13

Discrete-Time Adaptive Predictive Control with Asymptotic Output Tracking

Chenguang Yang[1] and Hongbin Ma[2]
[1]*University of Plymouth*
[2]*Beijing Institute of Technology*
[1]*United Kingdom*
[2]*China*

1. Introduction

Nowadays nearly all the control algorithms are implemented digitally and consequently discrete-time systems have been receiving ever increasing attention. However, as to the development of nonlinear adaptive control methods, which are generally regarded as smart ways to deal with system uncertainties, most researches are conducted for continuous-time systems, such that it is very difficult or even impossible to directly apply many well developed methods in discrete-time systems, due to the fundamental difference between differential and difference equations for modeling continuous-time and discrete-time systems, respectively. Even some concepts for discrete-time systems have very different meaning from those for continuous-time systems, e.g., the "relative degrees" defined for continuous-time and discrete-time systems have totally different physical explanations Cabrera & Narendra (1999). Therefore, nonlinear adaptive control of discrete-time systems needs to be further investigated.

On the other hand, the early studies on adaptive control were mainly concerning on the parametric uncertainties, i.e., unknown system parameters, such that the designed control laws have limited robustness properties, where minute disturbances and the presence of nonparametric model uncertainties can lead to poor performance and even instability of the closed-loop systems Egardt (1979); Tao (2003). Subsequently, robustness in adaptive control has been the subject of much research attention for decades. However, due to the difficulties associated with discrete-time uncertain nonlinear system model, there are only limited researches on robust adaptive control to deal with nonparametric nonlinear model uncertainties in discrete-time systems. For example, in Zhang et al. (2001), parameter projection method was adopted to guarantee boundedness of parameter estimates in presence of small nonparametric uncertainties under certain wild conditions. For another example, the sliding mode method has been incorporated into discrete-time adaptive control Chen (2006). However, in contrast to continuous-time systems for which a sliding mode controller can be constructed to eliminate the effects of the general uncertain model nonlinearity, for discrete-time systems, the uncertain nonlinearity is normally required to be of small growth rate or globally bounded, but sliding mode control is yet not able to completely compensate for the effects of nonlinear uncertainties in discrete-time. As a matter of fact, unlike in continuous-time systems, it is much more difficulty in discrete-time systems to deal with nonlinear uncertainties. When the size of the uncertain nonlinearity is larger than a certain level, even a simple first-order discrete-time system cannot be globally stabilized Xie & Guo (2000). In an early work on discrete-time adaptive systems, Lee (1996) it is also pointed out that when there is large parameter time-variation, it may be impossible to construct a global stable control even for a first order system. Moreover, for discrete-time systems, most existing robust approaches only guarantee the closed-loop stability in the presence of the nonparametric model uncertainties, but are not able to improve control performance by complete compensation for the effect of uncertainties.

Towards the goal of complete compensation for the effect of nonlinear model uncertainties in discrete-time adaptive control, the methods using output information in previous steps to compensate for uncertainty at current step have been investigated in Ma et al. (2007) for first order system, and in Ge et al. (2009) for high order strict-feedback systems. We will carry forward to study adaptive control with nonparametric uncertainty compensation for *NARMA system* (nonlinear auto-regressive moving average), which comprises a general nonlinear discrete-time model structure and is one of the most frequently employed form in discrete-time modeling process.

2. Problem formulation

In this chapter, *NARMA system* to be studied is described by the following equation

$$y(k+n) = \sum_{i=1}^{n} \theta_i^T \phi_i(\underline{y}(k+n-i)) + \sum_{j=1}^{m} g_j u(k-m+j) + \nu(z(k-\tau)) \tag{1}$$

where $y(k)$ and $u(k)$ are output and input, respectively. Here

$$\underline{y}(k) = [y(k), y(k-1), \dots, y(k-n+1)]^T \tag{2}$$

$$\underline{u}(k) = [u(k-1), u(k-2), \dots, u(k-m+1)]^T \tag{3}$$

and $z(k) = [\underline{y}^T(k), \underline{u}^T(k-1)]^T$. And for $i = 1, 2, \cdots, n$, $\phi_i(\cdot) : R^n \to R^{p_i}$ are known vector-valued functions, $\theta_i^T = [\theta_{i,1}, \dots, \theta_{i,p_i}]$, and g_j are unknown parameters. And the last term $\nu(z(k-\tau))$ represents the nonlinear model uncertainties (which can be regarded as unmodeled dynamics uncertainties) with unknown time delay τ satisfying $0 \le \tau_{\min} \le \tau \le \tau_{\max}$ for known constants $\tau_{\min}$ and $\tau_{\max}$. The control objective to make sure the boundedness of all the closed-loop signals while to make the output $y(k)$ *asymptotically* track a given bounded reference $y^*(k)$.

Time delay is an active topic of research because it is frequently encountered in engineering systems to be controlled Kolmanovskii & Myshkis (1992). Of great concern is the effect of time delay on stability and asymptotic performance. For continuous-time systems with time delays, some of the useful tools in robust stability analysis have been well developed based on the Lyapunov's second method, the Lyapunov-Krasovskii theorem and the Lyapunov-Razumikhin theorem. Following its success in stability analysis, the utility of Lyapunov-Krasovskii functionals were subsequently explored in adaptive control designs for continuous-time time delayed systems Ge et al. (2003; 2004); Ge & Tee (2007); Wu (2000); Xia et al. (2009); Zhang & Ge (2007). However, in the discrete-time case there dos not exist a counterpart of Lyapunov-Krasovskii functional. To resolve the difficulties associated with unknown time delayed states and the nonparametric nonlinear uncertainties, an augmented

states vector is introduced in this work such that the effect of time delays can be canceled at the same time when the effects of nonlinear uncertainties are compensated.
In the *NARMA system* described in (1), we can see that there is a "relative degree" n which can be regarded as response delay from input to output. Thus, the control input at the kth step, $u(k)$, will actually only determine the output at n-step ahead. The n-step ahead output $y(k+n)$ also depends on the following future outputs:

$$y(k+1), y(k+2), \ldots, y(k+n-2), y(k+n-1) \tag{4}$$

and ideally the controller should also incorporate the information of these states. However, dependence on these future states will make the controller non-causal!
If system (1) is linear, e.g., there is no nonlinear functions ϕ_i, we could find a so called Diophantine function by using which system (1) can be transformed into an n-step predictor where $y(k+n)$ only depends on outputs at or before the k-th step. Then, linear adaptive control can be designed under *certainty equivalence principal* to emulate a deadbeat controller, which forces the n-step ahead future output to acquire a desired reference value. However, transformation of the nonlinear system (1) into an n-step predictor form would make the known nonlinear functions and unknown parameters entangled together and thus not identifiable. Thus, we propose *future outputs prediction*, based on which adaptive control can be designed properly.
Throughout this chapter, the following notations are used.

- $\|\cdot\|$ denotes the Euclidean norm of vectors and induced norm of matrices.
- Z_t^+ represents the set of all integers which are not less than a given integer t.
- $\mathbf{0}_{[\mathbf{q}]}$ stands for q-dimension zero vector.
- $A := B$ means that A is defined as B.
- $(\hat{\ })$ and $(\tilde{\ })$ denote the estimates of unknown parameters and estimate errors, respectively.

3. Assumptions and preliminaries

Some reasonable assumptions are made in this section on the system (1) to be studied. In addition, some useful lemmas are introduced in this section to facilitate the later control design.

Assumption 3.1. *In system (1), the functional uncertainty $\nu(\cdot)$, satisfies Lipschitz condition, i.e., $\|\nu(\varepsilon_1) - \nu(\varepsilon_2)\| \leq L_\nu \|\varepsilon_1 - \varepsilon_2\|$, $\forall \varepsilon_1, \varepsilon_2 \in R^n$, where $L_\nu < \lambda^*$ with λ^* being a small number defined in (58). The system functions $\phi_i(\cdot)$, $i = 1, 2, \ldots, n$, are also Lipschitz functions with Lipschitz coefficients L_j.*

Remark 3.1. *Any continuously derivable function is Lipschitz on a compact set, refer to Hirsch & Smale (1974) and any function with bounded derivative is globally Lipschitz. As our objective is to achieve global asymptotic stability, it is not stringent to assume that the nonlinearity is globally Lipschitz.*
In fact, Lipschitz condition is a common assumption for nonlinearity in the control community Arcak et al. (2001); Nešić & Laila (July, 2002); Nešić & Teel (2006); Sokolov (2003). In addition, it is usual in discrete-time control to assume that the uncertain nonlinearity is of small Lipschitz coefficient Chen et al. (2001); Myszkorowski (1994); Zhang et al. (2001); Zhu & Guo (2004). When the Lipschitz coefficient is large, discrete-time uncertain systems are not stabilizable as indicated in Ma (2008); Xie & Guo (2000); Zhang & Guo (2002). Actually, if the discrete-time

models are derived from continuous-time models, the growth rate of nonlinear uncertainty can always be made sufficient small by choosing sufficient small sampling time.

