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Vibration Analysis of Beams with and without Cracks Using the Composite Element Model

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1. Introduction

Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges, tall buildings, and robot arms.

The vibration of Euler–Bernoulli beams with one step change in cross-section has been well studied. Jang and Bert (1989) derived the frequency equations for combinations of classical end supports as fourth order determinants equated to zero. Balasubramanian and Subramanian (1985) investigated the performance of a four-degree-of-freedom per node element in the vibration analysis of a stepped cantilever. De Rosa (1994) studied the vibration of a stepped beam with elastic end supports. Recently, Koplow et al. (2006) presented closed form solutions for the dynamic response of Euler–Bernoulli beams with step changes in cross section.

There are also some works on the vibration of beams with more than one step change in cross-section. Bapat and Bapat (1987) proposed the transfer matrix approach for beams with n-steps but provided no numerical results. Lee and Bergman (1994) used the dynamic flexibility method to derive the frequency equation of a beam with n-step changes in cross-section. Jaworski and Dowell (2008) carried out a study for the free vibration of a cantilevered beam with multiple steps and compared the results of several theoretical methods with experiment.

A new method is presented to analyze the free and forced vibrations of beams with either a single step change or multiple step changes using the composite element method (CEM) (Zeng, 1998; Lu & Law, 2009). The correctness and accuracy of the proposed method are verified by some examples in the existing literatures. The presence of cracks in the structural components, for instance, beams can have a significant influence on the dynamic responses of the whole structure; it can lead to the catastrophic failure of the structure. To predict the failure, vibration monitoring can be used to detect changes in the dynamic responses and/or dynamic characteristics of the structure. Knowledge of the effects of cracks on the vibration of the structure is of importance. Efficient techniques for the forward analysis of cracked beams are required. To this end, the composite element method is then extended for free and forced vibration analysis of cracked beams.

The principal advantage of the proposed method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points

and the whole beam can be treated as a uniform beam. Moreover, the presented work can easily be extended to cracked beams with an arbitrary number of non-uniform segments.

2. Theory

2.1 Introduction to Composite Element Method (CEM)

The composite element is a relatively new tool for finite element modeling. This method is basically a combination of the conventional finite element method (FEM) and the highly precise classical theory (CT). In the composite element method, the displacement field is expressed as the sum of the finite element displacement and the shape functions from the classical theory. The displacement field of the CEM can be written as

$$u_{CEM}(x,t) = u_{FEM}(x,t) + u_{CT}(x,t) \tag{1}$$

where $u_{FEM}(x,t)$ and $u_{CT}(x,t)$ are the individual displacement fields from the FEM and CT, respectively.

Taking a planar beam element as an example, the first term of the CEM displacement field can be expressed as the product of the shape function vector of the conventional finite element method $N(x)$ and the nodal displacement vector q

$$u_{FEM}(x,t) = N(x)q(t) \tag{2}$$

where $q(t) = [v_1(t), \theta_1(t), v_2(t), \theta_2(t)]^T$ and 'v' and 'θ' represent the transverse and rotational displacements, respectively.

The second term $u_{CT}(x,t)$ is obtained by the multiplication of the analytical mode shapes with a vector of N coefficients c (also called the c degrees-of-freedom or c-coordinates).

$$u_{CT}(x,t) = \sum_{i=1}^{N} \varphi_i(x)c_i(t) \tag{3}$$

where φ_i ($i=1,2,…N$) is the analytical shape function of the beam. Different analytical shape functions are used according to the boundary conditions of the beam.

Like the FEM, the CEM can be refined using the *h*-refinement technique by increasing the number of finite elements. Moreover, it can also be refined through the *c*-refinement method, by increasing the number of shape functions. Here, we apply the *c*-refinement from the CEM, where the beam needs only to be discretized into one element. This will reduce the total number of degrees-of-freedom in the FEM.

The displacement field of the CEM for the Euler-Bernoulli beam element can be written from Equations (1) to (3) as

$$u_{CEM}(x,t) = S(x)Q(t) \tag{4}$$

where $S(x) = [N_1(x), N_2(x), N_3(x), N_4(x), \phi_1(x), \phi_2(x),...,\phi_N(x)]$ is the generalized shape function of the CEM, $Q(t) = [v_1(t), \theta_1(t), v_2(t), \theta_2(t), c_1(t), c_2(t),...,c_N(t)]^T$ is the vector of generalized displacements, and N is the number of shape functions used from the classical theory.

2.2 Vibration analysis for stepped beams without crack

Figure 1 shows the sketch of a beam with n steps, the height of the beam $d(x)$ with n step changes in cross section is expressed as

$$d(x) = \begin{cases} d_1 & 0 \le x < L_1 \\ d_2 & L_1 \le x \le L_2 \\ \quad \vdots \\ d_n & L_{n-1} \le x \le L_n \end{cases} \tag{5}$$

