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# Robust H∞ PID Controller Design Via LMI Solution of Dissipative Integral Backstepping with State Feedback Synthesis

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## 1. Introduction

PID controller has been extensively used in industries since 1940s and still the most often implemented controller today. The PID controller can be found in many application areas: petroleum processing, steam generation, polymer processing, chemical industries, robotics, unmanned aerial vehicles (UAVs) and many more. The algorithm of PID controller is a simple, single equation relating proportional, integral and derivative parameters. Nonetheless, these provide good control performance for many different processes. This flexibility is achieved through three adjustable parameters of which values can be selected to modify the behaviour of the closed loop system. A convenient feature of the PID controller is its compatibility with enhancement that provides higher capabilities with the same basic algorithm. Therefore the performance of a basic PID controller can be improved through judicious selection of these three values.

Many tuning methods are available in the literature, among with the most popular method the Ziegler-Nichols (Z-N) method proposed in 1942 (Ziegler & Nichols, 1942). A drawback of many of those tuning rules is that such rules do not consider load disturbance, model uncertainty, measurement noise, and set-point response simultaneously. In general, a tuning for high performance control is always accompanied by low robustness (Shinskey, 1996). Difficulties arise when the plant dynamics are complex and poorly modeled or, specifications are particularly stringent. Even if a solution is eventually found, the process is likely to be expensive in terms of design time. Varieties of new methods have been proposed to improve the PID controller design, such as analytical tuning (Boyd & Barrat, 1991; Hwang & Chang, 1987), optimization based (Wong & Seborg, 1988; Boyd & Barrat, 1991; Astrom & Hagglund, 1995), gain and phase margin (Astrom & Hagglund, 1995; Fung et al., 1998). Further improvement of the PID controller is sought by applying advanced control designs (Ge et al., 2002; Hara et al., 2006; Wang et al., 2007; Goncalves et al., 2008).

In order to design with robust control theory, the PID controller is expressed as a state feedback control law problem that can then be solved by using any full state feedback robust control synthesis, such as Guaranteed Cost Design using Quadratic Bound (Petersen et al., 2000), H∞ synthesis (Green & Limebeer, 1995; Zhou & Doyle, 1998), Quadratic Dissipative Linear Systems (Yuliar et al., 1997) and so forth. The PID parameters selection by

transforming into state feedback using linear quadratic method was first proposed by Williamson and Moore in (Williamson & Moore, 1971). Preliminary applications were investigated in (Joelianto & Tomy, 2003) followed the work in (Joelianto et al., 2008) by extending the method in (Williamson & Moore, 1971) to $H_\infty$ synthesis with dissipative integral backstepping. Results showed that the robust $H_\infty$ PID controllers produce good tracking responses without overshoot, good load disturbance responses, and minimize the effect of plant uncertainties caused by non-linearity of the controlled systems.

Although any robust control designs can be implemented, in this paper, the investigation is focused on the theory of parameter selection of the PID controller based on the solution of robust $H_\infty$ which is extended with full state dissipative control synthesis and integral backstepping method using an algebraic Riccati inequality (ARI). This paper also provides detailed derivations and improved conditions stated in the previous paper (Joelianto & Tomy, 2003) and (Joelianto et al., 2008). In this case, the selection is made via control system optimization in robust control design by using linear matrix inequality (LMI) (Boyd et al., 1994; Gahinet & Apkarian, 1994). LMI is a convex optimization problem which offers a numerically tractable solution to deal with control problems that may have no analytical solution. Hence, reducing a control design problem to an LMI can be considered as a practical solution to this problem (Boyd et al., 1994). Solving robust control problems by reducing to LMI problems has become a widely accepted technique (Balakrishnan & Wang, 2000). General multi objectives control problems, such as $H_2$ and $H_\infty$ performance, peak to peak gain, passivity, regional pole placement and robust regulation are notoriously difficult, but these can be solved by formulating the problems into linear matrix inequalities (LMIs) (Boyd et al., 1994; Scherer et al., 1997)).

The objective of this paper is to propose a parameter selection technique of PID controller within the framework of robust control theory with linear matrix inequalities. This is an alternative method to optimize the adjustment of a PID controller to achieve the performance limits and to determine the existence of satisfactory controllers by only using two design parameters instead of three well known parameters in the PID controller. By using optimization method, an absolute scale of merits subject to any designs can be measured. The advantage of the proposed technique is implementing an output feedback control (PID controller) by taking the simplicity in the full state feedback design. The proposed technique can be applied either to a single-input-single-output (SISO) or to a multi-inputs-multi-outputs (MIMO) PID controller.

The paper is organised as follows. Section 2 describes the formulation of the PID controller in the full state feedback representation. In section 3, the synthesis of $H_\infty$ dissipative integral backstepping is applied to the PID controller using two design parameters. This section also provides a derivation of the algebraic Riccati inequality (ARI) formulation for the robust control from the dissipative integral backstepping synthesis. Section 4 illustrates an application of the robust PID controller for time delay uncertainties compensation in a network control system problem. Section 5 provides some conclusions.

## 2. State feedback representation of PID controller

In order to design with robust control theory, the PID controller is expressed as a full state feedback control law. Consider a single input single output linear time invariant plant described by the linear differential equation

$$\dot{x}(t) = Ax(t) + B_2 u(t)$$
$$y(t) = C_2 x(t) \tag{1}$$

with some uncertainties in the plant which will be explained later. Here, the states $x \in R^n$ are the solution of (1), the control signal $u \in R^1$ is assumed to be the output of a PID controller with input $y \in R^1$. The PID controller for regulator problem is of the form

$$u(t) = K_1 \int_0^t y(t)d(t) + K_2 y(t) + K_3 \frac{d}{dt} y(t) \tag{2}$$

which is an output feedback control system and $K_1 = K_p / T_i$, $K_2 = K_p$, $K_3 = K_p T_d$ of which $K_p$, $T_i$ and $T_d$ denote proportional gain, time integral and time derivative of the well known PID controller respectively. The structure in equation (2) is known as the standard PID controller (Astrom & Hagglund, 1995).
The control law (2) is expressed as a state feedback law using (1) by differentiating the plant output $y$ as follows

$$y = C_2 x$$
$$\dot{y} = C_2 A x + C_2 B_2 u$$
$$\ddot{y} = C_2 A^2 x + C_2 A B_2 u + C_2 B_2 \dot{u}$$

This means that the derivative of the control signal (2) may be written as

$$(1 - K_3 C_2 B_2)\dot{u} - (K_3 C_2 A^2 + K_2 C_2 A + K_1 C_2)x - (K_3 C_2 A B_2 + K_2 C_2 B_2)u = 0 \tag{3}$$

Using the notation $\hat{K}$ as a normalization of $K$, this can be written in more compact form

$$\hat{K} = [\hat{K}_1 \quad \hat{K}_2 \quad \hat{K}_3] = (1 - K_3 C_2 B_2)^{-1}[K_1 \quad K_2 \quad K_3] \tag{4}$$

or $\hat{K} = cK$ where $c$ is a scalar. This control law is then given by

$$\dot{u} = \hat{K}[C_2^T \quad A^T C_2^T \quad (A^2)^T C_2^T]^T x + \hat{K}[0 \quad B_2^T C_2^T \quad B_2^T A^T C_2^T]^T u \tag{5}$$

