We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,300

Open access books available

172,000

International authors and editors

190M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Some Complicating Effects in the Vibration of Composite Beams

Metin Aydogdu¹, Vedat Taskin¹, Tolga Aksencer¹,
Pinar Aydan Demirhan¹ and Seckin Filiz²

¹*Trakya University Department of Mechanical Engineering, Edirne,*

²*Trakya University Natural Sciences Institute, Edirne,
Turkey*

1. Introduction

In the last 50-60 years, use of composite structures in engineering applications has increased. Due to this fact many studies have been conducted related with composite structures (such as: shells, plates and beams).

Bending, buckling and free vibration analysis of composite structures has taken considerable attention. Beams are one of these structures that are used in mechanical, civil and aeronautical engineering applications (such: robot arms, helicopter rotors and mechanisms). Considering these applications free vibration problem of the composite beams are studied in the previous studies. Kapania & Raciti, 1989 investigated the nonlinear vibrations of un-symmetrically laminated composite beams. Chandashekhara et al., 1990 studied the free vibration of symmetric composite beams. Chandrashekhara & Bangera, 1993 investigated the free vibration of angle-ply composite beams by a higher-order shear deformation theory. They used the shear flexible finite element method. Krishnaswamy et al., 1992 solved the generally layered composite beam vibration problems. Chen et al., 2004 used the state-space based differential quadrature method to study the free vibration of generally laminated composite beams. Solution methods for composite beam vibration problems depend on the boundary conditions, some analytical (Chandrashekhara et al., 1990, Abramovich, 1992, Krishnaswamy et al., 1992, Abramovic & Livshits, 1994, Khdeir & Reddy, 1994, Eisenberger et al., 1995, Marur & Kant, 1996, Kant et al., 1998, Shi & Lam, 1999, Yildirim et al., 1999, Yildirim, 2000, Matsunaga, 2001, Kameswara et al., 2001, Banerjee, 2001, Chandrashekhara & Bangera, 1992, Ramtekkar et al., 2002, Murthy et al., 2005, Arya, 2003, Karama et al., 1998, Aydogdu, 2005, 2006) solution procedures have been used.

Many factors can affect the vibrations of beams, in particular the attached springs and masses, axial loads and dampers. This type of complicating effects is considered in the vibration problem of isotropic beams. Gürgöze and his colleagues studied vibration of isotropic beam with attached mass, spring and dumpers (Gürgöze, 1986, Gürgöze, 1996, Gürgöze & Erol, 2004). Vibration of Euler-Bernoulli beam carrying two particles and several particles investigated by Naguleswaran, 2001, 2002. Nonlinear vibrations of beam-mass system with different boundary conditions are investigated by Ozkaya & Pakdemirli, 1999, Ozkaya et. al., 1997. They used multiscale perturbation technique in their solutions.

It is interesting to note that, although mass or spring attached composite beams are used or can be used in some engineering applications, their vibration problem is not generally considered in the previous studies. Vibration of symmetrically laminated clamped-free beam with a mass at the free end is studied by Chandrashekhara & Bangera, 1993.

The aim of present study is to fill this gap. Therefore in this study vibration of composite beams with attached mass or springs is studied. After deriving equations of motion different boundary conditions, lamination angles, attached mass or spring are considered in detail.

2. Equation of motion

In this study, equations of motion of composite beams will be derived from Classical Laminated Plate Theory (CLPT). For CLPT following displacement field is generally assumed:

$$\begin{aligned} U(x, z; t) &= u(x, t) - zw_{,x} \\ V(x, z; t) &= v(x, t) - zw_{,y} \\ W(x, z; t) &= w(x, t) \end{aligned} \quad (1)$$

where U, V and W are displacement components of a point of the plate in the x, y and z directions respectively and u, v and w are the displacement components of a point of the beam in the midplane again in the x, y and z directions respectively. The comma after a letter denotes partial derivative with respect to x and y . The Hooke's law can be written in the following form using CLPT:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2)$$

where σ_x and σ_y are the in-plane normal stress components in the x and y directions respectively, τ_{xy} is the shear stress in the x - y plane, $\varepsilon_x, \varepsilon_y$ and γ_{xy} are normal strains and shear strain respectively and Q_{ij} are the reduced transformed rigidities (Jones, 1975). These strains are defined in the following form:

$$\varepsilon_x = \frac{\partial U}{\partial x}, \quad \varepsilon_y = \frac{\partial V}{\partial y}, \quad \gamma_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \quad (3)$$

Applying Hamilton principle leads to the following equations of motion for laminated composite plate.

$$\begin{aligned} N_{x,x} + N_{xy,y} &= \rho u_{,tt} \\ N_{xy,x} + N_{y,y} &= \rho v_{,tt} \\ M_{x,xx} + 2M_{xy,xy} + M_{y,yy} &= \rho w_{,tt} \end{aligned} \quad (4)$$

where the force and moment resultants are defined in the following form.