Assumption 3.2. *In system (1), the control gain coefficient g_m of current instant control input $u(k)$ is bounded away from zero, i.e., there is a known constant $\underline{g}_m > 0$ such that $|g_m| > \underline{g}_m$, and its sign is known a priori. Thus, without loss of generality, we assume $g_m > 0$.*

Remark 3.2. *It is called unknown control direction problem when the sign of the control gain is unknown. The unknown control direction problem of nonlinear discrete-time system has been well addressed in Ge et al. (2008); Yang et al. (2009) but it is out the scope of this chapter.*

Definition 3.1. *Chen & Narendra (2001) Let $x_1(k)$ and $x_2(k)$ be two discrete-time scalar or vector signals, $\forall k \in Z_t^+$, for any t.*

- *We denote $x_1(k) = O[x_2(k)]$, if there exist positive constants m_1, m_2 and k_0 such that $\|x_1(k)\| \leq m_1 \max_{k' \leq k} \|x_2(k')\| + m_2, \ \forall k > k_0$.*
- *We denote $x_1(k) = o[x_2(k)]$, if there exists a discrete-time function $\alpha(k)$ satisfying $\lim_{k\to\infty} \alpha(k) = 0$ and a constant k_0 such that $\|x_1(k)\| \leq \alpha(k) \max_{k' \leq k} \|x_2(k')\|$, $\forall k > k_0$.*
- *We denote $x_1(k) \sim x_2(k)$ if they satisfy $x_1(k) = O[x_2(k)]$ and $x_2(k) = O[x_1(k)]$.*

Assumption 3.3. *The input and output of system (1) satisfy*

$$u(k) = O[y(k+n)] \tag{5}$$

Assumption 3.3 implies that the system (1) is bounded-output-bounded-input (BOBI) system (or equivalently minimum phase for linear systems).
For convenience, in the followings we use $O[1]$ and $o[1]$ to denote bounded sequences and sequences converging to zero, respectively. In addition, if sequence $y(k)$ satisfies $y(k) = O[x(k)]$ or $y(k) = o[x(k)]$, then we may directly use $O[x(k)]$ or $o[x(k)]$ to denote sequence $y(k)$ for convenience.
According to Definition 3.1, we have the following proposition.

Proposition 3.1. *According to the definition on signal orders in Definition 3.1, we have following properties:*

(i) $O[x_1(k+\tau)] + O[x_1(k)] \sim O[x_1(k+\tau)]$, $\forall \tau \geq 0$.

(ii) $x_1(k+\tau) + o[x_1(k)] \sim x_1(k+\tau)$, $\forall \tau \geq 0$.

(iii) $o[x_1(k+\tau)] + o[x_1(k)] \sim o[x_1(k+\tau)]$, $\forall \tau \geq 0$.

(iv) $o[x_1(k)] + o[x_2(k)] \sim o[|x_1(k)| + |x_2(k)|]$.

(v) $o[O[x_1(k)]] \sim o[x_1(k)] + O[1]$.

(vi) If $x_1(k) \sim x_2(k)$ *and* $\lim_{k\to\infty} \|x_2(k)\| = 0$, *then* $\lim_{k\to\infty} \|x_1(k)\| = 0$.

(vii) If $x_1(k) = o[x_1(k)] + o[1]$, *then* $\lim_{k\to\infty} \|x_1(k)\| = 0$.

(viii) Let $x_2(k) = x_1(k) + o[x_1(k)]$. *If* $x_2(k) = o[1]$, *then* $\lim_{k\to\infty} \|x_1(k)\| = 0$.

Proof. See Appendix A. ■

Lemma 3.1. *Goodwin et al. (1980) (Key Technical Lemma) For some given real scalar sequences $s(k)$, $b_1(k)$, $b_2(k)$ and vector sequence $\sigma(k)$, if the following conditions hold:*

(i) $\lim_{k\to\infty} \frac{s^2(k)}{b_1(k)+b_2(k)\sigma^T(k)\sigma(k)} = 0$,

(ii) $b_1(k) = O[1]$ *and* $b_2(k) = O[1]$,

(iii) $\sigma(k) = O[s(k)]$.

Then, we have

a) $\lim_{k\to\infty} s(k) = 0$, *and b)* $\sigma(k)$ *is bounded.*

Lemma 3.2. *Define*

$$Z(k) = [z(k-\tau_{\max}), \ldots, z(k-\tau), \ldots, z(k-\tau_{\min})] \tag{6}$$

and

$$l_k = \arg\min_{l\le k-n} \|Z(k) - Z(l)\| \tag{7}$$

such that

$$Z(l_k) = [z(l_k-\tau_{\max}), \ldots, z(l_k-\tau), \ldots, z(l_k-\tau_{\min})] \tag{8}$$

and

$$\Delta Z(k) = Z(k) - Z(l_k) \tag{9}$$

Then, if $\|Z(k)\|$ *is bounded we have* $\|\Delta Z(k)\| \to 0$ *as well as* $\|\nu(z(k-\tau)) - \nu(z(l_k-\tau))\| \to 0$.
Proof. Given the definition of l_k in (7), it has been proved in Ma (2006); Xie & Guo (2000) that the boundedness of sequence $Z(k)$ leads to $\|\Delta Z(k)\| \to 0$. As $0 \le \|\nu(z(k-\tau)) - \nu(z(l_k-\tau))\| \le \|\Delta Z(k)\|$, it is obvious that $\|\nu(z(k-\tau)) - \nu(z(l_k-\tau))\| \to 0$ as $k\to\infty$. ■

According to the definition of $\Delta Z(k)$ in (9) and Assumption 3.1, we see that

$$|\nu(z(k-\tau)) - \nu(z(l_k-\tau))| \le L_\nu \|\Delta Z(k))\| \tag{10}$$

The inequality above serves as a key to compensate for the nonparametric uncertainty, which will be demonstrated later.

4. Future output prediction

In this section, an approach to predict the future outputs in (4) is developed to facilitate control design in next section. To start with, let us define an auxiliary output as

$$y_a(k+n-1) = \sum_{i=1}^{n} \theta_i^T \phi_i(\underline{y}(k+n-i)) + \nu(z(k-\tau)) \tag{11}$$

such that (1) can be rewritten as

$$y(k+n) = y_a(k+n-1) + \sum_{j=1}^{m} g_j u(k-m+j) \tag{12}$$

It is easy to show that

$$\begin{aligned} y_a(k+n-1) &= y_a(k+n-1) - y_a(l_k+n-1) + y_a(l_k+n-1) \\ &= \sum_{i=1}^{n} \theta_i^T [\phi_i(\underline{y}(k+n-i)) - \phi_i(\underline{y}(l_k+n-i))] - \sum_{j=1}^{m} g_j u(l_k-m+j) \\ &\quad + y(l_k+n) + (\nu(z(k-\tau)) - \nu(z(l_k-\tau)) \end{aligned} \tag{13}$$

For convenience, we introduce the following notations

$$\Delta\phi_i(k+n-i) = \phi_i(\underline{y}(k+n-i)) - \phi_i(\underline{y}(l_k+n-i)) \tag{14}$$

$$\Delta u(k-m+j) = u(k-m+j) - u(l_k-m+j)$$

$$\Delta\nu(k-\tau) = \nu(z(k-\tau)) - \nu(z(l_k-\tau)) \tag{15}$$

for $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$.
Combining (12) and (13), we obtain

$$y(k+n) = \sum_{i=1}^{n} \theta_i^T \Delta\phi_i(k+n-i) + \sum_{j=1}^{m} g_j \Delta u(k-m+j) + y(l_k+n) + \Delta\nu(k-\tau) \tag{16}$$

Step 1:
Denote $\hat{\theta}_i(k)$ and $\hat{g}_j(k)$ as the estimates of unknown parameters θ_i and g_j at the kth step, respectively. Then, according to (16), one-step ahead future output $y(k+1)$ can be predicted at the kth step as

$$\begin{aligned}\hat{y}(k+1|k) = &\sum_{i=1}^{n} \hat{\theta}_i^T(k-n+2)\Delta\phi_i(k-i+1) + \sum_{j=1}^{m} \hat{g}_j(k-n+2)\Delta u(k-m+j-n+1)\\ &+y(l_{k-n+1}+n)\end{aligned} \tag{17}$$

Now, based on $\hat{y}(k+1|k)$, we define

$$\Delta\hat{\phi}_1(k+1|k) = \phi_1(\underline{\hat{y}}(k+1|k)) - \phi_1(\underline{y}(l_{k-n+2}+n-1)) \tag{18}$$

which will be used in next step for prediction of two-step ahead output and where

$$\underline{\hat{y}}(k+1|k) = [\hat{y}(k+1|k), y(k), \ldots, y(k-n+2)]^T \tag{19}$$

Step 2: By using the estimates $\hat{\theta}_i$(k) and $\hat{g}_j(k)$ and according to (16), the two-step ahead future output $y(k+2)$ can be predicted at the kth step as

$$\begin{aligned}\hat{y}(k+2|k) = &\hat{\theta}_1^T(k-n+3)\Delta\hat{\phi}_1(k+1|k) + \sum_{i=2}^{n} \hat{\theta}_i^T(k-n+3)\Delta\phi_i(k-i+2)\\ &+\sum_{j=1}^{m} \hat{g}_j(k-n+3)\Delta u(k-m+j-n+2) + y(l_{k-n+2}+n)\end{aligned} \tag{20}$$

Then, by using $\hat{y}(k+1|k)$ and $\hat{y}(k+2|k)$, we define

$$\Delta\hat{\phi}_1(k+2|k) = \phi_1(\underline{\hat{y}}(k+2|k)) - \phi_1(\underline{y}(l_{k-n+3}+n-1))$$

$$\Delta\hat{\phi}_2(k+1|k) = \phi_2(\underline{\hat{y}}(k+1|k)) - \phi_2(\underline{y}(l_{k-n+3}+n-2)) \tag{21}$$

which will be used for prediction in next step and where

$$\underline{\hat{y}}(k+2|k) = [\hat{y}(k+2|k), \hat{y}(k+1|k), y(k), \ldots, y(k-n+3)]^T \tag{22}$$

