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A Warning to the Human-Factors Engineer: False Derivations of Riesz's Weber Fraction, Piéron's Law, and Others Within Norwich et al.'s Entropy Theory of Perception

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1. Introduction

The Entropy Theory of Perception of Professor K.H. Norwich and various collaborators spans 1975 to 2010. Thirty-five years is a surprisingly long publication life for a mathematical model of perception. Its longevity is no doubt related to its unusual ability to provide derivations, *from pure theory*, of a large cadre of well-established empirical relations of psychophysics, referred to by Norwich et al. as „laws". Norwich et al.'s work involves computational biology, because they always started their derivations using the same base equation, whose utility was justified through curve-fitting to various kinds of empirical data (see Norwich, 1993). Norwich et al. intended the scope of their theory to be vast, and so the present paper focuses on just one particular set of derivations. Few people offer first-principles derivations of natural relations, making such derivations all the more demanding of our attention.

At „Fechner Day 96", the 12th Annual Meeting of the International Society for Psychophysics, W. Wong and K.H. Norwich introduced „Weber fraction and reaction time from the neural entropy". Therein they showcased what appeared to be two long-overdue breakthroughs: the first-principles derivation of Riesz's equation for auditory just-noticeable intensity differences (Riesz, 1928), and the first-principles derivation of Piéron's empirical relation for the minimum time required for a human subject to signal their perception of a newly presented auditory or visual stimulus (Piéron, 1952). The Wong and Norwich breakthroughs were allowed by a „universal model of single-unit sensory receptor action" presented in a previous paper (Norwich & Wong, 1995). The present paper summarizes the latter work, as an introduction to Norwich et al.'s Entropy Theory, and then proceeds to scrutinize the Wong and Norwich (1996) derivations. In so doing, it reveals that, unfortunately, the algebra of Wong and Norwich (1996) conceals hidden assumptions, unjustified simplifications, and outright mistakes, which had not been brought to public attention. The problems prove to be uncorrectable. This is all the more important because several of the derivations produced in Wong and Norwich (1996) were presented again later, in Wong and Figueiredo (2002). The problems with the Wong and Norwich (1996) and Wong and Figueiredo (2002) derivations are instructional, as they illustrate just how easily

misleading outcomes can arise in the kind of theory work that underlies computational biology. They also show what happens when models are made which ignore the warning by William of Ockham (c. 1285-1349) that „entities must not be multiplied beyond necessity".

2. The empirical equations for which Wong and Norwich (1996) provided first-principles derivations

Let us briefly review the empirical relations that Wong and Norwich (1996) claim to derive, starting with that of Riesz (1928). First, some crucial experimental details. Riesz's subjects listened to two simultaneous sinusoidal pressure waveforms whose frequencies were sufficiently close to produce an audible oscillating intensity apparently at a single carrier frequency, the well-known phenomenon called „beating". Early experiments by Riesz had revealed that listeners were most sensitive to a frequency of oscillation – the „beat frequency" – of three per second. Therefore, listeners indicated their threshold intensity change, ΔI, for just barely detecting three beats/second, as a function of the base intensities of the component sinusoids. The dependence of beat-detection threshold upon base intensity I was plotted by Riesz, who then fitted an equation to the plot. Riesz's empirical equation was

$$\frac{\Delta I}{I} = S_\infty + \left(S_0 - S_\infty\right)\left(\frac{I_0}{I}\right)^r, \quad \text{where} \quad S_\infty, S_0, I_0, r > 0. \tag{1}$$

The terms S_∞, S_0, r are empirical constants required to have no physical units, constants for which Riesz supplied approximate empirical relations as a function of tone frequency. The term $(\Delta I)/I$ is well-known in psychophysics as the *Weber fraction*. Eq. 1 will be referred to as Riesz's Weber fraction or, as done in various Entropy Theory papers, as *Riesz's Law*.

Wong and Norwich (1996) (and later Wong and Figueiredo, 2002) also claimed to derive an empirical relation for auditory absolute detection thresholds.[1] Using I_{th} as the threshold stimulus intensity, I_∞ as the detection threshold for a stimulus that is infinitely long (or its equivalent practical duration), t as the actual stimulus duration, and „a" as an empirical constant (whose value may be dependent upon tone frequency), the threshold relation of interest is

$$I_{th} = \frac{I_\infty}{1 - e^{-a\,t}}, \quad \text{where} \quad I_\infty, a > 0. \tag{2}$$

In this equation, „I" refers to physical intensity, not decibels (taking ten times the logarithm to base 10 of I gives decibels). From Eq. 2, Wong and Norwich derived *Bloch's Law*, which states that for short light flashes, the stimulus intensity multiplied by the stimulus duration yields the same brightness, at absolute detection threshold and above: $I \cdot t = \text{constant}$, phrased by Wong and Norwich as $\Delta I \cdot \Delta t = \text{constant}$ (e.g., Brindley, 1952).

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[1] Wong and Norwich (1996) attributed the empirical relation to Zwislocki (1960); later Wong and Figueiredo (2002) attributed it to Plomp and Bouman (1959) as well as to Zwislocki (1960). Plomp and Bouman (1959), in turn, noted that the equation had already been published by Feldtkeller and Oetinger (1956).

Wong and Norwich also derived Piéron's empirical relation for reaction time (Piéron, 1952), which is as follows. Using $t_{r,\min}$ as the shortest possible time of reaction, and with empirical constants „A" and „n", *Piéron's Law* is

$$t_r = t_{r,\min} + \frac{A}{I^b}, \quad \text{where} \quad t_{r,\min}, A, b > 0 \tag{3}$$

(Piéron, 1952, pp. 352-353).

In order to understand how Wong and Norwich (1996) derived the preceding psychophysical laws, an earlier paper, Norwich and Wong (1995), had to be consulted. Norwich and Wong (1995) reviewed the basic ideas of the Entropy Theory of Perception, and that summary is reproduced *in brevia* below.