$$(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) dz \quad (5)$$

$$(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z dz \quad (6)$$

These force and moment results can also be written in the following form:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} u_{,x} \\ v_{,y} \\ u_{,x} + v_{,y} \\ -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{bmatrix} \quad (7)$$

where extensional, coupling and bending rigidities are defined as follows:

$$\begin{aligned} A_{ij} &= \int_{-h/2}^{h/2} Q_{ij}^{(k)} dz \\ B_{ij} &= \int_{-h/2}^{h/2} Q_{ij}^{(k)} z dz \\ D_{ij} &= \int_{-h/2}^{h/2} Q_{ij}^{(k)} z^2 dz \end{aligned} \quad (8)$$

Now, consider a laminated composite beam with length L , width b and thickness h . Equations of motion of laminated composite beams can be derived from Eq.(4) assuming $N_y = N_{xy} = M_y = M_{xy} = 0$.

$$\begin{aligned} N_{x,x} &= \rho u_{,tt} \\ M_{x,xx} &= \rho w_{,tt} \end{aligned} \quad (9)$$

Eq.(7) can be inverted in the following form:

$$\begin{bmatrix} u_{,x} \\ \epsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* & B_{11}^* & B_{12}^* & B_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* & B_{12}^* & B_{22}^* & B_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* & B_{16}^* & B_{26}^* & B_{66}^* \\ B_{11}^* & B_{12}^* & B_{16}^* & D_{11}^* & D_{12}^* & D_{16}^* \\ B_{12}^* & B_{22}^* & B_{26}^* & D_{12}^* & D_{22}^* & D_{26}^* \\ B_{16}^* & B_{26}^* & B_{66}^* & D_{16}^* & D_{26}^* & D_{66}^* \end{bmatrix} \begin{bmatrix} N_x \\ 0 \\ 0 \\ M_x \\ 0 \\ 0 \end{bmatrix} \quad (10)$$

where $A_{ij}^*, B_{ij}^*, D_{ij}^*$ are the members of inverse of rigidity matrix given in Eq.(7). Eq.(10) can be written in the following form.

$$\begin{aligned} u_{,x} &= A_{11}^* N_x + B_{11}^* M_x \\ w_{,xx} &= B_{11}^* N_x + D_{11}^* M_x \end{aligned} \quad (11)$$

Eq.(11) can be solved in term of N_x and M_x .

$$M_x = \frac{B_{11}^*}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} u_{,x} + \frac{A_{11}^*}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} w_{,xx} \quad (12)$$

$$N_x = \frac{B_{11}^* u_{,x} - B_{11}^{*2} \left[\frac{B_{11}^*}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} u_{,x} + \frac{A_{11}^*}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} w_{,xx} \right]}{A_{11}^* B_{11}^*} \quad (13)$$

Inserting equations (12)-(13) in equation (9) yields to:

$$\begin{aligned} \bar{A} \frac{\partial^2 u}{\partial x^2} + \bar{B} \frac{\partial^3 w}{\partial x^3} &= \rho \frac{\partial^2 u}{\partial t^2} \\ \bar{D} \frac{\partial^4 w}{\partial x^4} + \bar{B} \frac{\partial^3 u}{\partial x^3} &= -\rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (14)$$

Where $\bar{A}$, $\bar{B}$ and $\bar{D}$ are defined in the following form.

$$\bar{A} = \frac{1}{A_{11}^*} - \frac{(B_{11}^*)^2}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} \quad (15)$$

$$\bar{B} = -\frac{B_{11}^*}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} \quad (16)$$

$$\bar{D} = \frac{A_{11}^*}{[(B_{11}^*)^2 - (A_{11}^* D_{11}^*)]} \quad (17)$$

Eqs. (14) are the equations of motion of generally laminated composite beam for the assumptions $N_y = N_{xy} = M_y = M_{xy} = 0$. Boundary conditions of the generally laminated composite beams can be written in the following form:

$$\begin{aligned} S : w = M_x = N_x &= 0 \\ C : w = w_{,x} = u &= 0 \\ F : M_x = Q_x = N_x &= 0 \end{aligned} \quad (18)$$

2.1 Symmetrically laminated composite beams

For symmetrically laminated composite beams coupling terms B_{ij} 's are zero. Then Eq. (14) takes the following form.

$$\bar{D} \frac{\partial^4 w}{\partial x^4} = -\rho \frac{\partial^2 w}{\partial t^2} \quad (19)$$

General solution of Eq.(19) can be written in the following form:

$$w(x) = A \sin(\Omega x) + B \cos(\Omega x) + C \sinh(\Omega x) + D \cosh(\Omega x) \quad (20)$$

Where A,B,C and D are undetermined coefficients, $\Omega^4 = \rho \omega^2 L^4 / E_2 h^3$ is non-dimensional frequency parameter. Using boundary conditions given in Eq.(18) following Eigenvalue determinants are obtained for different boundary conditions:

H-H boundary condition:

Following condition exists between undetermined coefficients given in Eq.(20): B=D=0

$$\begin{vmatrix} \sin(\Omega) & \sinh(\Omega) \\ -\Omega^2 \sin(\Omega) & \Omega^2 \sinh(\Omega) \end{vmatrix} = 0 \quad (21)$$

C-H boundary condition:

