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Convergence and Collision Avoidance in Formation Control: A Survey of the Artificial Potential Functions Approach

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1. Introduction

Multi-agent Robots Systems (MARS) can be defined as sets of autonomous robots coordinated through a communication system to achieve cooperative tasks. During the last 20 years, MARS have found a wide range of applications in terrestrial, spatial and oceanic explorations emerging as a new research area (Cao et al. (1997)). Some advantages can be obtained from the collective behavior of MARS. For instance, the kind of tasks that can be accomplished are inherently more complex than those a single robot can accomplish. Also, the system becomes more flexible and fault-tolerant (Yamaguchi (2003)). The range of applications includes toxic residues cleaning, transportation and manipulation of large objects, alertness and exploration, searching and rescue tasks and simulation of biological entities behaviors (Arai et al. (2002)). The study of MARS extends the classical problems of single robots with new issues like motion coordination, task decomposition and task assignment, network communications, searching and mapping, etc. Therefore, the study of MARS encompass distributed systems, artificial intelligence, game theory, biology, ethology, economics, control theory, etc.

Motion coordination is an important research area of MARS, specifically formation control (Chen & Wang (2005)). The main goal is to coordinate a group of mobile agents or robots to achieve a desired formation pattern avoiding inter-agent collisions at the same time. The formation strategies are decentralized because it is assumed that every agent measures the position of a certain subset of agents and, eventually, it detects the position of other agents when a minimal allowed distance is violated and collision danger appears. Thus, the main intention is to achieve desired global behaviors through local interactions (Francis et al. (2004)). Also, the decentralized approaches offer greater autonomy for the robots, less computational load in control implementations and its applicability to large scale groups (Do (2007)). According to Desai (2002); Muhammad & Egerstedt (2004), the possible inter-agent communications and the desired relative position of every agent with respect to the others can be represented by a Formation Graph (FG). The application of different FG's to the same group of robots produces different dynamics on the team behavior. In the literature, some special FG topologies are chosen and the convergence to the desired formation and non-collision is analyzed for any number of robots.

A decentralized formation strategy must comply with two fundamental requirements: Global convergence to the desired formation and inter-agent collision avoidance (Cao et al. (1997)).

The standard methodology of Artificial Potential Functions consists in applying the negative gradient of a mixture of Attractive (APF) and Repulsive Potential Functions (RPF) as control inputs to satisfy the convergence and non-collision properties, respectively (Leonard & Fiorelli (2001); Tanner & Kumar (2005)). The APF's are designed according the desired inter-robot distances and steer all agents to the desired formation. The RPF's are based on functions of the distance of a pair of agents. In a decentralized non-collision strategy, a local RPF tends to infinity when two agents collide and vanishes smoothly until the minimal allowed distance is reached. A control law based on APF's only guarantees the global convergence to the formation pattern, however inter-robot collision can occur. The addition of RPF's guarantees the non-collision. However, the main drawback of mixing APF and RPF is the appearance of equilibria where the composite vector field vanishes and the robots can get trapped at undesired equilibrium points. Therefore, a proof of global convergence to the desired formation for all initial conditions becomes quite involved because the analysis to calculate these equilibria and the trajectories which do not converge to the desired formation is very complex (Do (2006)).

This chapter analyzes the global convergence and non-collisions strategies of formation strategies based on Artificial Potential Functions on the context of formation graphs for the case of point robots or omnidirectional robots. In Section 2, we present a literature review about different formation strategies with emphasis on the approaches that modify the original Artificial Potential Functions method to ensure convergence and collision avoidance at the same time. Section 3 establishes a formal problem statement and the basic concepts of FG's. The standard methodology of APF's and RPF's and the complexity of the computation of equilibria is presented in Section 4. Some contributions to the literature of formation control based on Artificial Potential Functions are presented in Sections 5 and 6 with the analysis of the centroid of positions and the design of a new repulsive vector field that improve the performance of the robots' trajectories. Finally, in Section 7, the control laws are extended to for the case of unicycle-type robots with numerical simulations and Section 8 presents some experiments using a setup composed by two o three unicycle-like robots and a computer vision system to estimate positions and orientations of the robots within the workspace.

2. State of the art

Formation control is presented in most of MARS applications because generally is required a coordination control to obtain a strategic displacement or posture of the robots within the workspace to achieve a common work (Chen & Wang (2005)). For instance, on surveillance and exploration tasks, is required that robots move forward on a specific formation pattern to maximize their detection capacities and eventually, reconfigure this pattern if some robot breaks down (Balch & Arkin (1998)). In the case of manipulation of large objects (Arai et al. (2002)), the robots must achieve strategic team positions to carry an object within the workspace (Asahiro et al. (1999); Cao et al. (1997)).

Chen & Wang (2005) suggest to consider the formation control as a regulation problem, a well known concept in control theory. As mentioned above, the goal is the design of decentralized schemes based on the following assumptions: a) every agent knows its desired position on the group but not the goals of the others and b) every robot knows only the position of a certain subset of robots to converge to the desired formation (Dimarogonas, Kyriakopoulos & Theodorakatos (2006); Feddema et al. (2002)). However, there is no general consensus to delimit the decentralized schemes. The most accepted idea is to identify the degree of decentralization of the formation control. For instance, the case of zero decentralization (or

full centralization) consists on team robots where every agent knows the positions and goals of the others. The next level arises when the agents know the positions of the others but not their goals and the maximum degree of decentralization is the case where the agents share the minimum information to converge to the desired formation.

Formation control schemes can be classified into two categories. First, the behavior-based schemes come from the study of animal behaviors where the agents are formed following simple behavior rules, as maintaining a distance between neighbors, swarm intelligence and self-organization, aggregation, flocks, hunter-prey system, etc. (Balch & Arkin (1998); Reynolds (1987); Spears et al. (2004); Yamaguchi (2003)). This scheme considers to all agents with the same sensing capacities and generally converge to formation patterns without a specific position for every agent. The second scheme is related to model-based behaviors or emergent behaviors on the context of FG's. Some tools of graph theory and linear systems are used to analyze the closed-loop system (Desai (2002); Desai et al. (1998); Fax & Murray (2004); Olfati-Saber & Murray (2002); Tanner (2004)). Similar to FG's, the focus on geometric patterns is found in works as Marshall et al. (2004). Other model-based behaviors are mentioned in Chen & Wang (2005), as nonlinear servomechanisms, genetic algorithms, Distributed Artificial Intelligence, attractive forces of particles, etc.

Some works in the literature are related to the analysis of convergence without considering the inter-robot collisions. For instance, the analysis of *agreement problem* or *consensus problem* (Dimarogonas & Kyriakopoulos (2006b); Francis et al. (2004); Olfati-Saber & Murray (2003)) establishes the minimum conditions for the convergence of the robots to a common point considering sensing capacities limited to a certain influence area. Other works are based on graph theory and linear systems to prove the convergence for some typical cases of FG's, described bellow (see Baillieul & McCoy (2007); Fax & Murray (2004); Muhammad & Egerstedt (2004)). Finally, works as (Dimarogonas & Kyriakopoulos (2006a); Hendrickx et al. (2007); Lafferriere et al. (2004); Li & Chen (2005b); Swaroop & Hedrick (1996); Tanner et al. (2002; 2004)) complement the convergence analysis with the study of formation infeasibility, formation rigidity and formation stability. For instance, the formation infeasibility studies the conditions of the desired vectors of position in a FG that eliminate the equilibrium points and consequently the possibility of the convergence to the desired formation. The *chain stability* and rigidity analyze the disturbance propagation in a group of robots when has achieved the desired formation. The *leader-formation stability* studies how the leader behavior affects the formation of all robots.

On the other hand, the non-collision strategies include reactive schemes based on simple behavior-based rules (Ando et al. (1999); Balch & Arkin (1998); Egerstedt & Hu (2001); Reynolds (1987)), hybrid architectures (Cao et al. (2003); Das & Fierro (2003); Mallapragada et al. (2006)), physics-based and swarms techniques (Spears et al. (2004)) or repulsive forces based on Artificial Potential Functions or repulsive vector fields (Ogren & Leonard (2003); Rimon & Koditschek (1992); Schneider & Wildermuth (2005)). As mentioned above, decentralized RPF's appear only within the influence zone of every robot, equivalently, every agent does not know the position of other agents unless there is danger of collision. Due to the possible scenarios of collision for the general case, Do (2006) establishes that a proof of convergence to the desired formation for all initial conditions becomes quite involved by the appearance of undesired equilibria. In Dimarogonas & Kyriakopoulos (2006a), it is shown the complexity analysis of decentralized RPF's applied to all FG's with bidirectional communication. However, the convergence analysis discards the undesired equilibria.

