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On the Transport Phenomena in Composite Materials using the Fractal Space-Time Theory

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1. Introduction

A new way to analyze the dynamics of the physical systems is to consider that the particle movements take place on continuous but non-differentiable curves, i.e. on fractals. Then, the complexity of these dynamics is substituted by fractality. There are some fundamental arguments which can justify such hypothesis: i) by interaction, the trajectory is no longer everywhere differentiable. The "uncertainty" in tracking the particle is eliminated by means of the fractal approximation of motion; ii) the complex dynamical systems, which display chaotic behavior, are recognized to acquire self-similarity and manifest strong fluctuations at all possible scales. Every type of "elementary" process of motion induces both spatio-temporal scales and the associated fractals. Moreover, the movement complexity is directly related to the fractal dimension: the fractal dimension increases as the movement becomes more complex. Different definitions were given for the fractal dimension (Kolmogorov dimension, Hausdorff dimension, etc.), but once we choose the fractal-type dimension in the study of motion we must work with it until the end.

Therefore, considering that the complexity of the physical processes (from the system's interactions) is replaced by fractality (situation in which the particle movements take place on fractal curves), it is no longer necessary to use notions as collision time, mean free path, etc., i.e., the whole classical "arsenal" of quantities from the dynamics of physical systems. Then, the physical systems will behave as a special interaction-less "fluid" by means of geodesics in a fractal space-time. The theory which treats the interactions in the previously mentioned manner is the Scale Relativity (SR).

The SR is based on a generalization of Einstein's principle of relativity to scale transformations. Namely, "one redefines space-time resolutions as characterizing the state of reference systems scale, in the same way as speed characterizes their state of motion. Then one requires that the laws of physics apply whatever the state of the reference system, of motion (principle of motion-relativity) and of scale (principle of SR). The principle of SR is mathematically achieved by the principle of scale-covariance, requiring that the equations of physics keep their simplest form under transformations of resolution".

Another way of analyzing the system dynamics by means of the "fractals" is given by the Transfinite Physics (TP). The Transfinite Physics theory uses the Cantorian geometry as a working method. This geometry is a compromise between the discrete and the continuum. It is not simply discrete. It is transfinite discrete and has the cardinality of the continuum

although it is not continuous. Seen from a far it looks as if it were continuous. The result was startling because it is possible to simulate four dimensionality using infinitely many weighted Cantor sets. This created a geometry and topology for space-time that is similar to that of radiation and obeys the same statistical distribution, namely a discrete gamma distribution which is known in physics as Planck distribution. El Naschie defined this space-time as $\varepsilon^{(\infty)}$ space-time.

Many applications of the fractal space-time and particularly of $\varepsilon^{(\infty)}$ space-timewere given in references. In the present paper, using the extended model of the Scale Relativity theory, the transport phenomena (speed and temperature fields) in composite materials are analyzed.

2. A short reminder of the Nottale's scale relativity theory in correspondence with Cresson's mathematical procedures

Let us suppose that the motion of particles take place on continuous but non-differentiable curves (fractal curves). The non-differentiability, according with Cresson's mathematical procedures and Nottale's physical principles, implies the followings:

i. a continuous and a non-differentiable curve (or almost nowhere differentiable) is explicitly scale dependent, and its length tends to infinity, when the scale interval tends to zero. In other words, a continuous and non-differentiable space is fractal, in the general meaning given by Mandelbrot to this concept;
ii. there is an infinity of fractals curves (geodesics) relating any couple of its points (or starting from any point), and this is valid for all scales;
iii. the breaking of local differential time reflection invariance. The time-derivative of a function F can be written two-fold:

$$\frac{dF}{dt} = \lim_{dt\to 0} \frac{F(t+dt)-F(t)}{dt} = \lim_{dt\to 0} \frac{F(t)-F(t-dt)}{dt} \tag{1}$$

Both definitions are equivalent in the differentiable case. In the non-differentiable situation these definitions fail, since the limits are no longer defined. "In the framework of scale relativity, the physics is related to the behavior of the function during the "zoom" operation on the time resolution δt, here identified with the differential element dt (substitution principle), which is considered as an independent variable. The standard function $F(t)$ is therefore replaced by a fractal function $F(t,dt)$, explicitly dependent on the time resolution interval, whose derivative is undefined only at the unobservable limit $dt \to 0$". As a consequence, this lead us to define the two derivatives of the fractal function as explicit functions of the two variables t and dt,

$$\frac{d_+F}{dt} = \lim_{dt\to 0_+} \frac{F(t+dt,dt)-F(t,dt)}{dt}$$
$$\frac{d_-F}{dt} = \lim_{dt\to 0_-} \frac{F(t,dt)-F(t-dt,dt)}{dt} \tag{2a,b}$$

The sign, +, corresponds to the forward process and, -, to the backward process;

iv. the differential of a fractal function $F(t,dt)$ can be expressed as the sum of two differentials, one which is not scale-dependent, $dF'(t)$, and the other dependent on it, $dF''(t,dt)$, therefore

$$dF(t,dt) = dF'(t) + dF''(t,dt) \tag{3}$$

Particularly, the differential of the generalized coordinates, $d_{\pm}\boldsymbol{X}(t,dt)$, can be decomposed as follows

$$d_{\pm}\boldsymbol{X}(t,dt) = d_{\pm}\boldsymbol{x}(t) + d_{\pm}\boldsymbol{\xi}(t,dt) \tag{4a,b}$$

where $d_{\pm}\boldsymbol{x}(t)$ is the "classical part" and $d_{\pm}\boldsymbol{\xi}(t,dt)$ is the "fractal part". Starting from here, multiplying by dt^{-1} and using the substitutions

$$\boldsymbol{V}_{\pm} = \frac{d_{\pm}\boldsymbol{X}}{dt},\ \boldsymbol{v}_{\pm} = \frac{d_{\pm}\boldsymbol{x}}{dt},\ \boldsymbol{u}_{\pm} = \frac{d_{\pm}\boldsymbol{\xi}}{dt} \tag{5a-c}$$

we obtain the velocity field

$$\boldsymbol{V}_{\pm} = \boldsymbol{v}_{\pm} + \boldsymbol{u}_{\pm}; \tag{6a,b}$$

v. the fractal part of F, i.e. F'', satisfies the relation

$$\left|F''(t) - F''(t')\right| \approx \left|t - t'\right|^{\delta} \tag{7}$$

where δ depends on the fractal dimension D_F (for detail see references).
Particularly, the differential of the "fractal part" of $d_{\pm}\boldsymbol{X}$, becomes

$$d_{\pm}\xi_i \sim dt^{\frac{1}{D_F}} \tag{8a,b}$$

or more, as an equality relation (fractal equation):

$$\left(\frac{d_{\pm}\xi_i}{\lambda}\right) = \left(\frac{dt}{\tau}\right)^{\frac{1}{D_F}} \tag{9a,b}$$