It is assumed that the beam has aligned neutral axis, the flexibility of the beam $EI(x)$ can be expressed as

$$EI(x) = \begin{cases} \dfrac{wd_1^3}{12} & 0 \le x < L_1 \\[2mm] \dfrac{wd_2^3}{12} & L_1 \le x \le L_2 \\[2mm] \quad \vdots \\[2mm] \dfrac{wd_n^3}{12} & L_{n-1} \le x \le L_n \end{cases} \tag{6}$$

where w is the width of the beam. For the stepped beam with misaligned neutral axes, the expression of $EI(x)$ can not expressed simply as shown in Equation (6).
The beam mass per unit length is

$$m(x) = \begin{cases} \rho w d_1 & 0 \le x < L_1 \\ \rho w d_2 & L_1 \le x \le L_2 \\ \quad \vdots \\ \rho w d_n & L_{n-1} \le x \le L_n \end{cases} \tag{7}$$

where ρ is the mass density of the beam.
The elemental stiffness matrix of the stepped beam can be obtained from the following equation

$$\mathbf{K_e} = \int_0^L \frac{d^2 S^T}{dx^2} EI(x) \frac{d^2 S}{dx^2} dx = \begin{bmatrix} [k_{qq}] & [k_{qc}] \\ [k_{cq}] & [k_{cc}] \end{bmatrix} \tag{8}$$

where the submatrix $[k_{qq}]$ corresponds to the element stiffness matrix from the FEM for the stepped beam; the submatrix $[k_{qc}]$ corresponds to the coupling terms of the q-dofs and the c-dofs; submatrix $[k_{cq}]$ is a transpose matrix of $[k_{qc}]$, and the submatrix $[k_{cc}]$ corresponds to the c-dofs and is a diagonal matrix.
The consistent elemental mass matrix can be expressed as

$$\mathbf{M_e} = \int_0^L S(x)^T m(x) S(x) dx = \begin{bmatrix} [m_{qq}] & [m_{qc}] \\ [m_{cq}] & [m_{cc}] \end{bmatrix} \tag{9}$$

where the submatrix $[m_{qq}]$ corresponds to the elemental mass matrix from the FEM for the stepped beam; the submatrix $[m_{qc}]$ corresponds to the coupling terms of the q-dofs and the c-dofs; submatrix $[m_{cq}]$ is a transpose matrix of $[m_{qc}]$, and the submatrix $[m_{cc}]$ corresponds to the c-dofs and is a diagonal matrix.

After introducing the boundary conditions, this can be performed by setting the associated degrees-of-freedom in the systematic stiffness matrix **K** to be a large number, say, 10^{12}, the governing equation for free vibration of the beam can be expressed as

$$(\mathbf{K} - \omega^2 \mathbf{M})V = 0 \tag{10}$$

where **K** and **M** are system stiffness and mass matrices, respectively, ω is the circular frequency, from which and the natural frequencies are identified. The ith normalized mode shapes of the stepped beam can be expressed as

$$\Psi_i = \sum_{i=1}^{4} N_i V_i + \sum_{i=1}^{N} \varphi_i V_{i+4} \tag{11}$$

The equation of motion of the forced vibration of the beam with n steps when expressed in terms of the composite element method is

$$\mathbf{M}\ddot{Q} + \mathbf{C}\dot{Q} + \mathbf{K}Q = f(t) \tag{12}$$

where **M** and **K** are the system mass and stiffness matrices, which are the same as those shown in Equation (10), **C** is the damping matrix which represents a Rayleigh damping model, say, $\mathbf{C} = a_1\mathbf{M} + a_2\mathbf{K}$, a_1 and a_2 are constants to be determined from two modal damping ratios. For an external force $F(t)$ acting at the location x_F from the left support, the generalized force vector $f(t)$ can be expressed as

$$f(t) = \begin{bmatrix} N_1(x_F) & N_2(x_F) & N_3(x_F) & N_4(x_F) & \phi_1(x_F) & \cdots & \phi_n(x_F) \end{bmatrix}^T F(t) \tag{13}$$

The generalized acceleration $\ddot{Q}$, velocity $\dot{Q}$ and displacement Q of the stepped beam can be obtained from Equation (12) by direct integration. The physical acceleration $\ddot{u}(x,t)$ is obtained from

$$\ddot{u}(x,t) = [S(x)]^T \ddot{Q} \tag{14}$$

The physical velocity and displacement can be obtained in a similar way, i.e.

$$\dot{u}(x,t) = [S(x)]^T \dot{Q}, \tag{15a}$$

$$u(x,t) = [S(x)]^T Q \tag{15b}$$

2.3 The crack model

Numerous crack models for a cracked beam can be found in the literature. The simplest one is a reduced stiffness (or increased flexibility) in a finite element to simulate a small crack in the element (Pandey et al., 1991; Pandy & Biswas, 1994). Another simple approach is to divide the cracked beam into two beam segments joined by a rotational spring that

represents the cracked section (Rizos et al., 1990; Chaudhari & Maiti, 2000). Christides and Barr (1984) developed the one-dimensional vibration theory for the lateral vibration of a cracked Euler-Bernoulli beam with one or more pairs of symmetric cracks.

According to Christides and Barr(1984), the variation of bending stiffness $EI_d(x)$ along the cracked beam length takes up the form of

$$EI_d(x) = \frac{EI_0}{1 + (c-1)\exp(-2\alpha|x - x_c|/d)}$$ (16)

where E is the Young's modulus of the beam, $I_0 = wd^3/12$ is the second moment of area of the intact beam, $c = 1/(1-C_r)^3$, $C_r = d_c/d$ is the crack depth ratio and d_c and d are the depth of crack and the beam, respectively, x_c is the location of the crack. α is a constant which governs the rate of decay and it is estimated by Christides and Barr from experiments to be 0.667. According to Lu and Law (2009), this parameter needs to be adjusted to be 1.426.