In the literature, there exist different approaches to modify the original Artificial Potential Functions method to ensure convergence and collision avoidance. For example, in (Dimarogonas & Kyriakopoulos (2005); Dimarogonas, Loizou, Kyriakopoulos & Zavlanos (2006); Gennaro & Jadbabaie (2006); Olfati-Saber & Murray (2002); Tanner & Kumar (2005)), composed functions or navigation functions are designed with attractive and repulsive behavior to eliminate the undesired equilibria. The drawback is that the non-collision strategy becomes centralized because it requires full-knowledge of the system. Also, most of these functions are high-order with a corresponding high computational cost for the implementation on real robots. Do (2006) demonstrates that the undesired equilibrium points are unstable (saddle point) for the case of the complete FG. Exploiting the unstable behavior of these equilibria, some approaches propose small disturbances in order to agents escape of these equilibria using online strategies such that virtual obstacle method (Lee & Park (2003; 2004); Li & Chen (2005a); Ogren & N.E. Leonard (2004)), instantaneous goal approach (Ge & Fua (2005)), etc. However, the previous strategies are patch-type and they do not include formal proofs about the convergence to the desired formation. On the other hand, the use of non-smooth vector fields can rule out the existence of undesired equilibria. Some works about discontinuous vector fields in formation control are (Hernandez-Martinez & Aranda-Bricaire (2009b); Loizou & Kyriakopoulos (2002); Loizou, Tannert, Kumar & Kyriakopoulos (2003); Loizou, Tannert & Kyriakopoulos (2003); Tanner (2004); Yao et al. (2006)). The analysis falls on the control of variable structure systems (Itkis (1976)). In most works, the repulsive discontinuous forces are designed heuristically and no formal proofs are presented (for instance, Barnes et al. (2007); Kim et al. (2005)).

Finally, other strategies of non-collision are listed in Chen & Wang (2005) like the predictive model control, social potential fields, fuzzy logic and neural networks. In these schemes, a hierarchical control scheme is proposed, where the higher level coordinates reactive collision avoidance actions. A few works deal about the non-adequacy on communication and the delay effects on the formation stability.

3. Problem statement and formation graphs

Inspired in (Chen & Wang (2005); Francis et al. (2004); Tanner & Kumar (2005)), a general definition of formation control for point robots or omnidirectional robots is established as follows:

Denote by $N = \{R_1, ..., R_n\}$, a set of n agents moving in plane with positions $z_i(t) = [x_i(t), y_i(t)]^T, i = 1, ..., n$. The kinematic model of each agent or robot R_i is described by

$$\dot{z}_i = u_i, \quad i = 1, ..., n, \tag{1}$$

where $u_i = [u_{i1}, u_{i2}]^T \in \Re^2$ is the velocity of i-th robot along the X and Y axis . Let $N_i \subseteq \{z_1, ..., z_n\}$, $N_i \neq \varnothing$, $i = 1, ..., n$. denote the subset of positions of the agents which are detectable for R_i. Let $c_{ji} = [h_{ji}, v_{ji}]^T, \forall j \in N_i$ denote a vector which represents the desired position of R_i with respect to R_j in a particular formation. Thus, we define the desired relative position of every R_i in the formation by

$$z_i^* = \varphi_i(N_i) = \frac{1}{n_i} \sum_{j \in N_i} \left(z_j + c_{ji} \right), i = 1, ..., n \tag{2}$$

where n_i is the cardinality of N_i. Thus, the desired relative position of R_i can be considered as a combination of the desired positions of z_i with respect to the positions of all elements of N_i. Let $d/2$ be the radius of the closed ball that every agent occupies within the workspace.

Problem Statement. The control goal is to design a control law $u_i(t) = f_i(N_i(t))$ for every robot R_i, such that

- $\lim_{t \to \infty}(z_i - z_i^*) = 0$, $i = 1, ..., n$. (convergence to the desired formation) and
- $\|z_i(t) - z_j(t)\| \neq 0$, $\forall t \geq 0$, $i \neq j$ (collision avoidance).

According to (Desai (2002); Muhammad & Egerstedt (2004)), the desired relative positions of a group of agents on a desired formation can be represented by a FG defined by

Definition 1. *A Formation Graph $G = \{Q, E, C\}$ is a triplet that consists in (i) a set of vertices $Q = \{R_1, R_2, ..., R_n\}$ related to the team members, (ii) a set of edges $E = \{(j, i) \in Q \times Q\}, i \neq j$ containing pairs of nodes that represent inter-agent communications, therefore $(j, i) \in E$ iff $j \in N_i$ and (iii) a set of vectors $C = \{c_{ji}\}, \forall (j, i) \in E$ that specify the desired relative position between agents i and j, i.e. $z_i - z_j = c_{ji} \in \Re^2, \forall i \neq j, j \in N_i$ in a desired formation pattern.*

If $(i, j) \in E$, then the vertices i and j are called *adjacent*. The degree g_i of the $i - th$ vertex is defined as the number of its adjacent vertices. A path from vertex i to j is a sequence of distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. The underlying graph of a FG, is the graph where $\forall (i, j) \in E$, is added a new edge (j, i), if it does not appear on the original FG. The underlying graph is always an undirected graph. If there is a path between any two vertices of the underlying graph of FG, then the FG is said to be *connected*. Thus, a FG is said to be well defined if it satisfied the following conditions: (1) the graph is connected, (2) there are no conflicts in the desired vectors of positions, in the sense that if $c_{ij}, c_{ji} \in C$, then $c_{ij} = -c_{ji}$ and (3) the desired vectors of positions establish a closed-formation, i.e., if there exist the vectors $c_{jm_1}, c_{m_1 m_2}, c_{m_2 m_3}, ..., c_{m_r j}$, then they must satisfy:

$$c_{jm_1} + c_{m_1 m_2} + c_{m_2 m_3} + ... + c_{m_r j} = 0. \tag{3}$$

The previous condition establishes that some position vectors form closed-polygons. The *Laplacian matrix* of a FG captures many fundamental topological properties of the graph and it is defined bellow.

Definition 2. *The* Laplacian matrix *of a FG G is the matrix*

$$\mathcal{L}(G) = \Delta - A_d \tag{4}$$

where $\Delta = diag[g_1, ..., g_n]$, where g_i is the degree of the vertex i, $A_d = \in \Re^{n \times n}$ is called the adjacency matrix *with elements*

$$a_{ij} = \begin{cases} 1, & \text{if } (j, i) \in E \text{ (or equivalently } c_{ji} \in C) \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

For a connected FG, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is $[1, ..., 1]^T \in \Re^n$. Fig. 1 shows an example of FG. The vertices are represented by circles and the arrows are the vectors c_{ji}. The circled elements of the Laplacian matrix are the degrees g_i. It is clear that $g_i = n_i, i = 1, ..., n$.

A FG is said to be *directed* if $\forall (j, i) \in E$, then $(i, j) \notin E$ (or $j \in N_i$ implies $i \notin N_j$), *undirected* if $\forall (j, i) \in E$ then $\forall (i, j) \in E$ (or $j \in N_i$ implies $i \in N_j$) and *mixed* otherwise. For instance, the FG of fig. 1 is mixed. For the case of undirected FG, the Laplacian is always a symmetric semidefinite positive matrix.

