$

Thus, we obtain from Eq. (95) that

$$2K\tilde{E}ex'+\frac{\alpha}{\Gamma}=-\frac{\ddot{u}}{2}x'+\beta-\dot{h}(t')-\frac{\dot{u}^2}{4},\qquad h(t')=\left(\beta-\frac{\alpha}{\Gamma}-\frac{1}{4}\upsilon^2\right)t'-\frac{4}{3}(K\tilde{E}e)^2(t')^3+e\upsilon K\tilde{E}(t')^2 \tag{98}$$

Substituting Eq. (98) into Eqs. (92) - (93), we obtain:

$$\theta=\left(-2K\tilde{E}et'+\frac{1}{2}\upsilon\right)x'+\left(\beta-\frac{\alpha}{\Gamma}-\frac{1}{4}\upsilon^2\right)t'-\frac{4}{3}(K\tilde{E}e)^2(t')^3+e\upsilon K\tilde{E}(t')^2 \tag{99}$$

Finally, substituting the Eq. (99) into Eq. (96), we can get

$$\frac{\partial^2\tilde{\varphi}}{\partial\xi^2}-\beta\tilde{\varphi}+\frac{\lambda}{\Gamma}\tilde{\varphi}^3=0 \tag{100}$$

When $\beta>0$, the solution of. Eq. (100) is of the form

$$\tilde{\varphi}=\sqrt{\frac{2\beta\Gamma}{\lambda}}\sec h\left(\sqrt{\beta}\xi\right) \tag{101}$$

Thus [80-81]

$$\phi = \sqrt{\frac{2\beta\Gamma}{\lambda}}\sec h\left[\sqrt{\beta}\left(\sqrt{\frac{2m\Gamma}{\hbar^2}}x + \frac{2eKEt^2 - \upsilon t - d}{\hbar}\right)\right]\times$$
$$\exp\left\{i\left[\left(\frac{-2eKEt}{\hbar} + \frac{\upsilon}{2}\right)\sqrt{\frac{2m\Gamma}{\hbar^2}}x + \left(\beta - \frac{\alpha}{\Gamma} - \frac{1}{4}\upsilon^2\right)\frac{t}{\hbar} - \frac{4(eK\tilde{E})^2 t^3}{3\hbar^3} + \frac{\upsilon Ke\tilde{E}t^2}{\hbar}\right]\right\} \quad (102)$$

This is also a soliton solution, but its shape, amplitude and velocity have been changed relatively as compared to that of Eq. (87). It can be shown that Eq. (102) does indeed satisfy Eq. (85). Thus, equation (85) has a soliton solution. It can also be shown that this solition solution is stable.

6.2 The properties of soliton motion of the superconductive electrons

For the solution of Eq. (102), we may define a generalized time-dependent wave number, $k = \frac{\partial\theta}{\partial x'} = \frac{\upsilon}{2} - 2K\tilde{E}et'$ and a frequency

$$\omega = -\frac{\partial\theta}{\partial t'} = 2K\tilde{E}ex' - \left(\beta - \frac{\alpha}{\Gamma} - \frac{1}{4}\upsilon^2\right) + e(K\tilde{E}e)^2(t')^2 -$$
$$2K\tilde{E}e\upsilon t' = 2K\tilde{E}ex' - \beta - \frac{\alpha}{\Gamma} + k^2 \quad (103)$$

The usual Hamilton equations for the superconductive electron (soliton) in the macroscopic quantum systems are still valid here and can be written as [80-81] $\frac{dk}{dt'} = -\frac{\partial\omega}{\partial x'}\Big|_k = -2K\tilde{E}e$, then the group velocity of the superconductive electron is

$$\upsilon_g = \frac{dx'}{dt'} = \frac{\partial\omega}{\partial k}\Big|_{x'} = 2\left(\frac{\upsilon}{2} - 2K\tilde{E}et'\right) = \upsilon - 4K\tilde{E}et' \quad (104)$$

This means that the frequency ω still represents the meaning of Hamiltonian in the case of nonlinear quantum systems. Hence, $\frac{d\omega}{dt'} = \frac{d\omega}{dk}\Big|_{x'}\frac{dk}{dt'} + \frac{\partial\omega}{\partial x'}\Big|_k\frac{dx'}{dt'} = 0$, as seen in the usual stationary linear medium.

These relations in Eqs. (103)-(104) show that the superconductive electrons move as if they were classical particles moving with a constant acceleration in the invariant electric-field, and that the acceleration is given by $-4K\tilde{E}e$. If $\upsilon > 0$, the soliton initially travels toward the overdense region, it then suffers a deceleration and its velocity changes sign. The soliton is then reflected and accelerated toward the underdense region.The penetration distance into the overdense region depends on the initial velocity υ.