Following condition exists between undetermined coefficients given in Eq.(20): D=-B, C=-A:

$$\begin{vmatrix} \sin(\Omega) - \sinh(\Omega) & \cos(\Omega) - \cosh(\Omega) \\ -\Omega^2 \sin(\Omega) - \Omega^2 \sinh(\Omega) & -\Omega^2 \cos(\Omega) - \Omega^2 \cosh(\Omega) \end{vmatrix} = 0 \quad (22)$$

C-C boundary condition:

Following condition exists between undetermined coefficients given in Eq.(20): D=-B, C=-A:

$$\begin{vmatrix} \sin(\Omega) - \sinh(\Omega) & \cos(\Omega) - \cosh(\Omega) \\ \Omega \cos(\Omega) - \Omega \cosh(\Omega) & -\Omega \sin(\Omega) - \Omega \sinh(\Omega) \end{vmatrix} = 0 \quad (23)$$

C-F boundary condition:

Following condition exists between undetermined coefficients given in Eq.(20): D=-B, C=-A

$$\begin{vmatrix} -\Omega^2 \sin(\Omega) - \Omega^2 \sinh(\Omega) & -\Omega^2 \cos(\Omega) - \Omega^2 \cosh(\Omega) \\ -\Omega^3 \cos(\Omega) - \Omega^3 \cosh(\Omega) & -\Omega^3 \sin(\Omega) - \Omega^3 \sinh(\Omega) \end{vmatrix} = 0 \quad (24)$$

F-F boundary condition:

Following condition exists between undetermined coefficients given in Eq.(20): D=B, C=A:

$$\begin{vmatrix} -\Omega^2 \sin(\Omega) & \Omega^2 \sinh(\Omega) \\ -\Omega^2 \cos(\Omega) & \Omega^2 \cosh(\Omega) \end{vmatrix} = 0 \quad (25)$$

H-F boundary condition:

Following condition exists between undetermined coefficients given in Eq.(20): B=D=0:

$$\begin{vmatrix} -\Omega^2 \sin(\Omega) + \Omega^2 \sinh(\Omega) & -\Omega^2 \cos(\Omega) + \Omega^2 \cosh(\Omega) \\ -\Omega^3 \cos(\Omega) + \Omega^3 \cosh(\Omega) & \Omega^3 \sin(\Omega) + \Omega^3 \sinh(\Omega) \end{vmatrix} = 0 \quad (26)$$

Solution of each determinant equation given in Eq.(21)-Eq.(26) gives frequency parameter of symmetrically laminated composite beams.

2.2 Symmetrically laminated beams with attached mass or spring

Now consider a symmetrically laminated composite beam with attached mass or spring (figure 1). Where η is length of first part of the beam. In order to investigate vibration of two portion composite beam Eq.(20) is written for each portion in the following form:

$$\begin{aligned} w_1(x) &= A_1 \sin(\Omega x) + B_1 \cos(\Omega x) + C_1 \sinh(\Omega x) + D_1 \cosh(\Omega x) \\ w_2(x) &= A_2 \sin(\Omega x) + B_2 \cos(\Omega x) + C_2 \sinh(\Omega x) + D_2 \cosh(\Omega x) \end{aligned} \quad (27)$$

Fig. 1. Composite beam with attached mass (a) and spring (b).

Continuity conditions of the beam at $x = \eta$ can be written in the following form:

$$\begin{aligned} w_1(\eta, t) &= w_2(\eta, t), \\ \dot{w}_1(\eta, t) &= \dot{w}_2(\eta, t) \\ w_1''(\eta, t) &= w_2''(\eta, t), \\ w_1'''(\eta, t) - w_2'''(\eta, t) + \alpha_m \Omega^4 w_1(\eta, t) &= 0 \quad (\text{mass}) \\ \Omega^3 w_1'''(\eta, t) - \Omega^3 w_2'''(\eta, t) - \alpha_s w_1(\eta, t) &= 0 \quad (\text{spring}) \end{aligned} \quad (28)$$

Where dimensionless mass and spring parameter are defined in the following form:

$$\alpha_m = \frac{M}{\rho_0 L}, \quad \alpha_s = \frac{kL}{AE}$$

Using boundary conditions Eq.(18) and continuity conditions Eq.(28) following equations are obtained for different boundary conditions and composite beams with attached mass and spring at different position.

H-H boundary condition:

Following condition exists between undetermined coefficients given in Eq.(27): $B_1 = D_1 = 0$:

$$\begin{vmatrix} 0 & 0 & S(\Omega) & C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ 0 & 0 & -S(\Omega) & -C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ S(\eta\Omega) & Sh(\eta\Omega) & -S(\eta\Omega) & -C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ C(\eta\Omega) & Ch(\eta\Omega) & -C(\eta\Omega) & -S(\eta\Omega) & -Ch(\eta\Omega) & -S(\eta\Omega) \\ -S(\eta\Omega) & Sh(\eta\Omega) & S(\eta\Omega) & C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ A61 & A62 & A63 & A64 & A65 & A66 \end{vmatrix} = 0 \quad (29)$$