Fig. 2 shows some examples of FG topologies commonly found in the literature and their respective Laplacian matrices. For instance, Do (2006) analyzes the convergence of the
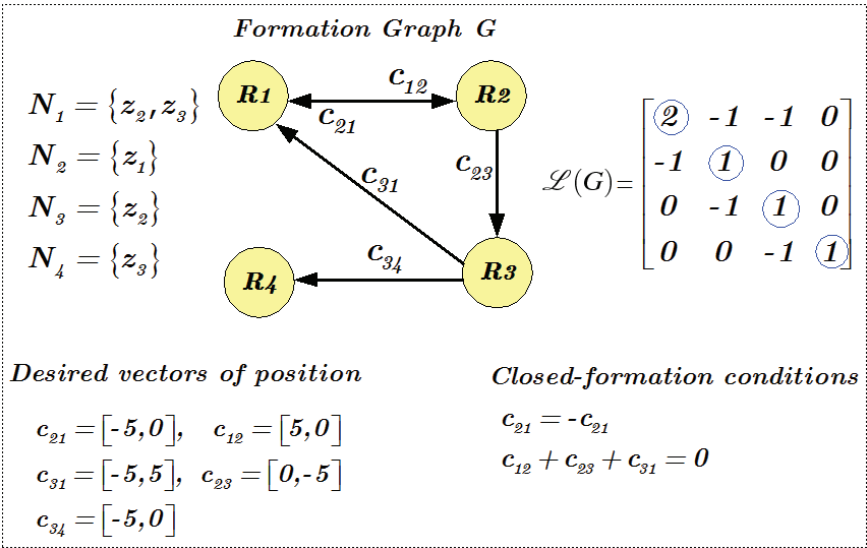
Fig. 1. Example of a Formation Graph

complete FG shown in Fig. 2a, where every robot senses the position of the others. The directed cyclic pursuit FG (fig. 2b) is studied in Francis et al. (2004), where the robot i pursues the robot $i + 1$ and the n-th robot pursues the first making a closed-chain configuration. A variant of the cyclic pursuit with bidirectional communication (undirected FG) is shown in Fig. 2c and analyzed in (Hernandez-Martinez & Aranda-Bricaire (2008b)). An analysis of the convergence for all undirected FG's is presented in (Dimarogonas & Kyriakopoulos (2006a)). The FG of leader-followers is analyzed in (Hernandez-Martinez & Aranda-Bricaire (2008a)) for the case of the FG centered on a virtual leader (Fig. 2d) and (Hernandez-Martinez & Aranda-Bricaire (2009a)) for the open-chain or convoy configuration (Fig. 2e). Other approaches of leader-followers schemes are found in (Desai et al. (2001); Leonard & Fiorelli (2001); Tanner et al. (2004)) including virtual leaders, i.e., robots that does not physically exist but they are emulated in order to improve the performance of the system.
For completeness, the following definition is introduced

Definition 3. *The centroid of positions $\bar{z}(t)$ is the mean of the positions of all robots in the group, i.e.*

$$\bar{z}(t) = \frac{1}{n}(z_1(t) + ... + z_n(t)) \tag{6}$$

4. Control strategy based on APF's and RPF's

For system (1), APF's are defined by

$$\gamma_i = \sum_{j \in N_i} \|z_i - z_j - c_{ji}\|^2, \ \ \forall j \in N_i, \ \ i = 1, ..., n \tag{7}$$

The functions γ_i are always positives and reach their minimum ($\gamma_i = 0$) when $z_i - z_j = c_{ji}$, $i = 1, ..., n, j \in N_i$. Then, a control law based on APF's only is defined as

$$u_i = -\frac{1}{2}k\left(\frac{\partial \gamma_i}{\partial z_i}\right)^T, i = 1, ...n, \ \ k > 0. \tag{8}$$
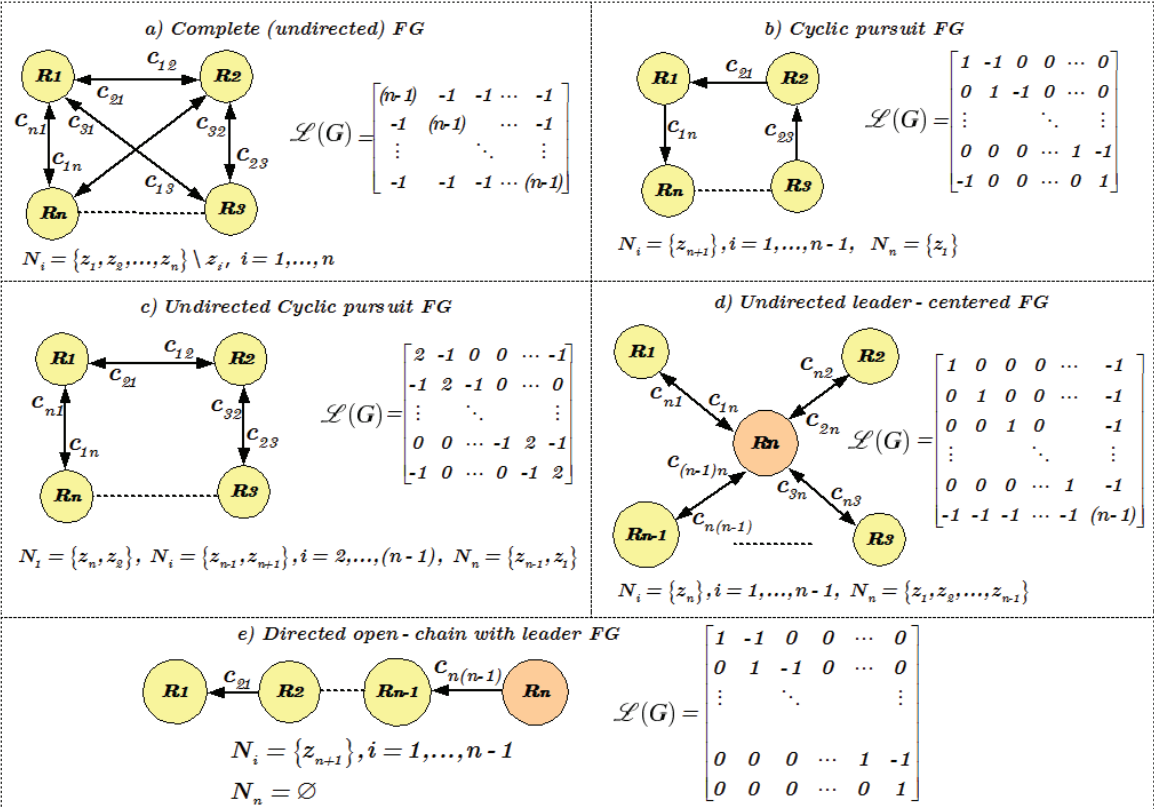
Fig. 2. Topologies of Formation Graphs

The closed-loop system (1)-(8) has the form

$$\dot{z} = -k((\mathcal{L}(G) \otimes I_2)z - c), \tag{9}$$

where $\mathcal{L}(G)$ is the Laplacian matrix of the FG, $z = [z_1,...,z_n]^T$, $\otimes$ denotes the Kronecker product (Dimarogonas & Kyriakopoulos (2006a)), I_2 is the 2×2 identity matrix and $c = \left[\sum_{j \in N_1} c_{j1},...,\sum_{j \in N_n} c_{jn}\right]^T$.

In (Hernandez-Martinez & Aranda-Bricaire (2010)) it is shown that in the closed-loop system (1)-(8) the agents converge exponentially to the desired formation, i.e. $\lim_{t \to \infty}(z_i - z_i^*) = 0$, $i = 1,...,n$, if the desired formation is based on a well-defined FG. The proof is based on the Laplacian Matrix and the Gershgorin circles Theorem (Bell (1972)).

Fig. 3 shows an example of the convergence to the desired formation with $n = 4$, $k = 1$ using the FG and desired vectors of positions given by Fig. 1. The initial positions in Fig. 3a (denoted by circles) are $z_1(0) = [0, -1]$, $z_2(0) = [-1, 0]$, $z_3(0) = [-4, -1]$ and $z_4(0) = [1, -3]$. We observe that the formation errors show in Fig. 3b converge to zero and therefore, all agents converge to the desired formation. The eigenvalues of $-k\mathcal{L}(G)$ are given by $0, -1, -2, -2$.

Note that the control strategies based on APF's guarantee the convergence to the desired formation. However, inter-robot collision can occur. The underlying idea of using RPF's is that every robot considers to all the others robots as mobile obstacles. The square of the distance between two robots is given by $\beta_{ij} = \|z_i - z_j\|^2$, $\forall i, j \in N$, $i \neq j$. Then, the robots R_j in danger of collision with R_i belong to the set

