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Propagation of Electron Waves in Monolayer Graphene and Optical Simulations with Negative-Zero-Positive Index Metamaterials

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1. Introduction

The analogies between phenomena occurring in two different physical systems open a route to find new effects or to translate solution on techniques or devices, and quite often help to understand both systems better (Dragoman & Dragoman, 2004). In particular, electronic analogues of many optical behaviors such as reflection, refraction (Gaylord & Brennan, 1989), focusing (Sivan et al., 1990; Spector et al., 1990; van Houten et al., 1988), collimation (Molenkamp et al., 1990), and interference (Ji et al., 2003; Yacoby et al., 1994) have been achieved in two-dimensional electron gas (2DEG) enabling the systems as a basic platform to study foundation problems in quantum mechanics as well as quantum information processing. The close relation between optics and electronics results from the fact that the electrons act as wave due to the ballistic transport properties of a highly mobility 2DEG created in semiconductor heterostructures (Palevski et al., 1989). As a result, there is a growing interest in the design and development of devices based on electron wave propagation, which has given rise to a research field described as electron wave optics (Datta, 1996; Gaylord et al., 1991).

Over the past six years, monolayer graphene has attracted much attention (Beenakker, 2008; Castro Neto et al., 2009) since the graphitic sheet of one-atom thickness has been fabricated experimentally by A. K. Geim *et al.* in 2004 (Novoselov et al., 2004). The valence electron dynamics in such a truly two-dimensional (2D) material is governed by a massless Dirac equation. Thus graphene has many unique electronic and transport properties (Beenakker, 2008; Castro Neto et al., 2009), including half-integer and unconventional quantum Hall effect (Zhang et al., 2005), observation of minimum conductivity (Novoselov et al., 2005), and Klein tunneling (Katsnelson et al., 2006).

The great progress on graphene has brought the analogy between ballistic electrons and light propagations to a new level, and has lead to the Dirac electron wave optics. In the regard, various electron optics phenomena, such as focusing (Cheianov et al., 2007), collimation (Park et al., 2008), subwavelength optics (Darancet et al., 2009), Bragg reflection (Ghosh et

al., 2009), and Goos-Hänchen effect (GH) (Beenakker et al., 2009; Zhao et al., 2010) provide different ways to control the electronic wave propagations in different graphene-based electron devices. One of the most important discoveries is that Cheianov et al. (Cheianov et al., 2007) have recently demonstrated the negative refraction at the n-p graphene interface when the incidence angle is less than the critical angle for total reflection and proposed the electron focusing effect as a "perfect lens" in metamaterial. However, the propagations of electron waves become quite different, when the evanescent waves are considered in total reflection. Zhao and Yelin (Zhao et al., 2010) have once studied that the electron beam will experience the so-called GH shift at the graphene interface, when the incidence angle is larger than the critical angle. C. W. J. Beenakker et al. (Beenakker et al., 2009) have further found that the GH effect at a *n*-*p* interface in graphene doubles the degeneracy of the lowest propagating mode. In addition, following the seminal paper on the negative refraction and electron focusing, Garcia-Pomar et al. (Pomar et al., 2008) have also proposed an n-p-n graphene transistor to realize the valley beam splitter or collimation. So the manipulation of electron beam propagation can benefit from all these optical-like phenomena by applying an external electric or magnetic field to alter the flowing of electrons (Wang and Liu, 2010).

From a somewhat different but relevant perspective, a growing interest has appeared regarding the simulations of Dirac equation and relativistic effects by many controllable physical setups, for instance, cold atom in optical lattice (Zhu et al., 2007), spin-1/2 particles in single trapped ion (Lamata et al., 2007), ultrarelativistic atom with internal energy levels in a tripod configuration (Juzeliunas et al., 2008), and light in fiber Bragg gratings (Longhi, 2010). It is worthwhile to point out that R. Gerritsma et al. (Gerritsma et al., 2010) have lately implemented for the first time a quantum optical simulation of a tunable relativistic quantum mechanical system. In the optical field, the Dirac point (DP) in photonic crystals for the Bloch states is also of significance and interest, based on the similarity of the photonic bands of the two-dimensional photonic crystals (2DPCs) with the electronic bands of solids. Consequently, the conical diffraction (Peleg et al., 2007), "pseudodiffusive" scaling (Sepkhanov et al., 2007), and photon's Zitterbewegung (Zhang, 2008) have been found near the DP in photonic crystals. As we know that, compared to solids, atomic and quantum optical systems, pure optical systems offer clean and easy controlled way to test theoretical predictions. Stimulated by the realization of the DP with a double-cone structure for optical field in the negative-zero-positive index metamaterial (NZPIM) (Wang et al., 2009, a;b), the pseudodiffusive property, Zitterbewegung effect, Bragg-like reflection, and the unique GH effect (Chen et al., 2009, b) inside the NZPIM slab are extensively investigated. Taking into account the close relation between Klein paradox and negative refraction (Guney & Meyer, 2009), the novel phenomena in the NZPIM will definitely motivate the further work to simulate many exotic phenomena in graphene with relatively simple optical system.

This chapter presents a review on the propagation of electron waves in monolayer graphene and optical simulations with NZPIM. The chapter is organized as follow. In Sec. 2, the transmission gap (corresponding to Bragg-like reflection) and GH effect are discussed in single graphene barrier. Similar to the transmission gap, the zero-averaged wavenumber gap associated with new DP in monlayer graphene superlattice is also studied in Sec. 3 by the transfer matrix method. In Sec. 4, the optical simulations of the relevant phenomena mentioned above are made by the NZPIM with the DP. Finally, we make brief conclusion in Sec. 5.

2. Transmission gap, Bragg-like reflection and Goos-Hänchen effect in monolayer graphene barrier

As we know, one of the most interesting phenomena is the perfect transmission, in particular, for normal incidence, through arbitrarily high and wide graphene barriers, referred to as Klein tunneling (Katsnelson et al., 2006). Therefore, such important property may lead to the design of various graphene-based device. Until recently, the transport properties of massless Dirac fermions, including Klein tunneling and perfect transmission, have been extensively studied in single graphene barriers (Katsnelson et al., 2006), n-p-n junctions (Cheianov et al., 2007), graphene-based double barriers (Perira et al., 2007) and even graphene superlattice (Bai et al., 2007; Barbier et al., 2008; 2010; Bliokh et al., 2009). In the meanwhile, inhomogeneous magnetic fields on the nanometer scale have been suggested to circumvent the Klein tunneling and produce confined graphene-based structures (Anna et al., 2009; Martino et al., 2007). It was found that the angular range of the transmission through monolayer and bilayer graphene with magnetic barrier structures can be efficiently controlled and gives the possibility to construct a direction-dependent wave vector filter (Masir et al., 2008, a;b).

In the following section, we shall firstly investigate the transmission properties of Dirac-like electron waves in single monolayer graphene barrier, when the ballistic electrons are obliquely incident on the monolayer graphene barrier. It is shown that the transmission at non-zero incidence angle has a gap, which can be considered as Bragg-like phenomenon. This controllable transmission gap is quite different from the perfectly transparent for the angles close to the normal incidence (Katsnelson et al., 2006) and does result from the evanescent waves in two cases of classical motion and Klein tunneling due to the effect of parallel wave vector. Based on the tunable transmission gap (Chen & Tao, 2009), GH shifts for Dirac fermions in transmission through a two-dimensional (2D) monolayer graphene barrier can be enhanced by transmission resonances, and can also be negative as well as positive. So these lateral shifts, associated with the transmission gap and Bragg-like reflection, lead to the significant difference from the conventional GH shift in total reflection.

2.1 Transmission and reflection in the graphene barrier

We consider the incident electron wave propagates with Fermi energy *E* at angle ϕ with respective to the *x* axis through a single 2D graphene barrier, as shown in Fig. 1, where V_0 and *d* are the height and width of the potential barrier, respectively. From the view point of its electronic properties, graphene is a two-dimensional zero-gap semiconductor with the linear dispersion relation, $E = \hbar k_F v_F$, thus the low-energy quasi-particles are formally described by the Dirac-like Hamiltonian (Katsnelson et al., 2006), $\hat{H}_0 = -i\hbar v_F \sigma \nabla$, $v_F \approx 10^6 m \cdot s^{-1}$ is the Fermi velocity, k_F is the Fermi wave vector, and $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices. So the wave function of the incident electrons is assumed to be

$$\Psi_{in}(x,y) = \begin{pmatrix} 1\\ se^{i\phi} \end{pmatrix} e^{i(k_x x + k_y y)},\tag{1}$$

the wave function of the transmitted electrons can be expressed as follows,

$$\Psi_t(x,y) = t \begin{pmatrix} 1\\ se^{i\phi} \end{pmatrix} e^{i(k_x x + k_y y)},$$
(2)

where s = sgn(E), $k_x = k_F \cos \phi$ and $k_y = k_F \sin \phi$ are the perpendicular and parallel wave vector components outside the barrier. According to the boundary conditions, transmission coefficient is determined by



$$t \equiv \frac{1}{f}e^{i\varphi} = \frac{1}{\cos(q_x d) - i(ss'\sec\phi\sec\theta - \tan\phi\tan\theta)\sin(q_x d)},$$
(3)

where $s' = sgn(E - V_0)$, $q_x = (k_F'^2 - k_y^2)^{1/2}$, $k_F' = |E - V_0|/\hbar v_F$, and $\theta = \arctan(k_y/q_x)$. The critical angle ϕ_c for total reflection can be defined by

$$\phi_c = \arcsin\left(\frac{|V_0 - E|}{E}\right),\tag{4}$$

so that when $\phi > \phi_c$, the wave function in the propagating case becomes evanescent wave by replacing q_x by $i\kappa$, where $\kappa = (k_y^2 - k_F'^2)^{1/2}$. Next, we will discuss the transmission in two cases of Klein tunneling ($E < V_0$) and classical motion ($E > V_0$) to show the unique transmission properties of electron waves in monolayer graphene barrier.