3. Background: The Entropy Theory of Perception

Norwich and Wong (1995) summarized the Entropy Theory of Perception, as follows. They first explained that „All modalities of sensation conduct information from the outside world to the nervous system of the sensating organism" (*ibid.*, p. 84). They also explained that, in crafting the Entropy Theory, „We also utilize Shannon's doctrine that information represents uncertainty or entropy which has been dispelled" (*ibid.*, p. 84). The Shannon in question is Claude Shannon, author of „A mathematical theory of communication" (Shannon, 1948), the paper that launched Information Theory as a major theme within communications science. Norwich and Wong continue: „That is, immediately following the start of a sensory stimulus, the receptor is maximally uncertain about the intensity of its stimulus signal and, hence, has received very little information. As time proceeds, the receptor loses uncertainty (that is, decreases its information theoretical entropy), and, therefore, gains information" (*ibid.*, p. 84). The entropy in question, which Norwich and Wong represented by the symbol H, was then developed by Norwich and Wong from theory and assumptions, synopsized over several pages of algebra that need not be repeated here. Finally, Norwich and Wong introduced „The fundamental assumption of the entropy theory of sensation" (*ibid.*, p. 86), which was „that the impulse frequency F in the primary afferent neuron issuing from a sensory receptor is in direct proportion to the entropy *H*, that is, *F = kH*" (*ibid.*, p. 86; original italics). Note that H has no physical units, so that any physical units of F must be those of k. Norwich and Wong then made the implicit assumption that F also represents the sensation experienced by the organism, albeit with a probably different value (and a lack of physical units) for k.

As Norwich and Wong (1995, p. 86) explained, „We take the receptor to be *sampling* its stimulus signal". The number of samples taken was assigned to symbol „m", the „receptor memory", where by conjecture m = αt for some positive constant α. Within the Entropy Theory, it was always understood that $m \geq 0$. Altogether, using the symbol β' to represent „a constant parameter of unknown value but greater than zero" (*ibid.*, p. 87), the sensation F as a function of I, the intensity of a steady stimulus, was given by what Norwich and Wong called the „seminal" equation

$$F = \frac{1}{2} k \ln \left(1 + \frac{\beta' I^n}{t} \right), \quad \text{such that} \quad t \geq t_0, \text{ and } t_0, \beta', n > 0, \tag{4}$$

where t_0 = the time required for one sampling, and β' and n are unknowns.

This equation is identified elsewhere by Norwich and co-authors as the Entropy Equation. Norwich and Wong noted a failing of Eq. 4, viz., it predicts that all sensation disappears over time, i.e. that there is complete adaptation to a maintained stimulus. Empirically, however, sensory adaptation is not always complete. Therefore a maximum value of t, called t_{max}, was proposed which, when substituted into Eq. 4, allows a non-zero asymptotic value of sensation. Norwich and Wong also introduced greater sophistication to their model by changing their equation for m, the number of samples of the stimulus taken by the sensory receptor, as will now be described.

4. A necessary preamble: The Entropy Theory under the „relaxation model" for receptor memory (Norwich & Wong, 1995)

Without providing any physiological rationale, Norwich and Wong (1995) conjectured that

$$\frac{dm}{dt} = - a\left(m - m_{eq}\right), \quad \text{where } m_{eq} = m(\infty), \ a > 0 \tag{5}$$

(Norwich & Wong, 1995, Eq. 24; Wong & Norwich, 1996, Eq. 2). The unknown constant „a" is the same one as in Eq. 2, as will be shown. Norwich and Wong also introduced the unknown constants m_{eq}^0 and q and declared (without proof) that m was a power function of intensity,

$$m_{eq} = m_{eq}^0 I^q \quad \text{where} \quad m_{eq}^0, \ q > 0 \tag{6}$$

(Norwich & Wong 1995, Eq. 28). Note that the superscript 0 is not an exponent. Norwich and Wong then stated, again without proof, that if the receptor memory at the start of a new steady stimulus is called $m(0)$, then

$$m(t) = m(0) e^{-at} + m_{eq}\left(1 - e^{-at}\right), \quad \text{where} \quad m(t) \geq 0 \tag{7}$$

(Norwich & Wong, Eq. 25). Norwich and Wong then introduced (without explanation) the constant β''. Altogether, sensation now follows

$$F = \frac{1}{2} k \ln\left(1 + \frac{\beta'' I^n}{m(0) e^{-at} + m_{eq}\left(1 - e^{-at}\right)}\right) \tag{8}$$

(Norwich & Wong, Eq. 33), which allows for „cases of successively applied step functions in stimulus intensity" (Norwich & Wong, p. 95).[2] Note that when Eq. 6 for m_{eq} is substituted

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[2] β'' appears to have the same meaning and value as β' in Eq. 4, which begs the question of why Norwich and Wong changed the notation. The reader will find that the Entropy Theory papers contain many such confusing changes in notation, none of them apparently needed, and none of them explained.

into Eq. 8, the latter contains 7 unknowns: k, β'', n, $m(0)$, m_{eq}^0, q, and a. Eq. 8 was the major result presented by Norwich and Wong.

Norwich and Wong (1995) noted that „for the case where the receptor is initially de-adapted (or adapted to a very small stimulus)", $m(0) \cong 0$ (*ibid.*, p. 94), so that

$$m(t) = m_{eq}\left(1 - e^{-at}\right),\tag{9}$$

where $m_{eq} > 0$ and   where $\left(1 - e^{-at}\right) \geq 0$ for $t \geq 0$

(Norwich & Wong, 1995, Eq. 26). Eq. 9 was an element of „our new, generalized entropy equation for the single, step stimulus" (*ibid.*, p. 94),

$$F = \frac{1}{2}k\ln\left(1 + \frac{\beta'' I^n}{m_{eq}\left(1 - e^{-at}\right)}\right),\quad \text{where}\quad k,\ \beta'',\ n,\ m_{eq},\ \left(1 - e^{-at}\right)\ > 0\tag{10}$$

(Norwich & Wong, 1995, Eq. 27). Here I and t are the physical quantities; all of the other letters represent unknown constants. As time increases, $m \rightarrow m_{eq}$ so that F approaches a nonzero limit, as Norwich and Wong (1995) had desired.

