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Sliding Mode Control System for Improvement in Transient and Steady-state Response

Takao Sato, Nozomu Araki, Yasuo Konishi, Hiroyuki Ishigaki
University of Hyogo
Japan

1. Introduction

This chapter discusses design methods for improving sliding mode control system (Chern & Wu, 1992b; Sato, 2010; Utkin, 1977). Variable structure control (VSC) can be easily applied to nonlinear systems and is robust to plant parameter variation or load disturbance because of the existence of a sliding mode. Hence, it has been applied to various systems (e.g., an inverted pendulum system, a magnetic levitation system and robot manipulators (Ashrefiuon & Whitman, 2010; Bandal & Vernekar, 2010; Zergeroglu & Tatlicioglu, 2010)).

VSC methods employing integral compensation have been proposed to achieve servo tracking in the presence of load disturbance or plant parameter variation (Chern & Wu, 1991; 1992a;b). Robust tracking servo can be attained with a controller using integral compensation but the integral action causes phase lag, which deteriorates control performance. However, proportional compensation can adjust the gain property without changing the phase property. Hence, if control systems are designed to use proportional compensation as well as integral compensation, control performance can be further improved. Therefore, this chapter discusses a method for designing a sliding mode controller using both proportional and integral compensations. Hence, this method has higher potential than conventional methods (Chern & Wu, 1991; 1992a;b). In particular, robust servo tracking in steady state is achieved by using integral compensation, and transient response is enhanced by using proportional compensation. Hence, both responses are improved. In conventional methods, to determine the switching plane and the integral gain, a quadratic function is minimized by using the optimal linear regulator technique (Chern & Wu, 1992b) or the characteristics equation of a closed-loop system is assigned to have desired eigenvalues (Chern & Wu, 1991; 1992a). The design methods discussed in this chapter employ the optimal linear regulator technique to determine an optimal switching plane, proportional gain and integral gain to stabilize a closed-loop system.

To demonstrate the potential of these design methods, the designed variable structure controllers are applied to an inverted pendulum system that has been developed to study bifurcations and chaos (Kameoka, 2003; Sato et al., 2005; 2006). Because of the existence of unknown disturbances and unmodeled factors, its exact dynamic characteristics cannot be obtained. Hence, desired control performance cannot be attained if the system is controlled by using a controller based on a variable structure configuration. The potential of the design methods is confirmed by applying these methods to this system, as shown by simulation and

experimental results. Note that the main purpose of this chapter is not to control chaos but to develop a new method for designing a variable structure controller for improving control performance in the presence of load disturbance or plant parameter variation.

This chapter is organized as follows. In Section 2, three control systems are designed: a design method using integral compensation (2.1), proportional compensation (2.2) and both proportional and integral compensations (2.3). Section 3 gives simulation and experimental results to evaluate three method methods. Finally, concluding remarks and future works are given.

2. Design of Sliding Mode Control Systems

Consider a controlled system described as

$$\dot{x}_i = x_{i+1} \quad (i = 1, \cdots, n-1) \tag{1}$$

$$\dot{x}_n = -\sum_{i=1}^{n} a_i x_i + bu - f_d \tag{2}$$

where $x_i (i = 1, \cdots, n)$, u and f_d are the state variable, the control input and the disturbance, respectively. x_1 is the plant output, and $a_i (i = 1, \cdots, n)$ and b are the plant parameters.

To have the plant output converge to its reference input without steady-state error, a method with integral compensation (Chern & Wu, 1992b) is designed as described in 2.1, and a design method using proportional compensation and a method using both proportional and integral compensations (Sato, 2010) are designed as described in 2.2 and 2.3, respectively. For the simplicity of description, this study deals with the case of $n = 2$.

2.1 Control with integral compensation
Chern & Wu (1992b) proposed an integral variable structure controller to achieve servo tracking.