2.4 Vibration analysis for beams with crack(s)

The elemental stiffness matrix of the cracked beam can be obtained from the following equation

$$\mathbf{K_e} = \int_0^L \frac{d^2 S^T}{dx^2} EI_d(x) \frac{d^2 S}{dx^2} dx = \begin{bmatrix} [k_{qq}] & [k_{qc}] \\ [k_{cq}] & [k_{cc}] \end{bmatrix}$$ (17)

It is assumed that the existence of crack does not affect the elemental mass matrix, the elemental mass matrix can be expressed in the similar way with the intact beam

$$\mathbf{M_e} = \int_0^L S(x)^T m(x) S(x) dx = \begin{bmatrix} [m_{qq}] & [m_{qc}] \\ [m_{cq}] & [m_{cc}] \end{bmatrix}$$ (18)

The equation of motion of the forced vibration of a cracked beam with n cracks when expressed in terms of the composite element method is

$$\mathbf{M}\ddot{Q} + \mathbf{C}\dot{Q} + \mathbf{K}(x_{L_1}, d_{c_1}, ..., x_{L_i}, d_{c_i}, ... x_{L_n}, d_{c_n})Q = f(t)$$ (19)

3. Applications Information

3.1 Free and forced vibration analysis for beam without crack

3.1.1 Free vibration analysis for a free-free beam with a single step

The free vibration of the free-free beam studied in Koplow et al. (2006) is restudied using the CEM and the results are compared with those in Koplow et al. Figure 2 shows the geometry of the beam under study. The material has a mass density of $\rho = 2830 kg/m^3$, and a Young's modulus of $E = 71.7 GPa$. In the CEM when 350 numbers of c-dofs are used, the first three natural frequencies are converged. The first three natural frequencies of the beam are 291.9Hz, 1176.2Hz and 1795.7Hz, respectively. The calculated natural frequencies from the CEM are very close to the experimental values in Koplow et al. when the test is measured at location A in Figure 2, which are 291Hz, 1165Hz and 1771Hz, respectively. The relative

errors between the CEM and the experimental values of the three natural frequencies are 0.31%, 0.96% and 1.39%, respectively. This shows the proposed method is accuracte.

3.1.2 Free vibration analysis for a cantilever beam with a several steps

The cantilever beam studied in Jaworski and Dowell (2008) is restudied to further check the accuracy and effectiveness of the proposed method. Figure 3 shows the dimensions of the beam under study. The parameters of the beam under study are: $E = 60.6 GPa$ and $\rho = 2664 kg / m^3$. In the CEM model of the beam, the beam is discretized into one element and 350 terms of c-dofs are used in the calculation. The first and second flapwise (out-of-plane) bending mode frequencies are calculated to be 10.758 Hz and 67.553 Hz, and the first chordwise (in-plane) bending mode frequency is 54.699 Hz. The results from the CEM agree well with the theoretical results in Jaworski and Dowell using Euler-Bernoulli theory, as shown in Table 1.

3.1.3 Forced vibration analysis for a cantilever beam with two steps

In this section, the forced vibration analysis for the stepped beam is investigated. The dynamic responses of the beam under external force are obtained from the CEM and the results are compared with those from the FEM. Figure 4 shows the cantilever beam under study. The parameters of the beam under study are $E = 69.6 GPa$ and $\rho = 2700 kg / m^3$. A sinusoidal external force is assumed to act at free end of the beam with a magnitude of 1 N and at a frequency of 10 Hz. The time step is 0.005 second in calculating the dynamic response. The Rayleigh damping model is adopted in the calculation with 0.01 and 0.02 as the first two modal damping ratios. In the CEM model, the beam is discretized into one element and 350 c-dofs are used in the calculation of the dynamic responses. Figure 5 shows the displacement response, velocity response and acceleration response at the free end of the beam. In order to check the accuracy of the responses from the CEM, a forced vibration analysis for the beam is conducted using the FEM. The beam is discretized into 90 Euler-Bernoulli beam elements with a total of 182 dofs. The corresponding responses from the FEM and the CEM are compared in Figure 5. This indicates the accuracy of the proposed method for forced vibration of multiple stepped beam. Figure 6 gives a close view between the responses from two methods. From this figure, one can see that the two time histories in every subplot are virtually coincident indicating the excellent agreement between the time histories.

3.2 Free and forced vibration analysis for beam with crack
3.2.1 Free vibration analysis for a uniform cantilever beam with a single crack

An experimental work in Sinha et al. (2002) is re-examined. The geometric parameters of the beam are: length 996mm, width 50mm, depth 25mm, material properties of the beam are: Young's modulus $E = 69.79 GPa$, mass density $\rho = 2600 kg / m^3$. The beam is discretized into one element and ten shape functions are used in the calculation with the total degrees-of-freedom in the CEM equals 14, while the total degrees-of-freedom in the finite element model is 34 for the beam in Sinha et al. The crack depth in the beams varies in three stages of 4mm, 8mm and 12mm. The comparison of predicted natural frequencies of the beam from the proposed model and those in Sinha et al. and the experimental results are shown in Table 2. The proposed model, in general, gives better results than the model in Sinha et al.

since the latter crack model is a linear approximation of the theoretical crack model of Christides and Barr.

3.2.2 Free vibration analysis for a cantilever beam with multiple cracks

The last beam above is studied again with a new crack introduced. The first crack is at 595mm from the left end with a fixed crack depth of 12mm while the second crack is at 800mm from the left end with the crack depth varying from 4mm to 12mm in step of 4mm. Table 3 gives the first five natural frequencies of the beam by CEM method and compares with those from Sinha et al. and the experimental measurement. The results from CEM are found closer to the experimental prediction than those in Sinha et al. The above comparisons show that the CEM approach of modeling a beam with crack(s) is accurate for the vibration analysis. A significant advantage of the model is the much lesser number of DOFs in the resulting finite element model of the structure.

3.2.3 Forced vibration analysis for a cracked simply supported beam

The forced vibration analysis for a simply supported cracked beam is conducted in this section. The effects of the presence of crack on the dynamic response of the beam is investigated. The parameters of the beam under study are taken as: Young's modulus $E = 28\,\mathrm{GPa}$, width w=200mm, depth d=200mm, length L=8.0m, mass density $\rho = 2500 kg / m^3$. Two cases are investigated in the following.