$$M_i = \{R_j \in N \mid \beta_{ij} \leq d^2\}, i = 1,...,n, \tag{10}$$
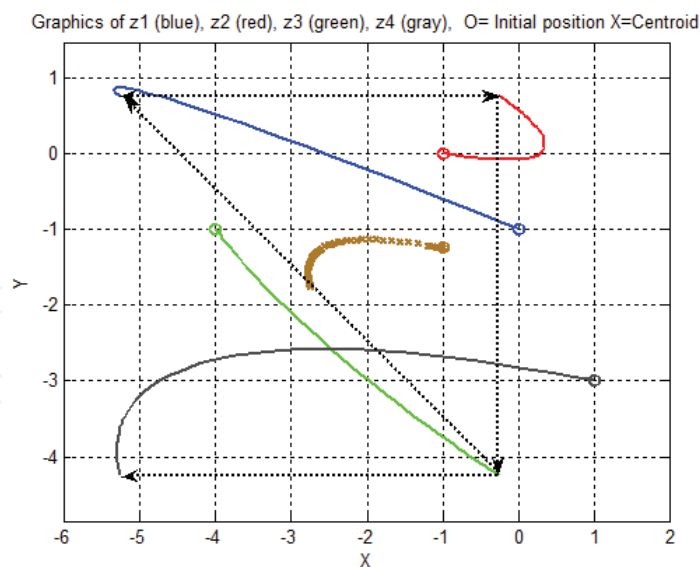
(a) Trajectories of the agents in the plane
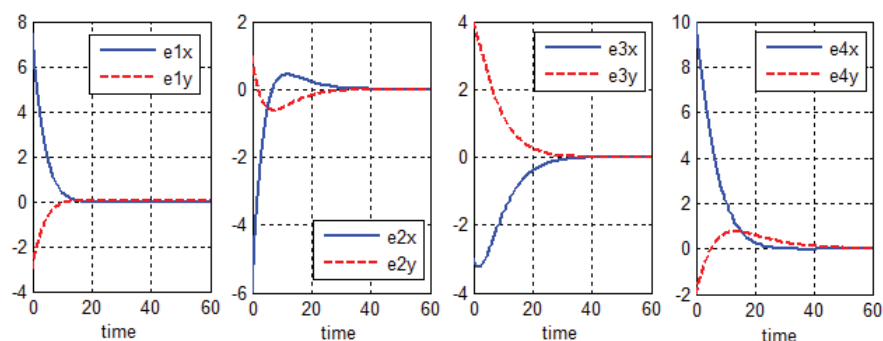
(b) Formation errors

Fig. 3. Formation control using the FG of fig. 1

where d is the diameter of the influence zone. In general, the set M_i changes in time due to the motion of agents. Then, a formation control law with collision avoidance based on APF's and RPF's is defined by

$$u_i = -\frac{1}{2}k\frac{\partial \gamma_i}{\partial z_i} - \sum_{j \in M_i} \frac{\partial V_{ij}}{\partial z_i}, i = 1, ..., n \tag{11}$$

where γ_i is the APF defined by (7) and $V_{ij}(\beta_{ij})$ is a RPF (between the pair of agents R_i and R_j) that satisfy the following properties:

1. V_{ij} es monotonously increasing when $\beta_{ij} \leq d^2$ and $\beta_{ij} \to 0$.

2. $\lim_{\beta_{ij} \to 0} V_{ij} = \infty$.

3. $V_{ij} = 0$ for $\beta_{ij} \geq d^2$, $\frac{\partial V_{ij}}{\partial z_i} = 0$ for $\beta_{ij} = d^2$.

The last condition establishes that every V_{ij} appears smoothly only within the influence area of the robot R_i. Also, it ensures that

$$\sum_{j \in M_i} \frac{\partial V_{ij}}{\partial z_i} = \sum_{j \neq i} \frac{\partial V_{ij}}{\partial z_i}. \tag{12}$$

A common function that satisfies the previous properties was proposed by Khatib (Rimon & Koditschek (1992)) as

$$V_{ij} = \begin{cases} \eta \left(\frac{1}{\beta_{ij}} - \frac{1}{d^2} \right)^2, & \text{if } \beta_{ij} \leq d^2 \\ 0, & \text{if } \beta_{ij} > d^2 \end{cases} \tag{13}$$

where $\eta > 0$. The following functions also comply with the RPF's properties.

$$V_{ij} = \begin{cases} \eta \left(\frac{1}{\beta_{ij}} - \frac{1}{d^2} \right)^r, & \text{if } \beta_{ij} \leq d^2 \\ 0, & \text{if } \beta_{ij} > d^2 \end{cases}, r = 2,3,4... \tag{14}$$

$$V_{ij} = \begin{cases} \eta \left(\frac{(\beta_{ij} - d^2)^2}{\beta_{ij}} \right), & \text{if } \beta_{ij} \leq d^2 \\ 0, & \text{if } \beta_{ij} > d^2 \end{cases} \tag{15}$$

Note that, in general, it is possible to rewrite $\frac{\partial V_{ij}}{\partial z_i} = 2 \frac{\partial V_{ij}}{\partial \beta_{ij}}(z_i - z_j)$. Since $\beta_{ij} = \beta_{ji}$, it is satisfied that $V_{ij} = V_{ji}$ and $\frac{\partial V_{ij}}{\partial \beta_{ij}} = \frac{\partial V_{ji}}{\partial \beta_{ji}}$, $\forall i \neq j$. This ensures that the RPF's complies with the following antisymmetry property:

$$\frac{\partial V_{ij}}{\partial z_i} = -\frac{\partial V_{ji}}{\partial z_j}, \quad \forall i \neq j. \tag{16}$$

Fig. 4 shows the trajectories of three agents under the control law (11) using Khatib's RPF (13) for the case of cyclic pursuit FG (Fig. 4a) and the case of undirected cyclic pursuit FG (Fig. 4b). The initial conditions and the desired formation (horizontal line) are the same in both simulations. The agents' trajectories in Fig. 4 are modified to avoid collision. Observe that the application of different FG's to the same number of robots produces a different behavior in the closed-loop system. Note that the centroid of positions (denoted by X) in Fig. 4 remains constant for all $t \geq 0$ unlike Fig. 3, where it does not remain constant within the workspace. This property is interesting because, regardless of the individual goals of the agents, the dynamics of the team behavior remains always centered on the position of the centroid. The time-invariance of the centroid of positions is studied in Section 5. This property is inherent to the structure of the Laplacian matrix and the antisymmetry of the RPF's.

As mentioned before, the main drawback of mixing APF's y RPF's is that the agents can get trapped at undesired equilibrium points. In Dimarogonas & Kyriakopoulos (2006a), the calculation of these equilibrium points, for the case of any undirected FG, is obtained solving the equation

$$(k\mathcal{L}(G) + 2R) \otimes I_2 z = kc \tag{17}$$

where $\mathcal{L}(G)$ is the Laplacian matrix of the undirected FG, $c = [c_1, ..., c_n]$ with $c_i = \sum_{j \in N_i} c_{ji}$ and

$$(R)_{ij} = \begin{cases} \sum_{j \neq i} \frac{\partial V_{ij}}{\partial \beta_{ij}}, & \text{if } i = j \\ -\frac{\partial V_{ij}}{\partial \beta_{ij}}, & \text{if } i \neq j \end{cases}$$

For instance, analyzing the simplest case of formation with two robots R_1 and R_2, where $N_1 = \{z_2\}$ y $N_2 = \{z_1\}$, Eq. (17) reduces to

$$\left(k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + 2 \begin{bmatrix} \frac{\partial V_{12}}{\partial \beta_{12}} & -\frac{\partial V_{12}}{\partial \beta_{12}} \\ -\frac{\partial V_{21}}{\partial \beta_{21}} & \frac{\partial V_{21}}{\partial \beta_{21}} \end{bmatrix} \right) \otimes I_2 \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = k \begin{bmatrix} c_{21} \\ c_{12} \end{bmatrix}. \tag{18}$$
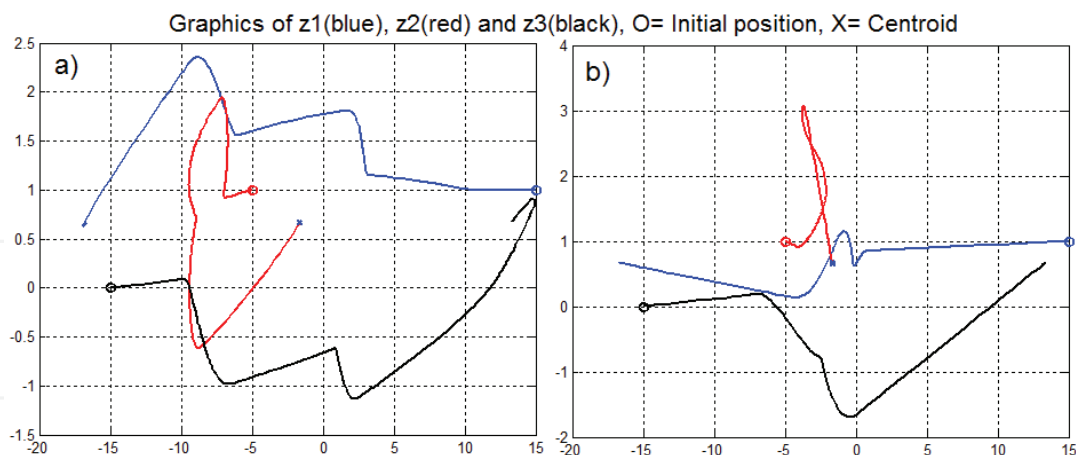
Fig. 4. Trajectories of robots in plane considering non-collision a) cyclic pursuit FG, b) undirected cyclic pursuit FG