From the above studies we see that the time-dependent motion of superconductive electrons still behaves like a soliton in non-equilibrium state of superconductor. Therefore, we can conclude that the electrons in the superconductors are essentially a soliton in both time-independent steady state and time-dependent dynamic state systems. This means that the soliton motion of the superconductive electrons causes the superconductivity of material. Then the superconductors have a complete conductivity and nonresistance property

because the solitons can move over a macroscopic distances retaining its amplitude, velocity, energy and other quasi- particle features. In such a case the motions of the electrons in the superconductors are described by a nonlinear Schrödinger equations (52), or (53) or (80) or (82) or (84). According to the soliton theory, the electrons in the superconductors are localized and have a wave-corpuscle duality due to the nonlinear interaction, which is completely different from those in the quantum mechanics. Therefore, the electrons in superconductors should be described in nonlinear quantum mechanics[16-17].

7. The transmission features of magnetic-flux lines in the Josephson junctions

7.1 The transmission equation of magnetic-flux lines

We have learned that in a homogeneous bulk superconductor, the phase $\theta(\vec{r},t)$ of the electron wave function $\phi(\vec{r},t)=f(\vec{r},t)e^{i\theta(\vec{r},t)}$ is constant, independent of position and time. However, in an inhomogeneous superconductor such as a superconductive junction discussed above, θ becomes dependent of $\vec{r}$ and t. In the previous section, we discussed the Josephson effects in the S-N-S or S-I-S, and SNIS junctions starting from the Hamiltonican and the Ginzburg-Landau equations satisfied by $\phi(\vec{r},t)$, and showed that the Josephson current, whether dc or ac, is a function of the phase change, $\varphi=\Delta\theta=\theta_1-\theta_2$. The dependence of the Josephson current on φ is clearly seen in Eq. (78) . This clearly indicates that the Josephson current is caused by the phase change of the superconductive electrons. Josephson himself derived the equations satisfied by the phase difference φ, known as the Josephson relations, through his studies on both the dc and ac Josephson effects. The Josephson relations for the Josephson effects in superconductor junctions can be summarized as the following,

$$J_s=J_m\sin\varphi,\quad \hbar\frac{\partial\varphi}{\partial t}=2eV\ ,\ \hbar\frac{\partial\varphi}{\partial x}=2ed'\overline{H}_y/c\ ,\ \hbar\frac{\partial\varphi}{\partial y}=2ed'\overline{H}_x/c \tag{105}$$

where d' is the thickness of the junction. Because the voltage V and magnetic field $\vec{H}$ are not determined, equation (105) is not a set of complete equations. Generally, these equations are solved simultaneously with the Maxwell equation $\nabla\times\vec{H}=(4\pi/c)\vec{J}$. Assuming that the magnetic field is applied in the xy plane, i.e. $\vec{H}=(\overline{H}_x,\overline{H}_y,0)$, the above Maxwell equation becomes

$$\frac{\partial}{\partial x}\overline{H}_y(x,y,t)-\frac{\partial}{\partial y}\overline{H}_x(x,y,t)=\frac{4\pi}{c}J(x,y,t) \tag{106}$$

In this case, the total current in the junction is given by $J=J_s(x,y,t)+J_n(x,y,t)+J_d(x,y,t)+J_0$

In the above equation, $J_s(x,y,t)$ is the superconductive current density, $J_n(x,y,t)$ is the normal current density in the junction (J_n =V/R(V) if the resistance in the junction is R(V) and a voltage V is applied at two ends of the junction), $J_d(x,y,t)$ is called a displacement current and it is given by $J_d=CdV(t)/dt$, where C is the capacity of the junction, and J_0 is a constant current density. Solving the equations in Eqs,(102) and (106) simultaneously, we can get

$$\nabla^2\varphi - \frac{1}{v_0^2}\left(\frac{\partial^2\varphi}{\partial t^2} - \gamma_0\frac{\partial\varphi}{\partial t}\right) = \frac{1}{\lambda_J^2}\sin\varphi + I_0 \tag{107}$$

where $v_0 = \sqrt{c^2/4\pi Cd'}, \gamma_0 = 1/RC,\ \lambda_J = \sqrt{c^2\hbar/4\pi d'e^*}, I_0 = 4e^*\pi J_0/c^2\hbar, e^* = 2e$.
Equation (107) is the equation satisfied by the phase difference. It is a Sine-Gordon equation (SGE) with a dissipative term. From Eq.(105), we see that the phase difference φ depends on the external magnetic field $\vec{H}$, thus the magnetic flux in the junction $\Phi' = \int \vec{H}ds = \oint \vec{A}\bullet d\vec{l} = \frac{c\hbar}{c^*}\oint \varphi dl$ can be specified in terms of φ, where $\vec{A}$ is vector potential of electromagnetic field, $d\vec{l}$ is line element of vortex lines. Equation (107) represents transmission of superconductive vortex lines. It is a nonlinear equation. Therefore, we know clearly that the Josephson effect and the related transmission of the vortex line, or magnetic flux, along the junctions are also nonlinear problems. The Sine-Gordon equation given above has been extensively studied by many scientists including Kivshar and Malomed[39-40]. We will solve it here using different approaches.

