24

# Registration between Multiple Laser Scanner Data Sets

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## 1. Introduction

Laser scanners provide a three-dimensional sampled representation of the surfaces of objects with generally a very large number of points. The spatial resolution of the data is much higher than that of conventional surveying methods. Since laser scanners have a limited field of view, it is necessary to collect data from several locations in order to obtain a complete representation of an object. These data must be transformed into a common coordinate system. This procedure is called the registration of point clouds.

In terms of input data, registration methods can be classified into two categories: one is the registration of two point clouds from different scanner locations, so-called pair-wise registration (Rusinkiewicz & Levoy, 2001), and the other is simultaneous registration of multiple point clouds (Pulli, 1999; Williams & Bennamoun, 2001). However, the global registration of multiple scans is more difficult because of the large nonlinear search space and the huge number of point clouds involved.

Commercial software typically uses separately scanned markers that can be automatically identified as corresponding points. Akca (2003) uses the special targets attached onto the object(s) as landmarks and their 3-D coordinates are measured with a theodolite in a ground coordinate system before the scanning process. Radiometric and geometric information (shape, size, and planarity) are used to automatically find these targets in point clouds by using cross-correlation, the dimension test and the planarity test.

According to the automatic registration problems, several efforts have been made to avoid the use of artificial markers. One of the most popular methods is the Iterative Closest Point (ICP) algorithm developed by Besl & McKay(1992) and Chen & Medioni (1992). ICP operates two point clouds and an estimate of the aligning rigid body transform. It then iteratively refines the transform by alternating the steps of choosing corresponding points across the point clouds, and finding the best rotation and translation that minimizes an error metric based on the distance between the corresponding points. One key to this method is to have a good priori alignment. That means, for partially unorganized and overlapping points, if there is lack of good initial alignment, many other ICP variant don't work well because it becomes very hard to find corresponding points between the point clouds. Also, although a considerable amount of work on registration of point clouds from laser scanners, it is difficult to understand convergence behavior related to different starting conditions, and error metrics. Many experiment showed that the rate of convergence of ICP heavily relies on the choice of the corresponding point-pairs, and the distance function.

There are kinds of variants of ICP method to enhance the convergence behavior according to different error metrics, and point selection strategies. There are two common distance metrics. First error metric (Besl, 1992) is the Euclidian distance between the corresponding points, but it is highly time consuming due to the exhaustive search for the nearest point. Another one is the point-to-plane distance that described in (Chen,1991, Rusinkiewicz & Levoy ,2001), which uses the distance between a point and a planar approximation of the surface at the corresponding point. They found the point-to-plane distance to perform better than other distance measures. If there are a good initial position estimation and relatively low noise, ICP method with the point-to-plane metric has faster convergence than the point-to-point one. However, when those conditions cannot be guaranteed, the point-to-plane ICP is prone to fail (Gelfand et al.,2003).

The iterative closest compatible point (ICCP) algorithm has been proposed in order to reduce the search space of the ICP algorithm (Godin et al., 2001). In the ICCP algorithm, the distance minimization is performed only between the pairs of points considered compatible on basis of their viewpoint invariant attributes (curvature, color, normal vector, etc.). Invariant features coming from points, lines and other shapes and objects, moment invariants can be applied widely from classification, identification and matching tasks. Sharp(2002) presented the feature-based ICP method that also called ICP using Invariant Feature(ICPIF), which chooses nearest neighbor correspondences by a distance metric that represented a scaled sum of the positional and feature distances. Compared with traditional ICP, ICPIE converges to the goal state in fewer iterations, and doesn't need for a user supplied initial estimate. In order to overcome different point densities, noise, and partial overlap, Trummer(2009), extending an idea known form 2-dimensional affine point pattern matching, presented a non-iterative method to optimally assign invariant features that are calculated from the 3-dimentional surface without local fitting or matching. Compared to Sharp, this method doesn't use any initial solution.

As for line-based and plane-based literature, Bauer(2004) proposed a method for the coarse alignment of 3D point clouds using extracted 3D planes that they both are visible in each scan, which leads to reduce the number of unknown transform parameters from six to three. Remain unknowns can be calculated by an orthogonal rectification process and a simple 2D image matching process. Stamos and Allen (2002) illustrated partial task for range-to-range registration where conjugate 3D line features for solving the transformation parameters between scans were manually corresponded. Stamos and Leordeanu (2003) developed an automated feature-based registration algorithm which searches line and plane pairs in 3D point cloud space instead of 2D intensity image space. The pair-wise registrations generate a graph, in which the nodes are the individual scans and the edges are the transformations between the scans. Then, the graph algorithm registers each single scan related to a central pivot scan. Habib et al. (2005) utilized straight-line segments for registering LIDAR data sets and photogrammetric data sets though. Gruen and Akca (2005) developed the least squares approach tackling surface and curve matching for automatic co-registration of point clouds. Hansen (2007) presented a plane-based approach that the point clouds are first split into a regular raster and made a gradual progress for automatic registration. Jian (2005) presented a point set registration using Gaussian mixture models as a natural and simple way to represent the given point sets. Rabbani et al. (2007) integrated modeling and global registration where start out extracting geometric information for automatic detection and fitting of simple objects like planes, spheres, cylinders, and so forth, to register the scans. In Jaw(2007),an approach for registering ground-based LiDAR point clouds using overlapping scans based on 3D line features, is presented, where includes three major parts: a 3D line feature extractor, a 3D line feature matching mechanism, and a mathematical model for simultaneously registering ground-based point clouds of multi-scans on a 3D line feature basis. Flory(2010)illustrated surface fitting and registration of point clouds using approximations of the unsigned distance function to B-spline surfaces and point clouds.

Nowadays, most of the laser scanners can supply intensity information in addition to the Cartesian coordinates for each point, or an additional camera may be used to collect texture. Therefore, further extension can simultaneously match intensity information and geometry under a combined estimation model. Kang (2009) presented an approach to automatic Image-based Registration (IBR) that refers to search for corresponding points based on the projected panoramic reflectance imagery that converted from 3D point clouds. In Roth (1999), a registration method based on matching the 3-D triangles constructed by 2-D interest points that are extracted from intensity data of each range image, was presented.

## 2. Iterative Closest Point (ICP)

### 2.1 Basic Concepts of ICP

The purpose of registration is to bring multiple image of the surveyed object in to the same coordinate system so that information from different views or different sensors can be integrated. Let $f$ denote the imaging process, $I$ is the image and $O$ is the surveyed object, we can get:

$$I = f(O) \tag{1}$$

Considering the currently used surveying method, such as photogrammetry and laser scanning, we can say that $f$ is an injection. What we have known is image fragments of the real world , and the destination of registration is then fitting all those fragments together. Mathematically, it is to find the relations between those $f$ s. Concretely, the aim of registration is to find the rigid transformation T that can bring any pair of corresponding points $(p_i, q_j)$ from the surfaces of two shapes $P$ and $Q$ representing the same point of the surveyed object into coincidence. The rigid transformation $T$ is determined by

$$E(P,Q) = \iint_{\Omega} d(Tp(u,v),\ \ q(f(u,v), g(u,v)))^2 dudv = 0 \tag{2}$$

Where $d$ is a distance function used to calculate the distance between two points, assume $d$ is the Euclidian distance function, then

$$E(P,Q) = \iint_{\Omega} \left\| Tp(u,v) - q(f(u,v), g(u,v)) \right\|^2 dudv = 0 \tag{3}$$

This function is usually used as the cost function in registration algorithms. For registration of point sets with point-to-point distance, it can be represented as discrete form:

$$\forall p_i \in P, \exists q_j \in Q \mid e_i = \left\| Tp_i - q_j \right\| = 0 \tag{4}$$

From the discrete representation we can directly get the solution idea of registration problem, finding corresponding points, in fact this can be treated as an absolute orientation problem without scaling which has been well solved by Horn(1997). He used unit quaternion to represent the rotation which simplifies this problem to a linear one and reduces this problem to a eigenvector problem, the rotation is estimated from finding the largest eigenvector of a covariance matrix, then translation can be found by the difference of the masses of two point clouds. More generally we can extend this to other features such as line segments, planar patches and some even more complex basic geometric models like spheres, cylinders and toruses (Rabbani, 2007).