Written as

$$d_{\pm}\xi_i = \frac{\lambda}{\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{1}{D_F}\right)-1} dt \tag{10a,b}$$

equations (9a,b) imply the temporal scales δt and τ, and the length scale λ, respectively. The significances of the time dt and τ result from the Random Walk (Brownian motion) or its generalization, Levy motion. The differential time dt is identified with the resolution time ("substitution principle"), $\delta t \equiv dt$, while τ corresponds to the fractal – non-fractal transition time. λ is a characteristic length, for example of Planck's or de Broglie's type (for details see references);

vi. by the relation (10 a, b) the velocity field $V_{\pm}^{i}$ becomes

$$V_{\pm}^{i} = v_{\pm}^{i} + u_{\pm}^{i} = v_{\pm}^{i} + \frac{\lambda}{\tau}\left(\frac{\tau}{dt}\right)^{1-\left(\frac{1}{D_F}\right)} \tag{11a,b}$$

The transition scale τ yields two distinct behaviors of the speed, depending on the resolution at which it is considered, since $V_{\pm}^{i} \to v_{\pm}^{i}$ when $dt >> \tau$, and $V_{\pm}^{i} \to u_{\pm}^{i}$, when $dt << \tau$;

vii. the local differential time reflection invariance is recovered by combining the two derivatives, d_{+}/dt and d_{-}/dt, in the complex operator

$$\frac{\hat{\partial}}{\partial t} = \frac{1}{2}\left(\frac{d_{+} + d_{-}}{dt}\right) - \frac{i}{2}\left(\frac{d_{+} - d_{-}}{dt}\right) \tag{12}$$

We call this procedure "an extension by differentiability" (Cresson's extension).
Applying this operator to the "position vector" yields a complex speed

$$\begin{aligned} \boldsymbol{V} = \frac{\hat{\partial}\boldsymbol{X}}{\partial t} &= \frac{1}{2}\left(\frac{d_{+}\boldsymbol{X} + d_{-}\boldsymbol{X}}{dt}\right) - \frac{i}{2}\left(\frac{d_{+}\boldsymbol{X} - d_{-}\boldsymbol{X}}{dt}\right) = \frac{\boldsymbol{V}_{+} + \boldsymbol{V}_{-}}{2} - i\frac{\boldsymbol{V}_{+} - \boldsymbol{V}_{-}}{2} = \\ &= \frac{1}{2}\left[\left(\boldsymbol{v}_{+} + \boldsymbol{v}_{-}\right) + \left(\boldsymbol{u}_{+} + \boldsymbol{u}_{-}\right)\right] - \frac{i}{2}\left[\left(\boldsymbol{v}_{+} - \boldsymbol{v}_{-}\right) + \left(\boldsymbol{u}_{+} - \boldsymbol{u}_{-}\right)\right] = \boldsymbol{v} - i\boldsymbol{u} \end{aligned} \tag{13}$$

with

$$\begin{aligned} \boldsymbol{v} &= \frac{\boldsymbol{V}_{+} + \boldsymbol{V}_{-}}{2} = \frac{1}{2}\left[\left(\boldsymbol{v}_{+} + \boldsymbol{v}_{-}\right) + \left(\boldsymbol{u}_{+} + \boldsymbol{u}_{-}\right)\right] \\ u &= \frac{\boldsymbol{V}_{+} - \boldsymbol{V}_{-}}{2} = \frac{1}{2}\left[\left(\boldsymbol{v}_{+} - \boldsymbol{v}_{-}\right) + \left(\boldsymbol{u}_{+} - \boldsymbol{u}_{-}\right)\right] \end{aligned} \tag{14a,b}$$

The real part, v, of the complex speed represents the standard classical speed which is differentiable and independent of resolution, while the imaginary part, u, is a new quantity arising from fractality, which is non-differentiable and resolution-dependent. In the usual classical limit, $dt >> \tau$,

$$\boldsymbol{v}_{+} = \boldsymbol{v}_{-} = \overline{v}\ ,\ \boldsymbol{u}_{+} = \boldsymbol{u}_{-} = 0 \tag{15a,b}$$

so that

$$\boldsymbol{V} = \overline{\boldsymbol{v}}\ ,\ \boldsymbol{u} = 0 \tag{16}$$

In the limit, $dt << \tau$,

$$\boldsymbol{v}_{+} = \boldsymbol{v}_{-} = 0\ ,\ \boldsymbol{u}_{+} = \boldsymbol{u}_{-} = \overline{\boldsymbol{u}} \tag{17a,b}$$

and

$$\boldsymbol{V} = \overline{\boldsymbol{u}}\ ,\quad \boldsymbol{u} = 0\ ; \tag{18}$$

viii. "in order to account for the infinity of geodesics in the bundle, for their fractality and for the two valuedness of the derivative which all come from the non-differentiable geometry of the space-time continuum, one therefore adopts a generalized statistical fluid like description, where instead of a classical deterministic speed or of a classical fluid speed field, one uses a doublet of fractal functions of spaces coordinates and time which are also explicit functions of resolution time". Thus, the average values of the quantities must be considered in the previously mentioned sense. Particularly, the average of $d_{\pm}\boldsymbol{X}$ is

$$\left\langle d_{\pm}\boldsymbol{X}\right\rangle = d_{\pm}\boldsymbol{x} \tag{19}$$

with

$$\left\langle d_{\pm}\boldsymbol{\xi}\right\rangle = 0 \tag{20a,b}$$

ix. in such an interpretation, the "particles", are identified with the geodesics themselves. As a consequence, any measurement is interpreted as a sorting out (or selection) of the geodesics by the measuring device.

3. Extended model of the scale relativity

Let us now assume that the curves describing the movement (continuous but non-differentiable) is immersed in a 3-dimensional space, and that X of components X^i (i = $\overline{1,3}$) is the position vector of a point on the curve. Let us also consider a function $f(\boldsymbol{X},t)$ and expand its total differential up to the third order:

$$d_{\pm}f = \frac{\partial f}{\partial t}dt + \nabla f \cdot d_{\pm}\boldsymbol{X} + \frac{1}{2}\frac{\partial^2 f}{\partial X^i \partial X^j}d_{\pm}X^i d_{\pm}X^j + \frac{1}{6}\frac{\partial^3 f}{\partial X^i \partial X^j \partial X^k}d_{\pm}X^i d_{\pm}X^j d_{\pm}X^k \tag{21a,b}$$

where only the first three terms were used in the Nottale's theory (i.e. second order terms in the equation of motion).
The relations (21a,b) are valid in any point of the space manifold and also for the points X on the fractal curve which we have selected in relations (21a,b).
From here, the forward and backward average values of this relation, using the notations $dX_{\pm}^{\ i} = d_{\pm}X^i$, take the form:

$$\left\langle d_{\pm}f\right\rangle = \left\langle \frac{\partial f}{\partial t}dt\right\rangle + \left\langle \nabla f \cdot d_{\pm}\boldsymbol{X}\right\rangle + \frac{1}{2}\left\langle \frac{\partial^2 f}{\partial X^i \partial X^j}d_{\pm}X^i d_{\pm}X^j\right\rangle + \frac{1}{6}\left\langle \frac{\partial^3 f}{\partial X^i \partial X^j \partial X^k}d_{\pm}X^i d_{\pm}X^j d_{\pm}X^k\right\rangle \tag{22a,b}$$