Effect of crack depth on the dynamic response

An impulsive force is assumed to act at mid-span of the beam with a magnitude of 100*N*, the force starts to act on the beam from the beginning and lasts for 0.1 second. The time step is 0.002 *s* in calculating the dynamic response. Rayleigh damping model is adopted in the calculation with 0.01 and 0.02 as the first two modal damping ratios.

Figure 7 shows comparison on the acceleration response at the 1/4 span of the beam for different crack depth. The crack is assumed to be at the mid-span of the beam. From this figure, one can see that the crack depth has significant effect on the dynamic response of the beam.

Effect of crack location on the dynamic response

Figure 8 shows comparison on the acceleration response at the 1/4 span of the beam for different crack locations with a fixed crack depth $d_c / d = 0.3$. The crack is assumed to be at the 0.1 L, 0.2L, 0.3L, 0.4L, and 0.5L of the beam. From this figure, one can see that the response changes with the crack location.

These studies show that the effect of the crack on the dynamic response is significant, so it is feasible to identify crack from measured structural dynamic responses.

4. Conclusion

The composite element method is proposed for both free and forced vibration analyses of beams with multiple steps. As the composite beam element is of a one-element-one-member configuration, modeling with this type of element would not need to take into account the discontinuity between different parts of the beam. The accuracy of this new composite element has been compared satisfactorily with existing results. One advantage of the

method proposed is that it can be extended easily to deal with beams consisting of an arbitrary number of non-uniform segments. Regarding the free and forced vibration analysis for cracked beam using composite element, modelling with this type of element would allow the automatic inclusion of interaction effect between adjacent local damages in the finite element model. The accuracy of the present method has been compared satisfactory with existing model and experimental results.
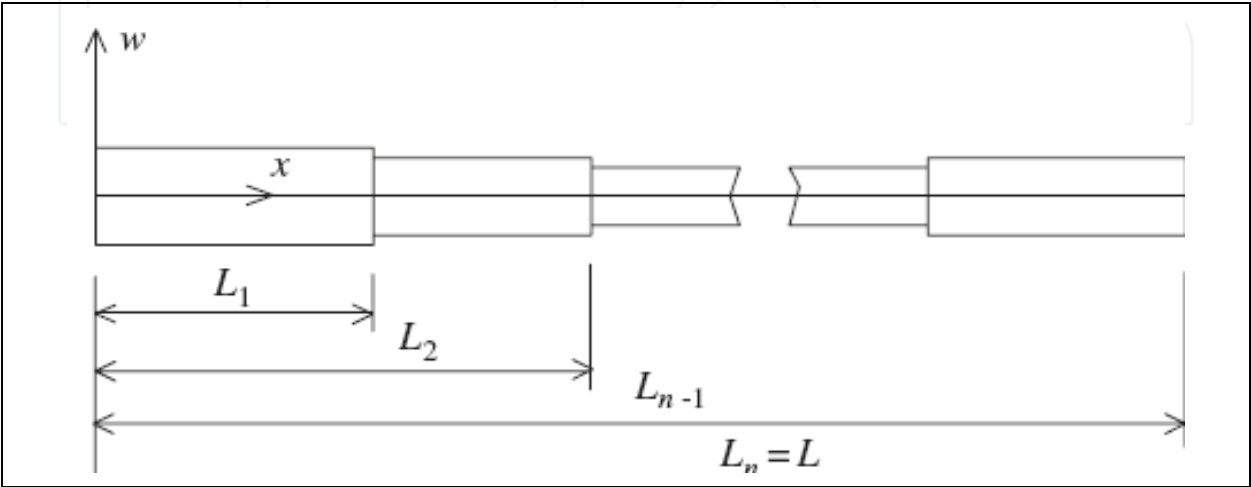