Fig. 2. (Color online) Transmission gap as the function of incident energy *E*, where d = 80 nm, $V_0 = 120$ meV, solid and dashed curves correspond to $\phi = 25^\circ$ and $\phi = 15^\circ$, respectively. *Case 1*: Klein tunneling (*ss'* = -1). The transmission probability *T* can be given by Eq. (3),

$$(k_{\mu}^{2} + k_{F}k_{F}^{\prime})^{2}$$
 $(k_{\mu}^{2} + k_{F}k_{F}^{\prime})^{2}$

$$T \equiv |t|^{2} = \left[\cos^{2}(q_{x}d) + \frac{(k_{y}^{2} + k_{F}k_{F})^{2}}{k_{x}^{2}q_{x}^{2}}\sin^{2}(q_{x}d)\right]^{-1}.$$
(5)

It is clear that the angular-dependent transmission probability *T* becomes equal to 1 under the resonance condition, $q_x d = N\pi$, $(N = 0, \pm 1, ...)$. However, when the angle of incidence satisfies $\phi > \phi'_c$, where the critical angle (4) tends to $\phi'_c = \sin^{-1} (V_0/E - 1)$, with the

width of transmission gap is given by

necessary condition $E < V_0 < 2E$, the tunneling of the Dirac-like electron through the monolayer graphene barrier occurs, thus the transmission probability damps exponentially in the following form: $T \approx 4k_x^2 \kappa^2 e^{-2\kappa d} / [k_x^2 \kappa^2 + (k_y^2 + k_F k_F')^2]$.

Case 2: Classical motion (ss' = 1). The transmission probability can be rewritten as

$$T = \left[\cos^2(q_x d) + \frac{(k_y^2 - k_F k_F')^2}{k_x^2 q_x^2} \sin^2(q_x d)\right]^{-1}.$$
 (6)

Similarly, when the incidence angle ϕ is less than the critical angle, $\phi_c'' = \sin^{-1} (1 - V_0/E)$ given by the critital angle (4), the transmission probability *T* in this case depends periodically on the width *d* of barrier. On the contrary, when $\phi > \phi_c''$, wave vector q_x becomes imaginary number, thus the transmission probability tends to $T \approx 4k_x^2\kappa^2 e^{-2\kappa d}/[k_x^2\kappa^2 + (k_y^2 - k_Fk_F')^2]$. Figure 2 shows that the transmission as the function of incidence energy *E* has a gap, where d = 80nm, $V_0 = 120meV$, solid and dashed curves correspond to $\phi = 25^\circ$ and $\phi = 10^\circ$. The energy region of the transmission gap, $V_0 - \hbar v_F k_y < E < V_0 + \hbar v_F k_y$, since $q_x^2 < 0$. Thus the

$$\Delta E = 2\hbar v_F k_{\nu},\tag{7}$$

which depends strongly on the parallel wave vector k_y , and can be controlled by the incidence angle, as shown in Fig. 2, where the center of transmission gap is $E = V_0$. It is clear that the transmission gap becomes narrower with the decrease of the incidence angle, and even vanishes at normal incidence. Since the transmission gap results from the evanescent wave in two cases of Klein tunneling and classical motion, the incidence angle, the height and width of potential barrier play important roles in the transmission gap. Especially, the transmission gap will become deeper with increasing the barrier width, due to the decrease of the the decay factor $\exp(-2\kappa d)$ in the evanescent case. Here we shall point out that the transmission gap can be tunable by changing the gate-voltage, which can realize the electron wave energy filter by the shift of transmission gap.

The tunable transmission gap implies Bragg-like reflection, which usually happens only in the single potential barrier, instead of multiple barriers. It turns out that Dirac-like electrons can perfectly reflected by the graphene barrier in the region of energy. This perfect reflection is similar to but different from the Bragg reflection in magnetic barrier in graphene (Ghosh et al., 2009). Actually, this Bragg-like phenomenon discussed here can also be applied to select electron energy by the reflection window.

Next, we will study the total conductance (G) and Fano factor (F) in the single graphene barrier. Using the Büttiker formula (Datta, 1996), the total conductance of the system at zero temperature is given by

$$G = G_0 \int_{-\pi/2}^{\pi/2} T(E_F, E_F \sin \phi) \cos \phi d\phi, \qquad (8)$$

with the Fermi energy E_F and the units of conductance $G_0 = (2e^2/\hbar)(\ell/\pi\hbar v_F)$, where ℓ is the length of the structure along the *y* direction. Furthermore, we can also study the Fano factor (F) (Tworzydlo et al., 2006), which is given by

$$F = \frac{\int_{-\pi/2}^{\pi/2} T(1-T) \cos \phi d\phi}{\int_{-\pi/2}^{\pi/2} T \cos \phi d\phi}.$$
(9)



Fig. 3. (Color online) Conductance G/G_0 and Fano factor F as the function of incident energy, where d = 80 nm (blue solid line), d = 50 nm (green dashed line), d = 30 nm (red dotted line), and other parameters are the same as in Fig. 2.

Figure 3 illustrates the dependence of the conductance and Fano factor on the incidence energy *E*, where the physical parameters are the same as in Fig. 2. Obviously, it is shown that the visible kinks of the conductance due to transmission resonance are closely related to the quasi-bound state. More importantly, all conductance curves indicate a pronounced forbidden region, which is the region of almost zero conductance corresponding to the transmission gap. In addition, the behavior of Fano factor resembles Bragg reflection. Also the interesting point is that Fano factor will approach the maximum value 1/3 in the transmission gap. In a word, the transmission gap has great effect on the electron transport in monolayer graphene, but also provides a novel phenomenon to design various electron wave devices.

2.2 Quantum Goos-Hänchen shift

As mentioned above, Cheianov *et al.* (Cheianov et al., 2007) have found the negative refraction and electron focusing in graphene p-n junction, when the incidence angle is less than the critical angle for total reflection. However, when the incidence angle is larger than the critical angle, the quantum GH effect in total reflection has been investigated (Beenakker et al., 2009; Zhao et al., 2010) at a p-n interface in graphene, which is analogous to the lateral shift of the light beam totally reflected from a dielectric interface (Goos & Hänchen, 1947; 1949). Here we would like to discuss the negative and positive GH shifts for Dirac fermions in transmission through a 2D monolayer graphene barrier, based on the tunable transmission gap. To calculate the lateral shifts, we consider the incidence beam as

 $\Psi_{in}(x,y) = \int_{\infty}^{\infty} dk_y A(k_y) e^{i(k_x x + k_y y)} \begin{pmatrix} 1 \\ s e^{i\phi} \end{pmatrix},$

where the angular-spectrum distribution $A(k_y)$ is sharpen distribution around k_{y0} , which can be simply assumed to be Gaussian function $A(k_y) = w_y \exp[-(w_y^2/2)(k_y - k_{y0})^2]$ with $w_y = w \sec \phi_0$, and the half width of the incident beam at waist w. Accordingly, the transmitted beam can also expressed as

$$\Psi_t(x,y) = \int_{\infty}^{\infty} dk_y t(k_y) A(k_y) e^{i(k_x x + k_y y)} \begin{pmatrix} 1\\ s e^{i\phi} \end{pmatrix},$$
(11)

(10)

For the well-collimated beam, that is, $\delta \phi = \lambda_f / (\pi w) \ll 1$, the lateral shifts can be defined, according to the stationary phase method (Beenakker et al., 2009; Zhao et al., 2010), as

$$s_t = -\frac{\partial \varphi}{\partial k_{y0}},\tag{12}$$

where the subscript 0 denotes the values taken at $k_y = k_{y0}$ corresponding to the central incidence angle $\phi = \phi_0$. It is noted that the lateral shifts of up and down components are both the same as s_t in transmission at the interface x = d, whereas on the total reflection the shifts of up and down components are different due to the phase in the spinor wave function, and the GH shift is thus defined as their average value in the literature (Beenakker et al., 2009).



Fig. 4. (Color online) Dependence of the lateral shifts in the propagating case on the barrier's width *d*, where $\phi_0 = 20^\circ$, $V_0 = 120$ meV, *d* is re-scaled to $q_{x0}d$, E = 80 meV (solid line), and E = 200 meV (dashed line).

Figure 4 shows the most impressive behavior that in the propagating case the lateral shift can be negative for Klein tunneling, $E < V_0$, and also be enhanced by the transmission resonances, whereas for classical motion, $E > V_0$, the shift is always large and positive. On the contrary, when the incidence angle ϕ_0 is larger than the critical angle ϕ_c , the lateral shifts become in the order of Fermi wavelength due to the evanescent wave, which is similar to those in total reflection at a single graphene interface (Beenakker et al., 2009; Zhao et al., 2010). Instead of the enhancement by the transmission resonances shown in Fig. 4, Fig. 5 illustrates that the lateral shifts for Klein tunneling and classical motion saturate respectively to negative and positive constants with increasing the barrier's width in the evanescent case, where (a) $\phi_0 = 40^\circ$ and (b) 70° (which are both larger than the critical angle for total reflection).

Similarly, we shall discuss the properties of the lateral shifts in two cases of Klein tunneling and classical motion.

Case 1: Klein tunneling (ss' = -1). When the incidence angle ϕ_0 is less than the critical angle ϕ_c , where $\phi'_c = \sin^{-1} (V_0/E - 1)$, the lateral shifts are given by

$$s_t = \frac{d \tan \phi_0}{f_0^2} \left\{ \left[2 + \left(\frac{k_0^2}{k_{x0}^2} + \frac{k_0^2}{q_{x0}^2} \right) \right] \frac{\sin(2q_{x0}d)}{2q_{x0}d} - \frac{k_0^2}{q_{x0}^2} \right\},\tag{13}$$

where $k_0 = (k_F k'_F + k_{y0}^2)^{1/2}$. Obviously, lateral shifts obtained above can be positive as well as negative because of $|\sin(2q_{x0}d)/(2q_{x0}d)| \le 1$. So for the thin barrier $d \to 0$, that is, the lateral

shifts can be positive, while the lateral shifts become negative for an enough thick barrier. It is also interesting that the negative lateral shifts can be enhanced by the transmission resonances. The exotic behaviors of negative and positive shifts are analogous to those of the transmitted light beam though a left-handed metamaterial slab (Chen et al., 2009, b).