With spring

$$A_{61} = -\Omega^3 C(\eta\Omega) - \alpha_s S(\eta\Omega)$$

$$A_{62} = \Omega^3 Ch(\eta\Omega) - \alpha_s Sh(\eta\Omega)$$

$$A_{63} = \Omega^3 C(\eta\Omega)$$

$$A_{64} = -\Omega^3 S(\eta\Omega)$$

$$A_{65} = -\Omega^3 Ch(\eta\Omega)$$

$$A_{66} = -\Omega^3 Sh(\eta\Omega)$$

With mass

$$A_{61} = -C(\eta\Omega) + \alpha_m \Omega S(\eta\Omega)$$

$$A_{62} = Ch(\eta\Omega) + \alpha_m \Omega Sh(\eta\Omega)$$

$$A_{63} = Ch(\eta\Omega)$$

$$A_{64} = -S(\eta\Omega)$$

$$A_{65} = -Ch(\eta\Omega)$$

$$A_{66} = -Sh(\eta\Omega)$$

H-C boundary condition:

Following condition exists between undetermined coefficients given in Eq.(27): $B_1 = D_1 = 0$:

$$\begin{vmatrix} 0 & 0 & S(\Omega) & C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ 0 & 0 & C(\Omega) & -S(\Omega) & Ch(\Omega) & Sh(\Omega) \\ S(\eta\Omega) & Sh(\eta\Omega) & -S(\eta\Omega) & -C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ C(\eta\Omega) & Ch(\eta\Omega) & -C(\eta\Omega) & -S(\eta\Omega) & -Ch(\eta\Omega) & -S(\eta\Omega) \\ -S(\eta\Omega) & Sh(\eta\Omega) & S(\eta\Omega) & C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{vmatrix} = 0 \quad (30)$$

With mass

$$A_{61} = -C(\eta\Omega) + \alpha_m \Omega S(\eta\Omega)$$

$$A_{62} = Ch(\eta\Omega) + \alpha_m \Omega Sh(\eta\Omega)$$

$$A_{63} = C(\eta\Omega) \quad A_{64} = -S(\eta\Omega)$$

$$A_{65} = -Ch(\eta\Omega) \quad A_{66} = -Sh(\eta\Omega)$$

With spring

$$A_{61} = -\Omega^3 C(\eta\Omega) - \alpha_s S(\eta\Omega)$$

$$A_{62} = \Omega^3 Ch(\eta\Omega) - \alpha_s Sh(\eta\Omega)$$

$$A_{63} = \Omega^3 C(\eta\Omega) \quad A_{64} = -\Omega^3 S(\eta\Omega)$$

$$A_{65} = -\Omega^3 Ch(\eta\Omega) \quad A_{66} = -\Omega^3 Sh(\eta\Omega)$$

C-C boundary condition:

Following condition exists between undetermined coefficients given in Eq.(27): $D_1 = -B_1$, $C_1 = -A_1$:

$$\begin{vmatrix} 0 & 0 & S(\Omega) & C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ 0 & 0 & C(\Omega) & -S(\Omega) & Ch(\Omega) & Sh(\Omega) \\ S(\eta\Omega) - Sh(\eta\Omega) & C(\eta\Omega) - Ch(\eta\Omega) & -S(\eta\Omega) & -C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ C(\eta\Omega) - Ch(\eta\Omega) & -S(\eta\Omega) - Sh(\eta\Omega) & -C(\eta\Omega) & -S(\eta\Omega) & -Ch(\eta\Omega) & -S(\eta\Omega) \\ -S(\eta\Omega) - Sh(\eta\Omega) & -C(\eta\Omega) - Ch(\eta\Omega) & S(\eta\Omega) & C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{vmatrix} = 0 \quad (31)$$

With mass

$$\begin{aligned} A61 &= -C(\eta\Omega) - Ch(\eta\Omega) + \alpha_m \Omega S(\eta\Omega) - \alpha_m \Omega Sh(\eta\Omega) \\ A62 &= S(\eta\Omega) - Sh(\eta\Omega) + \alpha_m \Omega C(\eta\Omega) - \alpha_m \Omega Ch(\eta\Omega) \\ A63 &= C(\eta\Omega) & A64 &= -S(\eta\Omega) \\ A65 &= -Ch(\eta\Omega) & A66 &= -Sh(\eta\Omega) \end{aligned}$$

With spring

$$\begin{aligned} A61 &= -\Omega^3 C(\eta\Omega) - \Omega^3 Ch(\eta\Omega) - \alpha_s S(\eta\Omega) + \alpha_s Sh(\eta\Omega) \\ A62 &= \Omega^3 S(\eta\Omega) - \Omega^3 Sh(\eta\Omega) - \alpha_s C(\eta\Omega) + \alpha_s Ch(\eta\Omega) \\ A63 &= \Omega^3 C(\eta\Omega) & A64 &= -\Omega^3 S(\eta\Omega) \\ A65 &= -\Omega^3 Ch(\eta\Omega) & A66 &= -\Omega^3 Sh(\eta\Omega) \end{aligned}$$

C-F boundary condition:

