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Localization Error: Accuracy and Precision of Auditory Localization

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1. Introduction

The act of localization is the estimation of the true location of an object in space and is characterized by a certain amount of inherent uncertainty and operational bias that results in estimation errors. The type and size of the estimation errors depend on the properties of the emitted sound, the characteristics of the surrounding environment, the specific localization task, and the abilities of the listener.

While the general idea of localization error is straightforward, the specific concepts and measures of localization error encountered in the psychoacoustic literature are quite diverse and frequently poorly described, making generalizations and data comparison quite difficult. In addition, the same concept is sometimes described in different papers by different terms, and the same term is used by different authors to refer to different concepts. This variety of terms and metrics used with inconsistent semantics can easily be a source of confusion and may cause the reader to misinterpret the reported data and conclusions.

A fundamental property of localization estimates is that in most cases they are angular and thus represent circular (spherical) variables, which in general cannot be described by a linear distribution as assumed in classical statistics. The azimuth and elevation of the sound source locations define an ambiguous conceptual sphere, which can only be fully analyzed with the methods of spherical statistics. However, these methods are seldom used in psychoacoustic studies, and it is not immediately clear to what degree they should be utilized. In many cases, localization estimates may, in fact, be correctly analyzed using linear methods, but neither the necessary conditions for nor the limitations of linear methods have been clearly stated.

In sum, localization error is a widely used and intuitively simple measure of spatial uncertainty and spatial bias in the perception of sound source location, but both a common terminology for its description and a broad understanding of the implications of its circular character are lacking. Some of the issues related to these topics are discussed in the subsequent sections. The presented concepts and explanations are intended to clarify some existing terminological ambiguities and offer some guidance as to the statistical treatment of localization error data. The focus of the discussion is on issues related to localization judgments, with only marginal attention given to distance estimation judgments which deserve to be the object of a separate article.

2. Basis of auditory localization

Spatial hearing provides information about the acoustic environment; about its geometry and physical properties and about the locations of sound sources. Sound localization generally refers to the act or process of identifying the direction toward a sound source on the basis of sound emitted by the source (see discussion of this definition in Section 3). For living organisms, this is a sensory act based on the perceived auditory stimulation. In the case of machine localization, it is an algorithmic comparison of signals arriving at various sensors. The sound can be either the main product of the source or a by-product of its operation. The act of sound localization when performed by living organisms can also be referred to as *auditory localization*, and this term is used throughout this chapter.

The localization ability of humans depends on a number of anatomical properties of the human auditory system. The most important of these is the presence of two entry points to the auditory system (the external ears) that are located on opposite sides of the human head. Such a configuration of the auditory input system causes a sound coming at the listener from an angle to have a different sound intensity and time of arrival at each ear. The difference in sound intensity is mainly caused by the acoustic shadow and baffle effects of the head and results in a lower sound intensity at the ear located farther away from the sound source (Strutt, 1876; Steinhauser, 1879). The difference in time of arrival is caused by the difference in the distance the sound has to travel to each of the ears (Strutt, 1907; Wilson and Myers, 1908). These differences are normally referred to as the *interaural intensity difference* (IID) and the *interaural time difference* (ITD). In the case of continuous pure tones and other periodic signals the term *interaural phase difference* (IPD) is used in place of ITD since such sounds have no clear reference point in time. The IID and ITD (IPD) together are called the *binaural localization cues*. The IID is the dominant localization cue for high frequency sounds, while the ITD (IPD) is the dominant cue for low frequency sounds (waveform phase difference). The ITD (IPD) is additionally an important cue for high frequency sounds because of differences in the waveform envelope delay (group delay) (Henning, 1974; 1980; McFadden & Pasanen, 1976).

Binaural cues are the main localization mechanisms in the horizontal plane but are only marginally useful for vertical localization or front-back differentiation. This is due to spatial ambiguity caused by head symmetry and referred to as the *cone of confusion* (Wallach, 1939). The *cone of confusion* is the imaginary cone extending outward from each ear along the interaural axis that represents sound source locations producing the same interaural differences. Although asymmetry in ear placement on the head and in the shape of the pinnae provides some disambiguation, the sound source positions located on the surface of the cone of confusion cannot be identified using binaural cues and can only be resolved using spectral cues associated with the directional sound filtering of the human body. These cues are called monaural cues as they do not depend on the presence of two ears.

Monaural cues result from the shadowing and baffle effects of the pinna and the sound reflections caused by the outer ear (pinna and tragus), head, and upper torso (Steinhauser, 1879; Batteau, 1967; Musicant & Butler, 1984; Lopez-Poveda & Meddis, 1996). These effects and reflections produce peaks and troughs in the sound spectrum that are unique for each sound source location in space relative to the position of the listener (Bloom, 1977; Butler & Belendiuk, 1977; Watkins, 1978).

Monaural cues and the related Interaural Spectrum Difference (ISD) also help binaural horizontal localization (Jin et al., 2004; Van Wanrooij & Van Opstal, 2004), but they are most

critical for vertical localization and front-back differentiation. The spectral cues that are the most important for accurate front-back and up-down differentiation are located in the 4-16 kHz frequency range (e.g., Langendijk & Bronkhorst, 2002). Spatial localization ability in both horizontal and vertical planes is also dependent on slight head movements, which cause momentary changes in the peak-and-trough pattern of the sound spectrum at each ear (Young, 1931; Wallach, 1940; Perrett & Noble, 1997; Iwaya et al., 2003), visual cues, and prior knowledge of the stimulus (Pierce, 1901; Rogers & Butler, 1992). More information about the physiology and psychology of auditory localization can be found elsewhere (e.g., Blauert, 1974; Yost & Gourevitch, 1987; Moore, 1989; Yost et al., 2008; Emanuel & Letowski (2009).

3. Terminology, notation, and conventions

The broad interest and large number of publications in the field of auditory localization has advanced our knowledge of neurophysiologic processing of spatial auditory signals, the psychology of spatial judgments, and environmental issues in determining the locations of sound sources. The authors of various experimental and theoretical publications range from physiologists to engineers and computer scientists, each bringing their specific expertise and perspective. The large number of diversified publications has also led to a certain lack of consistency regarding the meaning of some concepts. Therefore, before discussing the methods and measures used to describe and quantify auditory localization errors in Section 5, some key concepts and terminological issues are discussed in this and the following section.

Auditory spatial perception involves the perception of the surrounding space and the locations of the sound sources within that space on the basis of perceived sound. In other words, auditory spatial perception involves the perception of sound spaciousness, which results from the specific volume and shape of the surrounding space, and the identification of the locations of the primary and secondary (sound reflections) sound sources operating in the space in relation to each other and to the position of the listener.

In very general terms, auditory spatial perception involves four basic elements:

• Horizontal localization (azimuth, declination)
• Vertical localization (elevation)
• Distance estimation
• Perception of space properties (spaciousness)

The selection of these four elements is based on a meta-analysis of the literature on spatial perception and refers to the traditional terminology used in psychoacoustic research studies on the subject matter. It seems to be a logical, albeit obviously arbitrary, classification.