Considering Khatib's RPF given by (13), Eq. (18) is rewritten as the following system of nonlinear simultaneous equations:

$$k\,(z_1 - z_2) - \delta \frac{4\eta}{\|z_1 - z_2\|^4} \left(\frac{1}{\|z_1 - z_2\|^2} - \frac{1}{d^2} \right) (z_1 - z_2) = kc_{21}$$
$$k\,(z_2 - z_1) - \delta \frac{4\eta}{\|z_1 - z_2\|^4} \left(\frac{1}{\|z_1 - z_2\|^2} - \frac{1}{d^2} \right) (z_2 - z_1) = kc_{12} \tag{19}$$

where $\delta = \begin{cases} 1, & \text{if } \beta_{12} \le d^2 \\ 0, & \text{if } \beta_{12} > d^2 \end{cases}$. The system of equations (19) is of the sixth order. However, for this particular case, clearing the term $\delta \frac{4\eta}{\beta_{12}^2} \left(\frac{1}{\beta_{12}} - \frac{1}{d^2} \right)$, it comes out that

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2 - v_{21}}{x_1 - x_2 - h_{21}} = \frac{y_2 - y_1 - v_{12}}{x_2 - x_1 - h_{12}}. \tag{20}$$

The interpretation of Eq. (20) is that, at the undesired equilibrium point, the agents R_1 and R_2 are placed on the same line as their desired positions (Fig. 5). This undesired equilibrium point is generated because both agents mutually cancel its motion when they try to move to the opposite side.

To analyze the relative position of agents R_1 and R_2, define the variables

$$p = x_1 - x_2, \quad q = y_1 - y_2 \tag{21}$$

The phase plane that represents the dynamics of these variables is shown in Fig. 6 for $k = 1$, $\eta = 10$, $d = 6$ and $c_{21} = [-3, -3]$. Off the influence zone (denote by a circle) there exists only the effect of the attractive forces generated by the APF's. Within the influence zone (inside the circle), the repulsive forces generated by the RPF's are added smoothly to the attractive forces. When $[p, q] = [0, 0]$ the distance between agents is zero and the RPF's tend to infinity. Two equilibrium points are seen in Fig. 6. One of them corresponds to the desired formation (stable node) and the other one corresponds to the undesired equilibrium point (saddle). For the case of more than two agents a similar analysis is impossible.

In general, the solution of the equation (17) is a highly complex nonlinear problem depending on the Laplacian structure and the quantity of possible combinations of RPF's that appear on these equilibria. Also, it is difficult to find general expressions similar to (17) for directed or mixed FG.
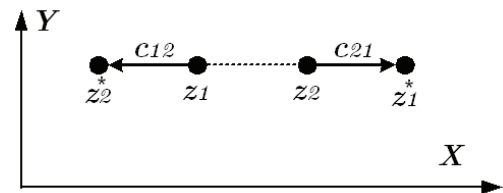
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Fig. 5. Position of two robots in a undesired equilibrium point
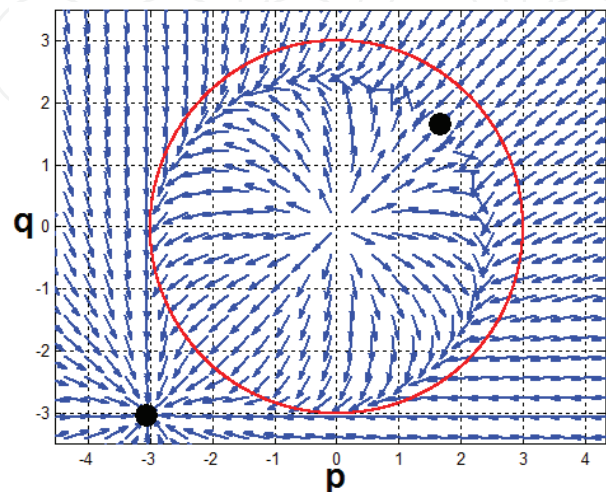

Fig. 6. Phase plane of coordinates (p, q) using the Khatib's RPF

5. Analysis of the centroid of positions

The next result was previously reported in (Hernandez-Martinez & Aranda-Bricaire (2010)) for the case of formation strategies based on APF's only. In this section, the result is extended to the case of control law (11) which mixes APF's and RPF's.

Proposition 1. *Consider the system (1) and the control law (11). Suppose that $k > 0$ and the desired formation is based on a well defined FG. Then, in the closed-loop system (1)-(11), the centroid of positions remains constant, i.e. $\bar{z}(t) = \bar{z}(0), \forall t \geq 0$ iff the FG topology satisfies the condition*

$$[1, ..., 1]\mathcal{L}(G) = [0, ..., 0]. \tag{22}$$

Proof. The dynamics of every agent R_i in the closed-loop system (1)-(11) can be written as

$$\dot{z}_i = -k\left(g_i z_i - \sum_{j \in N_i} z_j - \sum_{j \in N_i} c_{ji}\right) - \sum_{j \in M_i} \frac{\partial V_{ij}}{\partial z_i}, i = 1, ..., n \tag{23}$$

Using the property (12), note that

$$\dot{z}_i = -k\left(g_i z_i - \sum_{j \in N_i} z_j - \sum_{j \in N_i} c_{ji}\right) - \sum_{j \neq i} \frac{\partial V_{ij}}{\partial z_i}, i = 1, ..., n \tag{24}$$

Then, the dynamics of the centroid of positions is given by

$$\dot{\bar{z}}(t) = \frac{1}{n}\sum_{i=1}^{n} \dot{z}_i = -\frac{k}{n}\left(\sum_{i=1}^{n} g_i z_i - \sum_{i=1}^{n}\sum_{j \in N_i} z_j - \sum_{i=1}^{n}\sum_{j \in N_i} c_{ji}\right) - \frac{1}{n}\sum_{i=1}^{n}\sum_{j \neq i} \frac{\partial V_{ij}}{\partial z_i} \tag{25}$$

Due to the FG satisfies the closed-formation condition (3), then $\sum_{i=1}^{n} \sum_{j \in N_i} c_{ji} = 0$ and using the antisymmetry property given by (16) then $\sum_{i=1}^{n} \sum_{j \neq i} \frac{\partial V_{ij}}{\partial z_i} = 0$. Thus, equation (25) can be reduced to

$$\dot{z}(t) = -\frac{k}{n} \left(\sum_{i=1}^{n} g_i z_i - \sum_{i=1}^{n} \sum_{j \in N_i} z_j \right) = -\frac{k}{n} \sum_{i=1}^{n} \left(g_i z_i - \sum_{j \in N_i} z_j \right) \tag{26}$$

The term $\left(g_i z_i - \sum_{j \in N_i} z_j \right)$, $i = 1, ..., n$ corresponds to the $i - th$ element of the column vector $(\mathcal{L}(G) \otimes I_2) z$. Thus, Eq. (26) is the sum of the elements of $(\mathcal{L}(G) \otimes I_2) z$ multiplied by $-\frac{k}{n}$. Therefore, Eq. (26) is equivalent to

$$\dot{z}(t) = -\frac{k}{n} \left([1, ..., 1] \left(\mathcal{L}(G) \otimes I_2 \right) z \right) \tag{27}$$

At this point, it is clear that $\dot{z}(t) = 0$, $\forall t \geq 0$ iff condition (22) holds. Under these conditions, the centroid of positions is established by the initial positions of the robots, i.e. $\bar{z}(t) = \bar{z}(0)$ and remains constant $\forall t \geq 0$.

□

All the undirected graphs, the cyclic pursuit FG and some mixed FG satisfy the condition (22). Recall the numerical simulation of Fig. 4a (three robots in directed cyclic pursuit FG). Observe that

$$[1, 1, 1] \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = [0, 0, 0]. \tag{28}$$

As for the case of Fig. 4b (three robots in undirected cyclic pursuit FG)

$$[1, 1, 1] \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} = [0, 0, 0]. \tag{29}$$

Therefore, as seen before, the centroid of position in both simulations remains stationary for all $t \geq 0$.

