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Magnetoexciton Binding Energy in Graphene Bilayers

Zlatko Koinov
The University of Texas at San Antonio
San Antonio, Texas, USA

1. Introduction

A lot of experimental and theoretical studies in recent years are focusing on the unusual relativistically, kinematic properties of the electronic states in graphene predicted theoretically decades ago (Haldane, 1988; Semenoff, 1984). Later, it was confirmed that the graphitic monolayer have anomalous relativistically properties (Novoselov et al., 2005; 2004). Because electrons and holes in a graphene behave like massless Dirac particles, there is a number of unusual properties, such as high charge carrier mobility (Novoselov et al., 2005), the graphene's conductivity never falls below a minimum value (Nomura & MacDonald, 2007; Ziegler, 2006), and an anomalous quantum Hall effect.(Zhang et al., 2005)
Bilayer graphene systems, where carriers in one layer are electrons and carriers in the other are holes, have been considered as ideal candidates for observing superfluid properties at room temperatures (Lozovik & Sokolik, 2008; Min et al., 2008; Zhang & Jorlecar, 2008). It is expected that the excitons will behave as neutral bosons at low densities, and therefore, they can undergo Bose-Einstein condensation (BEC) when the interlayer distance is comparable to the distance between the particles within each layer. However, when we separate electrons and holes by introducing a dielectric between them we reduce the exciton binding energy, and so the critical temperature for condensation decreases. A possible way to increase the binding energy is to apply magnetic field perpendicular to the layers. As we shall see, the calculations predict the existence of a condensate of magnetoexcitons with superfluid properties under the Kosterlitz-Thouless critical temperature T_{KT} which in graphene bilayers decreases in the limit of large interlayer separation as $T_{KT} \sim B^{-2}$ (as $T_{KT} \sim B^{-1/2}$ in the limit of small interlayer separation).
Magnetoexcitons are bound states between two charged fermions (an electron from the conductive band and a hole from the valence band) in the presence of a magnetic field. The calculations are much more complicated compare to the corresponding calculations in the absence of a magnetic field because even a small transverse exciton velocity (or small transverse wave vector $\mathbf{Q}$) will induce an electric field in the rest frame of the exciton. This electric field will push the electron and the hole apart, so the magnetoexciton binding energy must decrease as the transverse velocity increases. In other words, the magnetic field induces a coupling between the center-of-mass and the relative internal motions, and therefore, the correct description of excitons in a strong magnetic field should take into account this coupling effect.

Turning our attention to magnetoexciton dispersion in non-relativistic systems, such as coupled quantum wells (CQW's) with parabolic dispersions ($E_{c,v} = \hbar^2 k^2/2m_{c,v}$) we find that the following Hamiltonian (Shevchenko, 1997)

$$\widehat{H} = -\frac{\hbar^2}{2\mu}\nabla^2_{\mathbf{r}} + \frac{\imath e\gamma\hbar}{2\mu c}(\mathbf{B}\times\mathbf{r}).\nabla_{\mathbf{r}} + \frac{e^2B^2}{8\mu c^2}\mathbf{r}^2 - V(\mathbf{r}+\mathbf{R}_0)$$

is used to obtain the magnetoexciton dispersion. Here μ is the exciton reduced mass, $\gamma = (m_v - m_c)/(m_c + m_v)$, $\mathbf{R}_0 = R^2\mathbf{Q}_0$, where $\mathbf{Q}_0 = (-Q_y, Q_x, 0)$, and $R = (\hbar c/eB)^{1/2}$ is the magnetic length. $V(\mathbf{r}) = e^2/(\varepsilon_0\sqrt{|\mathbf{r}|^2 + d^2})$ represents the electron-hole Coulomb attraction screened by the dielectric constant ϵ_0. Since the Coulomb term in the Hamiltonian is the only term which depends on the exciton momentum $\mathbf{Q} = (Q_x, Q_y, 0)$, the magnetoexciton dispersion does not depend on the electron and hole masses and the magnetoexciton mass is determined only by Coulomb interaction. In strong magnetic fields one can apply the lowest Landau level (LLL) approximation. In the LLL approximation the magnetoexciton mass M_{CQW} and the binding energy E_{CQW} are as follows:

$$\frac{M_{CQW}}{M_{2D}} = \left[(1+\frac{d^2}{R^2})e^{\left(\frac{d^2}{2R^2}\right)}Erfc\left(\frac{d}{\sqrt{2}R}\right) - \sqrt{\frac{2}{\pi}}\frac{d}{R}\right]^{-1}$$

$$E_{CQW} = E_b \exp\left(\frac{d^2}{2R^2}\right)Erfc\left(\frac{d}{\sqrt{2}R}\right).$$