Fig. 5. (Color online) Dependence of the lateral shifts in the evanescent case on the barrier's width *d*, where (a) $\phi_0 = 40^\circ$ and (b) $\phi_0 = 70^\circ$, *d* is re-scaled to $\kappa_0 d$, E = 80 meV (solid line), and E = 200 meV (dashed line).

On the contrary, when $\phi_0 > \phi'_c$, the lateral shifts become

$$s_t = \frac{d \tan \phi_0}{f_0^2} \left\{ \left[2 + \left(\frac{k_0^2}{\kappa_0^2} - \frac{k_0^2}{k_{x0}^2} \right) \right] \frac{\sinh(2\kappa_0 d)}{2\kappa_0 d} + \frac{k_0^2}{\kappa_0^2} \right\}.$$
 (14)

In the limit of opaque barrier, $\kappa_0 d \rightarrow \infty$, the lateral shifts tend to the constants as follows,

$$s_t = \frac{\tan \phi_0}{\kappa_0} \frac{2k_{x0}^2 \kappa_0^2 - k_0^2 (k_{x0}^2 - \kappa_0^2)}{k_{x0}^2 \kappa_0^2 - k_0^4},\tag{15}$$

which are proportional to $1/\kappa_0$, and imply that the shifts in the evanescent case are in the same order of electron wavelength as GH shifts in a single graphene interface (Beenakker et al., 2009; Zhao et al., 2010). More interestingly, the saturated lateral shifts are negative when the incidence angle satisfies $\phi'_c < \phi_0 < \phi^*$, where the critical angle is defined by $\phi^* = \arcsin \sqrt{\sin \phi'_c}$. But the shifts in this case will become positive when $\phi_0 > \phi^*$. The sign change of the lateral shifts described by Fig. 5 (b) appears at the incidence angle $\phi_0 = \phi^*$, which is similar to the result of the quantum GH effect in graphene, taking the pseudospin degree into account (Beenakker et al., 2009).

Case 2: classical motion (ss' = 1). When the incidence angle is less than the critical angle for total reflection, $\phi_0 < \phi_c''$, the lateral shifts can be written as

$$s_t = \frac{d \tan \phi_0}{f_0^2} \left\{ \left[2 - \left(\frac{k_0'^2}{k_{x0}^2} + \frac{k_0'^2}{q_{x0}^2} \right) \right] \frac{\sin(2q_{x0}d)}{2q_{x0}d} + \frac{k_0'^2}{q_{x0}^2} \right\},\tag{16}$$

where $k'_0 = (k_F k'_F - k^2_{y0})^{1/2}$. Similarly, the lateral shifts for classical motion also depend periodically on the barrier's width, and can be enhanced by the transmission resonances.

However, these lateral shifts in classical motion are always positive. When $\phi_0 > \phi_c''$, the shifts in the evanescent case will become

$$s_t = \frac{d \tan \phi_0}{f_0^2} \left\{ \left[2 - \left(\frac{k_0'^2}{\kappa_0^2} - \frac{k_0'^2}{k_{x0}^2} \right) \right] \frac{\sinh(2\kappa_0 d)}{2\kappa_0 d} - \frac{k_0'^2}{\kappa_0^2} \right\}.$$
 (17)

Then the lateral shifts in the limit, $\kappa_0 d \rightarrow \infty$, are given by

$$s_t = \frac{\tan \phi_0}{\kappa_0} \frac{2k_{x0}^2 \kappa_0^2 + k_0'^2 (k_{x0}^2 - \kappa_0^2)}{k_{x0}^2 \kappa_0^2 - k_0'^4},$$
(18)

which are always positive constants.



Fig. 6. (Color online) The lateral shifts as the function of incident energy *E*, where d = 80 nm, and $V_0 = 120$ meV, $\phi_0 = 25^{\circ}$ (red solid line), $\phi_0 = 20^{\circ}$ (blue dotted line), and 15° (orange dot-dashed line).

Based on the properties in two cases of Klein tunneling and classical motion, the lateral shifts as the function of incidence energy *E* are shown in Fig. 6. It is shown that the lateral shifts are closely related to the transmission gap $\Delta E = 2\hbar k_y v_F$, as descried in last subsection. Figure 6 indicates that the lateral shifts change the sign near the DP $E = V_0$, and can also be enhanced by the transmission resonances near the boundaries of energy gap. Actually, the sign change of the shifts does result from the fact that the DP $E = V_0$ means the transition between Klein tunneling ($E < V_0$) and classical motion ($E > V_0$), which correspond to the negative and positive group velocities, respectively.

Obviously, the lateral shifts considered here are quite different from the GH shifts in total reflection. The lateral shifts can be enhanced by the transmission resonances, thus can be easily modulated by various parameters such as potential heights and incidence angles. Actually, the periodical dependence of negative and positive lateral shifts on the gap provides an efficient way to modulate the spatial position in a fixed graphene barrier, which is useful for the manipulation of electron beam propagation in graphene (Wang and Liu, 2010).

3. Electronic Band gaps and transport properties inside graphene superlattices

Most recently, there have been a number of interesting theoretical investigations on the graphene supperlatices with periodic potential structures, which can be generated by different

methods. As a matter of fact, it is well known that the superlattice are very successful in controlling the electronic structures of many convectional semiconductor material [For review, see Ref. (Tsu, 2005)]. The peculiar properties of localized superlattice states including such as the effect of the Bragg confinement of electron in coupled superlattice and the formation of above-barrier quasi-bound states at the junction of superlattices are of significance for both the fundamental research and various devices (Steslicka et al., 2002). In this section, we will present the result on a new DP which is exactly located at the energy corresponds to the zero-averaged wavenumber inside the one-dimensional (1D) periodic potentials. It is emphasized here that the gap for the zero-averaged wavenumber is quite different from the Bragg gap, which is analogous to the case of the 1DPCs containing left-handed and right-handed materials (Bliokh et al., 2009; Wang & Zhu, 2010, b).

3.1 Transfer Matrix method for mono-layer graphene superlattices

First of all, we will develop the transfer matrix method for such system in this subsection. The Hamiltonian of a low-energy electron moving inside a mono-layer graphene in the presence of the electrostatic potential V(x), which only depends on the coordinate x, is given by

$$\hat{H} = v_F \sigma \cdot \hat{p} + V(x)\hat{I},\tag{19}$$

where $\hat{p} = (p_x, p_y) = (-i\hbar \frac{\partial}{\partial x}, -i\hbar \frac{\partial}{\partial y})$ is the momentum operator with two components, \hat{l} is a 2 × 2 unit matrix. This Hamiltonian acts on a state expressed by a two-component pseudospinor $\Psi = (\tilde{\psi}_A, \tilde{\psi}_B)^T$, where $\tilde{\psi}_A$ and $\tilde{\psi}_B$ are the smooth enveloping functions for two triangular sublattices in graphene. Due to the translation invariance in the *y* direction, the wave functions $\tilde{\psi}_{A,B}(x, y)$ can be written as $\tilde{\psi}_{A,B}(x, y) = \psi_{A,B}(x)e^{ik_y y}$. Therefore, from Eq. (19), we obtain

$$\frac{d\psi_A}{dx} - k_y \psi_A = ik\psi_B,\tag{20}$$

$$\frac{d\psi_B}{dx} + k_y \psi_B = ik\psi_A,\tag{21}$$

where $k = [E - V(x)]/\hbar v_F$ is the wavevector inside the potential V(x). Obviously, when E < V(x), the wavevector inside the barrier is opposite to the direction of the electron's velocity, which is relevant to the negative refraction in the graphene (Cheianov et al., 2007).

In what follows, we assume that the potential V(x) is comprised of periodic potentials of square barriers as shown in Fig. 7. Inside the *j* th potential, $V_j(x)$ is a constant, therefore, from Eqs. (20) and (21), we can obtain

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$$\frac{d^2\psi_A}{dx^2} + (k_j^2 - k_y^2)\psi_A = 0,$$
(22)

$$\frac{d^2\psi_B}{dx^2} + (k_j^2 - k_y^2)\psi_B = 0.$$
(23)

Here the subscript "*j*" denotes the quantities in the *j* th potential. The solutions of Eqs. (22) and (23) are the following forms: $\psi_A(x) = ae^{iq_jx} + be^{-iq_jx}$ and $\psi_B(x) = ce^{iq_jx} + de^{-iq_jx}$, where $q_j = \operatorname{sign}(k_j)\sqrt{k_j^2 - k_y^2}$ is the *x* component of the wavevector inside the *j* th potential V_j for $k_j^2 > k_y^2$, otherwise $q_j = i\sqrt{k_y^2 - k_j^2}$; and *a* (*c*) and *b* (*d*) are the amplitudes of the forward and backward propagating spinor components. Following the calculations in the literature (Wang



Fig. 7. (Color online) (a) Schematic representation of the finite periodic potentials of square barriers in x - y plane. Grey regions denote the electrodes to apply the periodic potentials on the graphene, where θ_0 (θ_e) denotes the incidence (exit) angles of the carriers passing through the graphene superlattice. In the inset, θ_A (θ_B) denotes the angles of the carriers in the barriers *A* and *B* for the cases with $V_B < E < V_A$. (b) The whole profiles of the periodic potentials applied on the monolayer graphene.

