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# Performance Analysis of Maximal Ratio Diversity Receivers over Generalized Fading Channels

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## 1. Introduction

Diversity reception, is an efficient communication receiver technique for mitigating the detrimental effects of multi-path fading in wireless mobile channels at relatively low cost. Diversity combining is the technique applied to combine the multiple received copies of the same information bearing signal into a single improved signal, in order to increase the overall signal-to-noise ratio (SNR) and improve the radio link performance. The most important diversity reception methods employed in digital communication receivers are maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) (Simon & Alouini, 2005). Among these well-known diversity techniques, MRC is the optimal technique in the sense that it attains the highest SNR of any combining scheme, independent of the distribution of the branch signals since it results in a maximum-likelihood receiver (Simon & Alouini, 2005).

Of particular interest is the performance analysis of MRC diversity receivers operating over generalized fading channels, as shown by the large number of publications available in the open technical literature. The performance of MRC diversity receivers depends strongly on the characteristics of the multipath fading envelopes. Recently, the so-called $\eta$-$\mu$ fading distribution that includes as special cases the Nakagami-$m$ and the Hoyt distribution, has been proposed as a more flexible model for practical fading radio channels (Yacoub, 2007). The $\eta$-$\mu$ distribution fits well to experimental data and can accurately approximate the sum of independent non-identical Hoyt envelopes having arbitrary mean powers and arbitrary fading degrees (Filho & Yacoub, 2005).

In the context of performance evaluation of digital communications over fading channels this distribution has been used only recently. Representative past works can be found in (Asghari et al., 2010; da Costa & Yacoub, 2007; 2008; Ermolova, 2008; 2009; Morales-Jimenez & Paris, 2010; Peppas et al., 2009; 2010). For example, in (da Costa & Yacoub, 2007), the average channel capacity of single branch receivers operating over $\eta$-$\mu$ channels was derived. In (da Costa & Yacoub, 2008), expressions for the moment generating function (MGF) of the above mentioned channel were provided. Based on these results, the average bit error probability (ABEP) of coherent binary phase shift keying (BPSK) receivers operating over $\eta$-$\mu$ fading channels was obtained. Furthermore, in (da Costa & Yacoub, 2009), using an approximate yet highly accurate expression for the sum of identical $\eta$-$\mu$ random variables, infinite series representations for the Outage Probability and ABEP of coherent and non

coherent digital modulations for MRC and EGC receivers are presented. The derived infinite series were given in terms of Meijer-G functions (Prudnikov et al., 1986, Eq. (9.301)).

In this chapter we present a thorough performance analysis of MRC diversity receivers operating over non-identically distributed $\eta$-$\mu$ fading channels. The performance metrics of interest is the average symbol error probability (ASEP) for a variety of $M$-ary modulation schemes, the outage probability (OP) and the average channel capacity. The well known MGF approach (Simon & Alouini, 2005) is used to derive novel closed-form expressions for the ASEP of $M$-ary phase shift keying ($M$-PSK), $M$-ary differential phase shift keying ($M$-DPSK) and general order rectangular quadrature amplitude modulation (QAM) using MRC diversity in independent, non identically distributed (i.n.i.d) fading channels. The derived ASEP expressions are given in terms of Lauricella and Appell hypergeometric functions which can be easily evaluated numerically using their integral or converging series representation (Exton, 1976). Furthermore, in order to offer insights as to what parameters determine the performance of the considered modulation schemes under the presence of $\eta$-$\mu$ fading, a thorough asymptotic performance analysis at high SNR is performed. A probability density function (PDF) -based approach is used to derive useful performance metrics such as the OP and the channel capacity. To obtain these results, we provide new expressions for the PDF of the sum of i.n.i.d squared $\eta$-$\mu$ random variables. The PDF is given in three different formats: An infinite series representation, an integral representation as well as an accurate closed form expression. It is shown that our newly derived expressions incorporate as special cases several others available in the literature, namely those for Nakagami-$m$ and Hoyt fading.

## 2. System and Channel model

We consider an $L$-branch MRC receiver operating in an $\eta$-$\mu$ fading environment. Assuming that signals are transmitted through independently distributed branches, the instantaneous SNR at the combiner output is given by

$$\gamma = \sum_{\ell=1}^{L} \gamma_\ell \tag{1}$$

where $\gamma_\ell$ is the instantaneous SNR of the $\ell$-th branch.

The moment generating function (MGF) of $\gamma$, defined as $\mathcal{M}_\gamma(s) = \mathbb{E}\langle\exp(-s\gamma)\rangle$, with the help of (Ermolova, 2008, Eq. (6)) can be expressed as :

$$\mathcal{M}_\gamma(s) = \prod_{i=1}^{L} \int_0^\infty \exp(-s\gamma_i) f_{\gamma_i}(\gamma_i) d\gamma_i = \prod_{i=1}^{L} (1 + A_i s)^{-\mu_i} (1 + B_i s)^{-\mu_i} \tag{2}$$

where $A_i = \frac{\overline{\gamma_i}}{2\mu_i(h_i - H_i)}$ and $B_i = \frac{\overline{\gamma_i}}{2\mu_i(h_i + H_i)}$, $i = 1 \cdots L$.

In the following analysis, we first address the error performance of the considered system using an MGF-based approach. Moreover, the outage probability and the average channel capacity will be addressed using a PDF-based approach.

## 3. Error rate performance analysis

In this Section, we make use of the MGF-based approach for the performance evaluation of digital communication over generalized fading channels (Alouini & Goldsmith, 1999b; Simon & Alouini, 1999; 2005) to derive the ASEP of a wide variety of modulation schemes when used in conjunction with MRC.

### 3.1 $M$-ary PSK

The ASEP of $M$-ary PSK signals is given by (Simon & Alouini, 2005, Eq. (5.78))

$$P_s(e) = \mathcal{I}_1 + \mathcal{I}_2 \tag{3}$$

where

$$\mathcal{I}_1 = \frac{1}{\pi} \int_{\pi/2}^{\pi-\pi/M} \mathcal{M}_\gamma \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta, \quad \mathcal{I}_2 = \frac{1}{\pi} \int_0^{\pi/2} \mathcal{M}_\gamma \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta \tag{4}$$

and $g_{PSK} = \sin^2(\pi/M)$. For the integral $\mathcal{I}_1$ by performing the change of variable $x = \cos^2 \theta / \cos^2(\pi/M)$ and after some necessary manipulations, one obtains:

$$\mathcal{I}_1 = \frac{\cos\left(\frac{\pi}{M}\right)}{2\pi} \int_0^1 x^{-\frac{1}{2}} \left(1 - x \cos^2 \frac{\pi}{M}\right)^{-\frac{1}{2}} \prod_{i=1}^L \left(1 + \frac{A_i g_{PSK}}{1 - x \cos^2 \frac{\pi}{M}}\right)^{-\mu_i} \left(1 + \frac{B_i g_{PSK}}{1 - x \cos^2 \frac{\pi}{M}}\right)^{-\mu_i} dx$$

$$= \frac{1}{2\pi} \cos\left(\frac{\pi}{M}\right) \mathcal{M}_\gamma(g_{PSK}) \int_0^1 x^{-1/2} \left(1 - x \cos^2 \frac{\pi}{M}\right)^{2\sum_{i=1}^L \mu_i - 1/2}$$

$$\times \prod_{i=1}^L \left(1 - \frac{\cos^2 \frac{\pi}{M}}{1 + A_i g_{PSK}} x\right)^{-\mu_i} \prod_{i=1}^L \left(1 - \frac{\cos^2 \frac{\pi}{M}}{1 + B_i g_{PSK}} x\right)^{-\mu_i} dx$$

$$= \frac{1}{\pi} \cos\left(\frac{\pi}{M}\right) \mathcal{M}_\gamma(g_{PSK}) F_D^{(2L+1)} \left( \frac{1}{2}, \frac{1}{2} - 2\sum_{i=1}^L \mu_i, \mu_1, \cdots \mu_L, \mu_1, \cdots \mu_L; \frac{3}{2}; \cos^2 \frac{\pi}{M} \right.$$