Here $Erfc(x)$ is the complementary error function, $M_{2D} = 2^{3/2}\varepsilon_0\hbar^2/(\sqrt{\pi}e^2R)$ and $E_b = \sqrt{\pi}e^2/(\sqrt{2}\varepsilon_0 R)$ are the magnetoexciton mass and the two-dimensional ($d = 2$) magnetoexciton binding energy, respectively.

Strictly speaking, the excitons are bound states between two charged fermions, and therefore, the appropriate framework for the description of the bound states is the Bethe-Salpeter (BS) formalism (Salpeter & Bethe, 1951). In the case of parabolic band quantum-well structures we find that beyond the LLL approximation, the BS equation contains an extra term (BS term) (Koinov, 2008). This term takes into account the transitions to the Landau levels with indexes $n \geq 1$. The contributions to the magnetoexciton binding energy and mass can be obtained by applying a variational procedure. In the non-relativistic case the results are as follows: in a strong magnetic field, the ground-state energy is very close to that obtained by means of the Schrödinger equation, but the magnetoexciton dispersion is determined by the BS term rather than the electron-hole Coulomb term in the Schrödinger equation.

Since the unique electronic behaviors of graphene is a result of the unusual quantum-relativistic characteristics of the so-called Dirac fermions, we shall study magnetoexciton binding energy in graphene bilayers embedded in a dielectric by applying the relativistic BS equation in the LLL approximation (Koinov, 2009). Several non-trivial effects produced by magnetic fields have been recently predicted in quantum field theories. For example, in the massless QED, the analysis based on the BS equation has predicted that the external constant magnetic field generates an energy gap (dynamical mass) in the spectrum of massless fermions for any arbitrary weak attractive interaction between fermions (Gusynin et al., 1994; 1995). It is expected that the effect is model independent (universal), because the physical reason of this effect lies in the dimensional reduction in the dynamics of fermion pairing in the presence of a the constant magnetic field.

In what follows we examine how both the magnetoexciton binding energy and magnetoexciton mass in graphene bilayer systems vary with the magnetic field and the separation d between the layers in the LLL approximation. The solution of this problem is related to the continued activities to observe superfluid properties of excitons in a bilayer graphene. It is expected that the excitons will behave as neutral bosons at low densities, and therefore, they can undergo Bose-Einstein condensation when the interlayer distance is comparable to the distance between the particles within each layer. The condensate of neutral bosons (excitons) should have superfluid properties under the Kosterlitz-Thouless critical temperature T_{KT}. It is worth mentioning that the calculations done by treating the Coulomb interaction as a perturbation (Berman et al., 2008) provide in the LLL approximation *a number of extra terms which do not exist in the case of CQW's*. From a general point of view, we have to expect that the binding energy is *exactly* four times higher than E_{CQW}, while the magnetoexciton mass is *exactly* four times lower than M_{CQW}. The physical reason for the above statement lies in the fact that in the LLL approximation we have a dimensional reduction in the dynamics of the electron-hole pairing from two space variables plus a time variable to zero space variable and a time variable. Because of this $2+1 \rightarrow 0+1$ reduction the results should be insensitive to the type of the band dispersion. The factor four is due to the four-component-spinor description used in the relativistic case.

2. Bethe-Salpeter equation

The system under consideration is made from two graphene sheets embedded in a dielectric and separated by distance d. Each of the two graphene layers has two Dirac-like linear dispersion $\hbar v_F k$ bands centered at two non-equivalent points $\mathbf{K}$ and $\mathbf{K\prime}$, where v_F is the Fermi velocity of electrons in graphene. Since the layers are embedded in a dielectric, there is no hopping of π-electrons between the layers. There is a potential difference $\pm V_g/2$ (gate voltage) applied to each of the two layers which allows us to adjust the charge density in the layers. We assume that the potential difference is chosen in a manner that the electrons are in the top layer (pseudospin index $\tau = 1$) and the same number of holes in the bottom layer ($\tau = 2$).