A direction judgment toward a sound source located in space is an act of localization and can be considered a combination of both horizontal and vertical localization judgments. Horizontal and vertical localization judgments are direction judgments in the corresponding planes and may vary from simple left-right, up-down, and more-less discriminations, to categorical judgments, to the absolute identifications of specific directions in space. A special form of localization judgments for phantom sound sources located in the head of the listener is called *lateralization*. Therefore, the terms lateralization and localization refer respectively to judgment of the internal and external positions of sound sources in reference to the listener's head (Yost & Hafter, 1987; Emanuel & Letowski, 2009).

Similarly to localization judgments, distance judgments may have the form of discrimination judgments (closer-farther), relative numeric judgments (half as far – twice as far), or absolute numeric judgments in units of distance. In the case of two sound sources located at different distances from the listener, the listener may estimate their relative difference in distance using the same types of judgments. Such relative judgments are referred to as *auditory distance difference* or *auditory depth* judgments.

Both distance and depth judgments are less accurate than angular localization judgments and show large intersubject variability. In general, perceived distance *PD* is a power function of the actual distance *d* and can be described as

$$PD = kd^a ,$$ (1)

where *a* and *k* are fitting constants dependent on the individual listener. Typically *k* is close to but slightly smaller than 1 (k=0.9), and *a* is about 0.4 but varies widely (0.3-0.8) across listeners (Zahorik et al., 2005).

The above differentiation between localization and distance estimation is consistent with the common interpretation of auditory localization as the act of identifying the direction toward the sound source (White, 1987; Morfey, 2001; Illusion, 2010). It may seem, however, inconsistent with the general definition of localization which includes distance estimation (APA, 2007; Houghton Mifflin, 2007). Therefore, some authors who view distance estimation as an inherent part of auditory localization propose other terms, e.g., direction-of-arrival (DOA) (Dietz et al., 2010), to denote direction-only judgments and distinguish them from general localization judgments.

The introduction of a new term describing direction-only judgments is intended to add clarity to the language describing auditory spatial perception. However, the opposite may be true since the term *localization* has a long tradition in the psychoacoustic literature of being used to mean the judgment of direction. This meaning also agrees with the common usage of this term. Therefore, it seems reasonable to accept that while the general definition of localization includes judging the distance to a specific location, it does not mandate it, and in its narrow meaning, localization refers to the judgment of direction. In this context, the term localization error refers to errors in direction judgment, and the term distance estimation error to errors in distance estimation.

Spaciousness is the perception of being surrounded by sound and is related to the type and size of the surrounding space. It depends not only on the type and volume of the space but also on the number, type, and locations of the sound sources in the space. Perception of spaciousness has not yet been well researched and has only recently become of more scientific interest due to the rapid development of various types of spatial sound recording and reproduction systems and AVR simulations (Griesinger, 1997). The literature on this subject is very fragmented, inconsistent, and contradictory. The main reason for this is that unlike horizontal localization, vertical localization, and distance estimation judgments, which are made along a single continuum, spaciousness is a multidimensional phenomenon without well defined dimensions and one that as of now can only be described in relative terms or using categorical judgments.

The two terms related to spaciousness that are the most frequently used are *listener envelopment* (LEV) and *apparent source width* (ASW). Listener envelopment describes the degree to which a listener is surrounded by sound, as opposed to listening to sound that happens "somewhere else". It is synonymous to *spatial impression* as defined by Barron and

Marshall (1981). Some authors treat both these terms as synonymous to spaciousness, but spaciousness can exist without listener envelopment. The ASW is also frequently equated with spaciousness, but such an association does not agree with the common meanings of both *width* and *spaciousness* and should be abandoned (Griesinger, 1999). The concept of ASW relates more to the size of the space occupied by the active sound sources and should be a subordinate term to spaciousness. Thus, LEV and ASW can be treated as two complementary elements of spaciousness (Morimoto, 2002). Some other correlated or subordinate aspects of spaciousness are panorama (a synonym of ASW), perspective, ambience, presence, and warmth.

Depending on the task given to the listener there are two basic types of localization judgments:

- Relative localization (discrimination task)
- Absolute localization (identification task)

Relative localization judgments are made when one sound source location is compared to another, either simultaneously or sequentially. Absolute localization judgments involve only one sound source location that needs to be directly pointed out. In addition, absolute localization judgments can be made on a continuous circular scale and expressed in degrees (°) or can be restricted to a limited set of preselected directions. The latter type of judgment occurs when all the potential sound source locations are marked by labels (e.g., number), and the listener is asked to identify the sound source location by label. The actual sound sources may or may not be visible. This type of localization judgment, in which the identification data are later expressed as selection percentages, i.e., the percent of responses indicating each (or just the correct) location, is referred to throughout this chapter as *categorical localization*.

From the listener's perspective, the most complex and demanding judgments are the absolute localization judgments, and they are the main subject of this chapter. The other two types of judgments, discrimination judgments and categorization judgments, are only briefly described and compared to absolute judgments later in the chapter.

In order to assess the human ability to localize the sources of incoming sounds, the physical reference space needs to be defined in relation to the position of the human head. This reference space can be described either in the rectangular or polar coordinate system. The rectangular coordinate system *x, y, z* is the basis of Euclidean geometry and is also called the Cartesian coordinate system. In the head-oriented Cartesian coordinate system the *x, y,* and *z* axes are typically oriented as left-right (west-east), back-front (south-north), down-up (nadir-zenith), respectively. The east, front, and up directions indicate the positive ends of the scales.

The Euclidean planes associated with the Cartesian coordinate system are the vertical lateral (x-z), the vertical sagittal (y-z), and the horizontal (x-y) planes. The main reference planes of symmetry for the human body are:

- Median sagittal (midsagittal) plane: y-z plane
- Frontal (coronal) lateral plane: x-z plane
- Axial (transversal, transaxial) horizontal plane: x-y plane

The relative orientations of the sagittal and lateral planes and the positions of the median and frontal planes are shown in Figure 2. The virtual line passing though both ears in the frontal plane is called the *interaural axis*. The ear closer to the sound source is termed the ipsilateral ear and the ear farther away from the sound source is the contralateral ear.
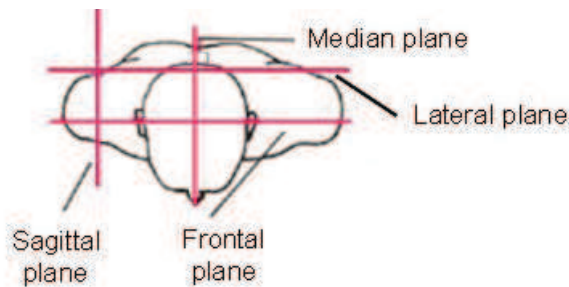
Fig. 1. Main reference planes of the human body. The axial plane is parallel to the page.

The median (midsagittal) plane is the sagittal plane (see Figure 1) that is equidistant from both ears. The frontal (coronal) plane is the lateral plane that divides the listener's head into front and back hemispheres along the interaural axis. The axial (transversal) plane is the horizontal plane of symmetry of the human body. Since the axial plane is not level with the interaural axis of human hearing, the respective plane, called the *visuoaural plane* by Knudsen (1982), is referred to here as the *hearing plane*, or as just the horizontal plane.