6. Repulsive vector fields based on unstable focus

Current research focuses on the design of RPF's that provide a better performance of the closed-loop system. Following this line of thought, the following Repulsive Vector Field (RVF) is proposed

$$\psi_{ij} = \begin{cases} V_{ij} \begin{bmatrix} \left(x_i - x_j \right) - \left(y_i - y_j \right) \\ \left(x_i - x_j \right) + \left(y_i - y_j \right) \end{bmatrix}, & \text{if } \beta_{ij} \leq d^2 \\ 0, & \text{if } \beta_{ij} > d^2 \end{cases} \tag{30}$$

where $V_{ij}(\beta_{ij})$ is a RPF. Note that the repulsive vector field is a clockwise unstable focus scaled by the function V_{ij} and centered at the position of another robot which appears only if a danger
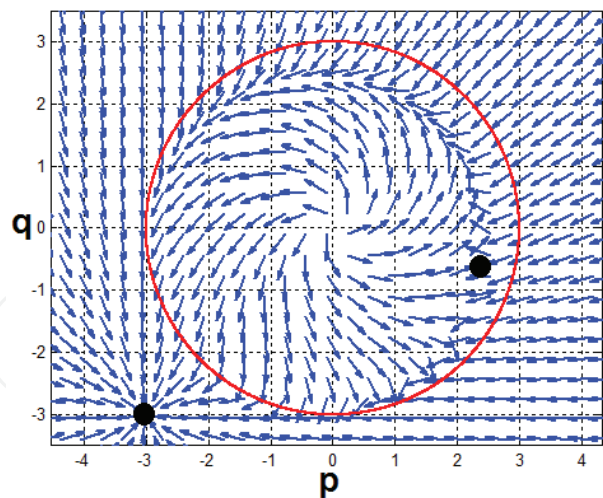
Fig. 7. Phase plane of coordinates (p, q) with RVF

of collision appears. It is interesting to point out that the vector field (30) is not obtained as the gradient of any scalar function. Using the previous RVF, we define a control law given by

$$u_i = -\frac{1}{2} k \frac{\partial \gamma_i}{\partial z_i} + \sum_{j \in M_i} \psi_{ij}, i = 1, ..., n \tag{31}$$

Note that the control law uses directly the RVF and not any partial derivative of a RPF. Therefore, the function V_{ij} is simpler to design than a standard RPF since it is only required that V_{ij} and not necessarily $\frac{\partial V_{ij}}{\partial z_i}$ vanish when $\beta_{ij} = d^2$.

In this new situation, the phase plane, for the case of two robots, of the variables (p, q) defined in (21) is shown in Fig. 7. Note that the closed-loop system (1)-(31) still displays the problem of undesired equilibria. However, the RVF provides best performance of the agent's trajectories than the classical RPF's. This comparison will be addressed by numerical simulations in Section 7.

7. Extension to the case of unicycles

In this section, the control laws developed so far are extended to the case of unicycle-type robot formations. The kinematic model of each agent or robot R_i, as shown in Fig. 8 is given by

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_i \\ w_i \end{bmatrix}, \ i = 1, ..., n \tag{32}$$

where u_i is the linear velocity of the midpoint of the wheels axis and w_i is the angular velocity of the robot. A celebrated result by (Brockett (1983)) states that the dynamical system (32) can not be stabilized by continuous and time-invariant control law. Because of this restriction, we will analyze the dynamics of the coordinates $\alpha_i = (p_i, q_i)$ shown in Fig. 8 instead coordinates (x_i, y_i). The coordinates α_i are given by

$$\alpha_i = \begin{bmatrix} p_i \\ q_i \end{bmatrix} = \begin{bmatrix} x_i + \ell \cos(\theta_i) \\ y_i + \ell \sin(\theta_i) \end{bmatrix}, i = 1, ..., n. \tag{33}$$
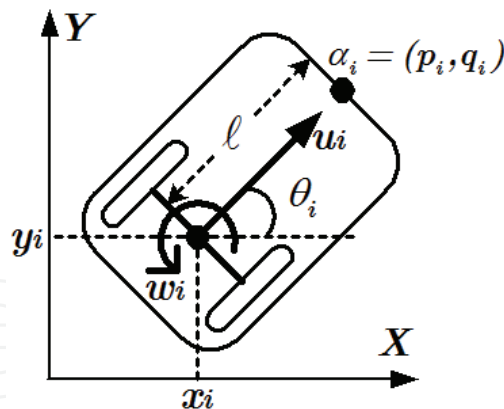
Fig. 8. Kinematic model of unicycles

The dynamics of (33) is obtained as

$$\dot{\alpha}_i = A_i(\theta_i)[u_i, w_i]^T, \quad A_i(\theta_i) = \begin{bmatrix} \cos\theta_i & -\ell\sin\theta_i \\ \sin\theta_i & \ell\cos\theta_i \end{bmatrix}, i = 1, ..., n \tag{34}$$

where the so-called decoupling matrix $A_i(\theta_i)$ is non-singular. The idea of controlling coordinates α_i instead of the center of the wheels axis is frequently found in the mobile robot literature in order to avoid singularities in the control law.
Following the formation control strategy with collisions avoidance presented on Section 4, the desired position of R_i, related to the coordinates α_i, is given by

$$\alpha_i^* = \frac{1}{g_i} \sum_{j \in N_i} (\alpha_j + c_{ij}), i = 1, ..., n \tag{35}$$

Then, a formation control strategy with non-collision, similar to (11) is defined as

$$\begin{bmatrix} v_i \\ w_i \end{bmatrix} = A_i^{-1}(\theta_i) \left(-\frac{1}{2} k \frac{\partial \tilde{\gamma}_i}{\partial \alpha_i} - \sum_{j \in M_i} \frac{\partial \tilde{V}_{ij}}{\partial \alpha_i} \right), i = 1, ..., n \tag{36}$$

where $\tilde{\gamma}_i = \sum_{j \in N_i} \tilde{\gamma}_{ij}$ with $\tilde{\gamma}_{ij}$ similar to the case of point robots but related to coordinates α_i and $\tilde{V}_{ij}$ is a RPF also related to α_i. The dynamics of the coordinates α_i for the closed-loop system (32)-(36) is given by

$$\dot{\alpha}_i = -\frac{1}{2} k \frac{\partial \tilde{\gamma}_i}{\partial \alpha_i} - \sum_{j \in M_i} \frac{\partial \tilde{V}_{ij}}{\partial \alpha_i}, i = 1, ...n \tag{37}$$

It is clear that the dynamics of coordinates α_i is the same than the case of point robots. Thus, the analysis of convergence and non-collision is reduced to the case of omnidirectional robots presented before.

Remark 1. *The control law (36) steers the coordinates α_i to a desired position. However, the angles θ_i remain uncontrolled. These angles do not converge to any specific value. Thus, the control law (36) is to be considered as a formation control without orientation.*