7.2 The transmission features of magnetic-flux lines

Assuming that the resistance R in the junction is very high, so that $J_n \to 0$, or equivivalently $\gamma_0 \to 0$, setting also $I_0 = 0$, equation (107) reduces to

$$\nabla^2\varphi - \frac{1}{v_0^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{1}{\lambda_J^2}\sin\varphi \tag{108}$$

Define $X = x/\lambda_J, T = v_0 t/\lambda_J$, then in one-dimension, the above equation becomes

$$\frac{\partial^2\varphi}{\partial X^2} - \frac{\partial^2\varphi}{\partial T^2} = \sin\varphi$$

which is the 1D Sine-Gordon equation. If we further assume that $\varphi = \varphi(X,T) = \varphi(\theta')$ with

$$\theta' = X' - X_0' - vT', X' = X/\sqrt{hc/2LI_0e}, T' = T/\sqrt{2I_0e/hc}$$

it becomes $(1-v^2)\varphi_{\theta'}^2(\theta') = 2(A' - \cos\varphi)$,where A' is a constant of integration. Thus

$$\int_{\varphi_0}^{\varphi(\theta')}[(A' - \cos\varphi)]^{-1/2}d\varphi = \sqrt{2}\delta\nu\theta'$$

where $\nu = 1/\sqrt{1-v^2}, \delta = \pm 1$. Choosing A'=1, we have

$$\int_{\pi}^{\varphi(\theta')}[\sin(\varphi/2)]^{-1/2}d\varphi = 2\nu\theta'$$

A kink soliton solution can be obtained as follows $\pm\nu\theta' = \ln[\tan(\varphi/2)], 'or$ $\varphi(\theta') = 4\tan^{-1}[\exp(\pm\nu\theta')]$. Thus yields

$$\varphi(X',T') = 4\tan^{-1}\{\exp[\delta\nu(X' - X_0' - vT')]\} \tag{109}$$

From the Josephson relations, the electric potential difference across the junction can be written as

$$V = \frac{\hbar}{2e}\frac{d\varphi}{dT'} = \frac{\varphi_0}{2c\pi}\frac{d\varphi}{dT'} = 2\delta v\nu\sqrt{\frac{2I_o e}{\hbar c^2}}\frac{\varphi_0}{2c\pi}\sec h[\nu(X'-X_0^{'}-vT')]$$

where $\varphi_0 = \pi\hbar c = 2\times 10^{-7} Gauss/cm^{-2}$ 'is a quantum fluxon, c is the speed of light. A similar expression can be derived for the magnetic field

$$\overline{H}_z = \frac{\hbar}{2e}\frac{d\varphi}{dX'} = \frac{\varphi_0}{2c\pi}\frac{d\varphi}{dX'} = \pm 2\delta\nu\sqrt{\frac{2I_o e}{\hbar c^2}}\frac{\varphi_0}{2c\pi}\sec h[\nu(X'-X_0^{'}-vT')]$$

We can then determine the magnetic flux through a junction with a length of L and a cross section of 1 cm2. The result is

$$\Phi' = \int_{-\infty}^{\infty} H_x(x,t)dx = B_0^{'}\int_{-\infty}^{\infty} H_x(X',T')dX' = \delta\varphi_0$$

Therefore, the kink ($\delta = +1$) carries a single quantum of magnetic flux in the extended Josephson junction. Such an excitation is often called a fluxon, and the Sine-Gordon equation or Eq.(107) is often referred to as transmission equation of quantum flux or fluxon. The excitation corresponding to $\delta = -1$ is called an antifluxon. Fluxon is an extremely stable formation. However, it can be easily controlled with the help of external effects. It may be used as a basic unit of information.

This result shows clearly that magnetic flux in superconductors is quantized and this is a macroscopic quantum effect as mentioned in Section 1. The transmission of the quantum magnetic flux through the superconductive junctions is described by the above nonlinear dynamic equation (107) or (108).The energy of the soliton can be determined and it is given by $E = 8m^2/\beta$, where $m^2/\beta = 1/\lambda_J^2$.

