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# Stability Criterion and Stabilization of Linear Discrete-time System with Multiple Time Varying Delay

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## 1. Introduction

The control of discrete systems with time-varying delays has been researched extensively in the last few decades. Especially in recent years there are increasing interests in discrete-time systems with delays due to the emerging fields of networked control and network congestion control (Altman & Basar 1999; Sichertiu et al., 2003; Boukas & Liu 2001). Stability problem for linear discrete-time systems with time-delays has been studied in (Kim & Park 1999; Song & Kim 1998; Mukaidani et al., 2005; Chang et al., 2004; Gao et al., 2004). These results are divided into delay-independent and delay-dependent conditions. The delay-independent conditions are more restrictive than delay-dependent conditions. In general, for discrete-time systems with delays, one might tend to consider augmenting the system and convert a delay problem into a delay-free problem (Song & Kim 1998; Mukaidani et al., 2005). The guaranteed cost control problem for a class of uncertain linear discrete-time systems with both state and input delays has been considered in (Chen et al., 2004). Recently, in (Boukas, 2006) new LMI-based delay-dependent sufficient conditions for stability have been developed for linear discrete-time systems with time varying delay in the state. In these papers above the time-varying delay of discrete systems is assumed to be unique in state variables.

On the other hand, in practice there always exist multiple time-varying delays in state variables, especially in network congestion control. Control problems of linear continuous-time systems with multiple time-varying delays have been studied in (Xu 1997). Quadratic stabilization for a class of multi-time-delay discrete systems with norm-bounded uncertainties has been studied in (Shi et al., 2009).

To the best of author's knowledge, stabilization problem of linear discrete systems has not been fully investigated for the case of multiple time-varying delays in state, and this will be the subject of this paper. This paper address stabilization problem of linear discrete-time systems with multiple time-varying delays by a memoryless state feedback. First, stability analysis conditions of these systems are given in the form of linear matrix inequalities (LMIs) by a Lyapunov function approach. It provides an efficient numerical method to analyze stability conditions. Second, based on the LMIs formulation, sufficient conditions of stabilization problem are derived by a memoryless state feedback. Meanwhile, robust

stabilization problem is considered based on these formulations and they are numerically tractable.

## 2. Problem statement

Considering the dynamics of the discrete system with multiple time-varying time delays as

$$x_{k+1} = Ax_k + \sum_{i=1}^N A_{di}x_{k-d_{ki}} + Bu_k, x_k = \phi_k, \quad k \in [-\bar{d}_{\max}, \dots, 0], \quad (1)$$

where  $x_k \in \mathfrak{R}^n$  is the state at instant  $k$ , the matrices  $A \in \mathfrak{R}^{n \times n}$ ,  $A_{di} \in \mathfrak{R}^{n \times n}$  are constant matrices,  $\phi_k$  represents the initial condition, and  $d_{ki}$  are positive integers representing multiple time-varying delays of the system that satisfy the following:

$$\underline{d}_i \leq d_{ki} \leq \bar{d}_i, \quad i = 1, \dots, N, \quad (2)$$

where  $\underline{d}_i$  and  $\bar{d}_i$  are known to be positive and finite integers, and we let

$$\bar{d}_{\max} = \max(\bar{d}_i), i = 1, \dots, N.$$

The aim of this paper is to establish sufficient conditions that guarantee the stability of the class of system (1). Based on stability conditions, the stabilization problem of this system (1) will be handled, too. The control law is given with a memoryless state-feedback as:

$$u_k = Kx_k, \quad x_k = \phi_k, \quad k = 0, -1, \dots, -\bar{d}_i,$$

where  $K$  is the control gain to be computed.

## 3. Stability analysis

In this section, LMIs-based conditions of delay-dependent stability analysis will be considered for discrete-time systems with multiple time-varying delays. The following result gives sufficient conditions to guarantee that the system (1) for  $u_k = 0, k \geq 0$  is stable.