& Zhu, 2010 , b), we obtain the relation between $\binom{\psi_A(x_{j-1})}{\psi_B(x_{j-1})}$ and $\binom{\psi_A(x_{j-1}+\Delta x)}{\psi_B(x_{j-1}+\Delta x)}$ can be finally written as:

$$\begin{pmatrix} \psi_A(x_{j-1} + \Delta x) \\ \psi_B(x_{j-1} + \Delta x) \end{pmatrix} = M_j(\Delta x, E, k_y) \begin{pmatrix} \psi_A(x_{j-1}) \\ \psi_B(x_{j-1}) \end{pmatrix},$$
(24)

where the matrix M_i is given by

$$M_{j}(\Delta x, E, k_{y}) = \begin{pmatrix} \frac{\cos(q_{j}\Delta x - \theta_{j})}{\cos \theta_{j}} & i\frac{\sin(q_{j}\Delta x)}{\cos \theta_{j}} \\ i\frac{\sin(q_{j}\Delta x)}{\cos \theta_{j}} & \frac{\cos(q_{j}\Delta x + \theta_{j})}{\cos \theta_{j}} \end{pmatrix}.$$
(25)

It is easily to verify the equality: $det[M_j] = 1$. Here we would like to point out that in the case of $E = V_j$, the transfer materix (25) should be recalculated with the similar method and it is given by

$$M_j(\Delta x, E, k_y) = \begin{pmatrix} \exp(k_y \Delta x) & 0\\ 0 & \exp(-k_y \Delta x) \end{pmatrix}.$$
 (26)

With help of the above equations (25) and (26), we manage to build up the boundary condition in order to obtain the transmission and reflection coefficients. As shown in Fig. 7, we assume that the Dirac fermions of *E* is incident from the region $x \le 0$ at any incidence angle θ_0 . In this region, the wave function is the superposition of the incident and reflected waves, so we have the following equation:

$$\begin{pmatrix} \psi_A(x_e)\\ \psi_B(x_e) \end{pmatrix} = \mathbf{X} \begin{pmatrix} \psi_A(0)\\ \psi_B(0) \end{pmatrix},$$
(27)

with

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \sum_{j=1}^{N} M_j(w_j, E, k_y).$$
(28)

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After some algebraic calculations, we find the reflection and transmission coefficients given by (Wang et al., 2010; Wang & Chen, 2010)

$$r(E,k_y) = \frac{(x_{22}e^{i\theta_0} - x_{11}e^{i\theta_e}) - x_{12}e^{i(\theta_e + \theta_0)} + x_{21}}{(x_{22}e^{-i\theta_0} + x_{11}e^{i\theta_e}) - x_{12}e^{i(\theta_e - \theta_0)} - x_{21}},$$
(29)

$$t(E,k_y) = \frac{2\cos\theta_0}{(x_{22}e^{-i\theta_0} + x_{11}e^{i\theta_e}) - x_{12}e^{i(\theta_e - \theta_0)} - x_{21}},$$
(30)

where we have used the property of det[X] = 1. In the following subsection, we will discuss the properties of the electronic band structure and transmission for the graphene-based periodic potentials of square barriers.

3.2 New Dirac point and zero-averaged wavenumber gap

In this section, we would like to discuss some unique properties of the band structures in the graphene-based periodic-potential systems by using the above transfer matrix method.

3.2.1 Infinite periodic structure

Firstly, let us investigate the electron's bandgap for an infinite periodic structure $(AB)^N$, where the periodic number *N* tends to infinity. The magnitude and width of the potential *A* (*B*) are with the electrostatic potential $V_{A(B)}$ and width $w_{A(B)}$, as shown in Fig. 7. According to the Bloch's theorem, the electronic dispersion at any incident angle follows the relation

$$2\cos[\beta_x\Lambda] = 2\cos[q_Aw_A + q_Bw_B] + \frac{[2\cos(\theta_A - \theta_B) - 2]}{\cos\theta_A\cos\theta_B}\sin(q_Aw_A)\sin(q_Bw_B).$$
(31)

Here $\Lambda = w_A + w_B$ is the length of the unit cell. When the incident energy of the electron satisfies $V_B < E < V_A$, we have $\theta_A < 0$, $q_A < 0$, $\theta_B > 0$, and $q_B > 0$ for the propagating modes. The angles for θ_A and θ_B are schematically shown in the inset of Fig. 7 (a). Then if $q_A w_A = -q_B w_B$, the above equation (31) becomes

$$\cos[\beta_x \Lambda] = 1 + \frac{[1 - \cos(2\theta_A)]}{\cos^2 \theta_A} |\sin(q_A w_A)|^2.$$
(32)

This equation indicates that, when $q_A w_A = -q_B w_B \neq m\pi$ and $\theta_A \neq 0$, there is no real solution for β_x , i.e., existing a bandgap; Additionally, this bandgap will be close at normal incident case ($\theta_A = 0$) from Eq. (32). Therefore, the location of the touching point of the bands is exactly given by $q_A w_A = -q_B w_B$ at $\theta_A = 0$, i. e., $k_A w_A = -k_B w_B$, or $(V_A - E) w_A = (E - V_B) w_B$, which is consistent with the condition $q_A w_A = \pm \Omega_B$ at $\theta_A = 0$ for the location of the touch point of the band in the limiting case of a periodic δ -barrier structures, called as the Kronig-Penney model (Barbier et al., 2009) or the Dirac comb (Arovas et al., 2010) with $w_B \to 0$ with $V_B w_B \to \pm \hbar v_F \Omega_B$ finite (Ω_B is a dimensionless positive quantity). Therefore the above condition for the periodic δ -barrier structures with $w_B \to 0$ and $V_B w_B \to \pm \hbar v_F \Omega_B$ is simplified into $k_A w_A = \pm \Omega_B$.

Figure 8 shows clearly that a band gap opens exactly at energy E = 25meV under the inclined incident angles (i.e., $k_y \neq 0$), where the condition $q_A w_A = -q_B w_B \neq m\pi$ is satisfied. At the case of normal incidence ($\theta_A = \theta_B = 0$), the upper and lower bands linearly touch together and form a new double-cone DP. The location of the new DP is governed by the condition, $k_A w_A = -k_B w_B$, or $(V_A - E) w_A = (E - V_B) w_B$. For the graphene-based periodic-barrier structure with $V_A \neq 0$ and $V_B = 0$, the distribution of the periodic potentials as an example is



Fig. 8. (Color online) Electronic band structures for (a) $w_A = w_B = 20$ nm, (b) $w_A = w_B = 30$ nm and (c) $w_A = w_B = 40$ nm, with $V_A = 50$ meV and $V_B = 0$. The dashed lines denote the "light cones" of the incident electrons, and the dot line denotes the location of the new DP.



Fig. 9. (Color online) Electronic band structures for (a) $w_A/w_B = 1$, (b) $w_A/w_B = 3/2$, and (c) $w_A/w_B = 2$, with $V_A = 50$ meV, $V_B = 0$ and $w_B = 20$ nm in all cases. The dashed lines denote the locations of the new DP.

shown in Fig. 7 (b), and in this case the new DP is exactly located at $E = V_A/(1 + w_B/w_A)$. It turns out that the location of the new DP has nothing to do with the lattice constants; and the position of the opened gap associated with the new DP is not only independent of the lattice constants but also is weakly dependent on the incident angles. Figure 9 further illustrates that the locations of both the new DP and the opened gap are determined by the ratio of w_A/w_B for the cases with the fixed heights V_A of the potentials. From the above discussions, we find that the volume-averaged wavenumber at the energy of the new DP is zero, therefore such an opened gap associated with the new DP may be called as *the zero-averaged wavenumber gap*, which is very similar to that in the 1DPCs containing the left-handed metamaterials (Li et al., 2003), where the so-called zero (volume) averaged index gap is independent of the lattice constant but only dependent on the ratio of the thicknesses of the right- and left-handed materials. For a special case of the graphene-based periodic-potential structures with $V_A = -V_B$ and $w_A = w_B$, the usual DP is located at the energy E = 0 with $k_y = 0$. This result is

the same as the discussion in Ref. (Barbier et al., 2010), and it is also similar to that for the cos-type modulated electric-potential structure in Refs. (Brey & Fertig, 2009; Ho et al., 2009). As a matter of fact, the properties of these novel zero-averaged wavenumber states are similar to those of the zero-energy states in the previous work (Brey & Fertig, 2009), and semimetallic properties are induced due to the effect of the modulated electric potential (Ho et al., 2009).

3.2.2 Finite periodic-barrier systems

Next, we turn to the study on the transmission, conductance and Fano factor in the finite periodic-barrier system. To illustrate the robust transmission gap, we plot the transmission as a function of incident energy in Fig. 9 for the finite structures, for example, $(AB)^{25}$, where (a) different lattice constants with a fixed ratio $w_A/w_B = 1$ and an incident angle $\theta_0 = 10^\circ$ and (b) different incident angles with the fixed lattice parameters $w_A = w_B = 30$ nm. Compared to the novel zero-averaged wavenumber gap, the other bandgap structures, that is, Bragg gap, are not only dependent on the lattice constants but also strongly dependent on different angles. Further calculations can demonstrate that the higher opened gap is destroyed by strong disorder, but the zero-averaged wavenumber gap survives. The robustness of the zero-averaged wavenumber gap comes from the fact that the zero-averaged wavenumber solution remains invariant under disorder. It should be emphasized again that the position of the zero-averaged wavenumber gap near the new DP is insensitive to both the incidence angles and the disorder. Moreover, we can also consider the effect of a defect barrier on the property of the electron's transport inside the zero-averaged wavenumber gap. One can compare the two cases of the defect modes respectively occurring inside the zero-averaged wavenumber gap and the higher bandgaps. As discussed in Ref. (Wang & Zhu, 2010, b), the location of the defect mode inside the zero-averaged wavenumber gap is very weakly dependent on the incident angle but the defect mode in the higher bandgap is strongly sensitive to the incident angle.