However, the boundary conditions must be considered for real superconductors. Various boundary conditions have been considered and studied. For example, we can assume the following boundary conditions for a 1D superconductor, $\varphi_x(0,t) = \varphi_x(L,t) = 0$. Lamb[47] obtained the following soliton solution for the SG equation (108)

$$\varphi(x,t) = 4\tan^{-1}[h(x)g(t)] \tag{110}$$

where h and g are the general Jacobian elliptical functions and satisfy the following equations

$$[h(x)]^2 = a'h^4 + (1+b')h^2 - c', \qquad [g(x)]^2 = c'h^4 + b'h^2 - a'$$

with a', b', and c' being arbitrary constants. Coustabile et al. also gave the plasma oscillation, breathing oscillation and vortex line oscillation solutions for the SG equation under certain boundary conditions. All of these can be regarded as the soliton solution under the given conditions.

Solutions of Eq.(108) in two and three-dimensional cases can also be found[80-81]. In two-dimensional case, the solution is given by

$$\varphi(X,Y,T) = 4\tan^{-1}[\frac{g(X,Y,T)}{f(X,Y,T)}] \tag{111}$$

where $X = x/\lambda_J, Y = y/\lambda_J, T = v_0 t/\lambda_J$, $f = 1 + a(1,2)e^{y_1+y_2} = a(2,3)e^{y_2+y_3}a(3,1)e^{y_1+y_3}$,

$$g = e^{y_1} + e^{y_2} + a(1,2)a(2,3)a(3,1)e^{y_1+y_2+y_3}$$

and

$$y_1 = p_i X + q_i Y - \Omega_i \tau - y_i^0, p_i^2 + q_i^2 - \Omega_i^2 = 1, (i = 1,2,3)$$

$$a(i,j) = \frac{(p_i - p_j)^2 + (q_i - q_j)^2 + (\Omega_i - \Omega_j)^2}{(p_i + p_j)^2 + (q_i + q_j)^2 + (\Omega_i + \Omega_j)^2}, (1 \le i \le j \le 3)$$

In addition, P_i, q_i and Ω_i satisfy $\det\begin{vmatrix} p_1 & q_1 & \Omega_1 \\ p_2 & q_2 & \Omega_2 \\ p_3 & q_3 & \Omega 3 \end{vmatrix} = 0$. In the three-dimensional case, the solution is given by

$$\varphi(X,Y,Z,T) = 4\tan^{-1}[\frac{g(X,Y,Z,T)}{f(X,Y,Z,T)}],, \tag{112}$$

where X, Y , and T are similarly defined as in the 2D case given above, and $Z = z/\lambda_J$. The functions f and g are defined as

$$f = dX_2 e^{y_1+y_2} + dY_3 e^{y_2+y_3} + dZ_3 e^{y_1+y_3} + 1, \quad g = e^{y_1} + e^{y_2} + e^{y_3} + dX_2 dY_3 dZ_3 e^{y_1+y_2+y_3},$$

$$y_i = a_{i1}X + a_{i2}Y + a_{i3} + b_i T - C_i, {a_{i1}}^2 + {a_{i2}}^2 + {a_{i3}}^2 - b_i^2 = 1, (i = X,Y,Z)$$

with

$$d(i,j) = \frac{\sum_{k=1}^{3}[(a_{ik} - a_{jk})^2 - (b_i - b_j)^2]}{\sum_{k}^{3}[(a_{ik} + a_{jk})^2 - (b_i + b_j)^2]}, (1 \le j \le 3),$$

here y_3 is a linear combination of y_1 and y_2 , i.e., $y_3 = \alpha y_1 + \beta y_2$.
We now discuss the SG equation with a dissipative term $\gamma_0 \partial\varphi/\partial t$. First we make the following substitutions to simplify the equation

$$X = x/\lambda_J, T = v_0 t/\lambda_J = t/\omega_J, a = \gamma_0 \lambda_J^2/v_0, B' = I_0 \lambda_J^2.$$

In terms of these new parameters, the 1D SG equation (107) can be rewritten as

$$\frac{\partial^2 \varphi}{\partial X^2} - \frac{\partial^2 \varphi}{\partial T^2} - a\frac{\partial \varphi}{\partial T} = \sin\varphi + B' \tag{113}$$

The analytical solution of Eq.(113) is not easily found. Now let

$$\alpha = \frac{1-v_0^2}{a^2 v_0^2}, \eta = \frac{1}{\sqrt{\alpha}} \frac{X - v_0 T}{a v_0}, q' = \frac{a v_0}{\sqrt{1-v_0^2}}, \varphi = \pi + \varphi' \tag{114}$$

Equation (113) then becomes

$$\frac{\partial^2 \varphi}{\partial \eta^2} + q' \frac{\partial \varphi}{\partial \eta} + \sin\varphi - B' = 0 \tag{115}$$