Continuing the procedure above, we have three-step ahead future output prediction and so on so forth until the $(n-1)$-step ahead future output prediction as follows:
Step ($n-1$): The $(n-1)$-step ahead future output is predicted as

$$\begin{aligned}\hat{y}(k+n-1|k) = & \sum_{i=1}^{n-2} \hat{\theta}_i^T(k)\Delta\hat{\phi}_i(k+n-1-i|k) + \sum_{i=n-1}^{n} \hat{\theta}_i^T(k)\Delta\phi_i(k-(i-(n-1))) \\ & + \sum_{j=1}^{m} \hat{g}_j(k)\Delta u(k-m+j-1) + y(l_{k-1}+n)\end{aligned} \tag{23}$$

where

$$\Delta\hat{\phi}_i(k+l|k) = \phi_i(\underline{\hat{y}}(k+l|k)) - \phi_i(\underline{y}(l_{k-n+i+l}+n-i)) \tag{24}$$

for $i = 1, 2, \ldots, n-2$ and $l = 1, 2, \ldots, n-i-1$.
The prediction law of future outputs is summarized as follows:

$$\begin{aligned}\hat{y}(k+l|k) = & \sum_{i=1}^{l-1} \hat{\theta}_i^T(k-n+l+1)\Delta\hat{\phi}_i(k+l-i|k) + \sum_{i=l}^{n} \hat{\theta}_i^T(k-n+l+1)\Delta\phi_i(k-(i-l)) \\ & + \sum_{j=1}^{m} \hat{g}_j(k)\Delta u(k-m-n+l+j) + y(l_{k-n+l}+n)\end{aligned} \tag{25}$$

for $l = 1, 2, \ldots, n-1$.

Remark 4.1. *Note that $\hat{\theta}_i(k-n+l+1)$ and $\hat{g}_j(k-n+l+1)$ instead of $\hat{\theta}_i(k)$ and $g_j(k)$ are used in the prediction law of the l-step ahead future output. In this way, the parameter estimates appearing in the prediction of $\hat{y}(k+l|k)$ and $\hat{y}(k+l|k+1)$ are at the same time step, such that the analysis of prediction error will be much simplified.*

Remark 4.2. *Similar to the prediction procedure proposed in Yang et al. (2009), the future output prediction is defined in such a way that the j-step prediction is based on the previous step predictions. The prediction method Yang et al. (2009) is further developed here for the compensation of the effect of the nonlinear uncertainties $\nu(z(k-\tau))$. With the help of the introduction of previous instant l_k defined in (7), it can been seen that in the transformed system (16) that the output information at previous instants is used to compensate for the effect of nonparametric uncertainties $\nu(z(k-\tau))$ at the current instant according to (15).*

The parameter estimates in output prediction are obtained from the following update laws

$$\begin{aligned}\hat{\theta}_i(k+1) &= \hat{\theta}_i(k-n+2) - \frac{a_p(k)\gamma_p\Delta\phi_i(k-i+1)\tilde{y}(k+1|k)}{D_p(k)} \\ \hat{g}_j(k+1) &= \hat{g}_j(k-n+2) - \frac{a_p(k)\gamma_p\Delta u(k-m+j-n+1)\tilde{y}(k+1|k)}{D_p(k)} \\ & i = 1, 2, \ldots, n, \;\; j = 1, 2, \ldots, m\end{aligned} \tag{26}$$

with

$$\begin{aligned}\tilde{y}(k+1|k) &= \hat{y}(k+1|k) - y(k+1)\\ D_p(k) &= 1 + \sum_{i=1}^{n} \|\Delta\phi_i(k-i+1)\|^2 + \sum_{j=1}^{m} \Delta u^2(k-m+j-n+1) \quad (27)\\ a_p(k) &= \begin{cases} 1 - \frac{\lambda\|\Delta Z(k-n+1)\|}{|\tilde{y}(k+1|k)|}, \\ \qquad \text{if } |\tilde{y}(k+1|k)| > \lambda\|\Delta Z(k-n+1)\| \\ 0 \quad \text{otherwise} \end{cases} \quad (28)\\ \hat{\theta}_i(0) &= \mathbf{0}_{[q]},\ \hat{g}_j(0) = 0 \quad (29)\end{aligned}$$

where $0 < \gamma_p < 2$ and λ can be chosen as a constant satisfying $L_\nu \le \lambda < \lambda^*$, with λ^* defined later in (58).

Remark 4.3. *The dead zone indicator $a_p(k)$ is employed in the future output prediction above, which is motivated by the work in Chen et al. (2001). In the parameter update law (38), the dead zone implies that in the region $|\tilde{y}(k+1|k)| \le \lambda\|\Delta Z(k-n+1)\|$, the values of parameter estimates at the $(k+1)$-th step are same as those at the $(k+n-2)$-th step. While the estimate values will be updated outside of this region. The threshold of the dead zone will converge to zero because $\lim_{k\to\infty}\|\Delta Z(k-n+1)\| = 0$, which will be guaranteed by the adaptive control law designed in the next section. The similar dead zone method will also be used in the parameter update laws of the adaptive controller in the next section.*

With the future outputs predicted above, we can establish the following lemma for the prediction errors.

Lemma 4.1. *Define $\tilde{y}(k+l|k) = \hat{y}(k+l|k) - y(k+l)$, then there exist constant c_l such that*

$$|\tilde{y}(k+l|k)| = o[O[y(k+l)]] + \lambda\Delta_s(k,l), \quad l = 1,2,\ldots,n-1 \quad (30)$$

where

$$\Delta_s(k,l) = \max_{1\le k'\le l}\{\|\Delta Z(k-n+k')\|\} \quad (31)$$

Proof. See Appendix B. ■

5. Adaptive control design

By introducing the following notations

$$\begin{aligned}\bar{\theta} &= [\theta_1^T, \theta_2^T, \ldots, \theta_n^T]^T\\ \bar{\phi}(k+n-1) &= [\Delta\phi_1(\underline{y}(k+n-1)), \Delta\phi_2(\underline{y}(k+n-2)), \ldots, \Delta\phi_n(\underline{y}(k))]^T\\ \bar{g} &= [g_1, g_2, \ldots, g_m]^T\\ \bar{u}(k) &= [\Delta u_1(k-m+1), \Delta u_2(k-m+2), \ldots, \Delta u_m(k)]^T \quad (32)\end{aligned}$$

we could rewrite (16) in a compact form as follows:

$$y(k+n) = \bar{\theta}^T\bar{\phi}(k+n-1) + \bar{g}^T\bar{u}(k) + y(l_k+n) + \Delta\nu(k-\tau) \quad (33)$$

Define $\hat{\bar{\theta}}(k)$ and $\hat{\bar{g}}(k)$ as estimate of $\bar{\theta}$ and $\bar{g}$ at the kth step, respectively, and then, the controller will be designed such that

$$y^*(k+n) = \hat{\bar{\theta}}^T(k)\hat{\bar{\phi}}(k+n-1) + \hat{\bar{g}}(k)^T\bar{u}(k) + y(l_k+n) \tag{34}$$

Define the output tracking error as

$$e(k) = y(k) - y^*(k) \tag{35}$$

A proper parameter estimate law will be constructed using the following dead zone indicator which stops the update process when the tracking error is smaller than a specific value

$$a_c(k) = \begin{cases} 1 - \frac{\lambda\|\Delta Z(k-n)\| + |\beta(k-1)|}{|e(k)|}, \text{if } |e(k)| > \lambda\|\Delta Z(k-n)\| + |\beta(k-1)| \\ 0 \quad \text{otherwise} \end{cases} \tag{36}$$

where

$$\beta(k-1) = \hat{\bar{\theta}}^T(k-n)(\hat{\bar{\phi}}(k-1) - \bar{\phi}(k-1)) \tag{37}$$

and λ is same as that used in (28).
The parameter estimates in control law (34) are calculated by the following update laws:

$$\begin{aligned} \hat{\bar{\theta}}(k) &= \hat{\bar{\theta}}(k-n) + \frac{\gamma_c a_c(k)\bar{\phi}(k-1)}{D_c(k)}e(k) \\ \hat{\bar{g}}(k) &= \hat{\bar{g}}(k-n) + \frac{\gamma_c a_c(k)\bar{u}(k-n)}{D_c(k)}e(k) \end{aligned} \tag{38}$$

with

$$D_c(k) = 1 + \|\bar{\phi}(k-1)\|^2 + \|\bar{u}(k-n)\|^2 \tag{39}$$

and $0 < \gamma_c < 2$.

Remark 5.1. *To explicitly calculate the control input from (34), one can see that the estimate of g_m, $\hat{g}_m(k)$, which appears in the denominator, may lead to the so called "controller singularity" problem when the estimate $\hat{g}_m(k)$ falls into a small neighborhood of zero. To avoid the singularity problem, we may take advantage of the a priori information of the lower bound of g_m, i.e. $\underline{g}_m$, to revise the update law of $\hat{g}_m(k)$ in (38) as follows:*

$$\hat{\bar{g}}'(k) = \hat{\bar{g}}(k-n) + \frac{\gamma_c a_c(k)\bar{u}(k-n)}{D_c(k)}e(k)$$

$$\hat{\bar{g}}(k) = \begin{cases} \hat{\bar{g}}'(k), & \textit{if } \hat{g}'_m(k) > \underline{g}_m \\ \hat{\bar{g}}_r(k) & \textit{otherwise} \end{cases} \tag{40}$$

$$\tag{41}$$

where

$$\begin{aligned} \hat{\bar{g}}'(k) &= [\hat{g}'_1(k), \hat{g}'_2(k), \ldots, \hat{g}'_m(k)] \\ \hat{\bar{g}}_r(k) &= [\hat{g}'_1(k), \hat{g}'_2(k), \ldots, \underline{g}_m]^T \end{aligned} \tag{42}$$

In (40), one can see that in case where the estimate of control gain $\hat{g}_m(k)$ falls below the known lower bound, the update laws force it to be at least as large as the lower bound such that the potential singularity problem will be solved.