Fig. 1. Sketch of the stepped free-free beam with n segments
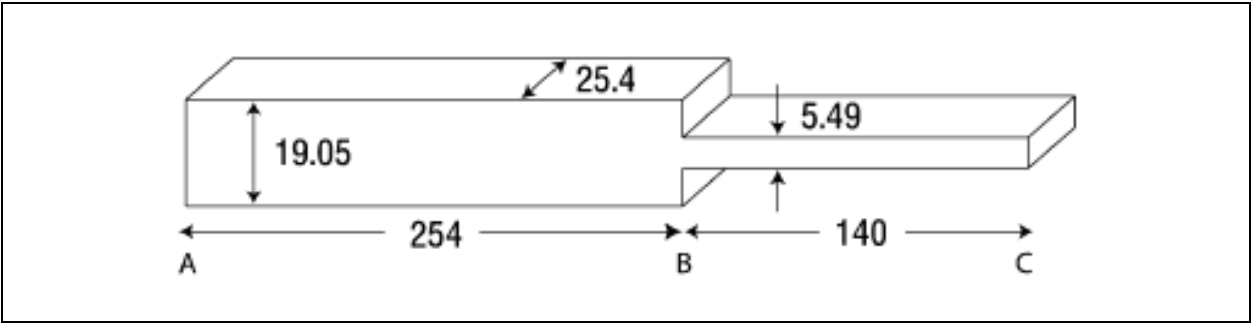
Fig. 2. Sketch of the stepped free-free beam in Koplow et al. (2006). Dimension in millimetre
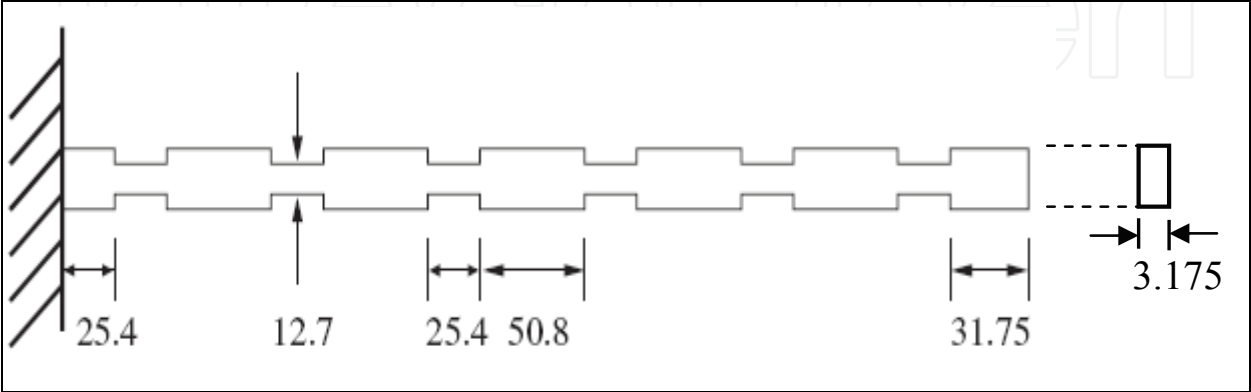
Fig. 3. Cantilever beam in Jaworski and Dowell (2008) with up and down steps. Dimension in millimeter
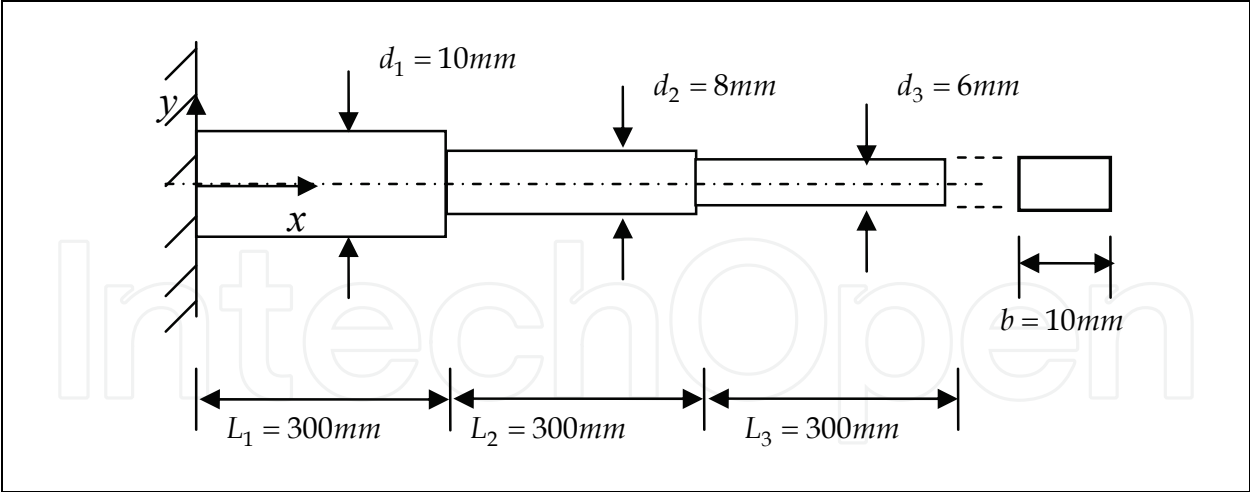
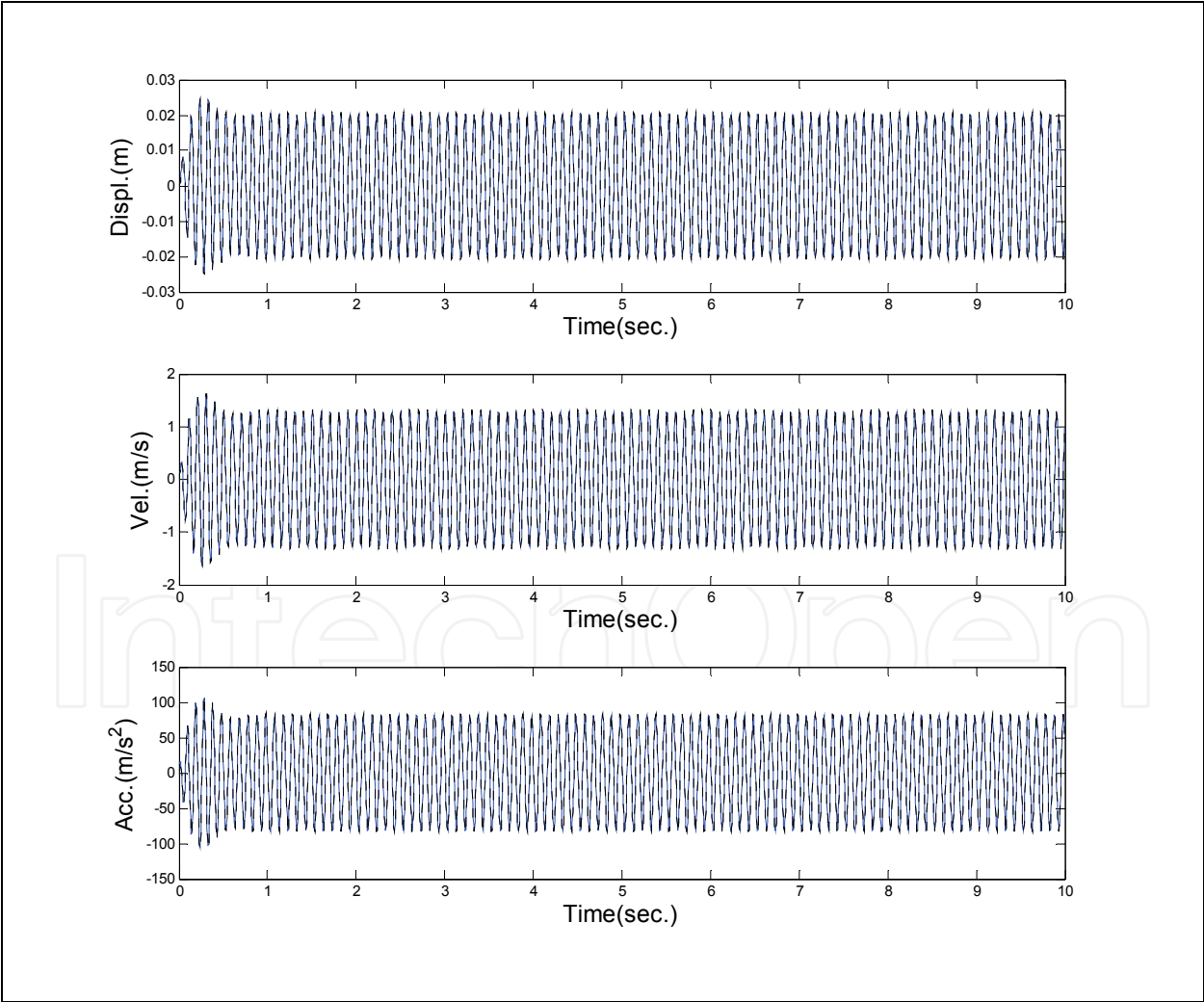Fig. 4. Sketch of the stepped cantilever beam (dimensions are not scaled)