The unit cell of graphene has two atoms, A and B, each belonging to the different sublattice. The operator $\psi^{(\tau)\dagger}_{\sigma,A,\alpha}(\mathbf{r})$ ($\psi^{(\tau)\dagger}_{\sigma,B,\alpha}(\mathbf{r})$) creates an electron of spin $\sigma = \uparrow, \downarrow$ on the atom A (atom B) of the unit cell in layer τ defined by the position vector $\mathbf{r}$. We introduce four component spinors:

$$\Psi^{(\tau)}_{\sigma}(\mathbf{r}) = \begin{pmatrix} \psi^{(\tau)}_{\sigma,A,\mathbf{K}}(\mathbf{r}) \\ \psi^{(\tau)}_{\sigma,B,\mathbf{K}}(\mathbf{r}) \\ \psi^{(\tau)}_{\sigma,B,\mathbf{K}'}(\mathbf{r}) \\ \psi^{(\tau)}_{\sigma,A,\mathbf{K}'}(\mathbf{r}) \end{pmatrix}, \tag{1}$$

$$\overline{\Psi}^{(\tau)}_{\sigma}(\mathbf{r}) = \Psi^{(\tau)\dagger}_{\sigma}(\mathbf{r})\gamma^0,$$

where the following representation of the Dirac matrices is chosen:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$
$$\gamma^2 = \begin{pmatrix} 0 & -\imath & 0 & 0 \\ -\imath & 0 & 0 & 0 \\ 0 & 0 & 0 & \imath \\ 0 & 0 & \imath & 0 \end{pmatrix}. \tag{2}$$

In continuum approximation the non-interacting quasiparticles in the layers are described by the Hamiltonian:

$$H_0 = \sum_{\sigma,\tau} \int d^2\mathbf{r} \overline{\Psi}_\sigma^{(\tau)}(\mathbf{r}) \widehat{H}^{(\tau)} \Psi_\sigma^{(\tau)}(\mathbf{r}), \tag{3}$$

where

$$\begin{aligned} \widehat{H}^{(\tau)} &= v_F \left(\gamma^1 \widehat{p}_x + \gamma^2 \widehat{p}_y \right), \\ \widehat{p}_x &= -\imath\hbar \frac{\partial}{\partial x}, \\ \widehat{p}_y &= -\imath\hbar \frac{\partial}{\partial y}. \end{aligned}$$

The action that describes the non-interacting quasiparticles in a layer τ is:

$$S_0^{(\tau)} = \int d^2\mathbf{r} dt \overline{\Psi}_\sigma^{(\tau)}(\mathbf{r}, t) \left[\gamma^0 \imath\hbar \frac{\partial}{\partial t} - v_F \left(\gamma^1 \widehat{p}_x + \gamma^2 \widehat{p}_y \right) \right] \Psi_\sigma^{(\tau)}(\mathbf{r}, t) \tag{4}$$

In the presence of a perpendicular magnetic field $\mathbf{B} = (0, 0, B)$ and a potential difference $\pm V_g/2$ (gate voltage) applied to each of the two layers, the action (4) assumes the form:

$$S_0^{(\tau)} = \int d^2\mathbf{r} dt \overline{\Psi}_\sigma^{(\tau)}(\mathbf{r}, t) \left[\gamma^0 \left(\imath\hbar \frac{\partial}{\partial t} - V_g^{(\tau)} \right) - v_F \left(\gamma^1 \widehat{\pi}_x + \gamma^2 \widehat{\pi}_y \right) \right] \Psi_\sigma^{(\tau)}(\mathbf{r}, t), \tag{5}$$

where $\widehat{\pi}_{x(y)} = \widehat{p}_{x(y)} \mp (e/c)\mathbf{A}_{x(y)}(\mathbf{r})$, and $\mathbf{A}(\mathbf{r}) = (1/2)\mathbf{B} \times \mathbf{r}$ is the vector potential in a symmetric gauge.

In what follows we assume that the interaction between an electron with a position vector $\mathbf{r}_1$ from the top layer ($\tau = 1$) and a hole with a position vector $\mathbf{r}_2$ from the bottom layer ($\tau = 2$) is described by the Coulomb potential $V(\mathbf{r}_1 - \mathbf{r}_2) = e^2/\varepsilon_0 \sqrt{|\mathbf{r}_1 - \mathbf{r}_2|^2 + d^2}$.

Instead of two position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, we introduce the center-of-mass $\mathbf{R} = \alpha(\mathbf{r}_1 + \mathbf{r}_2)$ and the relative $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ coordinates ($\alpha = 1/2$).