6. Main results and closed-loop system analysis

The performance of the adaptive controller designed above is summarized in the following theorem:

Theorem 6.1. *Under adaptive control law (34) with parameter estimation law (38) and with employment of predicted future outputs obtained in Section 4, all the closed-loop signals are guaranteed to be bounded and, in addition, the asymptotic output tracking can be achieved:*

$$\lim_{k\to\infty} |y(k) - y^*(k)| = 0 \tag{43}$$

To prove the above theorem, we proceed from the expression of output tracking error. Substitute control law (34) into the transformed system (33) and consider the definition of output tracking error in (35), then we have

$$e(k) = -\tilde{\bar{\theta}}^T(k-n)\bar{\phi}(k-1) - \tilde{\bar{g}}^T(k-n)\bar{u}(k-n) - \beta(k-1) + \Delta\nu(k-n-\tau) \tag{44}$$

where $\tilde{\bar{\theta}}(k) = \hat{\bar{\theta}}(k) - \bar{\theta}$ and $\tilde{\bar{g}}(k) = \hat{\bar{g}}(k) - \bar{g}\,\Delta\nu(k-n-\tau)$ satisfies

$$\|\Delta\nu(k-n-\tau)\| \le \lambda\|\Delta Z(k-n)\| \tag{45}$$

From the definition of dead zone indicator $a_c(k)$ in (36), we have

$$a_c(k)[|e(k)|(\lambda\|\Delta Z(k-n)\| + |\beta(k-1)|) - e^2(k)] = -a_c^2(k)e^2(k) \tag{46}$$

Let us choose a positive definite Lyapunov function candidate as

$$V_c(k) = \sum_{l=k-n+1}^{k} (\|\tilde{\bar{\theta}}(l)\|^2 + \|\tilde{\bar{g}}(l)\|^2) \tag{47}$$

and then by using (46) the first difference of the above Lyapunov function can be written as

$$\begin{aligned}
&\Delta V_c(k) = V_c(k) - V_c(k-1) \\
&\le \tilde{\bar{\theta}}^T(k)\tilde{\bar{\theta}}(k) - \tilde{\bar{\theta}}^T(k-n)\tilde{\bar{\theta}}(k-n) + \tilde{\bar{g}}^2(k) - \tilde{\bar{g}}^2(k-n) \\
&= [\|\bar{\phi}(k-1)\|^2 + \|\bar{u}(k-n)\|^2]\frac{a_c^2(k)\gamma_c^2 e^2(k)}{D_c^2(k)} \\
&\quad +[\tilde{\bar{\theta}}^T(k-n)\bar{\phi}(k-1) + \tilde{\bar{g}}^T(k-n)\bar{u}(k-n)]e(k)\frac{2a_c(k)\gamma_c}{D_c(k)} \\
&\le \frac{a_c^2(k)\gamma_c^2 e^2(k)}{D_c(k)} - \frac{2a_c(k)\gamma_c e^2(k)}{D_c(k)} \\
&\quad + \frac{2a_c(k)\gamma_c|e(k)|(\lambda\|\Delta Z(k-n)\| + |\beta(k-1)|)}{D_c(k)} \\
&\le -\frac{\gamma_c(2-\gamma_c)a_c^2(k)e^2(k)}{D_c(k)}
\end{aligned} \tag{48}$$

Noting that $0 < \gamma_c < 2$, we have the boundedness of $V_c(k)$ and consequently the boundedness of $\hat{\bar{\theta}}(k)$ and $\hat{\bar{g}}(k)$. Taking summation on both hand sides of (48), we obtain

$$\sum_{k=0}^{\infty} \gamma_c(2-\gamma_c)\frac{a_c^2(k)e^2(k)}{D_c(k)} \le V_c(0) - V_c(\infty)$$

which implies

$$\lim_{k\to\infty} \frac{a_c^2(k)e^2(k)}{D_c(k)} = 0 \tag{49}$$

Now, we will show that equation (49) results in $\lim_{k\to\infty} a_c(k)e(k) = 0$ using Lemma 3.1, the main stability analysis tool in adaptive discrete-time control. In fact, from the definition of dead zone $a_c(k)$ in (36), when $|e(k)| > \lambda\|\Delta Z(k-n)\| + |\beta(k-1)|$, we have

$$a_c(k)|e(k)| = |e(k)| - \lambda\|\Delta Z(k-n)\| - |\beta(k-1)| > 0$$

and when $|e(k)| \leq \lambda\|\Delta z(k-n)\| + |\beta(k-1)|$, we have

$$a_c(k)|e(k)| = 0 \geq |e(k)| - \lambda\|\Delta Z(k-n)\| - |\beta(k-1)|$$

Thus, we always have

$$|e(k)| - \lambda\|\Delta Z(k-n)\| - |\beta(k-1)| \leq a_c(k)|e(k)| \tag{50}$$

Considering the definition of $\beta(k-1)$ in (37) and the boundedness of $\hat{\hat{\theta}}(k)$, we obtain that $\beta(k-1) = o[O[y(k)]]$.
Since $y(k) \sim e(k)$, we have $\beta(k-1) = o[O[e(k)]]$. According to the Proposition 3.1, we have

$$\begin{aligned}
|y(k)| &\leq C_1 \max_{k'\leq k}\{|e(k')|\} + C_2 \\
&\leq C_1 \max_{k'\leq k}\{|e(k')| - \lambda\|\Delta Z(k'-n)\| - |\beta(k'-1)| \\
&\quad +\lambda\|\Delta Z(k'-n)\| + |\beta(k'-1)|\} + C_2 \\
&\leq C_1 \max_{k'\leq k}\{a_c(k')|e(k')|\} + \lambda C_1 \max_{k'\leq k}\{\|\Delta Z(k'-n)\|\} + C_1 \max_{k'\leq k}\{|\beta(k'-1)|\} \\
&\quad +C_2,\ \forall k \in Z_{-n}^{+}
\end{aligned} \tag{51}$$

According to Lemma 4.1 and Assumption 3.1, there exits a constant c_β such that

$$|\beta(k+n-1)| \leq o[O[y(k+n-1)]] + \lambda c_\beta \Delta_s(k, n-1) \tag{52}$$

Considering $\Delta Z(k)$ defined in (9) and $\Delta_s(k,m)$ defined in (31), Lemma (3.3), and noting the fact $l_k \leq k-n$, there exist constants $c_{z,1}$, $c_{z,2}$, $c_{s,1}$ and $c_{s,2}$ such that

$$\Delta Z(k-n) \leq c_{z,1} \max_{k'\leq k}\{|y(k')|\} + c_{z,2} \tag{53}$$

$$\begin{aligned}
\Delta_s(k, n-1) &= \max_{1\leq k'\leq n-1}\{\|Z(k-n+k') - Z(l_{k-n+k'})\|\} \\
&\leq c_{s,1} \max_{k'\leq k}\{|y(k'+n-1)|\} + c_{s,2}
\end{aligned} \tag{54}$$

According to the definition of $o[\cdot]$ in Definition 3.1, and (52), (54), it is clear that $\forall k \in Z_{-n}^{+}$

$$\begin{aligned}
|\beta(k+n-1)| &\leq o[O[y(k+n-1)]] + \lambda c_\beta \Delta_s(k, n-1) \\
&\leq (\alpha(k)c_{\beta,1} + \lambda c_\beta c_{s,1}) \max_{k'\leq k}\{|y(k'+n-1)|\} + \alpha(k)c_{\beta,2} + \lambda c_\beta c_{s,2}
\end{aligned} \tag{55}$$

where $\lim_{k\to\infty}\alpha(k)=0$, and $c_{\beta,1}$ and $c_{\beta,2}$ are positive constants. Since $\lim_{k\to\infty}\alpha(k)=0$, for any given arbitrary small positive constant ϵ_1, there exists a constants k_1 such that $\alpha(k)\le\epsilon_1$, $\forall k>k_1$. Thus, it is clear that

$$|\beta(k+n-1)|\le(\epsilon_1 c_{\beta,1}+\lambda c_\beta c_{s,1})\max_{k'\le k}\{|y(k'+n-1)|\}+\epsilon_1 c_{\beta,2}+\lambda c_\beta c_{s,2},\ \forall k>k_1 \quad (56)$$

From inequalities (51), (53), and (56), it is clear that there exist an arbitrary small positive constant ϵ_2 and constants C_3 and C_4 such that

$$\max_{k'\le k}\{|y(k')|\}\le C_1\max_{k'\le k}\{a_c(k')|e(k')|\}+(\lambda C_3+\epsilon_2)\max_{k'\le k}\{|y(k')|\}+C_4,\ k>k_1 \quad (57)$$

which implies the existence of a small positive constant

$$\lambda^*=\frac{1-\epsilon_2}{C_3} \quad (58)$$

such that

$$\max_{k'\le k}\{|y(k')|\}\le\frac{C_1}{1-\lambda C_3-\epsilon_2}\max_{k'\le k}\{a_c(k')|e(k')|\}+\frac{C_4}{1-\lambda C_3-\epsilon_2},\ k>k_1 \quad (59)$$

holds for all $\lambda<\lambda^*$, where $C_3=(\bar{c}_c c_{z,1}+c_\beta c_{s,1})C_1$, $\epsilon_2=\epsilon_1 c_{\beta,1}C_1$ and $C_4=C_2+\epsilon_1 c_{\beta,2}C_1+\lambda\bar{c}_c c_{z,2}C_1+\lambda c_\beta c_{s,2}C_1$. Note that inequality (59) implies $y(k)=O[a_c(k)e(k)]$. From $\bar{\phi}(y(k+n-1))$ defined in (32) and Assumption 3.1, it can be seen that $\bar{\phi}(y(k-1))=O[y(k-1)]$. According to the definition of $D_c(k)$ in (39), $y(k)\sim e(k)$, $l_{k-n}\le k-2n$, the boundedness of $y^*(k)$, and (53), we have

$$\begin{aligned}D_c^{\frac{1}{2}}(k)&\le 1+\|\bar{\phi}(k-1)\|+|\bar{u}(k-n)|\\&=O[y(k)]=O[a_c(k)e(k)]\end{aligned}$$