5. A missing derivation of the Norwich and Wong (1995) equation for receptor memory

Unfortunately, Norwich and Wong (1995) did not clarify the origin of the equation labeled above as Eq. 7. That equation is clearly crucial to the new Entropy Equation, Eq. 8, so let us try to understand Eq. 7, for example by solving Eq. 5. Some boundary conditions must be presumed, so let us reasonably presume that m has some minimum $m_0 = m(t_0)$ where $t_0 \geq 0$. We obtain

$$\int_{m_0}^{m(t)} \frac{dm}{m - m_{eq}} = -a\int_{t_0}^{t} dt,\qquad \therefore\ m(t) = m_0\, e^{-a\left(t - t_0\right)} + m_{eq}\left(1 - e^{-a\left(t - t_0\right)}\right).\tag{11}$$

Eq. 11 allows $m_{eq} = m(\infty)$, as required from Eq. 5. The term $m_0 = m(t_0)$ will be dealt with below. If the first sample is taken at $t = 0$, such that the minimum number of samples occurs at *t = 0*, then $t_0 = 0$. Eq. 11 then becomes

$$m(t) = m_0\, e^{-at} + m_{eq}\left(1 - e^{-at}\right),\quad t \geq 0,\tag{12}$$

which resembles Eq. 7. (Recall that Norwich and Wong defined t_0 as the time required for one sampling, a definition that was repeated by Wong and Norwich, 1996, p. 432.)

6. The Entropy Equation of Wong and Norwich (1996)

The Norwich and Wong (1995) new, generalized Entropy Equation for the single, step stimulus (Eq. 10) has a problem that has existed from the start of the Entropy Theory

(Norwich, 1975). That is, $F \to \infty$ as $t \to 0$. This problem was finally addressed by Wong and Norwich (1996). Bearing in mind their own conjecture that F is proportional to H, Wong and Norwich rewrote the entropy H by introducing $I(t)$, „an intensity input which may vary with time" (Wong & Norwich, 1996, p. 429), and δI, „an internal signal greater than threshold" (*ibid*.), to give

$$H = \frac{1}{2} \ln\left(1 + \frac{\beta \cdot (I(t) + \delta I)^{p}}{m(t)}\right), \quad where \quad \beta, t, I(t), \delta I, p, m(t) > 0 \tag{13}$$

(Wong & Norwich, 1996, Eq. 1). Wong and Norwich explained that „$m(t)$ represents the dynamic memory required to store stimulus samples drawn by the receptor" (Wong & Norwich, 1996, p. 430). The purpose of δI was not explained.

Eq. 13 displayed a number of oddities. For example, Wong and Norwich replaced the exponent n of Eq. 4 with the exponent p. The two are equivalent, but no rationale was offered for the change in notation. Wong and Norwich (1996, p. 429) also made the mysterious statement that Eq. 13 was proposed in Norwich and Wong (1995), where in fact it does not appear. They also made the remarkable declaration that Eq. 13 „is capable of accounting for almost all sensory phenomena, empirical laws, and rules of thumb relating the firing rate of the primary afferent neuron to the intensity and duration of the sensory stimulus" (*ibid*.). Wong and Norwich also declared that Eq. 13 has „five parameters" (*ibid*.). However, a closer look reveals 7 unknowns: β, p, δI, $m_0 = m(t_0)$, m_{eq}^0, q, and a. Finally, the origin of δI was not explained until later, by Wong and Figueiredo (2002). As they noted (*ibid*., p. ICAD02-2), „δI is a term that accounts for the non-zero fluctuations at the receptor level in the absence of a signal" (original italics). Adopting δI helps to remove the infinity in H which occurs as $t \to 0$, as will be made apparent in the following explanations. Wong and Norwich (1996) then dealt with an experimental paradigm in which the subject must „detect a continuous increment in intensity" for which „the initial pedestal of duration τ may be considered much longer than the duration of increment, Δt [*sic*]. Indeed, τ may be made great enough to insure complete neural adaptation" (Wong & Norwich 1996, p. 431; original italics). Without proof, Wong and Norwich presented the relevant solution of Eq. 5,

$$m(t') = (I + \delta I)^{q} e^{-a(t' - \tau)} + (I + \Delta I + \delta I)^{q}\left(1 - e^{-a(t' - \tau)}\right), \tag{14}$$

where presumably $\tau > 0$. Comparison of Eq. 14 to Eq. 7 implies that m for the pedestal alone follows

$$m(t) = m_0 e^{-a(t - t_0)} + (I + \delta I)^{q}\left(1 - e^{-a(t - t_0)}\right), \quad t \geq t_0, \tag{15}$$

and that m_{eq} for the pedestal-plus-increment obeys

$$m_{eq}^0 = 1, \quad m_{eq} = (I + \Delta I + \delta I)^{q}. \tag{16}$$

7. The derivation of $\Delta I/I$ by Wong and Norwich (1996)

Wong and Norwich (1996) did not explain why they replaced t by $t'-\tau$. It can eventually be understood by following the principal argument offered by Wong and Norwich, that the intensity increment ΔI of duration Δt starting at time τ „can be detected if and only if"

$$H\left(I+\Delta I,\tau+t_0\right)-H\left(I+\Delta I,\tau+\Delta t\right)\geq \Delta H \quad \text{for the interval } \{\tau,\tau+\Delta t\}. \tag{17}$$

Wong and Norwich replaced „$\geq$" by „$=$" in Eq. 17, and set out to evaluate the resulting equality using Eqs. 13 and 14. Note that Eq. 17 inherently contains 7 parameters whose values are unknown: β, δI, p, $m_0 = m(t_0)$, q, a, and ΔH. Nonetheless, Wong and Norwich (1996) adopted the assumptions that $\Delta H << 1$, that $\Delta I << (I+\delta I)$, and that $t_0 << \Delta t$. Wong and Norwich consequently used series expansions to first order in ΔH and $\Delta I/(I+\delta I)$. The series expansion to first order in ΔH was not shown by Wong and Norwich; it gives

$$e^{2\Delta H} \approx 1+2\Delta H. \tag{18}$$

The series expansion to first order in $\Delta I/(I+\delta I)$ was also not shown by Wong and Norwich; it yields

$$\left(I+\Delta I+\delta I\right)^p = \left(I+\delta I\right)^p\left(1+\frac{\Delta I}{I+\delta I}\right)^p \approx \left(I+\delta I\right)^p\left(1+\frac{p\Delta I}{I+\delta I}\right). \tag{19}$$