**Theorem 1:** For a given set of upper and lower bounds  $\underline{d}_i, \bar{d}_i$  for corresponding time-varying delays  $d_{ki}$ , if there exist symmetric and positive-definite matrices  $P_1 \in \mathfrak{R}^{n \times n}, Q_i \in \mathfrak{R}^{n \times n}$  and  $R_i \in \mathfrak{R}^{n \times n}, i = 1, \dots, N$  and general matrices  $P_2$  and  $P_3$  such that the following LMIs hold:

$$M = \begin{bmatrix} \sum_{i=1}^N Q_i + \sum_{i=1}^N (\bar{d}_i - \underline{d}_i) R_i + P_1 - A^T P_2 - P_2^T A & * & * & * & \dots & * \\ P_2 - P_3^T A & P_1 + P_3 + P_3^T & * & * & \dots & * \\ -A_{d1}^T P_2 & -A_{d1}^T P_3 & -Q_1 & * & \dots & * \\ -A_{d2}^T P_2 & -A_{d2}^T P_3 & 0 & -Q_2 & * & \vdots \\ \vdots & \vdots & \vdots & 0 & \ddots & * \\ -A_{dN}^T P_2 & -A_{dN}^T P_3 & 0 & \dots & 0 & -Q_N \end{bmatrix} < 0 \quad (3)$$

$$Q_i < R_i$$

Terms denoted by \* are deduced by symmetry. Then the system (1) is stable.

Proof: Consider the following change of variables:

$$x_{k+1} = y_k, \quad 0 = -y_k + Ax_k + \sum_{i=1}^N A_{di}x_{k-d_{ki}} \quad (4)$$

Define  $\tilde{x}_k = [x_k^T \quad y_k^T \quad x_{k-d_1}^T \quad \cdots \quad x_{k-d_N}^T]^T$ , and consider the following Lyapunov-Krasovskii candidate functional:

$$V(\tilde{x}_k) = V_1(\tilde{x}_k) + V_2(\tilde{x}_k) + V_3(\tilde{x}_k) \quad (5)$$

with

$$V_1(\tilde{x}_k) = \tilde{x}_k^T E^T P \tilde{x}_k,$$

$$V_2(\tilde{x}_k) = \sum_{i=1}^N \sum_{l=k-d_{ki}}^{k-1} x_l^T Q_i x_l$$

and

$$V_3(\tilde{x}_k) = \sum_{i=1}^N \sum_{l=-d_i+1}^{k-1} \sum_{m=k+l-1} x_m^T R_i x_m,$$

where  $Q_i > 0$  and  $R_i > 0$ , and  $E$  and  $P$  are, respectively, singular and nonsingular matrices with the following forms:

$$E = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 & 0 & \cdots & 0 \\ P_2 & P_3 & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I \end{bmatrix}$$

where  $P_1$  is a symmetric and positive-definite matrix.

The difference  $\Delta V(\tilde{x}_k)$  is given by

$$\Delta V(\tilde{x}_k) = \Delta V_1(\tilde{x}_k) + \Delta V_2(\tilde{x}_k) + \Delta V_3(\tilde{x}_k) \quad (6)$$

Let us now compute  $\Delta V_1(\tilde{x}_k)$ :

$$\begin{aligned} \Delta V_1(\tilde{x}_k) &= V_1(\tilde{x}_{k+1}) - V_1(\tilde{x}_k) = \tilde{x}_{k+1}^T E^T P \tilde{x}_{k+1} - \tilde{x}_k^T E^T P \tilde{x}_k \\ &= y_k^T P_1 y_k - x_k^T P_1 x_k = y_k^T P_1 y_k - 2 \begin{bmatrix} x_k^T & 0 & 0 & \cdots & 0 \end{bmatrix} P_1 \begin{bmatrix} \frac{1}{2} x_k \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{aligned}$$

which has the following equivalent formulation using the fact that

$$0 = -y_k + Ax_k + \sum_{i=1}^N A_{di}x_{k-d_{ki}} \text{ as}$$

$$\Delta V_1(\tilde{x}_k) = \tilde{x}_k^T \left[ \begin{array}{c} \left[ \begin{array}{cccccc} 0 & 0 & 0 & \dots & 0 \\ 0 & P_1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] - P^T \left[ \begin{array}{cccccc} \frac{1}{2}I & 0 & 0 & \dots & 0 \\ A & -I & A_{d1} & \dots & A_{dN} \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{array} \right] - \left[ \begin{array}{cccc} \frac{1}{2}I & A^T & 0 & \dots & 0 \\ 0 & -I & 0 & \dots & 0 \\ 0 & A_{d1}^T & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & A_{dN}^T & 0 & \dots & 0 \end{array} \right] P \end{array} \right] \tilde{x}_k \quad (7)$$