In the polar system of coordinates, the reference dimensions are d (distance or radius), θ (declination or azimuth), and φ (elevation). Distance is the amount of linear separation between two points in space, usually between the observation point and the target. The angle of declination (azimuth) is the horizontal angle between the medial plane and the line connecting the point of observation to the target. The angle of elevation is the vertical angle between the hearing plane and the line from the point of observation to the target. The Cartesian and polar systems are shown together in Figure 2.
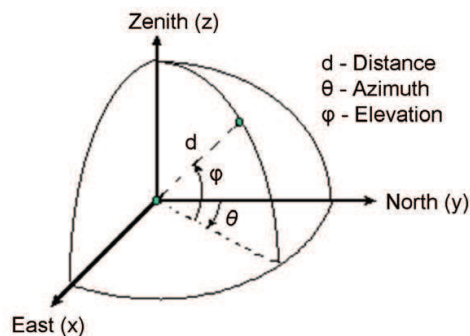

Fig. 2. Commonly used symbols and names in describing spatial hearing coordinates.

One advantage of the polar coordinate system over Cartesian coordinate system is that it can be used in both Euclidean geometry and the spherical, non-Euclidean, geometry that is useful in describing relations between points on a closed surface such as a sphere. In auditory perception studies two spherical systems of coordinates are used. They are referred to as the *single-pole system* and the *two-pole system*. Both are shown in Figure 3.

The head-oriented single-pole system is analogous to the planetary coordinate system of longitudes and latitudes. In the two-pole system, both longitudes and latitudes are represented by series of parallel circles. The single-pole system is widely used in many fields of science. However, in this system the length of an arc between two angles of azimuth depends on elevation. The two-pole system makes the length of the arc between two angles of azimuth the same regardless of elevation. Though less intuitive, this system may be convenient for some types of data presentation (Knudsen, 1982; Makous &
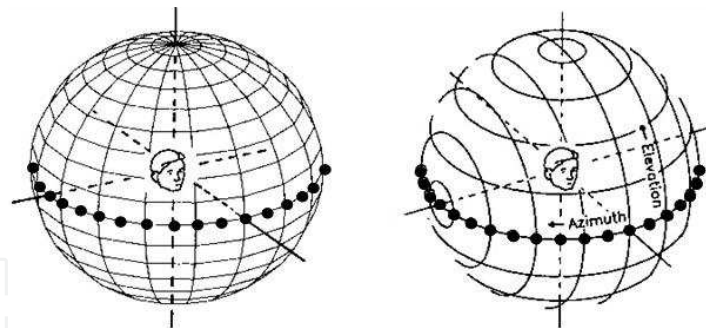
Fig. 3. Single-pole (left) and two-pole (right) spherical coordinate systems. Adapted from Carlile (1996).

Middlebrooks, 1990). Since both these systems share the same concepts of azimuth and elevation, it is essential that the selection of the specific spherical coordinate system always be explicit (Leong & Carlile, 1998).

It should also be noted that there are two conventions for numerically labeling angular degrees that are used in scientific literature: the *360° scheme* and the *±180° scheme*. There are also two possibilities for selecting the direction of positive angular change: clockwise (e.g., Tonning, 1970) or counterclockwise (e.g., Pedersen & Jorgensen, 2005).

The use of two notational schemes is primarily a nuisance that necessitates data conversion in order to compare or combine data sets labeled with different schemes. However, converting angles that are expressed differently in the two schemes from one scheme to the other is just a matter of either adding or subtracting 360°.

In the case of localization studies, where differences between angles are the primary consideration, the ±180° labeling scheme is overwhelmingly preferred. First, it is much simpler and more intuitive to use positive and negative angles to describe angular difference. Second, and more importantly, the direct summing and averaging of angular values can only be done with angles that are contained within a (numerically) continuous range of 180°, such as ±90°. If the 360° scheme is used, then angles to the left and right of 0° (the reference angle) cannot be directly added and must be converted into vectors and added using vector addition.

Less clear is the selection of the positive and negative directions of angular difference. However, if the ±180° scheme is used, the absolute magnitude of angular values is the same regardless of directionality, which is another reason to prefer the ±180° scheme. Under the 360° scheme, the clockwise measurement of any angle other than 180° will have a different magnitude than that same angle measured counterclockwise, i.e., 30° in the clockwise direction is 330° in the counterclockwise direction.

In mathematics (e.g., geometry) and physics (e.g., astronomy), a displacement in a counterclockwise direction is considered positive, and a displacement in a clockwise direction is considered negative. In geometry, the quadrants of the circle are ordered in a counterclockwise direction, and an angle is considered positive if it extends from the *x* axis in a counterclockwise direction. In astronomy, all the planets of our solar system, when observed from above the Sun, rotate and revolve around the Sun in a counterclockwise direction (except for the rotation of Venus).

However, despite the scientific basis of the counterclockwise rule, the numbers on clocks and all the circular measuring scales, including the compass, increase in a clockwise direction, effectively making it the positive direction. This convention is shown in Figure 2

and is accepted in this chapter. For locations that differ in elevation, the upward direction from a 0° reference point in front of the listener is normally considered as the positive direction, and the downward direction is considered to be the negative direction.

4. Accuracy and precision of auditory localization

The human judgment of sound source location is a noisy process laden with judgment uncertainty, which leads to localization errors. Auditory localization error (LE) is the difference between the estimated and actual directions toward the sound source in space. This difference can be limited to difference in azimuth or elevation or can include both (e.g., Carlile et al., 1997). The latter can be referred to as compound LE.

Once the localization act is repeated several times, LE becomes a statistical variable. The statistical properties of this variable are generally described by *spherical statistics* due to the spherical/circular nature of angular values ($\theta = \theta + 360°$). However, if the angular judgments are within a ±90° range (as is often the case in localization judgments, after disregarding front-back reversals), the data distribution can be assumed to have a linear character, which greatly simplifies data analysis. Front-back errors should be extracted from the data set and analyzed separately in order to avoid getting inflated localization error (Oldfield & Parker, 1984; Makous & Middlebrooks, 1990; Bergault, 1992; Carlile et al., 1997). Some authors (e.g. Wightman & Kistler, 1989) mirror the perceived reverse locations about the interaural axis prior to data analysis in order to preserve the sample size. However, this approach inflates the power of the resulting conclusions. Only under specific circumstances and with great caution should front-back errors be analyzed together with other errors (Fisher, 1987). The measures of linear statistics commonly used to describe the results of localization studies are discussed in Section 5. The methods of spherical (circular) statistical data analysis are discussed in Section 6.

The linear distribution used to describe localization judgments, and in fact most human judgment phenomena, is the normal distribution, also known as the Gaussian distribution. It is a purely theoretical distribution but it well approximates distributions of human errors, thus its common use in experiments with human subjects. In the case of localization judgments, this distribution reflects the random variability of the localizations while emphasizing the tendency of the localizations to be centered on some direction (ideally the true sound source direction) and to become (symmetrically) less likely the further away we move from that central direction.