The basic assumption in our BS formalism is that the electron-hole bound states are described by the BS wave function (BS amplitude). This function determines the probability amplitude to find the electron at the point $\mathbf{r}_1$ at the moment t_1 and the hole at the point $\mathbf{r}_2$ at the moment t_2. The BS amplitude depends on the relative internal time $t - t'$ and on the "center-of-mass" time:

$$\Phi^{\mathbf{Q}}(\mathbf{r}, \mathbf{R}; t, t') = \exp\left(-\frac{\imath E(\mathbf{Q})\alpha}{\hbar}(t + t') \right) \phi^{\mathbf{Q}}(\mathbf{r}, \mathbf{R}; t - t'), \tag{6}$$

where $E(\mathbf{Q})$ is the exciton dispersion. The BS equation for the equal-time BS amplitude in the center-of-mass and reduced coordinates is (Kouzakov & Studenikin, 2005):

$$\begin{aligned}\Phi^{\mathbf{Q}}(\mathbf{r},\mathbf{R};t,t) = \int d^2\mathbf{r}'d^2\mathbf{R}'dt'G^{(1)}(\mathbf{R}+\alpha\mathbf{r},\mathbf{R}'+\alpha\mathbf{r}';t-t')\gamma^0\times \\ G^{(2)}(\mathbf{R}'-\alpha\mathbf{r}',\mathbf{R}-\alpha\mathbf{r};t'-t)\gamma^0V(\mathbf{r}')\Phi^{\mathbf{Q}}(\mathbf{r}',\mathbf{R}';t',t').\end{aligned} \tag{7}$$

The Fourier transforms of the electron and hole propagators $G^{(\tau)}(\mathbf{r},\mathbf{r}';t)$ are define in terms of the Dirac four component spinors $\psi^{\kappa}(\mathbf{r})$ and the corresponding eigenvalues $E_n = \hbar v_F\sqrt{2n}/R$ (Kouzakov & Studenikin, 2005):

$$G^{(\tau)}(\mathbf{r},\mathbf{r}';\omega) = \sum_{\kappa}\frac{\psi^{\kappa}(\mathbf{r})\overline{\psi}^{\kappa}(\mathbf{r}')}{\hbar\omega - E_n \pm \imath 0^+} \tag{8}$$

Here we keep only the positive energy pole contributions, $n = 0,1,2,...$, and $\kappa = (n, j_z, \sigma)$, where j_z is the z component of the total angular momentum.
When the translation symmetry is broken by the magnetic field, the Green's functions can be written as a product of phase factors and translation invariant parts. The phase factor depends on the gauge. In the symmetric gauge the Green's functions are:

$$G^{(\tau)}(\mathbf{r},\mathbf{r}';\omega) = \exp\left[\imath\frac{e}{\hbar c}\mathbf{r}.\mathbf{A}(\mathbf{r}')\right]\widetilde{G}^{(\tau)}(\mathbf{r}-\mathbf{r}';\omega). \tag{9}$$

The broken translation symmetry requires a phase factor for the BS amplitude:

$$\phi^{\mathbf{Q}}(\mathbf{r},\mathbf{R};\Omega) = \exp\left[\imath\frac{e}{\hbar c}\mathbf{r}.\mathbf{A}(\mathbf{R})\right]\chi^{\mathbf{Q}}(\mathbf{r},\mathbf{R};\Omega). \tag{10}$$

The BS equation (7) admits translation invariant solution of the form:

$$\chi^{\mathbf{Q}}(\mathbf{r},\mathbf{R};\omega) = \exp\left[-\imath\left(\mathbf{Q}.\mathbf{R}\right)\right]\widetilde{\chi}^{\mathbf{Q}}(\mathbf{r};\omega). \tag{11}$$