This equation is the same as that of a pendulum being driven by a constant external moment and a frictional force which is proportional to the angular displacement. The solution of the latter is well known, generally there exists an stable soliton solution[80-81]. Let $Y = d\varphi'/d\eta$, equation (115) can be written as

$$\frac{\partial Y}{\partial \eta} + q' Y + \sin\varphi' - B' = 0 \tag{116}$$

For $0 < B' < 1$, we can let $B' = \sin\varphi_0 (0 < \varphi_0 < \pi/2)$ and $\varphi' = -\pi - \varphi_0 + \varphi_1$, then, equation (116) becomes

$$Y \frac{\partial Y}{\partial \eta} = -q' Y + \sin\varphi_0 + \sin(\varphi_1 - \varphi_0) \tag{117}$$

Expand Y as a power series of φ_1, i.e., $Y = \sum_n c_n \varphi_1^n$, and inserting it into Eq.(117), and comparing coefficients of terms of the same power of φ_1 on both sides, we get

$$c_1 = -\frac{q'}{2} \pm \sqrt{\frac{q'^2}{4} + \cos\varphi_0}, c_2 = -\frac{1}{q' + 3c_1} \frac{\sin\varphi_0}{2}, \; c_3 = \frac{1}{q' + 4c_1}(-2c_2^2 - \frac{\cos\varphi_0}{6}), c_4 = \frac{1}{q' + 5c_1}(-5c_2 c_3 - \frac{\sin\varphi_0}{24}) \tag{118}$$

and so on. Substituting these $c_n's$ into $Y = d\varphi'/d\eta = \sum_n c_n \varphi_1^2$, the solution of φ_1 may be found by integrating $\eta = \int d\varphi_1 / \sum_n c_n \varphi_1^2$. In general, this equation has soliton solution or elliptical wave solution. For example, when $d\varphi'/d\eta = c_1\varphi_1 + c_2\varphi_1^2 + c_3\varphi_1^3$ it can be found that

$$\eta = \frac{2}{\sqrt{A-C}} F(\sqrt{\frac{A-B}{A-C}}, \sin^{-1}(\sqrt{\frac{A-\varphi_1}{A-B}}))$$

where $F(k, \varphi_1)$ is the first Legendre elliptical integral, and A, B and C are constants. The inverse function φ_1 of $F(k, \varphi_1)$ is the Jacobian amplitude $\varphi_1 = amF$. Thus,

$$\sin^{-1}(\sqrt{\frac{A-\varphi_1}{A-B}}) = am\sqrt{\frac{A-C}{A-B}}\eta \quad \text{or} \quad \sqrt{\frac{A-\varphi_1}{A-B}}) = sn(\sqrt{\frac{A-C}{A-B}}\eta)$$

where snF is the Jacobian sine function. Introducing the symbol cscF = 1/snF, the solution can be written as

$$\varphi_1 = A - (A-B)[\csc(\sqrt{\frac{A-C}{A-B}}\eta)]^2 \tag{119}$$

This is a elliptic function. It can be shown that the corresponding solution at $|\eta| \to \infty$ is a solitary wave.

It can be seen from the above discussion that the quantum magnetic flux lines (vortex lines) move along a superconductive junction in the form of solitons. The transmission velocity v_0 can be obtained from $h = \alpha v_0 \sqrt{1 - v_0^2}$ and c_n in Eq. (118) and it is given by $v_0 = 1 / \sqrt{1 + [\alpha / h(\varphi_0)]^2}$.

That is, the transmission velocity of the vortex lines depends on the current I_0 injected and the characteristic decaying constant α of the Josephson junction. When α is finite, the greater the injection current I_0 is, the faster the transmission velocity will be; and when I_0 is finite, the greater the α is, the smaller the v_0 will be, which are realistic.

8. Conclusions

We here first reviewed the properties of superconductivity and macroscopic quantum effects, which are different from the microscopic quantum effects, obtained from some experiments. The macroscopic quantum effects occurred on the macroscopic scale are caused by the collective motions of microscopic particles , such as electrons in superconductors, after the symmetry of the system is broken due to nonlinear interactions. Such interactions result in Bose condensation and self-coherence of particles in these systems. Meanwhile, we also studied the properties of motion of superconductive electrons, and arrived at the soliton solutions of time-independent and time-dependent Ginzburg-Landau equation in superconductor, which are, in essence, a kind of nonlinear Schrödinger equation. These solitons, with wave-corpuscle duality, are due to the nonlinear interactions arising from the electron-phonon interaction in superconductors, in which the nonlinear interaction suppresses the dispersive effect of the kinetic energy in these dynamic equations, thus a soliton states of the superconductive electrons, which can move over a macroscopic distances retaining the energy, momuntum and other quasiparticle properties in the systems, are formed. Meanwhile, we used these dynamic equations and their soliton solutions to obtain, and explain, these macroscopic quantum effects and superconductivity of the systems. Effects such as quantization of magnetic flux in superconductors and the Josephson effect of superconductivity junctions,thus we concluded that the superconductivity and macroscopic quantum effects are a kind of nonlinear quantum effects and arise from the soliton motions of superconductive electrons. This shows clearly that studying the essences of macroscopic quantum effects and properties of motion of microscopic particles in the superconductors has important significance of physics.