2.1.1 Design of control law with integral compensation
Error variable z is defined as:

$$\dot{z} = r - x_1 \tag{3}$$

where r is the desired state of x_1 and is set by a user. Switching function σ is chosen as:

$$\sigma = S_1(x_1 - K_I z) + x_2 \tag{4}$$

where S_1 is a constant, and constant K_I is referred to as an integral gain. Equation (4) is differentiated with respect to t, and $\dot{\sigma}$ is calculated as:

$$\dot{\sigma} = S_1(\dot{x}_1 - K_I \dot{z}) + \dot{x}_2 \tag{5}$$

Substituting equations (1) and (2) into equation (5), the next equation is obtained as:

$$\dot{\sigma} = S_1(x_2 - K_I(r - x_1)) - a_1 x_1 - a_2 x_2 + bu - f_d \tag{6}$$

The dynamic characteristics of the switching function are assigned by the differential equation:

$$\dot{\sigma} = -Q_s \text{sat}(\sigma) - K_s f(\sigma) \tag{7}$$

where Q_s and K_s are arbitrary positive integers, and sat means saturation and is defined as:

$$\text{sat}(\sigma) = \begin{cases} 1 & (\sigma > L) \\ \dfrac{\sigma}{L} & (|\sigma| \leq L) \\ -1 & (\sigma < -L) \end{cases} \tag{8}$$

$\sigma f(\sigma) > 0$ is required because the condition for existence of a sliding mode is $\lim_{\sigma \to 0} \sigma \dot{\sigma} < 0$ (Utkin, 1977). Hence, $f(\sigma)$ is set as $f(\sigma) = \sigma$. Then, equation (7) is rewritten as:

$$\dot{\sigma} = -Q_s \text{sat}(\sigma) - K_s \sigma \tag{9}$$

Based on equations (6) and (9), a control law is derived as:

$$u = [-S_1(x_2 - K_I(r - x_1)) + a_1 x_1 + a_2 x_2 + f_d - Q_s \text{sat}(\sigma) - K_s \sigma] / b \tag{10}$$

2.1.2 Design of switching surface and integral gain

While in the sliding mode, the use of $\sigma = 0$ yields:

$$x_2 = -S_1(x_1 - K_I z) \tag{11}$$

Equation (11) is substituted into equation (1), and the following equation is obtained.

$$\dot{x}_1 = -S_1(x_1 - K_I z) \tag{12}$$

Then,

$$x = Ax + Bv + Er$$
$$v = Sx$$

where

$$x = \begin{bmatrix} z \\ x_1 \end{bmatrix}, A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, S = \begin{bmatrix} S_1 K_I & -S_1 \end{bmatrix}$$

The optimal gain of S is found by means of the optimal linear regulator technique (Chern & Wu, 1992b), and it is derived by minimizing quadratic index I given as:

$$I = \frac{1}{2} \int_{t_s}^{\infty} (x^T Q^T x + vRv) dt \tag{13}$$

where $Q = Q^T > 0$ and $R > 0$ are a weighting matrix and a weighting parameter, and t_s is the time from when the sliding mode begins (Anderson & Moore, 1971). Weighting matrix Q

can be chosen as:

$$Q = D^T D$$

where D is a $1 \times n$ vector and pair (A, D) is observable. Then, the solution that minimizes the quadratic index is given as:

$$S = -R^{-1}B^T P$$

where P is the solution of the Riccati equation given as:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{14}$$

2.2 Control with proportional compensation

A controller employing proportional compensation is designed as described herein before a variable structure controller employing both proportional and integral compensations to be discussed in 2.3 (Sato, 2010).

The controller designed in this section cannot achieve robust servo tracking, but in comparison to the controllers employing integral compensation designed as described in 2.1 and 2.3, the effectiveness of proportional compensation in variable structure control can be confirmed.

2.2.1 Design of control law with proportional compensation

Switching function σ is defined as:

$$\sigma = S_1(x_1 - r) + x_2 \tag{15}$$

Equation (15) can be differentiated. Hence,

$$\dot{\sigma} = S_1\dot{x}_1 + \dot{x}_2$$

Based on equations (1) and (2), the equation given above is rewritten as:

$$\dot{\sigma} = S_1 x_2 - a_1 x_1 - a_2 x_2 + bu - f_d$$

Using equations (9) and the above equation, a control law is obtained as:

$$S_1 x_2 - a_1 x_1 - a_2 x_2 + bu - f_d = -Q_s \text{sat}(\sigma) - K_s \sigma \tag{16}$$

2.2.2 Design of switching surface and proportional gain

While in the sliding mode ($\sigma = 0$), equation (15) is rewritten as:

$$x_2 = -S_1(x_1 - r) \tag{17}$$

Using equation (17), equation (1) is rewritten as:

$$\dot{x}_1 = -S_1(x_1 - r)$$

Then,

$$\dot{x} = Ax + Bv + Er$$
$$v = Sx$$

where

$$x = x_1, \quad A = 0, \quad B = -1, \quad E = S_1, \quad S = S_1$$

Using the Riccati equation (14), control parameter S_1 is decided.