$$\left. , \frac{\cos^2 \frac{\pi}{M}}{1 + A_1 g_{PSK}}, \cdots, \frac{\cos^2 \frac{\pi}{M}}{1 + A_L g_{PSK}}, \frac{\cos^2 \frac{\pi}{M}}{1 + B_1 g_{PSK}} \cdots \frac{\cos^2 \frac{\pi}{M}}{1 + B_L g_{PSK}} \right)$$

$$\tag{5}$$

where $F_D^{(n)}(v, k_1, \cdots, k_n; c; z_1, \cdots, z_n)$ is the Lauricella multiple hypergeometric function of $n$ variables defined as (Prudnikov et al., 1986, Eq. (7.2.4.57)), (Prudnikov et al., 1986, Eq. (7.2.4.15)):

$$F_D^{(n)}(v, k_1, \cdots, k_n; c; z_1, \cdots, z_n) = \frac{\Gamma(c)}{\Gamma(c - v)\Gamma(v)} \int_0^1 x^{v-1}(1 - x)^{c-v-1} \prod_{i=1}^n (1 - z_i x)^{-k_i} dx$$

$$= \sum_{l_1, l_2, \cdots, l_n = 0}^\infty \frac{(v)_{l_T}}{(c)_{l_T}} \prod_{i=1}^n \frac{(k_i)_{l_i}}{\Gamma(l_i + 1)} z_i^{l_i}, \quad |z_i| < 1 \tag{6}$$

where $l_T = \sum_{i=1}^n l_i$, $(\alpha)_\beta = \Gamma(\alpha + \beta)/\Gamma(\alpha)$ is the Pochhammer symbol. The integral in (6) exists for $\Re\{c - v\} > 0$ and $\Re\{v\} > 0$ where $\Re\{\cdot\}$ denotes the real part. If $n = 2$, this function reduces to the Appell hypergeometric function $F_1$ (Prudnikov et al., 1986, Eq. (7.2.1.41)) whereas if $n = 1$, it reduces to the Gauss hypergeometric function $_2F_1$. It is seen from (5) that the conditions for series convergence and integral existence of $F_D^{(2L+1)}$ are satisfied.

For the integral $\mathcal{I}_2$ by performing the change of variable $x = \cos^2 \theta$ and after some manipulations we obtain:

$$\mathcal{I}_2 = \frac{\mathcal{M}_\gamma(g_{PSK})}{2\pi} \int_0^1 x^{-\frac{1}{2}} (1-x)^{2\sum_{i=1}^{L}\mu_i-\frac{1}{2}} \prod_{i=1}^{L} \left(1 - \frac{1}{1+A_i g_{PSK}}x\right)^{-\mu_i} \left(1 - \frac{1}{1+B_i g_{PSK}}x\right)^{-\mu_i} dx$$

$$= \frac{\Gamma\left(2\sum_{i=1}^{L}\mu_i + 1/2\right)}{2\sqrt{\pi}\Gamma(2\sum_{i=1}^{L}\mu_i + 1)} \mathcal{M}_\gamma(g_{PSK}) F_D^{(2L)} \left(\frac{1}{2}, \mu_1, \cdots \mu_L, \mu_1, \cdots \mu_L; 2\sum_{i=1}^{L}\mu_i + 1; \frac{1}{1+A_1 g_{PSK}}, \right.$$

$$\left. \cdots, \frac{1}{1+A_L g_{PSK}}, \frac{1}{1+B_1 g_{PSK}}, \cdots, \frac{1}{1+B_L g_{PSK}}\right) \tag{7}$$

For the case of BPSK ($M = 2, g_{PSK} = 1$), it can be observed that $\mathcal{I}_1 = 0$ and therefore the expression for the ASEP is reduced to the following compact form:

$$P_s(e) = \frac{\Gamma\left(2\sum_{i=1}^{L}\mu_i + 1/2\right)}{2\sqrt{\pi}\Gamma(2\sum_{i=1}^{L}\mu_i + 1)} \mathcal{M}_\gamma(1) F_D^{(2L)} \left(\frac{1}{2}, \mu_1, \cdots \mu_L, \mu_1, \cdots \mu_L; 2\sum_{i=1}^{L}\mu_i + 1; \frac{1}{1+A_1}, \right.$$

$$\left. \cdots, \frac{1}{1+A_L}, \frac{1}{1+B_1}, \cdots, \frac{1}{1+B_L}\right) \tag{8}$$

For independent and identically distributed (i.i.d.) fading channels, where $h_i = h$, $H_i = H$, $\mu_i = \mu$, $A_i = A$ and $B_i = B$, using the integral representation of $F_D^{(2L+1)}$ and $F_D^{(2L)}$, the following simplified forms are obtained:

$$\mathcal{I}_{1_{iid}} = \frac{1}{\pi} \cos\left(\frac{\pi}{M}\right) \mathcal{M}_\gamma(g_{PSK}) F_D^{(3)} \left(\frac{1}{2}, \frac{1}{2} - 2L\mu, L\mu, L\mu; \frac{3}{2}; \cos^2\frac{\pi}{M}, \frac{\cos^2\frac{\pi}{M}}{1+A g_{PSK}}, \frac{\cos^2\frac{\pi}{M}}{1+B g_{PSK}}\right) \tag{9}$$

$$\mathcal{I}_{2_{iid}} = \frac{\Gamma(2L\mu + 1/2)}{2\sqrt{\pi}\Gamma(2L\mu + 1)} \mathcal{M}_\gamma(g_{PSK}) F_1 \left(\frac{1}{2}, L\mu, L\mu; 2L\mu + 1; \frac{1}{1+A g_{PSK}}, \frac{1}{1+B g_{PSK}}\right) \tag{10}$$

For the special case of Hoyt fading channels ($\mu = 0.5$) and no diversity ($L = 1$), the expression for the ASEP of $M$-ary PSK is reduced to a previously known result (Radaydeh, 2007, Eq. (18)). Finally, for Nakagami-$m$ fading channels and i.i.d. branches, using (10) and (Prudnikov et al., 1986, Eq. (7.2.4.60)), a previously known result may be obtained (Adinoyi & Al-Semari, 2002, Eq. (6)).

### 3.2 $M$-ary DPSK

The ASEP of $M$-ary DPSK signals is given by (Simon & Alouini, 2005, Eq. (8.200))

$$P_s(e) = \frac{2}{\pi} \int_0^{\pi/2 - \pi/2M} \mathcal{M}_\gamma\left(\zeta \sin^2\frac{\pi}{M}\right) d\theta \tag{11}$$

where $\zeta = \left(1 + \cos\frac{\pi}{M} - 2\cos\frac{\pi}{M}\sin^2\theta\right)^{-1}$. Applying the transformation $x = \sin^2\theta / \cos^2(\pi/2M)$ and after some algebraic manipulations the ASEP of $M$-ary DPSK can be expressed as:

$$P_s(e) = \frac{1}{\pi} \cos\left(\frac{\pi}{2M}\right) \mathcal{M}_\gamma(f(M)) \int_0^1 x^{-1/2} \left(1 - x\cos^2\frac{\pi}{2M}\right)^{-1/2} \left(1 - x\cos\frac{\pi}{M}\right)^{2\sum_{i=1}^L \mu_i}$$

$$\times \prod_{i=1}^L \left(1 - \frac{\cos\frac{\pi}{M}}{1 + A_i f(M)}x\right)^{-\mu_i} \left(1 - \frac{\cos\frac{\pi}{M}}{1 + B_i f(M)}x\right)^{-\mu_i} dx$$

$$= \frac{2}{\pi} \cos\left(\frac{\pi}{2M}\right) \mathcal{M}_\gamma(f(M)) F_D^{(2L+2)} \left(\frac{1}{2}, \frac{1}{2}, -2\sum_{i=1}^L \mu_i, \mu_1, \mu_1, \cdots, \mu_L, \mu_L; \frac{3}{2}; \cos^2\frac{\pi}{2M},\right.$$

$$\left. \cos\frac{\pi}{M}, \frac{\cos\frac{\pi}{M}}{1 + A_1 f(M)}, \frac{\cos\frac{\pi}{M}}{1 + B_1 f(M)}, \cdots, \frac{\cos\frac{\pi}{M}}{1 + A_L f(M)}, \frac{\cos\frac{\pi}{M}}{1 + B_L f(M)}\right)$$

$$(12)$$

where $f(M) = \frac{\sin^2(\pi/M)}{2\cos^2(\pi/2M)}$.