Then, applying Lemma 3.1 to (49) yields

$$\lim_{k\to\infty}a_c(k)e(k)=0 \quad (60)$$

From (59) and (60), we can see that the boundedness of $y(k)$ is guaranteed. It follows that tracking error $e(k)$ is bounded, and the boundedness of $u(k)$ and $z(k)$ in (75) can be obtained from (5) in Lemma 3.3, and thus all the signals in the closed-loop system are bounded. Due to the boundedness of $z(k)$, by Lemma 3.2, we have

$$\lim_{k\to\infty}\|\Delta Z(k)\|=0 \quad (61)$$

which further leads to

$$\lim_{k\to\infty}\|\Delta_s(k,n-1)\|=0 \quad (62)$$

Next, we will show that $\lim_{k\to\infty}a_c(k)e(k)=0$ implies $\lim_{k\to\infty}e(k)=0$. In fact, considering (52) and noting that $y(k)\sim e(k)\sim e(k)$, it follows that

$$|\beta(k-1)|\le o[O[e(k)]]+\lambda c_\beta\Delta_s(k-n,n-1) \quad (63)$$

which yields

$$|e(k)| - |\beta(k-1)| + \lambda c_\beta \Delta_s(k-n, n-1) \geq |e(k)| - o[O[e(k)]] \\ \geq (1-\alpha(k)m_1)|e(k)| - \alpha(k)m_2 \qquad (64)$$

according to Definition 3.1, where m_1 and m_2 are positive constants, and $\lim_{k\to\infty}\alpha(k) = 0$. Since $\lim_{k\to\infty}\alpha(k) = 0$, there exists a constant k_2 such that $\alpha(k) \leq 1/m_1$, $\forall k > k_2$. Therefore, it can be seen from (64) that

$$|e(k)| - |\beta(k-1)| + \lambda c_\beta \Delta_s(k-n, n-1) + \alpha(k)m_2 \geq (1-\alpha(k)m_1)|e(k)| \geq 0, \ \forall k > k_2 \qquad (65)$$

From (50), it is clear that

$$|e(k)| - |\beta(k-1)| + \lambda c_\beta \Delta_s(k-n, n-1) + \alpha(k)m_2 \\ \leq a_c(k)|e(k)| + \lambda\|\Delta Z(k-n)\| + \lambda c_\beta \Delta_s(k-n, n-1) + \alpha(k)m_2 \qquad (66)$$

which implies that $\lim_{k\to\infty} e(k) = 0$ according to (60)-(62), and (65), which further yields $\lim_{k\to\infty} e(k) = 0$ because of $e(k) \sim e(k)$. This completes the proof. ■

Remark 6.1. *The underlying reason that the asymptotic tracking performance is achieved lies in that the uncertain nonlinear term $\nu(k-n-\tau)$ in the closed-loop tracking error dynamics (44) will converge to zero because $\lim_{k\to\infty}\|\Delta Z(k)\| = 0$ as shown in (61).*

7. Further discussion on output-feedback systems

In this section, we will make some discussions on the application of control design technique developed before to nonlinear system in lower triangular form. The research interest of lower triangular form systems lies in the fact that a large class of nonlinear systems can be transformed into strict-feedback form or output-feedback form, where the unknown parameters appear linearly in the system equations, via a global parameter-independent diffeomorphism. In a seminal work Kanellakopoulos et al. (1991), it is proved that a class of continuous nonlinear systems can be transformed to lower triangular parameter-strict-feedback form via parameter-independent diffeomorphisms. A similar result is obtained for a class of discrete-time systems Yeh & Kokotovic (1995), in which the geometric conditions for the systems transformable to the form are given and then the discrete-time backstepping design is proposed. More general strict-feedback system with unknown control gains was first studied for continuous-time systems Ye & Jiang (1998), in which it is indicated that a class of nonlinear triangular systems T_{1S} proposed in Seto et al. (1994) is transformable to this form. The discrete-time counterpart system was then studied in Ge et al. (2008), in which discrete Nussbaum gain was exploited to solve the unknown control direction problem. In addition to strict-feedback form systems, output-feedback systems as another kind of lower-triangular form systems have also received much research attention. The discrete-time output-feedback form systems have been studied in Zhao & Kanellakopoulos (2002), in which a set of parameter estimation algorithm using orthogonal projection is proposed and it guarantees the convergence of estimated parameters to their true values in finite steps. In Yang et al. (2009), adaptive control solving the unknown control direction problem has been developed for the discrete-time output-feedback form systems.
As mentioned in Section 1, NARMA model is one of the most popular representations of nonlinear discrete-time systemsLeontaritis & Billings (1985). In the following, we are going to

show that the discrete-time output-feedback forms systems are transformable to the NARMA systems in the form of (1) so that the control design in this chapter is also applicable to the systems in the output-feedback form as below:

$$\begin{cases} x_i(k+1) = \theta_i^T\phi_i(x_1(k)) + g_i x_{i+1}(k) + v_i(x_1(k)),\ i = 1,2,\ldots,n-1 \\ x_n(k+1) = \theta_n^T\phi_n(x_1(k)) + g_n u(k) + v_n(x_1(k)) \\ y(k) = x_1(k) \end{cases} \tag{67}$$

where $x_i(k) \in R$, $i = 1,2,\ldots,n$ are the system states, $n \geq 1$ is system order; $u(k) \in R$, $y(k) \in R$ is the system input and output, respectively; θ_i are the vectors of unknown constant parameters; $g_i \in R$ are unknown control gains and $g_i \neq 0$; $\phi_i(\cdot)$, are known nonlinear vector functions; and $v_i(\cdot)$ are nonlinear uncertainties.
It is noted that the nonlinearities $\phi i(\cdot)$ as well as $v_{i(\cdot)}$ depend only on the output $y(k) = x_1(k)$, which is the only measured state. This justifies the name of "output-feedback" form.
According to Ge et al. (2009), for system (67) there exist prediction functions $F_{n-i}(\cdot)$ such that $y(k+n-i) = F_{n-i}(\underline{y}(k), \underline{u}(k-i))$, $i = 1,2,\ldots,n-1$, where

$$\underline{y}(k)=[y(k), y(k-1),\ldots, y(k-n+1)]^T \tag{68}$$

$$\underline{u}(k-i)=[u(k-i), u(k-i-1),\ldots, u(k-n+1)]^T \tag{69}$$

By moving the ith equation $(n-i)$ step ahead, we can rewrite system (67) as follows

$$\begin{cases} x_1(k+n) = \theta_1^T\phi_1(y(k+n-1)) + g_1x_2(k+n-1) + v_1(y(k+n-1)) \\ x_2(k+n-1) = \theta_2^T\phi_2(y(k+n-2)) + g_2x_3(k+n-2) + v_2(y(k+n-2)) \\ \quad\vdots \\ x_n(k+1) = \theta_n^T\phi_n(y(k)) + g_n u(k) + v_n(y(k)) \end{cases} \tag{70}$$

Then, we submit the second equation to the first and obtain

$$\begin{aligned} x_1(k+n) = {} & \theta_1^T\phi_1(y(k+n-1)) + g_1\theta_2^T\phi_2(y(k+n-2)) \\ & + g_1g_2x_3(k+n-2) + v_1(y(k+n-1)) + g_1v_2(F_{n-2}(\underline{y}(k), \underline{u}(k-2)) \end{aligned} \tag{71}$$

Continuing the iterative substitution, we could finally obtain

$$y(k+n) = \sum_{i=1}^{n} \theta_{fi}^T\phi_i(y(k+n-i)) + gu(k) + \nu(z(k)) \tag{72}$$

where

$$\begin{aligned} &\theta_{f_1} = \theta_1,\ \theta_{f_i} = \theta_i\prod_{j=1}^{i-1} g_j,\ \ i = 2,3,\ldots,n \\ &g_{f_1} = 1,\ \ g_{f_i} = \prod_{j=1}^{i-1} g_j,\ \ i = 2,3,\ldots,n,\ \ g = \prod_{j=1}^{n} g_j \end{aligned} \tag{73}$$

and

$$\nu(z(k)) = \sum_{i=1}^{n} g_{f_i}\nu_i(z(k)),\ \ z(k) = [\underline{y}^T(k), \underline{u}^T(k-1)]^T \tag{74}$$

with

$$\nu_i(z(k)) = v_i(y(k+n-i)) = v_i(F_{n-i}(\underline{y}(k), \underline{u}(k-i))), \ \ i = 1, 2, \ldots, n-1,$$
$$\nu_n(z(k)) = v_n(y(k)) \tag{75}$$

with $z(k)$ defined in the same manner as in (1). Now, it is obvious that the transformed output-feedback form system (72) is a special case of the general NARMA model (1).