The Fourier transform of the function $\widetilde{\chi}^{\mathbf{Q}}(\mathbf{r};\omega)$ satisfies the following BS equation:

$$\begin{aligned}\widetilde{\chi}^{\mathbf{Q}}(\mathbf{k};\omega) = \int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{d^2\mathbf{p}}{(2\pi)^2}d^2\mathbf{R}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}e^{-\imath(\mathbf{q}+\mathbf{Q}).\mathbf{R}} \\ \widetilde{G}^{(1)}\left(\frac{1}{2}\mathbf{q}+\mathbf{k}-\frac{e}{\hbar c}\mathbf{A}(\mathbf{R});\hbar\omega+\alpha(E-V_g)\right)\gamma^0\times \\ \widetilde{G}^{(2)}\left(-\frac{1}{2}\mathbf{q}+\mathbf{k}-\frac{e}{\hbar c}\mathbf{A}(\mathbf{R});\hbar\omega-\alpha(E-V_g)\right)\gamma^0V\left(\mathbf{p}-\left[\mathbf{k}-\frac{2e}{\hbar c}\mathbf{A}(\mathbf{R})\right]\right)\widetilde{\chi}_{\mathbf{Q}}(\mathbf{p};\Omega),\end{aligned} \tag{12}$$

where $\widetilde{G}^{(\tau)}(\mathbf{k};\hbar\omega)$ are the Fourier transforms of $\widetilde{G}^{(\tau)}(\mathbf{r};\hbar\omega)$.
In the effective-mass approximation the exact fermion Green's functions $G^{(\tau)}$ are replaced by the corresponding propagator of the free fermions. The translation invariant parts of the free fermion propagators can be decomposed over the Landau level poles (Gorbar et al., 2002):

$$\widetilde{G}^{(\tau)}(\mathbf{k};\hbar\omega) = 2\imath \sum_{n=0}^{\infty} (-1)^n e^{-R^2\mathbf{k}^2} \frac{\hbar\omega\gamma^0 f_1(k) + f_2(\mathbf{k})}{\hbar^2\omega^2 - 2n\hbar v_F^2 eB/c},$$
$$f_1(k) = \frac{1}{2}(1 - \imath\gamma^1\gamma^2)L_n(2R^2k^2) - \frac{1}{2}(1 + \imath\gamma^1\gamma^2)L_{n-1}(2R^2k^2),$$
$$f_2(\mathbf{k}) = 2v_F\hbar(k_x\gamma^1 + k_y\gamma^2)L^1_{n-1}(2R^2k^2).$$

Here $L^1_n(x)$ are the generalized Laguerre polynomials, $L^1_{-1}(x) = L_{-1}(x) = 0$ and $L_n(x)$ are the Laguerre polynomials. In strong magnetic fields the probability for transitions to the excited Landau levels due to the Coulomb interaction is small. Thus, the contributions to the Green's functions from the excited Landau levels is negligible, and therefore, one can apply the LLL approximation, where we keep only $n = 0$ term:

$$\widetilde{G}^{(1)}(\mathbf{k};\hbar\omega) \approx \imath \exp\left(-R^2\mathbf{k}^2\right) \frac{\gamma^0(1 - \imath\gamma^1\gamma^2)}{\hbar\omega + \imath 0^+},$$
$$\widetilde{G}^{(2)}(\mathbf{k};\hbar\omega) \approx \imath \exp\left(-R^2\mathbf{k}^2\right) \frac{\gamma^0(1 - \imath\gamma^1\gamma^2)}{\hbar\omega - \imath 0^+}. \tag{13}$$

The infinitesimal imaginary parts in our case reflect the fact that there are holes in layer number 2 (in electron-hole representation poles of the holes are above the real axis) and electrons in layer number 1.
The solution of the BS equation (12) in the LLL approximation can be written in the following form:

$$\widetilde{\chi}^{\mathbf{Q}}(\mathbf{k};\omega) = \exp\left[-R^2\mathbf{k}^2 - \imath\mathbf{R}_0.\mathbf{k}\right] \Phi_E(\omega). \tag{14}$$

Here $\Phi_E(\omega)$ is a 4×4 matrix. Thus, the LLL approximation reduces the problem from $2 + 1$-dimensions to $0 + 1$-dimension problem. The matrix $\Phi_E(\omega)$ and the magnetoexciton dispersion $E(\mathbf{Q})$ are determined by the solutions of the following equation:

$$\Phi_E(\omega) = -I(|\mathbf{Q}|) \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \frac{\gamma^0(1 - \imath\gamma^1\gamma^2)\gamma^0\Phi_E(\Omega)\gamma^0(1 - \imath\gamma^1\gamma^2)\gamma^0}{\left(\hbar\omega + \alpha(E - V_g) + \imath 0^+\right)\left(\hbar\omega - \alpha(E - V_g) - \imath 0^+\right)}. \tag{15}$$