Fig. 9 shows a numerical simulation of the agents' trajectories and their relative distances of the closed-loop system (32)-(36) using the cyclic pursuit FG and the Khatib's RFP. In Fig. 9a, the continuous line represents the trajectories of coordinates (p_i, q_i) and the dashed line represents coordinates (x_i, y_i) of every robot. The value d_{ij} is the actual distance between agents i and j. The design parameters and the initial conditions are given by $k = 1$, $\eta = 2$, $d = 0.16$, $\ell = 0.038$, $[x_{10}, y_{10}, \theta_{10}] = [0.05, -0.05, -0.83]$, $[x_{20}, y_{20}, \theta_{20}] = [0.2, 0.14, 0.41]$, $[x_{30}, y_{30}, \theta_{30}] = [-0.05, 0.15, 2.2]$ and the desired formation is an equilateral triangle with side equal to 0.2. The agents converge to the desired formation avoiding collisions.
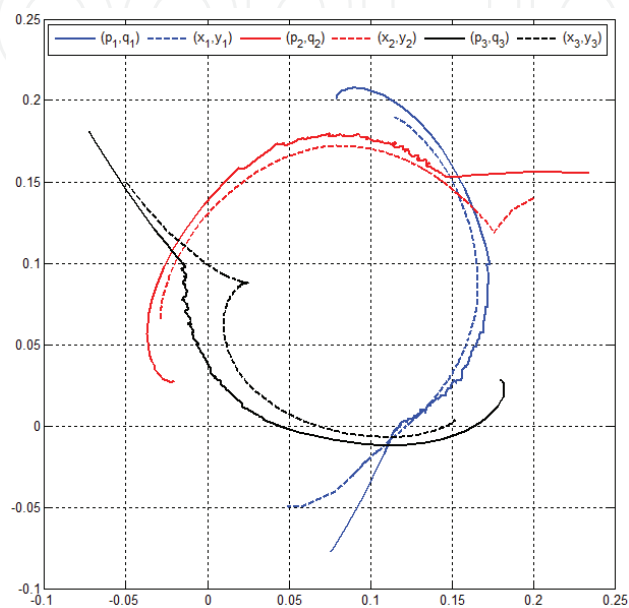
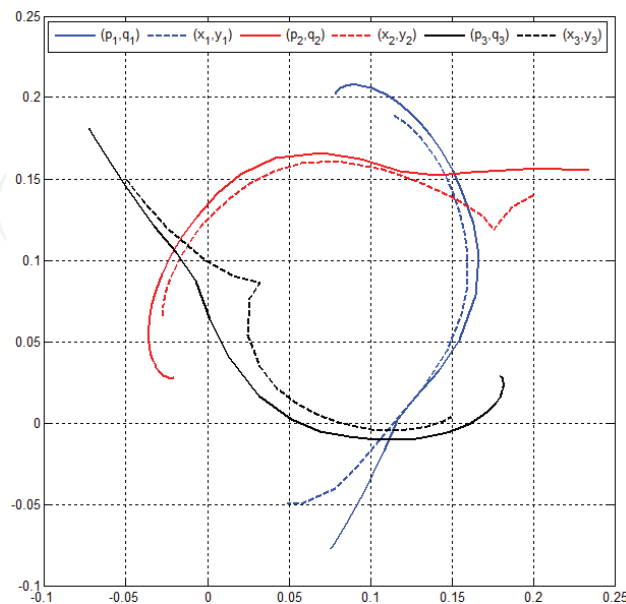(a) Trajectories of the robots in plane
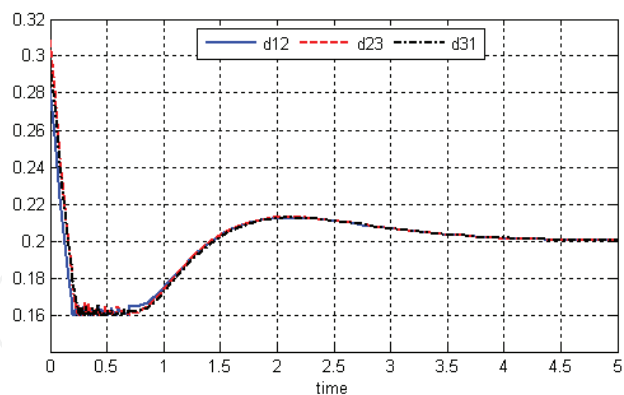
(b) Inter-robot distances

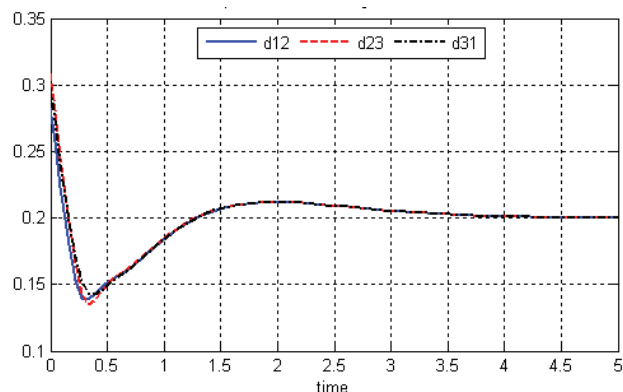Fig. 9. Formation control with non-collision using the Khatib's RPF.

On the other hand, extending the non-collision strategy of the RVF based on an unstable scaled focus for the case of unicycles, we obtain

$$\begin{bmatrix} v_i \\ w_i \end{bmatrix} = A_i^{-1}(\theta_i) \left(-\frac{1}{2} k \frac{\partial \tilde{\gamma}_i}{\partial \alpha_i} - \sum_{j \in M_i} \tilde{\psi}_{ij} \right) \qquad (38)$$

where $\tilde{\psi}_{ij}$ is similar to ψ_{ji} shown in (31) but related to coordinates α_i. Fig. 10 shows a numerical simulation for the closed-loop system (32)-(38) with the same parameters, desired formation and initial conditions than the previous case.

(a) Trajectories of the robots in plane

(b) Inter-robot distances

Fig. 10. Formation control with non-collision using the RVF.

To establish a comparison between the non-collision strategies, define an error performance index for coordinates α_i as follows:

$$J(e) = \frac{1}{t_f} \int_0^{t_f} \left(\sqrt{e_1^2 + e_2^2 + e_3^2} \right) dt \tag{39}$$

where $e_i = \|\alpha_i - \alpha_i^*\|$. Let $t_f = 5$, we obtain $J(e) = 0.1386$ for the case of the Khatib's RPF and $J(e) = 0.1173$ for the RVF strategy. Clearly, the the latter case presents the best results, which is reflected on less oscillations on the agents' trajectories while avoiding collisions.

8. Experimental work

This section presents some experiments of formation control with collision avoidance in an experimental setup consisting in three unicycle-type robots manufactured by Yujin (model: YSR-A) and a computer vision system composed by an UNIQ digital video camera (model: UF1000-CL) connected to an ARVOO video processor (model: Leonardo CL). The vision system captures and processes the position of two white circle marks placed on every robot (the marks represent the position of (x_i, y_i) and α_i) at 100 Hz rate. The position and orientation of each robot are obtained using this information . The images are processed in a Pentium4-based PC where the control actions u_i and w_i are also transformed into the desired angular velocities for the robot wheels using the parameters $\ell = 2.8$cm, $r = 2.2$cm and $L = 7.12$cm where r is the radius of the wheels and L is the distance between the two wheels. These commands are sent by a RF module to every robot.

Fig. 11 shows an experiment with Kathib's RPF strategy with $k = 0.2$, $\eta = 0.4$ and $d = 0.16$. The initial conditions (in meters and radians) are given by $[x_{10}, y_{10}, \theta_{10}] = [-0.0064, -0.2563, 1.5923]$, $[x_{20}, y_{20}, \theta_{20}] = [0.1813, 0.2136, -0.3955]$ and $[x_{30}, y_{30}, \theta_{30}] = [-0.2374, 0.1566, -1.1659]$. The desired formation is an equilateral triangle with side equal to 0.2.

Fig. 12 shows a second experiment with the RVF strategy using $k = 0.2$, $\eta = 1$ and $d = 0.16$. The desired formation is the same as in previous case. The initial conditions are given by $[x_{10}, y_{10}, \theta_{10}] = [0.0105, -0.2296, -0.8327]$, $[x_{20}, y_{20}, \theta_{20}] = [0.2047, 0.1445, 0.4030]$ and $[x_{30}, y_{30}, \theta_{30}] = [-0.2236, 0.1723, 2.1863]$. The simulation results are dashed lines whereas experimental results are continuous lines.

In both experiments, the control signals were normalized to

$$[\bar{v}_i, \bar{w}_i]^T = \frac{\mu}{\sqrt{\|F_i\|^2 + \varepsilon}} A_i^{-1}(\theta_i) F_i, i = 1, 2, 3 \tag{40}$$

where $\mu = 0.1$, $\varepsilon = 0.0001$, $F_i = -\frac{1}{2}k\frac{\partial \tilde{\gamma}_i}{\partial \alpha_i} - \sum_{j \in M_i} \frac{\partial \tilde{V}_{ij}}{\partial \alpha_i}$ for the first experiment with Kathib's RPF and $F_i = -\frac{1}{2}k\frac{\partial \tilde{\gamma}_i}{\partial \alpha_i} - \sum_{j \in M_i} \tilde{\psi}_{ij}$ for the second experiment with the RVF strategy. The normalization has two proposes. Firstly, to avoid actuator saturation for large values of $\|\alpha_i - \alpha_i^*\|$. Secondly, to compensate the adverse effects of friction and actuators' dead zone. We observe that the inter-agent distances converge to the desired value. However, the motion of coordinates α_i and (x_i, y_i) displays best performance in the case of the RVF strategy. Finally, fig. 13 shows the posture of the robots at final time recorded by the vision system in the second experiment. We observe that the front mark of every robot (coordinates α_i) converge to the desired formation.