Fig. 5. Forced vibration dynamic response comparison between the CEM and FEM(- Solid: CEM; -- Dashed: FEM)
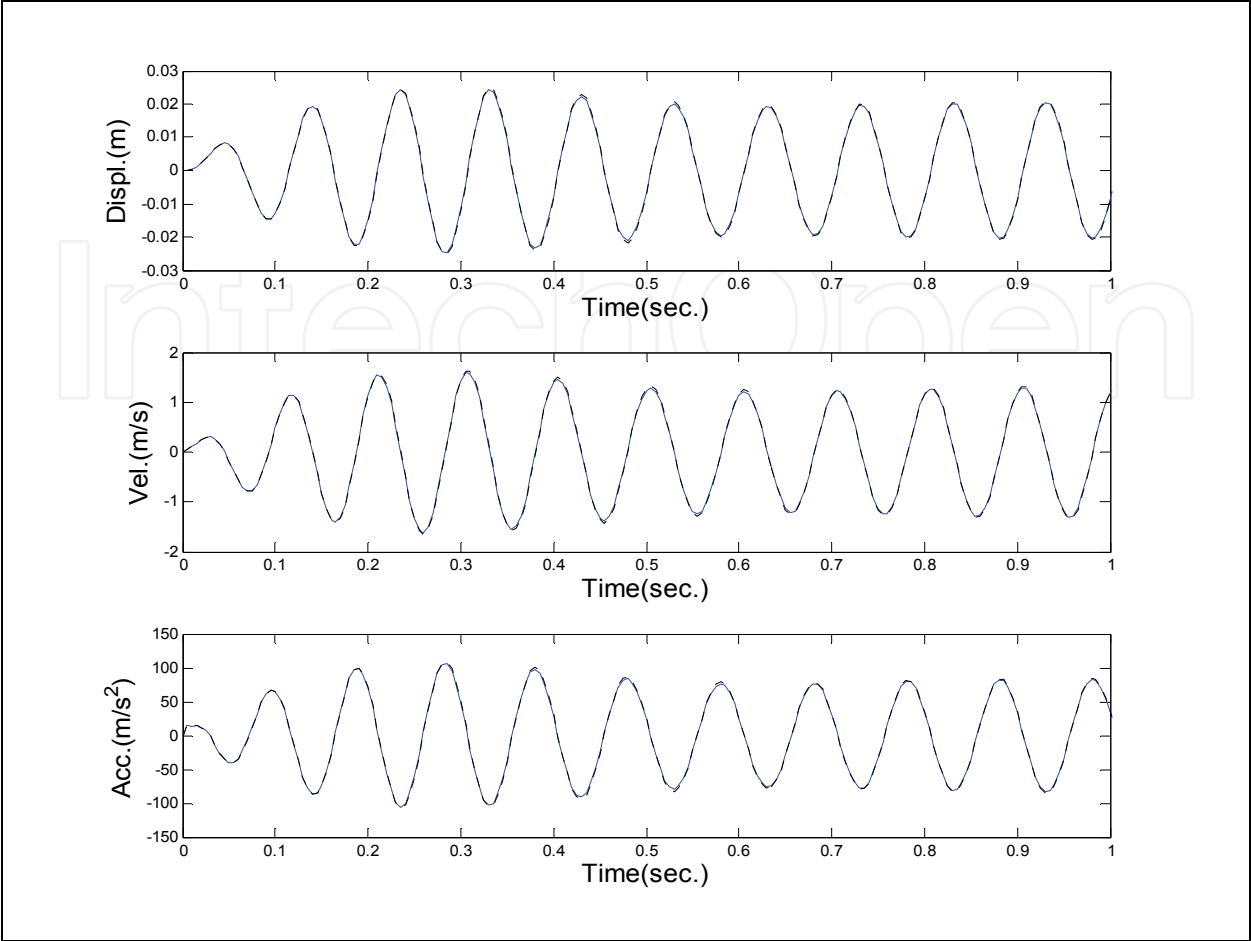
Fig. 6. Forced vibration dynamic response comparison between the CEM and FEM (a close view; - Solid: CEM, -- Dashed: FEM)
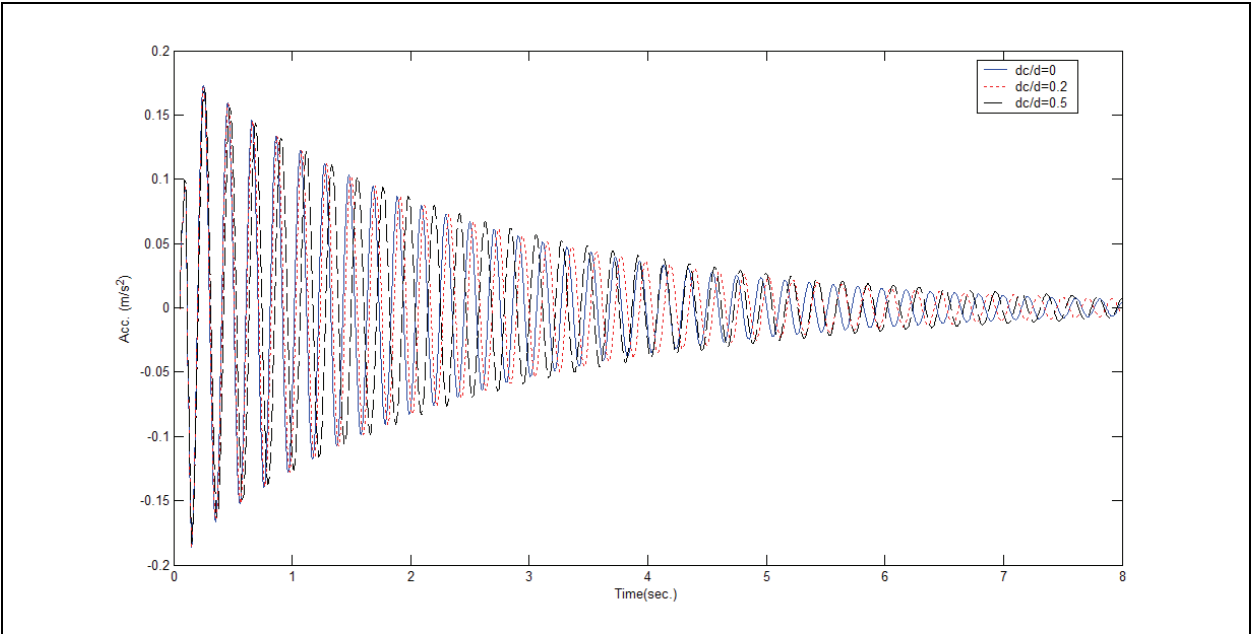