For binary DPSK signals ($M = 2$), using $F_D^{(N)}(v, k_1, k_2, \cdots, k_N; c; z, 0, \cdots, 0) = {}_2F_1(v, k_1; c; z)$ and ${}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z\right) = \arcsin(\sqrt{z})/\sqrt{z}$ (Prudnikov et al., 1986, Eq. (7.3.2.76)), the derived result is reduced to the well known expression for the error probability of binary DPSK signals over generalized fading channels, i.e. $P_{M=2}(e) = \frac{1}{2}\mathcal{M}_\gamma(1)$.

For the i.i.d. case, using the integral representation of the Lauricella function, a simplified expression of $P_s(e)$ may be obtained as:

$$P_s(e) = \frac{2\cos\left(\frac{\pi}{2M}\right)\mathcal{M}_\gamma(f(M))}{\pi} F_D^{(4)}\left(\frac{1}{2}, \frac{1}{2}, -2L\mu, L\mu, L\mu; \frac{3}{2};\right.$$

$$\left. \cos^2\frac{\pi}{2M}, \cos\frac{\pi}{M}, \frac{\cos\frac{\pi}{M}}{1 + Af(M)}, \frac{\cos\frac{\pi}{M}}{1 + Bf(M)}\right)$$

$$(13)$$

### 3.3 General order rectangular QAM

We consider a general order rectangular QAM signal which may be viewed as two separate Pulse Amplitude Modulation (PAM) signals impressed on phase-quadrature carriers. Let also $M_I$ and $M_Q$ be the dimensions of the in-phase and the quadrature signal respectively and $r = d_Q/d_I$ the quadrature-to-in-phase decision distance ratio, with $d_I$ and $d_Q$ being the in-phase decision and the quadrature decision distance respectively. For general order rectangular QAM, the ASEP is given by (Lei et al., 2007, Eq. (4))

$$P_s(e) = 2p\mathcal{J}(a) + 2q\mathcal{J}(b) - 4pq[\mathcal{K}(a, b) + \mathcal{K}(b, a)] \qquad (14)$$

where

$$\mathcal{J}(t) = \frac{1}{\pi}\int_0^{\pi/2} \mathcal{M}_\gamma\left(\frac{t^2}{2\sin^2\theta}\right) d\theta, \quad \mathcal{K}(u, v) = \frac{1}{2\pi}\int_0^{\pi/2-\arctan(v/u)} \mathcal{M}_\gamma\left(\frac{u^2}{2\sin^2\theta}\right) d\theta \quad (15)$$

and $p = 1 - 1/M_I$, $q = 1 - 1/M_Q$, $a = \sqrt{\frac{6}{(M_I^2-1)+r^2(M_Q^2-1)}}$, $b = \sqrt{\frac{6r^2}{(M_I^2-1)+r^2(M_Q^2-1)}}$. Using the result in (7), the integral $\mathcal{J}(t)$ can be easily evaluated in terms of the Lauricella functions as:

$$\mathcal{J}(t) = \frac{\Gamma\left(2\sum_{i=1}^{L}\mu_i + 1/2\right)}{2\sqrt{\pi}\Gamma(2\sum_{i=1}^{L}\mu_i + 1)}\mathcal{M}_{\gamma}(t^2/2)F_D^{(2L)}\left(\frac{1}{2},\mu_1,\cdots\mu_L,\mu_1,\cdots\mu_L; 2\sum_{i=1}^{L}\mu_i + 1; \frac{2}{2+A_1t^2},\right.$$
$$\left.\cdots,\frac{2}{2+A_Lt^2},\frac{2}{2+B_1t^2},\cdots,\frac{2}{2+B_Lt^2}\right)$$

$$(16)$$

To evaluate $\mathcal{K}(u,v)$ in (15), applying the transformation $x = 1 - (v^2/u^2)\tan^2\theta$ and after performing some algebraic and trigonometric manipulations, we obtain:

$$\mathcal{K}(u,v) = \frac{uv\mathcal{M}_{\gamma}(\frac{u^2+v^2}{2})}{4\pi(u^2+v^2)}\int_0^1 (1-x)^{2\sum_{i=1}^{L}\mu_i - \frac{1}{2}}\left(1 - \frac{u^2}{u^2+v^2}x\right)^{-1}$$
$$\times \prod_{i=1}^{L}\left(1 - x\frac{2+A_iu^2}{2+A_iu^2+A_iv^2}\right)^{-\mu_i}\left(1 - x\frac{2+B_iu^2}{2+B_iu^2+B_iv^2}\right)^{-\mu_i}dx$$
$$= \frac{uv\mathcal{M}_{\gamma}(\frac{u^2+v^2}{2})}{2\pi(u^2+v^2)\left(4\sum_{i=1}^{L}\mu_i + 1\right)}F_D^{(2L+1)}\left(1,1,\mu_1,\cdots,\mu_L,\mu_1,\cdots,\mu_L; 2\sum_{i=1}^{L}\mu_i + \frac{3}{2};\right.$$
$$\left.\frac{u^2}{u^2+v^2},\frac{2+A_1u^2}{2+A_1u^2+A_1v^2},\cdots,\frac{2+A_Lu^2}{2+A_Lu^2+A_Lv^2},\frac{2+B_1u^2}{2+B_1u^2+B_1v^2},\cdots,\frac{2+B_Lu^2}{2+B_Lu^2+B_Lv^2}\right)$$

$$(17)$$

For i.i.d. branches (16) reduces to:

$$\mathcal{J}_{iid}(t) = \frac{\Gamma(2L\mu + 1/2)}{2\sqrt{\pi}\Gamma(2L\mu + 1)}\mathcal{M}_{\gamma}(t^2/2)F_1\left(\frac{1}{2},L\mu,L\mu; 2L\mu + 1; \frac{2}{2+At^2},\frac{2}{2+Bt^2}\right) \qquad (18)$$

whereas (17) is reduced to:

$$\mathcal{K}_{iid}(u,v) = \frac{uv\mathcal{M}_{\gamma}(\frac{u^2+v^2}{2})}{2\pi(u^2+v^2)(4L\mu+1)}F_D^{(3)}\left(1,1,L\mu,L\mu; 2L\mu + \frac{3}{2}; \frac{u^2}{u^2+v^2},\frac{2+Au^2}{2+Au^2+Av^2},\right.$$
$$\left.\frac{2+Bu^2}{2+Bu^2+Bv^2}\right)$$

$$(19)$$

For the special case of Nakagami-$m$ channels, using (Prudnikov et al., 1986, Eq. (7.2.4.60)), (18) is reduced to a previously known result (Lei et al., 2007, Eq. 11). Moreover, using the infinite series representation of the Lauricella function (6), (Prudnikov et al., 1986, Eq. (7.2.4.60)) and after some necessary manipulations, (19) is reduced to a previously known result (Lei et al., 2007, Eq. 15).

### 3.4 Asymptotic error rate analysis at high SNR

The asymptotic performance analysis of a diversity system at high SNR region allows one to gain useful insights regarding the parameters determining the error performance. At high SNR, the ASEP of a digital communications system has been observed in certain cases to be approximated as (Simon & Alouini, 2005; Wang & Yannakis, 2003)

$$P_s(e) \simeq C_d\overline{\gamma}^{-d} \qquad (20)$$

where $C_d$ is referred to as the coding gain, and $d$ as the diversity gain. The diversity gain determines the slope of the ASEP versus average SNR curve, at high SNR, in a log-log scale. Moreover, $C_d$ (expressed in decibels) represents the horizontal shift of the curve in SNR relative to a benchmark ASEP curve of $P_s(e) \simeq \overline{\gamma}^{-d}$.