8. Study on periodic varying parameters

In this section we shall study the case where the parameters θ_i and g_j, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$ in (1) are periodically time-varying. The lth element of $\theta_i(k)$ is periodic with known period $N_{i,l}$ and the period of $g_i(k)$ is N_{gi}, i.e. $\theta_{i,l}(k) = \theta_{i,l}(k - N_{i,l})$ and $g_j(k) = g_j(k - N_{gj})$ for known positive constants $N_{i,l}$ and N_{gj}, $l = 1, 2, \ldots, p_i$.

To deal with periodic varying parameters, *periodic adaptive control* (PAC) has been developed in literature, which updates parameters every N steps, where N is a common period such that every period $N_{i,l}$ and N_{gj} can divide N with an integer quotient, respectively. However, the use of the common period will make the periodic adaptation inefficient. If possible, the periodic adaptation should be conducted according to individual periods. Therefore, we will employ the lifting approach proposed in Xu & Huang (2009).

Firstly, we define the augmented parametric vector and corresponding vector-valued nonlinearity function. As there are $N_{i,j}$ different values of the jth element of θ_i at different steps, denote an augmented vector combining them together by

$$\bar{\theta}_{i,l} = [\theta_{i,j,1}, \theta_{i,j,2}, \ldots, \theta_{i,j,N_{i,l}}]^T \tag{76}$$

with constant elements. We can construct an augmented vector including all p_i periodic parameters

$$\Theta_i = [\bar{\theta}_{i,1}^T, \bar{\theta}_{i,2}^T, \ldots, \bar{\theta}_{i,p_i}^T]^T = [\theta_{i,1,1}, \ldots, \theta_{i,1,N_{i,1}}, \ldots, \theta_{i,p_i,1}, \ldots, \theta_{i,p_i,N_{i,p_i}}]^T \tag{77}$$

with all elements being constant. Accordingly, we can define an augmented vector

$$\Phi_i(\underline{y}(k+n-1)) = [\bar{\phi}_{i,1}(\underline{y}(k+n-1)), \ldots, \bar{\phi}_{i,p_i}(\underline{y}(k+n-1))]^T \tag{78}$$

where $\bar{\phi}_{i,l}(\underline{y}(k+n-1)) = [0, \ldots, 0, \phi_i(\underline{y}(k+n-i)), 0, \ldots, 0]^T \in R^{N_{i,l}}$ and the element $\phi_i(k)$ appears in the qth position of $\bar{\phi}_{i,l}(\underline{y}(k+n-1))$ only when $k = sN_{i,l} + q$, for $i = 1, 2, \ldots, N_{i,l}$. It can be seen that n functions $\phi_i(k)$, rotate according to their own periodicity, $N_{i,l}$, respectively. As a result, for each time instance k, we have

$$\theta_i^T(k)\phi_i(\underline{y}(k+n-i)) = \Theta_i^T \Phi_i(\underline{y}(k+n-1)) \tag{79}$$

which converts periodic parameters into an augmented time invariant vector.

Analogously, we convert $g_i(k)$ into an augmented vector $\bar{g}_i = [g_{i,1}, g_{i,2}, \ldots, g_{i,N_{gj}}]$ and meanwhile define a vector

$$\varphi_j(k) = [0, \ldots, 0, 1, 0, \ldots, 0]^T \in R^{N_{gj}} \tag{80}$$

where the element 1 appears in the qth position of $\varphi_j(k)$ only when $k = sN_{gj} + q$. Hence for each time instance k, we have $g_j(k) = \bar{g}_j \varphi_j(k)$, i.e., $g_i(k)$ is converted into an augmented time-invariant vector.

Then, system (1) with periodic time-varying parameters $\theta_i(k)$ and $g_j(k)$ can be transformed into

$$y(k+n) = \sum_{i=1}^{n} \Theta_i^T \Phi_i(y(k+n-i)) + \sum_{j=1}^{m} \bar{g}_j \varphi_j(k) u(k-m+j) + \nu(z(k-\tau)) \tag{81}$$

such that the method developed in Sections 4 and 5 is applicable to (81) for control design.

9. Conclusion

In this chapter, we have studied *asymptotic tracking* adaptive control of a general class of *NARMA* systems with both parametric and nonparametric model uncertainties. The effects of nonlinear nonparametric uncertainty, as well as of the unknown time delay, have been compensated for by using information of previous inputs and outputs. As the *NARMA* model involves future outputs, which bring difficulties into the control design, a future output prediction method has been proposed in Section 4, which makes sure that the prediction error grows with smaller order than the outputs.
Combining the uncertainty compensation technique, the prediction method and adaptive control approach, a predictive adaptive control has been developed in Section 5 which guarantees stability and leads to *asymptotic tracking* performance. The techniques developed in this chapter provide a general control design framework for high order nonlinear discrete-time systems in *NARMA* form. In Sections 7 and 8, we have shown that the proposed control design method is also applicable to output-feedback systems and extendable to systems with periodic varying parameters.

10. Acknowledgments

This work is partially supported by National Nature Science Foundation (NSFC) under Grants 61004059 and 60904086. We would like to thank Ms. Lihua Rong for her careful proofreading.

11. Appendix A: Proof of Proposition 3.1

Only proofs of properties (ii) and (viii) are given below. Proofs of other properties are easy and are thus omitted here.
(ii) From Definition 3.1, we can see that $\|o[x(k)]\| \leq \alpha(k) \max_{k' \leq k+\tau} \|x(k')\|$, $\forall k > k_0, \tau \geq 0$, where $\lim_{k\to\infty} \alpha(k) = 0$. It implies that there exist constants k_1 and $\bar{\alpha}_1$ such that $\alpha(k) \leq \bar{\alpha}_1 < 1$, $\forall k > k_1$. Then, we have

$$\|x(k+\tau) + o[x(k)]\| \leq \|x(k+\tau)\| + \|o[x(k)]\| \leq (1+\bar{\alpha}_1) \max_{k' \leq k+\tau} \|x(k')\|, \forall k > k_1$$

which leads to $x(k+\tau) + o[x(k)] = O[x(k+\tau)]$. On the other hand, we have

$$\begin{aligned} \max_{k_1 < k' \leq k+\tau} \|x(k')\| &\leq \| \max_{k_1 < k' \leq k+\tau} x(k') + o[x(k)]\| + \|o[x(k)]\| \\ &\leq \| \max_{k_1 < k' \leq k+\tau} x(k') + o[x(k)]\| + \bar{\alpha}_1 \max_{k_1 < k' \leq k+\tau} \{\|x(k')\} \end{aligned}$$

and

$$\max_{k_1 < k' \leq k+\tau} \|x(k')\| \leq \frac{1}{1-\bar{\alpha}_1} \| \max_{k_1 < k' \leq k} x(k') + o[x(k')]\|, \forall k > k_1$$

which implies $x(k+\tau) = O[x(k) + o[x(k)]]$. Then, it is obvious that $x(k+\tau) + o[x(k)] \sim x(k)$.
(viii) First, let us suppose that $x_1(k)$ is unbounded and define $i_k = \arg\max_{i \le k} \|x_1(i)\|$. Then, it is easy to see that $i_k \to \infty$ as $k \to \infty$. Due to $\lim_{k\to\infty} \alpha(k) = 0$, there exist a constant k_2 such that $\alpha(i_k) \le \frac{1}{2}$ and $\|o[x_1(k)]\| \le \frac{1}{2} \max_{k' \le k} \|x_1(k')\|$, $\forall k > k_2$. Considering $x_2(k) = x_1(k) + o[x_1(k)]$, we have

$$\|x_2(i_k)\| = \|x_1(i_k) + o[x_1(i_k)]\| \ge \|x_1(i_k)\| - \|o[x_1(i_k)]\| \ge \frac{1}{2}\|x_1(i_k)\|, \ \forall k > k_2$$

which leads to $\|x_1(i_k)\| \le 2\|x_2(i_k)\|$, $\forall k \ge k_2$. Then, the unboundedness of $x_1(k)$ conflicts with $\lim_{k\to\infty} \|x_2(k)\| = 0$. Therefore, $x_1(k)$ must be bounded. Noting that $\alpha(k) \to 0$, we have

$$0 \le \|x_1(k)\| \le \|x_1(k) + o[x_1(k)]\| + \|o[x_1(k)]\| \le \|x_2(k)\| + \alpha(k) \max_{k' \le k} \|x_1(k')\| \to 0$$

which implies $\lim_{k\to\infty} \|x_1(k)\| = 0$.