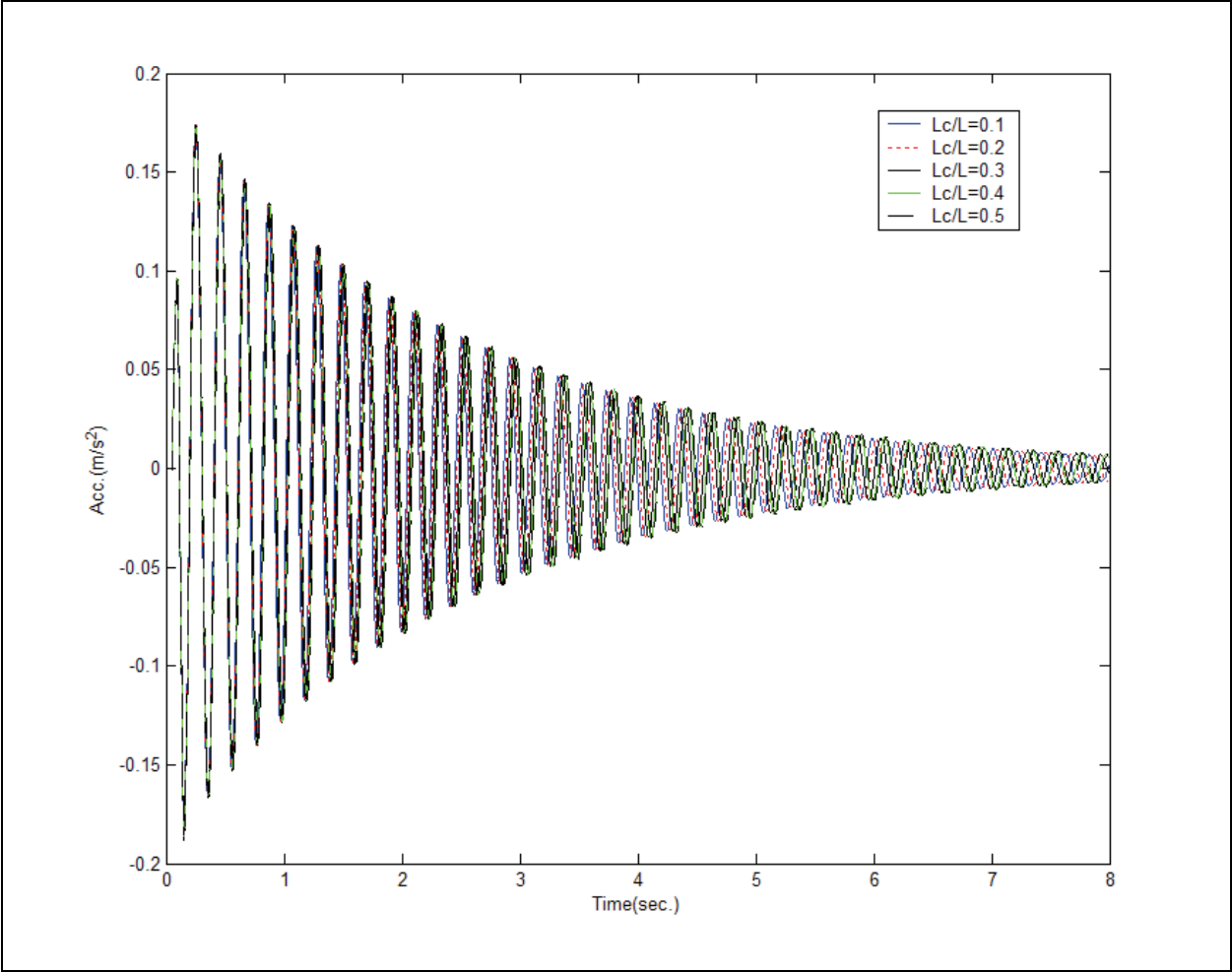Fig. 7. Comparison on the dynamic responses for different crack depth