We make the following stipulations: the mean values of the function f and its derivates coincide with themselves, and the differentials $d_{\pm}X^i$ and dt are independent, therefore the averages of their products coincide with the product of average. Thus equations (22a,b) become:

$$d_{\pm}f = \frac{\partial f}{\partial t}dt + \nabla f\left\langle d_{\pm}\boldsymbol{X}\right\rangle + \frac{1}{2}\frac{\partial^2 f}{\partial X^i \partial X^j}\left\langle d_{\pm}X^i d_{\pm}X^j\right\rangle + \frac{1}{6}\frac{\partial^3 f}{\partial X^i \partial X^j \partial X^k}\left\langle d_{\pm}X^i d_{\pm}X^j d_{\pm}X^k\right\rangle \tag{23a,b}$$

or more, using equations (4a,b) with the property (20a,b),

$$d_{\pm}f = \frac{\partial f}{\partial t}dt + \nabla f d_{\pm}\boldsymbol{x} + \frac{1}{2}\frac{\partial^2 f}{\partial X^i \partial X^j}\left(d_{\pm}x^i d_{\pm}x^j + \left\langle d\xi_{\pm}^i d\xi_{\pm}^j \right\rangle\right) + \frac{1}{6}\frac{\partial^3 f}{\partial X^i \partial X^j \partial X^k}\left(d_{\pm}x^i d_{\pm}x^j d_{\pm}x^k + \left\langle d\xi_{\pm}^i d\xi_{\pm}^j d\xi_{\pm}^k \right\rangle\right) \tag{24a,b}$$

Even the average value of the fractal coordinate, $d\xi_{\pm}^i$, is null (see (20a,b)), for the higher order of the fractal coordinate average the situation can be different. First, let us focus on the mean $\left\langle d\xi_{\pm}^i d\xi_{\pm}^j \right\rangle$. If $i \neq j$ this average is zero due the independence of $d\xi^i$ and $d\xi^j$. So, using (10a,b) we can write:

$$\left\langle d\xi_{\pm}^i d\xi_{\pm}^j \right\rangle = \pm\delta^{ij}\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} dt \tag{25a,b}$$

with

$$\delta^{ij} = \begin{cases} 1, & \text{if} \quad i = j \\ 0, & \text{if} \quad i \neq j \end{cases}$$

and we had considered that:

$$\begin{cases} \left\langle d\xi_{+}^i d\xi_{+}^j \right\rangle > 0 \text{ and } dt > 0 \\ \left\langle d\xi_{-}^i d\xi_{-}^j \right\rangle > 0 \text{ and } dt < 0 \end{cases}$$

Then, let us consider the mean $\left\langle d\xi_{\pm}^i d\xi_{\pm}^j d\xi_{\pm}^k \right\rangle$. If $i \neq j \neq k$ this average is zero due the independence of $d\xi^i$ on $d\xi^j$ and $d\xi^k$. Now, using equation (10a,b), we can write:

$$\left\langle d\xi_{\pm}^i d\xi_{\pm}^j d\xi_{\pm}^k \right\rangle = \delta^{ijk}\frac{\lambda^3}{\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1} dt \tag{26a,b}$$

with

$$\delta^{ijk} = \begin{cases} 1, & \text{if} \quad i = j = k \\ 0, & \text{if} \quad i \neq j \neq k \end{cases}$$

and we considered that:

$$\begin{cases} \left\langle d\xi_{+}^i d\xi_{+}^j d\xi_{+}^k \right\rangle > 0 \; and \; dt > 0 \\ \left\langle d\xi_{-}^i d\xi_{-}^j d\xi_{-}^k \right\rangle > 0 \; and \; dt < 0 \end{cases}$$

Then equations (24a,b) may be written under the form:

$$d_{\pm}f = \frac{\partial f}{\partial t}dt + \nabla f d_{\pm}\boldsymbol{x} + \frac{1}{2}\frac{\partial^2 f}{\partial X^i \partial X^j}d_{\pm}x^i d_{\pm}x^j \pm \frac{\partial^2 f}{\partial X^i \partial X^j}\delta^{ij}\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} dt + \frac{1}{6}\frac{\partial^3 f}{\partial X^i \partial X^j \partial X^k}d_{\pm}x^i d_{\pm}x^j d_{\pm}x^k + \frac{\partial^3 f}{\partial X^i \partial X^j \partial X^k}\delta^{ijk}\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1} dt \tag{27a,b}$$

If we divide by dt and neglect the terms which contain differential factors, equations (27a,b) are reduced to:

$$\frac{d_{\pm}f}{dt} = \frac{\partial f}{\partial t} + \boldsymbol{v}_{\pm}\nabla f \pm \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta f + \frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1}\nabla^3 f \qquad (28a,b)$$

with $\Delta = \sum_i \partial^2 / \partial X_i^2$ and $\nabla^3 = \sum_i \partial^3 / \partial X_i^3$. These relations also allows us to define of the operator,

$$\frac{d_{\pm}}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}_{\pm}\nabla \pm \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta + \frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1}\nabla^3 \qquad (29a,b)$$

Under the circumstances, let us calculate $(\hat{\partial}f/\partial t)$. Taking into account equations (29a,b), (12) and (13), we obtain:

$$\frac{\hat{\partial}f}{\partial t} = \frac{1}{2}\left[\frac{d_{+}f}{dt} + \frac{d_{-}f}{dt} - i\left(\frac{d_{+}f}{dt} - \frac{d_{-}f}{dt}\right)\right] = \frac{\partial f}{\partial t} + \boldsymbol{V}\cdot\nabla f - i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta f + \frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1}\nabla^3 f \qquad (30)$$

This relation also allows us to define the fractal operator:

$$\frac{\hat{\partial}}{\partial t} = \frac{\partial}{\partial t} + \boldsymbol{V}\cdot\nabla - i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta + \frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1}\nabla^3 \qquad (31)$$

We now apply the principle of scale covariance, and postulate that the passage from classical (differentiable) mechanics to the "fractal" mechanics which is considered here can be implemented by replacing the standard time derivative d/dt by the complex operator $\hat{\partial}/\partial t$ (this results is a generalization of the principle of scale covariance given by Nottale). As a consequence, we are now able to write the equation of geodesics (a generalization of the first Newton's principle) in a fractal space-time under its covariant form:

$$\frac{\hat{\partial}\boldsymbol{V}}{\partial t} = \frac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{V}\cdot\nabla\boldsymbol{V} - i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\boldsymbol{V} + \frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{(3/D_F)-1}\nabla^3\boldsymbol{V} = 0 \qquad (32)$$