Accordingly, the zero-averaged wavenumber gap has great effect on the electronic properties of graphene. Figure 10 shows the electronic conductance and its Fano factor as a function of energy inside the finite graphene-based superlattices. It is clearly seen that the conductance becomes minimal at the new DP of E = 25 meV, and the corresponding Fano factor has a value of 1/3. Near this new DP the conductance is almost decreasing linearly for the energy below the DP and increasing for the energy above the DP. More interestingly, the Fano factor may become larger than 1/3 for the higher gap (Bragg gap) due to the fact that the higher band gap is highly shifted to the higher energy at the non-zero incidence angles. All the results can be applicable to the gapped graphene superlattice, as we discussed in Ref. (Wang & Chen, 2010). From all above discussions, we can draw the conclusion that the physical meaning of the zero-averaged wavenumber gap in the graphene superlattice is very similar to the transmission gap in the monolayer graphene, as mentioned in last section. The transmission properties do result from the Klein tunneling in graphene, which is analogous to the negative refraction in the left-handed metamaterials. These phenomena can be applied to predict novel effect in electron wave optics, thus design various electron waves devices. Following the optical analogy in 1DPCs containing left-handed metamaterials (Wang & Zhu, 2010, a), the one thing that we can do is to investigate the negative and positive GH shifts in the graphene superlattice with the zero-averaged wavenumber gap, where the condition for the extra Dirac points in the graphene-based superlattices is the same as that for the band-crossing effect in 1DPCs consisted of left-handed material and right-handed material. More interesting optical analogies or simulations of the phenomena in graphene will be discussed in the next



Fig. 10. (Color online) Transmission probabilities of the finite periodic-potential structure $(AB)^{25}$ under (a) different lattice constants with a fixed ratio $w_A/w_B = 1$ and $\theta_0 = 10^\circ$ and (b) different incidence angles with the fixed lattice parameters $w_A = w_B = 30$ nm.



Fig. 11. (Color online) The dependence of the electronic conductance and Fano factor on the energy in the graphene superlattices with the periodic-barrier structures $(AB)^{25}$, where the parameters are $w_A = w_B = 20$ nm, $V_A = 50$ meV and $V_B = 0$ meV.

section, according to the link between Klein tunneling in graphene and negative refraction in metamaterial.

4. Optical simulations with negative-zero-positive index metamaterial

Compare to solids, optical systems offer clean and easy controlled way to test theoretical predictions. The experimental test in electronic systems is usually hindered by the difficulty to maintain system homogeneity. Our central study is to emphasize that the DP with the double-cone structure for the light field and its applications can be realized in a homogenous negative-zero-positive index (NZPI) medium, instead of the 2DPCs (Zhang, 2008).

4.1 Dirac point with double cones in optics

It is well known that that Maxwell's equations for light field reduce to Helmholtz equation, which could be written as $\wp E_z(x, y, \omega) + k^2(\omega)E_z(x, y, \omega) = 0$, with a wavenumber k and $\wp = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$ in two dimensional case for an homogenous material when the polarization of the light field is in the *z* direction. In general, the Helmholtz equation is written as the Dirac equation,

$$\begin{bmatrix} 0 & -i(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}) \\ -i(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}) & 0 \end{bmatrix} \Psi = k(\omega)\Psi,$$
(33)

where $\Psi = \begin{pmatrix} E_{z1}(x, y, \omega) \\ E_{z2}(x, y, \omega) \end{pmatrix}$ are the eigenfunctions of the electric fields with the same $k(\omega)$.

It is amazing that when the index of the homogenous optical medium varies from negative to zero and then to positive with frequency (Wang et al., 2009, a), $k(\omega_D) = 0$ and the higher-order terms in the series of $k(\omega) = k(\omega_D) + (\omega - \omega_D)/v_D + \beta(\omega - \omega_D)^2$ can be neglected, where $\omega_D > 0$ is the frequency of the DP (corresponding wavelength is $\lambda_D = 2\pi c/\omega_D$) and group velocity $v = (d\omega/dk)|_{\omega=\omega_D}$. In this case, the homogenous material with linear dispersion

$$k(\omega) = (\omega - \omega_D) / v_D, \tag{34}$$

is called as NZPI media. In this case, it is seen from the transmission spectral at point $L = 40\lambda_D$ that two bands touch each other forming a double-cone structure, see Fig. 12. Thus, the light transport near the DP obeys the massless Dirac equation as follows:

$$\begin{bmatrix} 0 & -i(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}) \\ -i(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}) & 0 \end{bmatrix} \Psi = \left(\frac{\omega - \omega_D}{v_D}\right) \Psi,$$
(35)

In fact, these two eigenfunctions near DP correspond to the two eigenvalues, $\pm |\omega - \omega_D|/v_D$ of $k(\omega)$, which can be respectively realized by NZPIM in the cases of $\omega > \omega_D$ and $\omega < \omega_D$. So the combination of these eigenfunctions for the same $k(\omega)$ will result in the same properties of transmission and reflection as those in 1DPCs, as discussed below. For example, we take the Drude model as the parameters for both the relative permittivity and permeability of the NZPIM (Wang et al., 2009, a;b): $\varepsilon_1(\omega) = 1 - \omega_{ep}^2/(\omega^2 + i\gamma_e\omega)$, $\mu_1(\omega) = 1 - \omega_{mp}^2/(\omega^2 + i\gamma_m\omega)$, where ω_{ep}^2 and ω_{mp}^2 are the electronic and magnetic plasma frequencies, and γ_e and γ_m are the damping rates relating to the absorption of the material. Here we can assume $\gamma_e = \gamma_e = \gamma \ll_{ep}^2, \omega_{mp}^2$. It is important that when $\omega_{ep} = \omega_{mp} = \omega_D$ and $\gamma = 0$ (no loss), then both $\varepsilon_1(\omega_D)$ and $\mu_1(\omega_D)$ may be zero simultaneously. In this case, we find $k(w_D) \approx 0$ and $v_D \simeq c/2$, where *c* is the light speed in vacuum (Wang et al., 2009, b). So far, we find out the condition to have the Dirac dispersion for light field in the homogenous media, and also provide the candidate material to implement it. Clearly, the difference from the DP in 2DPCs is that the eigenfunctions of the light fields in our system are not the Bloch states in periodic structures but the electromagnetic fields itself in the homogenous NZPIM.

4.2 Pseudodiffusive property and Zitterbewegung effect

As discussed in the above subsection, for NZPI media, we have the DP with a double-cone structure. When frequency ω is close to ω_D , owing to $k^2 = k_x^2 + k_y^2 \rightarrow 0$, k_x becomes an imaginary number for real k_y , so that the field along the *x* direction between the interval



Fig. 12. (Color online) (a) Schematic of a NZPIM slab. (b) Distribution of spectral transmittance for a slab with $L = 40\lambda_D$. Blue area denotes the high transmission and white area denotes the prohibition of light. The parameters of the NZPIM are $\omega_{ep} = \omega_{mp} = \omega_D = 10 \times 2\pi$ GHz and $\gamma = 10^{-5}$ GHz.

between 0 and *L* have the following relation: $t(L, k_y) = E(L)/E(0) = \exp(-|k_y|L)$. Then the total energy transmittance is

$$T_{total}(L,k_y) = \int_{-\infty}^{\infty} |t(L,k_y)|^2 dk_y = \frac{1}{L},$$
(36)

which implies that the propagation of light field at ω_D has the 1/L scaling law a main characteristic of diffusion phenomenon, which has been studied for electron transport in graphene (Katsnelson, 2006) and light transport in photonic crystals (Sepkhanov et al., 2007). Consider the 1/L scaling law in a semi-infinite NZPI media, as indicated in Fig. 12 (a). Such a structure may reduce but cannot completely eliminate the non-ideal interface effect x = 0. As a simple proof, ε_1 and μ_1 are assumed to be real. The transmission coefficient at x = Lis $t(k_y, \omega) = \alpha \exp(ik_x L)$, where $\alpha = 2(qq_m)^{1/2}/(q + q_m)$ is determined by the boundary condition, $q_m = k_{x1}/(\mu_1 k_0)$, $q_0 = (k_0^2 - k_y^2)^{1/2}/k_0$ for $k_0 > k_y$ and otherwise $q_0 = i(k_y^2 - k_0^2)^{1/2}/k_0$, where $k_0 = \omega/c$, k_{x1} is the x-component wave number in the metamaterial. Near the DP, we have $t(k_y, \omega) = \alpha \exp(-|k_y|L)$. Usually, α depends on k_y . For the large distance L, this function decreases quickly with increasing k_y . In this sense we assume that α does't depend on k_y (when an ideal interface is considered). Then the total transmittance is $|\alpha|^2/L$, which is different from Eq. (36) only by a value α due to the interface. Therefore the light transport near DP, namely $\omega = \omega_D$, is proportional to 1/L inside the NZPIM.

Figure 12 (b) shows the transmission spectrum at $L = 40\lambda_D$ insider the NZPIM, as shown in Fig. 12 (a). We see that both the upper and lower passbands touch at $\omega_D = 2\pi \times 10$ GHz and nearby the dispersion is linear. With increasing the distance *L*, the touch at ω_D is an ideal point. Note that $t(k_u, \omega)$ at the DP is close to one even if the metamaterial has a small absorption.

To demonstrate the 1/L scaling law near the DP, a characteristic quantity $\xi = S_r L$ is defined to describe the light propagation inside the medium, where $S_r \equiv S(x, y = 0)/S_0$ is a relative energy flow along the x axis, and $S_0 \equiv S(x = 0, y = 0)$ depends on the coupling strength. Instead of the semi-infinite structure, we would like to discuss the propagation of light through a homogenous slab system which is the realistic case. Figure 13 (a) shows the change of the characteristic quantity ξ as a function of the distance inside different slabs with different thicknesses d. It is clearly seen that as d increases, the change of ξ inside the slab approaches to the limit of $d \to \infty$ (i. e., the semi-infinite structure). For the finite thickness d, the value ξ always initially increases and then gradually decays in order to match the second boundary



Fig. 13. (Color online) Change of ξ as a function of the propagating position x inside the finite slab systems with different thicknesses, with the light frequency $\omega = \omega_D = 10$ GHz. The crossed points denote the exit positions x = d for different slabs. (b) Dependence of the energy flows S_0 on the slab thickness d. The incident beam is with $10\lambda_D$ and other parameters are the same as in Fig. 12 except for $\gamma = 10^{-4}$ GHz.

condition at x = d [the crossed point in Fig. 13(a)]. From Fig. 13 (a), it is expected that for a sufficient thick slab, the light energy transport obeys the 1/L scaling law. Meanwhile, from Fig. 13 (b), it is found that for the small d, the coupling strength of the light field inside the finite slab is strong (with large S_0). With the increasing of the slab thickness d, the value S_0 gradually decreases to the limit of $d \rightarrow 0$ (the semi-infinite structure). It indicates that the effect of the second interface on S_0 becomes weaker and weaker, therefore the light energy transport obeys the 1/L scaling law.