The normal distribution has the shape of a bell and is completely described in its ideal form by two parameters: the mean (μ) and the standard deviation (σ). The mean corresponds to the central value around which the distribution extends, and the standard deviation describes the range of variation. In particular, approximately 2/3 of the values (68.2%) will be within one standard deviation from the mean, i.e., within the range [$\mu - \sigma, \mu + \sigma$]. The mathematical formula and graph of the normal distribution are shown in Figure 4.

Based on the above discussion, each set of localization judgments can be described by a specific normal distribution with a specific mean and standard deviation. Ideally, the mean of the distribution should correspond with the true sound source direction. However, any lack of symmetry in listener hearing or in the listening conditions may result in a certain bias in listener responses and cause a misalignment between the perceived location of the sound source and its actual location. Such bias is called constant error (CE).
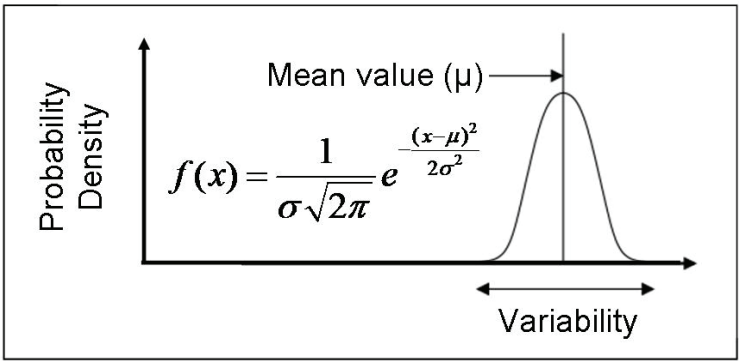
Fig. 4. Normal distribution. Standard deviation (σ) is the range of variability around the mean value ($\mu \pm \sigma$) that accounts for approximately 2/3 of all responses.

Another type of error is introduced by both listener uncertainty/imprecision and random changes in the listening conditions. This error is called random error (RE). Therefore, LE can be considered as being composed of two error components with different underlying causes: constant error (CE) resulting from a bias in the listener and/or environment and random error (RE) resulting from the inherent variability of listener perception and listening conditions. If LE is described by a normal distribution, CE is given by the difference between the true sound source location and the mean of the distribution (x_o) and RE is characterized by the standard deviation (σ) of the distribution.

The concepts of CE and RE can be equated, respectively, with the concepts of precision and accuracy of a given set of measurements. The definitions of both these terms, along with common synonyms (although not always used correctly), are given below:

Accuracy (constant error, systematic error, validity, bias) is the measure of the degree to which the measured quantity is the same as its actual value.

Precision (random error, repeatability, reliability, reproducibility) is the measure of the degree to which the same measurement made repeatedly produces the same results.

The relationship between accuracy and precision and the normal distribution from Figure 4 are shown in Figure 5.
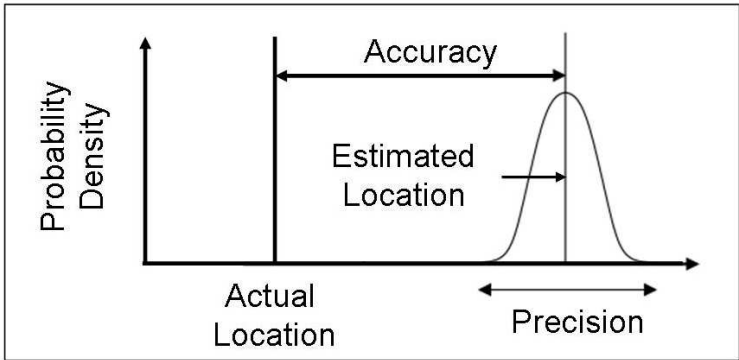

Fig. 5. Concepts of accuracy in precision in localization judgments.

Localization accuracy depends mainly on the symmetry of the auditory system of the listener, the type and behavior of the sound source, and the acoustic conditions of the surrounding space. It also depends on the familiarity of the listener with the listening conditions and on the non-acoustic cues available to the listener. For example, auditory localization accuracy is affected by eye position (Razavi et al., 2007). Some potential bias may also be introduced by the reported human tendency to misperceive the midpoint of the angular distance between two horizontally distinct sound sources. Several authors have reported the midpoint to be located 1° to 2° rightward (Cusak et al., 2001; Dufour et al., 2007; Sosa et al., 2010), although this shift may be modulated by listener handedness. For example, Ocklenburg et al. (2010) observed a rightward shift for left-handed listeners and a leftward shift for right-handed listeners.

Localization precision depends primarily on fluctuations in the listener's attention, the type and number of sound sources, their location in space, and the acoustic conditions of the surrounding space. In addition, both localization accuracy and precision depend to a great degree on the data collection methodology (e.g., direct or indirect pointing, verbal identification, etc). In general, the overall goodness-of-fit of the localization data to the true target location can be expressed in terms of error theory as (Bolshev, 2002) as:

$$p(\theta) = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{(CE^2 + RE^2)}} \ . \tag{2}$$

5. Linear statistical measures

The two fundamental classes of measures describing probability distributions are measures of central tendency and measures of dispersion. Measures of central tendency, also known as measures of location, characterize the central value of a distribution. Measures of dispersion, also known as measures of spread, characterize how spread out the distribution is around its central value. In general, distributions are described and compared on the basis of a specific measure of central tendency in conjunction with a specific measure of spread.

For the normal distribution, the mean (μ), a measure of central tendency, and the standard deviation (σ), a measure of dispersion, serve to completely describe (parametrize) the distribution. There is, however, no way of directly determining the true, actual values of these parameters for a normal distribution that has been postulated to characterize some population of judgments, measurements, etc. Thus these parameters must be estimated on the basis of a representative sample taken from the population. The sample arithmetic mean (x_0) and the sample standard deviation (SD) are the standard measures used to estimate the mean and standard deviation of the underlying normal distribution.

The sample mean and standard deviation are highly influenced by outliers (extreme values) in the data set. This is especially true for smaller sample sizes. Measures that are less sensitive to the presence of outliers are referred to as robust measures (Huber & Ronketti, 2009). Unfortunately, many robust measures are not very efficient, which means that they require larger sample sizes for reliable estimates. In fact, for normally distributed data (without outliers), the sample mean and standard deviation are the most efficient estimators of the underlying parameters.

A very robust and relatively efficient measure of central tendency is the median (ME). A closely related measure of dispersion is the median absolute deviation (MEAD), which is also very robust but unfortunately also very inefficient. A more efficient measure of dispersion that is however not quite as robust is the mean absolute deviation (MAD). Note that the abbreviation "MAD" is used in other publications to refer to either of these two measures. The formulas for both the standard and robust sample measures discussed above are given below in Table 1. They represent the basic measures used in calculating LE when traditional statistical analysis is performed.