The difference  $\Delta V_2(\tilde{x}_k)$  is given by

$$\Delta V_2(\tilde{x}_k) = V_2(\tilde{x}_{k+1}) - V_2(\tilde{x}_k) = \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^k x_l^T Q_i x_l - \sum_{i=1}^N \sum_{l=k-d_{ki}}^{k-1} x_l^T Q_i x_l$$

Note that

$$\begin{aligned} \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^k x_l^T Q_i x_l &= \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^{k-d_i} x_l^T Q_i x_l + \sum_{i=1}^N \sum_{l=k+1-d_i}^{k-1} x_l^T Q_i x_l + \sum_{i=1}^N x_k^T Q_i x_k \\ \sum_{i=1}^N \sum_{l=k-d_{ki}}^{k-1} x_l^T Q_i x_l &= \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^{k-1} x_l^T Q_i x_l + \sum_{i=1}^N x_{k-d_{ki}}^T Q_i x_{k-d_{ki}} \end{aligned}$$

Using this,  $\Delta V_2(\tilde{x}_k)$  can be rewritten as

$$\begin{aligned} \Delta V_2(\tilde{x}_k) &= \sum_{i=1}^N x_k^T Q_i x_k - \sum_{i=1}^N x_{k-d_{ki}}^T Q_i x_{k-d_{ki}} + \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^{k-d_i} x_l^T Q_i x_l \\ &+ \sum_{i=1}^N \sum_{l=k+1-d_i}^{k-1} x_l^T Q_i x_l - \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^{k-1} x_l^T Q_i x_l. \end{aligned} \quad (8)$$

For  $\Delta V_3(\tilde{x}_k)$ , we have

$$\begin{aligned} \Delta V_3(\tilde{x}_k) &= \sum_{i=1}^N \sum_{l=-\bar{d}_i+2}^{-\bar{d}_i+1} \sum_{m=k+l}^k x_m^T R_i x_m - \sum_{i=1}^N \sum_{l=-\bar{d}_i+2}^{-\bar{d}_i+1} \sum_{m=k+l-1}^{k-1} x_m^T R_i x_m \\ &= \sum_{i=1}^N \sum_{l=-\bar{d}_i+2}^{-\bar{d}_i+1} \left[ \sum_{m=k+l}^{k-1} x_m^T R_i x_m + x_k^T R_i x_k - \sum_{m=k+l}^{k-1} x_m^T R_i x_m - x_{k+l-1}^T R_i x_{k+l-1} \right] \\ &= \sum_{i=1}^N \sum_{l=-\bar{d}_i+2}^{-\bar{d}_i+1} [x_k^T R_i x_k - x_{k+l-1}^T R_i x_{k+l-1}] = \sum_{i=1}^N [(\bar{d}_i - \underline{d}_i) x_k^T R_i x_k - \sum_{l=k+1-\bar{d}_i}^{k-\underline{d}_i} x_l^T R_i x_l]. \end{aligned} \quad (9)$$

Note that  $\underline{d}_i \leq d_{ki} \leq \bar{d}_i$  for all  $i$ , we get

$$\begin{aligned} \sum_{i=1}^N \sum_{l=k+1-\underline{d}_i}^{k-1} x_l^T Q_i x_l &\leq \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^{k-1} x_l^T Q_i x_l, \quad \sum_{i=1}^N \sum_{l=k+1-d_{ki}}^{k-d_i} x_l^T Q_i x_l \leq \sum_{i=1}^N \sum_{l=k+1-\bar{d}_i}^{k-d_i} x_l^T Q_i x_l \\ \sum_{i=1}^N \sum_{l=k+1-\bar{d}_i}^{k-d_i} x_l^T Q_i x_l &\leq \sum_{i=1}^N \sum_{l=k+1-\bar{d}_i}^{k-d_i} x_l^T R_i x_l, \quad \text{since } Q_i < R_i. \end{aligned}$$