This means that the global complex acceleration field, $\hat{\partial}\boldsymbol{V}/\partial t$, depends on the local complex acceleration field, $\partial_t \boldsymbol{V}$, on the non-linearity (convective) term, $\boldsymbol{V}\cdot\nabla\boldsymbol{V}$, on the dissipative term, $\Delta\boldsymbol{V}$, and on the dispersive one, $\nabla^3\boldsymbol{V}$.
If the motions of the fractal fluid are irrotational, i.e. $\boldsymbol{\Omega} = \nabla\times\boldsymbol{V} = 0$ we can choose $\boldsymbol{V}$ of the form:

$$\boldsymbol{V} = \nabla\phi \qquad (33)$$

with ϕ a complex speed potential. Then, equation (32) becomes:

$$\frac{\partial \boldsymbol{V}}{\partial t}+\nabla\left(\frac{\boldsymbol{V}^2}{2}\right)-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}\Delta\boldsymbol{V}+\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{\left(3/D_F\right)-1}\nabla^3\boldsymbol{V}=0 \tag{34}$$

and more, by substituting equation (33) in equation (34), we have by integration,

$$\frac{\partial \varphi}{\partial t}+\frac{1}{2}(\nabla\phi)^2-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}\Delta\phi+\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{\left(3/D_F\right)-1}\nabla^3\phi=F(t) \tag{35}$$

with $F(t)$ a function of time only. We note that equation (34) has been reduced to a single scalar relation (35), i.e. a generalized Bernoulli (GB) type equation.
Let us choose the complex speed potential in the form:

$$\phi=-i\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}ln\psi \tag{36}$$

where ψ behaves both as speed potential and wave function. Then, ψ by means of equation (35) satisfies a generalized Schrödinger (GS) type equation:

$$\frac{\lambda^4}{4\tau^2}\left(\frac{dt}{\tau}\right)^{\left(4/D_F\right)-2}\Delta\psi+i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}\partial_t\psi+\left(i\frac{\lambda^5}{12\tau^2}\left(\frac{dt}{\tau}\right)^{\left(5/D_F\right)-2}(\nabla^3 ln\psi)+\frac{F(t)}{2}\right)\psi=0 \tag{37}$$

When the transport phenomenon in a fractal space-time implies the temperature fields, T, the heat transfer equation has the form

$$\frac{\hat{\partial}T}{\partial t}=\frac{\partial T}{\partial t}+\boldsymbol{V}\cdot\nabla T-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}\Delta T+\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{\left(3/D_F\right)-1}\nabla^3 T=0 \tag{37a}$$

4. The dissipative approximation of transport phenomenon in fractal structures and some applications. Extended fractal hydrodynamic model

Let us consider that the dissipative and convective effects are dominant in comparison with the dispersive ones. Consequently, the covariant form of the first Newton's principle in the fractal space-time is reduced to equation

$$\frac{\hat{\partial}\boldsymbol{V}}{\partial t}=\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}\cdot\nabla\boldsymbol{V}-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\Delta\boldsymbol{V}=0 \quad . \tag{38}$$

For $\boldsymbol{V}$ of the form (see relation (33) with (36))

$$\boldsymbol{V}=-i\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla ln\psi \tag{39}$$

equation (38) becomes a Navier-Stokes type equation,

$$\frac{\hat{\partial}\boldsymbol{V}}{\partial t}=\frac{\partial \boldsymbol{V}}{\partial t}+\nabla\left(\frac{\boldsymbol{V}^2}{2}\right)-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\Delta\boldsymbol{V}=0 \tag{40}$$

with an imaginary viscosity coefficient, ν

$$\nu=i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1} \tag{41}$$

while in terms of the ψ function, up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase, a "Schrödinger" type equation results

$$\frac{\lambda^4}{4\tau^2}\left(\frac{dt}{\tau}\right)^{\left(\frac{4}{D_F}\right)-2}\Delta\psi+i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\frac{\partial\psi}{\partial t}=0\,. \tag{42}$$

The presence of an imaginary viscosity coefficient specifies the followings: i) at macroscopic scale, the behavior of the fractal fluids is of viscoelastic type or hysteretic type. Such a result is in agreement with the opinions given in references: the fractal fluid can be described by Kelvin-Voight or Maxwell rheological model with complex structure coefficients (particularly, the imaginary viscosity coefficient (41)). Thus, such "materials" are endowed with "memory"; ii) at microscopic scale, the scalar field of the complex velocity has a stochastic behavior. Particularly, at Compton scale ($D=\lambda^2/2\tau=\hbar/2m_0$, with $\hbar$ the reduced Planck constant and m_0 the rest mass of the microparticle) and in the fractal dimension D_F =2 (the microparticle motion take place on Peano`s curves), the scalar field of the complex velocity is also a wave function such that, equation (42) takes the form of the "standard" Schrödinger equation:

$$\frac{\hbar^2}{2m_0}\Delta\psi+i\hbar\frac{\partial\psi}{\partial t}=0 \tag{43}$$

This means that, the Schrödinger equation results from a Navier-Stokes type equation with the imaginary viscosity coefficient, $\nu=i\hbar/2m_0$, for irrotational movements of a fractal fluid at Compton scale.

Let us choose the scalar function ψ in the form $\psi=\sqrt{\rho}e^{iS}$, with $\sqrt{\rho}$ being amplitude and S phase. Thus, the complex speed field (13), has the components

$$\boldsymbol{v}=\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla S,\qquad \boldsymbol{u}=\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla ln\,\rho\,. \tag{44a,b}$$

where $\mathbf{v}$ is the real (differentiable) part and $\mathbf{u}$ is the imaginary (non-differentiable or fractal) part.

The equations (44a, b) which define the components of the complex speed field of the fractal fluid are more general than those from Nottale's SR model. For the fractal dimension $D_F=2$, the Nottale's results are obtained:

$$\mathrm{v} = \frac{\lambda^2}{\tau}\nabla S, \qquad \mathrm{u} = \frac{\lambda^2}{2\tau}\nabla ln\,\rho \tag{45a,b}$$

Introducing (13) with (44a,b) in (40) and separating the real and imaginary parts, i.e. through the separation of the movements at differentiable scale from those at non-differentiable scale, we obtain

$$\begin{aligned} &\frac{\partial \boldsymbol{v}}{\partial t} + \nabla\left(\frac{\boldsymbol{v}^2}{2} - \frac{\boldsymbol{u}^2}{2} - \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}\right) = 0 \\ &\frac{\partial \boldsymbol{u}}{\partial t} + \nabla\left(\boldsymbol{v}\cdot\boldsymbol{u} + \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{v}\right) = 0 \end{aligned} \tag{46a,b}$$

With equation (45a,b), equation (46b) takes the form

$$\nabla\left(\frac{\partial ln\,\rho}{\partial t} + \mathrm{v}\cdot\nabla ln\,\rho + \nabla\cdot\mathrm{v}\right) = 0 \tag{47}$$

or, by integration with $\rho \neq 0$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{v}) = T(t) \tag{48}$$

with T(t) a function which depends only an time. In these condition, the equations (46a) and (48) with T(t)=0 become:

$$\begin{aligned} &m_0\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = -\nabla(Q) \\ &\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0 \end{aligned}, \tag{49a,b}$$

with Q the fractal potential,

$$Q = -m_0\frac{\lambda^4}{2\tau^2}\left(\frac{dt}{\tau}\right)^{\left(\frac{4}{D_F}\right)-2}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{m_0\boldsymbol{u}^2}{2} - m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}\ . \tag{50}$$

The fractal potential depends only on the imaginary part $\boldsymbol{u}$ of the complex speed field, $\boldsymbol{V}$, and it comes from the non-differentiability of the fractal space-time. Equation (49a), i.e. the momentum conservation law, and equation (49b), i.e. the probability density conservation law, form the fractal hydrodynamic model.

