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Common features of analog sampled-data and digital filters design

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1. Introduction

Cascade realization of the analog ARC- and digital filters shows more common features. These relationships are especially evident in comparison of sampled-data and digital filters, namely biquadratic sections used in cascade design. Aim of this chapter is thus to show, how to effectively use the mentioned relationships in the optimized design of both the sampled-data and digital filters.
Here the most important role play possible transformations between sampled-data and digital biquadratic section structures, application of the sensitivity concept in digital filter design and optimization of dynamic properties in the digital and sampled-data filters.
The switched-current (SI) circuits were chosen as an "analog counterpart" of the digital filters, with respect to their full compatibility to the digital VLSI-CMOS technologies, lower supply voltage and wide dynamic range. In addition, principle of SI-circuit signal processing is rather similar to the digital ones, therefore arises possibility to use a "digital prototype" for the SI filter design. On the other hand, some procedures applied in SI filter design can be successfully applied in the optimized design of digital filters, especially digital biquadratic sections.
Content of the chapter is divided into the following parts:
A short introduction to SI circuit theory and principles of operation. Although the theory of SI circuits has been described in detail in several publications – see e.g. Toumazou et al. (1993), Toumazou et al. (1996), we consider appropriate to shortly introduce the basic of operation of SI circuits for better understanding. The dynamic current mirror, memory cell, integrator and differentiator are presented as the main building blocks – i.e. blocks indispensable in filter design.
The next section presents a new universal algorithm suitable for symbolic analysis of all types of sampled-data filters. The original approach using "memory transistor" or "memory transconductor" has been introduced in Bičák et al. (1999), Martinek et al. (2003), Bičák & Hospodka (2006) and was applied in newly developed libraries for symbolic analysis PraSCan and PraCAn of the MAPLE program.

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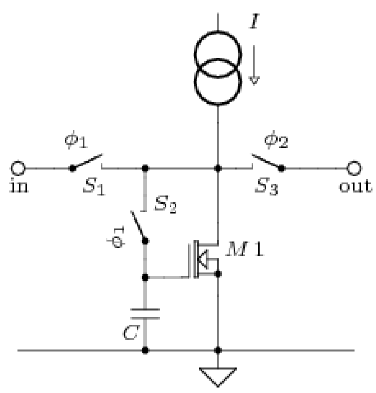
The main part of this chapter is an overview of possible biquad realization structures and follows the previous work Martinek & Tichá (2007). We turn attention to some aspects of the "digital prototype" approach in sampled-data biquads design. Here the first and second direct forms of the 2^{nd}-order digital filter were chosen as the prototypes. As a generalization of this approach the replacement of the memory cells in the basic structure by a simple BD integrator and differentiator is discussed. The structures obtained were compared in according to their sensitivity properties, an influence of SI building blocks losses and element values spread. The results obtained are demonstrated on the examples of the 2^{nd}-order biquad realizations.
The following section of the chapter is devoted to some auxiliary tools, suitable for digital- and sampled-data filters design.
The first concerns an application of sensitivity approach, a powerful tool in continuous-time biquadratic sections design. With respect to the discrete-time character of SI- and digital filters, the "equivalent sensitivities" are derived and used. A more detailed explanation of this approach has been published in Tichá (2006). The relevance of sensitivity computation in digital filter design can be more obvious, if we are aware of the correspondence between rounding errors in "digital area" and tolerances of element values in the "continuous-time area". Therein the sensitivities represent the measure for possible rounding without loss of the accuracy of the filter frequency response.
The second useful tool for filter optimized design is a symbolic analysis. The prospective approach leads via mathematical programs oriented to the symbolic mathematics. A suitable program for this purpose seems to be MAPLE, especially developed for symbolic computations. The symbolic analysis of analog circuits is supported in MAPLE by SYRUP library Riel (2007) and newly developed libraries PraSCan and PraCAn – see Bičák et al. (1999) and Bičák & Hospodka (2006). All the libraries represent simple, but very efficient universal tool for circuit analysis, similar to the SPICE program in numerical area. The mentioned libraries allow simple modeling of the basic building blocks of digital filters - i.e. memory cells, summers and multipliers. Usage of the extended library is demonstrated on the analysis of some typical examples of digital filters, represented by block diagrams. It is important to say, that the obtained transfer functions $H(z)$ can be easily post-processed in MAPLE environment and used for the optimized design of the simulated subsystems.
The final section summarizes the results achieved and the usefulness of the presented principles of optimized analog filter design usage in "digital area".

2. The basic of Switched-Currents technique

Switched-currents (SI), as the latest technique for sampled-data analogue circuits, play an important role in modern electronic system design. In comparison to switched-capacitor circuits, SI have some important advantages, particularly full compatibility to the digital VLSI-CMOS technologies, lower supply voltage and wider dynamic range, as mentioned in the previous section.
The basic SI-cell is shown in Fig 1. Switches $S_1 - S_3$ are controlled by 2-phase switching signal. A principle of operation corresponds to the current mirror - during phase ϕ_1 are switched S_1 and S_2 and circuit operates as the input of current mirror with low input resistance (input current $i_{in(nT)}$). The second phase ϕ_2 is a storage (or output) phase – S_3 is closed and output current $i_{out(nT+1/2)}$ flows into load. The function is characterized by equations Eq. (1) and (2).
To obtain transfer function $H(z) = z^{-1}$, it is necessary to use two basic cells connected in cascade, as shown in Fig. 2. Here is simultaneously shown, how to realize multiple outputs with different transfer gain constants.



$$i_{out(nT+1/2)} = i_{in(nT)} \tag{1}$$

$$H(z) = \frac{I_{out}(z)}{I_{in}(z)} = z^{-1/2} \tag{2}$$

Fig. 1. The basic SI-cell

Output terminal out 1 pertains to the "pure" memory cell, created by transistors M 1 and M 2 and switches S_1 to S_5. Outputs out 2 and out 3 combine the second basic cell (transistor M 2) together with transistors M 3 and M 4 creating "conventional" current mirrors. Such arrangement allows setting the gain constant $\alpha_{i,\, i=1,2}$ in the form (3) and (4), where W_k, L_k denote the channel width and length of transistor $M\,k$, $_{k=2,3,4}$. Note that ratios W/L can be normalized with respect to the channel parameters of the basic cell transistor - (in our case M 2).

$$H_2(z) = \frac{I_{out\,2}(z)}{I_{in}(z)} = \alpha_1\, z^{-1}\,; \qquad \alpha_1 = \frac{W_3/L_3}{W_2/L_2}\,, \tag{3}$$

$$H_3(z) = \frac{I_{out\,3}(z)}{I_{in}(z)} = \alpha_2\, z^{-1}\,; \qquad \alpha_2 = \frac{W_4/L_4}{W_2/L_2}\,. \tag{4}$$

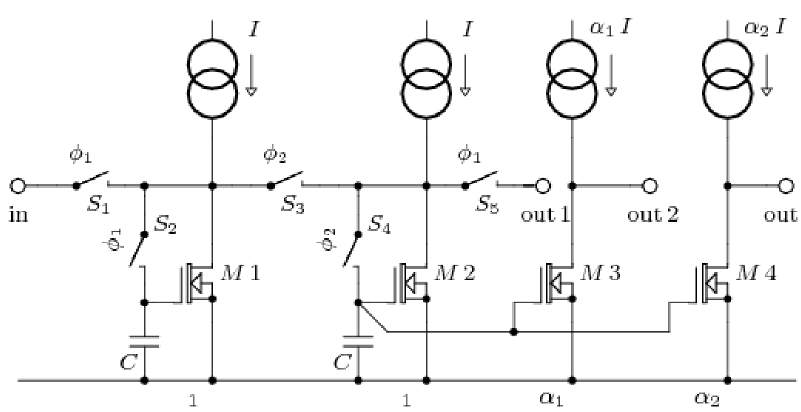


Fig. 2. Multiple-output SI memory cell

Higher-level blocks, as integrator and differentiator, can be derived from the memory cell by simple modification. In the case of integrator the output current samples are added to input, together with input signal. Resulting circuit diagram is shown in Fig. 3. Output signal is obtained under Eq. (5), corresponding to the "standard" backward-difference discrete integration. Corresponding transfer function is defined by Eq. (6).
If the switching phase of the switch S_1 is changed into ϕ_2, we obtain forward difference inverting integrator, whose transfer function is expressed by formula (7).

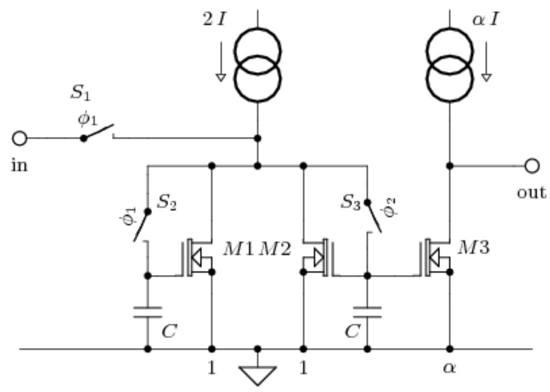


Fig. 3. Non-inverting BD integrator

$$i_{out(nT)} = i_{in(nT-1)} + i_{out(nT-1)} = \sum_{n=1}^{\infty} i_{in(nT)}\,; \tag{5}$$

$$H_{BD}(z) = \frac{I_{out}(z)}{I_{in}(z)} = \alpha\,\frac{z^{-1}}{1-z^{-1}}\,; \tag{6}$$

$$H_{FD}(z) = \frac{I_{out}(z)}{I_{in}(z)} = -\alpha\,\frac{1}{1-z^{-1}}\,. \tag{7}$$

In contrast to the SC- and continuous-time technique there are no problems with realization of differentiator SI building blocks. A simple example of Si-implementation is shown in Fig. 4. Similarly to an integrator, the differentiator was derived from the digital prototype using equation Eq. (8). Note that the simplified inverting BD differentiator ($\alpha = 1$) can be gained by removing the input current mirror (M1 and M2 transistors).