Measure Name	Symbol	Definition/Formula	Comments
Arithmetic Mean	x_o	$x_O = \dfrac{1}{n} \sum\limits_{i=1}^{n} x_i$	
Standard Deviation	SD	$SD = \sqrt{\dfrac{1}{n} \sum\limits_{i=1}^{n} (x_i - x_0)^2}$	V (variance) = SD^2.
Median	ME	middle value of responses	
Median Absolute Deviation	MEAD	middle value of the absolute deviations from the median	
Mean Absolute Deviation	MAD	$MAD = \dfrac{1}{n} \sum\limits_{i=1}^{n} |x_i - x_0|$	

Table 1. Basic measures used to estimate the parameters of a normal distribution.

Strictly speaking, the sample median estimates the population median, which is the midpoint of the distribution, i.e., half the values (from the distribution) are below it and half are above it. The median together with the midpoints of the two halves of the distribution on either side of the median divide the distribution into 4 four parts of equal probability. The three dividing points are called the 1st, 2nd, and 3rd quartiles (Q1, Q2 and Q3), with the 2nd quartile simply being another name for the median. Since the normal distribution is symmetric around its mean, its mean is also its median, and so the sample median can be used to directly estimate the mean of a normal distribution.

The median absolute deviation of a distribution does not coincide with its standard deviation, thus the sample median absolute deviation does not give a direct estimate of the population standard deviation. However, in the case of a normal distribution, the median absolute deviation corresponds to the difference between the 3rd and 2nd quartiles, which is proportional to the standard deviation. Thus for a normal distribution the relationship between the standard deviation and the MEAD is given by (Goldstein & Taleb, 2007):

$$\sigma \approx 1.4826(Q3 - Q2) = 1.4826(MEAD) \qquad (3)$$

The SD is the standard measure of RE, while the standard measure of CE is the mean signed error (ME), also called mean bias error, which is equivalent to the difference between the sample mean of the localization data (x_o) and the true location of the sound source. The unsigned, or absolute, counterpart to the ME, the mean unsigned error (MUE) is a measure of total LE as it represents a combination of both the CE and the RE. The MUE was used among others by Makous and Middlebrooks (1990) in analyzing their data. Another error

measure that combines the CE and RE is the root mean squared error (RMSE). The relationship between these three measures is given by the following inequality, where n is the sample size (Willmott & Matusuura, 2005).

$$|ME| \leq MUE \leq RMSE \leq \sqrt{n}\,MUE\,. \tag{4}$$

The RE part of the RMSE is given by the sample standard deviation (SD), but the RE in the MUE does not in general correspond to any otherwise defined measure. However, if each localization estimate is shifted by the ME so as to make the CE equal to zero, the MUE of the data normalized in this way is reduced to the sample mean absolute deviation (MAD). Since the MAD is not affected by linear transformations, the MAD of the normalized data is equal to the MAD of the non-normalized localizations and so represents the RE of the localizations. Thus, the MAD is also a measure of RE. For a normal distribution, the standard deviation is proportional to the mean absolute deviation in the following ratio (Goldstein & Taleb, 2007):

$$\sigma = \sqrt{\frac{\pi}{2}}\,MAD \approx 1.253(MAD) \tag{5}$$

This means that for sufficiently large sample sizes drawn from a normal distribution, the normalized MUE (=MAD) will be approximately equal to 0.8 times the SD. The effect of sample size on the ratio between sample MAD and sample SD for samples from a normal distribution in shown below in Fig. 6.
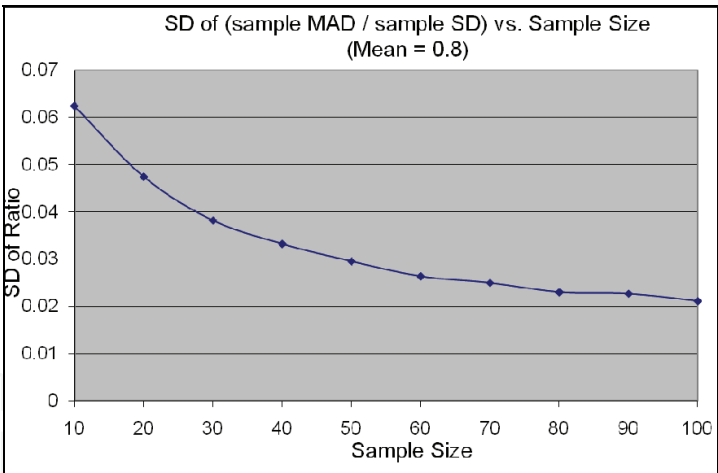

Fig. 6. The standard deviation of the ratios between sample MAD and sample SD for 1000 simulated samples plotted against the size of the sample.

Note that unlike the RMSE, which is equal to the square root of the sum of the squares of the CE (ME) and RE (σ), the MUE is not expressible as a function of CE (ME) and RE (MAD). The formulas for the error measures are given below in Table 2.

The formulas listed in Table 2 and the above discussion apply to normal or similar unimodal distributions. In the case of a multimodal data distribution, these measures are in general not applicable. However, if there are only a few modes that are relatively far apart, then these measures (or similar statistics) can be calculated for each of the modes using appropriate subsets of the data set. This is in particular applicable to the analysis of front-back errors, which tend to define a separate unimodal distribution.

Measure Name	Symbol	Type	Definition/Formula	Comments
Mean Error (Mean Signed Error)	ME	CE	$ME = \frac{1}{n} \sum_{i=1}^{n} (x_i - \eta) = x_0 - \eta$	
Mean Absolute Error (Mean Unsigned Error)	MUE	CE & RE	$MUE = \frac{1}{n} \sum_{i=1}^{n} |x_i - \eta|$	$|ME| \leq MUE \leq |ME| + MAD$
Root-Mean-Squared Error	RMSE	CE & RE	$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \eta)^2}$	$RMSE^2 = ME^2 + SD^2$
Standard Deviation	SD	RE	$SD = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - x_0)^2}$	
Mean Absolute Deviation	MAD	RE	$MAD = \frac{1}{n} \sum_{i=1}^{n} |x_i - x_0|$	

Table 2. Basic measures used to calculate localization error (η denotes true location of the sound source).

There is a continuing debate in the literature as to what constitutes a front-back error. Most authors define front-back errors as any estimates that cross the interaural axis (Carlile et al., 1997; Wenzel, 1999). Other criteria include errors crossing the interaural axis by more than 10º (Schonstein, 2008) or 15º (Best et al., 2009) or errors that are within a certain angle after subtracting 180º. An example of the last case is using a ±20º range around the directly opposite angle (position) which corresponds closely to the range of typical listener uncertainty in the frontal direction (e.g., Carlile et al., 1997). The criterion proposed in this chapter is that only estimates exceeding a ±150º error should be considered nominal front-back errors. This criterion is based on a comparative analysis of location estimates made in anechoic and less than optimal listening conditions.