In (Wang & Yannakis, 2003), the authors developed a simple and general method to quantify the asymptotic performance of wireless transmission in fading channels at high SNR values. In this work, it was shown that the asymptotic performance depends on the behavior of the PDF of the instantaneous channel power gain denoted by $\beta$, $p(\beta)$. Moreover, it was shown that if the MGF of $p(\beta)$ can be expressed for $s \to \infty$ as $|\mathcal{M}_\beta(s)| = C|s|^{-d} + o(|s|^{-d})$, a diversity gain equal to $d$ may be obtained. It is noted that we write $f(x) = o[g(x)]$ as $x \to x_0$ if $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$. We observe that (2) can be expressed as:

$$\mathcal{M}_\gamma(s) = \prod_{i=1}^{L} \left[ s^{-2} A_i B_i \left( 1 + \frac{1}{A_i s} \right) \left( 1 + \frac{1}{B_i s} \right) \right]^{-\mu_i} = C s^{-d} + o(s^{-d}) \qquad (21)$$

for $s \to \infty$, where $C = \prod_{i=1}^{L} (A_i B_i)^{-\mu_i} = \prod_{i=1}^{L} h_i^{\mu_i} \left( \frac{\overline{\gamma}_i}{2\mu_i} \right)^{-2\mu_i}$ and $d = 2 \sum_{i=1}^{L} \mu_i$. This interesting result shows that the diversity gain of the considered system depends on the number of the receive antennas $L$ as well as on the parameters $\mu_i$. For the special case of i.i.d. branches, it is obvious that a diversity gain equal to $2\mu L$ may be obtained.

### 3.4.1 Asymptotic ASEP of $M$-PSK

At high SNR, using the previously derived asymptotic expression for the MGF, $\mathcal{I}_1$ can be computed as:

$$\mathcal{I}_{1_{asym}} = \frac{C \cos \frac{\pi}{M}}{2 g_{PSK}^d \pi} \int_0^1 x^{-1/2} \left( 1 - x \cos^2 \frac{\pi}{M} \right)^{-1/2+d} dx = \frac{C \cos \frac{\pi}{M}}{\pi g_{PSK}^d} {}_2F_1 \left( \frac{1}{2} - d, \frac{1}{2}; \frac{3}{2}; \cos^2 \frac{\pi}{M} \right)$$
(22)

Also, a simplified asymptotic expression of $\mathcal{I}_2$ may be obtained as:

$$\mathcal{I}_{2_{asym}} = \frac{C}{g_{PSK}^d \pi} \int_0^{\pi/2} \sin^{2d} \theta d\theta = \frac{C \Gamma(d+1/2)}{2\sqrt{\pi} \Gamma(d+1) g_{PSK}^d} \qquad (23)$$

In Figure 1 the ASEP for BPSK modulation is plotted as a function of $\overline{\gamma}_1$ for constant $\eta$ equal to 2 and for various values of $\mu$ and $L$. An exponential power decay profile is considered, that is the average input SNR of the $l$-th branch is given by $\overline{\gamma}_l = \overline{\gamma}_1 \exp[-\delta(l-1)]$ where $\delta$ is the decay factor. We also assume $\delta = 0.5$. As expected, by keeping $\eta$ constant, an increase in $\mu$ and/or $L$ results in an improvement of the system performance. For comparison purposes, for $L = 1, 2$ and $3$, the ASEP of the BPSK modulation for the Hoyt fading channel ($\mu = 0.5$) has also been plotted versus the average input SNR per branch. It is obvious that the Hoyt channel results in the worst symbol error performance. The asymptotic performance of the BPSK ASEP expressions is also investigated and as it can be observed, for no diversity ($L = 1$) and small values of $\mu$, the high SNR asymptotic expressions yield accurate results even for low to medium SNR values. Moreover in Figure 2 the ASEP of $M$-PSK modulation is plotted as a function of the first branch average input SNR $\overline{\gamma}_1$, for the $\eta$-$\mu$ fading channel, Format 1 and for different values of $M$ and $L$, with $\eta_\ell = \eta = 2$ and $\mu_\ell = \mu = 1.5$, $\ell = 1, 2, 3$. As one can observe, an increase in the number of diversity branches from $L = 1$ to $L = 3$ significantly enhances the system's error performance. In the same figure, the exact and the asymptotic

ASEP results are also compared. As it is evident, the asymptotic results correctly predict the diversity gain, however for large $M$ and $L$, the predicted asymptotic behavior of the ASEP curves shows up at relatively high SNR (e.g. for $M = 16$, $L = 3$ we need $\overline{\gamma}_1 > 30\text{dB}$).
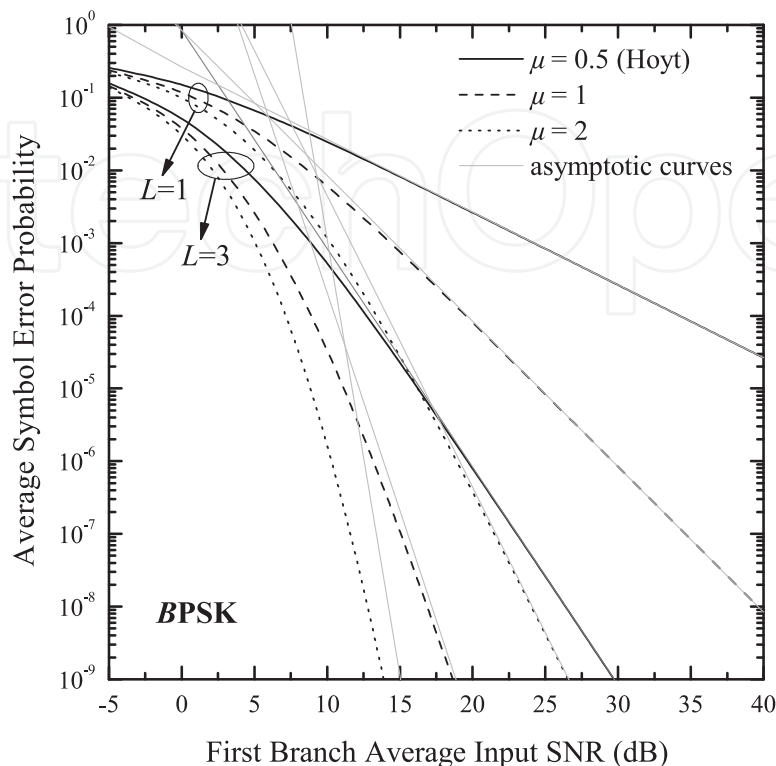


Fig. 1. Average Symbol Error Probability of BPSK receivers with MRC diversity ($L = 1, 3$) operating over n.i.d. $\eta$-$\mu$ fading channels (Format 1), for $\eta = 2$ and for different values of $\mu$, as a function of the First Branch Average Input SNR ($\delta = 0.5$)


### 3.4.2 Asymptotic ASEP of $M$-DPSK

For $M$-DPSK, by substituting (21) to (11) and using the same methodology for the evaluation of the corresponding integral, the following asymptotic expression of $P_s(e)$ may be obtained as:

$$
\begin{aligned}
P_s(e) &= \frac{C[f(M)]^{-d}}{\pi} \cos\left(\frac{\pi}{2M}\right) \int_0^1 x^{-1/2} \left(1 - x\cos^2\frac{\pi}{2M}\right)^{-1/2} \left(1 - x\cos\frac{\pi}{M}\right)^d dx \\
&= \frac{2C[f(M)]^{-d}}{\pi} \cos\left(\frac{\pi}{2M}\right) F_1\left(\frac{1}{2}, \frac{1}{2}, -d; \frac{3}{2}; \cos^2\frac{\pi}{2M}, \cos\frac{\pi}{M}\right)
\end{aligned}
\tag{24}
$$