The wave function of $\psi(\boldsymbol{r},t)$ is invariant when its phase changes by an integer multiple of 2π. Indeed, equation (44a) gives:

$$\oint m_0\boldsymbol{v}d\boldsymbol{r} = m_0\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\oint dS = 2\pi n m_0\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}, \quad n = 0,\ \pm 1,\ \pm 2,\ ... \tag{51}$$

a condition of compatibility between the extended SR hydrodynamic model and the wave mechanics. Particularly, at Compton scalar $\lambda^2/\tau = \hbar/m_0$ and for D_F=2 equation (51) takes the standard form

$$\oint \boldsymbol{p}\cdot d\boldsymbol{r} = nh \tag{52}$$

i.e. the Ehrenfest relation.

The set of equations (49a,b) represents a complete system of differential equations for the fields $\rho(\boldsymbol{r},t)$ and $\boldsymbol{v}(\boldsymbol{r},t)$; relation (51) relates each fractal hydrodynamic solution $(\rho,\boldsymbol{v})_n$ with the wave solution ψ in a unique way.

The field $\rho(\boldsymbol{r},t)$ is a probability distribution, namely the probability of finding the particle in the vicinity $d\boldsymbol{r}$ of the point $\boldsymbol{r}$ at time t,

$$dP = \rho d\boldsymbol{r}, \quad \iiint \rho dr = 1, \tag{53a,b}$$

the space integral being extended over the entire area of the system. Any time variation of the probability density $\rho(\boldsymbol{r},t)$ is accompanied by a probability current $\rho\boldsymbol{v}$ pointing towards or outwards, the corresponding field point $\boldsymbol{r}$ (equation (49b)).

The real velocity field $\boldsymbol{v}(\boldsymbol{r},t)$ (equation (49a)), varies with space and time similar to a hydrodynamic fluid placed in a fractal potential (50). The fractal fluid (in the sense of a statistical particles ensemble) exhibits, however, an essential difference compared to an ordinary fluid: in a rotation motion $\boldsymbol{v}(\boldsymbol{r},t)$ increases (decreases) with the distance from the center $\boldsymbol{r}$ decreasing (increasing) (equation (51)).

The expectation values for the real velocity field and the velocity operator $\hat{\boldsymbol{v}} = -i\left(\lambda^2/\tau\right)(dt/\tau)^{\left(\frac{2}{D_F}\right)-1}\nabla$ of wave mechanics are equal,

$$\langle \boldsymbol{v} \rangle = \iiint \rho \boldsymbol{v} d\boldsymbol{r} = \iiint \Psi^* \hat{\boldsymbol{v}} \Psi d\boldsymbol{r} = \langle \hat{\boldsymbol{v}} \rangle_{WM} \tag{54}$$

but in the higher-order, $|n|>2$, similar identities are invalid, namely $\langle \boldsymbol{v}^n \rangle \neq \langle \boldsymbol{v}^n \rangle_{WM}$. The expectation value for the 'fractal force' vanishes at all times (theorem of Ehrenfest), i.e.

$$\langle -\nabla Q \rangle = \iiint \rho(-\nabla Q) d\boldsymbol{r} = 0 \tag{55}$$

since

$$m_0 \frac{\lambda^4}{2\tau^2}\left(\frac{dt}{\tau}\right)^{\left(\frac{4}{D_F}\right)-2} \iiint \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right) d\boldsymbol{r} = m_0 \frac{\lambda^4}{4\tau^2}\left(\frac{dt}{\tau}\right)^{\left(\frac{4}{D_F}\right)-2} \oint \left(\rho \nabla \nabla \ln \rho\right) \cdot d\boldsymbol{\sigma} = 0 \tag{56}$$

Two types of stationary states are distinguished:

i. Dynamic states. For $\partial / \partial t = 0$ and $\boldsymbol{v} \neq 0$, i.e. at the differentiable scale, equations (49a,b) give

$$\nabla\left(\frac{m_0\boldsymbol{v}^2}{2}-\frac{m_0\boldsymbol{u}^2}{2}-m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}\right)=0 \qquad (57a,b)$$

$$\nabla\cdot(\rho\boldsymbol{v})=0$$

namely,

$$\frac{m_0\boldsymbol{v}^2}{2}-\frac{m_0\boldsymbol{u}^2}{2}-m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\cdot\nabla\cdot\boldsymbol{u}=E \qquad (58a,b)$$

$$\rho\boldsymbol{v}=\nabla\times\boldsymbol{F}$$

Consequently, the non-fractal inertia, $m_0\boldsymbol{v}\cdot\nabla\boldsymbol{v}$, and the fractal force, $-\nabla Q$, are in balance at every field point - equation (57a). The sum of the non-fractal kinetic energy, $m\boldsymbol{v}^2/2$, and fractal potential, Q, is invariant, i.e., equal to the integration constant $E\neq E(\boldsymbol{r})$ - equation (58a). $E\equiv<E>$ represents the total energy of the dynamic system. The probability flow density $\rho\boldsymbol{v}$ has no sources - equation (57b), i.e., its streamlines are closed - equation (58b).
In an external potential U the equation (58a) becomes:

$$\frac{m_0\boldsymbol{v}^2}{2}-\frac{m_0\boldsymbol{u}^2}{2}-m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}+U=E \qquad (59)$$

ii. Static states. For $\partial/\partial t=0$ and $\boldsymbol{v}=0$, i.e. at the non-differentiable scale, equations (49a,b) give

$$\nabla\left(-\frac{m_0\boldsymbol{u}^2}{2}-m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}\right)=0\,, \qquad (60)$$

i.e.

$$-\frac{m_0\boldsymbol{u}^2}{2}-m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}=E\ . \qquad (61)$$

Thus, the fractal force, $-\nabla Q$ has the zero value - equation (60). The fractal potential, Q, is invariant, i.e. equal to the integration constant $E\neq E(\boldsymbol{r})$ - equation (61). $E\equiv<E>$ represents the total energy of the static system. Equation (49b) is identically satisfied.
In an external potential U the equation (61) becomes:

$$-\frac{m_0\boldsymbol{u}^2}{2}-m_0\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\cdot\boldsymbol{u}+U=E \qquad (62)$$

As an illustration of the fractal hydrodynamic formalism, stationary and time-dependent fractal systems are further analyzed.

