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Low Complexity Fourier Transforms using Multiple Square Waves

Khoirul Anwar¹ and Minoru Okada²

¹*School of Information Science,
Japan Advanced Institute of Science and Technology (JAIST),*

²*Graduate School of Information Science,
Nara Institute of Science and Technology (NAIST)
Japan*

1. Introduction

Fourier Transform (FT) is widely applied in digital mobile cellular radio systems. The implementation requires low power consumption and smaller chip size. The primary factor of the FT applications is its chip complexity. The complexity is typically expressed in terms of number of adders, the number of multiplier, data storage and control complexity rather than the speed of operation.

The current divide and conquer technique in fast Fourier transform (FFT) reduces the number of operations in conventional discrete Fourier transform (DFT) by utilizing the advantage of complex twiddle factors instead of matrix multiplications (Oppenheim, 1990). The computation of DFT is decomposed into nested smaller DFTs which are computed separately and combined to give the final results. FFT reduces the number of multiplier which account of much of the chip area and power consumption in digital hardware design.

However, a pipeline FFT processor is characterized by real time continuous processing of an input data sequence. It is difficult to initiate the FFT operation until all of the N sampled data are taken. Another complexity issue is the arithmetic unit, especially multipliers, that requires larger area than a digital register. To meet real-time processing in FFT with size of N , the multiplicative complexity of $N \log_r N$ is required (r is generally the radix). It contributes the complexity of the processor and power consumption.

Another consideration of FFT is the data storage or memory for buffering the data and intermediate results of the real time computations. The butterfly at the first stage has to take the input data elements separated by N/r from the sequence. The required memory becomes another major chip area issue especially for large Fourier transform.

The facts expressed above need to be improved so that the amounts of power consumption, chip area and complexity are suitable especially for handheld transceiver. Since the power consumption is directly related to the number of complex multiplications, an algorithm to reduce or replace these multiplications is important.

In (Shattil and Nassar, 2002), a simple computation of Fourier transform using a square-wave is introduced. A mathematical derivation shows that it is possible to replace the

complex multiplication in Fourier transform by additions. However, the performance evaluation of the method in (Shattil and Nassar, 2002) is not available to make sure the effectiveness of the method.

In this paper, we propose double square-waves (DS) that completely replace complex multiplications by sampling and additions for Fourier and inverse Fourier transforms called DSFT. The proposed method only requires sampler, multiplier and filter to remove the harmonic components of square-wave. Our results confirm that DS-FT is applicable to any system that requires Fourier transforms such as orthogonal frequency division multiplexing (OFDM) (Nee and Prasad, 2000), multicarrier code division multiple access (MC-CDMA), FFT-based carrier interferometry spreading (Anwar and Yamamoto, 2006) and other techniques that requires FFT.

2. Important

This chapter presents a simple computation method for Fourier Transform (FT) and its inverse (IFT) by employing multiple square waves (MSW), whose complex multiplications are replaced by simple additions. Since the square wave is superposition of harmonic sinusoids, a simple mathematical derivation shows that fast Fourier transform (FFT) and its inverse can be performed by MSW with low computational complexity. MSW replaces the complex twiddle factor multiplications in FFT/IFFT by simple adding operation. The main parts of this chapter is adapted from (Takahashi et. al., 2007).

The orthogonality of FFT/IFFT is still kept, by which the bit-error-rate (BER) performance is satisfactory. Compared to the standard single square wave (SSW), our results confirm that excellent BER performance is achievable without error floor. Furthermore, the proposed multiple square wave for Fourier transform (MSW-FT) is free from restriction in its size (e.g. power of two, etc.) and is useful for signal processing of multi-carrier system, such as orthogonal frequency division multiplexing (OFDM), and multi-carrier code division multiple access (MC-CDMA), WiMAX, single carrier frequency division multiple access (SC-FDMA) and other frequency domain processing such as frequency domain turbo equalization. The proposed MSW-FT and MSW-IFT are less complex than FFT or IFFT, which is suitable to digital communication systems, where the power consumption constraint is considered.

3. System model

We consider an OFDM system as the model to evaluate the effectiveness of the proposed DS-FT. Fig. 1 describes the transceiver structure of OFDM system where its FFT is replaced by DS-FT. Inverse DS-FT, called DS-IFT, is located at the transmitter while DS-FT is located at the receiver. The N incoming data symbols are converted from serial to parallel. Then $(L - 1)N$ zeros are added to the center of the parallel data to obtain the oversampled signal, where L is the oversampling factor. The LN data symbols (with zero padding) are converted to time-domain signals using DS-IFT. After filtering, guard interval (GI) is inserted. The OFDM signals are then transmitted to the channel.