Registration is a chicken-and-egg problem, if correspondences are know an estimation can be obtained by minimizing the error between the correspondences, if an initial estimation is known we can generate matches by transforming one point set to the other. The first approach is usually used for coarse registration and the second is for refinement. The ICP method belongs to the second approach and it is an excellent algorithm for registration refinement.

Since the introduction of ICP method (Besl & McKay, 1992, Chen & Medioni, 1991), it has been the most popular method for alignment of various 3d shapes with different geometric representation. It is widely used for registration of point clouds, and there have been many kinds of variants of basic ICP algorithm. However the basic concepts or workflow of ICP method is the same. It starts with two meshes and an initial estimate of the aligning rigid-body transform, then it iteratively refines the transform by alternately choosing corresponding points in the meshes through the given initial estimation and finding the best translation and rotation that minimizes an error metric based on the distance between them. Intuitively the ICP method chooses the reference point p and the closest point q of surface Q as a point pair, that's why it is called iterative closest point algorithm. The basic workflow of ICP method is as below,

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**Input**: model $P$ and $Q$ with overlapping region, initial estimation of the rigid transformation $T_i$ which transform P to the coordinate system of $Q$ ;

- Select reference points $p_j$ on $P$ ;
- For every reference point $p_j$ ,transform the reference point to the coordinate system of $Q$ with T, the new point will be $p'_j = Tp_j$ ;
- For every transformed reference point $p'_j$, select the closest point $q_i$ on $Q$ which is closest to $p'_j$ ;
- Using the point pairs $(p_j, q_j)$ to calculate a new estimation of the rigid transform $T_{i+1}$
- Iteratively repeat this procedure until the difference between the two transformation $T_{i+1}$ and $T_i$ is little enough.

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Rusinkiewicz & Levoy(2001) gave a Taxonomy of ICP variants according to the methods used in the stages of the ICP method. Salvi et al.(2007) gave a survey of recent range image registration methods also include some ICP methods. Because the ICP method has to identify the closest point on a model to a reference point, and thus affect the quality of the point pairs and the error metric directly, various definitions of closest point such as point-to-point, point-to-(tangent) plane and point-to-projection were proposed for accuracy or efficiency]. As an optimization problem, the stability is very important; kinds of sampling

method, outlier detection method and robust estimate method were proposed for ICP method.
In this part, we will first discuss the usually used point-to-point, point-to-plane and point-to-projection error metric and introduce some method proposed for the improving of stability of the ICP method.

### 2.2 Closest point definition and finding

As the quality of alignment obtained by this algorithm depends heavily on choosing good pairs of corresponding points in the two datasets, one of the key problems of ICP method is how to define the closest point, it influences the convergence performance and speed and accuracy of the algorithm directly. In order to improve the ICP method various variants of ICP method have been developed with different definition of closest point. Rusinkiewicz and Levoy's survey about variants of ICP introduced a taxonomy of some of these methods. According to him, these methods can be classified into several groups based on their error metrics, including direct point-to-point method, methods using normal of the points, such as point-to-(tangent) plane methods. Additional information such as range camera information such as point-to-projection method and intensity or colors can be used to reduce the search effort of correspondent point pairs or to eliminate the ambiguities due to inadequate geometric information on the object surface . All of these methods can be accelerated with KD-Tree searching (Simon, 1996), Z-buffer (Benjemaa & Schmitt, 1999) or closest-point caching (Simon,1996, Nishino & Ikeuchi, 2002).
Besl's original ICP method aims at the registration of 3d shapes, so he proposed kinds of definition of closest point for a given point on various geometric representations. We focus on the point set representation. Let $Q$ be a point set with Nq points, d is an Euclidean distance calculator, the distance of closest point of $Q$ to $\vec{p}$ of point set $P$ equals to the distance between the closest point $\vec{q}_i$ of $Q$ to the point $\vec{p}$ ,

$$d(\vec{p},Q) = \min_{i\in\{1,2,\cdots,N_q\}} d(\vec{p},\vec{q}_i) \tag{5}$$

And the residual for each pair of points $(\vec{p}_i,\vec{q}_i)$ is

$$e_i = T\vec{p}_i - \vec{q}_i \tag{6}$$

Besl's original work used naive point-to-point error metric, adopting the Euclidian distance between corresponding points as the error metric. It belongs to explicit method which has to detect corresponding points on the other surface. And the problem is the low convergence speed of the algorithm for certain types of data sets and initial estimation.
Laser scanning sensors often combined with image sensors, it is easy to obtain both the range and color information of the surveyed object. Weik (1997) used intensity information to detect corresponding points between two range images. Godin et al.(1994) first proposed iterative closest compatible point (ICCP) method with additional intensity information, then adopted this method with invariant computation of curvatures (Godin et al., 1995), and introduced a method for the registration of attributed range images (Godin et al., 2001). He gave the distance between two attributed points in a (3+m) dimensional space with m additional attributes like color (m=3).

$$d(\vec{p},\vec{q}) = (\vec{p}_x - \vec{q}_x) + \sum_{i=1}^{m} \lambda_i (\vec{p}_a^i - \vec{q}_a^i) \tag{7}$$

Where $\vec{p}_x$ and $\vec{q}_x$ are positions of the two points, $\vec{p}_a^i$ and $\vec{q}_a^i$ are the ith components of the additional attributes. Johnson & Kang(1997) took both the distance and the intensity or color difference into account. He found the closest point in a space with six freedoms, three for coordinates in the Euclidian space, and three for the color space. The distance between two corresponding point p with position $(x_1^p, x_2^p, x_3^p)$ and color $(c_1^p, c_2^p, c_3^p)$ and $q$ with position $(x_1^q, x_2^q, x_3^q)$ and color $(c_1^q, c_2^q, c_3^q)$ is

$$d(\vec{p},\vec{q}) = [(x_1^p - x_1^q)^2 + (x_2^p - x_2^q)^2 + (x_3^p - x_3^q)^2 + \alpha_1(c_1^p - c_1^q)^2 + \alpha_2(c_2^p - c_2^q)^2 + \alpha_3(c_3^p - c_3^q)^2]^{\frac{1}{2}} \tag{8}$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ are scale factors that weigh the importance of color against the importance of shape. In order to eliminate the effect of shading which influence the intensity of the color. Johnson and Kang employ the YIQ color model which separate intensity ($Y$) from hue ($I$) and saturation($Q$) and make the scale of the Y channel one tenth the scale of the $I$ and $Q$ channels. Let $(c_1, c_2, c_3) = (y, i, q)$, then $\alpha = (\alpha_1, \alpha_2, \alpha_3) = (0.1, 1, 1)$

However the point-to-point error metric doesn't take the surface information into account, this point-point distance based error metric suffers from the inability to "slide" overlapping range images (Nishino & Ikeuchi,2002), and may converges to a local minimum with a noise data set. Pottman & Hofer(2002) did a research on squared distance function to curves and surfaces, according to their work the point-to-point distance is only good when the two surface are aligned with a long distance after the initial transformation. As we have mentioned, a good initial estimation is essential to avoid running into a local minimum position for ICP registration method, that means the distance between the two surfaces should be close. Pottman and Hofer indicated that the point-to-tangent plane distance is exactly the second order Taylor approximant of the distance between the two surfaces.