In Fig. 3 similar results for $M$-DPSK signal constellations are given with $\eta_\ell = \eta = 2$, $\mu_\ell = \mu = 1.5$ and $L = 1, 3$, $\delta = 0.5$. As in the case of coherent modulation, the performance of the considered system significantly improves as the number of diversity branches increases. Asymptotic results are also included and as it can be observed, the asymptotic approximation predicts the diversity gain correctly and provides good approximation to the exact error performance in the high SNR region, especially when $L$ and/or $\mu$ are small.
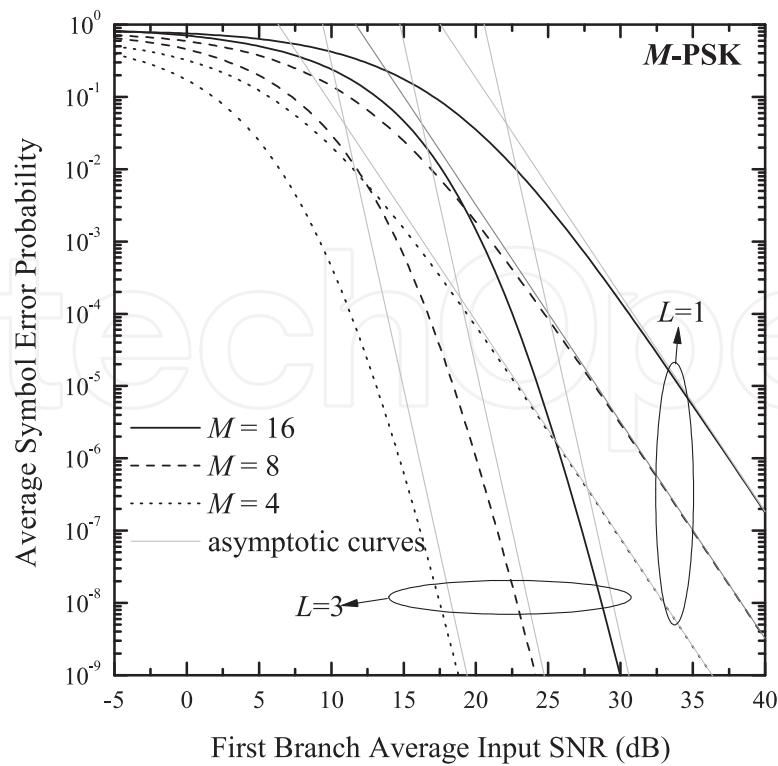
Fig. 2. Average Symbol Error Probability of $M$-PSK receivers with MRC diversity ($L = 1, 3$) operating over n.i.d. $\eta$-$\mu$ fading channels (Format 1), for different values of $M$, as a function of the First Branch Average Input SNR ($\eta = 2$, $\mu = 1.5$, $\delta = 0.5$)
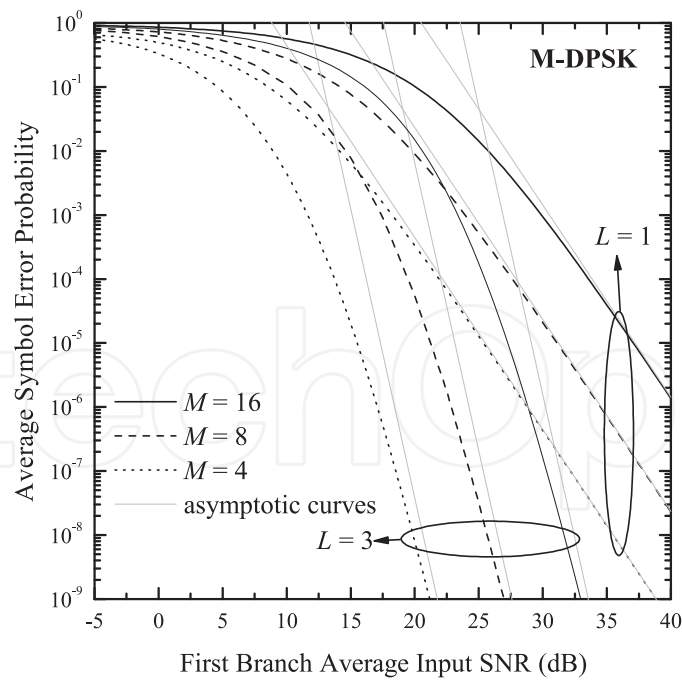


Fig. 3. Average Symbol Error Probability of $M$-DPSK receivers with MRC diversity ($L = 1, 3$) operating over n.i.d. $\eta$-$\mu$ fading channels (Format 1), for different values of $M$, as a function of the First Branch Average Input SNR ($\eta = 2$, $\mu = 1.5$, $\delta = 0.5$)

### 3.4.3 Asymptotic ASEP of general order rectangular QAM

For general order rectangular QAM and at high SNR values, the following asymptotic form for $\mathcal{J}(t)$ may be obtained:

$$\mathcal{J}_{asym}(t) = \frac{2^d C}{t^{2d}\pi}\int_0^{\pi/2}\sin^{2d}\theta d\theta = \frac{2^{d-1}C\Gamma(d+1/2)}{\sqrt{\pi}\Gamma(d+1)t^{2d}} \tag{25}$$

Moreover, using the integral representation of the $_2F_1$ function, a simplified asymptotic expression for $\mathcal{K}(u,v)$ may be obtained, as:

$$\begin{aligned}
\mathcal{K}_{asym}(u,v) &= \frac{2^{d-2}Cuv}{\pi(u^2+v^2)^{d+1}}\int_0^1(1-x)^{d-1/2}\left(1-\frac{u^2}{u^2+v^2}x\right)^{-d-1}dx \\
&= \frac{2^{d-1}Cuv}{\pi(u^2+v^2)^{d+1}(2d+1)}{}_2F_1\left(1,d+1;d+\frac{3}{2};\frac{u^2}{u^2+v^2}\right)
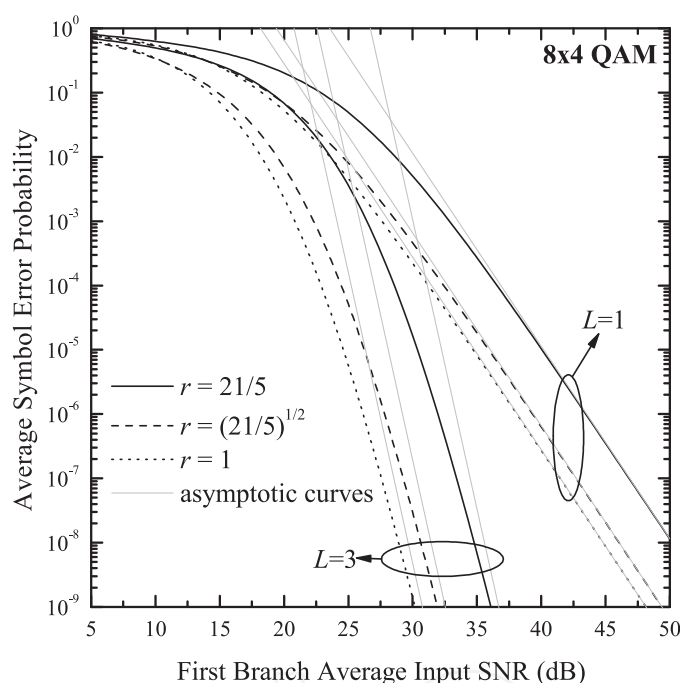\end{aligned} \tag{26}$$



Fig. 4. Average Symbol Error Probability of $8 \times 4$ QAM receivers with MRC diversity ($L = 1, 2, 3$) operating over $\eta$-$\mu$ fading channels (Format 1), for different values of $r$, as a function of the Average Input SNR per Branch ($\eta = 2$, $\mu = 1.5$, $\delta = 0.5$)

In Figure 4 the error performance of a $8 \times 4$ QAM system is illustrated for $\eta = 2$, and $\mu = 1.5$ with $\delta = 0.5$. The ASEP is plotted for different values of $L$ and $r$ and as it is obvious, the error performance significantly increases as $L$ increases. Also, it can be observed that the ASEP deteriorates substantially as $r$ increases, for all values of $\mu$. Moreover, asymptotic results have been included and as it is evident, the simplified asymptotic expressions yield accurate results at high SNR values, especially when $L$ and/or $r$ are small.