Denote $K_x = \hat{K}[C_2^T \quad A^T C_2^T \quad (A^2)^T C_2^T]^T$ and $K_u = \hat{K}[0 \quad B_2^T C_2^T \quad B_2^T A^T C_2^T]^T$, the block diagram of the control law (5) is shown in Fig. 1. In the state feedback representation, it can be seen that the PID controller has interesting features. It has state feedback in the upper loop and pure integrator backstepping in the lower loop. By means of the internal model principle (IMP) (Francis & Wonham, 1976; Joelianto & Williamson, 2009), the integrator also guarantees that the PID controller will give zero tracking error for a step reference signal.
Equation (5) represents an output feedback law with constrained state feedback. That is, the control signal (2) may be written as

$$u_a = K_a x_a \tag{6}$$

where

$$u_a = \dot{u} \,, \quad x_a = \begin{bmatrix} x \\ u \end{bmatrix}$$

$$K_a = \hat{K}\left[[C_2^T \quad A^TC_2^T \quad (A^2)^TC_2^T]^T \qquad [0 \quad B_2^TC_2^T \quad B_2^TA^TC_2^T]^T\right]$$

Arranging the equation and eliminating the transpose lead to

$$K_a = \hat{K}\begin{bmatrix} C_2 & 0 \\ C_2A & C_2B \\ C_2A^2 & C_2AB_2 \end{bmatrix} = \hat{K}\Gamma \tag{7}$$

The augmented system equation is obtained from (1) and (7) as follows

$$\dot{x}_a = A_ax_a + B_au_a \tag{8}$$

where

$$A_a = \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix}; \quad B_a = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
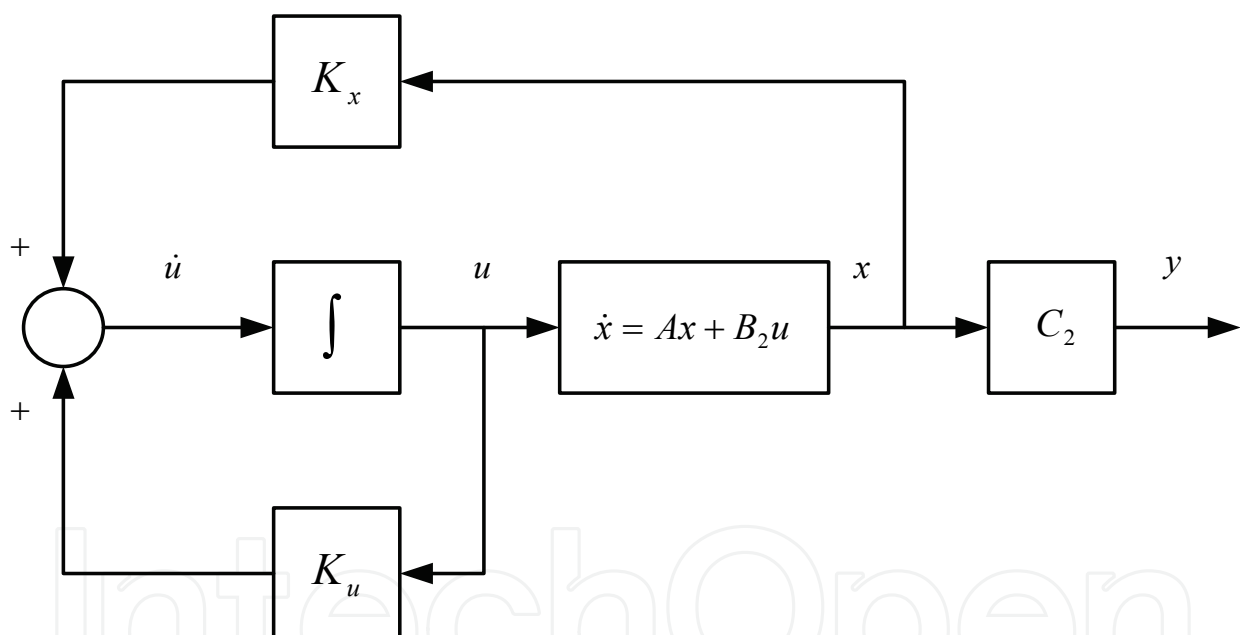


Fig. 1. Block diagram of state space representation of PID controller

Equation (6), (7) and (8) show that the PID controller can be viewed as a state variable feedback law for the original system augmented with an integrator at its input. The augmented formulation also shows the same structure known as the integral backstepping method (Krstic et al., 1995) with one pure integrator. Hence, the selection of the parameters of the PID controller (6) via full state feedback gain is inherently an integral backstepping control problems. The problem of the parameters selection of the PID controller becomes an optimal problem once a performance index of the augmented system (8) is defined. The parameters of the PID controller are then obtained by solving equation (7) that requires the inversion of the matrix $\Gamma$. Since $\Gamma$ is, in general, not a square matrix, a numerical method should be used to obtain the inverse.

For the sake of simplicity, the problem has been set-up in a single-input-single-output (SISO) case. The extension of the method to a multi-inputs-multi-outputs (MIMO) case is straighforward. In MIMO PID controller, the control signal has dimension $m$, $u \in R^m$ is assumed to be the output of a PID controller with input has dimension $p$, $y \in R^p$. The parameters of the PID controller $K_1$, $K_2$, and $K_3$ will be square matrices with appropriate dimension.

## 3. H∞ dissipative integral backstepping synthesis

The backstepping method developed by (Krstic et al., 1995) has received considerable attention and has become a well known method for control system designs in the last decade. The backstepping design is a recursive algorithm that steps back toward the control input by means of integrations. In nonlinear control system designs, backstepping can be used to force a nonlinear system to behave like a linear system in a new set of coordinates with flexibility to avoid cancellation of useful nonlinearities and to focus on the objectives of stabilization and tracking. Here, the paper combines the advantage of the backstepping method, dissipative control and H∞ optimal control for the case of parameters selection of the PID controller to develop a new robust PID controller design.

Consider the single input single output linear time invariant plant in standard form used in H∞ performance by the state space equation

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t), \quad x(0) = x_0 \\
z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\
y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t)
\end{aligned}
\tag{9}
$$

where $x \in R^n$ denotes the state vector, $u \in R^1$ is the control input, $w \in R^p$ is an external input and represents driving signals that generate reference signals, disturbances, and measurement noise, $y \in R^1$ is the plant output, and $z \in R^m$ is a vector of output signals related to the performance of the control system.

**Definition 1.**

A system is dissipative (Yuliar et al., 1998) with respect to supply rate $r(z(t), w(t))$ for each initial condition $x_0$ if there exists a storage function $V$, $V : R^n \to R^+$ satisfies the inequality

$$
V(x(t_0)) + \int_{t_0}^{t_1} r(z(t), w(t)) dt \geq V(x(t_1)), \forall (t_1, t_0) \in R^+, x_0 \in R^n
\tag{10}
$$

and $t_0 \leq t_1$ and all trajectories ($x, y, z$) which satisfies (9).

The supply rate function $r(z(t), w(t))$ should be interpreted as the supply delivered to the system. If in the interval $[t_0, t_1]$ the integral $\int_{t_0}^{t_1} r(z(t), w(t)) dt$ is positive then work has been done to the system. Otherwise work is done by the system. The supply rate determines not only the dissipativity of the system but also the required performance index of the control system. The storage function $V$ generalizes the notion of an energy function for a dissipative system. The function characterizes the change of internal storage $V(x(t_1)) - V(x(t_0))$ in any interval $[t_0, t_1]$, and ensures that the change will never exceed the amount of the supply into

the system. The dissipative method provides a unifying tool as index performances of control systems can be expressed in a general supply rate by selecting values of the supply rate parameters.