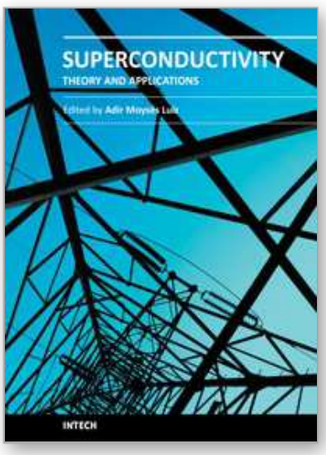
9. References

[1] Parks, R. D., Superconductivity, Marcel. Dekker, 1969.
[2] Rogovin, D. and M. Scully, Superconductiviand macroscopic quantum phenomena, Phys. Rep. 25(1976) 178.
[3] Rogovin, D., Electrodynamics of Josephson junctions，Phys. Rev. B11 (1975) 1906-108
[4] Abrikosov, A. A. and L. P. Gorkov, I.V. Dzyaloshinkii, Quantum field theoretical mothods in statistic phyics, Pregamon Press, Oxfordea, 1965
[5] Rogovin, D., Josephson tunneling: An example of steady-state superradiance,Phys. Rev. B12 (1975) 130-133.

[6] Ginzburg V L, Superconductivity, Superdiamagnetism, Superfluidity , Moscow: MIR Publ., 1987
[7] Ginzburg V L, Superconductivity, Moscow-Leningrad: Izd. Moscoew, AN SSSR, 1946
[8] Ginzburg, V. L.. Superconductivity and superfluidity *Phys.-Usp.* 40 (1997) 407
[9] Leggett, A. J., Macroscopic Effect of *P*- and *T*-Nonconserving Interactions in Ferroelectrics: A Possible Experiment? Phys. Rev. Lett. 41 (1978) 586
[10] Leggett, A. J., in Percolation, Localization and Superconductivity, eds. by A. M. Goldlinan,S. A. Bvilf, Plenum Press, New York, 1984. pp. 1- 41.
[11] Leggett, A. J., Low temperature physics, Springer, Berlin, 1991, pp. 1-93;
[12] Pang Xiao-feng, Investigations of properties of motion of superconductive electrons in superconductors by nonlinear quantum mechanics, J. Electronic Science and Technology of China, 6(2)(2008)205-211
[13] Pang, Xiao-feng, macroscopic quantum effects, Chinese J. Nature, 5 (1982) 254,
[14] Pang Xiao-feng, Investigations of properties and essences of macroscopic quantum effects in superconductors by nonlinear quantum mechanics, Nature Sciences, 2(1)(2007) 42
[15] Pang, Xiao-feng, Investigation of solutions of a time-dependent Ginzburg-Landau equation in superconductor by nonlinear quantumtheory, IEEE Compendex, 2009, 274-277, DOI: 10.1109/ ASEMD.2009.5306641(EI)
[16] Pang, Xiao-feng, Theory of Nonlinear Quantum Mechanics, Chongqing Press, Chongqing, 1994.p35-97
[17] Pang Xiao-feng Nonlinear Quantum Mechanics, Beijing, Chinese Electronic Industry Press, Beijing, 2009,p20-63
[18] Bardeen, L. N., L. N. Cooper and J. R. Schrieffer, Superconductivity theory, Phys. Rev. 108 (1957) 1175;
[19] Cooper, L. N., The bound electronic pairs in degenerated Fermi gas Phys. Rev. 104 (1956) 1189.
[20] Schrieffer, J. R., Superconductivity, Benjamin, New York, 1969.
[21] Schrieffer, J. R., Theory of Superconductivity, Benjamin, New York, 1964.
[22] Frohlich.H., Theory of superconductive states, Phys. Rev.79(1950)845;
[23] Frohlich.H., On superconductivity theory : one-dimensional case, Proc. Roy.Soc.A, 223(1954)296
[24] Josephson, B.D, Possible New Effects in Superconducting Tunnelling , Phys. Lett. 1 (1962) 251
[25] Josephson, B.D, Supercurrents through barriers, Adv. Phys. 14 (1965) 419.
[26] Josephson, B.D., Thesis, unpublised, Cambridge University (1964)
[27] Pang, Xiao-feng, The relation between the physical parameters and effective spectrum of phonon, Southwest Inst. For Nationalities, . 17 (1991) 1.
[28] Pang, Xiao-feng, On the solutions of the time-dependent Ginzburg-Landau equations for a superconductor in a weak field, J. Low Temp. Physics, 58(1985)333
[29] Pang, Xiao-feng, The isotope effects of superconductor, Chinese. J. Low Temp. Supercond.,No. 3 (1982) 62
[30] Pang, Xiao-feng, The properties of soliton motion for superconductivity electrons. Proc. ICNP, Shanghai, 1989, p139.
[31] Rayfield, G. W. and F. Reif, Evidence for the creation and motion of quantized vortex rings in superfluid helium, Phys. Rev. Lett. 11 (1963) 305.
[32] Perring, J. K., and T.H.R.Skyrme, A model unified field equation,Nucl. Phys. 31 (1962) 550.