At the receiver, first GI is removed, then the signals are converted to frequency domain signals by DS-FT. From the frequency-domain signals, padded zeros are removed. Finally, we obtain the data.

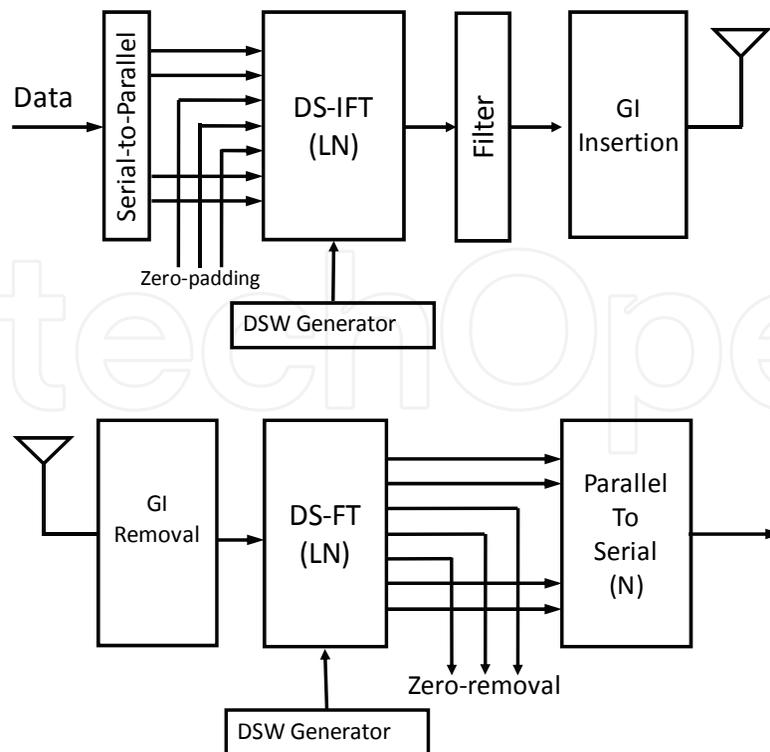


Fig. 1. System Model of OFDM WLAN where its FFT is replaced by DS-FT

4. Proposed double square-waves for Fourier transform

4.1 Square-wave model

The frequency-domain signal $X(f_n)$ converted from time-domain symbol x_k by discrete Fourier transform (DFT) is expressed by

$$\begin{aligned}
 X(f_n) &= \sum_{k=0}^{K-1} x_k e^{-j2\pi f_n k t_0} \\
 &= \sum_{k=0}^{K-1} x_k \Psi(f_n, t)
 \end{aligned}
 \tag{1}$$

where f_n is an n -th frequency component, K is the number of time-domain samples and t_0 is the interval of time-domain samples. The exponential function $\Psi(f_n, t)$ is expressed by Euler's theorem as

$$\begin{aligned}
 \Psi(f_n, t) &= e^{-j2\pi f_n t} \\
 &= \cos 2\pi f_n t - j \sin 2\pi f_n t,
 \end{aligned}
 \tag{2}$$

From (1) and (2), at least $K \times K$ complex multiplications are required. In addition, a lot of phases should be restored when performing the multiplications or additions. To replace a large number of multiplications, we propose to use square-waves which consist of only 2 levels of amplitude as a substitute for exponential function in DFT.

The single square-wave function for n -th frequency can be expressed as a sum of harmonic sinusoids as (Kreuzig, 1993)