Following condition exists between undetermined coefficients given in Eq.(27): $D_1 = -B_1$, $C_1 = -A_1$:

$$\begin{vmatrix} 0 & 0 & -S(\Omega) & -C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ 0 & 0 & -C(\Omega) & S(\Omega) & Ch(\Omega) & Sh(\Omega) \\ S(\eta\Omega) - Sh(\eta\Omega) & C(\eta\Omega) - Ch(\eta\Omega) & -S(\eta\Omega) & -C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ C(\eta\Omega) - Ch(\eta\Omega) & -S(\eta\Omega) - Sh(\eta\Omega) & -C(\eta\Omega) & -S(\eta\Omega) & -Ch(\eta\Omega) & -S(\eta\Omega) \\ -S(\eta\Omega) - Sh(\eta\Omega) & -C(\eta\Omega) - Ch(\eta\Omega) & S(\eta\Omega) & C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ A61 & A62 & A63 & A64 & A65 & A66 \end{vmatrix} = 0 \quad (32)$$

With mass

$$\begin{aligned} A61 &= -C(\eta\Omega) - Ch(\eta\Omega) + \alpha_m \Omega S(\eta\Omega) - \alpha_m \Omega Sh(\eta\Omega) \\ A62 &= S(\eta\Omega) - Sh(\eta\Omega) + \alpha_m \Omega C(\eta\Omega) - \alpha_m \Omega Ch(\eta\Omega) \\ A63 &= C(\eta\Omega) \\ A64 &= -S(\eta\Omega) \\ A65 &= -Ch(\eta\Omega) \\ A66 &= -Sh(\eta\Omega) \end{aligned}$$

With spring

$$\begin{aligned} A61 &= -\Omega^3 C(\eta\Omega) - \Omega^3 Ch(\eta\Omega) - \alpha_s S(\eta\Omega) + \alpha_s Sh(\eta\Omega) \\ A62 &= \Omega^3 S(\eta\Omega) - \Omega^3 Sh(\eta\Omega) - \alpha_s C(\eta\Omega) + \alpha_s Ch(\eta\Omega) \\ A63 &= \Omega^3 C(\eta\Omega) \\ A64 &= -\Omega^3 S(\eta\Omega) \\ A65 &= -\Omega^3 Ch(\eta\Omega) \\ A66 &= -\Omega^3 Sh(\eta\Omega) \end{aligned}$$

F-F boundary condition

Following condition exists between undetermined coefficients given in Eq.(27): $D_1 = B_1$, $C_1 = A_1$:

$$\begin{vmatrix} 0 & 0 & -S(\Omega) & -C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ 0 & 0 & -C(\Omega) & S(\Omega) & Ch(\Omega) & Sh(\Omega) \\ S(\eta\Omega) + Sh(\eta\Omega) & C(\eta\Omega) + Ch(\eta\Omega) & -S(\eta\Omega) & -C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ C(\eta\Omega) + Ch(\eta\Omega) & -S(\eta\Omega) + Sh(\eta\Omega) & -C(\eta\Omega) & S(\eta\Omega) & -Ch(\eta\Omega) & -Sh(\eta\Omega) \\ -S(\eta\Omega) + Sh(\eta\Omega) & -C(\eta\Omega) + Ch(\eta\Omega) & S(\eta\Omega) & C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ A61 & A62 & A63 & A64 & A65 & A66 \end{vmatrix} = 0 \quad (33)$$

With mass

$$\begin{aligned} A_{61} &= -C(\eta\Omega) - Ch(\eta\Omega) + \alpha_m \Omega S(\eta\Omega) + \alpha_m \Omega Sh(\eta\Omega) \\ A_{62} &= S(\eta\Omega) - Sh(\eta\Omega) + \alpha_m \Omega C(\eta\Omega) + \alpha_m \Omega Ch(\eta\Omega) \\ A_{63} &= C(\eta\Omega) & A_{64} &= -S(\eta\Omega) \\ A_{65} &= -Ch(\eta\Omega) & A_{66} &= -Sh(\eta\Omega) \end{aligned}$$

With spring

$$\begin{aligned} A_{61} &= -\Omega^3 C(\eta\Omega) + \Omega^3 Ch(\eta\Omega) - \alpha_s S(\eta\Omega) - \alpha_s Sh(\eta\Omega) \\ A_{62} &= \Omega^3 S(\eta\Omega) + \Omega^3 Sh(\eta\Omega) - \alpha_s C(\eta\Omega) + \alpha_s Ch(\eta\Omega) \\ A_{63} &= \Omega^3 C(\eta\Omega) & A_{64} &= -\Omega^3 S(\eta\Omega) \\ A_{65} &= -\Omega^3 Ch(\eta\Omega) & A_{66} &= -\Omega^3 Sh(\eta\Omega) \end{aligned}$$

H-F boundary condition:

Following condition exists between undetermined coefficients given in Eq.(27): $B_1=D_1=0$:

$$\begin{vmatrix} 0 & 0 & -S(\Omega) & -C(\Omega) & Sh(\Omega) & Ch(\Omega) \\ 0 & 0 & -C(\Omega) & S(\Omega) & Ch(\Omega) & Sh(\Omega) \\ S(\eta\Omega) & Sh(\eta\Omega) & -S(\eta\Omega) & -C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ C(\eta\Omega) & Ch(\eta\Omega) & -C(\eta\Omega) & S(\eta\Omega) & -Ch(\eta\Omega) & -Sh(\eta\Omega) \\ -S(\eta\Omega) & Sh(\eta\Omega) & S(\eta\Omega) & C(\eta\Omega) & -Sh(\eta\Omega) & -Ch(\eta\Omega) \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{vmatrix} = 0 \quad (34)$$