[33] Barenghi, C. F., R. J. Donnerlly and W. F.Vinen, Quantized Vortex Dynamics and Superfluid Turbulence, Springer, Berlin, 2001.
[34] Bogoliubov, N. N., Quantum statistics, Nauka, Moscow, 1949
[35] Bogoliubov, N. N., V. V. Toimachev and D. V. Shirkov, A New Method in the Theory of Superconductivity, AN SSSR, Moscow, 1958.
[36] London, F., superfluids Vol.1, Weley, New York 1950.
[37] de Gennes, P. G., Superconductivity of Metals and Alloys, W. A. Benjamin, New York, 1966.
[38] Suint-James, D., et al., Type-II Superconductivity, Pergamon, Oxford, 1966.
[39] Kivshar, Yu. S. and B. A. Malomed, Dynamics of solitons in nearly integrable systems，Rev. Mod. Phys. 61 (1989) 763.
[40] Kivshar, Yu. S., T. J. Alexander and S. K. Turitsy, Nonlinear modes of a macroscopic quantum oscillator， Phys. Lett. A 278(2001) 225.
[41] Bullough, R. K., N. M. Bogolyubov, V. S. Kapitonov, C. Malyshev, J. Timonen, A. V. J.Rybin, A.V., Vazugin, G.G. and Lindberg, M, Quantum integrable and nonintegrable nonlinear Schrodinger models for realizable Bose-Einstein condensation in d+1 dimensions (d=1,2,3)， Theor. Math.Phys. 134(2003)47
[42] Bullough, R. K. and P. T. Caudeey, Solitons, Plenum Press, New York, 1980.
[43] Huepe, C. and M. E. Brachet, *Scaling laws for vortical nucleation solutions in a model of superflow*，Physica D 140 (2000) 126.
[44] Sonin, E. B., Nucleation and creep of vortices in superfluids and clean superconductors,Physica，B210（1995）234-250
[45] Davydov.A.S. and V.N.Ermakov, Stability of a Superconducting Condensate of Bisolitons, Phys. Stat.Sol.B148(1988)305
[46] Landau, L. D. and E. M. Lifshitz, Quantum mechanics, Pergamon Press, Oxford, 1987
[47] Lamb, G. L., Analytical descriptions of ultrashort optical pulse propagation in a resonant medium，Rev. Mod. Phys. 43 (1971) 99.
[48] Ginzberg, V. L. and L. D. Landau, On the theory of superconductivity，Zh. Eksp. Theor. Fiz. 20 (1950) 1064;
[49] Ginzburg V.L., On superconductivity and superfluidity, Physics –Usp. 47 (2004) 1155 - 1170
[50] Ginzberg, V. L. and D. A. Kirahnits, Problems in High-Temperature Superconductivity, Nauka, Moscow, 1977.
[51] Gorkov, L. P., On the energy spectrum of superconductors, Sov. Phys. JETP 7(1958) 505.
[52] Gorkov, L. P., Microscopic derivation of the Ginzburg-Landau equation in the theory of superconductivity, Sov. Phys. JETP 9 (1959) 1364.
[53] Abrikosov, A. A., On the magnetic properties of superconductors of the second group，Zh. Eksp. Theor. Fiz. 32 (1957) 1442.
[54] Abrikosov, A. A. and L. P. Gorkov, Zh. Eksp. Theor. Phys. 39 (1960) 781;
[55] Valatin. J.G. , *Comments on the theory of superconductivity,* Nuovo Cimento7(1958)843
[56] Liu, W. S. and X. P. Li, BCS states as squeezed fermion-pair states, European Phys. J. D2 (1998) 1.
[57] Gross, E. F.,structure of a quantized vertex in boson systems, II Nuovo Cimento, 20 (1961) 454.
[58] Pitaevskii, L. P.. Vortex lines in an imperfect Bose gas. Sov. Phys. JETP-USSR, 13 (1961)451
[59] Pitaevskii, L. P. and Stringari, S. *Bose–Einstein Condensation*. Oxford, Clarendon Press, 2003