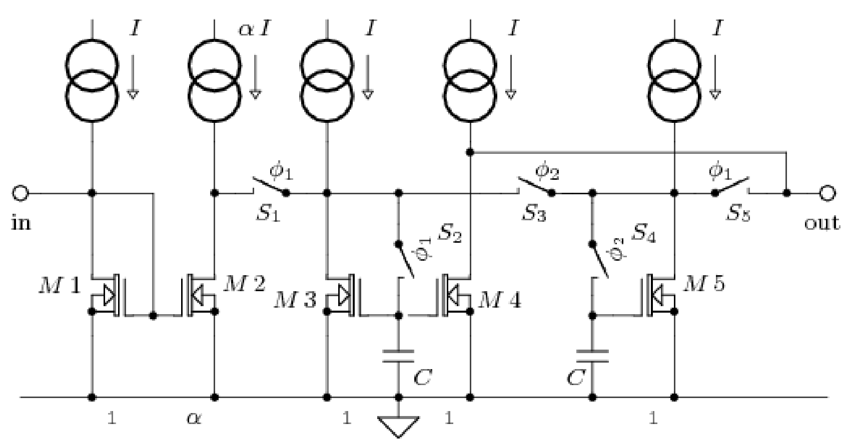


Fig. 4. Non-inverting BD differentiator

$$\frac{\partial\, i(t)}{\partial\, t} = \frac{\Delta\, i(t)}{\Delta\, t} = \frac{i_{nT} - i_{nT-1}}{T}\,; \tag{8}$$

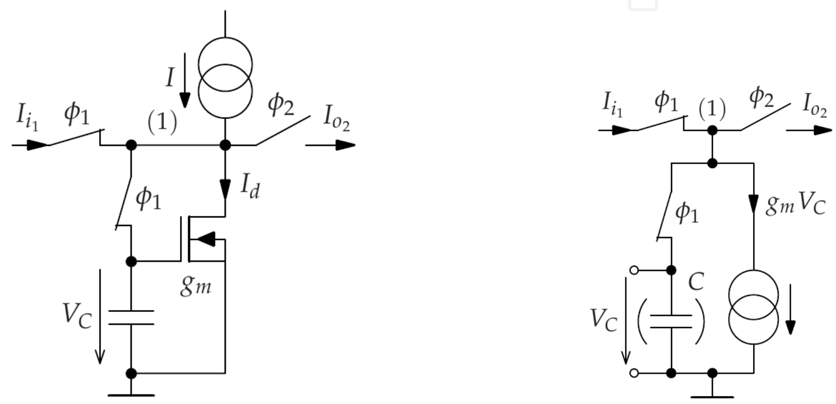
$$H_{BD}(z) = \frac{I_{out}(z)}{I_{in}(z)} = \alpha\,(1 - z^{-1})\,; \tag{9}$$

Similarly it is possible to create other SI-building blocks, suitable for current-mode signal processing. It is important to say, that presented schematics correspond to the simplest models of the “real” circuits, without discussion of their real implementation, behavior and further improvements. This is not topic of this chapter. More about SI-circuits and their applications can be found in Toumazou et al. (1993), Toumazou et al. (1996), Mucha (1999), Šubrt (2003), and others.

3. A symbolic analysis of SI circuits

This section describes method of SI circuit analysis based on modified algorithm for switched capacitor circuits, especially for symbolic analysis of idealized circuit. It made it more universal and useful - see Bičák et al. (1999).

The circuit description is based on modified nodal-charge equations - Kurth & Moschytz (1979); Yuan & Opal (2003); making possible to include resistive elements. The simple transformation of charges into currents is the main goal of the developed procedure. This leads to the correct evaluation of nodal voltages in the case of SI circuit. Modified capacitance matrix is possible to use for description of the switched-current (SI) basic cell and complex SI circuit by this way. Let us consider basic configuration of dynamic current-mirror shown in Fig. 5.



(a) Basic SI-cell (b) Linearized model

Fig. 5. Basic cell of SI circuits and linearized model.

To accomplish the starting conditions of the charge-voltage description, the SI cell is modeled by voltage controlled charge source (instead of current source) with transfer gain g_Q, memory capacitor C and ideal switches. The gain g_Q has the same numeric value as the transistor transconductance g_m, but different unit. Modified model is shown in Fig. 6.

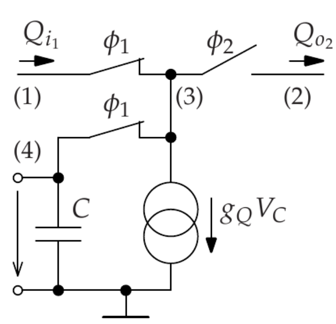


Fig. 6. Model of SI cell used for analysis by charge-voltage equations.

The resultant capacitance matrix of the SI cell model in Fig. 6 can be written in the following form

$$\begin{bmatrix} Q_{i_1} \\ \hline Q_{o_2} \\ 0 \end{bmatrix} = \left[\begin{array}{c|cc} C + g_Q & 0 & -z^{-1/2}C \\ \hline 0 & 0 & g_Q \\ -z^{-1/2}C & 0 & C \end{array}\right] \times \begin{bmatrix} V_{1_1} \\ \hline V_{2_2} \\ U_{4_l} \end{bmatrix} \tag{10}$$

The charge transfer from phase ϕ_1 to phase ϕ_2 is than

$$H_Q = \frac{Q_{o_2}}{Q_{i_1}} = -\frac{g_Q\, z^{-1/2}}{g_Q + C(1 - z^{-1})}. \tag{11}$$

The transfer function H_Q contains additional terms, corresponding "parasitic" changes of memory capacitor charge. This effect can be eliminated in idealized circuit description by minimizing capacitance C. When $C \to 0$, the equation (11) limits into the correct known formula (2)

$$H_{id} = \lim_{C \to 0} H = -z^{-1/2} \tag{12}$$

In fact, the described procedure corresponds to the charge $\to$ current transformation in the circuit description (in other words, "charge is divided by time"). In this case, the "starting" description of VCCS by voltage controlled charge source can be turned back ($g_Q \to g_m$)[1] and original nodal voltage-charge description changes into voltage-current equations. Note that presented transformation does not change the numeric value of VCCS gain (transconductance g_m).

It is important to say, the procedure of capacitance zeroing should be performed as the last step of transfer evaluation to avoid the complication in description of phase-to-phase energy transfer. The symbolic or special case of semi-symbolical analysis is necessary with respect to correct simulation result. This fact limits the described method of memory capacitor zeroing. This problem can be solved by special model of the SI cell shown in following figure, Fig. 7.

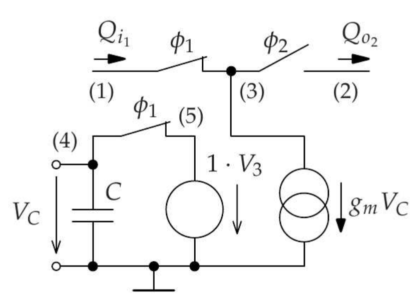


Fig. 7. Model of SI cell with separator.

This circuit can be described by following equations in matrix representation.

$$\left[\begin{array}{c} Q_{i_1} \\ 0 \\ \hline Q_{o_2} \\ 0 \\ 0 \end{array}\right] = \left[\begin{array}{cc|ccc} 0 & g_Q & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & g_Q & 0 \\ 0 & -z^{1/2}C_1 & 0 & C_1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right] \times \left[\begin{array}{c} V_{1_1} \\ V_{4_1} \\ \hline V_{2_2} \\ V_{4_2} \\ U_{5_2} \end{array}\right] \tag{13}$$

The same transfer function as in relation (12) is obtained by computation of Q_{o_2}/Q_{i_1} from this matrix.

This representation is possible to implement directly into the C-matrix for SC circuit description. By this way idealized SI circuit can be analyzed in programs for SC circuit analysis without symbolic formulation of results and without any limit calculation. Larger matrix is the certain disadvantage of the method.

[1] The transfer function does not include transconductances in this elementary example.

Direct description of SI cell can be applied in case of special program for idealized SI circuit analysis. Direct matrix representation of SI cell from Fig. 5 for switching in phase ϕ_1 and also in phase ϕ_2 has the following expressions in case of circuit switched in two phases.

$$\begin{array}{c||c|c} & V_{1_1} & V_{1_2} \\ \hline I_{i_1} & g_m & 0 \\ I_{i_2} & z^{-1/2} g_m & 0 \end{array} \quad \text{for } \phi_1, \qquad \begin{array}{c||c|c} & V_{1_1} & V_{1_2} \\ \hline I_{1_1} & 0 & z^{-1/2} g_m \\ I_{1_2} & 0 & g_m \end{array} \quad \text{for } \phi_2, \tag{14}$$

where $I_{1_2} = -I_{o_2}$ for circuit switched in phase ϕ_1 and $I_{1_1} = -I_{o_1}$ for circuit switched in phase ϕ_2.

Now the currents are used instead of charges – it is a case of modified node voltages method applied for circuit switched in two phases. In our case the circuit contains only one non-grounded node. It means the matrix has only 2×2 dimension. The memory effect is here described by current source controlled by voltage in phase ϕ_1 and phase ϕ_2 *with non zero transfer (transconductance) from one phase to the other* as can be seen from the above mentioned matrix form.

Presented procedure leads to the simple and easy description of SI structures and their effective analysis in both symbolic and numerical form.

4. Basic SI-biquad structures

This part intends to discuss some aspects of the "digital prototype" approach in sampled-data biquads design.

It is important to say, that many applications of SI technique in sampled-data filter design published from the nineties are mostly based on a two-integrator structure in the case of biquads, or operational simulation of LC-prototype – see e.g. Toumazou et al. (1993). But the principle of SI-circuit operation is rather similar to the digital ones, so there arises possibility to use a "digital prototype" for SI-filter design.

The first and second direct forms of the 2^{nd}-order digital filter were chosen as the prototypes. Firstly, the design using SI memory cells was considered; in this case the final circuit should preserve the dominant features of the prototype. As a generalization of this approach the replacement of the memory cells in the basic structure by a simple BD integrator and differentiator was investigated. The structures obtained were compared in according to their sensitivity properties, an influence of SI building blocks losses and circuit element values spread. The results are demonstrated on the examples of the typical 2^{nd}-order biquad realizations.

As mentioned, the selected prototypes are known as the first and the second direct-form digital filter structures, characterized by common transfer function (15) – see e.g. Antoniou (1979), Mitra (2005).

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{15}$$

After redrawing, following the SI technique, the block diagrams shown in Figs. 8 and 9 were obtained. Here the symbol *CM* denotes current copier (multiple-output current mirror), FB means SI building block, for the first time the SI memory cell. The transfer function coefficients are set by current copier gains a_i, b_i, as evident from Fig. 8 and Fig. 9.

With respect to the practical realization aspects, the direct-form 2 structure seems to be more suitable because of simpler input and output current copiers. Multiple outputs of the SI-building blocks do not mean design complications, as is shown in Fig. 2 – see Section 2.

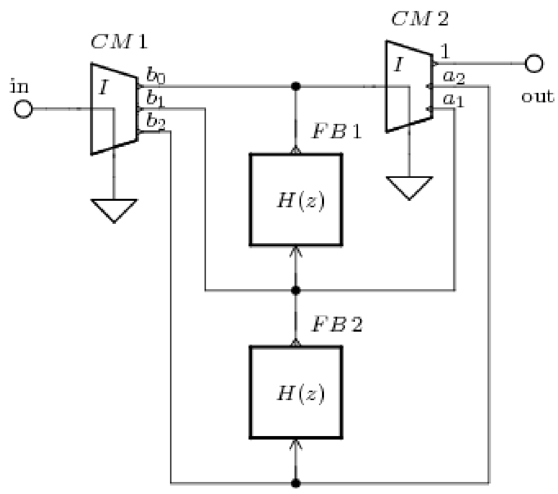


Fig. 8. Case I. SI circuit

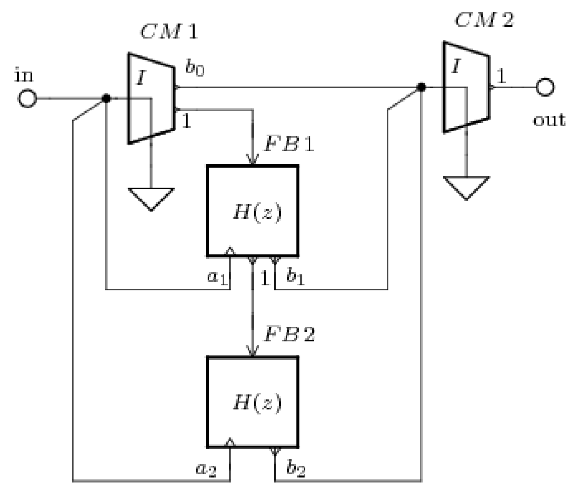


Fig. 9. Case II. SI circuit

To obtain a more complex overview about the circuits behavior, the following versions were considered:

1. The SI-FBs are realized by memory cells in compliance with the digital prototype. These are simple in the case of direct form 1, multiple-output under Fig. 2 in the case of direct form 2. The weighted outputs are set using changed W/L output transistor ratios.
2. Memory cells are replaced by non-inverting BD and FD integrators.
3. SI-FBs are realized by BD differentiators under Fig. 4, described by the transfer function $H(z) = \alpha\,(1 - z^{-1})$.

The following evaluative criteria were used for comparing all the considered structures:

- *Sensitivity properties:* With respect to the discrete-time character of SI circuits, the "equivalent sensitivity" approach has been applied. A more detailed explanation of this approach has been published in Ref. Tichá (2006), and it is shortly indicated in Section 5.
- *Losses influence:* The important imperfections of SI circuits are caused by parasitic output conductances of SI cells. In the following, these parasitics will be characterized by output conductance g_o or by ratio $x_g = \frac{g_m}{g_o}$, where g_m represents transistor transconductance.
- *Transistor parameters spread:* With respect to the technological limitations, the limits of spread $\alpha = W/L$ of transistors are crucial. In our considerations the maximum available spread is expected to be in the interval $\alpha_{max}/\alpha_{min} < 50$. In general, the given limit influences the maximum ratio of sampling frequency f_c to ω_{0eq}.

The necessary symbolic analysis were made using MAPLE libraries PraSCan and PraCAn, developed by Bičák & Hospodka (2006), Bičák et al. (1999) for symbolic and numerical analysis of sampled-data circuits.

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4.1 Results obtained

Sensitivity evaluation:

At first, let us consider the "original SI networks" under Figs. 8 and 9. The transfer function of both structures corresponds directly to the Eq. (15), and the sensitivity properties can be expressed using procedure described in Sec. 5 in the form (25) and (26), as the functions of parameters a_1, a_2. More suitable for practical design are the sensitivity functions of "continuous-time" $H(s)$ parameters ω_0, Q and sampling period T. In this case the sensitivities can be expressed by (29) and (30).

Evaluated sensitivity graphs of ω_{0eq}- and Q_{eq}-sensitivities on f_c/f_0 ratio in Fig. 10 and Fig. 11 show unsuitable values for higher x_c. This fact limits the use of such biquads to lower values of x_c.

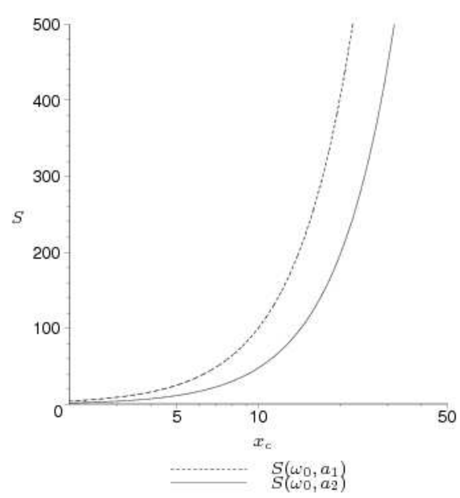


Fig. 10. $S_{a_i}^{\omega_{0eq}} = f(x_c)$

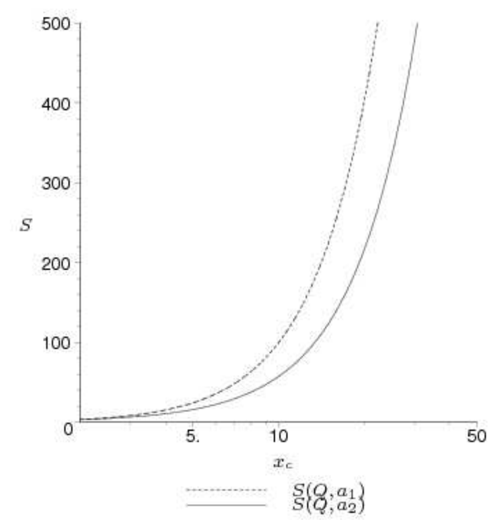


Fig. 11. $S_{a_i}^{Q_{eq}} = f(x_c)$

The modified structures containing integrators or differentiators show better sensitivity properties as is evident from Fig. 12 and Fig. 13. The graphs pertain to the non-inverting BD integrator version of Case I structure; similar behavior was found in versions based on FD integrators, mixed BD-FD integrator combinations or differentiator based circuits.

This behavior can be easily explained, because the introduced integrator- and differentiator-type structures are in fact the special cases of SFG or state-variable based biquad design.

Note that the ω_{0eq} and Q_{eq} sensitivities to the gain constants α_i, $_{i=1,2}$ of integrator- and differentiator-type building blocks are typically 0.5 - 1 and decrease to the limit value $S_{a_i}^{Q_{eq}} = 0.5$ for $x_c \gg 1$. Similar values were obtained in the case of ω_{0eq} sensitivities. Table 1 illustrates the sensitivity properties of the chosen Case I structure versions for starting parameters $f_0 = 2\,\text{kHz}$, $f_c = 48\,\text{kHz}$, $Q = 1/\sqrt{2}$.

Here symbol "M" denotes the "original" structure containing SI memory cells, "BD int" denotes the version using BD integrators and similarly "FD int" denotes the version using FD integrators. Case "FD+BD int" corresponds to the arrangement where FB1 block is implemented as the FD integrator and FB2 block as the BD integrator. The order of FBs is important, a changed arrangement results in increased sensitivities. The last row contains sensitivity values for a BD differentiator based circuit.

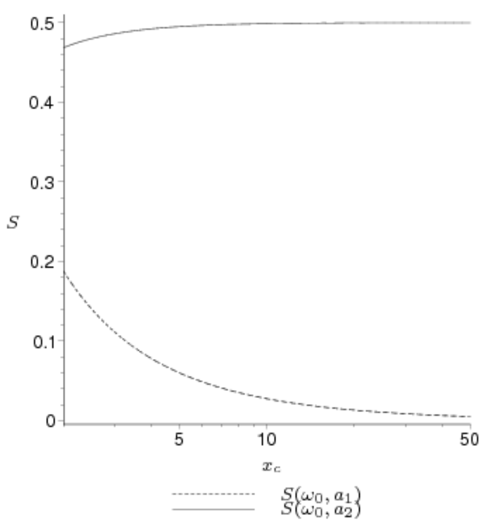


Fig. 12. $S_{a_i}^{\omega_{0eq}} = f(x_c)$

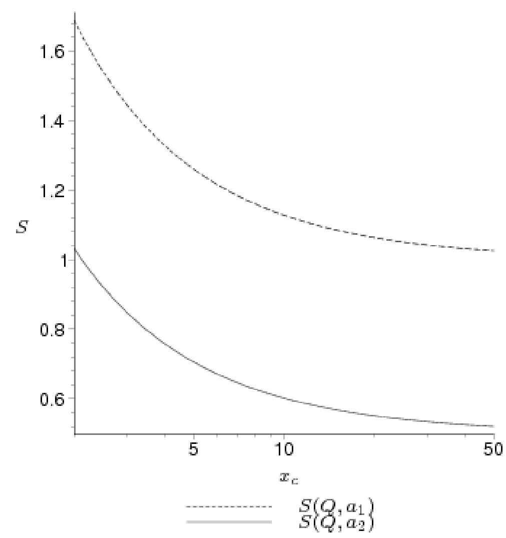


Fig. 13. $S_{a_i}^{Q_{eq}} = f(x_c)$

Type	$S_{a_1}^{\omega_{0eq}}$	$S_{a_2}^{\omega_{0eq}}$	$S_{a_1}^{Q_{eq}}$	$S_{a_2}^{Q_{eq}}$	$S_{\alpha_1}^{Q_{eq}}$	$S_{\alpha_2}^{Q_{eq}}$
M	-14.6	5.97	-14.1	8.42	-	-
BD int	0.109	0.491	-1.29	0.693	-0.601	0.693
FD int	-0.075	0.491	-0.739	0.323	-0.416	0.323
FD+BD int	-0.092	0.508	-0.907	0.491	-0.416	0.491
BD diff	-0.075	-0.416	-0.739	0.416	-0.323	0.416

Table 1. Sensitivity properties

Losses influence:
As mentioned, the finite output conductances of the basic SI cells and current copiers (current mirrors) are crucial in SI circuit design together with the number of blocks in the signal path. With regard to this, it is necessary to distinguish between the Case I and Case II structures. Some simulations showed slightly better behavior of the Case II arrangement. Simultaneously it is important to take into account the finite "on" resistance of switches. Especially differentiator-based circuits are sensitive to switch imperfections.
Table 2 documents typical frequency response errors for the realizations introduced in Table 1. Here the typical ratios $x_g = g_m/g_o = 200$ and r_{on} switches equal to the input resistance of current building blocks were considered.

Transistor parameters spread
This is markedly determined by the designed structure type and f_c/f_0 ratio. For illustration, let us assume the LP biquad designed under the same conditions documented in Table 1 and Table 2.
As is evident from Table 3, the maximum values spread shows the memory cell based version, the max-to-min ratio equals 114.3. The differentiator and integrator based versions are less demanding, the max-to-min ratio was evaluated from 48.5 to 69.9.

Type	$\bar{\varepsilon}$	ε_{max}	$\varepsilon(0)$	$\varepsilon(\omega_0)$
M-Case I	0.0346	0.426	0.426	0.176
M-Case II	0.0274	0.335	0.335	0.142
BD int Case I	0.0136	0.123	0.106	0.0853
BD int Case II	0.0147	0.139	0.126	0.0905
FD int Case I	0.0149	0.127	0.109	0.0915
BD diff Case I	0.0124	0.116	0.109	0.0458

Table 2. Frequency response errors

Note that the last versions have two free parameters α_1, α_2 which can be exploited for design optimization; unfortunately changes to these parameters do not allow any minimization of values spread.

Type	b_0	b_1	b_2	a_1	a_2
M	0.0143	0.285	0.0143	-1.635	0.692
BD int	0.0143	$\frac{0.057}{\alpha_1}$	$\frac{0.057}{\alpha_1 \alpha_2}$	$\frac{0.365}{\alpha_1}$	$\frac{0.057}{\alpha_1 \alpha_2}$
FD int	0.0206	$\frac{0.0824}{\alpha_1}$	$\frac{0.0824}{\alpha_1 \alpha_2}$	$-\frac{0.3626}{\alpha_1}$	$\frac{0.0824}{\alpha_1 \alpha_2}$
FD+BD int	0.0206	0	$\frac{0.0824}{\alpha_1 \alpha_2}$	$-\frac{0.445}{\alpha_1}$	$\frac{0.0824}{\alpha_1 \alpha_2}$
BD diff	1	$-\frac{1}{\alpha_1}$	$-\frac{0.25}{\alpha_1 \alpha_2}$	$\frac{4.402}{\alpha}$	$\frac{12.139}{\alpha_1 \alpha_2}$

Table 3. design parameters for $f_0 = 2\,\text{kHz}$

Type	b_0	b_1	b_2	a_1	a_2
M	0.00391	0.00781	0.00391	-1.816	0.831
BD int	0.00391	$\frac{0.0156}{\alpha_1}$	$\frac{0.0156}{\alpha_1 \alpha_2}$	$\frac{0.184}{\alpha_1}$	$\frac{0.0156}{\alpha_1 \alpha_2}$
FD int	0.0047	$\frac{0.0188}{\alpha_1}$	$\frac{0.0188}{\alpha_1 \alpha_2}$	$-\frac{0.184}{\alpha_1}$	$\frac{0.0156}{\alpha_1 \alpha_2}$
FD+BD int	0.0047	0	$\frac{0.0188}{\alpha_1 \alpha_2}$	$-\frac{0.203}{\alpha_1}$	$\frac{0.0188}{\alpha_1 \alpha_2}$
BD diff	1	$-\frac{1}{\alpha_1}$	$-\frac{0.25}{\alpha_1 \alpha_2}$	$\frac{9.804}{\alpha}$	$\frac{53.21}{\alpha_1 \alpha_2}$

Table 4. design parameters for $f_0 = 1\,\text{kHz}$

The influence of the f_c/f_0 ratio to the transistor parameters spread is demonstrated in Table 4, showing parameter changes for the lowered $f_0 = 1\,\text{kHz}$ from the previous design.

In this case the max-to-min ratio increases for the memory cell version to 464.4. The best result is obtained for the differentiator based circuit, where the max-to-min ratio equals 212.8. It is evident that such designs are hardly realizable and strongly require lower sampling frequency.

5. Sensitivity approach in discrete-time filters design

The sensitivity approach is a worthwile tool for the optimized design of analog continuous-time and sampled-data filters. Particularly the design of biquadratic sections for cascade realization of higher-order filters is significantly influenced by the sensitivity properties of the considered circuits. Mainly the sensitivities of ω_0- and Q- parameters to the filter elements changes serve as the effective criterion for suitable circuit structure selection and design optimization, because ω_0 and Q uniquely determine the frequency response shape.
The "main" sensitivities of the biquadratic transfer function $H(s)$ (16) are defined by formulas (17), where x_i means active and passive circuit elements. The ω_0 and Q parameters are defined by (18) as the functions of the real and imaginary parts σ_1, ω_1 of the complex-conjugate poles of the 2^{nd}-order biquadratic transfer function (16).

$$H(s) = \frac{k_2\, s^2 + k_1\, s + k_0}{s^2 + \frac{\omega_0}{Q}\, s + \omega_0^2} \tag{16}$$

$$S_{x_i}^{\omega_0} = \frac{\partial \omega_0}{\partial x_i}\,\frac{x_i}{\omega_0}\,; \qquad S_{x_i}^{Q} = \frac{\partial Q}{\partial x_i}\,\frac{x_i}{Q}\,; \tag{17}$$

$$\omega_0 = \sqrt{\sigma_1^2 + \omega_1^2}\,; \qquad Q = \frac{\omega_0}{2\,\sigma_1}\,. \tag{18}$$

Sensitivity concept is less usual in the field of the digital filters, because there is not a direct equivalent of the ω_0 and Q parameters in the s-plane to the similar parameters in z-plane. Nevertheless the relevance of sensitivity usage in digital filter design can be more obvious, if we are aware of the correspondence between rounding errors in "digital area" and tolerances of circuit element values in the "continuous-time" area. Here the sensitivities represent the measure for possible rounding without loss of the accuracy of the filter frequency response. Simultaneously, sensitivities can help to solve problems with the optimum choice of the realization structure with respect to the "non-standard" design conditions, e.g. in design of the digital filters and equalizers for audio signal processing.
To apply sensitivity approach in digital filter design effectively, it is necessary to formularize equivalent sensitivity parameters, transforming z-plane parameters into s-plane and evaluate them like functions of $H(z)$. Such a procedure, described in Tichá (2006), will be presented in the following.

5.1 Equivalent sensitivity evaluation

Let us assume "standard" 2^{nd}-order transfer function $H(z)$ in the form (19). The equivalent parameters ω_0 and Q can be obtained using an appropriate transformation of $H(z)$ into s-plane and comparison to the ordinary form of $H(s)$ under (16)

$$H(z) = \frac{b_0 + b_1\, z^{-1} + b_2\, z^{-2}}{1 - a_1\, z^{-1} - a_2\, z^{-2}}\;; \tag{19}$$

To obtain the generally valid relationship, the $z - s$ transformation should be symbolic. Using inverse bilinear transformation (20) of $H(z)$

$$z = \frac{2 + s\,T}{2 - s\,T} \tag{20}$$

we obtain equivalent $H_{eq}(s)$ in the form (21) and after formal rearrangement the final form (22) comparable to (16).

$$H_{eq}(s) = \frac{T^2\,(b_0 - b_1 + b_2)\,s^2 + 4\,T\,(b_0 - b_2)\,s + 4\,(b_0 + b_1 + b_2)}{T^2\,(1 + a_1 - a_2)\,s^2 \;\; + 4\,T\,(a_2 + 1)\,s + 4\,(1 - a_1 - a_2)}\,; \tag{21}$$

$$H_{eq}(s) = \frac{\frac{(b_0 - b_1 + b_2)}{1 + a_1 - a_2}\,s^2 + 4\,\frac{(b_0 - b_2)}{T(1 + a_1 - a_2)}\,s + 4\,\frac{b_0 + b_1 + b_2}{T^2(1 + a_1 - a_2)}}{s^2 + 4\,\frac{(a_2 + 1)}{T(1 + a_1 - a_2)}\,s + 4\,\frac{1 - a_1 - a_2}{T^2(1 + a_1 - a_2)}}\,. \tag{22}$$

A comparison of (22) to (16) gives

$$\omega_{0eq} = \frac{2}{T}\sqrt{\frac{1 - a_1 - a_2}{1 + a_1 - a_2}}\,; \tag{23}$$

$$Q_{eq} = \frac{\sqrt{(1 - a_2)^2 - a_1^2}}{2\,(1 + a_2)}\,. \tag{24}$$

Now it is possible to express the equivalent sensitivity of ω_{0eq} and Q_{eq} to the denominator coefficients a_1 and a_2 using formula (17). The symbolic form of the evaluated sensitivities is as follows

$$S_{a_1}^{\omega_0} = -\frac{a_1\,(1 - a_2)}{(1 - a_2)^2 - a_1^2}\,; \qquad S_{a_1}^{Q} = -\frac{{a_1}^2}{(1 - a_2)^2 - a_1^2}\,; \tag{25}$$

$$S_{a_2}^{\omega_0} = \frac{a_1\,a_2}{(1 - a_2)^2 - a_1^2}\,; \qquad S_{a_2}^{Q} = \frac{a_2\,\left[{a_1}^2 - 2\,(1 - a_2)\right]}{(1 + a_2)\,\left[(1 - a_2)^2 - a_1^2\right]}\,. \tag{26}$$

In some cases it is suitable to express the equivalent sensitivities as the functions of ω_0, Q and T, or $x_c = f_c/\omega_0$. To extend the expressions (25) - (26), it is necessary to transform coefficients a_1, a_2 into s-plane using backward bilinear transformation of $H(z)$ denominator. Doing this, the following expressions were gained:

$$a_1 = \frac{2\,(4 - \omega_0^2\,T^2)\,Q}{2\,\omega_0\,T + 4\,Q + \omega_0^2\,T^2\,Q}\,; \tag{27}$$

$$a_2 = -\frac{-2\,\omega_0\,T + \omega_0^2\,T^2\,Q + 4\,Q}{2\,\omega_0\,T + 4\,Q + \omega_0^2\,T^2\,Q}\,. \tag{28}$$

Applying (27) and (28) in Eqs. (25) to (26) we obtain the modified sensitivity expressions (29) – (30). The parameter x_c is defined by Eq. (31).

$$S_{a_1e}^{\omega_0} = -\frac{(16\,x_c^4 - 1)}{16\,x_c^2}\,; \qquad S_{a_1e}^{Q} = -\frac{(4\,x_c^2 - 1)^2}{16\,x_c^2}\,; \tag{29}$$

$$S_{a_2e}^{\omega_0} = \frac{x_c^2}{2} - \frac{x_c}{4\,Q} + \frac{1}{16\,x_c\,Q} - \frac{1}{32\,x_c^2}\,; \quad S_{a_2e}^{Q} = -\frac{1}{4} + \frac{x_c^2}{2} + \frac{(1 + 4x_c)\,(4Q^2 - 1)}{16\,Q\,x_c} + \frac{1}{32\,x_c^2}\,. \tag{30}$$

$$x_c = \frac{1}{T\,\omega_0} = \frac{f_c}{\omega_0} \tag{31}$$

The formulas obtained are valid directly for the 1^{st} and the 2^{nd} canonic direct form of the digital filters – see Laipert et al. (2000), Antoniou (1979), Mitra (2005) and others. For the other 2^{nd}-order structures it is necessary to express the transfer function $H(z)$ coefficients $a_i,\ b_i,\ _{i=0,1,2}$ (19) as the functions of the analyzed structure parameters. The practical use of this will be explained in the following parts.

5.2 Sensitivity properties of the direct canonic forms of digital filters

As mentioned, the sensitivity properties to the parameters of the 1^{st} and the 2^{nd} direct form of the digital 2^{nd}-order filters are straightly specified by above presented formulas, because the coefficients are determined by the multipliers and adders constants of the filter block diagram. The filter general sensitivity properties can be in this case characterized preferably by modified equations (29) and (30) as the functions of equivalent Q-factor and the ratio x_c given by eq. (31). The following figures Fig. 14 and Fig. 15 show the sensitivity $S_{a_{1,2}}^{\omega_{0eq}}$ and $S_{a_{1,2}}^{Q_{eq}}$ as functions of Q_{eq}.

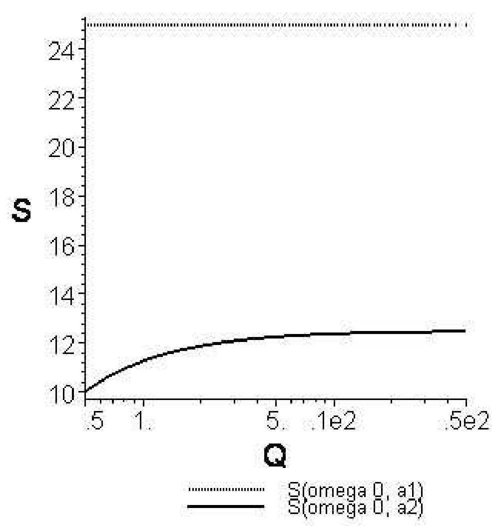


Fig. 14. $S_{a_{1,2}}^{\omega_0} = f(Q)$

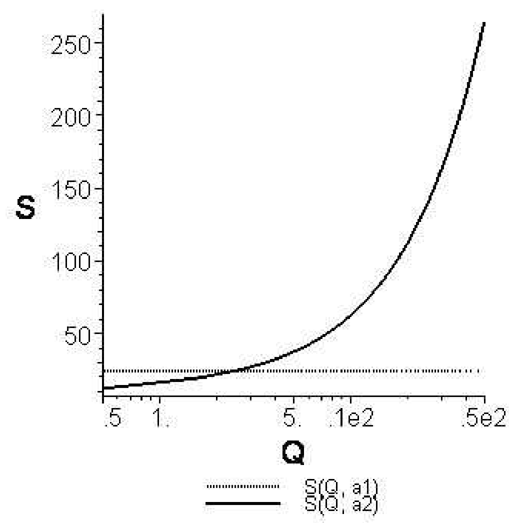


Fig. 15. $S_{a_{1,2}}^{Q} = f(Q)$

As evident, $S_{a_1}^{\omega_{0eq}}$ together with $S_{a_1}^{Q_{eq}}$ do not depend on Q-factor value, in contrast to the $S_{a_2}^{\omega_0}$ sensitivities. Note that sensitivities values are higher in comparison to the similar analogue realizations.

From the practical point-of-view the Figs. 16 and 17 are more important. Here the $S_{a_{1,2}}^{\omega_{0eq}}$ and $S_{a_{1,2}}^{Q_{eq}}$ sensitivities are depicted in dependence of ratio x_c, thus indirectly as the functions of ω_{0eq} and T. These sensitivities are significantly higher than the previous ones and rapidly increase for $x_c \geq 10$. This bears to the known fact, that direct forms of digital filters are less appropriate for such implementations, where the sampling frequency is relative high.

5.3 Digital filters derived from SFG graph

These filters are analogous to the continuous-time 2^{nd}-order filters designed on two-integrator feedback loop. A typical example of such a filter is shown in Fig. 18. Transfer function of this filter given by Eq. (32) was evaluated using modified SYRUP library in the mathematical program MAPLE – see Tichá & Martinek (2007).

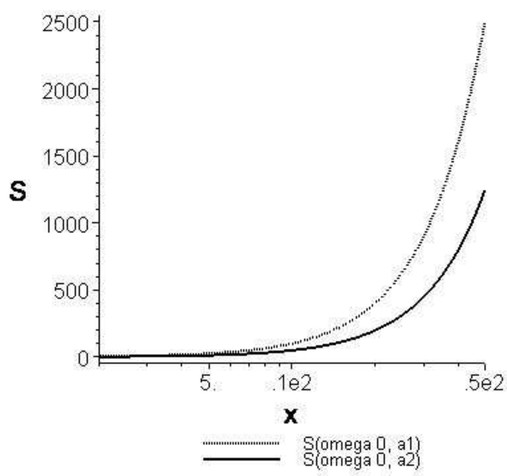


Fig. 16. $S^{\omega_0}_{a_{1,2}} = f(x)$

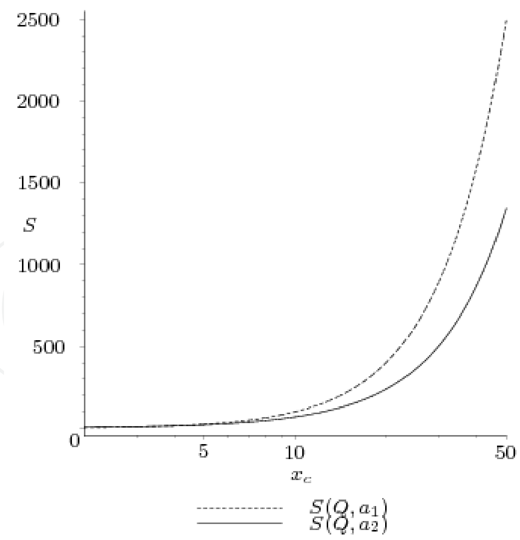


Fig. 17. $S^{Q}_{a_{1,2}} = f(x)$

A sensitivity evaluation was made according to the previous example. The results are as follows:

$$H(z) = \frac{a_5\, z^2 + (a_1 - a_5 + a_6)\, z - a_6}{(1 - a_4)\, z^2 - (2 + a_2 - a_4)\, z + 1}\,; \tag{32}$$

$$\omega_{0eq} = \frac{2}{T}\sqrt{-\frac{a_2}{4 + a_2 - 2\,a_4}}\,; \tag{33}$$

$$Q_{eq} = \frac{\sqrt{a_2\;(2\,a_4 - a_2 - 4)}}{2\,a_4}\,. \tag{34}$$

The corresponding sensitivities of ω_{0eq} and Q_{eq} to the $H(z)$ denominator coefficients a_i have the form (35) to (38), and the modified sensitivities the form (39) to (42). Note that parameter x_c is defined by Eq. (31)

$$S^{\omega_0}_{a_2} = \frac{2 - a_4}{4 + a_2 - 2\,a_4}\,; \tag{35}$$

$$S^{Q}_{a_2} = \frac{2 + a_2 - a_4}{4 + a_2 - 2\,a_4}\,; \tag{36}$$

$$S^{\omega_0}_{a_4} = \frac{a_4}{4 + a_2 - 2\,a_4}\,; \tag{37}$$

$$S^{Q}_{a_4} = -\frac{4 + a_2 - a_4}{4 + a_2 - 2\,a_4}\,; \tag{38}$$

$$S^{\omega_0}_{a_2 m} = \frac{1}{2} + \frac{1}{8\,x_c^2}\,; \tag{39}$$

$$S^{Q}_{a_2 m} = \frac{1}{2} - \frac{1}{8\,x_c^2}\,; \tag{40}$$

$$S^{\omega_0}_{a_4 m} = -\frac{1}{4\,x_c\,Q}\,; \tag{41}$$

$$S^{Q}_{a_4 m} = -1 + \frac{1}{4\,x_c\,Q}\,. \tag{42}$$

Similarly to the previous example the evaluated sensitivities can be presented as the functions of Q and x_c. The graphical representation of the functions $S^{\omega_0}_{a_i} = f(Q)$ and $S^{Q}_{a_i} = f(Q)$; $_{i=2,3,4}$ for given $x_c = 5$ is in Fig. 19. The graphs of functions $S^{\omega_0}_{a_i} = f(x_c)$ and $S^{Q}_{a_i} = f(x_c)$; $_{i=2,4}$ for $Q = 2$ are shown in Figs. 20.

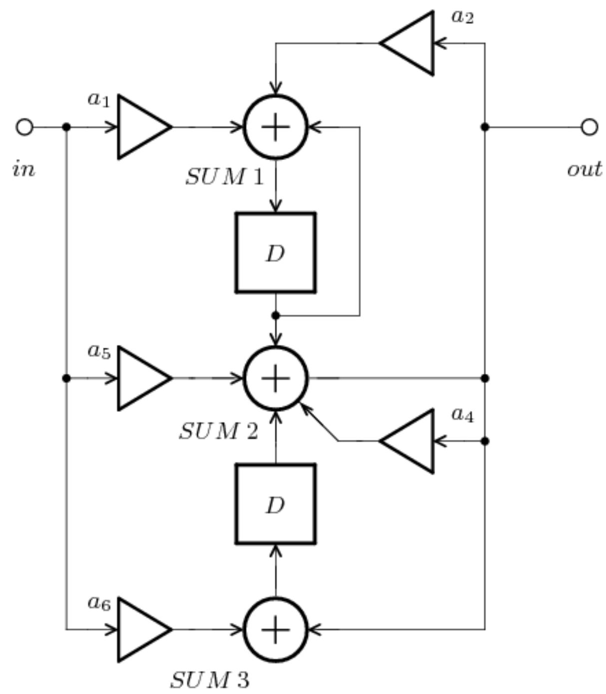
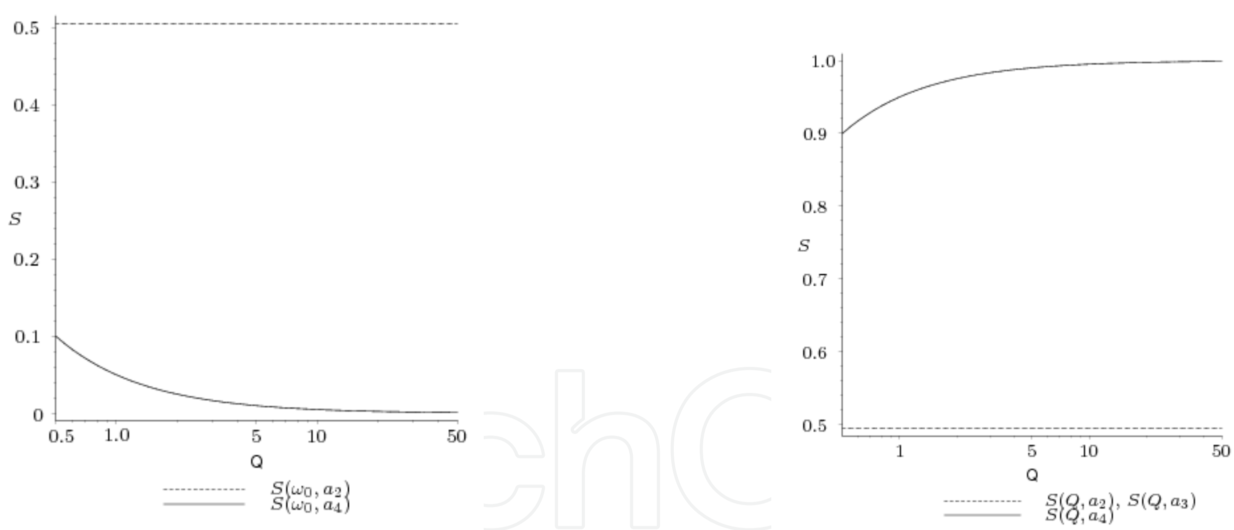


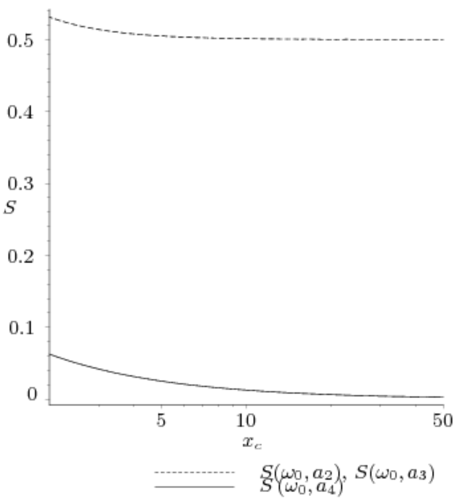
Fig. 18. Digital 2^{nd}-order integrator-based filter



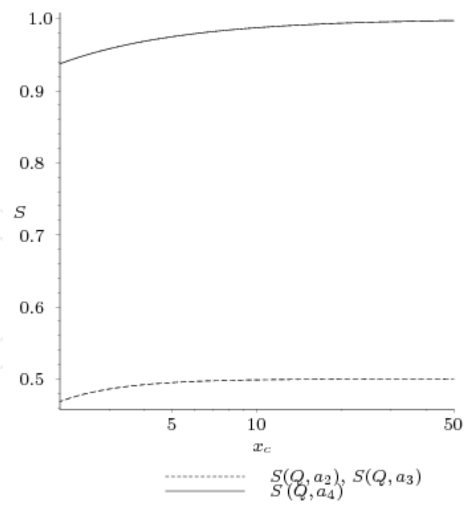
(a) $S_{a_{2,4}}^{\omega_0} = f(Q)$ (b) $S_{a_{2,4}}^{Q} = f(Q)$

Fig. 19. Sensitivities $S_{a_{2,4}}^{\omega_0} = f(Q)$ and $S_{a_{2,4}}^{Q} = f(Q)$ for $x_c = 5$.

In comparison to the direct-form structure all the sensitivities are considerably smaller and do not exceed unit value. It is important to emphasize the sensitivity independence from ratio x_c. It means that such a filter can be implemented successfully under non-standard conditions, where the limited word length or high ratio of ω_0 and f_c lead to the significant frequency response inaccuracy or filter instability.



(a) $S^{\omega_0}_{a_{2,4}} = f(x)$



(b) $S^{Q}_{a_{2,4}} = f(x)$

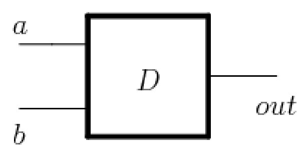
Fig. 20. Sensitivities $S^{\omega_0}_{a_{2,4}} = f(x_c)$ and $S^{Q}_{a_{2,4}} = f(x_c)$ for $Q = 2$.

6. A tool for symbolic analysis of digital filters

Symbolic and semi-symbolic analysis is considered to be an efficient tool for design and optimization of electrical and electronic circuits, not only analogue, but also digital. During the last period many specialized programs were developed for this purpose, but the most of them do not allow the direct post-processing of the results obtained. The more prospective approach is based on the use of mathematical programs oriented to the symbolic mathematics. Here the MAPLE program, especially developed for symbolic computations, seems to be the most suitable for this purpose. The symbolic analysis of analogue circuit is supported in MAPLE program by the SYRUP library Riel (2007). The SYRUP represents simple, but very efficient universal tool for circuit analysis, similar to the SPICE program in the circuit numerical analysis area.
As shown in the following, the SYRUP library can be easily adapted for the digital filters symbolic analysis as well. This assertion results from the fact, that circuit equations describing the digital filter block diagrams are very similar to the ones describing common analogue circuits. It leads to the direct use of the modified node-voltage equations method after completing the basic elements library. In contrast to the commonly used programs for circuit analysis, the input language of the SYRUP library is very flexible and allows to create models of the digital filter building block by a simple way.

6.1 The MAPLE-SYRUP library extension

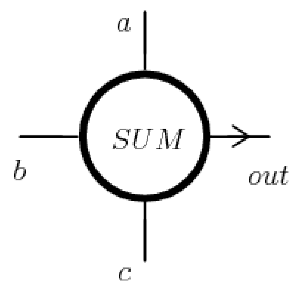
To analyze digital filter block diagrams using SYRUP, it is necessary to complete the basic set of circuit elements models. The most important "digital" building blocks are the delay element *D* and general multiple-input summing element *SUM*. The first of them is presented in Fig. 21 and the second in Fig. 22. Note that *A* in the summing element equation means summer gain; i.e. the multiplication operation can be included into this element. Nevertheless, the multiplication can be realized independently as well by some of "standard" library elements.



$$Y_{out}(z) = [X_a(z) + X_b(z)]\ z^{-1}$$

```
>  .subckt MEM out a b
>     Vout out 0
(v[a]+v[b])/z
>  .ends
```

Fig. 21. The Delay element model



$$Y_{out}(z) = A\ [X_a(z) + X_b(z) + X_c(z)]$$

```
>  .subckt SUM out a b c
>     Vout out 0
A*(v[a]+v[b]+v[c])
>  .ends
```

Fig. 22. The general summer model

All the mentioned blocks can be represented by sub-circuits, based on "voltage" description, as demonstrated by listings in SYRUP language – see Fig. 21 and 22. It is important to say that the multiple-input delay element model can be easily created, and, in this modified form it makes possible significant simplification of the block diagram and its description in the SYRUP input file.

6.2 Post-processing of the results

The MAPLE program environment offers an efficient processing of the symbolic terms including simplification of algebraic expressions, solution of the sets of symbolic or semi-symbolic equations, symbolic differentiation or integration and so forth. This gives facilities for effective post-processing of the symbolic analysis results, especially for the purpose of the analyzed networks optimized design. The following topics can be typically solved:

- *Derivation of the design formulas.*

  The "standard" procedure compares the given numerical transfer function with the symbolic one of the filter designed. It leads to the system of equations for unknown parameters of building blocks (usually multipliers). In the case of the direct form structures the design procedure is the simplest with respect to the canonical character of the solved filter. The general solution of design formulas for the uncanonical structures is not so simple and usually requires any auxiliary tool.

  Design of the IIR filters usually starts from the prewarped continuous-time transfer function $H(s)$, obtained using approximation procedure. Here the necessary $H(s) \rightarrow H(z)$ transformation can be integrated with the designed filter parameters computation, similarly to the design of analogue sampled-data filters. Especially for the 2^{nd}-order partial transfer functions it is easy to derive the direct formulas based on $H(s)$ parameters ω_0 and Q. The use for cascade realization of the higher-order digital filters is evident.

- *Sensitivity properties computations.*

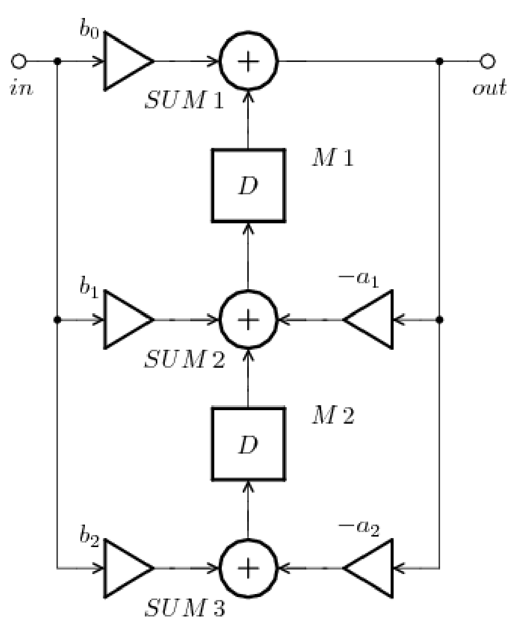
  The relevance of sensitivity computation in digital filter design can be more obvious, if we are aware of the correspondence between rounding errors in "digital area" and tolerances of element values in the "continuous-time area". Therein the sensitivities represent the measure for possible rounding without loss of the accuracy of the filter frequency response.

- *Optimization with respect to the building blocks parameter values spread, dynamics and sensitivity properties.*

  The dynamics optimization is important with respect to the data-overflow. The optimization is based on the partial transfer maxims comparison and their equalization with respect to the "main" transfer maximum. The optimization procedure can be supported by symbolic partial transfers computation and the critical parameter finding. As proved, symbolic analysis is the excellent tool for complex optimization solving all the mentioned criteria.

6.3 Examples

The usage of the extended library is demonstrated on the analysis of some typical examples of digital filters, represented by block diagrams. Note that the obtained transfer functions $H(z)$ can be easily post-processed in MAPLE environment and used for the optimized design of the simulated systems.
The simplest example of symbolic analysis seems to be the 2^{nd}-order digital filter direct form II. structure. The block diagram is shown in Fig. 23 and the SYRUP data file in the Fig. 24.



$$HK2 := \frac{b0\,z^2 + b1\,z + b2}{z^2 + a1\,z + a2}$$

Fig. 23. The 2^{nd}-order direct form II.

```
>  obvod5:= "
>  Vn   1  0
>  XS1  3  1  7 0 SUM(A=1)
>  XS2  7  6 11 0 SUM(A=1)
>  XM1  5  3  0   MEM
>  XM2 10  5  0   MEM
>  Ea1  6  0  5 0 -a1
>  Ea2 11  0 10 0 -a2
>  Eb0  4  0  3 0  b0
>  Eb1  8  0  5 0  b1
>  XS3  9 12  8 0 SUM(A=1)
>  Eb2 12  0 10 0  b2
>  XS4  2  4  9 0 SUM(A=1)
>  .subckt SUM out a b c
>  Vd out 0
A*(v[a]+v[b]+v[c])
>  .ends
>  .subckt MEM out a b
>  Vg out 0 (v[a]+v[b])/z
>  .ends
>  .end ":
```

Fig. 24. Data-file SYRUP

The presented structure does not require any special procedure for design formulas. On the other hand, it could be interesting to analyze the sensitivity properties.
The obtained expressions are suitable for the estimation of the "starting continuous-time parameters" influence to the digital filter parameters changes. As an example, the following graph in Fig. 25 illustrates the $S^{Q}_{a_1,a_2}$ sensitivity dependence on the Q-factor, when the ratio $x_c = \frac{f_c}{\omega_0}$ is set to $x_c = 5$. The graph in Fig. 26 presents the $S^{Q}_{a_1,a_2}$ sensitivities changes for fixed $Q = 2$ and variable x_c. This graph simultaneously explains the realization problems of direct-form structures in the case of relatively high sampling frequencies f_c. Similar results were gained in the case of $S^{\omega_0}_{a_1,a_2}$ sensitivities.

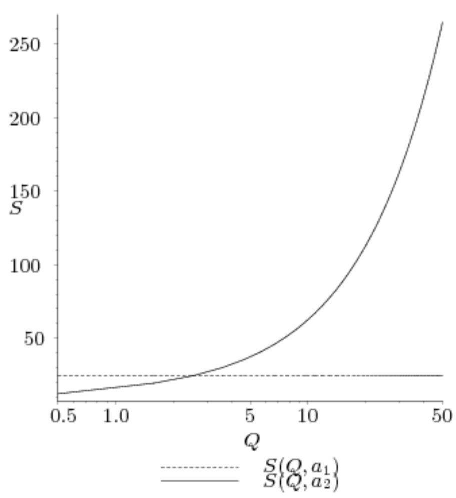


Fig. 25. $S^{Q}_{a_{1,2}} = f(Q)$

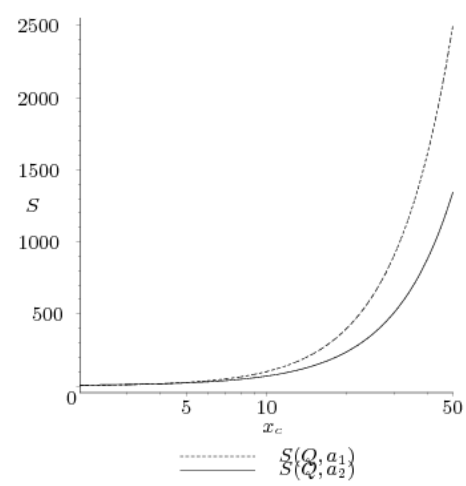


Fig. 26. $S^{Q}_{a_{1,2}} = f(x)$

Note that the formulas obtained are valid directly for the first and the second canonic direct form of the digital filters – see Mitra (2005), Laipert et al. (2000), Antoniou (1979) and others. For the other 2^{nd}-order structures it is necessary to express the transfer function $H(z)$ coefficients a_1 a_2 as the functions of the analyzed network parameters.

The second example presents the 2^{nd}-order allpass filter from Mitra (2005), based on lattice structure. The block diagram is showed in Fig. 27 and the computed symbolic transfer function in Fig. 28.
The following computations show better sensitivities of the analyzed filter in comparison to the direct-form structure; the symbolic expressions for the $S^{Q}_{k_1,k_2}$ and $S^{\omega_0}_{k_1,k_2}$ sensitivities were computed in the form

$$S^{\omega_0}_{k_1} = -\frac{k_1}{{k_1}^2 - 1}; \qquad S^{Q}_{k_1} = \frac{{k_1}^2}{{k_1}^2 - 1}; \tag{43}$$

$$S^{\omega_0}_{k_2} = 0; \qquad S^{Q}_{k_2} = -\frac{2\,k_2}{{k_2}^2 - 1}. \tag{44}$$

The numerical values for $\omega_0 = 2\pi * 1000$, $Q = 2$ and $x = 5$ are $S^{Q}_{k_1} = -24.50245745$, $S^{Q}_{k_2} = 10.07523914$ and $S^{\omega_0}_{k_1} = -24.99745744$.

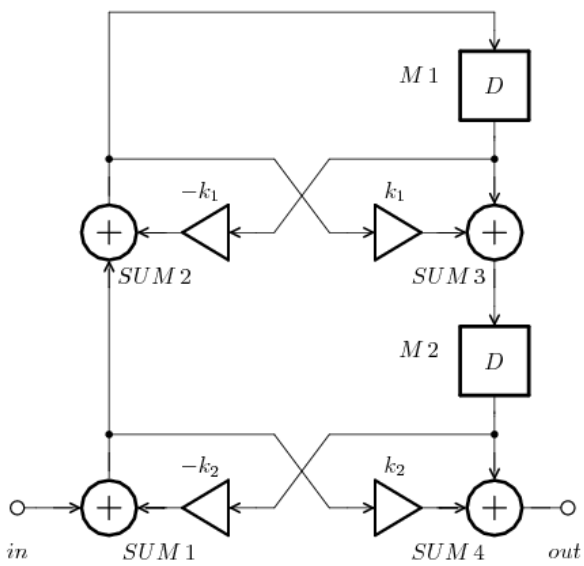


Fig. 27. The 2^{nd}-order all-pass.

```
> A9:= syrup(obvod9,ac):
> assign(A9):
> H9:= collect(v[11]/v[1],
> z,factor);
```

$$H9 := \frac{k_2\, z^2 + k_1\,(k_2+1)\, z + 1}{z^2 + k_1\,(k_2+1)\, z + k_2}$$

Fig. 28. The all-pass simulation result.

The third example introduces state-space structure from Mitra (2005) whose block diagram is in Fig. 29. This structure contains 9 unknown parameters, which represents 4 freedom degrees in design conditions. Symbolic transfer function is expressed by Eqs.(45)–(47)

$$H_{14} = \frac{NH_{14}}{DH_{14}} \tag{45}$$

where

$$NH_{14} = d\,z^2 + (c_1\,b_1 + c_2\,b_2 - d\,(a_{22}+a_{11}))\,z + d\,\Delta + (-c_1\,a_{22} + c_2\,a_{21})\,b_1 + (c_1\,a_{12} - c_2\,a_{11})\,b_2 \tag{46}$$

$$DH_{14} = z^2 - (a_{22}+a_{11})\,z + \Delta\,; \qquad \Delta = a_{11}\,a_{22} - a_{12}\,a_{21}\,. \tag{47}$$

The design conditions can be solved directly in the z-plane, or, after transformation to the s-plane. In this case, the transformed denominator receives the form (48)

$$DH_{14s} = s^2 + \frac{4\,(1-\Delta)\,s}{T\,(1+a_{11}+a_{22}+\Delta)} + \frac{4\,(1-a_{22}+\Delta-a_{11})}{T^2\,(1+a_{11}+a_{22}+\Delta)} \tag{48}$$

A comparison of Eq. (48) to the denominator of the standard form of $H(s)$ (16) allows easily to solve the expressions for ω_{0eq} and Q_{eq} parameters. Free parameters then are chosen with respect to the prescribed optimization criteria.

Similarly the other digital filters or their parts were analyzed as well; e.g. SFG-based 2^{nd}-order sections, published in Tichá (2006), equalizers for audio-signal processing, or a tunable 2^{nd}-order bandpass/bandstop filter structure. All the solved structures were evaluated with the excellent results and MAPLE environment was found as fully acceptable and sufficiently flexible for the required post-processing of the results obtained.

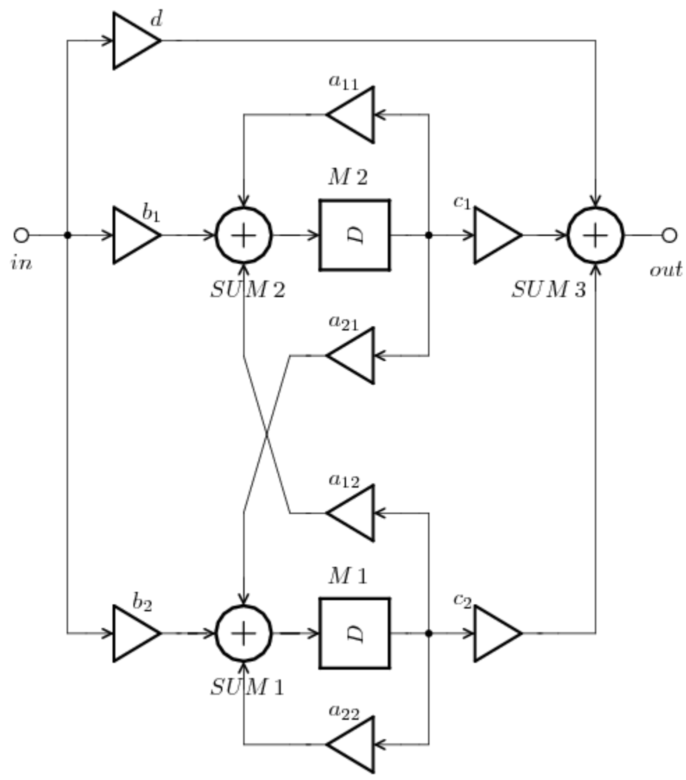


Fig. 29. The general state-space structure.

7. An example of digital filter design

7.1 Introduction

Digital filter design, especially based on cascade connection of the 2nd-order sections usually does not bring problems. But, in the case of non-standard operating conditions, e.g. too high ratio of the sampling-frequency-to-cut-off-filter-frequency, the "standard" direct-form structures fail to satisfy the given requirements. Here the usage of more sophisticated filter sections could be the possible solution. Nevertheless, such structures require more demanding design with respect to the inherency of free design parameters. The two-integrator based sections or state-space biquads introduced in Laipert et al. (2000), Antoniou (1979) or Mitra (2005) should serve as the examples. The design of such sections needs more complex approach, respecting not only the "basic" requirements, but also dynamics, sensitivity, building blocks parameters spread and others.

An efficient design of such filters should be based either on an rigorous mathematical description of the main parameters, or an effective global optimization procedure. This section describes the second way, where the Differential Evolutionary Algorithms were used as the powerful design tool. The reason is in good experience with DE algorithms usage in analog filter optimized design.

The method used is explained on a practical example of state-space 2nd-order IIR section design procedure. The DE algorithms were implemented in MAPLE mathematical program, allowing symbolical computations. Design includes the "basic" computation of the main filter parameters and multi-criteria optimization covering sensitivity properties, dynamics and partial blocks parameter spread. To accelerate necessary computations, filter transfer function, sensitivity expressions and other parameters were preprocessed in symbolic form using

SYRUP library. The symbolic analysis of digital filters using SYRUP was described in Tichá & Martinek (2007), sensitivity computations use the "equivalent sensitivity" approach presented at the last DT Workshop Tichá (2006).

7.2 Design conditions

Let us start by remembering the basic principle of biquad design. It is based on a comparison of a given transfer function $H(z)$ coefficients to the symbolically expressed coefficients of the designed circuit transfer function $H_s(z)$. The comparison leads to the system of design equations for unknown filter component values. Considering "standard" $H(z)$ notation in the form (49)

$$H(z) = \frac{NH(z)}{DH(z)} = \frac{n_2\, z^2 + n_1\, z + n_0}{z^2 + d_1\, z + d_0}\,, \tag{49}$$

$$\begin{aligned} H_s(z) = (d\, z^2 &+ (c_1\, b_1 + c_2\, b_2 - d\,(a_{11} + a_{22}))\, z + d\,(a_{11}\, a_{22} - a_{21}\, a_{12}) - c_2\, a_{11}\, b_2 \\ &- c_1\, a_{22}\, b_1 + c_1\, b_2\, a_{12} + c_2\, a_{21}\, b_1) / (z^2 - (a_{11} + a_{22})\, z + a_{11}\, a_{22} - a_{21}\, a_{12}) \end{aligned} \tag{50}$$

five equations for unknown filter component parameters are necessary. Provided that the filter structure is canonical, the solution of the design equations system is unique for five multiplier constants. If it be to the contrary, we have some freedom parameters on disposal which usually influence filter sensitivity properties, dynamic behavior and component values spread and can be set independently. They are suitable for the filter design optimization.
As mentioned, the complex design respecting all the additional optimization criteria is hardly solved by rigorous mathematical procedure. An application of the global optimization algorithms, in our case the differential evolutionary algorithm (DEA) was found to be simpler and more efficient way. Its usage is demonstrated on the example of the state-space biquad described in Mitra (2005), whose block diagram is shown in Fig. 29.
Symbolical analysis of the filter block diagram was performed in the previous Section 6 and the resulting transfer function is expressed in the Eqs. (45) - to - (48). It contains 9 unknown component parameters, which represent 4 freedom degrees in design conditions. It means, all the additional optimization criteria can be taken into account.

A "basic" design

is usually solved either directly by comparison of the corresponding coefficients of the given $H(z)$ and the symbolical $H_s(z)$ under (50) in the z-plane, or after $z \Leftrightarrow s$ transformation of the $H_s(z)$ to s-plane, similarly to the sampled-data biquad design procedure. Note that both ways are possible in MAPLE program environment, but the first is preferred with respect to the simpler design equations. In contrast to the mentioned procedures, the application of DE algorithm does not require creation of the design equations.

Sensitivity optimization

is based on equivalent ω_0 and Q sensitivities, discussed in Section 5.

Filter dynamics optimization

serves for equalization of the signal maxims inside filter structure. The critical points are usually inputs or outputs of delay elements and outputs of the summers and multipliers. In the case of the solved state-space biquad the outputs of delay elements D were considered.

Optimization requires an evaluation of partial transfers from filter input to the considered block outputs and their maximum magnitude. As sufficient was found to test partial transfer magnitudes at frequency corresponding to ω_{0eq} and their comparison to the "full" transfer magnitude value.

7.3 Algorithm used

Differential Evolutionary Algorithms applied previously in solution of the analog filter design presented e.g. in Tichá & Martinek (2005) were successfully used in the described tasks as well. To improve computation efficiency, a convergence accelerator using simplex built-in procedure was used. Objective function is critical for the optimum design and it was defined as follows

$$fit = w_e \sum_{i=0}^{5} \delta_i^2 + w_p \frac{m_{max}}{m_{min}} + w_s\, PPs + w_d\, PPd\,, \tag{51}$$

where δ_i means transfer function coefficient relative errors, PPs represents penalty function for sensitivity optimization defined as

$$PPs = \sum_{i=1}^{4} |S_{m_i}^{\omega_{0eq}}| + \sum_{i=1}^{4} |S_{m_i}^{Q_{eq}}|\,, \tag{52}$$

and PPd represents dynamics error

$$PPd = \sum_{i=1}^{2} \frac{max|(H(j\omega))|}{max|(H_{Di}(j\omega))|} - 1\,. \tag{53}$$

Parameters w_e, w_p, w_s and w_d characterize weights of objective function components.

7.4 Results

The described optimized design procedure was tested for more examples of biquadratic functions under different operating conditions. As the first example the band-pass section with equivalent parameters $f_0 = 1\,kHz$, $Q_{eq} = 5$, gain constant $h = 1$ and sampling frequency $f_c = 48\,kHz$ is introduced.

Design was made with respect to the sensitivity and building block parameters minimization, without other limitations. No free parameters were numerically defined.

The design results are:

$a_{11} = 0.9787125$, $a_{12} = -0.0564576$, $a_{21} = 0.290288$, $a_{22} = 0.9787125$, $b_1 = 0.0762136$, $b_2 = -0.1492225$, $c_1 = 0.150311$, $c_2 = -0.0917967$, $d = 0.0136064$.

Parameter values spread $\frac{m_{max}}{m_{min}} = 71.93$ and sensitivity values

$S_{a_{11}}^{\omega_0} = S_{a_{22}}^{\omega_0} = -0.8648$, $S_{a_{12}}^{\omega_0} = S_{a_{21}}^{\omega_0} = 0.4845$ $S_{a_{11}}^{Q} = S_{a_{22}}^{Q} = 36.85$, $S_{a_{12}}^{Q} = S_{a_{21}}^{Q} = 1.126$.

Transfer function coefficient errors were typically $\delta_i \approx 10^{-7}$.

DE algorithm parameters: Number of members in population typically $NP = 90 - 120$, control parameters $CR = 0.75$, $F = 0.8$. The results were obtained after approximately $100 - 200$ generations (iteration cycles).

It is important to say, similar other results were gained as well, with respect to more free parameters.

The second example concerns LP section design with similar parameters to the previous example: $f_0 = 1\,kHz$, $Q_{eq} = 5$, gain constant $h = 1$ and sampling frequency $f_c = 48\,kHz$. Here the dynamics optimization was preferred (of course with respect to the previously defined).

The design results are:
$a_{11} = 0.962724$, $a_{12} = 0.0892054$, $a_{21} = -0.186585$, $a_{22} = 0.994701$, $b_1 = 0.0442087e - 1$, $b_2 = -0.116697$, $c_1 = -0.994701$, $c_2 = -0.517322$, $d = 0.0120655e - 1$.
Parameter values spread $\frac{m_{max}}{m_{min}} = 82.44$ and sensitivity values
$S_{a_{11}}^{\omega_0} = -0.3957$, $S_{a_{22}}^{\omega_0} = -1.349$, $S_{a_{12}}^{\omega_0} = S_{a_{21}}^{\omega_0} = 0.4920$ $S_{a_{11}}^{Q} = 37.31$, $S_{a_{22}}^{Q} = 36.36$, $S_{a_{12}}^{Q} = S_{a_{21}}^{Q} = 1.143$.
Transfer function coefficient errors were similarly to the previous example typically $\delta_i \approx 10^{-7}$.
Filter dynamic behavior optimization gives all the partial frequency responses approximately equal with maximum error $\leq 1.8\,\mathrm{dB}$.

8. Conclusions

This chapter introduces some "non-standard" views to the sampled-data and digital filter properties and design. The main goals can be formulated as follows:
As shown, the digital filter direct form prototype can serve for a wider area of implementations. Comparing the implementation using SI memory cells to the modified ones based on simple BD or FD integrators and differentiators, the "exact" implementation shows problems with higher sensitivities and parameter values spread. On the other hand, an influence of SI-blocks parasitics is lower, especially the output conductances g_o cause less frequency shifts and can be respected in design procedure. One possible improvement would be to insert some free parameters into this circuit, e.g. optional gain of the memory cells, but this is a topic for further research.
Sensitivity concept and symbolic analysis are efficient tools for digital filter design, especially when "non-standard" design conditions are required. As shown, the equivalent sensitivity principle allows the appropriate selection of filter structure and, after re-computation, to check the acceptable word-length and ω_{0eq} to f_c ratio.
A new application area of the MAPLE program and its library SYRUP has been introduced. In contrast to the commonly used programs for digital filter design, the presented approach offers wider possibility in filter properties analysis and the evaluated results post-processing.
The last section aims at presenting new ways in "complex" design of digital and analog filters using stochastic algorithms. As shown, especially Differential Evolutionary Algorithms are very suitable tool for this purpose and give excellent results in multi-criteria design. Their use in digital filter design presented here is rather demonstrative, more complicated tasks can be successfully solved. The new in this approach is the conjoined application of more design criteria and possibility to prefer such criterion which is more important in particular design. The design procedure is implemented in mathematical program and this allows its easy modification and/or post-processing of the gained results if necessary.

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The new technology advances provide that a great number of system signals can be easily measured with a low cost. The main problem is that usually only a fraction of the signal is useful for different purposes, for example maintenance, DVD-recorders, computers, electric/electronic circuits, econometric, optimization, etc. Digital filters are the most versatile, practical and effective methods for extracting the information necessary from the signal. They can be dynamic, so they can be automatically or manually adjusted to the external and internal conditions. Presented in this book are the most advanced digital filters including different case studies and the most relevant literature.

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