We emphasize here that although the light field doesn't obey very well the 1/L scaling law inside the finite homogenous slab with small d, the static property of the light field is still clear demonstrated, as shown in Fig. 14. From Fig. 14, it turns out that the evolutions of the total electric field and the Poynting vector for a narrow Gaussian beam $E(k_y, 0) = W/\sqrt{2} \exp(-W^2 k_y^2/4)$ with a half-width W inside the finite slab system possess the diffusive property. The light fields near the DP have no phase delay and diffuse inside the medium.

In addition, we have also demonstrated the Zitterbewegung effect for optical pulses during the propagation inside the a homogenous NZPIM slab. As we know, Zitterbewgung effect refers to the interference between the positive- and negative-states in the relativistic electron's wave packet. The initial finite pulse is considered to be Gaussian shape both in the transverse spatial domain and in the longitudinal temporal domain. The dynamics of the finite optical



Fig. 14. (Color online) Evolutions of the total electric fields for a narrowed Gaussian beam passing through the finite slab system at $\omega_D = 10$ GHz with $W = 10\lambda_D$, where $\gamma = 0.05$ GHz other parameters are the same as in Fig. 12. The arrows denote the direction and relative magnitude of the energy flow \vec{S} .

pulse through the slab shows that before reaching the slab, the pulse propagates in the free space, and after entering the slab, it gradually diffuses and oscillates as the propagating distance increases. Thus, the output pulse at the exit of the slab has the oscillation with the characteristic frequency independent of the slab thickness. The physical origin of such oscillation comes from the interference between the upper and lower high-transmission bands near the DP of the NZPIM slab. Actually, the finite pulse can be divided into two parts: one $(\omega > \omega_D)$ belongs to the upper band and the other one $(\omega < \omega_D)$ belongs to the lower band (see Fig. 12), and each part undergoes the opposite phase change due to the opposite properties of the two bands. For instance, in the upper band the wave number k is positive so the phase shift is positive, while in the lower band the phase shift is negative due to negative k. Therefore the two parts interfere with each other, which leads to the oscillatory property. At the DP, the light field obeys the diffuse equation, thus the pulse behavior also has the diffusion property. Moreover, for the pulse with a fixed transverse spatial width W, the oscillation frequency is proportional to the pulse spectral width near the DP, and for the pulse with a fixed time duration, the oscillation frequency decreases with the increasing of W. From the experimental viewpoint, it is suggested that smaller transverse spatial width and/or shorter pulse duration of the finite pulse make the optical Zitterbewegung effect easier to be observed experimentally.

4.3 Optical Goos-Hänchen shift in Bragg-like reflection

In this subsection, we will take an example of the optical simulations in NZPIM with DP. Similar to the Sec. 2.2, we start to investigate the GH shifts of the light beam reflected from the slab of NZPI medium. Firstly, we assume that the incident plane wave is $E_z^{in}(x,y) = \exp[i(k_x x + k_y y)]$, where $k_x = k_0 \cos \theta_0$, $k_y = k_0 \sin \theta_0$, $k_0 = (\varepsilon_0 \mu_0)^{1/2} \omega/c$ is the wave number in the air, ε_0 and μ_0 are the relative permittivity and permeability of the air, the reflected and transmitted plane waves can be expressed by $E_z^{ref}(x,y) = r \exp[i(-k_x x + k_y y)]$ and

 $E_z^{tr}(x, y) = t \exp \{i[-k_x(x-d) + k_y y]\},$ where the reflection coefficient *r*

$$r = \frac{\exp(i\pi/2)}{4g^2} \left(\frac{\mu_0}{\mu_1} \frac{k_{1x}}{k_x} - \frac{\mu_1}{\mu_0} \frac{k_x}{k_{1x}}\right) \left[\sin 2k_{1x}d + i\left(\frac{\mu_1}{\mu_0} \frac{k_x}{k_{1x}} + \frac{\mu_0}{\mu_1} \frac{k_{1x}}{k_x}\right) \sin^2 k_{1x}d\right], \quad (37)$$

and the transmission coefficient is $t = e^{i\phi}/g$ with $ge^{i\phi} = \cos k_{1x}d + \frac{i}{2}\left(\frac{\mu_1}{\mu_0}\frac{k_x}{k_{1x}} + \frac{\mu_0}{\mu_1}\frac{k_{1x}}{k_x}\right)\sin k_{1x}d$,

 $k_{1x} = \sqrt{k_1^2 - k_y^2}$ and $k_1 = (\omega - \omega_D)/v_D$ near the DP. As mentioned above, the upper and lower bands have different properties. Thus, we will discuss the unique properties of reflection and transmission in two cases of $\omega > \omega_D$ and $\omega < \omega_D$. The details can be seen in the literature (Chen et al., 2009, b).

Similar to the transmission in graphene barrier, the transmission as the function of frequency ω has a gap. The only difference is that the width of transmission gap has the following form: $\Delta \omega = 2k_y v_D$. To avoid the repetition, we concentrate on the reflection case here. The reflection behaves Bragg-like reflection. Figure 15 indicates the dependence of corresponding reflection probability R on the wavelength $\lambda = 2\pi\omega/c$, where $\theta_0 = 20^\circ$ and $\omega_D = 10 \times 2\pi$ GHz. Solid and dashed line correspond to a = 100 mm and a = 10 mm. It is interesting that the light beam can be perfectly reflected by such single NZPIM slab at certain range of the wavelength. As indicated in Fig. 15, the wavelength window for perfect reflection will become narrower with the increase of the width of slab. These frequency or wavelength passing-bands in reflection discussed here are similar to but different from the Bragg reflection in the 1D PCs. This so-called Bragg-like reflection discussed here is exactly due to the linear Dirac dispersion, which results in the evanescent waves in two cases of $\omega > \omega_D$ and $\omega < \omega_D$, corresponding to the two eigenfunctions of electric fields with the same $k(\omega)$.



Fig. 15. (Color online) The reflection probability *R* as the function of the wavelength λ , where $\theta_0 = 20^\circ$, $\omega_D = 10 \times 2\pi$ GHz, d = 100 mm (red solid line), and d = 10 mm (blue dashed line).

Next, we have a look at the optical GH shifts inside the optical NZPIM slab. It is noted that the GH shift in transmission is the equal to that in reflection inside such symmetric slab configuration, because the values of the derivation of the phase shifts with respect to k_y are the same. Here we just only consider the GH shift in reflection.

Figure 16 demonstrates that the GH shifts can be positive and negative. Due to the properties of DP, it is reasonable that the GH shifts are negative in the case of $\omega < \omega_D$, while they are positive in the case of $\omega > \omega_D$ is positive. More interestingly, the GH shifts near the DP can change from positive to negative with the increase (decrease) of the wavelength (frequency). In

addition, it is also shown that the GH shifts near the DP have only the order of wavelength due to the evanescent waves. The smallness of the GH shifts are similar to those in total reflection or frustrate-total-internal-reflection structure (Chen et al., 2009, a).

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Fig. 16. (Color online) The GH shifts as the function of the wavelength, ω , where d = 100 mm, and other parameters are the same as in Fig. 15. Solid, dashed and dotted curves correspond to $\theta_0 = 30^\circ$, $\theta_0 = 20^\circ$, and $\theta_0 = 10^\circ$.

However, these GH shifts don't change sign with increasing the angle of incidence. This is similar to the quantum GH effect in graphene on the total reflection (Zhao et al., 2010), but it is quite different from the result given in the literature (Beenakker et al., 2009), where the sublattice (or "pseudospin") degree of freedom is considered. Anyway, we have managed to simulate the quantum GH in graphene barrier by the optical metamaterial with the DP.

4.4 Further work on optical Dirac point in metamaterials

Finally, we would like to point out other interesting work on the optical DP in metamaterials. Motivated by the realization of the optical Dirac dispersion in the homogenous NZPIM, we make a theoretical investigation on the properties of thermal emission in layered structures containing the NZPI medium (Wang et al., 2010). When the thermal emission frequency is close to the DP, the spectral hemisperical power of thermal emission in such a structure is strongly suppressed and the emission can become a high directional source with large spatial coherence.

In addition, the guided modes and nonlinear surface waves near the DP have been also studied, respectively (Shen et al., 2010, a;b). In the simple model of optical waveguide, our theoretical results show that due to the linear Dirac dispersion, the fundamental mode is absent when the angular frequency is smaller than the DP, while the behaviors of NZPIM waveguide are similar to the conventional dielectric waveguide when the angular frequency is larger than the DP. The unique properties of the guided modes are analogous to the propagation of electron waves in graphene waveguide (Zhang et al., 2009), corresponding to the classical motion and the Klein tunneling. It is amazing that electron guiding as the analogue of an optical fiber has been experimentally demonstrated in graphene by tuning the carrier type and density using local electrostatic fields (Williams et al., 2010). This timely work

will definitely simulate us to design various novel optic-like devices in graphene according to the Dirac-like properties of graphene.

5. Conclusion

In summary, there are a lot of optic-like phenomena of electron wave in graphene, which leads to novel Dirac electron wave devices. The field covered here is vast, and we pay special attention to work done by the authors, while making effort to to offer a global perspective. In this chapter, we have presented the propagation of electron waves in monolayer graphene and optical simulations with NZPIM. The specific electronic analogies of Bragg-like reflection (transmission gap), zero-averaged wavenumber gap, and GH effect have been respectively discussed in single and multiple monlayer graphene barriers. The key point is that the transmission gap has great effect on the electronic transports including electric conductance and Fano factor. More importantly, all these results suggest that the electron wave propagation and their optical counterparts in NZPIM not only give the deeper understanding of several exotic phenomena in graphene, but also predict richer phenomena in different physical systems.