The solution to Eq. 17 that was offered by Wong and Norwich was

$$\frac{\Delta I}{I} = \frac{2\Delta H}{q\left(1-e^{-a\Delta t}\right)}\left(1+\frac{\delta I}{I}\right)\left(1+\frac{1}{\beta\left(I+\delta I\right)^n}\right), \quad \text{where } n = p-q. \tag{20}$$

Note that Wong and Norwich re-introduced the term n, which they first used in Eq. 4 and then had replaced, without explanation, by the letter p. The present n was implied by Wong and Norwich to be a positive number, which will turn out to be very important. Eq. 20 has 6 unknowns - ΔH, q, a, δI, β, and n – and one supplied parameter, Δt. Wong and Norwich then held Δt constant and assumed that $\delta I << I$, so that δI can be set to zero, and arrived at

$$\frac{\Delta I}{I} = A\left(1+\frac{B}{I^n}\right) \tag{21}$$

$$\text{where } A = \frac{2\Delta H}{q\left(1-e^{-a\Delta t}\right)}, \quad B = \frac{1}{\beta}$$

(Wong & Norwich, 1996, Eq. 11; Wong & Figueiredo, 2002, Eq. 12) which „is identical to the empirical Knudsen-Riesz Weber fraction equation" (Wong & Norwich, 1996, p. 432).
Returning to Eq. 20, when $I \rightarrow 0$, such that $\delta I >> I$ and thus $\delta I/I >> 1$, Wong and Norwich obtained

$$\frac{\Delta I}{I} = \frac{C}{I},$$

(22)

$$\text{where } C = \frac{2\Delta H \delta I}{q\left(1 - e^{-a\Delta t}\right)}\left(1 + \frac{1}{\beta(\delta I)^n}\right)$$

(Wong & Norwich, 1996, Eq. 12; Wong & Figueiredo, 2002, Eq. 14). Wong and Norwich (1996) claimed that this relation characterizes the empirical Weber fraction „as the pedestal intensity is made smaller and smaller" (*ibid.*, p. 432), a phenomenon said to be „observed by many investigators" (*ibid.*, p. 433; no citations supplied). Eq. 20 can also be rearranged under the assignment *I=0*, giving

$$\Delta I = \frac{I_\infty}{1 - e^{-a\Delta t}},$$

(23)

$$\text{where } I_\infty = \frac{2\Delta H \delta I}{q}\left(1 + \frac{1}{\beta(\delta I)^n}\right)$$

(Wong & Norwich, 1996, Eq. 13; Wong & Figueiredo, 2002, Eq. 15), which Wong and Norwich recognized as Zwislocki's (1960) empirical relation for auditory absolute detection thresholds. (Note that Wong and Norwich implicitly adopted the old but unqualified notion that the absolute detection threshold is a just-noticeable intensity increment.) The reader may note the similarity of the constants C and I_∞; the latter was written out in Wong and Norwich, whereas the former was described only as „a constant" (*ibid.*, p. 433). Eqs. 22 and 23 are in fact the same, a point not made by Wong and Norwich.

Wong and Norwich (1996) then expanded the denominator of Eq. 23 to first order under the assumption that $\Delta t << 1/a$, so that Eq. 23 became $\Delta I \cdot \Delta t$=constant. The latter appeared as an unnumbered equation in Wong and Norwich (1996) and as Eq. 16 of Wong and Figueiredo (2002), and was described as Bloch's Law. Of course, under these circumstances, it can only describe threshold phenomena, not above-threshold phenomena also (viz. the original Bloch's Law).

8. A re-derivation of Δ*I/I*: some disagreements with Wong and Norwich (1996)

We may start with Eq. 17 and attempt to recreate the derivation described by Wong and Norwich. The resulting equation contains 10 elements: the 6 unknowns, ΔH, q, a, δI, β, and p, the 2 supplied parameters t_0 and Δt, the dependent variable (Δ*I*)/I, and the independent variable I. In order to achieve a solution resembling Eq. 20, a further assumption had to be made that was not mentioned by Wong and Norwich, viz., that t_0=0. This reduced the total number of elements from 10 to 9. Altogether, the solution found was

$$\frac{\Delta I}{I} = \frac{2\Delta H}{q\left(1 - e^{-a\Delta t}\right)}\left(1 + \frac{\delta I}{I}\right)\left(1 + \frac{1}{\beta(I + \delta I)^n}\right) \Bigg/ \left(1 - \frac{2\Delta H}{\beta(I + \delta I)^n} - \frac{2p\Delta H}{q\left(1 - e^{-a\Delta t}\right)}\right).$$

(24)

Despite the Wong and Norwich assumption that $\Delta H << 1$, Eq. 24 can only be reconciled with the Wong and Norwich solution, Eq. 20, if Wong and Norwich had simply chosen to ignore

the second and third terms in the denominator on the right-hand-side of Eq. 24. Such a simplification is unjustified, however, because q, a, δI, β, and p (= n+q) remain unknown. This is still a problem even if a range of values was specified for I (which was not done). Values for such unknowns had traditionally been obtained by Norwich and co-authors through fitting of equations to other people's data, but no such parameters were provided by Wong and Norwich. (In any case, curve-fitting is not measurement.)

Let us examine the derivations made by Wong and Norwich, but now using Eq. 24 in its entirety rather than Eq. 20. First, let us hold Δt constant and assume that $\delta I \ll I$, so that δI can be set to zero. This yields

$$\frac{\Delta I}{I} = A\left(1 + \frac{B}{I^n}\right) \Big/ \left(1 - Ap - \frac{2B\Delta H}{I^n}\right), \tag{25}$$

$$\text{where} \quad A = \frac{2\Delta H}{q\left(1 - e^{-a\Delta t}\right)}, \quad B = \frac{1}{\beta}.$$

This is not quite the equation for the „empirical Knudsen-Riesz Weber fraction". To recover that particular equation, the denominator in Eq. 25 would have to be unity alone.