In order to get a better registration result, point-to-plane metric is widely used (Chen & Medioni 1991; Dorai et al.,1997). Chen and Medioni employ a line-surface intersection method which can be subsumed in point-to-(tangent) plane methods. They treat the intersection point of the line $l$ passing through the point $\vec{p}$ directing to the normal of the surface $P$ at $\vec{p}$ as the closest point.

$$l = \{\vec{a} \mid \vec{n}_p \times (\vec{p} - \vec{a})\} \tag{9}$$

The green points indicate the real position of the correspondence, the red one is the identical point obtained from corresponding method. For point-to-point metric, it is the closest point to $p$. The calculating for point-to-plane metric is much more complex, we have to get the point $q$ which is the intersection of the normal vector at p of surface $P$ and the surface $Q$, then the distance from the point $p$ to the tangent plane of $Q$ at point q as the metric. The point-to-projection metric can be get easily from the projection of point $p$ onto surface $Q$ from the scanning station of $Q$.

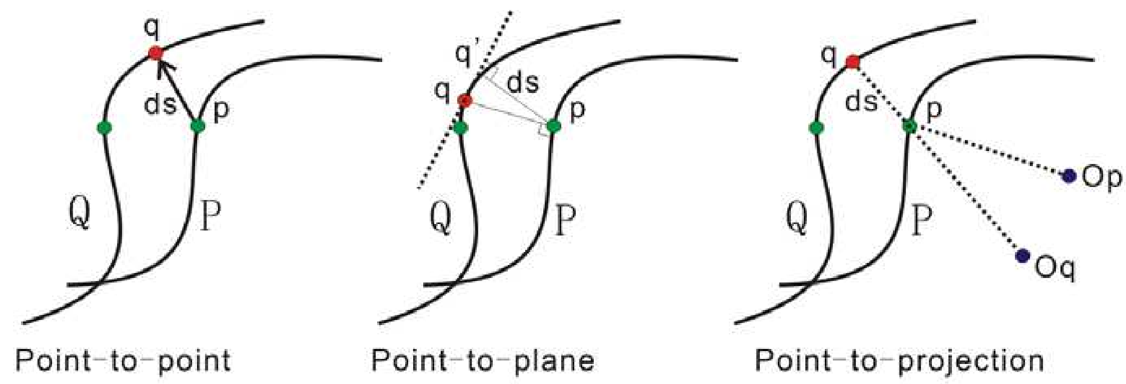


Fig. 1. From left to right are point-to-point point-to-plane and point-to-projection metric.

And the residual for each pair of points $(\vec{p}_i, \vec{q}_i)$ is

$$e_i = (T\vec{p}_i - \vec{q}_i) \cdot n_{\vec{p}_i} \tag{10}$$

By embedding the surface information into the error metric in this way, point-plane distance metric based methods tend to be robust against local minima and converge quickly with an accurate estimation. However the computation of point-plane distance is too expensive, given that we have gotten the initial estimation, we can take the advantage of the information to accelerate the closest point finding process, then point-to-projection methods using viewing direction to find the correspondence are proposed for efficiency (Benjemaa & Schmitt, 1999; Neugebauer, 1997; Blais & Levine, 1995 ).

The principal idea of Blais and Levine's points-to-projection method is to firstly project the point $\vec{p}$ backward to a 2D point $\vec{p}_Q$ on the predefined range image plane of the destination surface scanned at station $O_Q$, and then $\vec{p}_Q$ is forward projected to the destination surface $Q$ from the station $O_Q$ to get $\vec{q}$. The obvious drawback of this method is that the closest point between the two surfaces obtained can't represent the closeness of the two surfaces, consequently resulting in bad accuracy.

Park & Subbarao (2003) introduced the contractive projection point algorithm which take the advantage of both the point-to-plane metric's accuracy and point-to-projection metric's efficiency. He projects the source point to the destination surface then re-project the projection point to the normal vector of the source point, by iteratively applying the normal projection the point will converge to the intersection point of point-to-plane method.

As the Fig. 2 showed, first project $p_0$ onto a image plane viewed from the station $O_Q$, then find the intersection point $q_{p_0}$ of the line of sight from $O_Q$ to $p_0$ and the surface $Q$, and project this point onto the normal of the point $p_0$ of surface $P$ to get the point $p_1$. Iteratively do forward projection and normal projection to $p_i$ we can get the approximate correspondence of $p_0$.

Assume $M_Q$ is the perspective projection matrix of view $Q$ to the image plane $I_Q$, $T_Q$ is the transformation matrix from the camera coordinate system of view $Q$ to the world coordinate system, $I_Q$ is a 2d image plane with respect to the data set $Q$ which can be treated as a range image viewed from station $O_Q$. First back project $\vec{p}_0$ onto the image plane,

$$P_q = M_Q T_Q^{-1} \vec{p}_0 \tag{11}$$

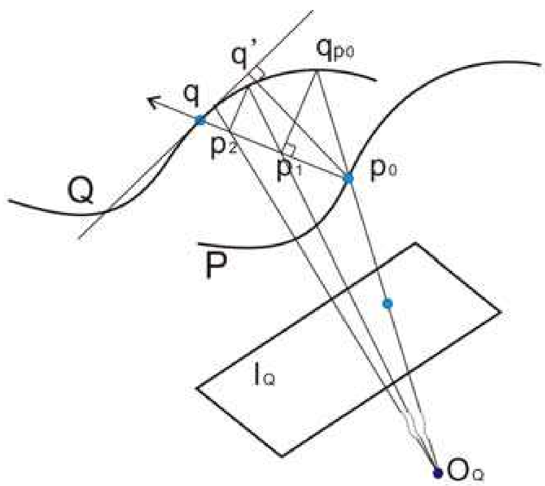


Fig. 2. Finding the closest point with the Park and Subbarao's method.