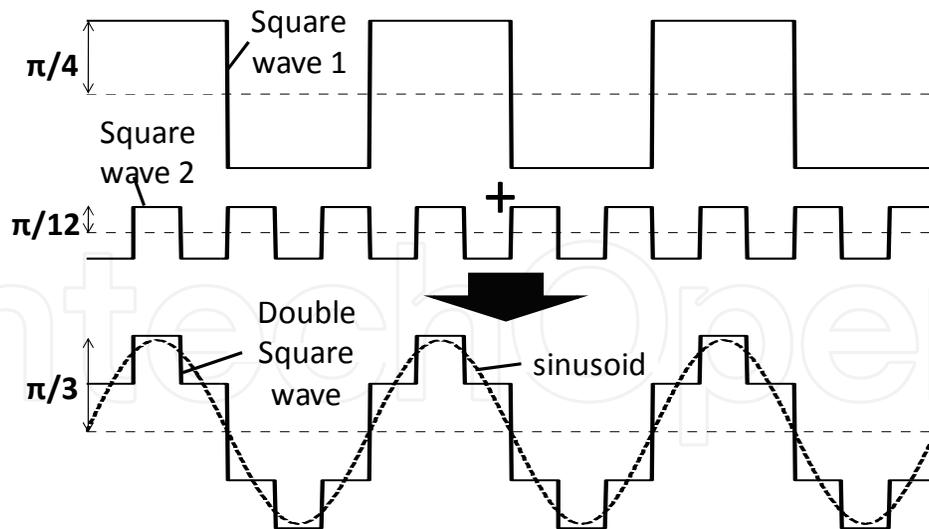


Fig. 2. Double Square-waves consists of $\pi/4$ and $\pi/12$ single square-waves

$$\begin{aligned}
 x_{ss}(2\pi f_n t) &= \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin((2k-1)2\pi f_n t)}{(2k-1)} \\
 &= \frac{4}{\pi} \left\{ \sin 2\pi f_n t + \frac{1}{3} \sin(3 \cdot 2\pi f_n t) + \dots \right\}.
 \end{aligned} \tag{3}$$

Here, (3) can be rewritten as

$$\begin{aligned}
 \sin 2\pi f_n t &= \frac{\pi}{4} x_{ss}(2\pi f_n t) - \frac{1}{3} \sin(3 \cdot 2\pi f_n t) \\
 &\quad - \frac{1}{5} \sin(5 \cdot 2\pi f_n t) - \frac{1}{7} \sin(7 \cdot 2\pi f_n t) - \dots \\
 &= \frac{\pi}{4} x_{ss}(2\pi f_n t) \\
 &\quad - \frac{1}{3} \left\{ \sin(3 \cdot 2\pi f_n t) + \frac{1}{3} \sin(9 \cdot 2\pi f_n t) + \dots \right\} \\
 &\quad - \frac{1}{5} \sin(5 \cdot 2\pi f_n t) - \frac{1}{7} \sin(7 \cdot 2\pi f_n t) - \dots \\
 &= \frac{\pi}{4} x_{ss}(2\pi f_n t) - \frac{1}{3} \left\{ \frac{\pi}{4} x_{ss}(3 \cdot 2\pi f_n t) \right\} \\
 &\quad - \frac{1}{5} \sin(5 \cdot 2\pi f_n t) - \frac{1}{7} \sin(7 \cdot 2\pi f_n t) - \dots.
 \end{aligned} \tag{4}$$

4.2 Order of truncation

Due to the hardware limitation, truncation is required in performing $\sin 2\pi f_n t$ in (4). This subsection discusses errors caused by the truncation of (4). We construct a sinusoid by some square-waves and measure the average error that shows how the signal is similar with the perfect sinusoid signal. The order of number of square-waves is 1, 2, 3, \dots , 12. The result is plotted in Fig. 3.

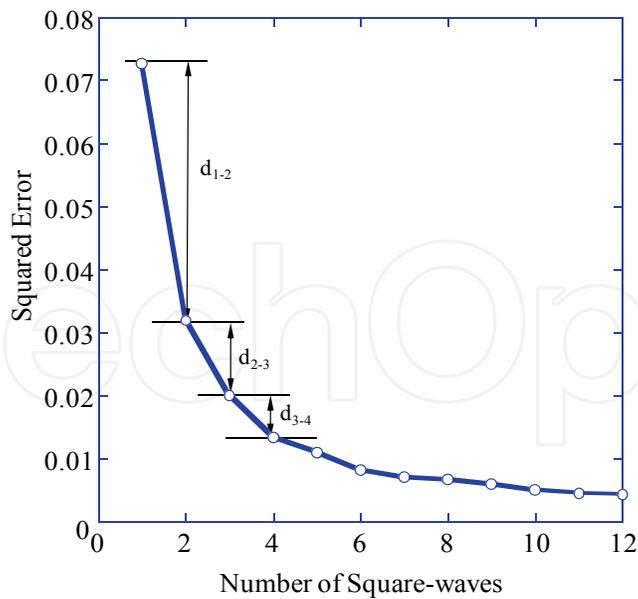


Fig. 3. Average squared-error of square-waves based sinusoid compared to a perfect sinusoid-waves

It is shown when we have a single square-wave, the average error is about 0.073 while it is 0.032 with a double squarewaves. The difference between a single square-wave and double square-waves, d_{1-2} , is about 0.041. That means double square-waves improve about 0.041 of average error. Increasing the number of square-waves more than 3 does not significantly reduce the average error, i.e. $d_{1-2} > d_{2-3} > d_{3-4}$.