## 4. A PDF-based approach for outage probability and channel capacity evaluation

The PDF of $\gamma$ in (1) may be obtained by taking the inverse Laplace transform of $\mathcal{M}_\gamma(s)$, i.e.

$$f_\gamma(\gamma) = \mathbb{L}^{-1}\{\mathcal{M}_\gamma(s); s; \gamma\} \tag{27}$$

where $\mathbb{L}^{-1}\{\cdot;s;t\}$ denotes inverse Laplace transform. It is noted that the PDF of $L$ i.i.d. $\eta$-$\mu$ channels is an $\eta$-$\mu$ distribution with parameters $\eta$, $L\mu$ and $L\overline{\gamma}$ (Yacoub, 2007). In the following analysis analytical expressions for $f_\gamma(\gamma)$ will be obtained. This statistical result is then applied to the evaluation of the outage probability and channel capacity of MRC receivers operating over $\eta$-$\mu$ channels.

### 4.1 Infinite series representation of the PDF of the sum of independent $\eta$-$\mu$ variates

We may observe that in (2), each factor of the form $(1 + sA_\ell)^{-\mu_\ell}$ is the MGF of a gamma-distributed random variable with parameters $\mu_\ell$ and $A_\ell$. Similarly, each factor of the form $(1 + sB_\ell)^{-\mu_\ell}$ is the MGF of a gamma-distributed random variable with parameters $\mu_\ell$ and $B_\ell$. Hence, the PDF of the sum of $L$ independent squared $\eta$-$\mu$ variates may be obtained as the PDF of the sum of $2L$ independent gamma variates with suitably defined parameters. Using Moschopoulos (1985) and (Alouini et al., 2001, eq. (2)), the PDF of $\gamma$ may be expressed as

$$f_\gamma(\gamma) = \prod_{j=1}^{L}(A_j B_j)^{-\mu_j} \sum_{k=0}^{\infty} \xi_k \frac{\gamma^{2\sum_{\ell=1}^{L}\mu_\ell+k-1}e^{-\frac{\gamma}{C_m}}}{C_m^k \Gamma\left(k + 2\sum_{\ell=1}^{L}\mu_\ell\right)}, \tag{28}$$

where $C_m = \min\{A_\ell, B_\ell\}$ and the coefficients $\xi_k$ may be recursively obtained as

$$\xi_{k+1} = \frac{1}{k+1}\sum_{i=1}^{k+1}\left[\sum_{j=1}^{L}\mu_j\left(1-\frac{C_m}{A_j}\right)^i + \sum_{j=1}^{L}\mu_j\left(1-\frac{C_m}{B_j}\right)^i\right]\xi_{k+1-i}, \quad k = 0, 1, 2, \ldots, \tag{29}$$

with $\xi_0 = 1$.

### 4.2 Integral representation of the PDF of the sum of independent $\eta$-$\mu$ variates

An alternative expression for the PDF of $\gamma$ may be obtained by using the Gil-Pelaez result (Gil-Pelaez, 1951) to obtain the inverse Laplace transform of (2). The cumulative distribution function (CDF) of $\gamma$ may be obtained as

$$F_\gamma(\gamma) = \mathbb{L}^{-1}\left\{\frac{\mathcal{M}_\gamma(s)}{s};s;\gamma\right\} = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\Im\{\mathcal{M}_\gamma(\jmath t)e^{-\jmath\gamma t}\}}{t}dt, \tag{30}$$

where $\jmath = \sqrt{-1}$, and $\Im\{\cdot\}$ is the imaginary part. By expressing $\mathcal{M}_\gamma(\jmath t)$ in polar form and substituting in (30), the CDF of $\gamma$ may be expressed as

$$F_\gamma(\gamma) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\sin\left\{\sum_{\ell=1}^{L}\mu_\ell[\arctan(A_\ell t) + \arctan(B_\ell t)] - t\gamma\right\}}{t\prod_{\ell=1}^{L}[(1 + t^2 A_\ell^2)(1 + t^2 B_\ell^2)]^{\frac{\mu_\ell}{2}}}dt, \tag{31}$$

The corresponding PDF may be obtained by taking the derivative of (31) with respect to $\gamma$, yielding

$$f_\gamma(\gamma) = \int_0^\infty \frac{\cos\left\{\sum_{\ell=1}^{L}\mu_\ell[\arctan(A_\ell t) + \arctan(B_\ell t)] - t\gamma\right\}dt}{\pi\prod_{\ell=1}^{L}[(1 + t^2 A_\ell^2)(1 + t^2 B_\ell^2)]^{\frac{\mu_\ell}{2}}}. \tag{32}$$

Both integrals can be numerically evaluated in an efficient way, for example by using the Gauss-Legendre quadrature rule (Abramovitz & Stegun, 1964, eq. (25.4.29)) over (32) or (31) (Efthymoglou et al., 1997). Another fast and computationally efficient means to evaluate such integrals is symbolic integration, using any of the well known software mathematical packages such as Maple or Mathematica.

### 4.3 A Closed-Form expression of the PDF of the sum of independent $\eta$-$\mu$ variates

A closed-form expression of the PDF of $\gamma$ may be obtained by making use of the following Laplace transform pair (Srivastava & L.Manocha, 1984, p. 259)

$$\mathbb{L}\{t^{\alpha-1}\Phi_2^{(n)}(b_1,\ldots b_n;\alpha;x_1t,\ldots,x_nt);s;t\} = \frac{\Gamma(\alpha)}{s^\alpha}\prod_{j=1}^{n}\left(1-\frac{x_j}{s}\right)^{-b_j}, \quad \alpha > 0, \qquad (33)$$

where $\Phi_2^{(L)}(\cdot)$ is the confluent Lauricella hypergeometric function defined in (Srivastava & L.Manocha, 1984, eq. 10, p. 62)

$$\Phi_2^{(n)}(b_1,\ldots b_n;\alpha;x_1,\ldots x_n) = \sum_{l_1,l_2,\cdots,l_n=0}^{\infty}\frac{(b_1)_{l_1}\ldots(b_n)_{l_n}}{(\alpha)_{l_1+\ldots+l_n}}\frac{x_1^{l_1}}{l_1!}\cdots\frac{x_n^{l_n}}{l_n!}, \qquad (34)$$

with $(\alpha)_\beta = \Gamma(\alpha+\beta)/\Gamma(\alpha)$ being the Pochhammer symbol. By comparing (33) and (2), the PDF of $\gamma$ may be obtained as

$$f_\gamma(\gamma) = \frac{\prod_{\ell=1}^{L}(A_\ell B_\ell)^{-\mu_\ell}\gamma^{2\sum_{j=1}^{L}\mu_\ell-1}}{\Gamma\left(2\sum_{\ell=1}^{L}\mu_\ell\right)}\Phi_2^{(2L)}(\mu_1,\mu_1,\cdots,$$

$$\mu_L,\mu_L,2\sum_{\ell=1}^{L}\mu_\ell,-\frac{\gamma}{A_1},-\frac{\gamma}{B_1}\cdots,-\frac{\gamma}{A_L},-\frac{\gamma}{B_L}\right). \qquad (35)$$

It is noted that the both the integral and infinite series representations for the pdf of $Y$ are much more convenient for accurate and efficient numerical evaluation than this accurate closed form, especially for large values of $L$, i.e. $L > 6$. (Efthymoglou et al., 2006). However, accurate results may be obtained by expressing the series in (34) as multiple integrals (Saigo & Tuan, 1992) that can be easily evaluated numerically (Efthymoglou et al., 2006).

## 5. Applications to the performance analysis of MRC diversity systems

In this section, based on our previously derived results the outage probability and the average channel capacity of MRC diversity systems will be derived. Moreover, an analytical expression for the average error probability for binary modulation schemes will be derived in closed-form using the PDF-based approach. As it will become evident both the PDF and the MGF-based approach yield the same results.

### 5.1 Outage probability

The outage probability is defined as the probability that the instantaneous SNR at the combiner output, $\gamma$, falls below a specified threshold $\gamma_{th}$, i.e. $P_{out}(\gamma_{th}) = Pr(\gamma < \gamma_{th}) = F_\gamma(\gamma_{th})$. Using (28) and with the help of (Gradshteyn & Ryzhik, 2000, eq. (8.356.3)), an infinite series representation of the outage probability may be obtained as

$$P_{out}(\gamma_{th}) = C_m^U\prod_{\ell=1}^{L}(A_\ell B_\ell)^{-\mu_\ell}\sum_{k=0}^{\infty}\xi_k\left[1-\frac{\Gamma\left(k+U,\frac{\gamma_{th}}{C_m}\right)}{\Gamma(k+U)}\right], \qquad (36)$$

where $U \triangleq 2 \sum_{\ell=1}^{L} \mu_{\ell}$. and $\Gamma(\cdot, \cdot)$ is the incomplete gamma function (Gradshteyn & Ryzhik, 2000, eq. 8.350.2). Moreover, using (31), an integral representation of the outage probability may readily be obtained as

$$P_{out}(\gamma_{th}) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\sin[V(t) - t\gamma_{th}]}{W(t)} dt, \tag{37}$$

where

$$V(t) \triangleq \sum_{\ell=1}^{L} \mu_{\ell} [\arctan(A_\ell t) + \arctan(B_\ell t)], \tag{38a}$$

$$W(t) \triangleq t \prod_{\ell=1}^{L} [\left(1 + t^2 A_\ell^2\right)\left(1 + t^2 B_\ell^2\right)]^{\frac{\mu_\ell}{2}}, \tag{38b}$$

Finally, by integrating (35) term-by-term, a closed form expression for $P_{out}(\gamma_{th})$ may be obtained as

$$P_{out}(\gamma_{th}) = \frac{\prod_{j=1}^{L}(A_\ell B_\ell)^{-\mu_\ell} \gamma_{th}^U}{\Gamma(1+U)} \Phi_2^{(2L)}\left(\mu_1, \ \mu_1, \cdots, \mu_L, \mu_L, 1+U, -\frac{\gamma_{th}}{A_1}, -\frac{\gamma_{th}}{B_1} \cdots, \right.$$
$$\left. -\frac{\gamma_{th}}{A_L}, -\frac{\gamma_{th}}{B_L}\right). \tag{39}$$
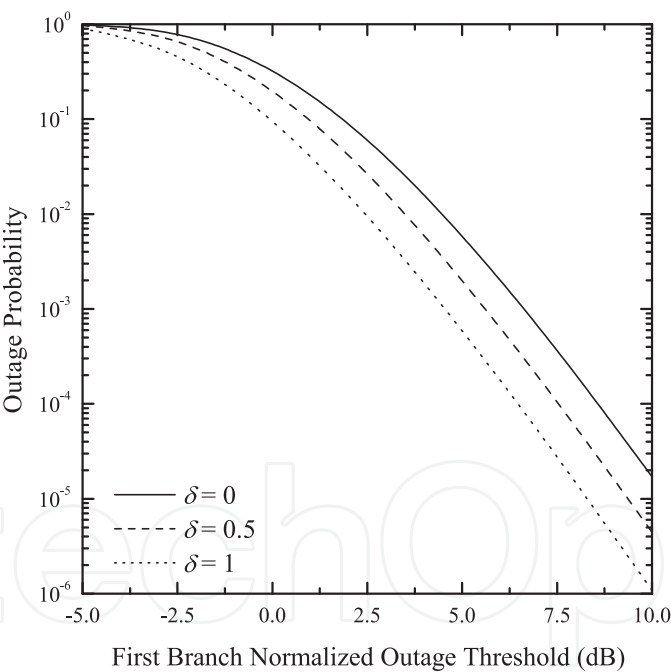


Fig. 5. Outage Probability of dual-branch MRC diversity receivers ($L = 2$) operating over $\eta$-$\mu$ fading channels (Format 1, $\eta = 2$, $\mu = 1.5$), for different values of $\delta$, as a function of the First Branch Normalized Outage Threshold

In Figure 5 the outage performance of a dual-branch MRC diversity system versus the first branch normalized outage threshold $\overline{\gamma_1}/\gamma_{th}$ illustrated for $\eta = 2$, and $\mu = 1.5$. An exponential power decay profile with $\delta = 0, 0.5, 1$ is considered. The outage probability is plotted for different values of $\delta$ and as it is obvious, the outage performance increases as $\delta$ decreases. Note that both the integral representation, given by (36) and the infinite series representation, given by (37) yield identical results.

### 5.2 Channel capacity

For fading channels, the ergodic channel capacity characterizes the long-term achievable rate averaged over the fading distribution and depends on the amount of available channel state information (CSI) at the receiver and transmitter Alouini & Goldsmith (1999a). Two adaptive transmission schemes are considered: Optimal rate adaptation with constant transmit power (ORA) and optimal simultaneous power and rate adaptation (OPRA). Under the ORA scheme that requires only receiver CSI, the capacity is known to be given by Alouini & Goldsmith (1999a)

$$\langle C \rangle_{ORA} = \frac{1}{\ln 2} \int_0^\infty f_\gamma(\gamma) \ln(1 + \gamma) d\gamma \tag{40}$$

In order to obtain an analytical expression of $\langle C \rangle_{ORA}$ for the considered DS-CDMA system, we first make use of the infinite series representations of the PDF of $\gamma$ given by (28). Then, by expressing the exponential and the logarithm in terms of Meijer-G functions (Prudnikov et al., 1986, Eq.(8.4.6.5)), (Prudnikov et al., 1986, Eq. (8.4.6.2)) and applying the result given in (Prudnikov et al., 1986, Eq. (2.24.1.1)), the following expression for the capacity may be obtained:

$$\langle C \rangle_{ORA} = \frac{C_m^U}{\ln 2} \prod_{\ell=1}^{L} (A_\ell B_\ell)^{-\mu_\ell} \sum_{k=0}^{\infty} \xi_k \frac{G_{3,2}^{1,3} \left[ C_m \left| \begin{matrix} 1,1,1-U-k \\ 1,0 \end{matrix} \right. \right]}{\Gamma(k+U)}, \tag{41}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer-G function (Gradshteyn & Ryzhik, 2000, Eq. (9.301)). For the OPRA scheme, the capacity is known to be given by (Alouini & Goldsmith, 1999a, Eq. (7))

$$\langle C \rangle_{OPRA} = \int_{\gamma_0}^\infty \log_2 \left( \frac{\gamma}{\gamma_0} \right) f_\gamma(\gamma) d\gamma, \tag{42}$$

where $\gamma_0$ is the cutoff SNR below which transmission is suspended. By substituting (28) to (42), expressing the logarithm and the exponential in terms of Meijer-G functions (Prudnikov et al., 1986, Eq.(8.4.6.5)), (Prudnikov et al., 1986, Eq. 8.4.3.1)) and with the help of (Prudnikov et al., 1986, Eq. (2.24.1.1)), $\langle C \rangle_{OPRA}$ may be obtained as

$$\langle C \rangle_{OPRA} = \frac{C_m^U}{\ln 2} \prod_{l=1}^{L} (A_l B_l)^{-\mu_l} \sum_{k=0}^{\infty} \xi_k \frac{G_{3,2}^{0,3} \left[ \frac{C_m}{\gamma_0} \left| \begin{matrix} 1,1,1-U-k \\ 0,0 \end{matrix} \right. \right]}{\Gamma(k+U)}. \tag{43}$$

In Figure 6 the average of a triple-branch MRC diversity system under the ORA transmission scheme, is illustrated versus $\overline{\gamma_1}$ for $\eta = 2$, and $\mu = 1.5$. An exponentially power decay profile with $\delta = 0, 0.5, 1$ is considered. The average channel capacity is plotted for different values of $\delta$ and as it is obvious, the capacity improves as $\delta$ decreases.

### 5.3 Average bit error probability

The conditional bit error probability $P_e(\gamma)$ in an AWGN channel may be expressed in unified form as

$$P_e(\gamma) = \frac{\Gamma(b, a\gamma)}{2\Gamma(b)} \tag{44}$$

where $a$ and $b$ are parameters that depend on the specific modulation scheme. For example, $a = 1$ for binary phase shift keying (BPSK) and $1/2$ for binary frequency shift keying (BFSK). Also, $b = 1$ for non-coherent BFSK and binary differential PSK (BDPSK) and $1/2$ for coherent
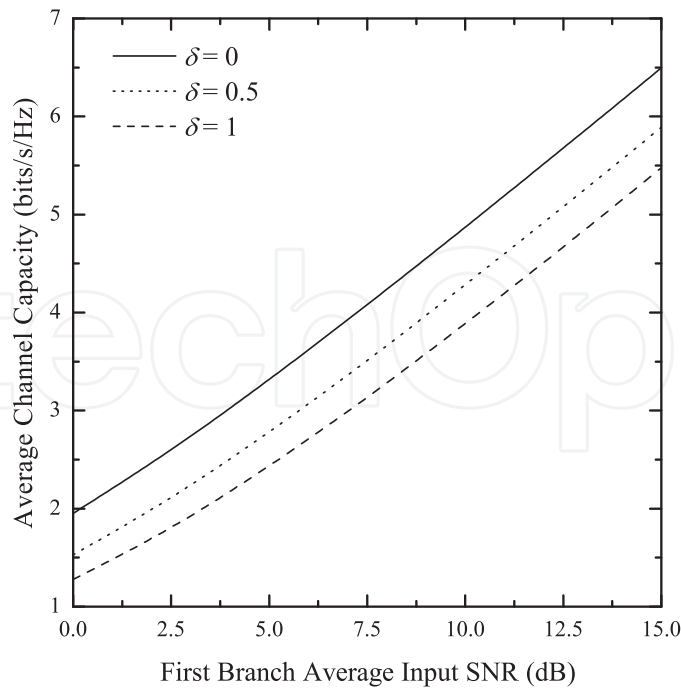
Fig. 6. Average Channel Capacity of triple-branch MRC diversity receivers ($L = 3$) operating over $\eta$-$\mu$ fading channels, (Format 1, $\eta = 2$, $\mu = 1.5$), under ORA policy, for different values of $\delta$, as a function of the First Branch Average Input SNR

BFSK/BPSK. The average bit error probability (ABEP) for the considered system may be obtained by averaging $P_e(\gamma)$ over the PDF of $\gamma$ i.e.,

$$\overline{P}_{be} = \int_0^\infty P_e(\gamma) f_\gamma(\gamma) d\gamma. \tag{45}$$

Using (28) in conjunction with (45) and with the help of (Gradshteyn & Ryzhik, 2000, eq. 6.455) the ABEP may be obtained as

$$\overline{P}_{be} = \frac{a^b C_m^{U+b}}{2\Gamma(b)} \prod_{\ell=1}^L (A_\ell B_\ell)^{-\mu_\ell} \sum_{k=0}^\infty {}_2F_1\left(1, k+U+b; k+U+1; \frac{1}{1+aC_m}\right)$$
$$\times \xi_k \frac{\Gamma(k+U+b)}{(1+aC_m)^{k+U+b}\Gamma(k+U+1)}, \tag{46}$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function (Prudnikov et al., 1986, eq. (7.2.1.1)). Also, by substituting (32) to (45), the ABEP is expressed as a two-fold integral. This expression may be simplified by performing integration by parts and after some algebraic manipulations as follows

$$\overline{P}_{be} = \frac{1}{2\pi} \int_0^\infty \left\{ \cos[V(t)] \frac{a^b \sin(b \arctan(t/a))}{(t^2+a^2)^{b/2}} + \sin[V(t)] \right.$$
$$\left. \left[ 1 - \frac{a^b \cos(b \arctan(t/a))}{(t^2+a^2)^{b/2}} \right] \right\} \frac{dt}{W(t)}. \tag{47}$$

This integral can be efficiently evaluated by means of the Gauss-Legendre quadrature integration rule or by symbolic integration. Finally, an alternative ABEP expression may

be obtained by substituting (35) to (45). By integrating the corresponding infinite series term-by-term and with the help of (Abramovitz & Stegun, 1964, eq. (6.5.37)), the ABEP may be obtained in closed form as

$$
\overline{P}_{be} = \frac{\Gamma\left(U+b\right)\prod_{\ell=1}^{L}(A_\ell B_\ell)^{-\mu_\ell}}{2a^U\Gamma(b)\Gamma(U+1)} F_D^{(2L)}\left(U+b,\mu_1,\mu_1,\cdots,\mu_L,\mu_L;U+1;\right.
$$
$$
\left. -\frac{1}{aA_1},-\frac{1}{aB_1}\cdots,-\frac{1}{aA_{M_L}},-\frac{1}{aB_{M_L}}\right)
$$
. (48)

This expression can be easily evaluated using the integral representation of the Lauricella function. This representation converges if $\left|\frac{1}{aA_\ell}\right| < 1$ and $\left|\frac{1}{aB_\ell}\right| < 1$, $\forall \ell = 1,\dots L$. To guarantee that these conditions are always be fulfilled, we may use the following identity (Exton, 1976, p. 286)

$$
F_D^{(n)}\left(a,b_1,\cdots,b_n;c;x_1,\cdots,x_n\right) = \left[\prod_{\ell=1}^{L}\left(1-x_\ell\right)^{-b_\ell}\right]
$$
$$
\times F_D^{(n)}\left(c-a,b_1,\cdots,b_n;c;\frac{x_1}{x_1-1},\cdots,\frac{x_n}{x_n-1}\right).
$$
(49)

Thus, (48) can be written as

$$
P_s(e) = \frac{\Gamma\left(2\sum_{i=1}^{L}\mu_i + b\right)}{2\Gamma(b)\Gamma(2\sum_{i=1}^{L}\mu_i + 1)} \mathcal{M}_\gamma(a) F_D^{(2L)}\left(b-1,\mu_1,\cdots\mu_L,\mu_1,\cdots\mu_L;2\sum_{i=1}^{L}\mu_i + 1;\frac{1}{1+aA_1},\right.
$$
$$
\left. \cdots,\frac{1}{1+aA_L},\frac{1}{1+aB_1},\cdots,\frac{1}{1+aB_L}\right)
$$
(50)

As it can easily be observed, for BPSK modulation ($a = 1$, $b = 1/2$), (50) reduces to (8), thus verifying the correctness of our analysis. Finally, it is worth mentioning that in Moschopoulos (1985), a proof for the uniform convergence of the series in (28) is provided and a bound for the truncation error is presented. Our conducted numerical experiments confirmed this bound on the truncation error and showed that infinite series converge steadily for all the scenarios of interest, a fact that was also established in Alouini et al. (2001).

## 6. Conclusions

In this chapter, a thorough performance analysis of MRC diversity receivers operating over $\eta$-$\mu$ fading channel was provided. Using the MGF-based approach, we derived closed-form expressions for a variety of $M$-ary modulation schemes. Moreover, in order to provide more insight as to which parameters affect the error performance, asymptotic expressions for the ASEP were derived. Based on these formulas, we proved that the diversity gain depends only on the parameter $\mu$ in each branch whereas $\eta$ affects only the coding gain. Furthermore, we provided three new analytical expressions for the PDF of the sum of non-identical $\eta$-$\mu$ variates. Such expressions are useful to assess the outage performance and the average channel capacity of MRC diversity receivers under different adaptive transmission schemes. Finally, based on this PDF-based analysis, alternative expressions for the error performance of MRC receivers are provided. Various numerically evaluating results are presented that illustrate the analysis proposed in this chapter.

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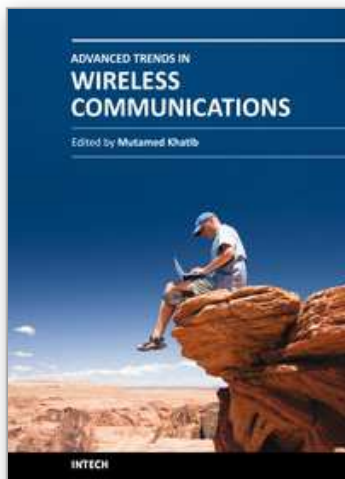
**Advanced Trends in Wireless Communications**

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Physical limitations on wireless communication channels impose huge challenges to reliable communication. Bandwidth limitations, propagation loss, noise and interference make the wireless channel a narrow pipe that does not readily accommodate rapid flow of data. Thus, researches aim to design systems that are suitable to operate in such channels, in order to have high performance quality of service. Also, the mobility of the communication systems requires further investigations to reduce the complexity and the power consumption of the receiver. This book aims to provide highlights of the current research in the field of wireless communications. The subjects discussed are very valuable to communication researchers rather than researchers in the wireless related areas. The book chapters cover a wide range of wireless communication topics.

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