The general quadratic supply rate function (Hill & Moylan, 1977) is given by the following equation

$$r(z,w) = \frac{1}{2}(w^T Q w + 2w^T S z + z^T R z) \tag{11}$$

where $Q$ and $R$ are symmetric matrices and

$$\bar{Q}(x) = Q + S D_{11}(x) + D_{11}^T(x) S^T + D_{11}^T(x) R D_{11}(x)$$

such that $\bar{Q}(x) > 0$ for $x \in R^n$ and $R \le 0$ and $\inf_{x \in R^n}\{\sigma_{\min}(\bar{Q}(x))\} = k > 0$. This general supply rate represents general problems in control system designs by proper selection of matrices $Q$, $R$ and $S$ (Hill & Moylan, 1977; Yuliar et al., 1997): finite gain (H$_\infty$) performance ($Q = \gamma^2 I$, $S = 0$ and $R = -I$); passivity ($Q = R = 0$ and $S = I$); and mixed H$_\infty$-positive real performance ($Q = \theta\gamma^2 I$, $R = -\theta I$ and $S = (1-\theta)I$ for $\theta \in [0,1]$).

For the PID control problem in the robust control framework, the plant ($\Sigma$) is given by the state space equation

$$\Sigma = \begin{cases} \dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t), x(0) = x_0 \\ \\ z(t) = \begin{pmatrix} C_1 x(t) \\ D_{12} u(t) \end{pmatrix} \end{cases} \tag{12}$$

with $D_{11} = 0$ and $\gamma > 0$ with the quadratic supply rate function for H$_\infty$ performance

$$r(z,w) = \frac{1}{2}(\gamma^2 w^T w - z^T z) \tag{13}$$

Next the plant ($\Sigma$) is added with integral backstepping on the control input as follows

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ u_a(t) &= \dot{u}(t) \\ z(t) &= \begin{pmatrix} C_1 x(t) \\ D_{12} u(t) \\ \rho u_a(t) \end{pmatrix} \end{aligned} \tag{14}$$

where $\rho$ is the parameter of the integral backstepping which act on the derivative of the control signal $\dot{u}(t)$. In equation (14), the parameter $\rho > 0$ is a tuning parameter of the PID controller in the state space representation to determine the rate of the control signal. Note that the standard PID controller in the state feedback representation in the equations (6), (7) and (8) corresponds to the integral backstepping parameter $\rho = 1$. Note that, in this design the gains of the PID controller are replaced by two new design parameters namely $\gamma$ and $\rho$ which correspond to the robustness of the closed loop system and the control effort.

The state space representation of the plant with an integrator backstepping in equation (14) can then be written in the augmented form as follows

$$\begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_a(t)$$

$$z(t) = \begin{bmatrix} C_1 & 0 \\ 0 & D_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix} u_a(t)$$

(15)

The compact form of the augmented plant ($\Sigma_a$) is given by

$$\dot{x}_a(t) = A_a x_a(t) + B_w w(t) + B_a u_a(t); x_a(0) = x_{a0}$$
$$z(t) = C_a x_a(t) + D_{1a} w(t) + D_{2a} u_a(t)$$

(16)

where

$$x_a = \begin{bmatrix} x \\ u \end{bmatrix}, \ A_a = \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix}, \ B_w = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \ B_a = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_a = \begin{bmatrix} C_1 & 0 \\ 0 & D_{12} \\ 0 & 0 \end{bmatrix}, \ D_{2a} = \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix}$$

Now consider the full state gain feedback of the form

$$u_a(t) = K_a x_a(t)$$

(17)

The objective is then to find the gain feedback $K_a$ which stabilizes the augmented plant ($\Sigma_a$) with respect to the dissipative function $V$ in (10) by a parameter selection of the quadratic supply rate (11) for a particular control performance. Fig. 2. shows the system description of the augmented system of the plant and the integral backstepping with the state feedback control law.
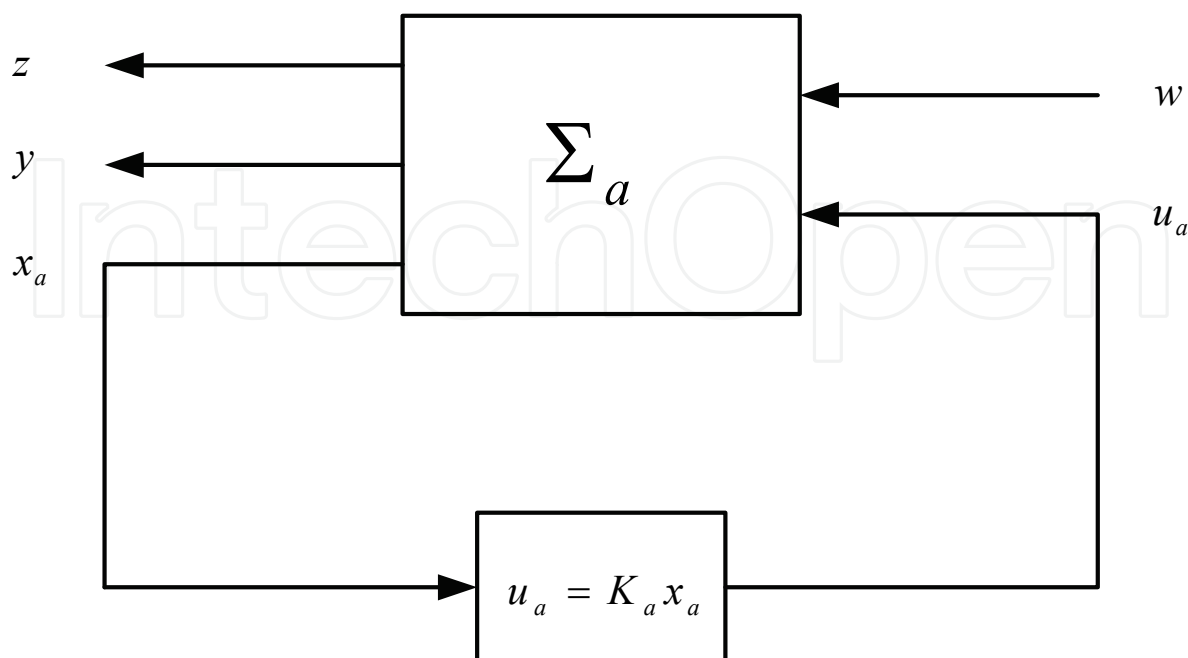


Fig. 2. System description of the augmented system

The following theorem gives the existence condition and the formula of the stabilizing gain feedback $K_a$.