On the other hand, using square-waves more than 3 will increase the computational complexity in hardware. We conclude that double square-waves is enough to keep lower error and hardware complexity.

4.3 Double square-wave transform

As a consequence of result in Subsection 4.2, it is reasonable to assume

$$-\frac{1}{5} \sin(5 \cdot 2\pi f_n t) - \frac{1}{7} \sin(7 \cdot 2\pi f_n t) - \dots \cong 0 \tag{5}$$

such that we obtain

$$\sin 2\pi f_n t \cong \frac{\pi}{4} \left\{ x_{ss}(2\pi f_n t) - \frac{1}{3} x_{ss}(3 \cdot 2\pi f_n t) \right\}. \tag{6}$$

From (6), it is shown that the sinusoid can be composed by combining two square-waves of different amplitudes and different periods. We call it as double square-wave (DS) signal and it is noted as $x_{ds}(2\pi f_n t)$.

Because additions require less computational complexity than subtractions, we modify the phase of second wave by π to prevent the subtraction. Then double square-wave function $x_{ds}(2\pi f_n t)$ is expressed by

$$x_{ds}(2\pi f_n t) = \frac{\pi}{4} \left\{ x_{ss}(2\pi f_n t) + \frac{1}{3} x_{ss}(3 \cdot 2\pi f_n t + \pi) \right\}. \tag{7}$$

The number of samples is 96.

Now, we can obtain the function $\Psi_{ds-ft}(f_n, t)$ for DS-FT and $\Psi_{ds-ift}(f_n, t)$ for DS-IFT as substitution of exponential function $\Psi(f_n, t)$ in (2) as

$$\Psi_{ds-ft}(f_n, t) = x_{ds}(2\pi f_n t + \frac{\pi}{2}) - j \cdot x_{ds}(2\pi f_n t), \quad (8)$$

$$\Psi_{ds-ift}(f_n, t) = x_{ds}(2\pi f_n t + \frac{\pi}{2}) + j \cdot x_{ds}(2\pi f_n t). \quad (9)$$

Finally, we can express the frequency-domain signal $X(f_n)$ as

$$X(f_n) \cong \sum_{k=0}^{K-1} x_k \Psi_{ds-ft}(f_n, kt_0), \quad (10)$$

and the time-domain signal $x(f_n)$ as

$$x(f_n) \cong \sum_{k=0}^{K-1} X_k \Psi_{ds-ift}(f_n, kt_0). \quad (11)$$

4.4 Computational complexity

The square-wave generator is simpler than the sinusoid generator because it uses digital logic. It doesn't need the complex analog multiplier and can be replaced by a simple hardware.

An inverter can be used to multiply the data by -1 , while multiplication of $+1$ is possible by copying the signal. Compared to the conventional single square-wave method, our proposed method needs an additional multiplication by $1/3$. However, multiplication by a constant is not too complex in a hardware. Therefore, multiplication by double square-waves is easier in hardware implementation than multiplying by a sinusoid.

5. Performance evaluation

This section evaluates signal resolution and BER performances using the proposed DS-FT compared to that of single square-wave Fourier transform (SS-FT) (Shattil and Nassar, 2002) (Bates et. al., 1970).

5.1 Signal resolution

Figures 4(a) and (b) show the signal resolution of a sinc function. The sinc waveform is represented by the dashed line that has been sampled in 96 samples with normalized amplitude. The sinc waveform by SS-FT is shown in Fig. 4(a), while that by DS-FT is shown in Fig. 4(b). It is shown that the resolution of SS-FT can not reach the maximum while the left and right parts of signals are too high. The sinc waveform represented by the proposed DS-FT has better quality than that of SS-FT.

5.2 BER performances evaluation

In this subsection, to confirm the effectiveness of the proposed method, we evaluate the BER performances of an OFDM system where its FFT and IFFT are replaced by DSFT and