9. Conclusion

Convergence to the desired formation and collision avoidance are the most important requirements on a formation control strategy. The analysis is complex because the control laws are decentralized considering the closed-loop behavior for any number of robots. Formation graphs are useful to describe the possible interaction between robots and provide mathematical tools for the analysis of the system. Although decentralized control strategies based on Artificial Potential Fields can be easily implemented, the complexity of the calculation of undesired equilibria remains an open problem.
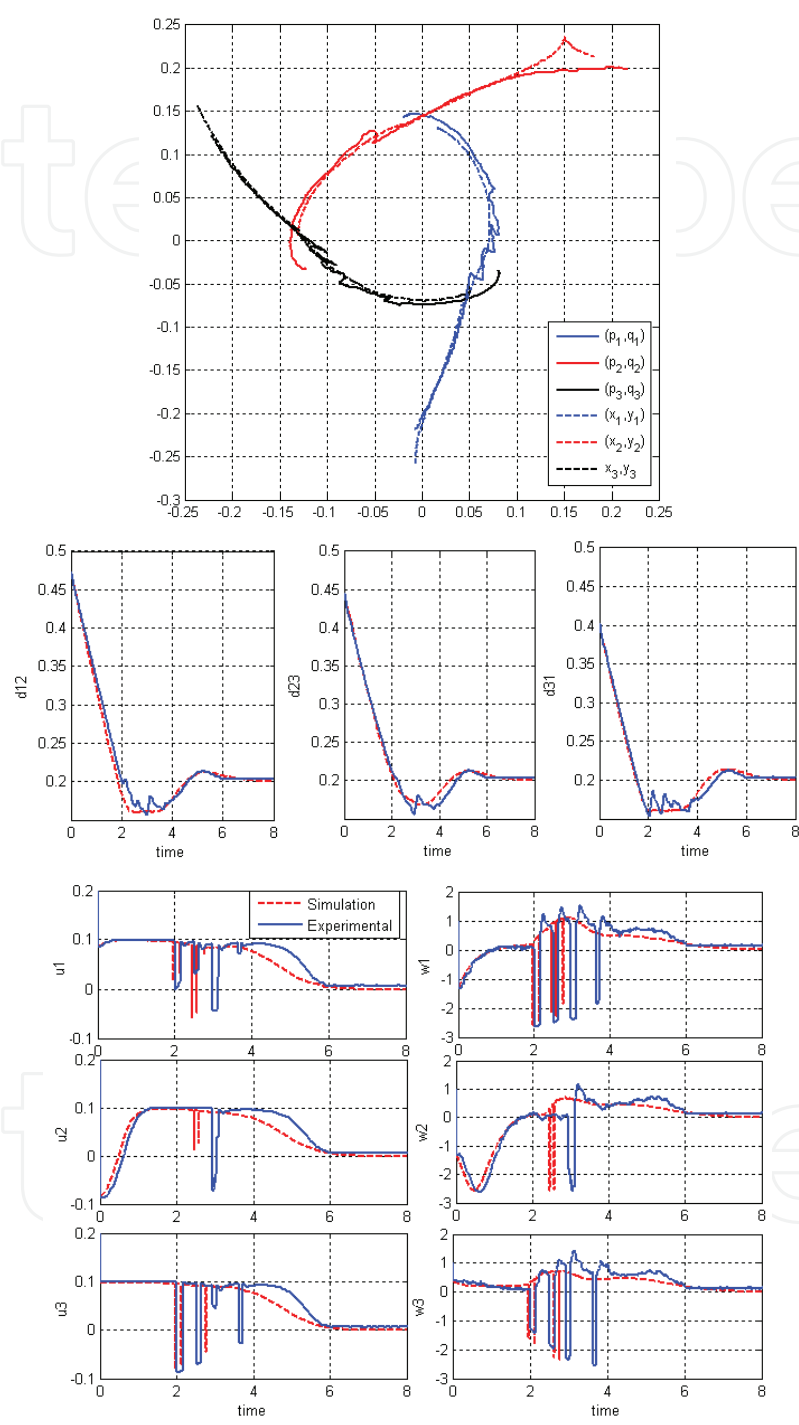
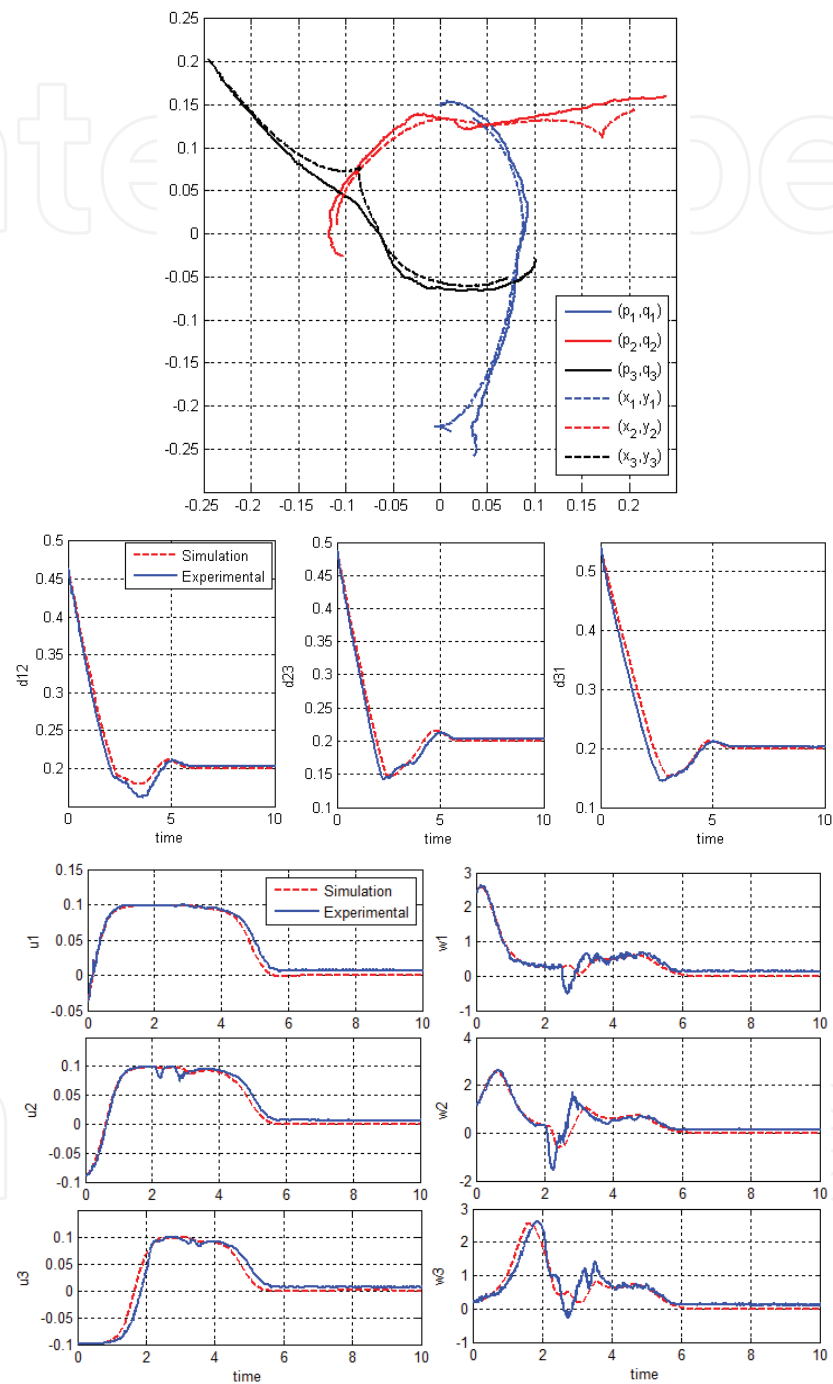Fig. 11. Experiment 1 using the standard RPF of Khatib

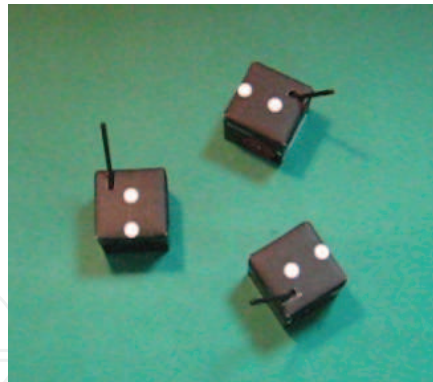Fig. 12. Experiment 2 using the RVF strategy

Fig. 13. Final positions of robots in the experiment 2

This chapter presents some alternatives to ensure convergence, modifying the standard design of potential functions. Also, we contribute to the state of art of formation control with the analysis of time invariance of the centroid of positions. This property is interesting because the dynamics of the team behavior remains centered on the position of the centroid, although every agent obeys a decentralized control strategy. Another contribution is a novel non-collision strategy based on RVF's instead of the repulsive forces from the negative gradient of a RPF. Numerical simulations and real-time experiments for the case of three unicycle-like robots show a better performance of the proposed strategies than the standard methodology.

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