2.3 Control with both proportional and integral compensations
A controller is designed using both proportional and integral compensations as described in this section (Sato, 2010).

2.3.1 Design of control law with both proportional and integral compensation
Switching function σ is defined as:

$$\sigma = S_1(x_1 - r - K_I z) + x_2 \tag{18}$$

and

$$\dot{\sigma} = S_1(\dot{x}_1 - K_I \dot{z}) + \dot{x}_2 \tag{19}$$

where equation (18) can be differentiated with respect to t. Because equation (19) is equivalent to equation (5), a control law is derived as:

$$u = [-S_1(x_2 - K_I(r - x_1)) + a_1 x_1 + a_2 x_2 + f_d - Q_s \text{sat}(\sigma) - K_s \sigma] / b \tag{20}$$

2.3.2 Design of switching surface and proportional and integral gains
Using $E = [1 \ S_1]^T$, the control parameters of this law are decided in the same way as 2.1.2.

3. Application

3.1 Controlled plant and controller design
The controlled object is an inverted pendulum, which is a nonlinear system (Kameoka, 2003). The model of the inverted pendulum system is illustrated in Fig. 1, and its motion equation is given as:

$$J\ddot{\theta} + C\dot{\theta} + K\theta - mgh \sin\theta = mha\omega^2 \cos\theta \sin\omega t + u \tag{21}$$

where θ and u are expressed as functions of t. The system parameters in the motion equation are shown in Table 1. In particular, the damping coefficient C depends on room air temperature and is sensitive to slight changes in surroundings because the damper is an air damper. The control objective is to control the pendulum rod at a specified angle. To this end, controllers were designed using sliding mode control, as described in 2.1, 2.2 and 2.3, respectively.
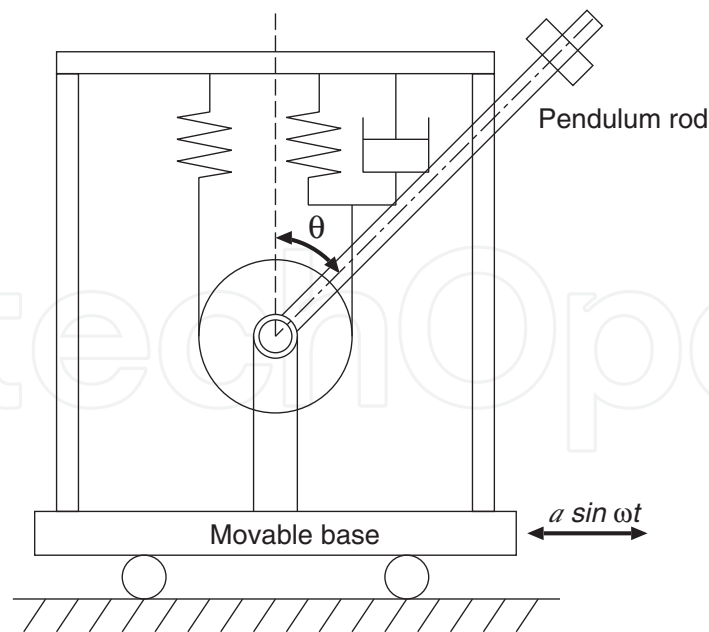
Fig. 1. Model of an inverted pendulum system

θ $[rad]$	angle of a pendulum rod
θ $[rad/s]$	angular velocity of a pendulum rod
J $[kgm^2]$	moment of inertia
C $[Ns \cdot m/rad]$	damping coefficient
K $[Nm/rad]$	spring modulus
m $[kg]$	mass of a pendulum rod
g $[m/s^2]$	gravity acceleration
h $[m]$	distance between the center of gyration and the center of gravity of a pendulum rod
a $[m]$	amplitude of oscillation
ω $[rad/s]$	angular frequency
u $[Nm]$	input torque

Table 1. System parameters in motion equation (21)

Three control methods derived in 2.1, 2.2 and 2.3 are applied to the inverted pendulum system, and their control results are compared. These control methods are designed to control a pendulum rod at a specified angle. Their control parameters are calculated on the basis of system parameters shown in Table 2 given by pre-experiments. The true parameter of a dumper is $K = 0.587$, but assuming that there is modeling error, three control laws are designed as $K = 0.5$.