Let us continue to follow the Wong and Norwich derivations. Now, letting $I \to 0$ gives

$$\frac{\Delta I}{I} = \frac{C'}{I}, \tag{26}$$

$$\text{where} \quad C' = \frac{2\Delta H \delta I}{q\left(1 - e^{-a\Delta t}\right)}\left(1 + \frac{1}{\beta(\delta I)^n}\right) \Big/ \left(1 - \frac{2\Delta H}{\beta(\delta I)^n} - \frac{2p\Delta H}{q\left(1 - e^{-a\Delta t}\right)}\right).$$

Eq. 26 has the same form as Eq. 22, but the C' of Eq. 26 is not the C of Eq. 22.

We now leave Eq. 26 and return to Eq. 24, in order to continue to follow the Wong and Norwich derivations, and set *I=0*. After some rearrangement, we obtain

$$\Delta I = \frac{K}{L \cdot \left(1 - e^{-a\Delta t}\right) - M}, \tag{27}$$

$$\text{where} \quad K = 2\Delta H \cdot \delta I \cdot \left(1 + \frac{1}{\beta(\delta I)^n}\right), \quad L = q\left(1 - \frac{2\Delta H}{\beta(\delta I)^n}\right), \quad M = 2p\Delta H.$$

This is not quite Zwislocki's (1960) equation for absolute detection thresholds, as Eq. 23 was claimed to be. Note that Eq. 27 is in fact the same as Eq. 26, just as Eq. 23 was the same as Eq. 22.

Finally, continuing from Eq. 27 and following the Wong and Norwich (1996) assumption that $\Delta t \ll 1/a$, we obtain

$$\Delta I \cdot \left(\Delta t - \frac{M}{aL}\right) = \frac{K}{aL}, \tag{28}$$

which is certainly not Bloch's Law, $\Delta I \cdot \Delta t = \text{constant}$.

9. The derivation of Piéron's Law by Wong and Norwich (1996)

In the Wong and Norwich (1996) derivation of Piéron's Law, they started fresh with Eq. 17, solving it for Δt under the conditions that (1) the pedestal (background) stimulus intensity was zero ($I=0$), so that the only stimulus intensity was ΔI; (2) $\Delta H<<1$; and (3) $\delta I<<\Delta I$. However, Wong and Norwich abandoned their earlier approximation $t_0\approx0$ (used above), although they did not explain why. They also expanded in "zeroth order" in $\delta I/\Delta I$. Altogether they obtained

$$\Delta t = \Delta t_{\min} \cdot \left(1 + \frac{1}{\zeta \cdot (\Delta I)^n} \right), \tag{29}$$

where $\Delta t_{\min} = \dfrac{2\Delta H}{a}\left(e^{a t_0} - 1\right)$ and ζ was unspecified.

As before, Wong and Norwich implied that $n = p - q$ is a positive number. When a constant representing physiological motor-response time was added to both sides of Eq. 29, the resulting equation was identified by Wong and Norwich as Piéron's Law for reaction time.

10. A re-derivation of Piéron's Law: some disagreements with Wong and Norwich (1996)

The present author attempted to retrace the Wong and Norwich derivation, starting right back with Eq. 17. It was immediately noted that „a" could only appear as a free parameter under the approximation, not mentioned by Wong and Norwich, that $e^{-a\Delta t} \approx 1 - a\Delta t$. Evaluating Eq. 17 using this approximation, and the approximations outlined by Wong and Norwich, leads to an equation containing 9 elements: the 6 unknowns, ΔH, q, a, δI, β, and p; the supplied parameter t_0; the dependent variable Δt; and the independent variable ΔI,

$$\Delta t = \frac{1}{a}\left(\frac{C_1 (\Delta I)^n + C_2}{C_3 (\Delta I)^n + C_4} \right), \tag{30}$$

where

$$C_1 = -\beta\left(1+2\Delta H\right)\left(e^{-a t_0}-1\right) + \beta\left[2\Delta H + \left(e^{-a t_0}-1\right)\left(1+2\Delta H\right)\right]\left(\frac{\delta I}{\Delta I}\right)^q,$$

$$C_2 = \left(\frac{\delta I}{\Delta I}\right)^q\left[\left(e^{-a t_0}-1\right) - \frac{\left(e^{-a t_0}-1\right)\left(1+2\Delta H\right)}{(\Delta I)^q} + 2\Delta H\left[1+\left(e^{-a t_0}-1\right)\right]\left(\frac{\delta I}{\Delta I}\right)^q\right],$$

$$C_3 = \beta\left[1-\left(\frac{\delta I}{\Delta I}\right)^q\right],$$

$$C_4 = 2\Delta H\left(e^{-a t_0} - 1\right) + \left(\frac{\delta I}{\Delta I}\right)^q \left[\begin{array}{l} \left(e^{-a t_0} - 1\right) - \dfrac{a}{(\Delta I)^q}\left[\left(1 + 2\Delta H\right)\left(2e^{-a t_0} - 1\right)\right] \\ + \left(\dfrac{\delta I}{\Delta I}\right)^q \left[2\Delta H\, e^{-a t_0}\right] \end{array}\right].$$

Eq. 30 is not the same as Eq. 29. If we assume, simply for the sake of exploration, that the term $(\delta I/\Delta I)^q$ is ignorable, then Eq. 30 simplifies to

$$\Delta t = -\left(1 + 2\Delta H\right)\left(e^{-a t_0} - 1\right) \Bigg/ \left(a\left[1 + \frac{2\Delta H\left(e^{-a t_0} - 1\right)}{\beta(\Delta I)^n}\right]\right). \tag{31}$$

This is still not Eq. 29. Note especially that in Eq. 31, n is in the denominator of a denominator, rendering n overall of opposite sign in Eq. 31 to that in Eq. 29. This difference is crucial; recall that n was defined by Wong and Norwich as being greater than zero, just like the exponent of Piéron's Law (Eq. 3), so that the exponent n in Eq. 29 is a positive number, as required. We can also now see a possible reason why Wong and Norwich abandoned their earlier approximation $t_0 \approx 0$; letting $t_0 = 0$ in Eq. 31 results in $\Delta t = 0$, which would imply instantaneous reactions.