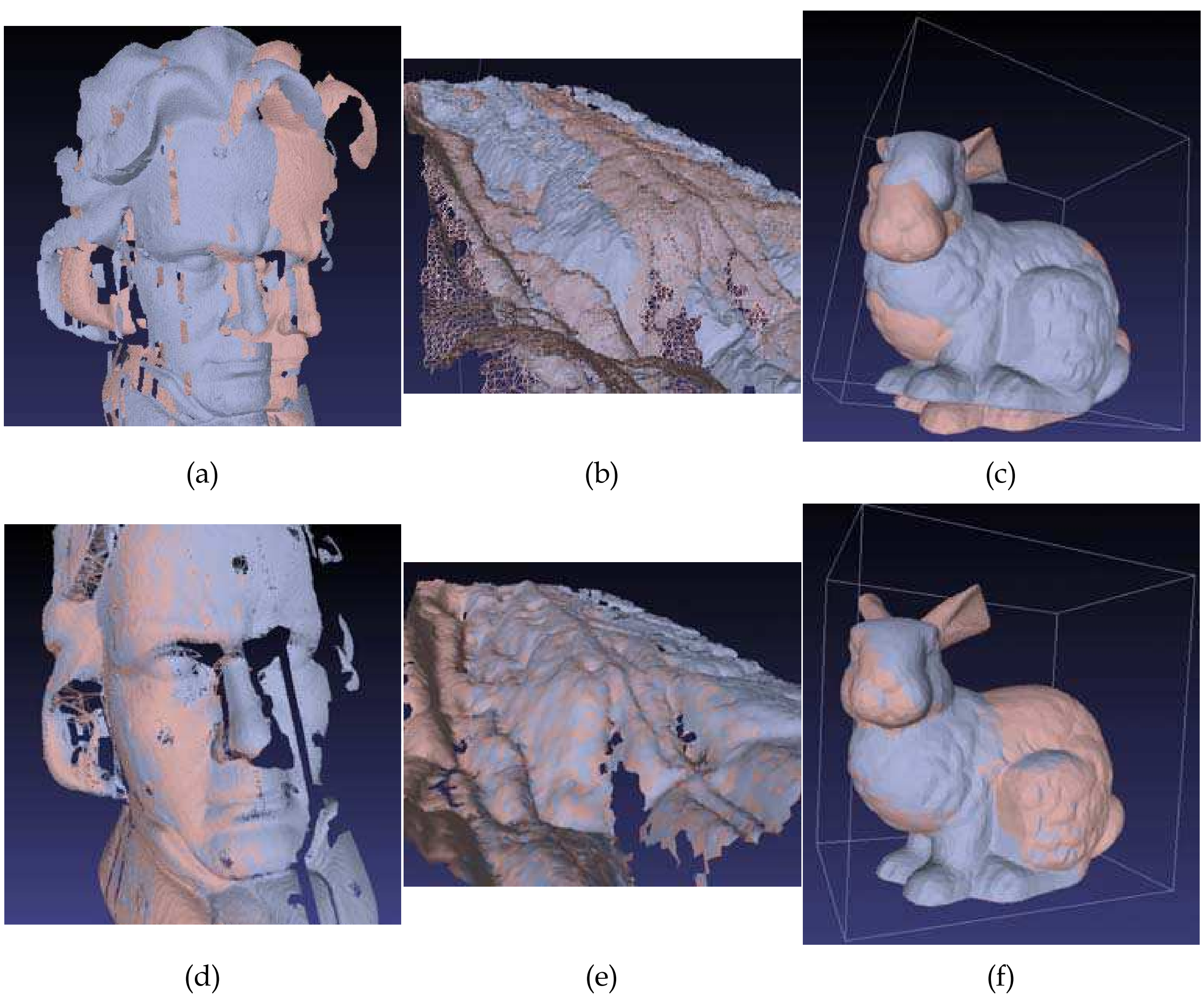


Fig. 3. The alignment with point-to-point ICP method. (a) ,(b)and (c) are initial transformed with the initial estimation of the transformed used as input in the ICP algorithm; (d) and (e) are aligned with the point-to-point ICP method, (f) is aligned with the point-to-plane ICP method.

Then forward-project the point $P_q$ onto the surface $Q$, we get point $q_{p_0}$. In fact $q_{p_0}$ is interpolated from the range image $I_Q$. Next is the so called normal projection which project the point $q_{p_0}$ onto the normal vector $\hat{p}$ at $\vec{p}_0$ to get the point $\vec{p}_1$,

$$\vec{p}_1 = \vec{p}_0 + (\vec{q}_{p_0} - \vec{p}_0)\bullet\hat{p}\bullet\hat{p} \tag{12}$$

By iterating the similar projections to $\vec{p}_1$, $\vec{p}_i$ will converge to the intersection point $\vec{q}_s$ of point-to-plane method if i goes to infinity. Experimentally this method is fast for both pair-wise registration and multi-view registration problems.

### 2.3 Techniques for stability

Though we can find corresponding points through above methods, the quality of alignment obtained by those algorithms depend heavily on choosing good pairs of corresponding points. If the input datasets are noised or bad conditioned, the correspondence selection methods and the outlier detection methods play a very important role in the registration problem. In fact correspondence selection and outlier detection essentially mean to the same thing which keep good matches and eliminate bad matches.

First, we will give some details of Gelfand et al.'s method (Gelfand et al., 2003) to analyze geometric stability of point-to-pane ICP method based on 6x6 co-variance matrixes. By decompose the rigid transformation T into rotation $R$ and translation $\vec{t}$, the residual of point-to-plane error metric can be written as

$$e_i = (T\vec{p}_i - \vec{q}_i)\cdot n_{\vec{p}_i} = (R\vec{p}_i + \vec{t} - \vec{q}_i)\cdot n_{\vec{p}_i} = (\vec{r}\times\vec{p}_i + \vec{t} - \vec{q}_i)\cdot n_{\vec{p}_i} \tag{13}$$

Where $\vec{r}$ is a $(3\times 1)$ vector of rotations around the x, y, and z axes, and $\vec{t}$ is the translation vector. Replace the functor d in formulation (1) instead of (13) then we can get

$$\begin{aligned} E &= \sum_{i=1}^{k}((R\vec{p}_i + \vec{t} - \vec{q}_i)\cdot n_i)^2 \\ &= \sum_{i=1}^{k}((\vec{p}_i - \vec{q}_i)\cdot n_i + \vec{r}\cdot(\vec{p}_i \times \vec{n}_i) + \vec{t}\cdot\vec{n}_i)^2 \end{aligned} \tag{14}$$

Given an increment $(\Delta\vec{r}^T\ \Delta\vec{t}^T)$ to the transformation vector $(\vec{r}^T\ \vec{t}^T)$, the point-to-plane distance will correspondingly changed by

$$\Delta d_i = [\Delta\vec{r}^T\ \Delta\vec{t}^T]\begin{bmatrix} p_i \times n_i \\ n_i \end{bmatrix} \tag{15}$$

By linearizing the residual functions, we will get the covariance matrix

$$C = \begin{bmatrix} p_1 n_1 & \cdots & p_k n_k \\ n1 & \cdots & nk \end{bmatrix}\begin{bmatrix} (p_1 n_1)^{\mathrm{T}} & (n_1)^T \\ \cdots & \cdots \\ (p_k n_k)^{\mathrm{T}} & (nk)^T \end{bmatrix} \tag{16}$$

Formulation (15) tells us if $p_i \times n_i$ is perpendicular to $\vec{r}$ and $n_i$ is perpendicular to $\vec{t}$, the distance to plane will always be zero, that means the error function (14) will not change.

This will cause problems when using the covariant matrix to calculate the transformation vector $(\vec{r}^T\ \vec{t}^T)$, because certain types of geometry may lead the matrix to be a singular one which means the answer will not be unique. So Gelfand et al. used the condition number of the covariance matrix as a measure of stability, and gave some simple shapes below which may cause problems.

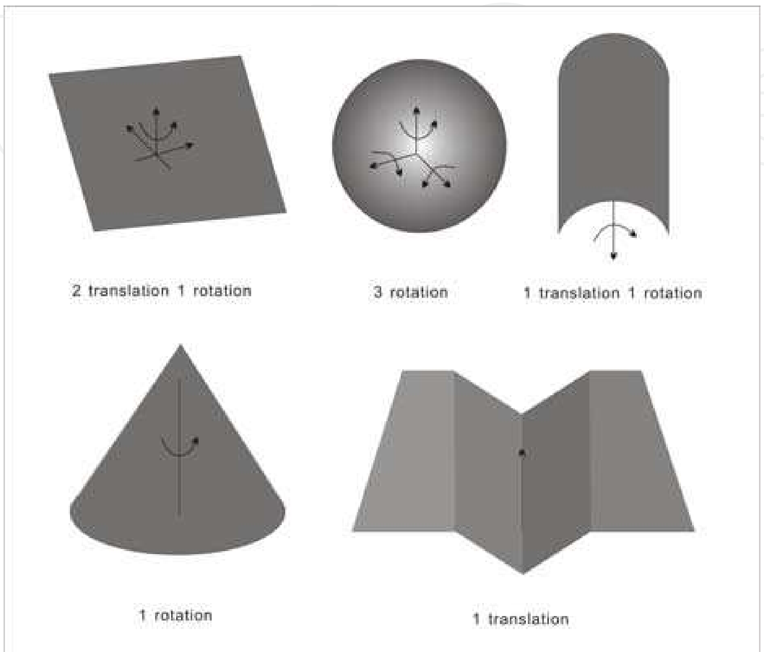


Fig. 4. Unstable shapes and the corresponding number and types of instability.