**Theorem 2.**

Given $\gamma > 0$ and $\rho > 0$. If there exists $X = X^T > 0$ of the following Algebraic Riccati Inequality

$$X \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} A^T & 0 \\ B_2^T & 0 \end{bmatrix} X - X \left( \rho^{-2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - \gamma^{-2} \begin{bmatrix} B_1 B_1^T & 0 \\ 0 & 0 \end{bmatrix} \right) X + \begin{bmatrix} C_1^T C_1 & 0 \\ 0 & D_{12}^T D_{12} \end{bmatrix} < 0 \quad (18)$$

Then the full state feedback gain

$$K_a = -\rho^{-2} B_a^T X = -\rho^{-2} \begin{bmatrix} 0 & 1 \end{bmatrix} X \quad (19)$$

leads to $||\, \Sigma \,||_\infty < \gamma$

**Proof.**

Consider the standard system (9) with the full state feedback gain

$$u(t) = Kx(t)$$

and the closed loop system

$$\dot{x}(t) = A^u x(t) + B_1 w(t), \quad x(0) = x_0$$
$$z(t) = C^u x(t) + D_{11} w(t)$$

where $D_{11} = 0$, $A^u = A + B_2 K$, $C^u = C_1 + D_{12} K$ is strictly dissipative with respect to the quadratic supply rate (11) such that the matrix $A^u$ is asymptotically stable. This implies that the related system

$$\dot{x}(t) = \tilde{A} x(t) + \tilde{B}_1 w(t), \quad x(0) = x_0$$
$$z(t) = \tilde{C}_1 x(t)$$

where $\tilde{A} = A^u - B_1 \bar{Q}^{-1} \bar{S} C^u$, $\tilde{B}_1 = B_1 \bar{Q}^{-1/2}$ and $\tilde{C}_1 = (\bar{S}^T \bar{Q}^{-1} \bar{S} - R)^{1/2} C^u$ has $H_\infty$ norm strictly less than 1, which implies there exits a matrix $X > 0$ solving the following Algebraic Riccati Inequality (ARI) (Petersen et al. 1991)

$$\tilde{A}^T X + X \tilde{A} + X \tilde{B}_1 \tilde{B}_1^T X + \tilde{C}_1^T \tilde{C}_1 < 0 \quad (20)$$

In terms of the parameter of the original system, this can be written as

$$(A^u)^T X + X A^u + [X B_1 - (C^u)^T \bar{S}^T] \bar{Q}^{-1} [B_1^T X - \bar{S} C^u] - (C^u)^T R C^u < 0 \quad (21)$$

Define the full state feedback gain

$$K = -\bar{E}^{-1} \left( \left( B_2 - B_1 \bar{Q}^{-1} \bar{S} D_{12} \right)^T X + D_{12} \bar{R} C_1 \right) \quad (22)$$

By inserting

$$A^u = A + B_2K, \quad C^u = C_1 + D_{12}K$$
$$\bar{S} = S + D_{11}^T R, \quad \bar{Q} = Q + SD_{11} + D_{11}^T S^T + D_{11}^T R D_{11}$$
$$\bar{R} = \bar{S}^T \bar{Q}^{-1} \bar{S} - R, \quad \bar{E} = D_{12}^T \bar{R} D_{12}$$
$$\bar{B} = B_2 - B_1 \bar{Q}^{-1} \bar{S} D_{12}, \quad \bar{D} = I - D_{12} \bar{E}^{-1} D_{12}^T \bar{R}$$

into (21) , completing the squares and removing the gain $K$ give the following ARI

$$\begin{aligned} &X(A - \bar{B}\bar{E}^{-1}D_{12}^T \bar{R} C_1 - B_1 \bar{Q}\bar{S}C_1) + (A - \bar{B}\bar{E}^{-1}D_{12}^T \bar{R} C_1 - B_1 \bar{Q}\bar{S}C_1)X - \\ &-X(\bar{B}\bar{E}^{-1}\bar{B}^T - B_1 \bar{Q}^{-1}B_1^T)X + C_1^T \bar{D}^T \bar{R}\bar{D}C_1 < 0 \end{aligned} \tag{23}$$

Using the results (Scherer, 1990), if there exists $X > 0$ satisfies (23) then $K$ given by (22) is stabilizing such that the closed loop system $A^u = A + B_2K$ is asymptotically stable.

Now consider the augmented plant with integral backstepping in (16). In this case, $D_{1a} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. Note that $D_{2a}^T C_a = 0$ and $D_{1a} = 0$. The H∞ performance is satisfied by setting the quadratic supply rate (11) as follow:

$$\bar{S} = 0, \quad \bar{R} = -R = I, \quad \bar{E} = D_{2a}^T \bar{R} D_{2a} = D_{2a}^T D_{2a}, \quad \bar{B} = B_a,$$
$$\bar{D} = I - D_{2a}(D_{2a}^T D_{2a})^{-1} D_{2a}^T$$

Inserting the setting to the ARI (23) yields

$$\begin{aligned} &X(A_a - B_a(D_{2a}^T D_{2a})^{-1} D_{2a}^T I C_a - B_w Q^{-1} 0 C_a) + \\ &+(A_a - B_a(D_{2a}^T D_{2a})^{-1} D_{2a}^T I C_a - B_w Q^{-1} 0 C_a)X - \\ &-X(B_a(D_{2a}^T D_{2a})^{-1} B_a^T - B_w Q^{-1} B_w^T)X + \\ &+(C_a^T (I - D_{2a}(D_{2a}^T D_{2a})^{-1} D_{2a}^T)^T x \quad (I - D_{2a}(D_{2a}^T D_{2a})^{-1} D_{2a}^T)C_a) < 0 \end{aligned}$$

The equation can then be written in compact form

$$XA_a + A_a^T X - X(\rho^{-2}B_a B_a^T - \gamma^{-2}B_w B_w^T)X + C_a^T C_a < 0 \tag{24}$$

this gives (18). The full state feedback gain is then found by inserting the setting into (22)

$$K_a = -\bar{E}^{-1}\left((B_a - B_w \bar{Q}^{-1}\bar{S}D_{2a})^T X - D_{2a}^T \bar{R} C_a\right)$$

that gives $||\Sigma||_\infty < \gamma$ (Yuliar et al., 1998; Scherer & Weiland, 1999). This completes the proof.

The relation of the ARI solution (8) to the ARE solution is shown in the following. Let the transfer function of the plant (9) is represented by

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix}$$

and assume the following conditions hold:
(A1). $(A, B_2, C_2)$ is stabilizable and detectable
(A2). $D_{22} = 0$

(A3). $D_{12}$ has full column rank, $D_{21}$ has full row rank

(A4). $P_{12}(s)$ and $P_{21}(s)$ have no invariant zero on the imaginary axis

From (Gahinet & Apkarian, 1994), equivalently the Algebraic Riccati Equation (ARE) given by the formula

$$X(A - \bar{B}\bar{E}^{-1}D_{12}^{T}\bar{R}C_1 - B_1\bar{Q}\bar{S}C_1) + (A - \bar{B}\bar{E}^{-1}D_{12}^{T}\bar{R}C_1 - B_1\bar{Q}\bar{S}C_1)X - X(\bar{B}\bar{E}^{-1}\bar{B}^{T} - B_1\bar{Q}^{-1}B_1^{T})X + C_1^{T}\bar{D}^{T}\bar{R}\bar{D}C_1 = 0 \tag{25}$$

has a (unique) solution $X_{\infty} \geq 0$, such that

$$A + B_2 K + B_1\bar{Q}^{-1}[B_1^{T}X - \bar{S}(C_1 + D_{12}K)]$$

is asymptotically stable. The characterization of feasible $\gamma$ using the Algebraic Riccati Inequality (ARI) in (24) and ARE in (25) is immediately where the solution of ARE ($X_{\infty}$) and ARI ($X_0$) satisfy $0 \leq X_{\infty} < X_0$, $X_0 = X_0^{T} > 0$ (Gahinet & Apkarian, 1994).