The extraction and separate analysis of front-back errors should not be confused with the process of trimming the data set to remove outliers, even though they have the same effect. Front-back errors are not outliers in the sense that they simply represent extreme errors. They represent a different type of error that has a different underlying cause and as such should be treated differently. Any remaining errors exceeding ±90º may be trimmed (discarded) or winsorized to keep the data set within the ±90º range. Winsorizing is a strategy in which the extreme values are not removed from the sample, but rather are replaced with the maximal remaining values on either side. This strategy has the advantage of not reducing the sample size for statistical data analysis. Both these procedures mitigate the effects of extreme values and are a way of making the resultant sample mean and standard deviation more robust.

The common primacy of the sample arithmetic mean and sample standard deviation for estimating the population parameters is based on the assumption that the underlying distribution is in fact perfectly normal and that the data are a perfect reflection of that distribution. This is frequently not the case with human experiments, which have numerous potential sources for data contamination. In general, this is evidenced by more values farther away from the mean than expected (heavier tails or greater kurtosis) and the presence of extreme values, especially for small data sets. Additionally, the true underlying

distribution may deviate slightly in other ways from the assumed normal distribution (Huber & Ronchetti, 2009).

It is generally desired that a small number of inaccurate results should not overly affect the conclusions based on the data. Unfortunately, this is not the case with the sample mean and standard deviation. As mentioned earlier the mean and, in particular, the standard deviation are quite sensitive to outliers (the inaccurate results). Their more robust counterparts discussed in this section are a way of dealing with this problem without having to specifically identify which results constitute the outliers as is done in trimming and winsorizing. Moreover, the greater efficiency of the sample SD over the MAD disappears with only a few inaccurate results in a large sample (Huber & Ronchetti, 2009). Thus, since there is little chance of human experiments generating perfect data and a high chance of the underlying distribution not being perfectly normal, the use of more robust measures for estimating the CE (mean) and RE (standard deviation) may be recommended.

It is also recommended that both components of localization error, CE and RE, always be reported individually. A single compound measure of error such as the RMSE or MUE is not sufficient for understanding the nature of the errors. These compound measures can be useful for describing total LE, but they should be treated with caution. Opinions as to whether RMSE or MUE provides the better characterization of total LE are divided. The overall goodness-of-fit measure given in Eq. 2 clearly uses RMSE as its base. Some authors also consider RMSE as "the most meaningful single number to describe localization performance" (Hartmann, 1983). However, others argue that MUE is a better measure than RMSE. Their criticism of RMSE is based on the fact that RMSE includes MUE but is additionally affected by the square root of the sample size and the distribution of the squared errors which confounds its interpretation (Willmott & Matusuura 2005).

6. Spherical statistics

The traditional statistical methods discussed above were developed for linear infinite distributions. These methods are in general not appropriate for the analysis of data having a spherical or circular nature, such as angles. The analysis of angular (directional) data requires statistical methods that are concerned with probability distributions on the sphere and circle. Only if the entire data set is restricted to a ±90° range can angular data be analyzed as if coming from a linear distribution. In all other cases, the methods of linear statistics are not appropriate, and the data analysis requires the techniques of a branch of statistics called spherical statistics.

Spherical statistics, also called directional statistics, is a set of analytical methods specifically developed for the analysis of probability distributions on spheres. Distributions on circles (two dimensional spheres) are handled by a subfield of spherical statistics called circular statistics. The fundamental reason that spherical statistics is necessary is that if the numerical difference between two angles is greater than 180°, then their linear average will point in the opposite direction from their actual mean direction. For example, the mean direction of 0° and 360° is actually 0°, but the linear average is 180°. Note that the same issue occurs also with the ±180° notational scheme (consider -150° and 150°). Since parametric statistical analysis relies on the summation of data, it is clear that something other than standard addition must serve as the basis for the statistical analysis of angular data. The simple solution comes from considering the angles as vectors of unit length and applying vector addition. The Cartesian coordinates X and Y of the mean vector for a set of vectors corresponding to a set of angles θ about the origin are given by:

$$X = \frac{1}{n} \sum_{i=1}^{n} \sin(\theta_i) \qquad (6)$$

and

$$Y = \frac{1}{n} \sum_{i=1}^{n} \cos(\theta_i) \qquad (7)$$

The angle θ_o that the mean vector makes with the X-axis is the mean angular direction of all the angles in the data set. Its calculation depends on the quadrant the mean vector is in:

$$\theta_o = \begin{cases} \tan^{-1}(Y/X) & X > 0 \\ \pi + \tan^{-1}(Y/X) & X < 0, Y \geq 0 \\ -\pi + \tan^{-1}(Y/X) & X < 0, Y < 0 \\ \pi/2 & X = 0, Y \geq 0 \\ -\pi/2 & X = 0, Y < 0 \end{cases} \qquad (8)$$

The magnitude of the mean vector is called the *mean resultant length* (R):

$$R = \sqrt{X^2 + Y^2} . \qquad (9)$$

R is a measure of concentration, the opposite of dispersion, and plays an important role in defining the circular standard deviation. Its magnitude varies from 0 to 1 with R = 1 indicating that all the angles in the set point in the same direction. Note that R = 0 not only for a set of angles that are evenly distributed around the circle but also for one in which they are equally divided between two opposite directions. Thus, like the linear measures discussed in the previous section, R is most meaningful for unimodal distributions.

One of the most significant differences between spherical statistics and linear statistics is that due the bounded range over which the distribution is defined, there is no generally valid counterpart to the linear standard deviation in the sense that intervals defined in terms of multiples of the standard deviation represent a constant probability independent of the value of the standard deviation. Clearly, as the circular standard deviation increases, fewer and fewer standard deviations are needed to cover the whole circle.

The circular counterpart to the linear normal distribution is known as the von Mises distribution (Fisher, 1993)

$$f(\theta, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\theta - \theta_0)}, \qquad (10)$$

where θ_o is the mean angle and $I_o(\kappa)$ the modified Bessel function of order 0. The κ parameter of the von Mises function is not a measure of dispersion, like the standard deviation, but, like R, is a measure of concentration. At $\kappa = 0$, the von Mises distribution is equal to the uniform distribution on the circle, while at higher values of κ the distribution becomes more and more concentrated around its mean. As κ continues to increases above 1, the von Mises distribution begins to more and more closely resemble a wrapped normal distribution, which is a linear normal distribution that has been wrapped around the circle

$$f(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} e^{-\frac{(\theta-\theta_0+2\pi k)^2}{2\sigma^2}} , \qquad (11)$$

where θ_o and σ are the mean and standard deviation of the linear distribution.

A reasonable approach to defining the circular standard deviation would be to base it on the wrapped normal distribution so that for a wrapped normal distribution it would coincide with the standard deviation of the underlying linear distribution. This can be accomplished due to the fact that for the wrapped normal distribution there is a direct relationship between the mean resultant length, R, and the underlying linear standard deviation

$$R = e^{\frac{-\sigma^2}{2}} . \qquad (12)$$

The above equality provides the general definition of the circular standard deviation as:

$$\sigma_C = \sigma = \sqrt{-2\ln(R)} . \qquad (13)$$

The sample circular mean direction and sample circular standard deviation can be used to describe any circular data set drawn from a normal circular distribution. However, if the angular data are within ±90°, or within any other numerically continuous 180° range, then linear measures can still be used. Since standard addition applies, the linear mean can be calculated, and it will be equal to the circular mean angle. The linear standard deviation will also be almost identical to the circular standard deviation as long as the results are not overly dispersed. In fact, the relationship between the linear standard deviation and the circular standard deviation is not so much a function of the the range of the data as of its dispersion. For samples drawn from a normal linear distribution, the two sample standard deviations begin to deviate slightly at about $\sigma = 30°$, but even at $\sigma = 60°$ the difference is not too great for larger sample sizes. Results from a set of simulations in which the two sample standard deviations were compared for 500 samples of size 10 and 100 are shown in Fig. 6. The samples were drawn from linear normal distributions with standard deviations randomly selected in the range $1° \leq \sigma \leq 60°$.