The design parameters of the control law employing just proportional compensation derived in 2.2 are set as:

$$Q = 10, \;\; R = 1, \;\; Q_s = 0.5, \;\; K_s = 1 \tag{22}$$

System parameter	Physical quantity
J	0.022 [kgm^2]
C	0.01 [Nsm/rad]
K	0.587 [Nm/rad]
m	0.547 [kg]
g	9.8 [m/s^2]
h	0.113 [m]
a	0.045 [m]
ω	1.34 [rad/s]

Table 2. Physical quantity of system parameters

Compensation method	S_1	K_I
Proportional (15)	3.2	-
Integral (4)	3.3	0.097
Proportional and Integral (18)	3.3	0.097

Table 3. Switching surface and proportional and integral compensators

The design parameters of the method employing just integral compensation derived in 2.1 are set as:

$$Q = \begin{bmatrix} 0.1 & 0 \\ 0 & 10 \end{bmatrix}, \quad R = 1, \quad Q_s = 0.5, \quad K_s = 1 \tag{23}$$

The design parameters of the control law with both the proportional and integral compensations derived in 2.3 are set to be the same as those obtained from equation (23). The calculated parameters of the switching surface and proportional and integral compensators are shown in Table 3. The reference angle of a pendulum rod is set to 10[degree], and parameter L of the saturation function (8) is set to 0.01. Control is started after 30[s].

An experimental setup is illustrated in Fig. 2. Because of the capability of a DC motor, the control input is limited as:

$$|u| < 0.0245$$

The resolution of an encoder is 0.18[degree].

To compare the control results of three control methods, a performance index is defined as:

$$E = \sum_{k=30/T_s}^{100/T_s} (r[k] - y[k])^2 \tag{24}$$

where T_s denotes the sampling interval and is set to 50[ms].

3.2 Simulation

Simulations have been conducted using the design parameters (22) and (23). The simulation results are shown in Fig. 3. The result for proportional compensation is shown in Fig. 3(a), and it is shown that the pendulum rod could not precisely follow the reference angle and steady-state error remains because of the modeling error, although its response is quick.
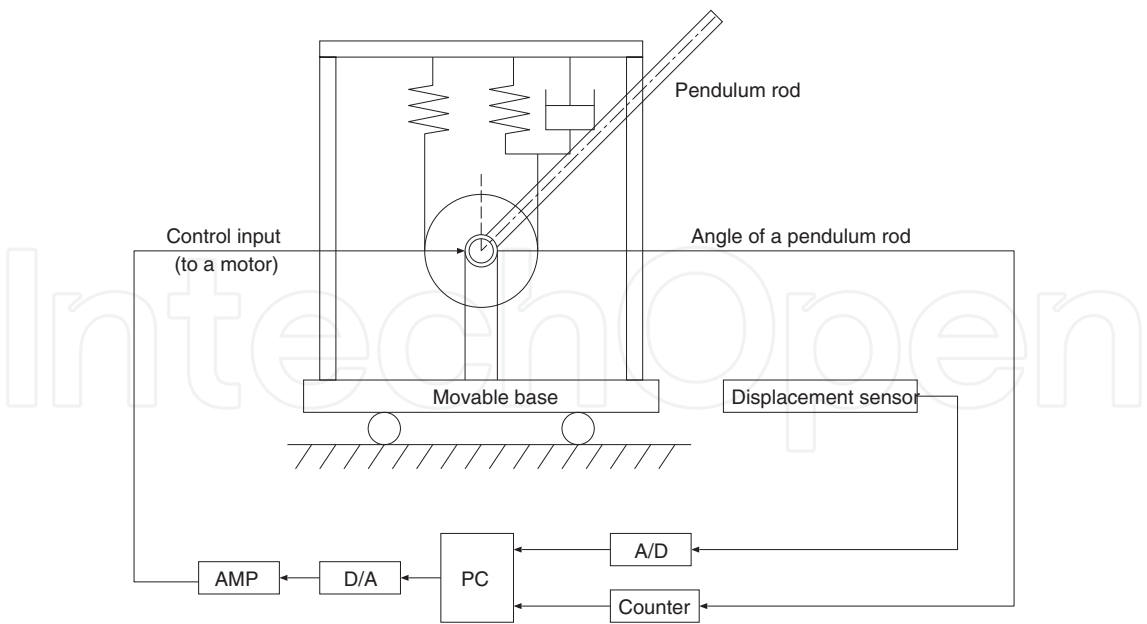
Fig. 2. Experimental setup