If the transport phenomenon implies temperature fields then, according with the fractal operator (31) in which we neglected the dispersive effects, the heat transfer equation becomes:

$$\frac{\hat{\partial}T}{\partial t}=\frac{\partial T}{\partial t}+\boldsymbol{V}\cdot\nabla T-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}\Delta T=0 \tag{63}$$

From here, by separating the real and imaginary parts and then by their summing, the usual heat transfer equations results:

$$\frac{\partial T}{\partial t}+\left(\mathrm{v}-\mathrm{u}\right)\cdot\nabla T=\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{\left(2/D_F\right)-1}\Delta T \tag{64}$$

5. The dispersive approximation of transport phenomenon in fractal structures. Some properties of matter

Let us consider that the dissipate effects can be neglected in comparison with the convective and dispersive ones. Then, through the equation (34), the microparticle movements are described by a generalized Korteweg-de Vries (GKdV) type equation

$$\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}\cdot\nabla\boldsymbol{V}+\frac{\lambda^3}{6\tau}\left(\frac{dt}{2}\right)^{\left(\frac{3}{D_F}\right)-1}\nabla^3\boldsymbol{V}=0 \tag{65}$$

By substituting (44a,b) in equation (65), and separating the real and the imaginary parts, we obtain the following system:

$$\begin{aligned}&\frac{\partial \boldsymbol{v}}{\partial t}+\nabla\left(\frac{\boldsymbol{v}^2}{2}-\frac{\boldsymbol{u}^2}{2}\right)+\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{3}{D_F}\right)-1}\nabla^3\boldsymbol{v}=0\\&\frac{\partial \boldsymbol{u}}{\partial t}+\nabla\left(\boldsymbol{v}\cdot\boldsymbol{u}\right)+\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{3}{D_F}\right)-1}\nabla^3\boldsymbol{u}=0\end{aligned} \tag{66a,b}$$

In the one-dimensional differentiable case, u=0 or $\rho=const.$, using the dimensionless parameters,

$$\overline{\phi}=(v/v_0)\,,\ \overline{\tau}=\omega_0 t\,,\ \overline{\xi}=k_0X \tag{67a-c}$$

and the normalizing condition

$$\frac{v_0k_0}{6\omega_0}=\frac{k_0^3}{\omega_0}\frac{\lambda^3}{6\tau}\left(\frac{dt}{\tau}\right)^{\left(\frac{3}{D_F}\right)-1}=1 \tag{68}$$

the equations (66a,b), takes the standard form of the KdV equation,

$$\partial_{\overline{\tau}}\overline{\phi}+6\overline{\phi}\partial_{\overline{\xi}}\overline{\phi}+\partial_{\overline{\xi}\overline{\xi}\overline{\xi}}\overline{\phi}=0 \tag{69}$$

Through the substitutions,

$$w(\theta) = \bar{\phi}(\bar{\xi}, \bar{\tau}),\ \theta = \bar{\xi} - v_f \bar{\tau} \tag{70}$$

equation (69), by double integration, becomes

$$\frac{1}{2} w^2 = F(w) = -(w^3 - \frac{v_f}{2} w^2 - gw - h) \tag{71}$$

with g, h two integration constants. If F(w) has real roots, they are of the form

$$e_1 = \bar{w} + 2a\left[\frac{E(s)}{K(s)} - \frac{1}{s^2}\right],\ e_2 = \bar{w} + 2a\left[\frac{E(s)}{K(s)} - 1\right],\ e_3 = \bar{w} + 2a\frac{E(s)}{K(s)} \tag{72a-c}$$

with

$$a = \frac{e_3 - e_2}{2},\ s^2 = \frac{e_3 - e_2}{e_3 - e_1},\ K(s) = \int_0^{\pi/2} (1 - s^2 \sin^2\varphi)^{-1/2} d\varphi,$$

$$E(s) = \int_0^{\pi/2} (1 - s^2 \sin^2\varphi)^{1/2} d\varphi \tag{73a-d}$$

$\bar{w}$ a reference value, and K(s), E(s) the complete elliptic integrals of s modulus. The stationary solution of equation (69) has the expression,

$$\bar{\phi}(\bar{\xi}, \bar{\tau}) = \bar{w} + 2a\left(\frac{E(s)}{K(s)} - 1\right) - 2a \cdot cn^2\left\{\frac{\sqrt{a}}{s}\left[\bar{\xi} - \left(6\bar{w} + 4a\left(\frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2}\right)\right)\bar{\tau} + \bar{\xi}_0\right]; s\right\} \tag{74}$$

where cn is the Jacobi's elliptic function of s modulus and $\bar{\xi}_0$ an integration constant. As a result, the one-dimensional oscillation modes of the speed field are of cnoidal type – and have the normalized wave length,

$$\lambda = 2sK(s) / \sqrt{a} \tag{75}$$

- see figure 1a, the normalized phase speed,

$$v_f = 6\bar{w} + 4a\left[3\left(E(s) / K(s)\right) - \left((1+s^2) / s^2\right)\right] \tag{76}$$

- see figure 1b, and the normalized group speed,

$$v_g = 6\bar{w} + 4a\left[3\frac{E(s)}{K(s)} - 1 - \frac{1}{s^2} + 3\frac{E^2(s) + 2(s-1)E(s)K(s) - (s-1)K^2(s)}{E(s)K(s) + K^2(s) - sK^2(s)} + \right.$$
$$\left. + \frac{2(s-1)K(s)}{s^2[E(s) + K(s) - sK(s)]}\right] \tag{77}$$

- see figure 1c.

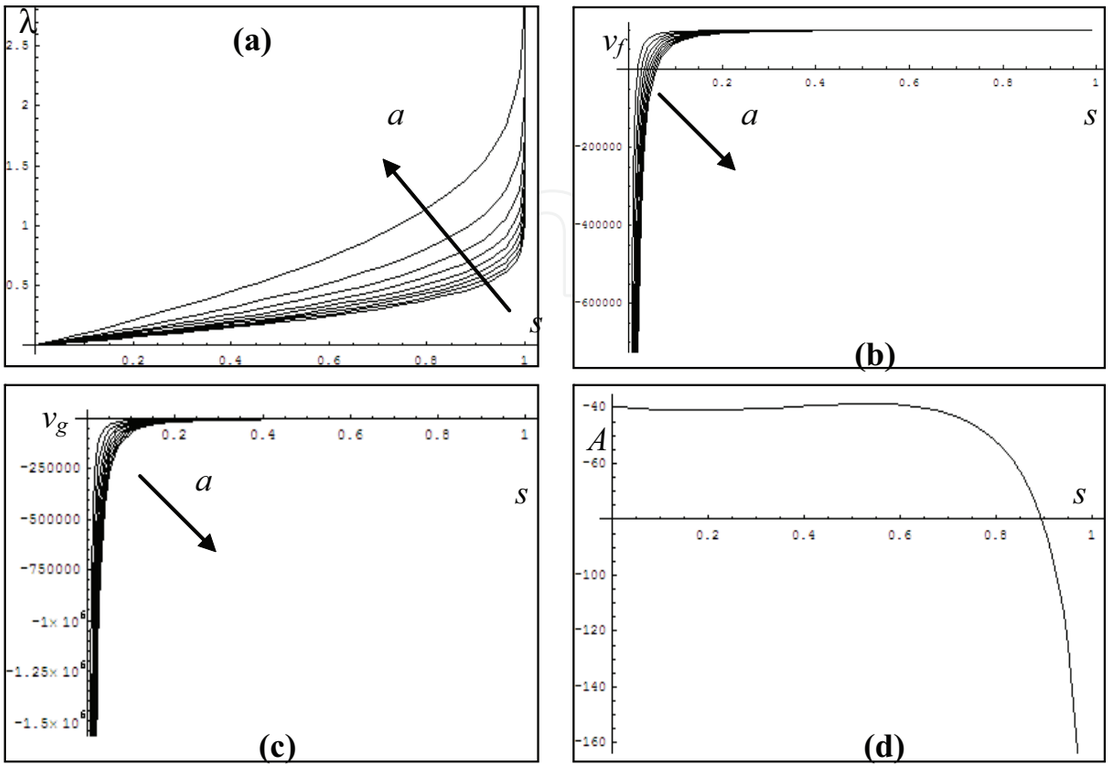


Fig. 1a-d. The dependences on s of the (a) normalized wave length λ, (b) normalized phase speed υ_f, (c) group velocity υ_g (various values of the parameter a), and (d) of the quantity A