Let us see what happens when we start from Eq. 30 under the assumption that $t_0 = 0$, but that $(\delta I/\Delta I)^q$ is *not* ignorable. Then

$$C_1 = 2\Delta H\beta\left(\frac{\delta I}{\Delta I}\right)^q, \quad C_2 = 2\Delta H\left(\frac{\delta I}{\Delta I}\right)^{2q},$$

$$C_4 = \left(\frac{\delta I}{\Delta I}\right)^q\left[-\frac{a}{(\Delta I)^q}\left(1 + 2\Delta H\right) + \left(\frac{\delta I}{\Delta I}\right)^q\left(2\Delta H\right)\right],$$

and Eq. 30 simplifies somewhat to

$$\Delta t = \frac{2\Delta H\left(\dfrac{\delta I}{\Delta I}\right)^q\left[\beta(\Delta I)^n + \left(\dfrac{\delta I}{\Delta I}\right)^q\right]}{a\left(\beta\left[1 - \left(\dfrac{\delta I}{\Delta I}\right)^q\right](\Delta I)^n + \left(\dfrac{\delta I}{\Delta I}\right)^q\left[-\dfrac{a}{(\Delta I)^q}\left(1 + 2\Delta H\right) + \left(\dfrac{\delta I}{\Delta I}\right)^q\left(2\Delta H\right)\right]\right)}, \tag{32}$$

which is still not Eq. 29.

It transpires there is indeed a way to obtain an equation of the form of Eq. 29 starting from Eq. 30, but some imagination is needed. Let us make the assumption that led to Eq. 31 – that is, that $(\delta I/\Delta I)^q$ is ignorable – and let us further assume the series approximation

$$\frac{1}{1 + (2\Delta H/\beta)\left(e^{-a t_0} - 1\right)(\Delta I)^{-n}} \cong 1 - \frac{(2\Delta H/\beta)\left(e^{-a t_0} - 1\right)}{(\Delta I)^n} \tag{33}$$

$$for \quad \left| \left(2\Delta H / \beta \right) \left(e^{-a\,t_0} - 1 \right) \left(\Delta I \right)^{-n} \right| < 1 .$$

The latter move transforms Eq. 31 to

$$\Delta t = \frac{\left(1 + 2\Delta H \right) \left(1 - e^{-a\,t_0} \right)}{a} \left[1 + \frac{2\Delta H \cdot \left(1 - e^{-a\,t_0} \right)}{\beta \cdot \left(\Delta I \right)^n} \right] , \tag{34}$$

which looks like Piéron's Law as expressed in Eq. 29 if $\left(1 - e^{-a\,t_0} \right) > 0$, which is guaranteed from Eq. 9 for any $t_0 > 0$, and also if Δt_{min} is defined as $\left(1 + 2\Delta H \right) \left(1 - e^{-a\,t_0} \right) / a$ and if ζ is defined as $\zeta = \beta / \left[2\Delta H \cdot \left(1 - e^{-a\,t_0} \right) \right]$. Wong and Norwich did not mention using the simplification that is shown here in Eq. 33; justifying that simplification would require knowing the values of the 5 unknowns ΔH, q, a, β, and p (=n+q), as well as the value of the supplied parameter t_0, and all relevant values of the independent variable ΔI. Wong and Norwich could not supply that knowledge, but apparently they made the approximation nonetheless.

Finally, the arbitrariness of the above derivation of Piéron's Law can be demonstrated by returning to Eq. 17 and re-simplifying it under some starting assumptions that are similar, and some that are different. That is, let us assume that $e^{-a\Delta t} \approx 1 - a\Delta t$, as used above, and also that $\delta I = 0$ (the equivalent of the later assumption, used above, that $(\delta I/\Delta I)^q$ is ignorable). Now, instead of following the Wong and Norwich conjecture that $e^{2\Delta H} \approx 1 + 2\Delta H$ (Eq. 18), made under the unsupported Wong and Norwich assumption $\Delta H \ll 1$, let us instead make the equally unsupported assumptions that

$$-1 < \frac{\beta \cdot \left(I + \Delta I + \delta I \right)^p}{m\left(\tau + t_0 \right)} , \frac{\beta \cdot \left(I + \Delta I + \delta I \right)^p}{m\left(\tau + \Delta t \right)} \le 1$$

so that

$$\ln \left(1 + \frac{\beta \cdot \left(I + \Delta I + \delta I \right)^p}{m\left(\tau + t_0 \right)} \right) \cong \frac{\beta \cdot \left(I + \Delta I + \delta I \right)^p}{m\left(\tau + t_0 \right)} , \tag{35a}$$

and that

$$\ln \left(1 + \frac{\beta \cdot \left(I + \Delta I + \delta I \right)^p}{m\left(\tau + \Delta t \right)} \right) \cong \frac{\beta \cdot \left(I + \Delta I + \delta I \right)^p}{m\left(\tau + \Delta t \right)} . \tag{35b}$$

After some algebra, and finally letting $I=0$,

$$\Delta t = - \left(e^{-a\,t_0} - 1 \right) \Bigg/ \left(a \left[1 + \frac{2\Delta H \left(e^{-a\,t_0} - 1 \right)}{\beta \left(\Delta I \right)^n} \right] \right) . \tag{36}$$

Note that only the lack of the multiplier $1+2\Delta H$ differentiates Eq. 36 from Eq. 31, whose derivation involved a partly different set of assumptions! This exercise should make clear the arbitrariness of the Wong and Norwich derivations.

Altogether, we can fairly say that Wong and Norwich did not actually derive Piéron's Law.

11. Summary

Wong and Norwich (1996) (repeated in Wong and Figueiredo, 2002) proposed an equation in seven unknowns to describe how the information-theoretic *entropy* of sensation will decrease over time for a quiescent sensory receptor suddenly exposed to a step base intensity followed by a superimposed step increment. Hypothetically, the increment can only be detected if the change in sensory entropy over the increment's duration equals or exceeds some minimum entropy change (of unknown size). When the entropy change equals the minimum, such that the intensity increment is just detectable, an equation in six unknowns emerges. Wong and Norwich rearranged that equation, under simplifying assumptions about the magnitudes of some of the unknowns, to give the hypothetical just-detectable intensity increment divided by the base intensity, the so-called Weber fraction. Further simplification from that point yielded an equation resembling an empirical relation proposed by Riesz (1928) for detection of beats. Similar manipulations by Wong and Norwich, but with the base intensity set to zero, gave an equation for the absolute detection threshold as a function of stimulus duration, which resembled an empirical equation of Zwislocki (1960). Simplifying that equation under yet another assumption about the values of the unknowns produced Bloch's Law, which states that stimulus duration multiplied by stimulus intensity is constant at the absolute detection threshold. A yet different set of assumptions about the unknowns was then made, from which Wong and Norwich obtained an equation relating the duration of a just-detectable stimulus to that stimulus' intensity, an equation said to be the empirical relation found by Piéron (1952) for reaction time to a stimulus as a function of stimulus intensity.

The Wong and Norwich (1996) derivations of empirical psychophysical relations from pure theory are remarkable. They deserve re-examination, and so the present author attempted to recreate the Wong and Norwich (1996) derivations. It first proved necessary to return to an earlier paper of theirs, Norwich and Wong (1995), in order to understand the origin of some of the Wong and Norwich (1996) starting equations. Following that, the derivation outlined by Wong and Norwich for the hypothetical Weber fraction was pursued. The Wong and Norwich version of the Weber fraction turns out to be missing two terms, terms that Wong and Norwich presumably ignored, perhaps in the hope that they would be too small to matter. Such a hope is unsupported because the parameters in the extra terms are all unknowns, and one term is intensity-dependent. The extra terms nullify the Wong and Norwich derivations of Riesz's Weber fraction, the Zwislocki relation, and Bloch's Law. Next to be pursued was the Wong and Norwich derivation of Piéron's Law, which relates the duration of a just-detectable stimulus to that stimulus' intensity. An equation eventually emerges whose intensity exponent is of opposite sign to that in Wong and Norwich's equation. The only way to arrive at Piéron's law is through a further unsupported assumption, one that Wong and Norwich did not even mention. Altogether, Wong and Norwich cannot fairly be said to have derived Piéron's Law.

12. Discussion

The Wong and Norwich (1996) derivations hid a number of conceptual problems that have not yet been mentioned because they were independent of the mathematical errors noted so far. Those conceptual problems are substantive and will now be described.

12.1 Hidden assumptions about receptor memory

In retrospect, the Wong and Norwich procedure of substituting Eq. 14 into Eq. 17 under the stimulus increment's starting time $t' = \tau + t_0$ and its finish time $t' = \tau + \Delta t$, followed by the unspoken assumption that $t_0 = 0$, is equivalent to using

$$m(t) = (I + \delta I)^q e^{-at} + (I + \Delta I + \delta I)^q \left(1 - e^{-at}\right) \tag{37}$$

for the number of samples associated with an increment of duration Δt starting at $t = 0$. Thus the number of samples at $t = 0$ is $(I + \delta I)^q$, which is the equilibrium number of samples for the pedestal stimulus alone, as can be seen by letting $t_0 = 0$ in Eq. 11. Letting $I = 0$, so that there is no pedestal stimulus, the starting number of samples for the increment ΔI is $(\delta I)^q$. This should equal the number that occurs just at the start of a stimulus of intensity ΔI, presented when the receptor is completely unadapted, such that the resting number of samples is zero. Thus $m_0 = m(t_0 = 0)$ from Eq. 9 must equal $(\delta I)^q$, although Wong and Norwich never say so. If the minimum nonzero number of samples is 1, then $(\delta I)^q = 1$, which is a hidden limit that makes δI and q covary. Hence, from Eq. 13, at the instant that a stimulus is applied (t=0) to an unadapted receptor, we have

$$F = kH = \frac{1}{2}k \ln\left(1 + \beta(\delta I)^p\right). \tag{38}$$

This initial value of sensation is not mentioned in Wong and Norwich (1996) or in Wong and Figueiredo (2002). F is hence *undefined*, rather than zero or constant, before the application of a stimulus to an unadapted receptor. This is another issue not mentioned in Wong and Norwich (1996) or in Wong and Figueiredo (2002). It seems to imply an extraordinary conclusion: that an unadapted receptor is nonetheless in a perpetual state of adaptation.

There is yet another unmentioned but significant issue. The intensity-dependence of detection thresholds for the kind of amplitude-modulated stimuli used by Riesz (1928) has not proven to be the same as the intensity-dependence of detection thresholds for step stimuli (see for example Wojtczak and Viemeister, 1999). Further, Riesz's modulation-detection thresholds fall monotonically with increase in base intensity. As such, they can be curve-fitted easily by the various versions of the Entropy Equation (above). In contrast, however, detection thresholds for step increments in single-frequency tones (e.g. Nizami et al., 2001, 2002; Nizami, 2006) or increments in auditory clicks (e.g., Nizami, 2005) do not fall monotonically with intensity; rather, they show, to a greater or lesser degree, a „mid-level hump" which cannot be easily fitted by the Entropy Equation.

12.2 Misassignment of stimulus duration as reaction time

In the Wong and Norwich derivation of Piéron's Law, they first defined Δt as the duration of a step in stimulus intensity. They then described it also as a property of a human observer, such as a reaction time! This duality is absurd on its face. Furthermore, the empirical dependence of reaction time upon stimulus intensity is always determined while stimulus duration is held constant, in order to remove duration as a possible confound. Hence Δt, defined as stimulus duration, would not vary. Norwich et al. (1989) presented an earlier derivation of Piéron's Law (repeated in Norwich, 1991) along the same lines as Wong and Norwich (1996), but using Eq. 4 rather than its later and more elaborate version, Eq. 8. The Norwich et al. (1989) derivation, like the later Wong and Norwich (1996) derivation, inappropriately equates reaction time to stimulus duration, and is therefore false.