The angle for integral compensation is shown in Fig. 3(b), and that for proportional & integral compensation is shown in Fig. 3(c). It can be seen that the steady-state error can be eliminated in the case that integral compensation is employed. However, the transient response of Fig. 3(c) is superior to that of Fig. 3(b) since the control error is quickly improved by proportional compensation. The scores of (24) are summarized in Table 4, and E_P, E_I and E_{PI} show the scores of the control methods employing proportional, integral, and proportional-integral compensations, respectively. It can be seen that E_P is worst because steady-state error remains. In the case that integral compensation is employed, steady-state error is eliminated by using integral compensation, and E_I is better than E_P. In the case that both proportional and integral compensations are employed, steady-state error can be eliminated by using integral compensation, and control error can be quickly improved by the proportional action. Hence, E_{PI} is the smallest.

Employed compensator	Control error E
Proportional	3.3×10^3 (E_P)
Integral	1.1×10^4 (E_I)
Proportional and Integral	5.4×10^2 (E_{PI})

Table 4. Control error of simulation results

3.3 Experiment
Experiments have been conducted using the same control laws as the simulation. In the experimental setup shown in Fig. 2, angular velocity $\dot{\theta}$ cannot be directly obtained. Hence, instead of its true value, the control input is calculated using an estimated value. The sampling interval T_s is set to be the same as that of the simulation. Experimental results are shown in Fig. 4. As the performance index (24), the control results are summarized in Table 5. The experimental results are similar to the simulation results. However, the obtained plant output, that is, the angle of a pendulum rod, is quantized, and furthermore,