Fig. 8. Comparison on the dynamic responses for different crack location

Mode	Present	Jaworski and Dowell (2008)						Experiment
	CEM	Rayleigh-Ritz	CMA	ANSYS				
	Euler	Euler	Euler	Euler	Timoshenko	2D Shell	3D Solid	
ω_{1B}	10.758	10.752	10.816	10.775	10.745	10.44	10.46	10.63
ω_{2B}	67.553	67.429	67.463	67.469	67.456	65.54	65.70	66.75
ω_{1C}	54.699	54.795	54.985	54.469	54.429	49.62	49.83	49.38

Table 1. Natural frequencies [Hz] comparison for the stepped beam in Jaworski and Dowell (2008)
Note: ω_{1B}, ω_{2B} are the first and second out-of-plane bending mode frequencies, respectively.
ω_{1c} denotes the first in-plane bending mode frequency.
CMA represents component modal analysis.

Mode	No crack		$d_{c1} = 4$ mm at $x_1 = 595$ mm			$d_{c1} = 8$ mm at $x_1 = 595$ mm			$d_{c1} = 12$ mm at $x_1 = 595$ mm		
	Exp.	Proposed	Exp.	Sinha et al.(2002)	Proposed	Exp.	Sinha et al.(2002)	Proposed	Exp.	Sinha et al.(2002)	Proposed
1	40.000	39.770	39.688	39.379	39.490	39.375	39.094	39.242	39.063	38.857	38.869
2	109.688	109.340	109.063	108.206	108.633	108.125	107.132	107.670	105.938	106.278	106.293
3	215.000	214.795	215.000	214.087	214.230	214.688	213.825	213.986	214.375	213.622	213.631
4	355.000	354.853	354.688	353.107	353.683	353.438	351.872	352.524	350.625	350.881	350.921
5	528.750	529.601	527.188	524.696	526.540	522.812	520.452	522.448	513.125	517.219	517.003

Table 2. Comparison of natural frequencies (Hz) of the aluminium free-free beam with one crack in Sinha et al.(2002)

Mode	No crack		$d_{c1} = 12$ mm at $x_1 = 595$ mm $d_{c2} = 4$ mm at $x_2 = 800$ mm			$d_{c1} = 12$ mm at $x_1 = 595$ mm $d_{c2} = 8$ mm at $x_2 = 800$ mm			$d_{c1} = 12$ mm at $x_1 = 595$ mm $d_{c2} = 12$ mm at $x_2 = 800$ mm		
	Exp.	Proposed	Exp.	Sinha et al.(2002)	Proposed	Exp.	Sinha et al.(2002)	Proposed	Exp.	Sinha et al.(2002)	Proposed
1	40.000	39.770	38.750	38.352	38.607	38.437	37.897	38.246	37.500	37.513	37.703
2	109.688	109.340	105.938	105.890	106.196	105.938	105.510	106.062	105.625	105.559	105.858
3	215.000	214.795	213.750	212.207	212.786	212.813	210.897	211.643	210.000	209.815	209.975
4	355.000	354.853	350.000	348.920	349.843	349.063	347.235	348.410	345.625	345.876	346.374
5	528.750	529.601	512.500	514.575	514.735	511.250	512.903	513.044	507.500	510.560	510.633

Table 3. Comparison of natural frequencies (Hz) of the aluminium free-free beam with two cracks in Sinha et al.(2002)

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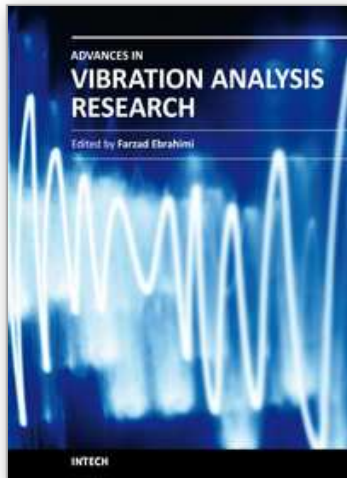
Advances in Vibration Analysis Research

Edited by Dr. Farzad Ebrahimi

Vibrations are extremely important in all areas of human activities, for all sciences, technologies and industrial applications. Sometimes these Vibrations are useful but other times they are undesirable. In any case, understanding and analysis of vibrations are crucial. This book reports on the state of the art research and development findings on this very broad matter through 22 original and innovative research studies exhibiting various investigation directions. The present book is a result of contributions of experts from international scientific community working in different aspects of vibration analysis. The text is addressed not only to researchers, but also to professional engineers, students and other experts in a variety of disciplines, both academic and industrial seeking to gain a better understanding of what has been done in the field recently, and what kind of open problems are in this area.

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