Finally, by using (7), (8) and (9) together with these inequalities, we obtain

$$\Delta V(\tilde{x}_k) \leq [x_k \quad y_k \quad x_{k-d1} \quad \cdots \quad x_{k-dN}]^T M \begin{bmatrix} x_k \\ y_k \\ x_{k-d1} \\ \vdots \\ x_{k-dN} \end{bmatrix} < 0,$$

where

$$M = \begin{bmatrix} \sum_{i=1}^N Q_i + \sum_{i=1}^N (\bar{d}_i - \underline{d}_i) R_i + P_1 - A^T P_2 - P_2^T A & * & * & * & \cdots & * \\ P_2 - P_3^T A & P_1 + P_3 + P_3^T & * & * & \cdots & * \\ -A_{d1}^T P_2 & -A_{d1}^T P_3 & -Q_1 & * & \cdots & * \\ -A_{d2}^T P_2 & -A_{d2}^T P_3 & 0 & -Q_2 & * & \vdots \\ \vdots & \vdots & \vdots & 0 & \ddots & * \\ -A_{dN}^T P_2 & -A_{dN}^T P_3 & 0 & \cdots & 0 & -Q_N \end{bmatrix} \quad (10)$$

This implies that the system is stable, and then the claim (3) can be established.  $\square$

*Remark:*

As to robust stability analysis of discrete time systems with polytopic-type uncertainties, robust stability analysis can be considered by the formulation above. When system state matrices in (1) are assumed as

$$[A(\lambda(k)) \quad A_{di}(\lambda(k))] = \sum_{j=1}^L \partial_j(k) [A_j \quad A_{dij}], \quad \partial_j(k) \geq 0, \quad \sum_{j=1}^L \partial_j(k) = 1.$$

Robust state feedback synthesis can be formulated as:

For a given set of upper and lower bounds  $\underline{d}_i, \bar{d}_i$  for corresponding time-varying delays  $d_{ki}$ , if there exist symmetric and positive-definite matrices  $P_1 \in \mathfrak{R}^{n \times n}, Q_i \in \mathfrak{R}^{n \times n}$  and  $R_i \in \mathfrak{R}^{n \times n}, i = 1, \dots, N$  and general matrices  $P_2$  and  $P_3$  such that the following LMIs hold:

$$\begin{bmatrix} \sum_{i=1}^N Q_i + \sum_{i=1}^N (\bar{d}_i - \underline{d}_i) R_i + P_1 - A_j^T P_2 - P_2^T A_j & * & * & * & \cdots & * \\ P_2 - P_3^T A_j & P_1 + P_3 + P_3^T & * & * & \cdots & * \\ -A_{d1j}^T P_2 & -A_{d1j}^T P_3 & -Q_1 & \ddots & & \\ -A_{d2j}^T P_2 & -A_{d2j}^T P_3 & 0 & -Q_2 & * & \vdots \\ \vdots & \vdots & \vdots & 0 & \ddots & * \\ -A_{dNj}^T P_2 & -A_{dNj}^T P_3 & 0 & \cdots & 0 & -Q_N \end{bmatrix} < 0 \quad (11)$$

$$Q_i < R_i \\ j = 1, \dots, L$$

Terms denoted by \* are deduced by symmetry. Then the system with poytopic-type uncertainties is stable.

#### 4. Stabilizability

The aim of this section is to design a memoryless state-feedback controller which stabilizes the system (1). When the memoryless state-feedback is substituted with plant dynamics (3), the dynamics of closed-loop system is obtained as

$$x_{k+1} = (A + BK)x_k + \sum_{i=1}^N A_{di}x_{k-d_{ki}}, \quad x_k = \phi_k, \quad k = 0, -1, \dots, -\bar{d}_i. \quad (12)$$

Note that stability analysis condition (3) is not convenient for us to design a memoryless state-feedback. By Schur Complement lemma, equivalent conditions of (3) are given easily to solve such a memoryless state-feedback which guarantees the closed-loop system (12) is stable. Due to

$$\begin{bmatrix} -Q_1 & 0 & \dots & 0 \\ 0 & -Q_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & -Q_N \end{bmatrix} < 0,$$