With mass

$$\begin{aligned} A_{61} &= -C(\eta\Omega) + \alpha_m \Omega S(\eta\Omega) \\ A_{62} &= Ch(\eta\Omega) + \alpha_m \Omega Sh(\eta\Omega) \\ A_{63} &= C(\eta\Omega) \\ A_{64} &= -S(\eta\Omega) \\ A_{65} &= -Ch(\eta\Omega) \\ A_{66} &= -Sh(\eta\Omega) \end{aligned}$$

With spring

$$\begin{aligned} A_{61} &= -\Omega^3 C(\eta\Omega) - \alpha_s S(\eta\Omega) \\ A_{62} &= \Omega^3 Ch(\eta\Omega) - \alpha_s Sh(\eta\Omega) \\ A_{63} &= \Omega^3 C(\eta\Omega) \\ A_{64} &= -\Omega^3 S(\eta\Omega) \\ A_{65} &= -\Omega^3 Ch(\eta\Omega) \\ A_{66} &= -\Omega^3 Sh(\eta\Omega) \end{aligned}$$

Solution of each determinant equation given in Eq.(29)-Eq.(34) gives frequency parameter of symmetrically laminated beams with attached point mass or spring at the different location of the beam.

3. Numerical results

In this section, firstly, numerical results are given for vibration of composite beams with or without attached mass or springs. In order to check validity of present results first five flexural vibration frequencies of laminated composite beams are compared with previous results (Reddy, 1997) and good agreement is observed between two results. After checking

validity of present formulation, vibration of composite beams with attached mass or spring is investigated for different boundary conditions. Material properties are chosen as: $E_1=25E_2$, $G_{12}=0.5E_2$ and $\nu_{12}=0.3$. Obtained parametrical results are given in figures. In order to completeness of present study, first five frequency of symmetric three layer ($\theta/-\theta/\theta$) composite beams are given in Fig.2. According to Fig. 2, dimensionless frequency parameters decrease with increasing lamination angle θ . This is due to decrease in rigidities D_{ij} with increasing θ . The frequency gap is narrowing for higher θ , so this type of beams should be carefully designed. Highest frequencies are obtained for C-C and F-F boundary conditions where as lowest one is obtained for C-F boundary condition. Variation of frequency ratio of composite beams with attached mass to composite beam without mass (Ω_m/Ω_0) is depicted in Fig.3 for different boundary conditions. According to this figure, ratio of frequencies is insensitive to lamination angle θ . The lowest frequencies generally are most affected by attached mass. Influence of attached mass is decreasing with increasing mode number. This fact can be explained by considering mode shapes of vibrating composite beams. For H-H, C-C, H-C and F-F beams $\eta=0.25$ is a nodal point for fourth frequency, therefore this frequency is not affected by attached mass as expected. Highest %40 and lowest %20 changes are observed for frequencies for different boundary conditions.

Fig. 2. Variation of frequency parameter of composite beam with lamination angle θ .

Fig. 3. Variation of frequency ratio of composite beam with lamination angle for $\alpha_m=1$ and $\eta=0,25$.

Variation of frequency ratio of composite beams with attached spring to composite beam without spring (Ω_s/Ω_0) is given in Fig.4 for different boundary conditions. According to this figure, ratio of frequencies is insensitive to lamination angle θ . Effect of attached spring on the frequency ratio is negligible for composite beams with at least one clamped edge. The beams with F-F and H-F boundary conditions are most affected by attached mass. For these boundary conditions spring behaves like a hinged boundary condition and decreases frequency of composite beam.

Fig. 4. Variation of frequency ratio of symmetric angle-ply composite beam with lamination angle for $\alpha_s=10$ and $\eta=0,25$.

In Fig. 5, variation of frequency ratio with α_m is given for three layer symmetric angle-ply ($30^\circ/-30^\circ/30^\circ$) composite beams. Increasing α_m decreases frequency of the composite beam. Different decreases are observed for different boundary conditions.

Fig. 5. Variation of frequency ratio of symmetric angle-ply composite beam (30°/-30°/30°) with α_m for $\eta=0,25$.

Variation of frequency ratio with α_s is given in Fig. 6 for three layer symmetric angle-ply (30°/-30°/30°) composite beams. Increasing α_s decreases frequency of the composite beam for F-F and H-F boundary conditions. For these two boundary conditions zero frequencies exist for rigid body motions. Attaching a spring prevents from rigid body motion and these

zero frequencies turn two none zero frequencies. Other boundary conditions are insensitive to increase of α_s for given range.

Fig. 6. Variation of frequency ratio of symmetric angle-ply composite beam (30°/-30°/30°) with α_s for $\eta=0,25$.

In Fig. 7, variation of frequency ratio of composite beam with η for $\alpha_m=1$ and $\eta=0.25$ are given for three layer symmetric angle-ply (30°/-30°/30°) composite beams. Generally, lower

frequencies are most affected by position of attached mass. Forth frequency is not affected by position of attached mass for boundary conditions other than F-F and F-H. This is due to nodal points coincides with position of attached masses.