12. Appendix B: Proof of Lemma 4.1

It follows from (16) and (17) that

$$\begin{aligned}
&\tilde{y}(k+1|k) = \hat{y}(k+1|k) - y(k+1) \\
= &\sum_{i=1}^{n} \tilde{\theta}_i^T(k-n+2)\Delta\phi_i(k-i+1) + \sum_{i=1}^{m} \tilde{g}_j(k-n+2)\Delta u(k-m+j-n+1) \\
&-\Delta v(k-n+1-\tau)
\end{aligned} \tag{82}$$

which results in

$$\begin{aligned}
&-\{\sum_{i=1}^{n} \tilde{\theta}_i^T(k-n+2)\Delta\phi_i(k-i+1) + \sum_{j=1}^{m} \tilde{g}_j(k-n+2)\Delta u(k-m+j-n+1)\}\tilde{y}(k+1|k) \\
= &-\{\tilde{y}(k+1|k) + \Delta v(k-n+1-\tau)\}\tilde{y}(k+1|k) \\
= &-\tilde{y}^2(k+1|k) - \Delta v(k-n+1-\tau)\tilde{y}(k+1|k) \\
\le &-\tilde{y}^2(k+1|k) + \lambda|\tilde{y}(k+1|k)|\|\Delta Z(k-n+1)\|
\end{aligned} \tag{83}$$

To prove the boundedness of all the estimated parameters, let us choose the following Lyapunov function candidate

$$V_p(k) = \sum_{l=k-n+2}^{k} \left(\sum_{i=1}^{n} \tilde{\theta}_i^2(l) + \sum_{j=1}^{m} \tilde{g}_j^2 \right) \tag{84}$$

Using the parameter update law (26), the difference of $V_p(k)$ is

$$\begin{aligned}
&\Delta V_p(k) = V_p(k+1) - V_p(k) \\
= &\sum_{i=1}^{n}[\tilde{\theta}_i^2(k+1) - \tilde{\theta}_i^2(k-n+2)] + \sum_{j=1}^{m}[\tilde{g}_j^2(k+1) - \tilde{g}_j^2(k-n+2)] \\
= &\frac{a_p^2(k)\gamma_p^2\tilde{y}^2(k+1|k)[\sum_{i=1}^{n} \|\Delta\phi_i(k-i+1)\|^2 + \sum_{j=1}^{m} \Delta u^2(k-m+j-n+1)]}{D_p^2(k)} - \frac{2a_p(k)\gamma}{D_p(k)} \times \\
&\{\sum_{i=1}^{n} \tilde{\theta}_i^T(k-n+2)\Delta\phi_i(k-i+1) + \sum_{j=1}^{m} \tilde{g}_j(k-n+2)\Delta u(k-m+j-n+1)\}\tilde{y}(k+1|k)
\end{aligned}$$

According to the definition of $D_p(k)$ in (27) and inequality (83), the difference of $V_p(k)$ above can be written as

$$\begin{aligned}\Delta V_p(k) \le & \frac{a_p^2(k)\gamma^2\tilde{y}^2(k+1|k)}{D_p(k)} - \frac{2a_p(k)\gamma\tilde{y}^2(k+1|k)}{D_p(k)} \\ & + \frac{2a_p(k)\gamma_p\lambda|\tilde{y}(k+1|k)|\|\Delta Z(k-n+1)\|}{D_p(k)} \\ = & \frac{a_p^2(k)\gamma_p^2\tilde{y}^2(k+1|k)}{D_p(k)} - \frac{2a_p^2(k)\gamma\tilde{y}^2(k+1|k)}{D_p(k)} \\ = & -\frac{a_p^2(k)\gamma_p(2-\gamma_p)\tilde{y}^2(k+1|k)}{D_p(k)} \end{aligned} \quad (85)$$

where the following equation obtained from the definition of dead zone (28) is used:

$$\begin{aligned}-2a_p^2(k)\gamma\tilde{y}^2(k+1|k) = & -2a_p(k)\gamma\tilde{y}^2(k+1|k) \\ & +2a_p(k)\gamma_p\lambda|\tilde{y}(k+1|k)|\|\Delta Z(k-n+1)\| \end{aligned} \quad (86)$$

Noting that $0 < \gamma_p < 2$, we can see from (85) that the difference of Lyapunov function $V_p(k)$, is non-positive and thus, the boundedness of $V_p(k)$ is guaranteed. It further implies the boundedness of $\hat{\theta}_i(k)$ and $\hat{g}_j(k)$. Thus, there exist finite constants b_{θ_i} and b_{g_j} such that

$$\|\hat{\theta}_i(k)\| \le b_{\theta_i},\ \hat{g}_j(k) \le b_{g_j},\ \ \forall k \in Z_{-n}^{+} \quad (87)$$

Taking summation on both hand sides of (85), we obtain

$$\sum_{k=0}^{\infty} \frac{a_p^2(k)\gamma(2-\gamma)\tilde{y}^2(k+1|k)}{D_p(k)} \le V_p(0) - V_p(\infty) \quad (88)$$

Note that the left hand side of inequality (88) is the summation of a non-decreasing sequence and thus the boundedness of $V_p(k)$ implies

$$\frac{a_p^2(k)\tilde{y}^2(k+1|k)}{D_p(k)} := \alpha(k) \to 0 \quad (89)$$

Noting that $l_{k-n+1} \le k - 2n + 1$ by (7) and considering Assumption 3.1, (5) in Lemma 3.3, we see that $D_p(k)$ in (27) satisfies

$$D_p^{\frac{1}{2}}(k) = O[y(k+1)] \quad (90)$$

From (89) and (90), we have

$$a_p(k)|\tilde{y}(k+1|k)| = \alpha^{\frac{1}{2}}(k)D_p^{\frac{1}{2}}(k) = o[D_p^{\frac{1}{2}}(k)] = o[O[y(k+1)]] \quad (91)$$

From the definition of dead zone in (28), when $|\tilde{y}(k+1|k)| > \lambda\|\Delta Z(k-n+1)\|$, we have

$$a_p(k)|\tilde{y}(k+1|k)| = |\tilde{y}(k+1|k)| - \lambda\|\Delta Z(k-n+1)\| > 0$$

while when $|\tilde{y}(k+1|k)| \leq \lambda \hat{c}_p(k-n+2)\|\Delta Z(k-n+1)\|$, we have

$$a_p(k)|\tilde{y}(k+1|k)| = 0 \geq |\tilde{y}(k+1|k)| - \lambda\|\Delta Z(k-n+1)\|.$$

In summary, the definition of dead zone in (28) guarantees the following inequality

$$|\tilde{y}(k+1|k)| \leq a_p(k)|\tilde{y}(k+1|k)| + \lambda \hat{c}_p(k-n+2)\|\Delta Z(k-n+1)\| \tag{92}$$

which together with (91), boundedness of the parameter estimates, and the definition of $\Delta_s(k,m)$ in (31) yields

$$|\tilde{y}(k+1|k)| \leq o[O[y(k+1)]] + \lambda c_1 \Delta_s(k,1) \tag{93}$$

with $c_1 = 1$. Now, let us analyze the two-step prediction error:

$$\begin{aligned}\tilde{y}(k+2|k) &= \hat{y}(k+2|k) - y(k+2) \\ &= \tilde{y}(k+2|k+1) + \breve{y}(k+2|k)\end{aligned} \tag{94}$$

where

$$\begin{aligned}\tilde{y}(k+2|k+1) &= \hat{y}(k+2|k+1) - y(k+2) \\ \breve{y}(k+2|k) &= \hat{y}(k+2|k) - \hat{y}(k+2|k+1)\end{aligned} \tag{95}$$

From (93), it is easy to see that

$$|\tilde{y}(k+2|k+1)| \leq o[O[y(k+2)]] + \lambda c_1 \Delta_s(k,2) \tag{96}$$

From (17), and (20), it is clear that $\breve{y}(k+2|k)$ in (95) can be written as

$$\begin{aligned}\breve{y}(k+2|k) &= \hat{y}(k+2|k) - \hat{y}(k+2|k+1) \\ &= \hat{\theta}_1^T(k-n+3)[\Delta\hat{\phi}(k+1|k) - \Delta\phi(k+1)]\end{aligned} \tag{97}$$

Using (93) and the Lipschitz condition of $\Delta\phi_i(\cdot)$ (or equivalently $\phi_i(\cdot)$) with Lipschitz coefficient L_i, we have

$$\|\Delta\hat{\phi}(k+1|k) - \Delta\phi(k+1)\| \leq L_1|\tilde{y}(k+1|k)| \leq o[O[y(k+1)]] + \lambda c_1 L_1 \Delta_s(k,1) \tag{98}$$

which yields

$$|\breve{y}(k+2|k)| \leq o[O[y(k+1)]] + \lambda L_1 b_{\theta_1} \Delta_s(k,1) \tag{99}$$

From (94), (96) and (99), it is clear that there exists a constant c_2 such that

$$|\tilde{y}(k+2|k)| \leq o[O[y(k+2)]] + \lambda c_2 \Delta_s(k,2) \tag{100}$$

Continuing the analysis above, for l-step estimate error $\tilde{y}(k+l|k)$, we have

$$\begin{aligned}\tilde{y}(k+l|k) &= \hat{y}(k+l|k) - y(k+l) \\ &= \breve{y}(k+l|k) + \tilde{y}(k+l|k+1)\end{aligned} \tag{101}$$

where

$$\begin{aligned}\tilde{y}(k+l|k+1) &= \hat{y}(k+l|k+1) - y(k+l) \\ \breve{y}(k+l|k) &= \hat{y}(k+l|k) - \hat{y}(k+l|k+1)\end{aligned} \tag{102}$$

For $(l-1)$-step estimate error $\tilde{y}(k+l-1|k)$, it can be seen that there exist constants $\tilde{c}_{l-1}$ and $\check{c}_{l-1}$ such that

$$\begin{aligned} |\tilde{y}(k+l-1|k)| &\leq o[O[y(k+l-1)]] + \lambda\tilde{c}_{l-1}\Delta_s(k,l-1) \\ |\check{y}(k+l-1|k)| &\leq o[O[y(k+l-2)]] + \lambda\check{c}_{l-1}\Delta_s(k,l-2) \end{aligned} \tag{103}$$

From (25) and (102), it is clear that $\check{y}(k+l|k)$ can be expressed as

$$\check{y}(k+l|k) = \sum_{i=1}^{l-1} \hat{\theta}_i^T(k-n+l+1)[\Delta\hat{\phi}(k+l-i|k) - \Delta\hat{\phi}(k+l-i|k+1)] \tag{104}$$

From (102), we have

$$\hat{y}(k+l-i|k) - \hat{y}(k+l-i|k+1) = \check{y}(k+l-i|k) \tag{105}$$

According to the Lipschitz condition of $\phi(\cdot)$ and (105), the following equality holds:

$$\sum_{i=1}^{l-1} \|\Delta\hat{\phi}(k+l-i|k) - \Delta\phi(k+l-i)\| \leq \max\{L_j\}_{1\leq j\leq l-1} \sum_{i=1}^{l-1} |\check{y}(k+l-i|k)| \tag{106}$$

From (93),(101)-(106), it follows that there exist constants c_l such that

$$|\tilde{y}(k+l|k)| \leq o[O[y(k+l)]] + \lambda c_l \Delta_s(k,l)$$

which completes the proof.