The equivalent formulation of (3) could be obtained as

$$\begin{bmatrix} \sum_{i=1}^N Q_i + \sum_{i=1}^N (\bar{d}_i - \underline{d}_i)R_i - P_1 - A^T P_2 - P_2^T A & * \\ P_2 - P_3^T A & P_1 + P_3 + P_3^T \end{bmatrix} \\ + \begin{bmatrix} -P_2^T A_{d1} & -P_2^T A_{d2} & \dots & -P_2^T A_{dN} \\ -P_3^T A_{d1} & -P_3^T A_{d2} & \dots & -P_3^T A_{dN} \end{bmatrix} \begin{bmatrix} -Q_1 & 0 & \dots & 0 \\ 0 & -Q_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & -Q_N \end{bmatrix}^{-1} \begin{bmatrix} -A_{d1}^T P_2 & -A_{d1}^T P_3 \\ -A_{d2}^T P_2 & -A_{d1}^T P_3 \\ \vdots & \vdots \\ -A_{dN}^T P_2 & -A_{d1}^T P_3 \end{bmatrix} < 0$$

If we denote by  $X$  the inverse of  $P$ , we have

$$X = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}, \quad X_1 = P_1^{-1},$$

$$0 = P_2 X_1 + P_3 X_2, \quad X_3 = P_3^{-1}.$$

$$X_1 = P_1^{-1}$$

Pre- and post multiplying the above LMI, respectively, by  $X^T$  and  $X$  and using these relations, we will get

$$\begin{bmatrix} \sum_{i=1}^N X_1^T Q_i X_1 + \sum_{i=1}^N (\bar{d}_i - d_i) X_1^T R_i X_1 - X_1 & * \\ X_2 - AX_1 & X_3 + X_3^T \end{bmatrix} + \begin{bmatrix} X_2^T \\ X_3^T \end{bmatrix} P_1 \begin{bmatrix} X_2 & X_3 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & \dots & 0 \\ A_{d1} & A_{d2} & \dots & A_{dN} \end{bmatrix} \begin{bmatrix} -Q_1 & 0 & \dots & 0 \\ 0 & -Q_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & -Q_N \end{bmatrix}^{-1} \begin{bmatrix} 0 & -A_{d1}^T \\ 0 & -A_{d1}^T \\ \vdots & \vdots \\ 0_2 & -A_{d1}^T \end{bmatrix} < 0$$

Let  $S_i = Q_i^{-1}$  and  $T_i = R_i^{-1}$ , we have

$$\begin{bmatrix} -X_1 & * & \dots & * & \dots & * & \dots & * & \dots & * \\ -AX_1 - BF + X_2 & X_3 + X_3^T & & \vdots & & & & & & \\ X_2 & X_3 & -X_1 & * & \vdots & \vdots & \vdots & & & \vdots \\ 0 & -S_1 A_{d1}^T & 0 & -S_1 & & & \ddots & * & & \\ 0 & -S_2 A_{d2}^T & \vdots & 0 & -S_2 & * & & & & * \\ \vdots & \vdots & 0 & \dots & 0 & \ddots & * & & & \vdots \\ 0 & -S_N A_{dN}^T & & & & \ddots & -S_N & & & \\ X_1 & 0 & \vdots & & & 0 & -S_1 & * & \ddots & \ddots \\ \vdots & \vdots & & & \dots & 0 & \ddots & & & \\ X_1 & 0 & \vdots & & & & \ddots & -S_N & & \\ \vdots & \vdots & & & \dots & \dots & 0 & -\frac{T_1}{\bar{d}_1 - d_1} & * & \\ \vdots & \vdots & & & & & & \ddots & \ddots & * \\ X_1 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & -\frac{T_N}{\bar{d}_N - d_N} \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} -X_1 & * & \dots & * & \dots & * & \dots & * & \dots & * \\ -AX_1 - BF + X_2 & X_3 + X_3^T & & \vdots & & & & & & \\ X_2 & X_3 & -X_1 & * & \vdots & \vdots & \vdots & & & \vdots \\ 0 & -S_1 A_{d1}^T & 0 & -S_1 & & & \ddots & * & & \\ 0 & -S_2 A_{d2}^T & \vdots & 0 & -S_2 & * & & & & * \\ \vdots & \vdots & 0 & \dots & 0 & \ddots & * & & & \vdots \\ 0 & -S_N A_{dN}^T & & & & \ddots & -S_N & & & \\ X_1 & 0 & \vdots & & & 0 & -S_1 & * & \ddots & \ddots \\ \vdots & \vdots & & & \dots & 0 & \ddots & & & \\ X_1 & 0 & \vdots & & & & \ddots & -S_N & & \\ \vdots & \vdots & & & \dots & \dots & 0 & -\frac{T_1}{\bar{d}_1 - d_1} & * & \\ \vdots & \vdots & & & & & & \ddots & \ddots & * \\ X_1 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & -\frac{T_N}{\bar{d}_N - d_N} \end{bmatrix} < 0 \quad (14)$$