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7. References

- Arovas, D. P.; Brey, L.; Fertig, H. A.; Kim, E.-A. & Ziegler K. (2010), Dirac spectrum in piecewise constant one-dimensional potentials, arXiv:1002.3655
- Bai C.-X. & Zhang, X.-D. (2007). Klein paradox and resonant tunneling in a graphene superlattice. *Phys. Rev. B*, Vol. 76, No. 7, Aug -2007, 075430 (1-4), ISSN 1098-0121
- Barbier, M.; Peeters, F. M.; Vasilopoulos, P. & Milton Pereira, Jr., J. (2008). Dirac and Klein-Gordon particles in one-dimensional periodic potentials. *Phys. Rev. B*, Vol. 77, No. 11, Mar -2008, 115446 (1-4), ISSN 1098-0121
- Barbier, M.; Vasilopoulos, P. & Peeters, F. M. (2009). Dirac electrons in a Kronig-Penney potential: Dispersion relation and transmission periodic in the strength of the barriers, *Phys. Rev. B*, Vol. 80, No. 11, Nov -2009, 205415 (1-5), ISSN 1098-0121
- Barbier, M.; Vasilopoulos, P. & Peeters, F. M. (2010). Extra Dirac points in the energy spectrum for superlattices on single-layer graphene. *Phys. Rev. B*, Vol. 81, No. 7, Feb -2010, 075438 (1-7), ISSN 1098-0121
- Beenakker, C. W. (2008). Colloquium: Andreev reflection and Klein tunneling in graphene.*Rev. Mod. Phys.*, Vol. 80, No. 4, Oct -2008, 1337 – 1354, ISSN 0034-6861

- Beenakker, C. W. J.; Sepkhanov, R. A.; Akhmerov, A. R. & Tworzydło, J. (2009). Quantum Goos-Hänchen effect in graphene. *Phys. Rev. Lett.*, Vol. 102, No. 14, Jan -2009, 146804 (1-4), ISSN 0031-9007
- Bliokh, Yury P.; Freilikher, V.; Savel'ev, S. & Nori, F. (2009). Transport and localization in periodic and disordered graphene superlattices. *Phys. Rev. B*, Vol. 79, No. 7, Feb -2009, 075123 (1-4), ISSN 1098-0121
- Brey, L. & Fertig, H. A. (2009), Emerging zero modes for graphene in a periodic potential, *Phys. Rev. Lett.*, Vol.103, No.4, Jul.-2009, 046809, (1-4), ISSN 0031-9007
- Castro Neto, A. H.; Guinea, F.; Peres, N. M. R.; Novoselov, K. S. & Geim, A. K. (2009). The electronic properties of graphene. *Rev. Mod. Phys.*, Vol. 81, No. 1, Jan -2009, 109 162, ISSN 0034-6861
- Cheianov, V. V.; Fal'ko, V. & Altshuler, B. L. (2007). The focusing of electron flow and a veselago lens in graphene p-n junctions. *Science*, Vol. 315, No. 5816, Mar -2007, 1252 – 1255, ISSN 0036-8075
- Chen, X.; Li, C.-F.; Wei, R.-R.; & Zhang, Y. (2009). Goos-Hänchen shifts in frustrated total internal reflection studied with wave-packet propagation. *Phys. Rev. A*, Vol. 80, No. 1, Jul.-2009, 015803 (1-4), ISSN 1050-2947
- Chen, X.; Wang, L.-G. & Li, C.-F. (2009). Transmission gap, Bragg-like reflection, and Goos-Hänchen shifts near the Dirac point inside a negative-zero-positive index metamaterial slab. *Phys. Rev. A*, Vol. 80, No. 4, Oct -2009, 043839 (1-5), ISSN 1050-2947
- Chen, X. & Tao, J.-W. (2009). Design of electron wave filters in monolayer graphene by tunable transmission gap. *Appl. Phys. Lett.*, Vol. 94, No. 26, Jun -2009, 262102 (1-3), ISSN 0003-6951
- Chen, X.; Tao, J.-W. & Ban, Y. (2010). Goos-Hänchen-like shifts for Dirac fermions in monolayer graphene barrier. arXiv:1004.0350v1 (to appear in Eur. Phys. J. B).
- Darancet, P.; Olevano, V. & Mayou, D. (2009). Coherent electronic transport through graphene constrictions: Subwavelength regime and optical analogy. *Phys. Rev. Lett.*, Vol. 102, No. 13, Mar -2009, 136803 (1-4), ISSN 0031-9007
- Datta, S. (1996). *Electronic Transport in Mesoscopic Systems*, Cambridge University Press, ISBN 0-521-41604-3, New York.
- Dragoman, D. & Dragoman, M. (2004). *Quantum-Classical Anologies*, Springer, ISBN 978-3-540-20147-2, Berlin.
- Dell'Anna L. & De Martino, A. (2009). Multiple magnetic barriers in graphene. *Phys. Rev. B*, Vol. 79, No. 4, Jan -2009, 045420 (1-4), ISSN 1098-0121
- Gaylord, T. K. & Brennan, K. F. (1989). Electron wave optics in semiconductors. J. Appl. Phys., Vol. 65, No. 2, Jan -1989, 814 – 820, ISSN 0021-8979
- Gaylord, T. K.; Glytsis, E. N.; Henderson, G. N.; Martin, K. P.; Walker, D. B.; Wilson, D. W. & Brennan, K. F. (1991). Quantum interference effects in semiconductors: a bibliography. *Proc. IEEE*, Vol. 79, No. 8, Aug -1991, 1159 1180, ISSN 0018-9219
- Gerritsma, R.; Kirchmair, G.; Zähringer, F.; Solano, E.; Blatt, R. & Roos, C. F. (2010). Quantum simulation of the Dirac equation. *Nature*, Vol. 463, Jan -2010, 68 71, ISSN 0028-0836
- Ghosh, S. & Sharma, M. (2009). Electron optics with magnetic vector potential barriers in graphene. J. Phys.: Condens. Matter, Vol. 21, No. 29, Jul -2009, 292204 (1-8), ISSN 0953-8984
- Goos, F. & Hänchen, H. (1947), Ein neuer und fundamentaler Versuch zur Totalreflexion. *Ann. Phys.* (*Leipzig*) Vol. 436, No. 7 8, Oct-1943, 333-346, ISSN 0003-3804

- Goos, F. & Hänchen, H. (1949), Neumessung des Strahlversetzungseffektes bei Totalreflexion. *Ann. Phys. (Leipzig)* Vol. 440, No. 3 5, Jul-1949, 251-252, ISSN 0003-3804
- Güney D. Ö. & Meyer, D. A. (2009). Negative refraction gives rise to the Klein paradox. *Phys. Rev. A*, Vol. 79, No. 6, Jun -2009, 063834 (1-4), ISSN 1050-2947
- Ho, J. H.; Chiu, Y. H.; Tsai, S. J. & Lin, M. F. (2009), Semimetallic graphene in a modulated electric potential, *Phys. Rev. B*, Vol.79, No.11, Mar.-2009, 115427, (1-7), ISSN 1098-0121
- Ji, Y.; Chung, Y.; Sprinzak, D.; Heiblum, M.; Mahalu, D. & Shtrikman, H. (2003). An electronic Mach-Zehnder interferometer. *Nature*, Vol. 422, Mar -2003, 415 – 418, ISSN 0028-0836
- Juzeliūnas, G.; Ruseckas, J.; Lingdberg, M.; Santos, L. & Öhberg, P. (2008). Quasirelativistic behavior of cold atoms in light fields. *Phys. Rev. A*, Vol. 77, No. 1, Jan -2008, 011802 (1-4), ISSN 1050-2947
- Katsnelson, M. I.; Novoselov, K. S. & Geim, A. K. (2006). Chiral tunnelling and the Klein paradox in graphene. *Nature Phys.*, Vol. 2, Feb -2006, 620 625, ISSN 1745-2473
- Katsnelson, M. I. (2006). Zitterbewegung, chirality, and minimal conductivity in graphene. *Eur. Phys. J. B* Vol.51, No. 2, May-2006, 157-160, ISSN 1434-6028
- Lamata, L.; León, J.; Schätz, T. & Solano, E. (2007). Dirac equation and quantum relativistic effects in a single trapped ion. *Phys. Rev. Lett.*, Vol. 98, No. 25, Jan -2007, 253005 (1-4), ISSN 0031-9007
- Li, J.; Zhou, L.; Chan, C. T. & Sheng, P. (2003), Photonic band gap from a stack of positive and negative index material, *Phys. Rev. Lett.*, Vol.90, No. 8, Feb-2003, 083901 (1-4), ISSN 0031-9007
- Longhi, S. (2010). Klein tunneling of light in fiber Bragg gratings. *Physics Research International*, Vol. 2010, No. 645106, July -2010, 645106 (1-5), ISSN 20902220
- De Martino, A.; Dell'Anna, L. & Egger, R. (2007). Magnetic confinement of massless Dirac fermions in graphene. *Phys. Rev. Lett.*, Vol. 98, No. 6, Feb -2007, 066802 (1-4), ISSN 0031-9007
- Masir, M. R.; Vasilopoulos, P. & Peeters, F. M. (2008). Wavevector filtering through single-layer and bilayer graphene with magnetic barrier structures. *Appl. Phys. Lett.*, Vol. 93, No. 24, Dec -2008, 242103 (1-3), ISSN 0003-6951
- Masir, M. R.; Vasilopoulos, P.; Matulis, A. & Peeters, F. M. (2008). Direction-dependent tunneling through nanostructured magnetic barriers in graphene. *Phys. Rev. B*, Vol. 77, No. 23, Jun -2008, 235443 (1-4), ISSN 1098-0121
- Molenkamp, L. W.; Staring, A. A. M.; Beenakker, C. W. J.; Eppenga, R.; Timmering, C. E.; Williamson, J. G.; Harmans, C. J. P. M. & Foxon, C. T. (1990). Electron-beam collimation with a quantum point contact. *Phys. Rev. B*, Vol. 41, No. 2, Jan -1990, 1274 (1-4), ISSN 1098-0121
- Novoselov, K. S.; Geim, A. K.; Morozov, S. V.; Jiang, D.; Zhang, Y.; Dubonos, S. V.; Grigorieva, I. V. & Firsov, A. A.(2004). Electric field effect in atomically thin carbon films. *Science*, Vol. 306, No. 5696, Oct -2004, 666 – 669, ISSN 0036-8075
- Novoselov, K. S.; Geim, A. K.; Morozov, S. V.; Grigorieva, I. V.; Dubonos, S. V. & Firsov, A. A. (2005). Two-dimensional gas of massless Dirac fermions in graphene. *Nature* (*London*), Vol. 438, Nov -2005, 197 200, ISSN 0028-0836
- Palevski, A.; Heiblum, M.; Umbach, C. P.; Knoedler, C. M.; Broers, A. N. & Koch, R. H. (1989). Lateral tunneling, ballistic transport, and spectroscopy in a two-dimensional electron gas. *Phys. Rev. Lett.*, Vol. 62, No. 15, Apr -1989, 1776 (1-4), ISSN 0031-9007