Then, the followings result: (i) Through the λ, τ coefficients, the parameter s becomes a 'measure' of 'charge' transport type in the considered matter. Thus, the solution (74), for $s = 0$, is reduced to one-dimensional harmonic waves, and for $s \to 0$ to one-dimensional waves packet. These two subsequences describe the 'charge' transport in a non-quasi-autonomous regime. For $s = 1$, the solution (74) becomes an one-dimensional soliton while for $s \to 1$ one dimensional solitons packet results. These last two subsequences describe 'charge' transport in a quasi-autonomous regime; (ii) By eliminating the parameter a from relations (75) and (76), one obtains,

$$(v_f - 6\overline{w})\lambda^2 = A(s)\,,\ A(s) = 16\left[3s^2E(s)K(s) - (1+s^2)K^2(s)\right] \qquad (78a,b)$$

where the quantity A(s) is plotted in figure 1d. We observe that for s = 0 ÷ 0.7, A(s) ≈ const. and consequently equation (78a) takes the form, $(v_f - 6\overline{w})\lambda^2 = const.$. Therefore, in the differentiable case, the 'charge' transport is controlled through the flowing regimes of the fractal fluid, and the separation between them is given by the 0.7 value of the parameter s; (iii) The previous results show through the normalized group speed (77) an increase of the 'charge' transport by means of quasi-autonomous structures. This theoretical result explain some "anomalies" that were experimentally observed in composite materials, e.g. the increase of the thermal conductance etc.

Let us study now the previous phenomenon in the non-differential case. This can be achieved by the substitutions, $\phi = \left(v_f/4\right) f^2$ and $i\eta = \left(v_f/4\right)^{1/2} \theta$ in equation (71). Moreover, this equation with $h = 0$, becomes, $\partial_{\eta\eta} f = f^3 - f$, i.e. a Ginzburg-Landau (GL) type equation. The followings result: (i) The η coordinate has dynamic significations and the variable f has probabilistic significance. The space-time becomes fractal. (ii) Since the general solution of GL equation can be expressed, with an adequate normalization and choice of the integration constants, by means of the elliptic function $f(\eta) = sn(\eta; s)$, then the 'charge' transport is controlled by the fractal potential (50)

$$Q = -(1/f)(d^2 f / d\eta^2) = \left(1 - f^2\right) = cn^2(\eta, s), \tag{79}$$

also through cnoidal oscillation modes. Thus, as in the previous differentiable case, $(s = 0, s \to 0)$ implies the non-quasi-autonoumous regime, while (s=1, $s \to 1$) implies the quasi-autonoumous regime; (iii) For $s = 1$ the general solution of GL equation is the fractal kink, $f_k(\eta) = tanh(\eta)$. In this case we can build a field theory with spontaneous symmetry breaking. The fractal kink spontaneously breaks the "vacuum" symmetry by tunneling, and generates coherent structures. This mechanism is similar with the one of superconductivity and can explain the properties of composites through the Cooper type pairs; (iv) The normalized fractal potential takes a very simple expression which is directly proportional with the density of states of the fractal fluid – see equation (79). When the density of states, f^2, becomes zero, i.e. in the absence of the vacuum symmetry spontaneous breaking, the fractal potential takes a finite value, $Q \to 1$. The fractal fluid is normal (it works in a non-quasi-autonomous regime) and there are no coherent structures in it. When f^2 becomes 1, i.e. in the presence of the vacuum symmetry spontaneous breaking, the fractal potential is zero,

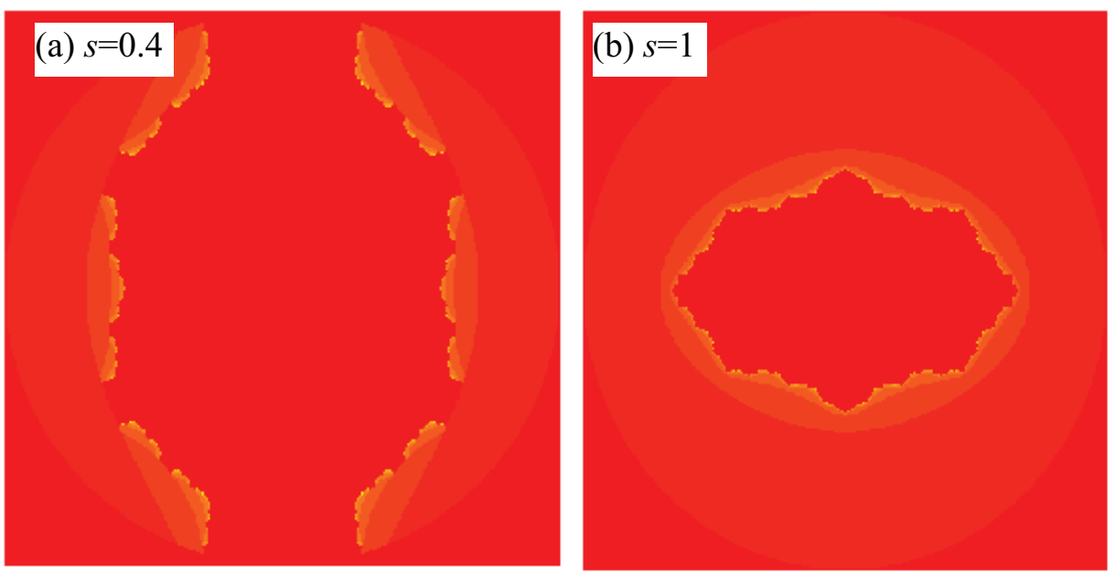


Fig. 2 a-b. The iterative map induced by the elliptic function cn^2 for various values of the s parameter

i.e. the entire quantity of energy of the fractal fluid is transferred to its coherent structures. Then the fractal fluid becomes coherent (it works in a quasi-autonomous regime). Therefore, one can assume that the energy from the fractal fluid can be stocked by transforming all the environment's entities into coherent structures and then 'freezing' them. The fractal fluid acts as an energy accumulator through the fractal potential (79); (v) the correlation between the differentiable and the non-differentiable scales implies the equivalence theorem in periods of two cn^2 elliptic functions. Then, the fractal space-time is of Cantor type.
Moreover, the 'charge' transport implies at any scale a fractal. Such result is obtained through the iterative map induced by the elliptic function cn^2 for various values of the s parameter – Figures 2a-b. For any object given in these figures the Hausdorff-Besicovitch theorem is respected. Evidently, all presented conclusions can be extended to the temperature field.

6. References

Agop, M., G. V. Munceleanu, O. Niculescu, T. Dandu-Bibire, *Physica Scripta* (in press).

Agop, M., C. Radu, T. Bontas, *Chaos Solitons and Fractals* 38, 5, 1243 (2008).

Agop, M., O. Niculescu, A. Timofte, L. Bibire, A. S. Ghenadi, A. Nicuta, C. Nejneru, G. V. Munceleanu, Non-differentiable mechanical model and its implications, I*nternational Journal of Theoretical Physics*, DOI: 10.1007/s10773-010-0330-5, (2008).