**Theorem 2:** For a given set of upper and lower bounds  $\underline{d}_i, \bar{d}_i$  for corresponding time-varying delays  $d_{ki}$ , if there exist symmetric and positive-definite matrices  $X_1 \in \mathfrak{R}^{n \times n}$ ,  $S_i \in \mathfrak{R}^{n \times n}$  and  $T_i \in \mathfrak{R}^{n \times n}$ ,  $i = 1, \dots, N$  and general matrices  $X_2$  and  $X_3$  such that LMIs below hold, the memoryless state-feedback gain is given by  $K = FX_1^{-1}$ .

Proof: Now we consider substituting system matrices of (12) into LMIs conditions (13), the LMIs-based conditions of the memoryless state-feedback problem can be obtained directly as (14).  $\square$

**Remark:**

When these time delays are constant, that is,  $d = \bar{d}_i = \underline{d}_i$ ,  $i = 1, \dots, N$ , theorem 2 is reduced to the following condition

$$\begin{bmatrix} -X_1 & * & \dots & * & \dots & * & * & \dots & \dots & * \\ -AX_1 + X_2 & X_3 + X_3^T & \vdots & \vdots & \dots & & & \dots & * & \vdots \\ X_2 & X_3 & X_1 & * & \dots & \vdots & \vdots & * & \dots & \vdots \\ 0 & -S_1 A_{d1}^T & 0 & -S_1 & & & & & \vdots & * \\ 0 & -S_2 A_{d2}^T & 0 & 0 & -S_2 & \ddots & * & & * & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots & & \vdots & \vdots \\ 0 & -S_N A_{dN}^T & 0 & 0 & \vdots & \vdots & -S_N & \vdots & \vdots & \vdots \\ X_1 & 0 & 0 & 0 & \dots & \dots & 0 & -S_1 & * & * \\ \vdots & 0 & \ddots & * \\ X_1 & 0 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & -S_N \end{bmatrix} < 0 \quad (15)$$

The condition above is delay-independent, which is more restrictive than delay-dependent conditions (14).

**Remark:**

When the time-varying delay of discrete systems is assumed to be unique in state variables, that is,  $N = 1$ , these results in theorem 2 could be reduced to those obtained in (Boukas, E. K., 2006).

**Remark:**

As to robust control problem of discrete time systems with polytopic-type uncertainties, robust state feedback synthesis can be considered by these new formulations. When system state matrices in (11) are assumed as

$$[A(\lambda(k)) \quad A_{di}(\lambda(k)) \quad B(\lambda(k))] = \sum_{j=1}^L \partial_j(k) [A_j \quad A_{dij} \quad B_j],$$

$$\partial_j(k) \geq 0, \quad \sum_{j=1}^L \partial_j(k) = 1,$$

Robust state feedback synthesis can be formulated as:

For a given set of upper and lower bounds  $\underline{d}_i, \bar{d}_i$  for corresponding time varying delays  $d_{ki}$ , if there exist symmetric and positive-definite matrices  $X_1 \in \mathfrak{R}^{n \times n}$ ,  $S_i \in \mathfrak{R}^{n \times n}$  and  $T_i \in \mathfrak{R}^{n \times n}$ ,  $i = 1, \dots, N$  and general matrices  $X_2$  and  $X_3$  such that LMIs (16) hold, the memoryless state-feedback gain is given by  $K = FX_1^{-1}$ .