- Park, C.-H.; Yang, L.; Son, Y.-W.; Cohen, M. L. & Louie, S. G. (2008). Electron beam supercollimation in graphene superlattices. *Nano Lett.*, Vol. 8, No. 9, Aug -2008, 2920 – 2924, ISSN 1530-6984
- Peraira, J. M.; Vasilopoulos, Jr., P. & Peeters, F. M. (2007). Graphene-based resonant-tunneling structures. *Appl. Phys. Lett.*, Vol. 90, No. 13, Mar -2007, 132122 (1-3), ISSN 0003-6951
- Garcia-Pomar, J. L.; Cortijo, A. & Nieto-Vesperinas, M. (2008). Fully valley-polarized electron beams in graphene. *Phys. Rev. Lett.*, Vol. 100, No. 23, Jun -2008, 236801 (1-4), ISSN 0031-9007
- Peleg, O.; Bartal, G.; Freedman,B.; Manela, O.; Segev, M. & Christodoulides, D. N. (2007). Conical diffraction and gap solitons in honeycomb photonic lattices. *Phys. Rev. Lett.*, Vol. 98, No. 10, Mar -2007, 103901 (1-4), ISSN 0031-9007
- Shen, M.; Ruan, L.-X. & Chen, X. (2010). Guided modes near the Dirac point in negative-zero-positive index metamaterial waveguide. *Opt. Express*, Vol. 18, No. 12, May -2010, 12779 – 12787, ISSN 1094-4087
- Shen, M.; Ruan, L.-X.; Chen, X.; Shi, J.-L.; Ding, H.-X.; Xi, N. & Wang Q. (2010). Nonlinear surface waves near the Dirac point in negative-zero-positive index metamaterial. *J. of Opt.* Vol. 12, N, 8, Aug- 2010, 085201 (1-5), ISSN 2040-8978
- Sivan, U.; Heiblum, M.; Umbach, C. P. & Shtrikman, H. (1990). Electrostatic electron lens in the ballistic regime. *Phys. Rev. B*, Vol. 41, No. 11, Apr -1990, 7937 (1-4), ISSN 1098-0121
- Spector, J.; Stormer, H. L.; Baldwin, K. W.; Pfeiffer, L. N. & West, K. W. (1990). Electron focusing in two-dimensional systems by means of an electrostatic lens. *Appl. Phys. Lett.*, Vol. 56, No. 13, Mar -1990, 1290 (1-3), ISSN 0003-6951
- Sepkhanov, R. A.; Bazaliy, Y. B. & Beenakker, C. W. J. (2007). Extremal transmission at the Dirac point of a photonic band structure. *Phys. Rev. A*, Vol. 75, No. 6, Jun -2007, 063813 (1-5), ISSN 1050-2947
- Steślicka, B.; Kucharczyk, R.; Akjoouj, A.; Rouhani, B. D.; Dobrzynski, L.; & Davison, S. G. (2002), Localised electronic states in semiconductor superlattices. *Surf. Sci. Rep.* Vol. 47, No.4 – 5, Aug-2002, 93-196, ISSN 0167-5729
- Tworzydlo, J.; Trauzettel B., Titov, M.; Rycerz, A.; & Beenakker, C. W. J. (2006). Sub-Poissonian shot noise in graphene. *Phys. Rev. Lett.* Vol. 96, No. 24, 246802 (1-4), Jun-2006, ISSN 0031-9007
- Tsu, R. (2005), Superlattice to Nanoelectronics, Elsevier, ISBN: 978-0-08-044377-5, Oxford.
- Van Houten, H.; van Wees, B. J.; Mooij, J. E.; Beenakker, C. W. J.; Williamson, J. G. & Foxon, C. T. (1988). Coherent electron focussing in a two-dimensional electron gas. *Europhys. Lett.*, Vol. 5, No. 8, Apr -1988, 721 – 725, ISSN 0295-5075
- Wang, L.-G.; Wang, Z.-G.; Zhang, J.-X. & Zhu, S.-Y. (2009). Realization of Dirac point with double cones in optics. *Opt. Lett.*, Vol. 34, No. 10, May -2009, 1510-2, ISSN 0146-9592
- Wang, L.-G.; Wang, Z.-G. & Zhu, S.-Y. (2009). Zitterbewegung of optical pulses near the Dirac point inside a negative-zero-positive index metamaterial. *EPL*, Vol. 86, No. 4, Jun -2009, 47008 – -5, ISSN 0295-5075
- Wang, L.-G. & Zhu, S.-Y. (2010), The reversibility of the Goos-Hänchen shift near the band-crossing structure of one-dimensional photonic crystals containing left-handed metamaterials, *Appl. Phys. B: Lasers and Optics*, Vol.98, No.2 – 3, Feb.-2010, 459-463, ISSN 0946-2171
- Wang, L.-G. & Zhu, S.-Y. (2010). Electronic band gaps and transport properties in graphene superlattices with one-dimensional periodic potentials of square barriers. *Phys. Rev. B*, Vol. 81, No. 20, May -2010, 205444 (1-9), ISSN 1098-0121

- Wang, L.-G.; Li, G.-X. & Zhu, S.-Y. (2010). Thermal emission from layered structures containing a negative-zero-positive index metamaterial. *Phys. Rev. B*, Vol. 81, No. 7, Jan -2010, 073105 (1-4), ISSN 1098-0121
- Wang, L.-G. & Chen, X. (2010). Robust zero-averaged wave-number gap inside gapped graphene superlattices. arXiv:1008.0504. (to appear in J. Appl. Phys.)
- Wang Z.-F. & Liu, F. (2010). Manipulation of electron beam propagation by hetero-dimensional graphene junctions. *ACS Nano*, Vol. 4, No. 4, Jan -2010, 2459 2465, ISSN 1936-0851
- White, C. T. & Todornov, T. N. (2001). Quantum electronics: Nanotubes go ballistic. *Nature*, Vol. 411, Jun -2001, 649 651, ISSN 0028-0836
- Williams, J. R.; Low, Tony; Lundstrom, M. S. & Marcus C. M. (2010). Fiber-optic and p-n guiding of electrons in graphene. arXiv:1008.3704
- Yacoby, A.; Heiblum, M.; Umansky, V.; Shtrikman, H. & Mahalu, D. (1994). Unexpected periodicity in an electronic double slit interference experiment. *Phys. Rev. Lett.*, Vol. 73, No. 23, Dec -1994, 3149 (1-4), ISSN 0031-9007
- Zhao, L. & Yelin, S. F. (2010). Proposal for graphene-based coherent buffers and memories. *Phys. Rev. B*, Vol. 81, No. 11, Mar -2010, 115441 (1-4), ISSN 1098-0121
- Zhang, Y.; Tan, Y.-W.; Stormer, H. L. & Kim, P. (2005). Experimental observation of the quantum Hall effect and Berry's phase in graphene. *Nature (London)*, Vol. 438, Nov -2005, 201 204, ISSN 0028-0836
- Zhang, X. (2008). Observing Zitterbewegung for photons near the Dirac point of a two-dimensional photonic crystal. *Phys. Rev. Lett.*, Vol. 100, No. 11, Mar -2008, 113903 (1-4), ISSN 0031-9007
- Zhang, F.-M.; He, Y. & Chen, X. (2009). Guided modes in graphene waveguides. *Appl. Phys. Lett.*, Vol. 94, No. 21, May -2009, 212105 (1-3), ISSN 0003-6951
- Zhu, S.-L.; Wang, B.-G. & Duan, L.-M. (2007). Simulation and detection of Dirac fermions with cold atoms in an optical lattice. *Phys. Rev. Lett.*, Vol. 98, No. 26, Jun -2007, 260402 (1-4), ISSN 0031-9007





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The Stone Age, the Bronze Age, the Iron Age... Every global epoch in the history of the mankind is characterized by materials used in it. In 2004 a new era in material science was opened: the era of graphene or, more generally, of two-dimensional materials. Graphene is the strongest and the most stretchable known material, it has the record thermal conductivity and the very high mobility of charge carriers. It demonstrates many interesting fundamental physical effects and promises a lot of applications, among which are conductive ink, terahertz transistors, ultrafast photodetectors and bendable touch screens. In 2010 Andre Geim and Konstantin Novoselov were awarded the Nobel Prize in Physics "for groundbreaking experiments regarding the two-dimensional material graphene". The two volumes Physics and Applications of Graphene - Experiments and Physics and Applications of Graphene - Theory contain a collection of research articles reporting on different aspects of experimental and theoretical studies of this new material.

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