13. References

Arcak, M., Teel, A. & Kokotovic, P. (2001). Robust nonlinear control of feedforward systems with unmodeled dynamics, *Automatica* **37**(2): 265–272.

Cabrera, J. B. D. & Narendra, K. S. (1999). Issues in the application of neural networks for tracking based on inverse control, *IEEE Transactions on Automatic Control* **44**(11): 2007–2027.

Chen, L. J. & Narendra, K. S. (2001). Nonlinear adaptive control using neural networks and multiple models, *Automatica* **37**(8): 1245–1255.

Chen, X. K. (2006). Adaptive sliding mode control for discrete-time multi-input multi-output systems, *Automatica* **42**(3): 427–435.

Chen, X. K., Fukuda, T. & Young, K. D. (2001). Adaptive quasi-sliding-mode tracking control for discrete uncertain input-output systems, *IEEE Transactions on Industrial Electronics* **48**(1): 216–224.

Egardt, B. (1979). *Stability of Adaptive Control*, Springer-Verlag, New York.

Ge, S. S., Hong, F. & Lee, T. H. (2003). Adaptive neural network control of nonlinear systems with unknown time delays, *IEEE Transactions on Automatic Control* **48**(11): 2004–2010.

Ge, S. S., Hong, F. & Lee, T. H. (2004). Adaptive neural control of nonlinear time-delay systems with unknow virtual control coefficients, *IEEE Transactions on Systems, Man, and Cybernetics, Part B* **34**(1): 499–516.

Ge, S. S. & Tee, K. P. (2007). Approximation-based control of nonlinear mimo time-delay systems, *Automatica* **43**(1): 31–43.

Ge, S. S., Yang, C., Dai, S.-L., Jiao, Z. X. & Lee, T. H. (2009). Robust adaptive control of a class of nonlinear strict-feedback discrete-time systems with exact output tracking, *Automatica* **45**(11): 2537–2545.

Ge, S. S., Yang, C. & Lee, T. H. (2008). Adaptive robust control of a class of nonlinear strict-feedback discrete-time systems with unknown control directions, *Systems & Control Letters* **57**(11): 888–895.

Goodwin, G. C., Ramadge, P. J. & Caines, P. E. (1980). Discrete-time multivariable adaptive control, *IEEE Transactions on Automatic Control* **25**(3): 449–456.

Hirsch, M. W. & Smale, S. (1974). *Differential Equations,Dynamical Systems, and Linear Algebra*, San Diego, CA:Acadamic Press, Inc.

Kanellakopoulos, I., Kokotovic, P. V. & Morse, A. S. (1991). Systematic design of adaptive controller for feedback linearizable systems, *IEEE Transactions on Automatic Control* **36**(11): 1241–1253.

Kolmanovskii, V. B. & Myshkis, A. (1992). *Applied theory of functional differential equations (Vol. 85)*, Kluwer Academic Publisher, Dordrecht.

Lee, T. H. (1996). Adaptive control of time-varying discrete-time systems: Stability and instability analysis for a first-order process, *Journal of Systems Engineering* **6**(1): 12–19.

Leontaritis, I. J. & Billings, S. A. (1985). Input-output parametric models for nonlinear systems, *International Journal of Control* **41**(2): 303–344.

Ma, H. B. (2006). *Capability and Limitation of Feedback Mechanism in Dealing with Uncertainties of Some Discrete-time Control Systems*, Phd thesis, Graduate School of Chinese Academy of Sciences, Beijing, China.

Ma, H. B. (2008). Further results on limitations to the capability of feedback, *International Journal of Control* **81**(1): 21–42.
URL: *http://dx.doi.org/10.1080/00207170701218333*

Ma, H. B., Lum, K. Y. & Ge, S. S. (2007). Adaptive control for a discrete-time first-order nonlinear system with both parametric and non-parametric uncertainties, *Proceedings of the 46th IEEE Conference on Decision and Control*, New Orleans, Louisiana USA, pp. 4839–4844.

Myszkorowski, P. (1994). Robust control of linear discrete-time systems, *Systems & Control Letters* **22**(4): 277–280.

Nešić, D. & Laila, D. S. (July, 2002). A note on input-to-state stabilization for nonlinear sampled-data systems, *IEEE Transactions on Automatic Control* **47**(7): 1153–1158.

Nešić, D. & Teel, A. R. (2006). Stabilization of sampled-data nonlinear systems via backstepping on their euler approximate model, *Automatica* **42**(10): 1801–1808.

Seto, D., Annaswamy, A. M. & Baillieul, J. (1994). Adaptive control of nonlinear systems with a triangular structure, *IEEE Transactions on Automatic Control* **39**: 1411–1428.

Sokolov, V. F. (2003). Adaptive suboptimal tracking for the first-order plant with Lipschitz uncertainty, *IEEE Transactions on Automatic Control* **48**(4): 607–612.

Tao, G. (2003). *Adaptive Control Design and Analysis*, John Wiley & Sons, Hoboken, NJ.

Wu, H. (2000). Adaptive stabilizing state feedback controllers of uncertaindynamical systems with multiple time delays, *IEEE Transactions on Automatic Control* **45**(9): 1697–1701.

Xia, Y., Fu, M. & Shi, P. (2009). *Analysis and Synthesis of Dynamical Systems with Time-Delays*, Vol. 387 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin Heidelberg.

Xie, L. L. & Guo, L. (2000). How much uncertainty can be dealt with by feedback?, *IEEE Transactions on Automatic Control* **45**(12): 2203–2217.

Xu, J.-X. & Huang, D. (2009). Discrete-time adaptive control for a class of nonlinear systems with periodic parameters: A lifting approach, *Proceedings of 2009 Asian Control Conference*, Hong Kong, pp. 678–683.

Yang, C., Ge, S. S. & Lee, T. H. (2009). Output feedback adaptive control of a class of nonlinear discrete-time systems with unknown control directions, *Automatica* **45**(1): 270–276.

Ye, X. & Jiang, J. (1998). Adaptive nonlinear design without *a priori* knowledge of control directions, *IEEE Transactions on Automatic Control* **43**(11): 1617–1621.

Yeh, P. C. & Kokotovic, P. V. (1995). Adaptive control of a class of nonlinear discrete-time systems, *International Journal of Control* **62**(2): 303–324.

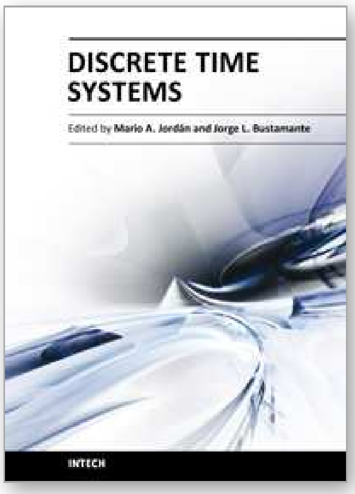
Zhang, T. & Ge, S. (2007). Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs, *Automatica* **43**(6): 1021–1033.

Zhang, Y., Wen, C. Y. & Soh, Y. C. (2001). Robust adaptive control of nonlinear discrete-time systems by backstepping without overparameterization, *Automatica* **37**(4): 551 – 558.

Zhang, Y. X. & Guo, L. (2002). A limit to the capability of feedback, *IEEE Transactions on Automatic Control* **47**(4): 687–692.

Zhao, J. & Kanellakopoulos, I. (2002). Active Identification for Discrete-Time Nonlinear Control-Part I: Output-Feedback Systems, *IEEE Transactions on Automatic Control* **47**(2): 210–224.

Zhu, Q. M. & Guo, L. Z. (2004). Stable adaptive neurocontrol for nonlinear discrete-time systems, *IEEE Transactions on Neural Networks* **15**(3): 653–662.



Discrete Time Systems
Edited by Dr. Mario Alberto JordÃ¡n

ISBN 978-953-307-200-5
Hard cover, 526 pages
Publisher InTech
Published online 26, April, 2011
Published in print edition April, 2011

Discrete-Time Systems comprehend an important and broad research field. The consolidation of digital-based computational means in the present, pushes a technological tool into the field with a tremendous impact in areas like Control, Signal Processing, Communications, System Modelling and related Applications. This book attempts to give a scope in the wide area of Discrete-Time Systems. Their contents are grouped conveniently in sections according to significant areas, namely Filtering, Fixed and Adaptive Control Systems, Stability Problems and Miscellaneous Applications. We think that the contribution of the book enlarges the field of the Discrete-Time Systems with signification in the present state-of-the-art. Despite the vertiginous advance in the field, we also believe that the topics described here allow us also to look through some main tendencies in the next years in the research area.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Chenguang Yang and Hongbin Ma (2011). Discrete-Time Adaptive Predictive Control with Asymptotic Output Tracking, Discrete Time Systems, Dr. Mario Alberto JordÃ¡n (Ed.), ISBN: 978-953-307-200-5, InTech, Available from: http://www.intechopen.com/books/discrete-time-systems/discrete-time-adaptive-predictive-control-with-asymptotic-output-tracking

open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821