The solution of (15) is given by:

$$\Phi_E(\omega) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \frac{1}{\left(\hbar\omega + \alpha(E - V_g) + \imath 0^+\right)\left(\hbar\omega - \alpha(E - V_g) - \imath 0^+\right)}, \tag{16}$$
$$E(\mathbf{Q}) = V_g - 4I(\mathbf{Q}).$$

Thus, in the LLL approximation, the magnetoexciton dispersion is determined by the Coulomb interaction term $I(\mathbf{Q}) = \int d^2\mathbf{r}\varphi^2_{00}(r)V(\mathbf{r} + \mathbf{R}_0)$, where $\varphi_{00}(r) = (\sqrt{2\pi}R)^{-1}\exp\left(-r^2/4R^2\right)$ is the ground-state wave function of an electron in a magnetic field. For small wave vectors we calculate:

$$E(\mathbf{Q}) \approx V_g - 4E_{CQW} + \frac{\hbar^2Q^2}{2M(B)}, \quad \frac{M(B)}{M_{CQW}} = \frac{1}{4}. \tag{17}$$

In graphene bilayer structures the magnetoexciton mass (binding energy) is four times lower (higher) than the corresponding magnetoexciton mass (binding energy) in coupled quantum wells with parabolic dispersion and the same d, ε_0 and B. In the limit of very small interlayer separation $d << R$ the asymptotical values of the binding energy and the effective magnetic mass of magnetoexciton in bilayer graphene are $4E_b$ and $M_{small}(B) = M_{2D}/4 \propto B^{1/2}$, respectively. In the limit of large interlayer separation $d >> R$ the asymptotical values of the magnetic mass is $M_{large}(B) = M_{2D}\pi^{1/2}d^3/(2^{7/2}R^3) \propto B^2$. As we mentioned above, under the certain critical temperature T_c a Bose-Einstein condensation might be expected to occur. The condensate of magnetoexcitons should have superfluid properties under the Kosterlitz-Thouless critical temperature T_{KT} (Kosterlitz & Thouless, 1973). A possible verification of our predictions could be based on the fact that at a fixed superfluid density the phase stiffness and the KT critical temperature, both are inversely proportional to the magnetoexciton mass, i.e. $T_{KT} \propto M^{-1}(B)$. Since the effective mass increases as a function of the magnetic field, one should expect that at a fixed superfluid density the KT critical temperature decreases in the limit of large interlayer separation as $T_{KT} \propto B^{-2}$ (as $T_{KT} \propto B^{-1/2}$ in the limit of small interlayer separation).

3. Conclusion

We have applied the relativistic BS formalism to the magnetoexcitons in a graphene structures. In the regime of a strong magnetic field the electrons and holes are confined primarily to the lowest Landau Level (LLL), and the Coulomb energy is much smaller than the exciton cyclotron energy. The LLL approximation greatly simplify the calculations, but we may ask whether the magnetoexciton dispersion will be significantly affected by the contributions from the *infinity number* of Landau levels with indexes $n \geq 1$ neglected in the LLL approximation. Going beyond the LLL approximation is an ambitious task (see, e.g. (Shabad & Usov, 2006)) which will be left as a subject of our future research.

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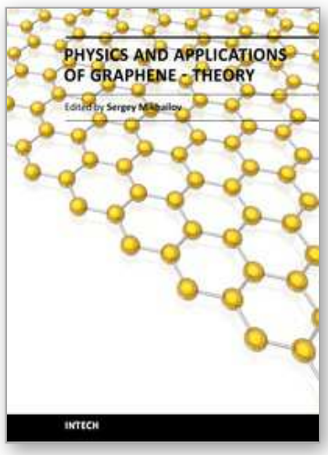
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The Stone Age, the Bronze Age, the Iron Age... Every global epoch in the history of the mankind is characterized by materials used in it. In 2004 a new era in material science was opened: the era of graphene or, more generally, of two-dimensional materials. Graphene is the strongest and the most stretchable known material, it has the record thermal conductivity and the very high mobility of charge carriers. It demonstrates many interesting fundamental physical effects and promises a lot of applications, among which are conductive ink, terahertz transistors, ultrafast photodetectors and bendable touch screens. In 2010 Andre Geim and Konstantin Novoselov were awarded the Nobel Prize in Physics "for groundbreaking experiments regarding the two-dimensional material graphene". The two volumes Physics and Applications of Graphene - Experiments and Physics and Applications of Graphene - Theory contain a collection of research articles reporting on different aspects of experimental and theoretical studies of this new material.

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InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

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