The original and early ICP methods (Besl & McKay, 1992, Chen & Medioni, 1991) usually chose all available points, some uniformly subsample the available points (Turk, 1994), others use random sampling method (Masuda et al.,1996). If too many points are chosen from featureless regions of the data, the algorithm converges slowly, finds the wrong pose, or even diverges, so color information is also used for point pairs selection (Weik, 1997). Normal-space sampling is another simple method of using surface features for selection (Johnson & Kang, 1997), it first buckets the points according to the normal vectors in angular space, then sample uniformly across the buckets. Rusinkiewicz & Levoy(2001) suggested another Normal-space sampling method which select points with large normals instead of uniformly sampling.

As sliding between datasets can be detected by analyzing the covariance matrix used for error minimization (Guehring, 2001), the adoption of weighting methods based on the contribution of point pairs to the covariance matrix is reasonable. Dorai et al(1997) improved Chen's method by weighting the residuals of those point pairs, extended it to an optimal weighted least-squares framework to weaken the contribution of the residual of outliers to the error function, as a point-to-plane method, the derivative of the standard deviation of point-to-plane distance is a good choice for the weighting factor. Simon(1997) iteratively adds and removes point-pairs to provide the best-conditioned covariance matrix, Gelfand et al. brought this method to the covariance-based sampling method. Assume $L_i$ and $x_i$ (i=1,…,6) are corresponding eigenvalues and eigenvectors of covariance matrix C, $n_{p_i}$ is the normal vector at $p_i$, and $v_i = [p_i \times n_{p_i}, n_{p_i}]$. Then, the magnitude of $v_i \cdot x_k$ represent the error the pair of points will bring in. The covariance-based sampling aims to minimize the sum of this value to all of chosen pairs over the six eigenvector. That is when choosing the nth pair we have to guarantee

$$\min\{V \mid \max\{\sum_{i=1}^{n} v_i \cdot x_k, (k = 1, \cdots, 6)\}\} \tag{17}$$

Methods with thresholds are also proposed. The distance thresholder of Schutz et al.(1998) regards the point pairs whose distance is much greater than the distance s between the centers of masses of the two point clouds as outliers, that is if $\left\| Tp_i - q_j \right\|^2 > (c \cdot s)^2$, then the point pair $(p_i, q_j)$ is an outlier, where c is an empirical threshold. Assuming the distances between corresponding point pairs are distributed as a Gaussian, Zhang(1994) introduced a statistical outlier classifier which examines the statistical distribution of unsigned point pair distances to dynamically estimate the threshold to distinguish which pairs are outliers. Dalley & Flynn (2002) did a comparison between original point-to-point ICP, point-to-point ICP with Schutz's and Zhang's classifier and point-to-plane ICP, he found that point-to-point ICP with Zhang's outlier detection method is sometimes even better than point-to-plane ICP method.

Generally the selection methods of point pairs, the outlier detection methods and threshold methods improve the stability of ICP method to certain conditions, this problem is far from well done, since no general method has been proposed to suit all of these bad conditions. The ultimate solution of these problems is to improve the quality of the datasets.

## 3. Feature based registration

The commonly used features for registration in 2D images are feature points, which are easy to find with various corner point detection methods, such as the very popular method Scale-invariant feature transform (SIFT)(Lowe, 2004). Contrarily point features of point cloud are hard to detect, instead line and plane features which can be easily extracted are widely used in feature based registration method. Line features are usually extracted from the intersection of plane features, plane features can be easily automatically extracted from point clouds with scan line or surface growing methods (Vosselman et al.,2004, Stamos, 2001). If we can found the matching between those features automatically the registration process can be automated without initial estimation or supply initial estimation for other algorithms, this is very important because fine registration methods such as ICP usually need initial estimation, if we can provide an initial transformation to the fine registration process, the whole procedure will be automated. Stamos's group used automatically extracted line features to do range-range image registration and range-2d image registration (Stamos & Leordeanu, 2003, Stamos, 2001, Chao & Stamos, 2005), they also proposed a circular feature based method (Chen & Stamos, 2006). He et al.(2005) introduced an interpretation tree based registration method with complete plane patches. Von Hansen(2007) grouped surface elements to large planes then adopted a generate-and-test strategy to find the transformation. Dold & Brenner(2006) and Pathak et al.(2010) brought geometric constraints into the matching of plane matches of the plane based registration method, and Makadia et al.(2006) proposed an extended Gaussian image based method to estimate the rotation automatically. Jaw & Chuang(2007) introduced a framework to integrate point, line and plane features together, and compared the integrated method with algorithms using such features separately, he found that the integrated method is much more stable than those separated ones. Rabbani et al.(2007) goes even far more, he integrated the modeling and registration together, simultaneously determined the shape and pose parameters of the objects as well as the registration parameters. In this chapter we will give the functional and stochastic models of point, line, and plane features based registration, and some of their properties.

### 3.1 Point features

Given a pair of point features $(\vec{p}_i, \vec{q}_i)$, Apply a rigid transformation T with a $3\times 3$ rotation $R$ and translation $\vec{t} = (t_x \ \ t_y \ \ t_z)^T$, we can get the mathematic model

$$v = R\vec{p}_i + \vec{t} - \vec{q}_i = 0 \tag{18}$$

In this model point pairs $(\vec{p}_i, \vec{q}_i)$ are measurements, $R$ and $\vec{t}$ are parameters we need to estimate, so this equation can be treated as an adjustment of condition equations with unknown parameters. To solve this problem we need at least three point pairs, one for translation the other two for rotation. Given one point pair $(\vec{p}_1, \vec{q}_1)$, we can fix the point $\vec{p}_1$ with the point $\vec{q}_1$, then the point cloud $P$ can rotate about $\vec{p}_1$, if another pair was known we $(\vec{p}_2, \vec{q}_2)$ can rotate $\vec{p}_2$ to $\vec{q}_2$, at this time the point cloud P can rotate about the line $l$ cross $\vec{p}_1$ and $\vec{p}_2$, the third pair $(\vec{p}_3, \vec{q}_3)$ will align the point cloud $P$ coincident with $Q$ by rotating about the line $l$ and fix $\vec{p}_3$ to $\vec{q}_3$.
There are various representation of rotation, such as Euler angle, $(\varphi, \omega, \kappa)$ system, roll-pitch-yaw system, rotation matrix, and quaternion system. Rotation matrix is the usually used representation for analysis, but it has nine elements, because there are only three freedoms for a rotation, there are six nonlinear conditions between the nine elements to form an orthogonal matrix which is troublesome in calculation. The first three methods represent the rotation with three angles; the difference between them is little in calculating process, so we will discuss them together as an angle system. The last quaternion method has been proven very useful in representing rotations due to several advantages above other representations; the most important one is that it varies continuously over the unit sphere without discontinuous jumps.
The point feature based method is the special case in absolute orientation without scaling, Horn [14] adopted quaternion representation of rotation into it and reduced it to a linear problem. Any rotation can be represented as a quaternion $q = [q_0 \ \ q_1 \ \ q_2 \ \ q_3]$, as rotation in $R^3$ has only three freedom, so the quaternion must be constrained by $\|q\| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1$. The relation between the quaternion and the rotation matrix is

$$R = \begin{bmatrix} 2q_0^2 + 2q_1^2 - 1 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 2q_0^2 + 2q_2^2 - 1 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 2q_0^2 + 2q_3^2 - 1 \end{bmatrix} \tag{19}$$

Horn found that the matrix

$$M = \sum_{i=1}^{n} \vec{p}_i \vec{q}_i^T = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \tag{20}$$

Whose elements are sum of product of matched point pairs contains all the information required to solve the least square problem.