12.3 The range of data described by the Wong and Norwich „laws"

Having read all that has been noted so far, some readers might still be tempted to believe that Wong and Norwich (1996) had derived psychophysical laws. If so, consider a crucial issue that has been saved for last: the range of data actually covered by the Wong and Norwich derivations. In deriving the Riesz Weber fraction, Wong and Norwich assumed that $\Delta H \ll 1$, that $\Delta I \ll (I+\delta I)$, and that $t_0=0$; they then held Δt constant and assumed that $\delta I \ll I$, operationally setting $\delta I=0$. The third of these assumptions was discussed above, and the first assumption has no obvious meaning. The second and fifth assumptions together combine to create the assumption that $\Delta I \ll I$, that is, that the Weber fraction is much less than unity. Empirically, however, ΔI in audition can be the same order of magnitude as I and can even exceed I. Under the Wong and Norwich assumption that $\Delta I \ll I$, the Weber fraction in decibels, defined as $10 \log_{10}[1+((\Delta I)/I)]$, will approach $10 \log_{10}1$, which is zero. Such infinitely fine auditory discrimination has never been recorded. Therefore, the Riesz Weber fraction derived by Wong and Norwich is unlikely to describe any real data.

The Zwislocki relation for absolute detection threshold was derived under the assumptions that $\Delta H \ll 1$, that $\Delta I \ll (I+\delta I)$, and that $t_0=0$, followed by the stipulation that $I=0$. Letting $\Delta I \ll (I+\delta I)$ and then $I=0$ amounts to the assumption that $\Delta I \ll \delta I$. For the resulting equation to be Zwislocki's threshold relation, as stated, then δI would have to be well above the absolute detection threshold. That notion was not noted by Wong and Norwich (1996).

The Wong and Norwich (1996) derivation of Bloch's Law continued from their derivation of Zwislocki's relation, under the further assumption that $\Delta t \ll 1/a$. Recall that „a" (Eq. 7) is an unknown constant said to characterize the rate at which samples build up in the sensory receptor's memory, and that Δt is the duration of the stimulus increment, here, the duration of the just-audible stimulus itself. Because "a" is unknown, there is no way of knowing under what circumstances $\Delta t \ll 1/a$ is obeyed, or indeed, whether there are any circumstances under which it is obeyed at all. For example, if $1/a$ is on the order of a few milliseconds, as would characterize a fast neuronal process in general, then Δt could well be too brief to represent any empirically detectable stimulus.

The Wong and Norwich derivation of Piéron's Law involved the assumptions that $I = 0$, $\Delta H \ll 1$, and $\delta I \ll \Delta I$. Regarding the last assumption, they in fact must have assumed that $\delta I = 0$, as mentioned. They also adopted the hidden assumption that $e^{-a\Delta t} \approx 1 - a \Delta t$. Altogether, then, there are limits upon ΔH, a, and Δt. Further, Wong and Norwich evidently used another hidden assumption, that $\left|(2\Delta H/\beta)\left(e^{-a t_0}-1\right)\left(\Delta I\right)^{-n}\right| < 1$. Along with the restrictions just mentioned, this places limits also upon β, t_0, n, and ΔI. Such a set of restrictions places mutual limits upon the values of Δt, Δt_{min}, ζ, ΔI, and n in Eq. 29. Piéron himself noted no such limitations (Piéron, 1952).

13. Conclusions

Wong and Norwich (1996) claimed to derive several important psychophysical laws, but in fact they did not. Their derivations involved indisputable mathematical errors. Those errors involved oversimplifications of Wong and Norwich's starting equation, the equation for the change in sensory entropy over the duration of an intensity increment. That equation has seven unknowns, about which a variety of assumptions were made in an effort to simplify the math, assumptions that were not justified by data or by theory. Indeed, one assumption

that was used in deriving Riesz's Weber fraction – that the starting time of an intensity increment was effectively zero - was then reversed in deriving Piéron's Law for reaction time, with no explanation given for the reversal. It turns out that without the reversal, the predicted reaction times are zero – a completely unrealistic situation.

The Wong and Norwich (1996) derivations also involve two serious conceptual errors. First, they made the hidden assumption of a nonzero receptor memory at the instant of the imposition of a stimulus to an unadapted receptor. That assumption is extraordinary, because it implies stimulus-driven neuronal activity in a quiescent receptor. They also made a second extraordinary assumption, viz., that the stimulus duration, a stimulus property, was identifiable with reaction time, an observer property. That assumption alone nullifies the Wong and Norwich derivation of Piéron's Law.

All of these problems remain unresolved, and resolution seems highly unlikely. Indeed, others have expressed profound doubts about the origin and meaning of the Entropy Equation itself, the equation on which Wong and Norwich (1996) ultimately based all of their algebra (e.g. MacRae, 1982; Ward, 1991; Laming, 1994; Ashby, 1995). Profound doubts have also been expressed elsewhere by the present author (Nizami 2008a, 2008b, 2009a, 2009b, 2009c, 2009d, 2009e, 2010).

Regardless, the flaws presently revealed stand alone, as a useful warning about the dangers of using equations in too many unknowns and then attempting to simplify those equations under arbitrary assumptions about the values of those unknowns. To their detriment, Wong and Norwich failed to heed the famous warning by William of Ockham (c. 1285-1349) that „entities must not be multiplied beyond necessity", inadvertently leaving a valuable lesson for the human-factors engineer who must likewise avoid needlessly complicated models of human perception.

14. Acknowledgements

15. References

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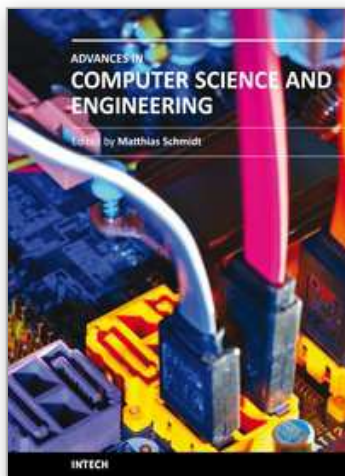
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