Agop, M., P. E. Nica, P. D. Ioannou, A. Antici, V. P. Paun, *Euro Physics Journal* D 49, 239 (2008).

Agop, M., P. E. Nica, S. Gurlui, C. Focsa, V. P. Paun, M. Colotin, *Euro Physics Journal* D 10.11.40/epjd/e2009_00304-5, (2008)

Agop, M., P. Nica, M. Gartu, *Gen. Relativ. Gravit*. 401, 35 (2008).

Argyris, J., C. Ciubotariu, G. Mattutis, *Chaos, Solitons and Fractals* 12, 1 (2001).

Bowman F; *Introduction to Elliptic Function with Applications;* London: English University Press, 1955.

Buzea, C. Gh., I. Rusu, V. Bulancea, Gh. Badarau, V. P. Paun, M. Agop, *Physics Letters A* (in press).

Buzea, C. Gh., C. Bejinariu, C. Boris, P. V. Vizureanu, M. Agop, *International Journal of Nonlinear Sciences and Numerical Simulations* 10 (11-12), 1399 (2009).

Célérier, M.N., L. Nottale, *J. Phys. A: Math. Gen*. 37, 931 (2004).

Cresson, J., F. Ben Adda, *C. R. Acad. Sci. Paris* 330, 261 (2000).

Cresson, J., F. Ben Adda, *J. Math. Anal. Appl*. 262, 721 (2001).

Cresson, J., *Chaos, Solitons and Fractals*, 14, 553 (2002).

Cresson, J., *J. Math. Phys.* 44, 4907 (2003).

Cresson, J., F. Ben Adda, *Chaos, Solitons and Fractals*, 19, 1323 (2004).

Cresson, J., *J. Math. Anal. Appl.* 307, 48 (2005).

Cresson, J., *Int. J. of Geometric Meth. in Mod. Phys*., 3, 1395 (2006).

Cresson, J., J-N. Denarie, *Lect. Notes in Physics*, Planat, Paris, 2000.

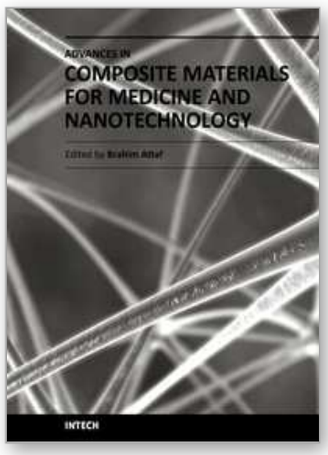
Cristescu, C.P., *Nonlinear Dynamics and Chaos in Science and Engineering*, Academy Publishing House, Bucharest, 2008.

Cristescu, C.P., B. Mereu, C. Stan, M. Agop, *Chaos Solitons and Fractals* 40, 975 (2009).

Da Rocha, D., L. Nottale, *Chaos, Solitons and Fractals* 16, 565 (2003).

El Naschie, M.S., O. E. Roessler, I. Prigogine, *Quantum Mechanics, Diffusion and Chaotic Fractal*, Elsevier, Oxford, 1995.

Feder, J., A. Aharony, *Fractals in Physics*, North Holland, Amsterdam, 1990.
Ferry, D. K.; Goodnick S. M.; *Transport in Nanostructures*; Cambridge: Cambridge University Press, 1997.
Gouyet, J.F., *Physique et Structures Fractals*, Masson, Paris, 1992.
Gurlui, S., M. Agop, M. Strat, G. Strat, S. Bacaita, A. Cerepaniuc, *Phys. Plasmas* 13, 063503 (2006).
Gurlui, S., M. Agop, P. Nica, M. Ziskind, C. Focsa, *Phys. Rev.* E 78, 026405 (2008).
Imry, Y.; *Introduction to Mesoscopic Physics*; Oxford: Oxford University Press,2002.
Ioannou, P.D., P. Nica, V. Paun, P. Vizureanu, M. Agop, *Physica Scripta* 78, 6, 065101.
Jackson, E. A.; *Perspectives in Nonlinear Dynamics*; Cambridge: Cambridge University Press, vol. I+II, 1991.
Madelbrot, B., *The Fractal Geometry of Nature*, Freeman, San Francisco, 1982.
Nelson, E., *Quantum Fluctuations*, Princeton Univ. Press, NY, 1985.
Nica, P. Vizureanu, M. Agop, S. Gurlui, C. Focsa, N. Forna, P. D. Ioannou, Z. Borsos, *Japanese Journal of Applied Physics 48, 066001 (2009).*
Niculescu, O., D. G, Dimitriu, V. P. Paun, P. D. Matasaru, D. Scurtu, M. Agop, *Phys. Plasmas* 17, 042305 (2010).
Nottale, L., *Chaos, Solitons and Fractals* 9, 1051 (1980).
Nottale, L., *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity*, World Scientific, Singapore, 1993.
Nottale, L., L'univers et la lumière, *Cosmologie classique et mirages gravitationnels*, Flammarion, Paris, 1993.
Nottale, L., *La Relativité dans tous ses états : du mouvements aux changements d'échelle*, Hachette, Paris, 1998.
Nottale, L., *Chaos, Solitons and Fractals* 10, 459 (1999).
Nottale, L., J. Chaline, P. Grou, *Les arbres de l'évolution*, Hachette, Paris, 2000.
Nottale, L., *Chaos, Solitons and Fractals* 16, 539 (2003).
Nottale, L., *Chaos, Solitons and Fractals* 25, 797 (2005).
Nottale, L., M. N. Célérier, T. Lehner, *J. Math. Phys.* 47, 032303 (2006).
Nottale, L., Ch. Auffray, *Progress in Biophysics and Molecular Biology* 97, 115 (2008).
Nottale, L., Scale relativity and fractal space-time: theory and applications, in *Proceedings of First International Conference on the Evolution and Development of the Universe*, Paris, France, 2008.
Nottale, L., J. Phys. A: *Mathematical and Theoretical* 42, 275306 (2009).
Poole, C.P.; Farach, H.A.; Creswick, R. J. *Superconductivity*; San Diego-New York-Boston-London-Sydney-Tokyo-Toronto: Academic Press, 1995.
Stan, C. P. Cristescu, D. Alexandroaie, M. Agop, *Chaos Solitons and Fractals* 41, 727 (2009).
Weibel, P., G. Ord, G. Rössler (Editors), *Space-time Physics and Fractality*, Springer, Vienna, New York, 2005.



Advances in Composite Materials for Medicine and Nanotechnology

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Due to their good mechanical characteristics in terms of stiffness and strength coupled with mass-saving advantage and other attractive physico-chemical properties, composite materials are successfully used in medicine and nanotechnology fields. To this end, the chapters composing the book have been divided into the following sections: medicine, dental and pharmaceutical applications; nanocomposites for energy efficiency; characterization and fabrication, all of which provide an invaluable overview of this fascinating subject area. The book presents, in addition, some studies carried out in orthopedic and stomatological applications and others aiming to design and produce new devices using the latest advances in nanotechnology. This wide variety of theoretical, numerical and experimental results can help specialists involved in these disciplines to enhance competitiveness and innovation.

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