(a) Proportional compensation

(b) Integral compensation
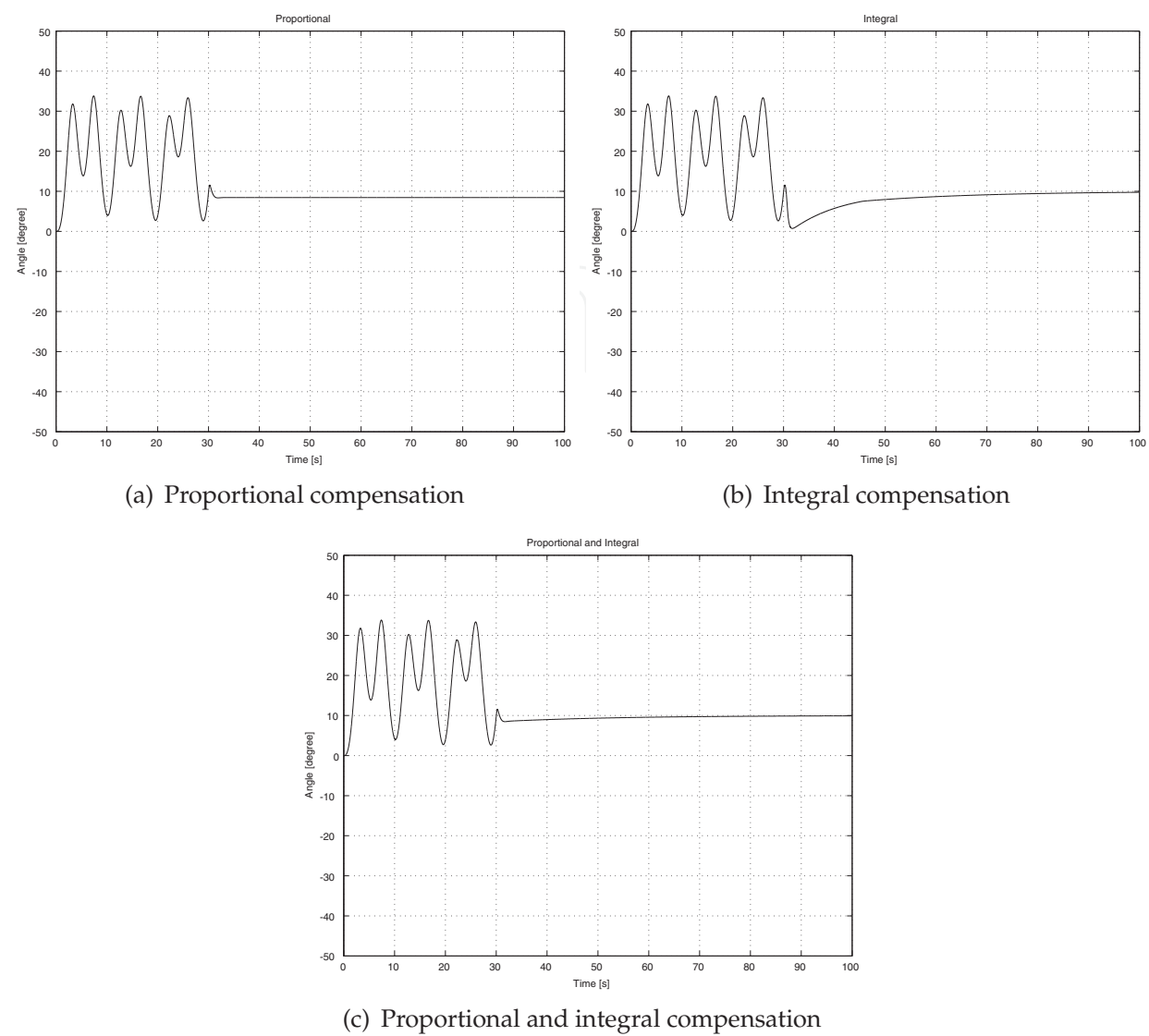

(c) Proportional and integral compensation

Fig. 3. Simulation results: angle

because its angular velocity cannot be directly obtained, an approximated value is employed instead of its true value. Hence, an obtained angular velocity is not accurate, and it usually vibrates. Therefore, a pendulum rod cannot be completely converged to a specified angle even if integral compensation is employed. However, the method using both proportional and integral compensations (E_{PI}) is better than the method employing just proportional compensation (E_P). Therefore, the effectiveness of the method using both proportional and integral compensations is confirmed.

Employed compensator	Control error E
Proportional	6.9×10^3 (E_P)
Integral	1.6×10^4 (E_I)
Proportional and Integral	6.3×10^3 (E_{PI})