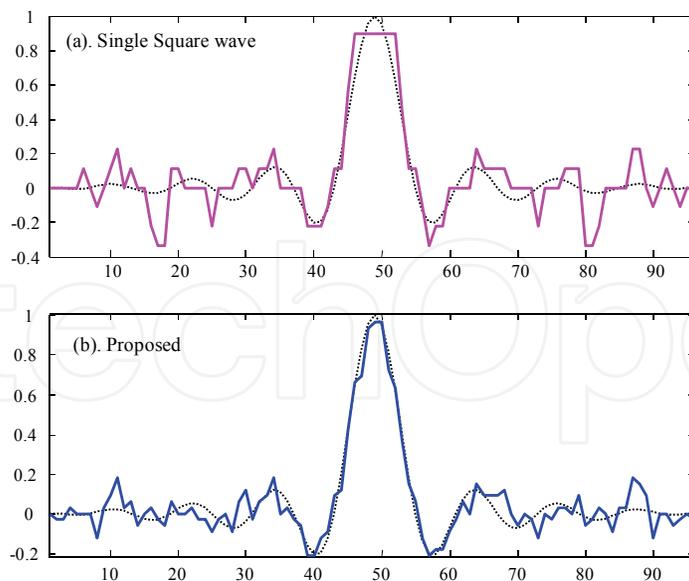


Fig. 4. Sinc waveform using (a) single square-wave and (b) double squarewaves

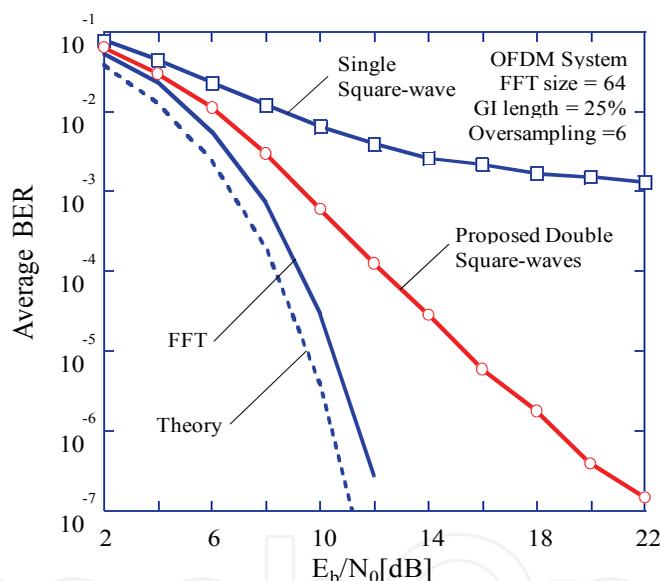


Fig. 5. BER performances of WLAN using FFT, SS-FT and the proposed DS-FT method

DS-IFT. Evaluation of DS-FT using signal resolution is not enough. Thus, evaluation of the BER is important to make sure the effectiveness of sampling and orthogonality guarantee. The parameters used for BER performance evaluation are shown in Table I which expected to be the condition of IEEE802.11a/g Wireless LAN system.

The modulation is QPSK for OFDM system with number of subcarrier is 52, as in Wireless LAN system. We use oversampling factor of 6 to observe efficiently the signal resolution. GI length is 25% of the symbol length. The overall simulation is performed in additive white Gaussian noise (AWGN) channel without error correction coding.

The BER performances are plotted in Fig. 5. The dashed line is a theoretical BER performance of QPSK symbol for reference. The BER of OFDM with FFT has degradation by about 1 dB as a consequence of guard interval (GI) insertion with length of 1/4 or 25% of the length of OFDM symbol. SS-FT has residual bit error at 1.5×10^{-3} . Increasing the number of

oversampling does not change the BER performance of SS-FT. The reason is that the orthogonality can not be kept by the SS-FT.

The proposed DS-FT does not have residual bit error (up to 10^{-7}) though it has BER degradation by about 2dB at BER level of 10^{-3} . Increasing the oversampling factor will increase the BER performance. But the oversampling factor enhancement should consider a practical reason related to the additional complexity.

Parameters	Value(s)
Modulation	QPSK
Number of Subcarriers	52
FFT size	64
GI Length	16 (25%)
Oversampling factor (L)	6
Channel	AWGN

Table 1. Simulation Parameters

6. Conclusion

In this paper, we propose DS-FT and evaluate it in the OFDM system. The DS-FT comprises double square-waves to simplify the Fourier transform computation with better signal resolution and BER performance compared to the Fourier transform using single square-wave. The double square-waves can be easily generated by two weighted single square waves with different periods. DS-FT contributes lower computational complexity of Fourier transform by replacing the complex multiplication with sampling, addition and filtering (only at the transmitter). Therefore, power consumption (related to the number of multiplication) and chip area (related to the memory) can be reduced by DS-FT with allowable performance degradation.

In our future work, we will consider the filter at the output of DS-FT to obtain a better signal resolution by completely removing the harmonic frequency components.

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University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
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