$$\begin{bmatrix}
 -X_1 & * & \dots & * & \dots & * & \dots & * & \dots & * \\
 -A_j X_1 - B_j F + X_2 & X_3 + X_3^T & & \vdots & & & & & & \\
 X_2 & X_3 & -X_1 & * & \vdots & \vdots & \vdots & & & \vdots \\
 0 & -S_1 A_{d1j}^T & 0 & -S_1 & & & \ddots & * & & \\
 0 & -S_2 A_{d2j}^T & \vdots & 0 & -S_2 & * & & & & * \\
 \vdots & \vdots & 0 & \dots & 0 & \ddots & * & & & \vdots \\
 0 & -S_N A_{dNj}^T & & & & \ddots & -S_N & & & \\
 X_1 & 0 & \vdots & & 0 & -S_1 & * & \ddots & \ddots & \vdots \\
 \vdots & \vdots & \dots & & & 0 & \ddots & & & \\
 X_1 & 0 & \vdots & & & \ddots & -S_N & & & \\
 \vdots & \vdots & & & \dots & & & 0 & -\frac{T_1}{d_1 - d_1} & * \\
 \vdots & \vdots & & & & & & & \ddots & \ddots \\
 X_1 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & -\frac{T_N}{d_N - d_N}
 \end{bmatrix} < 0 \quad (16)$$

$j = 1, \dots, L.$

### 5. Numerical example

To illustrate the usefulness of the previous theoretical results, let us give the following numerical examples.

Consider a discrete system with multiple time-varying delays  $N = 2$  as

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0.01 \end{bmatrix}$$

$$\text{and } A_{d2} = \begin{bmatrix} 0.02 & 0.25 \\ 0.10 & 0.01 \end{bmatrix}$$

with  $1 \leq d_1 \leq 2, 2 \leq d_2 \leq 3$ . Now the stabilization of this system will be considered with a memoryless state feedback.

Using Matlab LMI toolbox (P. Gahinet, et al., 1995), solving (21) we can get

$$X_1 = \begin{bmatrix} 1.36e-3 & 4.26e-3 \\ 4.26e-3 & 1.62e-2 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 7.31e-3 & 2.70e-2 \\ 1.95e-2 & 7.15e-2 \end{bmatrix}$$

$$\text{and } X_3 = \begin{bmatrix} -2.73e-4 & 1.89e-4 \\ -1.565e-3 & -3.42e-3 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 2.17e2 & 62.5 \\ 62.5 & 3.45e2 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1.64e2 & 47.8 \\ 47.8 & 8.56e2 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 4.04e2 & 1.22e2 \\ 1.22e2 & 6.32e2 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 2.84e2 & 96.6 \\ 96.6 & 1.03e3 \end{bmatrix}.$$

Therefore, a memoryless state-feedback gain is given by  $K = FX_1^{-1} = [2.0 \ 3.0]$ .

The closed-loop discrete-time system with multiple time-varying time delay is simulated in case of  $d_1 = 1, d_2 = 2$ ,  $d_1 = 1, d_2 = 3$ ,  $d_1 = 2, d_2 = 2$ , and  $d_1 = 2, d_2 = 3$ , respectively. And these results are illustrated in Figure 1, Figure 2, Figure 3 and Figure 4. These figures show that this system is stabilized by the state feedback.