Fig. 7. Variation of frequency ratio of symmetric angle-ply composite beam (30°/-30°/30°) with η for $\alpha_m=1$ and $\eta=0,25$.

Fig. 8. Variation of frequency ratio of symmetric angle-ply composite beam (30°/-30°/30°) with η for $\alpha_s=1$ and $\eta=0.25$.

Variation of frequency ratio of composite beam with η for $\alpha_s=1$ and $\eta=0.25$ are given in Fig. 8 for three layer symmetric angle-ply (30°/-30°/30°) composite beams. Similar to Fig. 7 generally lower frequencies are most affected by position of attached spring. Forth frequency is not affected by position of attached spring for boundary conditions other than F-F and F-H. This is due to nodal points coincides with position of attached spring.

In Fig. 9-10, variation of frequency parameter of composite beam with lamination angle for $\alpha_s=1$, $\alpha_m=1$ and $\eta=0.25$ for different number of layers (single, three and four layer) are given respectively. First frequencies are insensitive to number of layers but for the fourth frequencies higher frequencies are obtained with increasing number of layers.

Fig. 9. Variation of frequency parameter of symmetric angle-ply composite beam with lamination angle for $\alpha_s=1$ and $\eta=0.25$ for different number of layers.

Fig. 10. Variation of frequency parameter of symmetric angle-ply composite beam with lamination angle for $\alpha_m=1$ and $\eta=0,25$ for different number of layers.

In Fig 11-12 variation of frequency ratio of three layer cross-ply composite beams with η is given for $\alpha_m=1$ and $\alpha_s=1$ respectively. Generally similar behavior is observed with symmetric angle-ply and cross-ply composite beams.

Fig. 11. Variation of frequency ratio of symmetric cross-ply composite beam ($0^0/90^0/0^0$) with η for $\alpha_m=1$ and $\eta=0,25$.

4. Conclusion

In this study, vibration of laminated composite beams with attached mass or spring is studied using classical lamination theory. First five flexural frequencies of composite beams

Fig. 12. Variation of frequency ratio of symmetric cross-ply composite beam ($0^0/90^0/0^0$) with η for $\alpha_s=1$ and $\eta=0,25$.

are obtained for different boundary conditions, attached mass, spring and their different positions. It is obtained that attaching mass reduces frequency of composite beams whereas attaching spring increases frequency of composite beams. Some modes do not change depending on position of attached spring or mass. This study can be extended to anti-symmetric composite beams and shear deformation effects can be added in the future studies.

5. References

- Abramovich H. (1992) Shear deformation and rotary inertia effects of vibrating composite beams. *Composite Structures*, 20, 165-73.
- Abramovich H. & Livshits A. (1994) Free vibrations of non-symmetric cross-ply laminated composite beams, *Journal of Sound and Vibration*, 176, 5, 597-612.
- Arya H. (2003) A new zig-zag model for laminated composite beams: free vibration analysis. *Journal of Sound and Vibration*, 264, 2, 485-490.
- Aydogdu M. (2005) Vibration analysis of cross-ply laminated beams with general boundary conditions by Ritz method, *International Journal of Mechanical Sciences*, 47, 1740-1755.
- Aydogdu M. (2006) Free vibration analysis of angle-ply laminated beams with general boundary conditions, *Journal of Reinforced Plastics and Composites*, 25, 15, 1571-1583.
- Banerjee JR. (2001) Frequency equation and mode shape formulae for composite Timoshenko beams. *Composite Structures*, 51, 4, 381-388.
- Chandrashekhara, K., Krishnamurty, K. & Roy, S. (1990). Free vibration of composite beams including rotary inertia and shear deformation, *Composite Structures*, 14, 4, 269-279.
- Chandrashekhara, K. & Bangera, K.M. (1992). Free vibration of composite beams Using a refined shear flexible beam element, *Computers and Structures*, 43, 4, 719-727.
- Chandrashekhara K. & Bangera K. M. (1993) Vibration of symmetrically laminated clamped-free beam with a mass at the free end, *Journal of Sound and Vibration* 160, 1, 93-101.
- Chen, W.Q., Lv, C. F. & Bian, Z. G. (2004). Free Vibration Analysis of Generally Laminated Beams via State-space-based Differential Quadrature, *Composite Structures*, 63, 1, 417-425.
- Eisenberger M., Abramovich H. & Shulepov O. (1995), Dynamic stiffness analysis of laminated beams using a first order shear deformation theory, *Composite Structures*, 31, 4, 265-271.
- Gürgöze M. (1986) On the approximate determination of the fundamental frequency of a restrained cantilever beam carrying a tip heavy body Original Research Article, *Journal of Sound and Vibration*, 105, 3, 443-449.
- Gürgöze M. (1996) On the eigenfrequencies of a cantilever beam with attached tip mass and a spring-mass system, Original Research Article *Journal of Sound and Vibration*, 190, 2, 149-162.
- Gürgöze M. & Erol H. (2004) On laterally vibrating beams carrying tip masses, coupled by several double spring-mass systems., *Journal of Sound and Vibration*, 269, 1-2, 431-438.
- Jones R.M. (1975), *Mechanics of composite materials*. New York: Hemisphere Publishing Corporation,.
- Kapani, R.K. & Raciti, S. (1989). Recent advances in analysis of laminated beams and plates, Part II: Vibrations and Wave Propagation, *AIAA Journal*, 27, 935-946.
- Karama M, Abou Harb B, Mistou S & Caperaa S. (1998) Bending, buckling and free vibration of laminated composite with a transverse stress continuity model. *Composite Part B-Eng*, 29, 3, 223-234.
- Kameswara Rao M., Desai Y.M. & Chitnis M.R. (2001) Free vibrations of laminated beams using mixed theory. *Composite Structures*, 52, 2, 149-160.
- Kant T, Marur S.R. & Rao GS. (1998) Analytical solution to the dynamic analysis of laminated beams using higher order refined theory. *Composite Structures*, 40, 1, 1-9.