So, for angular data that are assumed to come from a reasonably concentrated normal distribution, as would be expected in most localization studies, the linear standard deviation can be used even if the data spans the full 360°, as long as the mean is calculated as the circular mean angle. This does not mean that localization errors greater than 120° (front-back errors) should not be excluded from the data set for separate analysis.

Once the circular mean has been calculated, the formulas in Table 2 in Section 5 can be used to calculate the circular counterparts to the other linear error measures. The determination of the circular median, and thus the MEAD, is in general a much more involved process. The problem is that there is in general no natural point on the circle from which to start ordering the data set. However, a defining property of the median is that for any data set the average absolute deviation from the median is less than for any other point. Thus, the circular median is defined on this basis. It is the (angle) point on the circle for which the average absolute deviation is minimized, with deviation calculated as the length of the shorter arc between each data point and the reference point. Note that a circular median does not necessarily always exist, as for example, for a data set that is uniformly distributed around the
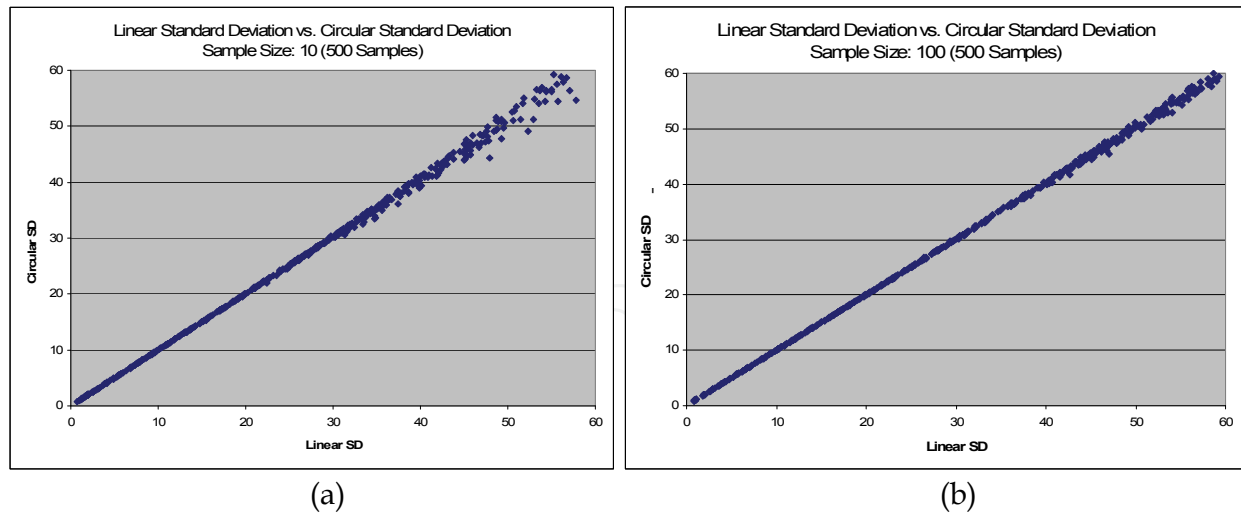
Fig. 6. Comparison of circular and linear standard deviations for 500 samples of (a) small (n=10) and (b) large (n=100) size.

circle (Mardia, 1972). If however, the range of the data set is less than 360° and has two clear endpoints, then the calculation of the median and MEAD can be done as in the linear case.

Two basic examples of circular statistics significance tests are the nonparametric Rayleigh z test and the Watson two sample U^2 test. The Rayleigh z test is used to determine whether data distributed around a circle are sufficiently random to assume a uniform distribution. The Watson two sample U^2 test can be used to compare two data distributions. Critical values for both tests and for many other circular statistics tests can be found in many advanced statistics books (e.g., Batschelet, 1981; Mardia, 1972; Zar, 1999; Rao and SenGupta, 2001). The special-purpose package Oriana (see http://www.kovcomp.co.uk) provides direct support for circular statistics as do add-ons such as SAS macros (e.g., Kölliker, M. 2005), A MATLAB Toolbox for Circular Statistics (Berens, 2009), and CircStat for S-Plus, R, and Stata (e.g., Rao and SenGupta, 2001).

7. Relative (discrimination) and categorical localization

The LE analysis conducted so far in this text was limited to the absolute identification of sound source locations in space. Two other types of localization judgments are relative judgments of sound source location (location discrimination) and categorical localization.

The basic measure of relative localization acuity is the minimum audible angle (MAA). The MAA, or *localization blur* (Blauert, 1974), is the minimum detectable difference in azimuth (or elevation) between locations of two identical but not simultaneous sound sources (Mills, 1958; 1972; Perrott, 1969). In other words, the MAA is the smallest perceptible difference in the position of a sound source. To measure the MAA, the listener is presented with two successive sounds coming from two different locations in space and is asked to determine whether the second sound came from the left or the right of the first one. The MAA is calculated as half the angle between the minimal positions to left and right of the sound source that result in 75% correct response rates. It depends on both frequency and direction of arrival of the sound wave. For wideband stimuli and low frequency tones, MAA is on the order of 1° to 2° for the frontal position, increases to 8-10° at 90° (Kuhn, 1987), and decreases again to 6-7° at the rear (Mills, 1958; Perrott, 1969; Blauert, 1974). For low frequency tones arriving from the frontal position, the MAA corresponds well with the difference limen (DL)

for ITD (~10 µs), and for high frequency tones, it matches well with the difference limen for IID (0.5-1.0 dB), both measured by earphone experiments. The MAA is largest for mid-high frequencies, especially for angles exceeding 40° (Mills, 1958; 1960; 1972). The vertical MAA is about 3-9° for the frontal position (e.g., Perrott & Saberi, 1990; Blauert, 1974).

The MAA has frequently been considered to be the smallest attainable precision (difference limen) in absolute sound source localization in space (e.g., Hartmann, 1983; Hartmann & Rakerd, 1989; Recanzone et al., 1998). However, the precision of absolute localization judgments observed in most studies is generally much poorer than the MAA for the same type of sound stimulus. For example, the average error in absolute localization for a broadband sound source is about 5° for the frontal and about 20° for the lateral position (Hofman & Van Opstal, 1998; Langendijk et al., 2001). Thus, it is possible that the acuity of the MAA, where two sounds are presented in succession, and the precision of absolute localization, where only a single sound is presented, are not well correlated and measure two different human capabilities (Moore et al., 2008). This view is supported by results from animal studies indicating that some types of lesions in the brain affect the precision of absolute localization but not the acuity of the MAA (e.g., Young et al., 1992; May, 2000). In another set of studies, Spitzer and colleagues observed that barn owls exhibited different MAA acuity in anechoic and echoic conditions while displaying similar localization precision across both conditions (Spitzer et al., 2003; Spitzer & Takahasi, 2006). The explanation of these differences may be the difference in the cognitive tasks and the much greater difficulty of the absolute localization task.