Let $N = \begin{bmatrix} S_{xx}+S_{yy}+S_{zz} & S_{yz}-S_{zy} & S_{zx}-S_{xz} \\ S_{yz}-S_{zy} & S_{xx}-S_{yy}-S_{zz} & S_{xy}+S_{yx} \\ S_{xy}-S_{yx} & S_{zx}+S_{xz} & -S_{xx}+S_{yy}-S_{zz} \end{bmatrix}$, the result unit quaternion will be the eigenvector corresponding to the most positive eigenvalue. Once the rotation is solved, the translation can be solved by calculating the direction from the mass of the rotated moving point set to the mass of the fixed point set.

The linear solution doesn't take the error of measurements into account, for better accuracy we will introduce the least square method below. No matter which representation is used, by taking the first-order partial derivatives of Eq. 18 with respect to the parameters, we can generally linearize Eq. 18 then adopt a least square method to get this problem solved. Because the coordinates of the feature points are also measurements which are not accurate, taking this into consideration we can even refine them by adding them to the parameters needed refined and take the first-order partial derivatives with respect to them. The initial value of the transformation can be obtained from the linear idea introduced above, then for every pair of points the general form of the linearized functional model is

$$A_{3\times 6} V_{6\times 1} + B_{3\times 6 or 3\times 7} \hat{x}_{6\times 1 or 7\times 1} + e_{3\times 1} = 0 \tag{21}$$

where $V$ is the incremental parameter vector of the coordinates of the pair of points, $\hat{x}$ is the incremental parameter vector of the parameters of the transformation, $e$ is the error vector of every pair of points.

Given n point pairs the functional model and statistical model will be

$$A_{3n\times 6} V_{6n\times 1} + B_{3n\times 6 or 3n\times 7} \hat{x}_{6\times 1 or 7\times 1} + e_{3\times 1} = 0 \tag{22}$$

$$D_{3n\times 3n} = \sigma_0^2 Q_{3n\times 3n} = \sigma_0^2 P_{3n\times 3n}^{-1} \tag{23}$$

where $\sigma_0$ is the mean error of unit weight along x, y, z directions for each pair of points, $Q$ is the covariance matrix, $P$ can be regarded as the weight matrix of each pair of points which indicate the matching degree along the three direction, if we assume the weight of all of the directions of a pair of points $(p_i, q_i)$ is the same, e.g. $w_i$, the weight matrix for each correspondence will be $P_i = diag(w_i, w_i, w_i)$, the matrix $P$ will be

$$P = diag(P_1, P_2, \cdots, P_n) \tag{24}$$

Apply condition adjustment with unknown parameters, we can get

$$\hat{x} = -N_{bb}^{-1} B^T N_{aa}^{-1} e \quad \text{where} \quad N_{aa} = AQA^T, N_{bb} = B^T N_{aa}^{-1} B \tag{25}$$

### 3.2 Line features

Line segments are very common in survey especially man-made features, they are more obvious and the matches between line features are more intuitive. Because the limitation of the resolution of the scanner, we can't find accurate point-to-point matches in the datasets, the line feature based method gets out of the hard work of finding point-to-point match because of more accurate and robust line features. The general expression of line is the intersection of two plane, however with such an expression this problem can be extended to the plane based method, so we choose six parameters $(x_0\ y_0\ z_0\ n_x\ n_y\ n_z)^T$ to define the line, three $(n_x\ n_y\ n_z)^T$ for the unit vector of the direction which has one constraint, the other three $(x_0\ y_0\ z_0)^T$ for position of the line. Lines in $R^3$ can be represented as

$$\frac{x-x_0}{n_x}=\frac{y-y_0}{n_y}=\frac{z-z_0}{n_z},(n_x^2+n_y^2+n_z^2=1) \tag{26}$$

For each line correspondence, there are two constraints, the first one is the transformed unit direction must be the same, secondly the already known point $(x_0^P\ y_0^P\ z_0^P)^T$ on the line feature $l_P$ detected from point set $P$ should be on the corresponding line $l_Q$ extracted from point set $Q$.

$$\vec{n}_Q = R\vec{n}_P \tag{27}$$

$$\frac{x'-x_0^Q}{n_x^Q}=\frac{y'-y_0^Q}{n_y^Q}=\frac{z'-z_0^Q}{n_z^Q},\ where\ \vec{x}'_Q=\begin{bmatrix}x'\\y'\\z'\end{bmatrix}=R\vec{x}_Q+\vec{t} \tag{28}$$

As the line feature correspondences are conjugate, we can get another function

$$\frac{x''-x_0^P}{n_x^P}=\frac{y''-y_0^P}{n_y^P}=\frac{z''-z_0^P}{n_z^P},\ where\ \vec{x}''_P=\begin{bmatrix}x'\\y'\\z'\end{bmatrix}=R^T(\vec{x}_P-\vec{t}) \tag{29}$$

If we regard $\vec{n}_P$ and $\vec{n}_Q$ as points, we can estimate the rotation with Horn's method directly just like the point based method, and then the translation can be also calculated directly from Eq. 28 and Eq.29.

For iterative least square method, the linearized functional model and statistic model are

$$A_{7n\times 12}V_{12n\times 1}+B_{7n\times 6 or 7n\times 7}\hat{x}_{6\times 1 or 7\times 1}+e_{7\times 1}=0 \tag{30}$$

$$D_{7n\times 7n}=\sigma_0^2 Q_{7n\times 7n}=\sigma_0^2 P_{7n\times 7n}^{-1} \tag{31}$$

For line feature based method, we need at least 2 pairs of nondegenerate line which are not parallel to get the transformation solved, given the first line correspondence, we can fix the line of the moving point cloud to the corresponding line of the fixed point cloud, the moving point cloud can still move along this line and rotate about this line, once the other nonparallel line correspondence is given, the moving point cloud is aligned to the fixed one.

### 3.3 Plane features

Plane is the most common feature in scene, and generally used in registration, for it is much more direct and accurate than point and line features. Just like what we have mentioned above, the line features based method also leads to the plane features based, since the line features are usually extracted from the intersection of detected planes, that is to say the general expression of line feature with the intersection of two plane is more accurate.
Each plane can be defined by three points or a plane equation with four parameters. usually there are numerous point measurements on each plane, estimating the function of the plane and applying them for the registration is much more reasonable.
The function of a plane can be represented as

$$n_x x + n_y y + n_z z + d = 0 \tag{32}$$

where $(n_x, n_y, n_z)$ is the unit normal of the plane.

For each pair of plane match, $(n_x^P, n_y^P, n_z^P, d^P)$ from the moving point cloud $P$ and $(n_x^Q, n_y^Q, n_z^Q, d^Q)$ from the fixed point cloud $Q$, we can get

$$n^Q - Rn^P = 0 \tag{33}$$

$$d^P - d^Q + (Rn^P) \cdot \vec{t} = 0 \tag{34}$$

Just like the line feature based method, the rotation can be also estimated directly, to get the translation, we can bring Eq. 33 to Eq.34, then

$$d^P - d^Q + n^Q \cdot \vec{t} = 0 \tag{35}$$

Which is a linear function which can be solved directly; we can use it to get an approximate estimation of the transformation.
The linearized functional model and statistic model of plane based method are

$$A_{6n\times 8} V_{8n\times 1} + B_{6n\times 6or7n\times 7} \hat{x}_{6\times 1or7\times 1} + e_{6\times 1} = 0 \tag{36}$$

$$D_{6n\times 6n} = \sigma_0^2 Q_{6n\times 6n} = \sigma_0^2 P_{6n\times 6n}^{-1} \tag{37}$$