[60] Elyutin P. V. and A. N. Rogovenko, Stimulated transitions between the self-trapped states of the nonlinear Schrödinger equation, Phys. Rev. E63 (2001) 026610.
[61] Elyutin, P. V., Buryak A V, Gubernov V V, Sammut R A and Towers I N, Interaction of two one-dimensional Bose-Einstein solitons: Chaos and energy exchange, Phys. Rev. E64 (2001) 016607.
[62] Pang, Xiao-feng, The properties of motion of superconductive electrons in superconductor, J. XinJiang Univ. (nature) 5(1988)33
[63] Pang Xiao-feng, The G-L theory of superconductivity bin magnetic-superconductor, Investigations of metal materials, 12(1986) 31
[64] Perez-Garcia, V. M., M. Michinel and H. Herrero, Bose-Einstein solitons in highly asymmetric traps, Phys. Rev. A57 (1998) 3837.
[65] London, F. and H. London, The electromagnetic equations of the superconductor, Proc. Roy. Soc. (London) A 149 (1935) 71
[66] Pang, Xiao-feng, Interpretation of proximity effect in superconductive junctions by G-L theory, J. Kunming Tech. Sci. Univ., 14. (1989) 78
[67] Pang, Xiao-feng, Properties of transmission of vortex lines along the superconductive junctions. J. Kunming Tech. Sci. Univ., 14 (1989) 83.
[68] Caradoc-Davies, B. M., R. J. Ballagh and K. Bumett, Coherent Dynamics of Vortex Formation in Trapped Bose-Einstein Condensates, Phys. Rev. Lett. 83 (1999) 895.
[69] Matthews, M. R., B. P. Anderson*, P. C. Haljan, D. S. Hall†, C. E. Wieman, and E. A. Cornell, Vortices in a Bose-Einstein Condensate, Phys. Rev. Lett. 83 (1999) 2498.
[70] Madison, K. W., F. Chevy, W. Wohlleben, and J. Dalibard , Vortex formation in a stirred Bose-Einstein condensate, Phys. Rev. Lett. 84 (2000) 806.
[71] Tonomura A, Kasai H, Kamimura O, Matsuda T, Harada K, Yoshida T, Akashi T, Shimoyama J, Kishio K, Hanaguri T, Kitazawa K, Masui T, Tajima S, Koshizuka N, Gammel PL, Bishop D, Sasase M, Okayasu S, Observation of structures of chain vortices inside anisotropic high- Tc superconductors, Phys. Rev. Lett. 88(2002) 237001
[72] Pang, Xiao-feng, Investigation of solutions of Josephson equation in three dimension superconductive junctions, J. Chinghai Normal Univ. Sin. No. 1 (1989) 37.
[73] Blackbunu, J. A., H.J.T. Smith and N.L. Rowell, Proximity effects and the generalized Ginzburg-Landau equation, Phys. Rev. B11 (1975) 1053.
[74] Jacobson, D. A., Ginzburg-Landau equations and the Josephson effect, Phys. Rev. B8 (1965) 1066;
[75] Pang, Xiao-feng, The Bose condensation properties of superconductive states, J. Science Exploration Sin. 1(4)(1986) 70.
[76] Pang, Xiao-feng, The features of coherent state of superconductive states, J. Southwest Inst. For Nationalities Sin. 17 (1991) 18.
[77] Dewitt, B. S., Superconductors and Gravitational Drag, Phys. Rev. Lett. 16 (1966) 1092
[78] Kusayanage.E, T. Kawashima and K.Yamafuji,Flux flow in nonideal type-II superconductor, J. Phys. Soc. Japan 33 (1972) 551.
[79] Cai, S. Y. and A. Bhattacharjee, Ginzburg-Landau equation: A nonlinear model for the radiation field of a free-electron laser, Phys. Rev. A43 (1991) 6934.
[80] Pang, Xiao-feng, Soliton Physics, Press of Sichuan Sci. and Tech., Chengdu, 2003
[81] Guo Bai-lin and Pang Xiao-feng, solitons, Chinese Science Press, Beijing, 1987



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Superconductivity was discovered in 1911 by Kamerlingh Onnes. Since the discovery of an oxide superconductor with critical temperature (Tc) approximately equal to 35 K (by Bednorz and Müller 1986), there are a great number of laboratories all over the world involved in research of superconductors with high Tc values, the so-called â€œHigh-Tc superconductorsâ€. This book contains 15 chapters reporting about interesting research about theoretical and experimental aspects of superconductivity. You will find here a great number of works about theories and properties of High-Tc superconductors (materials with Tc > 30 K). In a few chapters there are also discussions concerning low-Tc superconductors (Tc < 30 K). This book will certainly encourage further experimental and theoretical research in new theories and new superconducting materials.

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