Another method of determining LE is to ask listeners to specify the sound source location by selecting from a set of specifically labeled locations. These locations can be indicated by either visible sound sources or special markers on the curtain covering the sound sources (Butler et al., 1990; Abel & Banerjee, 1996). Such approaches restrict the number of possible directions to the predetermined target locations and lead to categorical localization judgments (Perrett & Noble, 1995). The results of categorical localization studies are normally expressed as percents of correct responses rather than angular deviations. The distance between the labeled target locations is the resolution of the localization judgments and describes the localization precision of the study. In addition, if the targets are only distributed across a limited region of the space, this may provide cues resolving potential front-back confusion (Carlile et al., 1997).

Although categorical localization was the predominant localization methodology in older studies, it is still used in many studies today (Abel & Banerjee, 1996; Vause & Grantham, 1999; Van Hosesel & Clark, 1999; Macaulay et al., 2010). Additionally, the Source Azimuth Identification in Noise Test (SAINT) uses categorical judgments with a clock-like array of 12 loudspeakers (Vermiglio et al., 1998) and a standard system for testing the localization ability of cochlear implant users is categorical with 8 loudspeakers distributed in symmetric manner in the horizontal plane in front of the listener with 15.5° of separation (Tyler & Witt, 2004).

In order to directly compare the results of a categorical localization study to an absolute localization study, it is necessary to extract a mean direction and standard deviation from the distribution of responses over the target locations. If the full distribution is known, then by treating each response as an indication of the actual angular positions of the selected target location, the mean and standard deviation can be calculated as usual. If only the percent of correct responses is provided, then as long as the percent correct is over 50%, a normal distribution z-Table (giving probabilities of a result being less than a given z-score) can be used to estimate the standard deviation. If d is the angle of target separation (i.e., the

angle between two adjacent loudspeakers), p the percent correct and z the z-score corresponding to *(p+1)/2*, then the standard deviation is given by

$$\sigma = \frac{d}{2z} \tag{14}$$

and the mean by the angular position of the correct target location. This is based on the assumption that the correct responses are normally distributed over the range delimited by the points half way between the correct loudspeaker and the two loudspeakers on either side. This range spans the angle of target separation (*d*) and thus *d/2* is the corresponding z-score for the actual distribution. The relationship between the standard z-score and the z-score for a normal distribution N(μ,σ) is given by:

$$z_{N(\mu,\sigma)} = \mu + \sigma \cdot z . \tag{15}$$

In this case, the mean, μ, is 0 as the responses are centered around the correct loudspeaker position, so solving for the standard deviation gives Equation 14. As an example, consider an array of loudspeakers separated by 15° and an 85% correct response rate for some individual speaker. The z-score for (1+.85)/2 = .925 is 1.44, so the standard deviation is estimated to be 7.5°/1.44 = 5.2°.

An underlying assumption in the preceding discussion is that the experimental conditions of the categorical judgment task are such that the listener is surrounded by evenly spaced target locations. If this is not the case, then the results for the extreme locations at either end may have been affected by the fact that there are no further locations. In particular this is a problem when the location with the highest percent of responses is not the correct location and the distribution is not symmetric around it. For example, this appears to be the case for the speakers located at ±90° in the 30° loudspeaker arrangement used by Abel & Banerjee (1996).

8. Summary

Judgments of sound source location as well as the resultant localization errors are angular (circular) variables and in general cannot be properly analyzed by the standard statistical methods that assume an underlying (infinite) linear distributions. The appropriate methods of statistical analysis are provided by the field of spherical or circular statistics for three- and two-dimensional angular data, respectively. However, if the directional judgments are relatively well concentrated around a central direction, the differences between the circular and linear measures are minimal, and linear statistics can effectively be used in lieu of circular statistics. The criteria under which the linear analysis of directional data is justified has been a focus of the present discussion. Some basic elements of circular statistics have been also presented to demonstrate the fundamental differences between the two types of data analysis. It has to be stressed that in both cases, it is important to differentiate front-back errors from other gross errors and analyze the front-back errors separately. Gross errors may then be trimmed or winsorized. Both the processing and interpretation of localization data becomes more intuitive and simpler when the ±180º scale is used for data representation instead of the 0-360º scale, although both scales can be successfully used.

In order to meaningfully interpret overall localization error, it is important to individually report both the constant error (accuracy) and random error (precision) of the localization judgments. Error measures like root mean squared error and mean unsigned error represent

a specific combination of these two error components and do not on their own provide an adequate characterization of localization error. Overall localization error can be used to characterizes a given set of results but does not give any insight into the underlying causes of the error.

Since the overall purpose of this chapter was to provide information for the effective processing and interpretation of sound localization data, the initial part of the chapter was devoted to differentiating auditory spatial perception from auditory localization and to summarizing the basic terminology used in spatial perception studies and data description. This terminology is not always consistently used in the literature and some standardization would be beneficial. In addition, prior to the discussion of circular data analysis, the most common measures used to describe directional data were compared, and their advantages and limitations indicated. It has been stressed that the standard statistical measures for assessing constant and random error are not robust measures, as they are quite susceptible to being overly influenced by extreme values in the data set. The robust measures discussed in this chapter are intended to provide a starting point for researchers unfamiliar with robust statistics. Given that localization studies, like many experiments involving human judgment, are apt to produce some number of outlying or inaccurate results, it may often be beneficial to utilize robust alternatives to the standard measures. In any case, researchers should be aware of this consideration.

All of the above discussion was related to absolute localization judgments as the most commonly studied form of localization. Therefore, the last section of the chapter deals briefly with location discrimination and categorical localization judgments. The specific focus of this section was to indicate how results from absolute localization and categorical localization studies could be directly compared and what simplifying assumptions are made in carrying out these types of comparisons.

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Sound source localization is an important research field that has attracted researchers' efforts from many technical and biomedical sciences. Sound source localization (SSL) is defined as the determination of the direction from a receiver, but also includes the distance from it. Because of the wave nature of sound propagation, phenomena such as refraction, diffraction, diffusion, reflection, reverberation and interference occur. The wide spectrum of sound frequencies that range from infrasounds through acoustic sounds to ultrasounds, also introduces difficulties, as different spectrum components have different penetration properties through the medium. Consequently, SSL is a complex computation problem and development of robust sound localization techniques calls for different approaches, including multisensor schemes, null-steering beamforming and time-difference arrival techniques. The book offers a rich source of valuable material on advances on SSL techniques and their applications that should appeal to researches representing diverse engineering and scientific disciplines.

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