The Algebraic Riccati Inequality (24) by Schur complement implies

$$\begin{bmatrix} A_a^{T}X + XA_a + C_a^{T}C_a & XB_a & XB_w \\ B_a^{T}X & \rho^2 I & 0 \\ B_w^{T}X & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{26}$$

Ther problem is then to find $X > 0$ such that the LMI given in (26) holds. The LMI problem can be solved using the method (Gahinet & Apkarian, 1994) which implies the solution of the ARI (18) (Liu & He, 2006). The parameters of the PID controller which are designed by using $H_{\infty}$ dissipative integral backstepping can then be found by using the following algorithm:

1.  Select $\rho > 0$
2.  Select $\gamma > 0$
3.  Find $X_0 > 0$ by solving LMI in (26)
4.  Find $K_a$ using (19)
5.  Find $\hat{K}$ using (7)
6.  Compute $K_1$, $K_2$ and $K_3$ using (4)
7.  Apply in the PID controller (2)
8.  If it is needed to achieve $\gamma$ minimum, repeat step 2 and 3 until $\gamma = \gamma_{min}$ then follows the next step

## 4. Delay time uncertainties compensation

Consider the plant given by a first order system with delay time which is common assumption in industrial process control and further assume that the delay time uncertainties belongs to an a priori known interval

$$Y(s) = \frac{1}{\tau s + 1} e^{-Ls} U(s), \ L \in [a, b] \tag{27}$$

The example is taken from (Joelianto et al., 2008) which represents a problem in industrial process control due to the implementation of industrial digital data communication via

ethernet networks with fieldbus topology from the controller to the sensor and the actuator (Hops et al., 2004; Jones, 2006, Joelianto & Hosana, 2009). In order to write in the state space representation, the delay time is approximated by using the first order Pade approximation

$$Y(s) = \frac{1}{\tau s + 1} \frac{-ds + 1}{ds + 1} U(s) \, , \ d = L / 2 \tag{28}$$

In this case, the values of $\tau$ and $d$ are assumed as follows: $\tau = 1$ s and $d_{nom} = 3$ s. These pose a difficult problem as the ratio between the delay time and the time constant is greater than one ($(d / \tau) > 1$). The delay time uncertainties are assumed in the interval $d \in [2,4]$.

The delay time uncertainty is separated from its nominal value by using linear fractional transformation (LFT) that shows a feedback connection between the nominal and the uncertainty block.
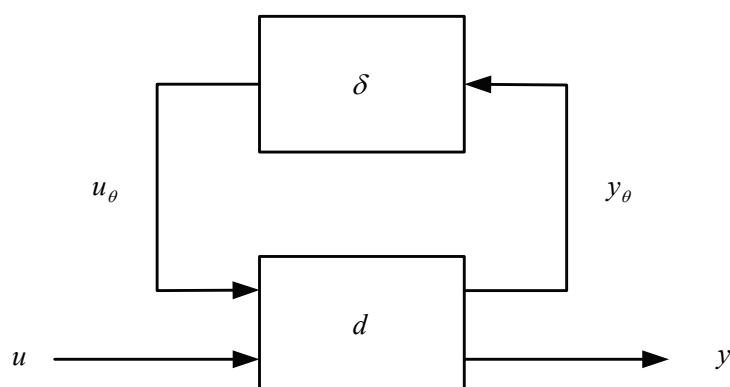


Fig. 3. Separation of nominal and uncertainty using LFT

The delay time uncertainties can then be represented as

$$d = \alpha d_{nom} + \beta \delta \, , \ -1 < \delta < 1$$

$$d = F_u \left( \begin{bmatrix} 0 & 1 \\ \beta & \alpha \end{bmatrix}, \delta \right)$$

After simplification, the delay time uncertainties of any known ranges have a linear fractional transformation (LFT) representation as shown in the following figure.
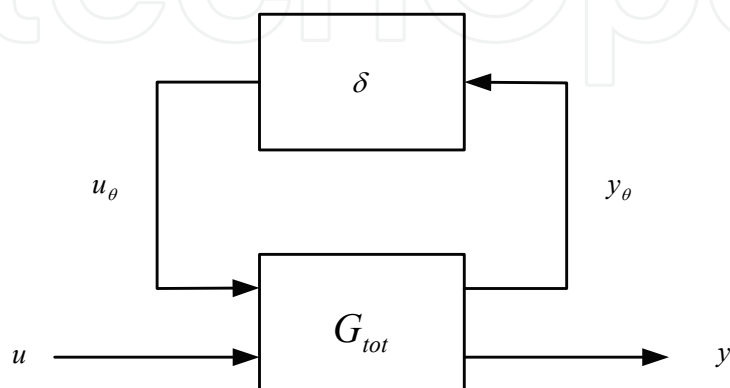


Fig. 4. First order system with delay time uncertainty

The representation of the plant augmented with the uncertainty is

$$G_{tot}(s) = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{29}$$

The corresponding matrices in (29) are

$$A_x = \begin{bmatrix} A_{x11} & 0 \\ 1 & -1 \end{bmatrix}, B_x = \begin{bmatrix} B_{x11} & B_{x2} \\ 0 & 1 \end{bmatrix}, C_x = \begin{bmatrix} C_{x11} & 0 \\ 0 & 1 \end{bmatrix}, D_x = \begin{bmatrix} D_{x11} & D_{x12} \\ 0 & 0 \end{bmatrix}$$

In order to incorporate the integral backstepping design, the plant is then augmented with an integrator as follows

$$A_a = \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} A_{x11} & 0 & B_{x11} \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_w = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} = \begin{bmatrix} B_{x11} \\ 0 \\ 0 \end{bmatrix},$$

$$B_a = \begin{bmatrix} 0 \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C_a = \begin{bmatrix} C_{x11} & 0 \\ 0 & D_{x12} \\ 0 & 0 \end{bmatrix},$$

$$D_{2a} = \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix}$$

The problem is then to find the solution $X > 0$ and $\gamma > 0$ of ARI (18) and to compute the full state feedback gain given by

$$u_a(t) = K_a x_a(t) = -\rho^{-2} \left( \begin{bmatrix} 0 & 1 \end{bmatrix} X \right) \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}$$

which is stabilizing and leads to the infinity norm $||\Sigma||_\infty < \gamma$.
The state space representation for the nominal system is given by

$$A_{nom} = \begin{bmatrix} -1.6667 & -0.6667 \\ 1 & 0 \end{bmatrix}, \ B_{nom} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ C_{nom} = \begin{bmatrix} -1 & 0.6667 \end{bmatrix}$$

In this representation, the performance of the closed loop system will be guaranteed for the specified delay time range with fast transient response (*z*). The full state feedback gain of the PID controller is given by the following equation

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \left(1 - K_3\begin{bmatrix} -1 & 0.6667 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} \hat{K}_1 \\ \hat{K}_2 \\ \hat{K}_3 \end{bmatrix}$$

For different $\gamma$, the PID parameters and transient performances, such as: settling time ($T_s$) and rise time ($T_r$) are calculated by using LMI (26) and presented in Table 1. For different $\rho$ but fixed $\gamma$, the parameters are shown in Table 2. As comparison, the PID parameters are also computed by using the standard H∞ performance obtained by solving ARE in (25). The results are shown Table 3 and Table 4 for different $\gamma$ and different $\rho$ respectively. It can be seen from Table 1 and 2 that there is no clear pattern either in the settling time or the rise time. Only Table 1 shows that decreasing $\gamma$ decreases the value of the three parameters. On the other hand, the calculation using ARE shows that the settling time and the rise time are decreased by reducing $\gamma$ or $\rho$. Table 3 shows the same result with the Table 1 when the value of $\gamma$ is decreased.