We need at least two nonparallel planes to estimate the rotation, and from Eq. 34 we can see that we need at least three planes in nondegenerate mode to estimate the translation. Given a plane correspondence we can fix these two planes together, then the moving point cloud can rotate about the norm of the plane and translate on the plane, once the other correspondence is given the rotation is determined, but the moving point cloud can still translate along the intersection line of the first two planes, so we need the third plane correspondence to get this resolved.

| | Measurements for each correspondence | Minimal number of correspondences | Num of functions for each correspondences |
|---|---|---|---|
| Point based | 6(3 for each point) | 3 | 3 |
| Line based | 12(6 for each line) | 2 | 7 |
| Plane based | 8(4 for eanch plane) | 3 | 6 |

Table 1. Summary and comparison of point, line and plane based registration

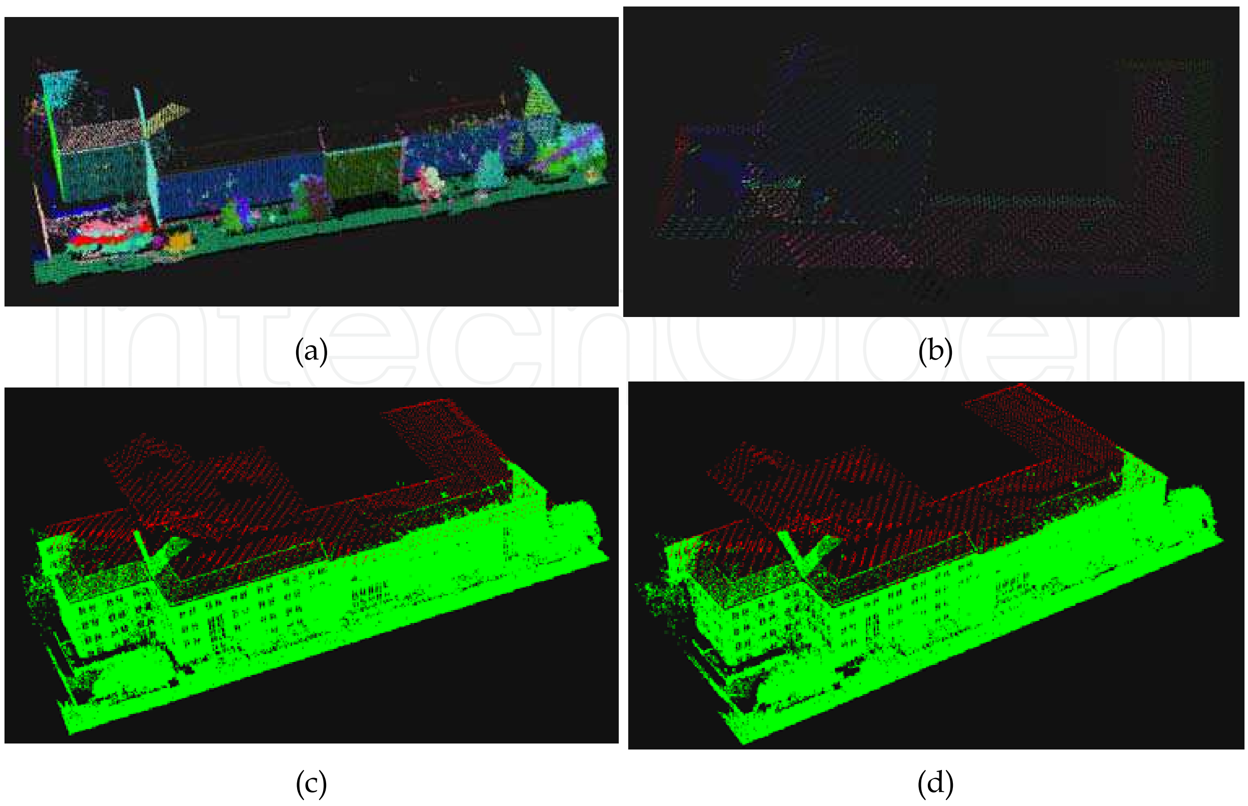


Fig. 5. The registration of ground laser based and airborne laser based point clouds of the civil building of Purdue University. (a) is the plane patches extracted from obtained by a ground laser; (b) is the plane patches extracted from obtained by a airborne laser. There is a great deal of difference of resolution between the two point cloud, the resolution of ground laser based is at least ten times higher than the airborne laser based one. (c) is the initial estimation from the approximate linear method; (d) is the refined alignment with point-to-plane ICP method.

### 3.4 Automatic matching of feature

We have introduced the basic concepts of feature based registration, however there is still a problem, which is how to automatically get corresponding features. Point features are the smallest elements in the 3D space, but they are so universal with little constraints and information that it is the very difficult to automatically find point matches. Line feature is also very difficult to find point matches, because the line are always extracted from the intersection of planes, thus the matching between line features can be deduced to plane matching problem. While planes have much information, including the plane equation and the points on this plane, so it can restrain the matching much better than point and line features.

Most of the automatic feature correspondence finding method use an exhaustively traversing method which taking all of the possible matches into consideration with some methods like geometry constraints(He et al.,2005), colors or intensity(Rabbani et al., 2007), statistic based method(Pathak et al.,2010) to cut off those obvious error matches and prune the searching space. We will generally introduce some of these methods below.

In He's work, he proposed a method with a interpretation tree and kinds of geometric constraints to generate plane correspondences. The interpretation tree includes all of the possible correspondences between two point sets. The ith layer of the interpretation tree are all possible features in point set $P$ corresponding to feature $q_i$ in point set $Q$.

The geometric constraints they introduced include area constraint, co-feature constraint and two matched feature constraint. The area constraint ensures the corresponding planes have almost the same area, and the co-feature constraint make the angle between a feature and its father is similar and ensure the difference of distance between the two features within a proper threshold. The two matched feature constraint assume the centroids of the corresponding planes are identical points, then two plane matches is enough for the registration; however it can't suit in such conditions where the planes are occluded and incomplete. But still we can generalize this constraint to three matched feature constraint according to the property described above, and stop the growing of the branch of the tree which has more than three matches.

Dold added boundary length constraint, bounding box constraint and Mean intensity value constraint to He's method. Boundary length constraint claims that the length of the boundary derived from the convex hull should be nearly the same, the ratio of the edges of the bounding box with one edge along the longest edge of the boundary is also proposed as an similarity between two corresponding planes, what's more, intensity information is also used in Dold's work.

Pathak introduced a statistic based method; the greatest difference between this method to the geometric based method above is the use of the covariance of the plane parameters and the covariance of the estimated transformation. The constraint or test used for pruning the searching space include size similarity test, given translation agreement test, odometry rotation agreement test, plane patch overlap test, cross angle test and parallel consistency test. The size similarity test is achieved by the use of the covariance matrix of the plane parameters which is proportional to the number of points in the plane. Given translation agreement test, odometry rotation agreement test need the initial value of the translation and rotation parameters which is not always available in practice, plane patch overlap test is to ensure that the estimated relative pose of the point sets have overlapped area, the cross angle test is similar with the geometric constraint which ensure the similarity of the angle between the correspondences, and the last parallel consistency test really can be classified to the cross angle test as a special situation where the cross angle should be zero. Size similarity test employs a threshold based method, each of the other tests generates a well-designed distribution, and uses an confidence interval to determine whether the correspondence is usable or not. Thorough introduction and derivation of this method can be found in Pathak's work(Pathak et al., 2010).

Makadia et al.(2006) proposed a reliable and efficient method which extended Gaussian images (EGI)(Horn, 1987). The rotation estimate is obtained by exhaustively traversing the space of rotations to find the one which maximizes the correlation between EGIs. it seems time consuming, but actually improves the efficiency by the use of the spherical harmonics of the Extended Gaussian Image and the rotational Fourier transform. Generally, the process is time-consuming, but by the use of the spherical harmonics of the Extended Gaussian Image and the rotational Fourier transform come up with by Makadia, the efficiency is improved remarkably. Once the rotation is estimated, the translation can also be estimated by maximizing the correlation between two point sets.