Table 5. Control error of experimental results

(a) Proportional compensation

(b) Integral compensation

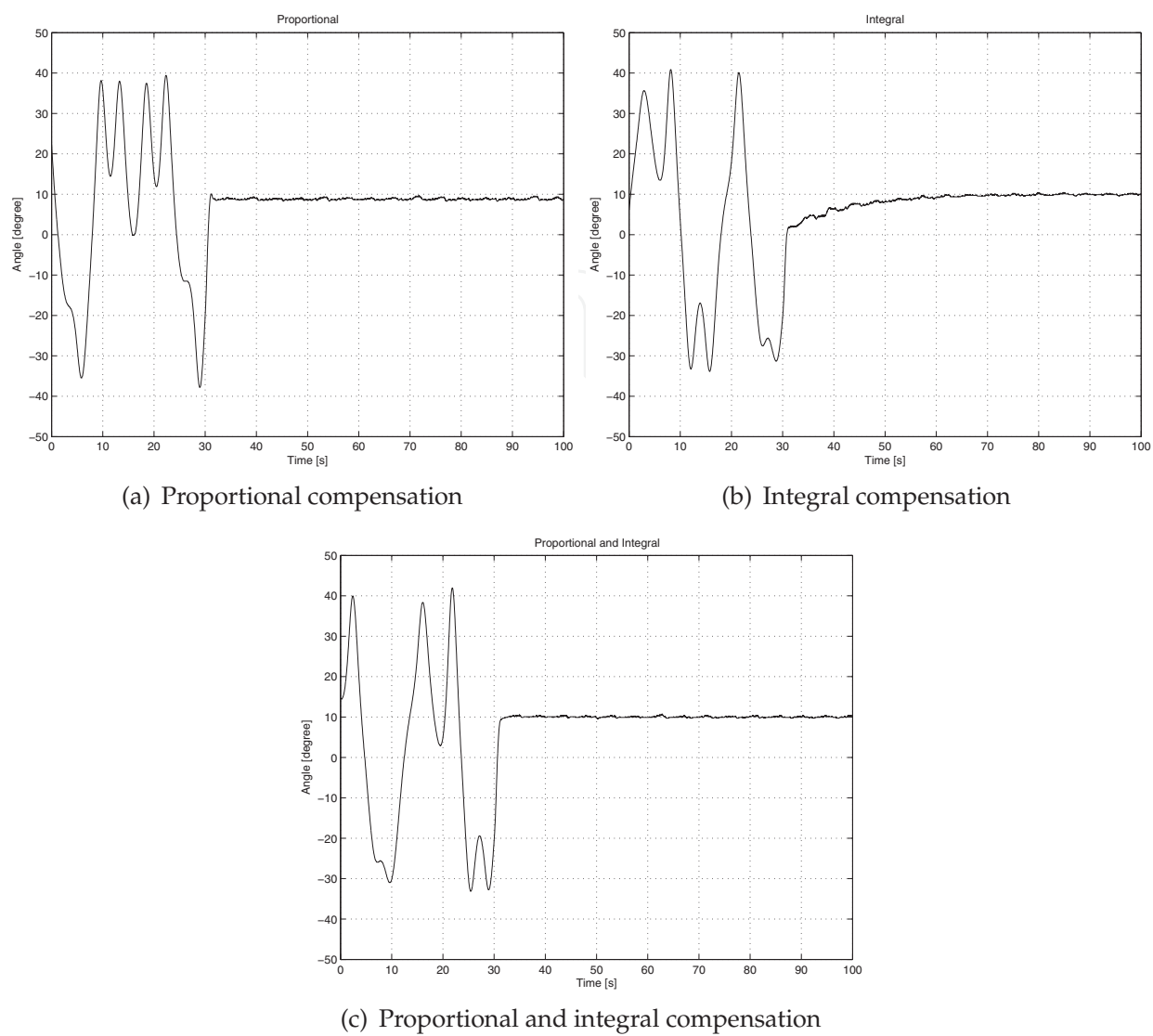(c) Proportional and integral compensation

Fig. 4. Experimental results: angle

4. Conclusion

This chapter has discussed new methods for designing a variable structure control. Chern & Wu (1992b) have designed a sliding mode controller employing integral compensation to achieve servo tracking. To improve the transient response, a sliding mode controller has been designed using proportional compensation as well as integral compensation. If a sliding mode controller is designed using proportional compensation, the transient response can be improved but the steady-state error remains. However, the problem can be resolved by the controller using both proportional and integral compensations, which is designed employing both proportional and integral compensations. Three methods have been applied to an inverted pendulum system, these control results have been compared compared.

In this chapter, the method using proportional and integral compensations has been derived, but approaches employing derivative compensation have the potential to improve control performance if the accurate angular velocity can be obtained because the derivative action leads phase in the whole frequency domain although it increases gain in the high-frequency

domain. However, in control of an inverted pendulum system, the derivative action cannot well work because an obtained angular velocity vibrates due to the quantization of the obtained output signal.

In this study, a controlled plant is assumed to be a single-rate control system, where both the plant output and the control input are sampled or updated at the same rate. However, if a control system is extended into a multirate system, where the sampling interval of the control input differs from the hold interval of the control input, the control performance can be enhanced (Bandal & Vernekar, 2010; Bandyopadhyay & Janardhanan, 2006; Inoue et al., 2007).

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The main objective of this monograph is to present a broad range of well worked out, recent application studies as well as theoretical contributions in the field of sliding mode control system analysis and design. The contributions presented here include new theoretical developments as well as successful applications of variable structure controllers primarily in the field of power electronics, electric drives and motion steering systems. They enrich the current state of the art, and motivate and encourage new ideas and solutions in the sliding mode control area.

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