| γ | ρ | $K_p$ | $K_i$ | $K_d$ | $T_r$ (s) | $T_s$ 5% (s) |
|------|------|--------|--------|--------|------|------|
| 0.1 | 1 | 0.2111 | 0.1768 | 0.0695 | 10.8 | 12.7 |
| 0.248 | 1 | 0.3023 | 0.2226 | 0.1102 | 8.63 | 13.2 |
| 0.997 | 1 | 0.7744 | 0.3136 | 0.2944 | 4.44 | 18.8 |
| 1.27 | 1 | 10.471 | 0.5434 | 0.4090 | 2.59 | 9.27 |
| 1.7 | 1 | 13.132 | 0.746 | 0.5191 | 1.93 | 13.1 |

Table 1. Parameters and transient response of PID for different $\gamma$ with LMI

| γ | ρ | $K_p$ | $K_i$ | $K_d$ | $T_r$ (s) | $T_s$ 5% (s) |
|------|------|--------|--------|--------|------|------|
| 0.997 | 0.66 | 11.019 | 0.1064 | 0.3127 | 39.8 | 122 |
| 0.997 | 0.77 | 0.9469 | 0.2407 | 0.3113 | 13.5 | 39.7 |
| 0.997 | 1 | 0.7744 | 0.3136 | 0.2944 | 4.44 | 18.8 |
| 0.997 | 1.24 | 0.4855 | 0.1369 | 0.1886 | 21.6 | 56.8 |
| 0.997 | 1.5 | 0.2923 | 0.0350 | 0.1151 | 94.4 | 250 |

Table 2. Parameters and transient response of PID for different $\rho$ with LMI

| γ | ρ | $K_p$ | $K_i$ | $K_d$ | $T_r$ (s) | $T_s$ 5% (s) |
|---|---|---|---|---|---|---|
| 0.1 | 1 | 0.2317 | 0.055 | 0.1228 | 55.1 | 143 |
| 0.248 | 1 | 0.2319 | 0.0551 | 0.123 | 55.0 | 141 |
| 0.997 | 1 | 0.2373 | 0.0566 | 0.126 | 53.8 | 138 |
| 1.27 | 1 | 0.2411 | 0.0577 | 0.128 | 52.6 | 135 |
| 1.7 | 1 | 0.2495 | 0.0601 | 0.1327 | 52.2 | 130 |

Table 3. Parameters and transient response of PID for different $\gamma$ with ARE

| γ | ρ | $K_p$ | $K_i$ | $K_d$ | $T_r$ (s) | $T_s$ 5% (s) |
|---|---|---|---|---|---|---|
| 0.997 | 0.66 | 0.5322 | 0.1396 | 0.2879 | 21.9 | 57.6 |
| 0.997 | 0.77 | 0.4024 | 0.1023 | 0.2164 | 29.7 | 77.5 |
| 0.997 | 1 | 0.2373 | 0.0566 | 0.126 | 39.1 | 138 |
| 0.997 | 1.24 | 0.1480 | 0.0332 | 0.0777 | 91.0 | 234 |
| 0.997 | 1.5 | 0.0959 | 0.0202 | 0.0498 | 150.0 | 383 |

Table 4. Parameters and transient response of PID for different $\rho$ with ARE
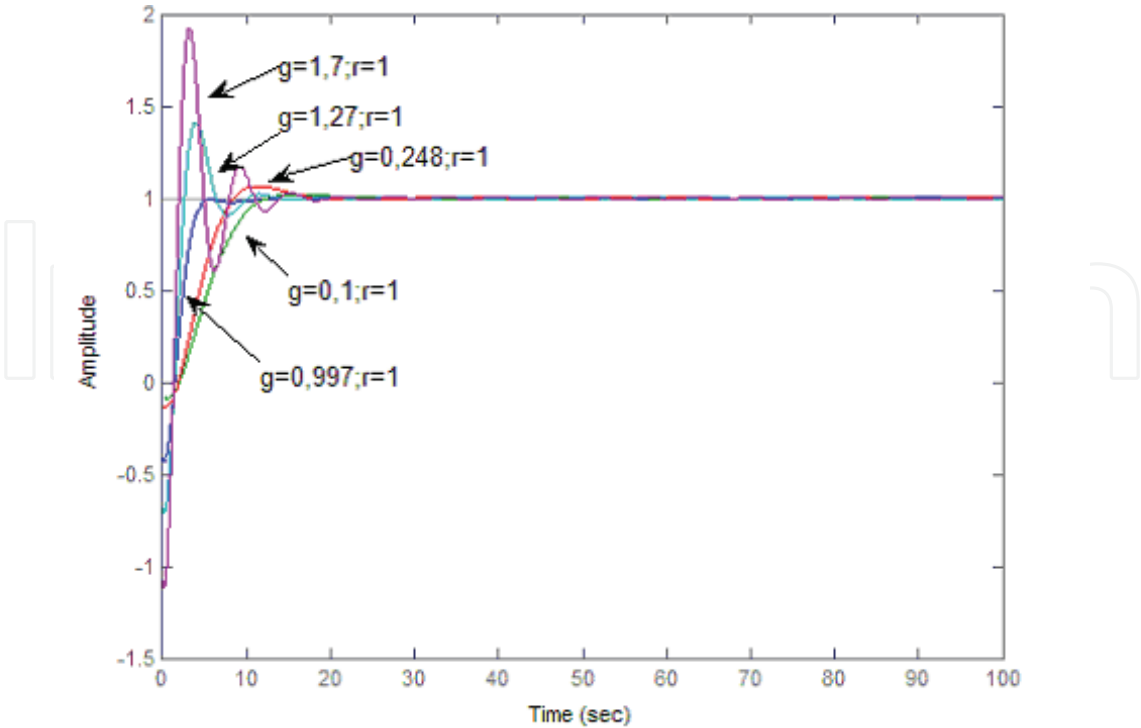