Another special method proposed by Aiger et al.(2008) adopted the invariability of certain ratios under affine transformation, and hence rigid transformation. This method extract all sets of coplanar 4-points from a point set that are approximately congruently related by rigid transforms to a given planar 4-points, and then test if the conjugate four-points satisfy the invariability of the such ratios.

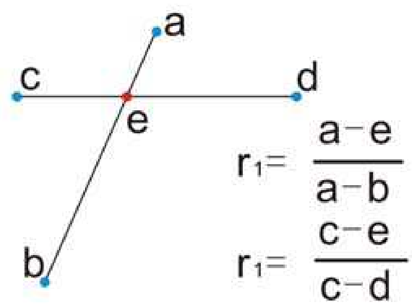


Fig. 6. e is the intermediate point e of the four-points {a,b,c,d}, the ratios $r_1$ and $r_2$ is invariant under rigid transformation.

## 4. Least square 3D surface matching

The Least Squares Matching(LSM) concept had been developed in parallel by Gruen (1984, 1985), Ackermann (1984) and Pertl (1984) for image matching which has been widely used in Photogrammetry due to its high level of flexibility, powerful mathematical model and its high accuracy. Gruen and Akca(2005) indicated that if 3D point clouds derived by any device or method represent an object surface, the problem should be defined as a surface matching problem instead of the 3D point cloud matching, and generalized the Least Squares (LS) image matching method to suit the surface matching mission. This method matches one or more 3D search surfaces to a 3D template surface through the minimizing of the sum of squares of the Euclidean distances between the surfaces. The unknown parameters are treated as stochastic quantities in this model with priori weights which can be used to eliminate gross erroneous and outliers, and this can improve the estimation very much. The unknown parameters of the transformation are estimated by the use of the Generalized Gauss–Markoff model of least squares. Because most of the point clouds in application just represent the surface of the scene, many registration missions can be treated as surface matching tasks. In this part we will give an introduction of the mathematical model of Least Square 3D surface matching which can be used for point cloud registration.

### 4.1 Mathematical model

Each surface can be represented of a function of three bases of the 3D world. For generalization the conjugate overlapping regions of the scene in the left and right surface can be represented as $f(x,y,z)$ and $g(x,y,z)$ no matter the representation of the surface is point cloud, triangle mesh, or any other methods. The mission is to make the two surfaces identical to each other, that is

$$f(x,y,z) = g(x,y,z) \tag{38}$$

Assume the errors caused by the sensor, the environment and the measure method are random errors, we can get the observation function as

$$f(x,y,z)-e(x,y,z)=g(x,y,z) \tag{39}$$

The sum of squares of the Euclidean distances between the template and the search surface elements can be used as the goal function of the least square minimization.

$$\sum\left\|\vec{d}\right\|^2=\min \tag{40}$$

Though the relation between two point clouds usually is a rigid transformation without scaling, Gruen and Akca use a seven-parameter similarity transformation to express the relationship of the conjugate surfaces. However the sole and core even the representation of this method will not change, the only difference is the parameter space. Adding a scaling parameter to the rigid transformation, the similarity transformation will be

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\vec{t}+sR\vec{x}_0=\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}+s\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \tag{41}$$

The formulation (39) can be linearized by Taylor expansion.

$$f(x,y,z)-e(x,y,z)=g^0(x,y,z)+\frac{\partial g^0(x,y,z)}{\partial x}dx+\frac{\partial g^0(x,y,z)}{\partial y}dy+\frac{\partial g^0(x,y,z)}{\partial z}dz \tag{42}$$

with
$$dx=\frac{\partial x}{\partial p}dp, dz=\frac{\partial x}{\partial p_i}dp, dz=\frac{\partial z}{\partial p}dp \tag{43}$$

Where $p=[t_x,t_y,t_z,s,\varphi,\omega,\kappa]^T$ is the parameter vector, get Eq. 41, 42 and 43 together we will obtain the linearized observation functions.

$$\begin{aligned} -e(x,y,z) &= g_x dt_x + g_y dt_y + g_y dt_y + (g_x a_{10} + g_y a_{20} + g_y a_{30})dm \\ &= (g_x a_{11} + g_y a_{21} + g_y a_{31})d\varphi + (g_x a_{12} + g_y a_{22} + g_y a_{32})d\omega \\ &= (g_x a_{13} + g_y a_{23} + g_y a_{33})d\kappa \end{aligned} \tag{44}$$

Then the general form of the observation model and the statistical model will be

$$-e=Ax-l \tag{45}$$

$$D=\sigma_0^2 Q=\sigma_0^2 P^{-1} \tag{46}$$

Until this step, we can say that it is almost the same with the point-to-point method, or point-to-point ICP because it also needs to find the correspondences with the initial estimation, and then iteratively improve the estimation of the transformation. To accelerate the searching of closest point, Akca and Gruen(2006) proposed an boxing structure, which partitions the search space into boxes, and correspondence is searched only in the box containing the point and its neighbors. What makes this method different from the ICP method is that the unknown transformation parameters are treated as stochastic quantities using proper a priori weights.

$$-e_b = Ix - l_b \tag{47}$$

$$D_b = \sigma_0^2 Q_b = \sigma_0^2 P_b^{-1} \tag{48}$$

where $I$ is an identity matrix, $l_b$ is a fictitious observation vector for the system parameters. The incremental vector and the standard deviation of the parameters can be got by

$$\hat{x} = (A^T PA + P_b)^{-1}(A^T Pl + P_b l_b) \tag{49}$$

$$\hat{\sigma}_0^2 = (v^T Pv + v_b^T P_b v_b) / r, \quad where \ v = A\hat{x} - l, v_b = I\hat{x} - l_b \tag{50}$$

Generally the contribution of this method is the generalized mathematical model which can suit to any kind of representation of the surface and the method of treating the unknown parameters as stochastic quantities which established the control of the estimating of the parameters.

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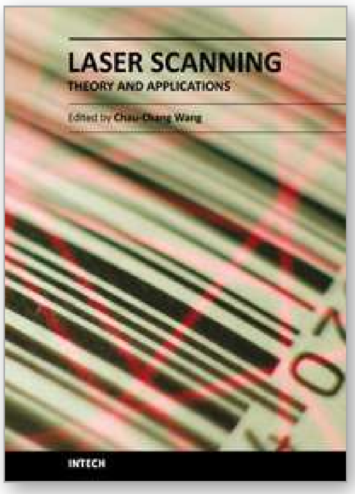


**Laser Scanning, Theory and Applications**

Edited by Prof. Chau-Chang Wang

ISBN 978-953-307-205-0
Hard cover, 566 pages
**Publisher** InTech
**Published online** 26, April, 2011
**Published in print edition** April, 2011

Ever since the invention of laser by Schawlow and Townes in 1958, various innovative ideas of laser-based applications emerge very year. At the same time, scientists and engineers keep on improving laser's power density, size, and cost which patch up the gap between theories and implementations. More importantly, our everyday life is changed and influenced by lasers even though we may not be fully aware of its existence. For example, it is there in cross-continent phone calls, price tag scanning in supermarkets, pointers in the classrooms, printers in the offices, accurate metal cutting in machine shops, etc. In this volume, we focus the recent developments related to laser scanning, a very powerful technique used in features detection and measurement. We invited researchers who do fundamental works in laser scanning theories or apply the principles of laser scanning to tackle problems encountered in medicine, geodesic survey, biology and archaeology. Twenty-eight chapters contributed by authors around the world to constitute this comprehensive book.