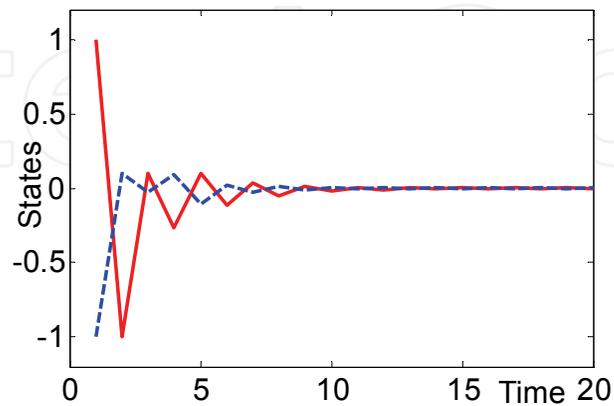


Fig. 1. The behavior of the states in case of  $d_1 = 1, d_2 = 2$

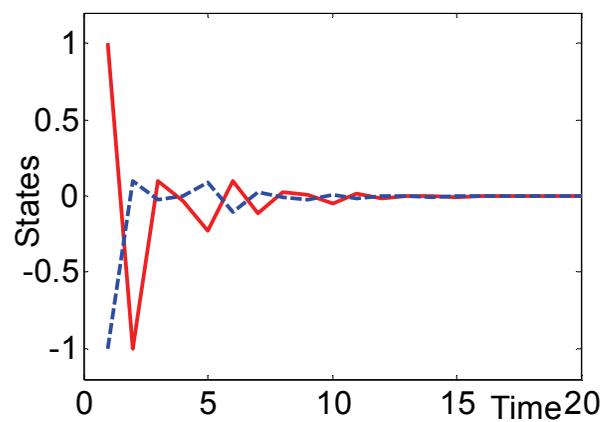


Fig. 2. The behavior of the states in case of  $d_1 = 1, d_2 = 3$

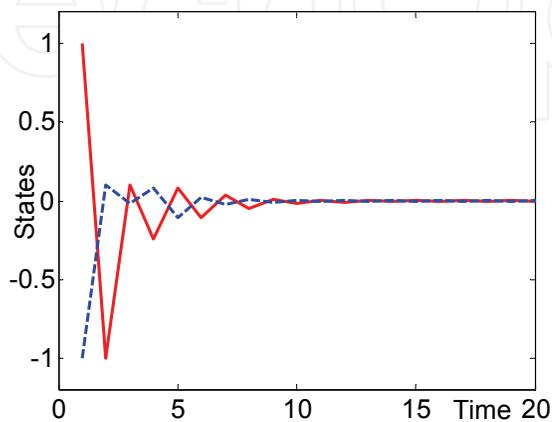


Fig. 3. The behavior of the states in case of  $d_1 = 2, d_2 = 2$

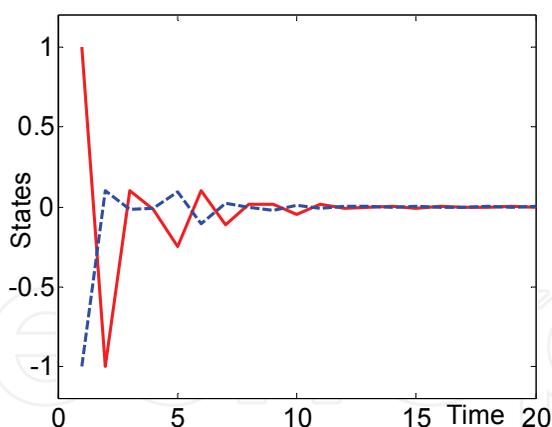


Fig. 4. The behavior of the states in case of  $d_1 = 2, d_2 = 3$

## 6. Conclusion

Stability Criterion and Stabilization for linear discrete-time systems with multiple time-varying delays have been considered. Main results have been given in terms of linear matrix inequalities formulation. It provided us an efficient numerical method to stabilize these systems. Based on these results, it can be also extended to the memory state feedback problem of these systems in the future research.

## 7. Acknowledgements

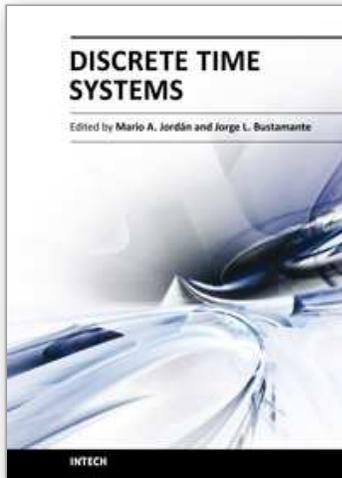
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## **Discrete Time Systems**

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Discrete-Time Systems comprehend an important and broad research field. The consolidation of digital-based computational means in the present, pushes a technological tool into the field with a tremendous impact in areas like Control, Signal Processing, Communications, System Modelling and related Applications. This book attempts to give a scope in the wide area of Discrete-Time Systems. Their contents are grouped conveniently in sections according to significant areas, namely Filtering, Fixed and Adaptive Control Systems, Stability Problems and Miscellaneous Applications. We think that the contribution of the book enlarges the field of the Discrete-Time Systems with signification in the present state-of-the-art. Despite the vertiginous advance in the field, we also believe that the topics described here allow us also to look through some main tendencies in the next years in the research area.

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