Fig. 5. Transient response for different $\gamma$ using LMI
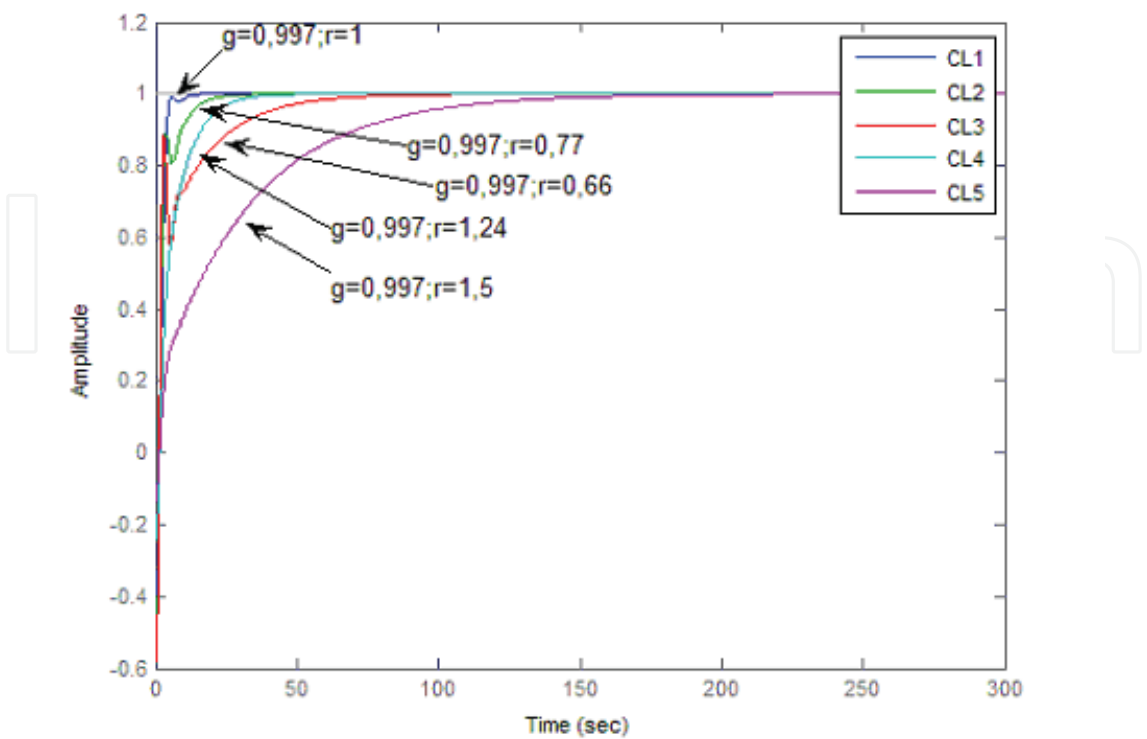
Fig. 6.  Transient response for different $\rho$ using LMI
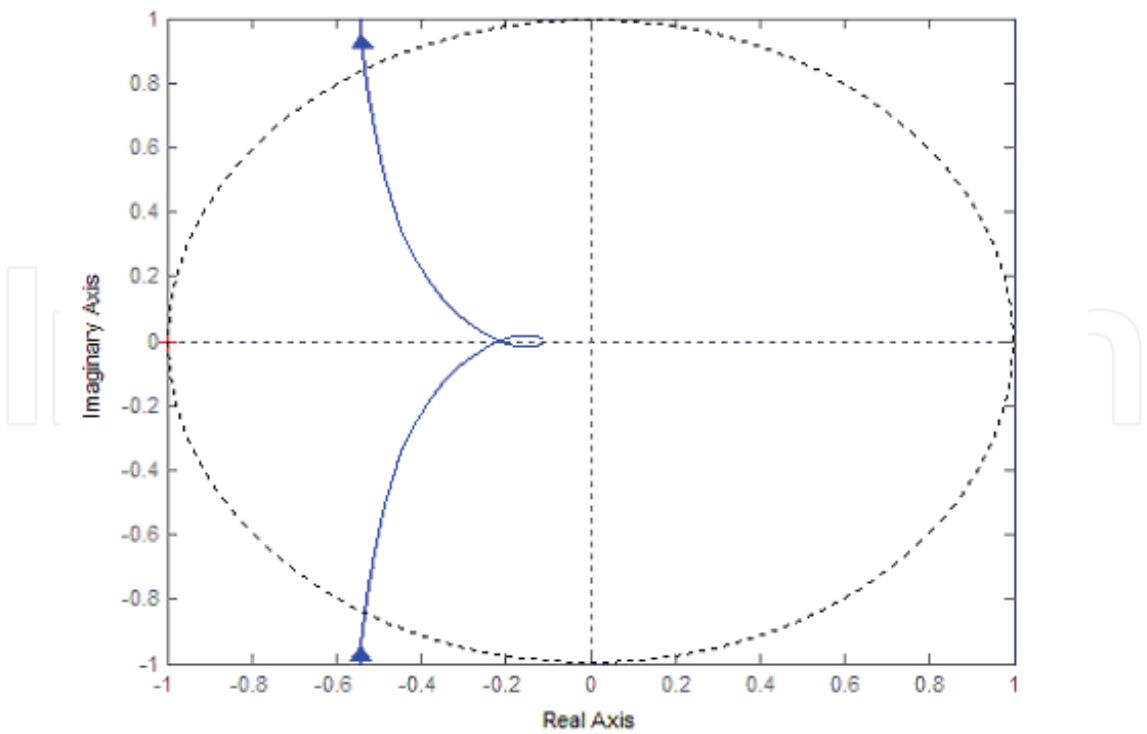


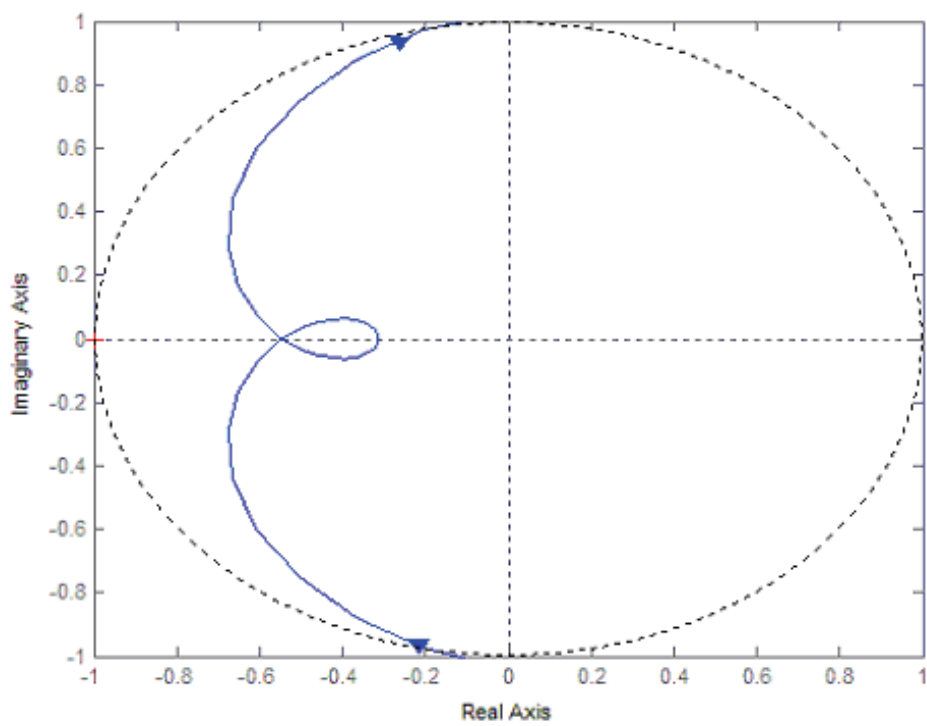Fig. 7. Nyquist plot $\gamma = 0.248$ and $\rho = 1$ using LMI

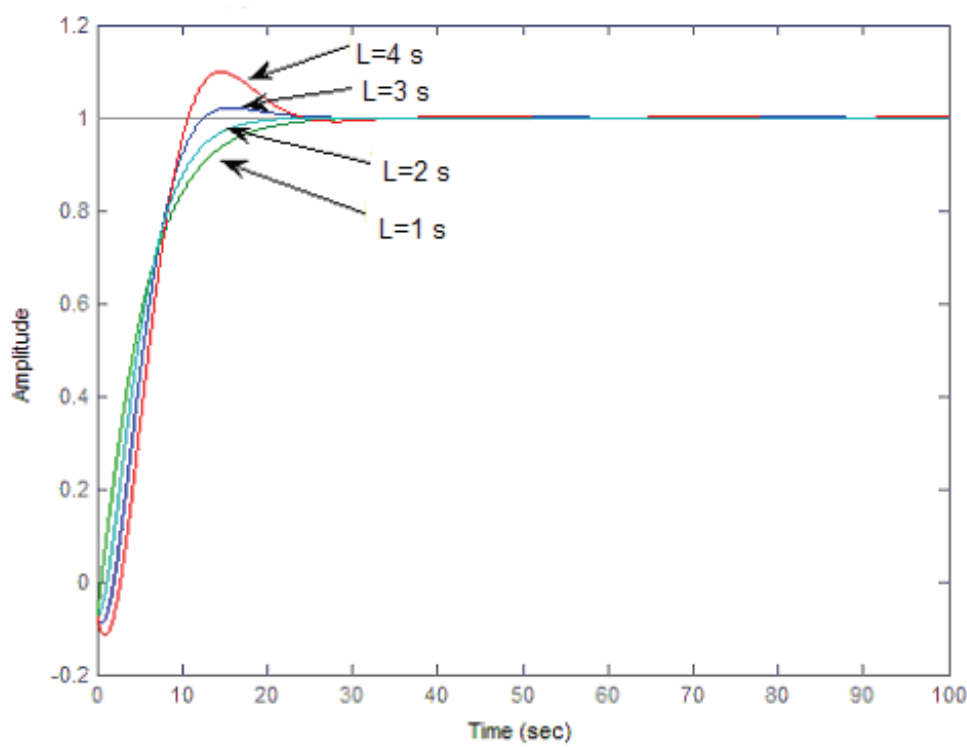Fig. 8. Nyquist plot $\gamma = 0.997$ and $\rho = 0.66$ using LMI



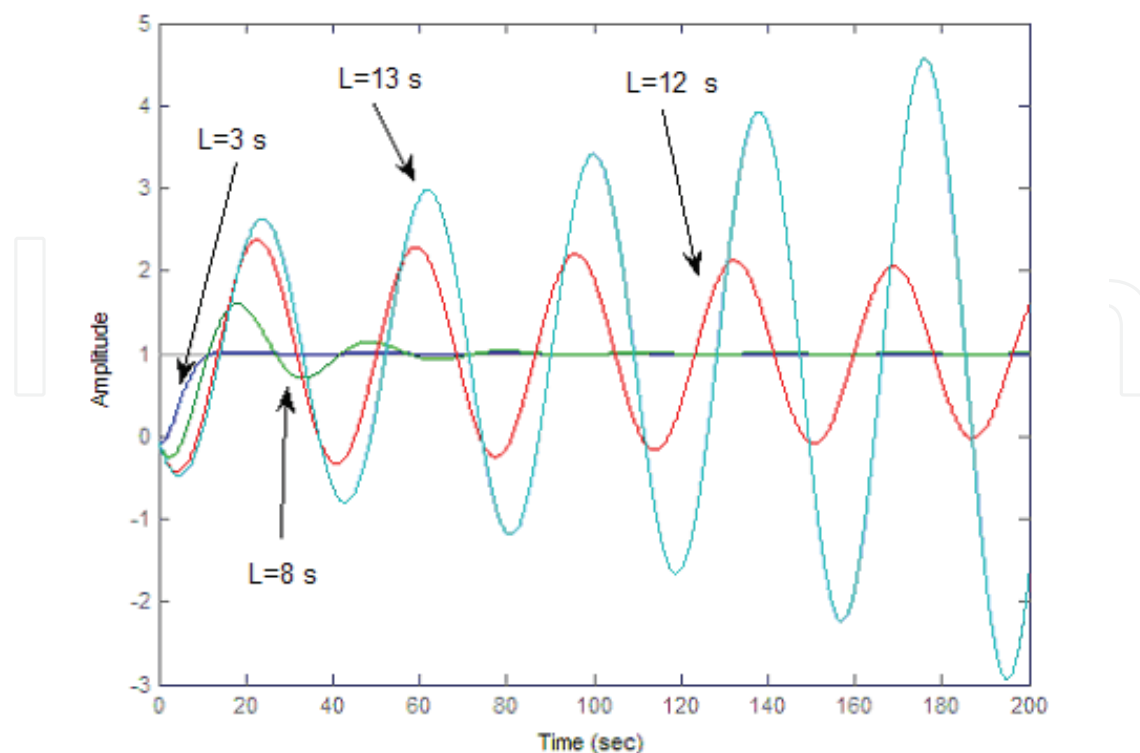Fig. 9. Transient response for different $d$ using LMI

Fig. 10. Transient response for different bigger $d$ using LMI

The simulation results are shown in Figure 5 and 6 for LMI, with $\gamma$ and $\rho$ are denoted by $g$ and $r$ respectively in the figure. The LMI method leads to faster transient response compared to the ARE method for all values of $\gamma$ and $\rho$. Nyquist plots in Figure 7 and 8 show that the LMI method has small gain margin. In general, it also holds for phase margin except at $\gamma = 0.997$ and $\rho = 1.5$ where LMI has bigger phase margin.

In order to test the robustness to the specified delay time uncertainties, the obtained robust PID controller with parameter $\gamma = 0.1$ and $\rho = 1$ is tested by perturbing the delay time in the range value of $d \in [1,4]$. The results of using LMI are shown in Figure 9 and 10 respectively. The LMI method yields faster transient responses where it tends to oscillate at bigger time delay. With the same parameters $\gamma$ and $\rho$, the PID controller is subjected to bigger delay time than the design specification. The LMI method can handle the ratio of delay time and time constant $L/\tau \leq 12$ s while the ARE method has bigger ratio $L/\tau \leq 43$ s. In summary, simulation results showed that LMI method produced fast transient response of the closed loop system with no overshoot and the capability in handling uncertainties. If the range of the uncertainties is known, the stability and the performance of the closed loop system will be guaranteed.

## 5. Conclusion

The paper has presented a model based method to select the optimum setting of the PID controller using robust H∞ dissipative integral backstepping method with state feedback synthesis. The state feedback gain is found by using LMI solution of Algebraic Riccati Inequality (ARI). The paper also derives the synthesis of the state feedback gain of robust H∞ dissipative integral backstepping method. The parameters of the PID controller are

calculated by using two new parameters which correspond to the infinity norm and the weighting of the control signal of the closed loop system.

The LMI method will guarantee the stability and the performance of the closed loop system if the range of the uncertainties is included in the LFT representation of the model. The LFT representation in the design can also be extended to include plant uncertainties, multiplicative perturbation, pole clustering, etc. Hence, the problem will be considered as multi objectives LMI based robust $H_\infty$ PID controller problem. The proposed approach can be directly extended for MIMO control problem with MIMO PID controller.

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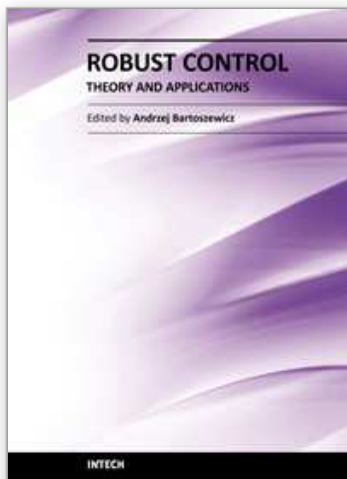
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Edited by Prof. Andrzej Bartoszewicz

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