- Khdeir A.A.& Reddy J.N. (1994) Free vibration of cross-ply laminated beams with arbitrary boundary conditions. *International Journal of Engineering Science*, 32, 12, 1971-1980.
- Krishnaswamy, K., Chandrashekhara, K. & Wu.W.Z.B. (1992). Analytical solutions to vibration of generally layered composite beams, *Composite Structures*, 14, 4, 269-279.
- Marur S.R., Kant T. (1996) Free vibration analysis of fiber reinforced composite beams using higher order theories and finite element modelling., *Journal of Sound Vibration*, 194, 3, 337-351.
- Matsunaga H. (2001) Vibration and buckling of multilayered composite beams according to higher order deformation theories., *Journal of Sound and Vibration*, 246, 1, 47-62.
- Murthy MVVS, Roy Mahapatra D, Badarinarayana K & Gopalakrishnan S. (2005) A refined higher order finite element for asymmetric composite beams. *Composite Structures*, 67, 1, 27-35.
- Nagulaeswaran S., (2001) Transverse vibrations of Euler-Bernoulli uniform beam carrying two particles in-span Original Research Article *International Journal of Mechanical Sciences*, 43, 12, 2737-2752.
- Naguleswaran S. (2002) Transverse vibrations of an Euler-Bernoulli uniform beam carrying several particles Original Research Article *International Journal of Mechanical Science*, 44, 12, 2463-2478.
- Ramtekkar GS, Desai YM & Shah A.H. (2002) Natural vibrations of laminated composite beams by using mixed finite element modelling., *Journal of Sound Vibration*. 257, 4, 635-651.
- Reddy, J.N., (1997). *Mechanics of Laminated Composite Plates and Shells Theory and Analysis*, Junuthula Narasimha (2nd Ed.), 182-186, CRC Press, ISBN: 0-8493-1592, USA.
- Özkaya E, Pakdemirli M & Öz H.R. (1997) Non-linear vibrations of a beam-mass system under different boundary conditions, *Journal of Sound and Vibration* 199, 4, 679-696.
- Özkaya E & Pakdemirli M.(1999) Non-linear vibrations of a beam-mass system with both Shi G. & Lam K.Y. (1999) Finite element vibration analysis of composite beams based on higher order beam theory. *Journal of Sound and Vibration*, 219, 4, 707-721.
- Yıldırım V., Sancaktar E. & Kırıl E. (1999) Comparison of the in-plane natural frequencies of symmetric cross-ply laminated beams based on the Bernoulli-Euler and Timoshenko Beam theories. *Journal of Applied Mechanics-T ASME*, 66, 2, 410-417.
- Yıldırım V. (2000) Effect of the longitudinal to transverse moduli ratio on the in-plane natural frequencies of symmetric cross-ply laminated beams by the stiffness method. *Composite Structures*, 50, 3, 319-326.

Advances in Vibration Analysis Research

Edited by Dr. Farzad Ebrahimi

ISBN 978-953-307-209-8

Hard cover, 456 pages

Publisher InTech

Published online 04, April, 2011

Published in print edition April, 2011

Vibrations are extremely important in all areas of human activities, for all sciences, technologies and industrial applications. Sometimes these Vibrations are useful but other times they are undesirable. In any case, understanding and analysis of vibrations are crucial. This book reports on the state of the art research and development findings on this very broad matter through 22 original and innovative research studies exhibiting various investigation directions. The present book is a result of contributions of experts from international scientific community working in different aspects of vibration analysis. The text is addressed not only to researchers, but also to professional engineers, students and other experts in a variety of disciplines, both academic and industrial seeking to gain a better understanding of what has been done in the field recently, and what kind of open problems are in this area.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Metin Aydogdu, Vedat Taskin, Tolga Aksencer, Pinar Aydan Demirhan and Seckin Filiz (2011). Some Complicating Effects in the Vibration of Composite Beams, *Advances in Vibration Analysis Research*, Dr. Farzad Ebrahimi (Ed.), ISBN: 978-953-307-209-8, InTech, Available from:
<http://www.intechopen.com/books/advances-in-vibration-analysis-research/some-complicating-effects-in-the-vibration-of-composite-beams>

INTech
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen