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Utilizing Monte Carlo Simulation for Valuation: the Case of Barrier Options under Stochastic Interest Rates

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1. Introduction

In this chapter we propose a Monte Carlo approach for pricing barrier options when analytical pricing formulas are unavailable. Barrier options are among the most commonly used options in the financial market and our approach should therefore be of interest. Based on numerical examples presented in the chapter, it seems likely that our approach reduces the computation time by a factor between 236,000 and 94,000,000.¹ To put this in perspective, assume we have a computer that uses one second to estimate the price using our proposed pricing algorithm. To obtain comparable price estimates using standard Monte Carlo simulations would require a computation time between three days and three years!

Plain vanilla put and call options give the owner the right to sell or buy an asset at a pre specified price at some future point in time. Barrier options are either of knock-out or knock-in type. If the price of the underlying asset crosses some barrier H , a knock-out option becomes worthless, i.e., the option contract is canceled. For a knock-in option the option is invoked when the underlying asset crosses the barrier. Thus, a knock-in option expires worthless if the price of the underlying asset never crosses the barrier during the option's life.

Analytical pricing formulas for barrier options are readily available when the value of the underlying asset follows a geometric Brownian motion and interest rates are deterministic. Here we have in mind a situation where the underlying asset follows a process that precludes the derivation of an analytical pricing formula for the option. In particular we focus on the situation where interest rates are stochastic. Other situations could be where the underlying asset follows more complicated price processes.

We propose a Monte Carlo approach to value the barrier option. Estimating the market value of barrier options by Monte Carlo simulations is known to be rather time consuming (see e.g., Broadie et al. (1997)). First, a relatively high number of simulations is needed in order to reduce the standard error of the price estimates. Second, since the barrier option is path-dependent, the whole price path for the underlying asset is needed to determine whether the barrier H has been crossed or not. Approximating the price path with few monitoring points, results in biased estimates. In fact, somewhat surprisingly many monitoring points is needed to get unbiased price estimates. Both these facts make estimation of barrier option

¹ Using other parameter values will change these factors, but we think they give a reasonable picture of the merits of our simulation approach.

prices by Monte Carlo simulation very computational intensive. Our simulation approach *simultaneously* handles these two issues. We exploit the analytical pricing formula for the option when the underlying asset follows a geometric Brownian motion and interest rates are deterministic, and include this price as a control variate in the simulations. This gives a significant reduction in the number of simulations needed to get estimates with a given level of the standard error. Also, and more important, by including the control variate the number of monitoring points needed to reduce the problem with biased estimates is considerably reduced. These two effects make the proposed Monte Carlo approach highly efficient. It is also easy to implement. Related use of the control variate technique for Asian options is considered in Fu et al. (1998) and for different price processes by Lindset & Lund (2007).

The chapter is organized as follows: In section 2 we present the economic set-up. In section 3 a short description of barrier options is given. The simulation approach is described in section 4, while the barrier option under stochastic interest rates is analyzed in section 5. Numerical examples are given in section 6 and the chapter is concluded in section 7.

2. The economic model and preliminaries

We assume a frictionless financial market with two primary traded assets; a non-dividend paying stock and a money market account.² We further assume that there exists a unique equivalent martingale measure Q , also known as the risk-neutral measure. The price dynamics of the stock under the equivalent martingale measure Q are given by³

$$dS_t = r_t S_t dt + \sigma(t)^\top S_t d\mathbf{W}_t^Q,$$

where r_t is the short-term interest rate at time t , $\sigma(t)$ is a d -dimensional, possibly time dependent volatility function, \mathbf{W}_t^Q is a standard d -dimensional Brownian motion under the equivalent martingale measure Q , and $^\top$ means transpose. In what follows we let $d = 2$.

It will be convenient to divide the time interval $[0, T]$ into N time periods of equal length. We let time period n be the interval $[t_{n-1}, t_n]$, where in particular $t_0 = 0$ and $t_N = T$. The accumulated log-return on the stock over the future time period $n \in \{1, 2, \dots, N\}$ is given by

$$\delta_n = \int_{t_{n-1}}^{t_n} \left(r_v - \frac{1}{2} \|\sigma(v)\|^2 \right) dv + \int_{t_{n-1}}^{t_n} \sigma(v)^\top d\mathbf{W}_v^Q,$$

where $\|\cdot\|$ is the Euclidean norm.

Let $f(t, s)$, $t \leq s$, be the instantaneous forward rate at time s prevailing at time t . Intuitively, we can think of $f(t, s)$ as the interest rate we can agree upon at time t to be paid on "very short-term" borrowing or received from "very short-term" deposits at time s . Adopting the framework of Heath et al. (1992), we have that the arbitrage free dynamics of the forward rate are given by

$$df(t, s) = \sigma_f(t, s)^\top \int_t^s \sigma_f(t, u) du dt + \sigma_f(t, s)^\top d\mathbf{W}_t^Q,$$

where $\sigma_f(t, s)$ is a time dependent d -dimensional volatility function. The short-term interest rate is obtained by setting $s = t$, i.e., $r_t = f(t, t)$. The money market account is an asset that accrues the short-term interest rate and has the following price dynamics

$$dM_t = r_t M_t dt, \quad M_0 = 1.$$

² Strictly speaking, we also assume that there is a continuum of zero-coupon bonds traded in the market.

³ Vectors and matrices are written in bold fonts.

Note that the accumulated log-return on the money market account over the future time period n is given by

$$\beta_n = \int_{t_{n-1}}^{t_n} r_v dv = -\ln F(0, t_{n-1}, t_n) + \frac{1}{2} \sigma_{\beta_n}^2 + \sum_{k=1}^{n-1} c_{k,n} \\ + \int_0^{t_{n-1}} \left(\int_{t_{n-1}}^{t_n} \sigma_f(v, u) du \right)^\top d\mathbf{W}_v + \int_{t_{n-1}}^{t_n} \left(\int_v^{t_n} \sigma_f(v, u) du \right)^\top d\mathbf{W}_v,$$

where

$$F(0, t_m, t_n) = \frac{B(0, t_n)}{B(0, t_m)}, \\ \sigma_{\beta_n}^2 = \int_0^{t_{n-1}} \left\| \int_{t_{n-1}}^{t_n} \sigma_f(v, u) du \right\|^2 dv + \int_{t_{n-1}}^{t_n} \left\| \int_v^{t_n} \sigma_f(v, u) du \right\|^2 dv,$$

and

$$c_{m,n} = \int_0^{t_{m-1}} \left(\int_{t_{m-1}}^{t_m} \sigma_f(v, u) du \right)^\top \left(\int_{t_{n-1}}^{t_n} \sigma_f(v, u) du \right) dv \\ + \int_{t_{m-1}}^{t_m} \left(\int_v^{t_m} \sigma_f(v, u) du \right)^\top \left(\int_{t_{n-1}}^{t_n} \sigma_f(v, u) du \right) dv.$$

Here $B(t, T)$ is the time t market value of a zero coupon bond maturing at time $T \geq t$ with unit face value, i.e., $B(T, T) = 1$, $\sigma_{\beta_n}^2$ is the variance of β_n , and $c_{m,n}$ is the covariance between β_m and β_n , $1 \leq m < n$.

Future cashflows (defined under the equivalent martingale measure Q) to be received from the options at time T are discounted back to present (time 0) with the discount factor $e^{-\sum_{i=1}^N \beta_i}$. To obtain numerical results, we need a closer functional specification for the volatility structure. Throughout the chapter we assume that

$$\sigma(t) = \sigma_S \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$\sigma_f(v, u) = \sigma e^{-\kappa(u-v)} \begin{bmatrix} \varphi \\ \sqrt{1-\varphi^2} \end{bmatrix},$$

where σ_S , σ , κ , and φ are constants. This specification corresponds to the model of Hull & White (1990), also known as the extended Vasicek (1977) model, and is Gaussian since the volatilities are only time dependent. Here κ is the force at which the short-term interest rate reverts to some long-term mean level.

For technical details, see e.g., Heath et al. (1992) and Amin & Jarrow (1992).

3. Barrier options

There are four types of plain barrier options:

1. down and out option

2. down and in option
3. up and out option
4. up and in option.

In this chapter we have chosen to focus on the first of these; a down and out option, or more precisely, a down and out call. The results we present apply equally well, given appropriate adjustments, for the other three options.

A down and out call is a regular call option with the extra feature that it is knocked out if the value of the underlying stock at some point in time during the option's life crosses the barrier H from above. When the option matures at time T , the payoff of the option is $B_T = \max(S_T - X, 0)\mathbb{I}$, where X is the exercise price and

$$\mathbb{I} = 1 \quad \text{if} \quad \min_{t \in [0, T]} S_t > H$$

and $\mathbb{I} = 0$ otherwise. Let B_0 be the time zero market value of the barrier option and C_0 the corresponding call value. It is then the case that $B_0 = C_0 - J_0$, where J_0 is the non-negative knock-out discount.⁴

Assuming constant interest rates, the market value of the barrier option can be calculated in closed form. Once we allow for stochastic interest rates, there is, to the best of our knowledge, no known analytical formula for the market value of the option. We therefore turn to numerical methods to estimate the market value. More precisely, we use Monte Carlo simulations.

As is well known in the literature (see e.g., Broadie et al. (1997)), estimating the market value of barrier options by Monte Carlo simulations can be extremely computational intensive. To see if the option is knocked out, and therefore has zero value, requires the entire sample path for the stock price over the option's life time to be observed. By simulation, the best we can do is to have discrete observations of the price path. We can therefore possibly miss observing where the stock price crosses the barrier (at least) twice between two monitoring points. Simulating many stock prices for each price path is more time consuming than simulating few stock prices. However, simulating few stock prices gives a higher probability of missing stock prices that cross the barrier. This results in problems with bias. For a knock-out option, the expected bias in the estimated option price is positive but decreases in the number of monitoring points. In figure 1 we illustrate the relative slow convergence rate in terms of number of monitoring points for the case with deterministic interest rates.⁵

Of course, if we only have one monitoring point (at the maturity date for the option, i.e., at time T), this corresponds to the plain European call option. As we see from figure 1, even 1,000 (10^3) and 10,000 (10^4) monitoring points lead to some bias.

4. The simulation procedure

Under the equivalent martingale measure Q , the stock price at time t_n , $n \in \{1, 2, \dots, N\}$ is given by

$$S_{t_n} = S_{t_{n-1}} e^{\delta_n}.$$

⁴ Expressions for B_0 , C_0 , and J_0 can be found in books on option pricing, for instance in Musiela & Rutkowski (1997).

⁵ The calculations are performed using Ox, see e.g., Doornik (1999).

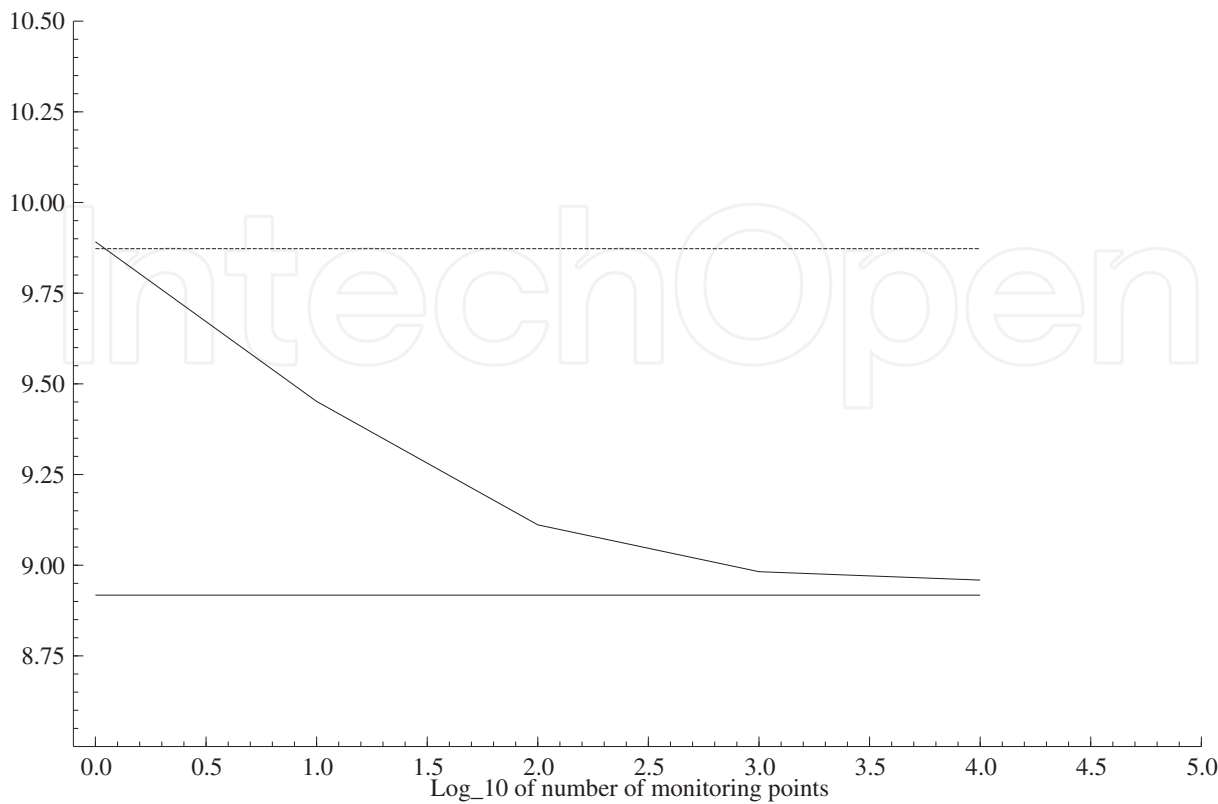


Fig. 1. The figure shows the estimated market value of the barrier option for different number of monitoring points when interest rates are deterministic (downward sloping line). The upper straight line shows the market value of the corresponding European option, while the lower straight line shows the market value of the barrier option (both estimated with analytical pricing formulas). Parameter values are: $S_0 = 100$, $X = 95$, $H = 90$, $r = 0.05$, $\sigma = 0.2$, and $T = 0.5$, where X is the exercise price and H is the knock out barrier. The prices are estimated using 1,000,000 simulations.

Notice that the log-return on the stock in period n can be written as

$$\delta_n = \underbrace{\beta_n}_{a_n} - \underbrace{\frac{1}{2} \int_{t_{n-1}}^{t_n} \|\sigma(v)\|^2 dv + \int_{t_{n-1}}^{t_n} \sigma(v)^\top d\mathbf{W}_v}_{b_n}.$$

Both a_n and b_n are random variables with Gaussian distributions. In order to simulate price paths for the stock and the discount factor, we *simultaneously* simulate all the $2N$ random variables a_n and b_n , $n \in \{1, 2, \dots, N\}$. To this end, we first calculate the variance-covariance matrix, Σ , for the $2N$ variables and Cholesky decompose this matrix into a new matrix \mathbf{A} , i.e.,

$$\Sigma = \mathbf{A}\mathbf{A}^\top.$$

Each of the variables has a deterministic part that we denote D . We include the $2N$ D s in a $2N$ -dimensional vector \mathbf{D} . Finally, letting \mathbf{R} be a vector containing all the random variables, we calculate \mathbf{R} as follows:

$$\mathbf{R} = \mathbf{D} + \mathbf{A}\boldsymbol{\varepsilon},$$

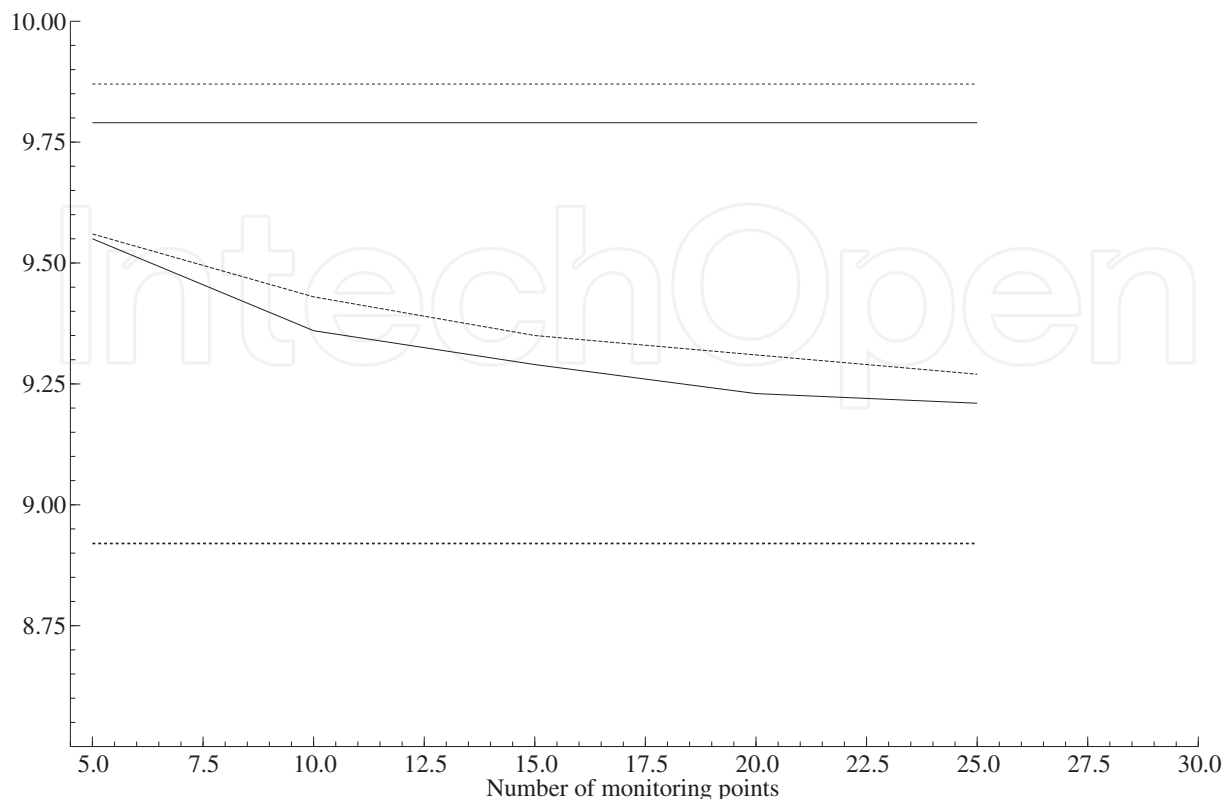


Fig. 2. The figure shows the estimated market value of the barrier option for different number of monitoring points (the two downward sloping lines). The dotted lines are option prices under deterministic interest rates, while the solid lines are under stochastic interest rates. The two straight lines at the top show the market values of European options, while the lower straight line shows the market value of the barrier option (these are estimated by analytical pricing formulas). Parameter values are: $S_0 = 100$, $X = 95$, $H = 90$, $\sigma_S = 0.2$, $\sigma = 0.03$, $\kappa = 0.1$, $\varphi = -0.5$ and $T = 0.5$. The initial term structure of interest rates is assumed flat and equal to 0.05. The prices are estimated using 1,000,000 simulations (they are also reported in table 1).

where ε is a $2N$ -dimensional vector with n th element $\varepsilon_n \sim \mathcal{N}(0,1)$ and where ε_n and ε_m , $n \neq m$, are independent.

5. Barrier options and stochastic interest rates

As already mentioned, we are not aware of any analytical pricing formulas for barrier options in the presence of stochastic interest rates, and a simulation approach for estimating the option value can therefore make sense. As figure 2 illustrates, also under stochastic interest rates the convergence rate is slow (note that we do not know the true value of the barrier option). Notice in particular that the differences between the price estimates under deterministic and stochastic interest rates are about the same for different number of monitoring points.

The idea in this chapter is to exploit the analytical pricing formula that exists under deterministic interest rates when we simulate under stochastic interest rates. We use the case with deterministic interest rates as a control variate for the estimation of the option value under stochastic interest rates. We benefit from the control variate in two ways.

First, for realistic parameter values the major part of the uncertainty in the state variable, i.e., in the future stock prices, comes from the b_n -parts, not the a_n -parts. Under deterministic interest rates the a_n -parts are of course non-random. Thus, future stock prices will be highly correlated under stochastic and deterministic interest rates, and so will also the discounted future payoffs from the options be. As we show later in the chapter, this gives a significant reduction in the standard errors of the price estimates.

Second, let g be the option price calculated under deterministic interest rates with the analytical pricing formula. Let further f_i and g_i be the i th simulated *unbiased* discounted payoff of the barrier option under stochastic and deterministic interest rates, respectively. The i th simulated value using the control variate then becomes

$$f_i(b) = f_i - b(g_i - g), \quad (1)$$

for some constant b . From figure 2 we remember that the price estimates behave somewhat similarly under deterministic and stochastic interest rates when the number of monitoring points changes. Let g'_i and f'_i be the i th simulated discounted payoff of the barrier options that are *biased* because too few monitoring points are used. Let further ϕ_i and γ_i be the bias under stochastic and deterministic interest rates, respectively. These biases are defined as $\phi_i = f'_i - f_i$ and $\gamma_i = g'_i - g_i$. The expected biases in the price estimates are $\phi = E_Q[\phi_i]$ and $\gamma = E_Q[\gamma_i]$. Let $f'_i(b)$ be the i th simulated value when using the control variate and the biased values f'_i and g'_i . We then have that

$$\begin{aligned} f'_i(b) &= f'_i - b(g'_i - g) \\ &= (f_i + \phi_i) - b((g_i + \gamma_i) - g) \\ &= f_i - b(g_i - g) + (\phi_i - b\gamma_i). \end{aligned}$$

In the special case where $\phi = b\gamma$, it is clear that $E_Q[f'_i(b)] = E_Q[f_i(b)]$ is an unbiased estimator for the value of the barrier option under stochastic interest rates. It seems difficult to analyze the bias analytically, and we therefore have to rely on numerical calculations. We show by numerical examples that the use of the control variate in the simulations makes it possible to reduce the number of monitoring points. The computation time increases quadratically in the number of monitoring points. Thus, including the control variate in the simulations has the potential to severely reduce the computation time. In figure 3 we illustrate the computation time as a function of the number of monitoring points.

The optimal b (see equation (1)) is usually found by regressing the simulated f_i s on the simulated g_i s. The optimality criterion is then to minimize the standard error of the price estimate. In our setting, as long as $|\phi - b\gamma| < \phi$, the control variate also helps reduce the problem with bias. The optimal choice of b is now more complicated; one has to balance increased speed because of variance reduction and because of bias reduction. How to balance this must be determined by the user of the algorithm. In this chapter we have for simplicity set $b = 1$.

6. Numerical examples

In this section we present numerical examples illustrating the use of our proposed pricing algorithm. As our base-case we use the following parameter values: $S_0 = 100$, $X = 95$, $H = 90$, $\sigma_S = 0.2$, $\sigma = 0.03$, $\kappa = 0.1$, $\varphi = -0.5$, and $T = 0.5$. The initial term structure of interest rates is assumed flat and equal to 0.05. In table 1 we present the results for the base-case parameters.

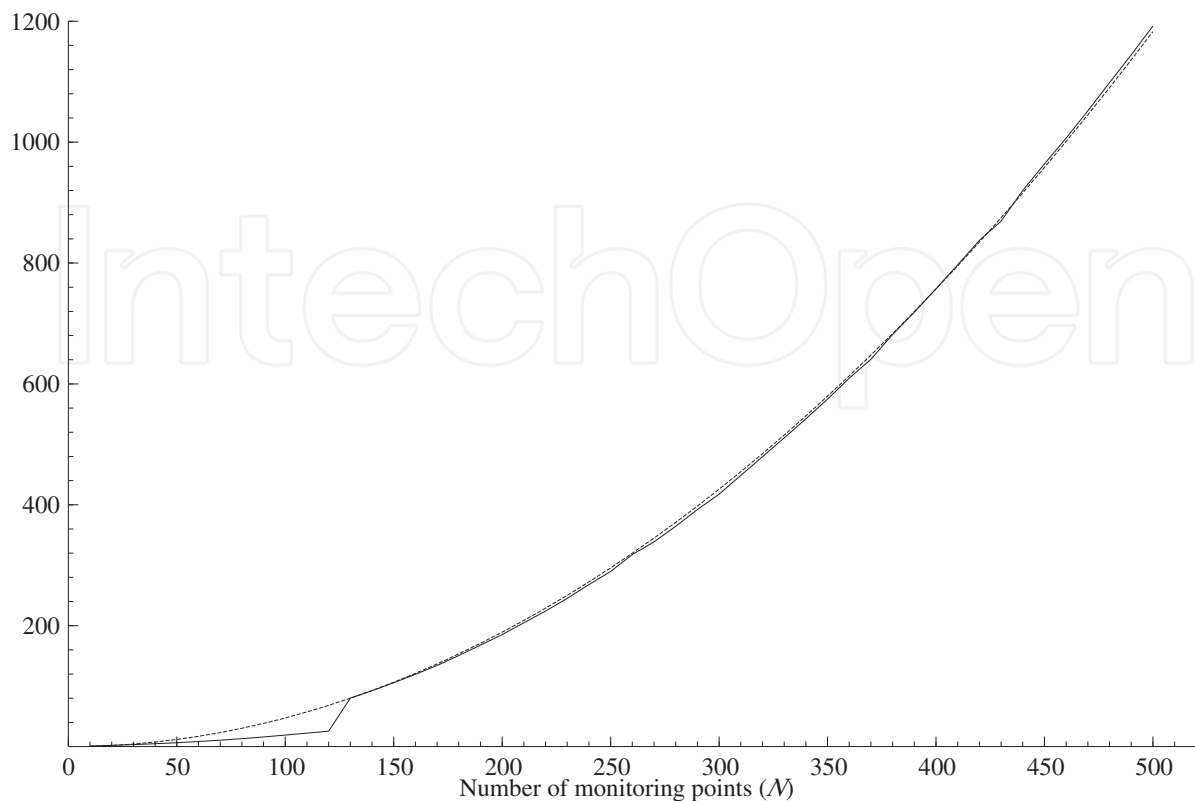


Fig. 3. The figure shows the normalized computation time (ct) for different number of monitoring points (solid line). The computation time for 10 monitoring points is set to one. The dotted line shows the fitted function $ct(N) = 0.004732N^2$ with $R^2 = 0.9995$.

We then change the parameter values that are related to the interest rates (σ (table 2), κ (table 3), and φ (table 4)).

The simulation results in table 1 indicate that in order to obtain estimates with standard errors of the same order of magnitude, we only have to do about 4,000 to 5,000 simulations when including the control variate, compared to 1,000,000 when using standard Monte Carlo simulations.⁶ Thus, in terms of standard errors, standard Monte Carlo simulation has about 200 to 250 times higher computation time than when the control variate is included. Furthermore, it seems like we can reduce the number of monitoring points from 500 to 10 or 20 and still obtain about the same price estimates. Keep in mind that we are not able to come up with a good benchmark price to compare our results with. For instance, with $T = 0.5$, the Cholesky decomposition failed for 600 monitoring points. However, given the similarity between the pricing problem under deterministic and stochastic interest rates and the results in figure 1, we may project that to get unbiased price estimates as much as 10,000 or more monitoring points are needed if raw Monte Carlo simulations are performed in the model with stochastic interest rates. When we take into account that the computation time grows quadratically in the number of monitoring points, we estimate that by *not* including the control variate in the simulations increases the computation time by a factor between

⁶ The squared ratio of the standard errors are about $(\frac{0.0113}{0.0007})^2 \approx \frac{1,000,000}{4,000}$.

Number of Observation Points	Deterministic Interest Rates		Stochastic Interest Rates		Stochastic Interest Rates Control Variate	
1	9.8727*	–	9.7901*	–	–	–
5	9.5563	(0.01137)	9.5460	(0.01123)	8.9019	(0.00119)
10	9.4273	(0.01140)	9.3626	(0.01127)	8.8438	(0.00078)
15	9.3472	(0.01142)	9.2898	(0.01129)	8.8322	(0.00069)
20	9.3061	(0.01143)	9.2320	(0.01130)	8.8294	(0.00067)
25	9.2688	(0.01143)	9.2096	(0.01133)	8.8279	(0.00067)
100	9.1066	(0.01147)	9.0432	(0.01135)	8.8274	(0.00070)
500	9.0215	(0.01150)	8.9266	(0.01136)	8.8269	(0.00073)
∞	8.9175*	–	–	–	–	–

* Estimated by analytical pricing formula.

Table 1. The table shows price estimates for barrier options for different number of monitoring points for the case with deterministic interest rates, stochastic interest rates, and for our proposed approach using the control variate. Parameter values are: $S_0 = 100$, $X = 95$, $H = 90$, $\sigma_S = 0.2$, $\sigma = 0.03$, $\kappa = 0.1$, $\varphi = -0.5$, and $T = 0.5$. The initial term structure of interest rates is assumed flat and equal to 0.05. The prices are estimated using 1,000,000 simulations. Standard errors are reported in parenthesis.

236,000 and 94,000,000!⁷ If we only use 5,000 simulations with 10 or 20 monitoring points, the calculations are fast (takes only a fraction of a second). As we can see from table 2, increasing the interest rate volatility to 0.08, which is much higher than what we observe in most economies, the option prices decrease and the standard errors for the cases where the control variate has been used increase. The explanation for the first observation is that increasing interest rate volatility actually leads to decreased volatility for the return on the stock under the equivalent martingale measure Q . The second observation follows because the stock price under deterministic and stochastic interest rates becomes less correlated. I.e., the interest rate volatility becomes more important. When the price of the two stocks becomes less correlated, the effect of using the control variate decreases and the standard errors therefore increase. Notice that the algorithm still reduces the bias and only using 10 or 20 monitoring points is likely to be sufficient. By comparing the results in table 1 and table 3 it is clear that doubling the value of the parameter κ from 0.1 to 0.2 has a negligible impact on the price estimates. Finally, from table 4 we see that by imposing a positive correlation between the stock return and the interest rates (i.e., by setting $\varphi = 0.5$), the option prices under stochastic interest rates increase. The reason is that the overall volatility in the stock return now increases. However, most importantly for our analysis is that the standard errors by including the control variate still are significantly reduced and that we also can use a significantly lower number of monitoring points. Finally, it should be mentioned that $\sigma = 0.2$ is a low parameter value if the underlying asset is a share of stock. This value would be more typical for a stock index where most of the idiosyncratic risk in individual stock returns is diversified away. By using a higher volatility,

⁷ The first number is estimated as $0.004732 \cdot 500^2 \cdot 200 = 236,600$, where the number 200 comes from the variance reduction. The second number assumes that 10,000 monitoring points are needed: $0.004732 \cdot 10,000^2 \cdot 200 = 94,640,000$.

Number of Observation Points	Deterministic Interest Rates		Stochastic Interest Rates		Stochastic Interest Rates Control Variate	
1	9.8727*	–	9.6694*	–	–	–
5	9.5563	(0.01137)	9.4194	(0.01102)	8.7743	(0.00169)
10	9.4273	(0.01140)	9.2313	(0.01105)	8.7149	(0.00146)
15	9.3472	(0.01142)	9.1578	(0.01108)	8.7007	(0.00141)
20	9.3061	(0.01143)	9.0994	(0.01108)	8.6986	(0.00143)
25	9.2688	(0.01143)	9.0765	(0.01112)	8.6950	(0.00143)
100	9.1066	(0.01147)	8.9053	(0.01112)	8.6917	(0.00146)
500	9.0215	(0.01150)	8.7993	(0.01114)	8.6883	(0.00148)
∞	8.9175*	–	–	–	–	–

* Estimated by analytical pricing formula.

Table 2. The table shows price estimates for barrier options for different number of monitoring points for the case with deterministic interest rates, stochastic interest rates, and for our proposed approach using the control variate. Parameter values are: $S_0 = 100$, $X = 95$, $H = 90$, $\sigma_S = 0.2$, $\sigma = 0.08$, $\kappa = 0.1$, $\varphi = -0.5$, and $T = 0.5$. The initial term structure of interest rates is assumed flat and equal to 0.05. The prices are estimated using 1,000,000 simulations. Standard errors are reported in parenthesis.

Number of Observation Points	Deterministic Interest Rates		Stochastic Interest Rates		Stochastic Interest Rates Control Variate	
1	9.8727*	–	9.7914*	–	–	–
5	9.5563	(0.01137)	9.5474	(0.01123)	8.9033	(0.00119)
10	9.4273	(0.01140)	9.3642	(0.01127)	8.8438	(0.00077)
15	9.3472	(0.01142)	9.2913	(0.01129)	8.8340	(0.00068)
20	9.3061	(0.01143)	9.2480	(0.01131)	8.8307	(0.00065)
25	9.2688	(0.01143)	9.1873	(0.01130)	8.8315	(0.00067)
100	9.1066	(0.01147)	9.0417	(0.01133)	8.8297	(0.00071)
500	9.0215	(0.01150)	8.9283	0.01136)	8.8285	(0.00072)
∞	8.9175*	–	–	–	–	–

* Estimated by analytical pricing formula.

Table 3. The table shows price estimates for barrier options for different number of monitoring points for the case with deterministic interest rates, stochastic interest rates, and for our proposed approach using the control variate. Parameter values are: $S_0 = 100$, $X = 95$, $H = 90$, $\sigma_S = 0.2$, $\sigma = 0.03$, $\kappa = 0.2$, $\varphi = -0.5$, and $T = 0.5$. The initial term structure of interest rates is assumed flat and equal to 0.05. The prices are estimated using 1,000,000 simulations. Standard errors are reported in parenthesis.

say 30%-50%, the b_n -parts increase in importance relative to the a_n -parts. Thus, using a higher (and more realistic) volatility benefits the relative merits of the pricing algorithm proposed in this chapter.

Number of Observation Points	Deterministic Interest Rates		Stochastic Interest Rates		Stochastic Interest Rates Control Variate	
1	9.8727*	–	9.9627*	–	–	–
5	9.5563	(0.01137)	9.7303	(0.01153)	9.0806	(0.00118)
10	9.4273	(0.01140)	9.5551	(0.01156)	9.0231	(0.00077)
15	9.3472	(0.01142)	9.4646	(0.01158)	9.0145	(0.00068)
20	9.3061	(0.01143)	9.4283	(0.01160)	9.0130	(0.00065)
25	9.2688	(0.01143)	9.3852	(0.01160)	9.0118	(0.00067)
100	9.1066	(0.01147)	9.2266	(0.01162)	9.0136	(0.00071)
500	9.0215	(0.01150)	9.1236	(0.01164)	9.0147	(0.00072)
∞	8.9175*	–	–	–	–	–

* Estimated by analytical pricing formula.

Table 4. The table shows price estimates for barrier options for different number of monitoring points for the case with deterministic interest rates, stochastic interest rates, and for our proposed approach using the control variate. Parameter values are: $S_0 = 100$, $X = 95$, $H = 90$, $\sigma_S = 0.2$, $\sigma = 0.03$, $\kappa = 0.1$, $\varphi = 0.5$, and $T = 0.5$. The initial term structure of interest rates is assumed flat and equal to 0.05. The prices are estimated using 1,000,000 simulations. Standard errors are reported in parenthesis.

7. Conclusions

We have in this chapter proposed an algorithm for pricing barrier options when analytical pricing formulas are unavailable. We have analyzed the special case where interest rates are stochastic and showed that our approach, compared to standard Monte Carlo simulations, reduces the computation time by a factor of 236,000 to 94,000,000 for realistic parameter values. Although we have focused on the case with stochastic interest rates, the approach we propose should also have potential for being used when the underlying asset follows more complicated price processes. We leave such extensions for future research.

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Applications of Monte Carlo Methods in Biology, Medicine and Other Fields of Science

Edited by Prof. Charles J. Mode

ISBN 978-953-307-427-6

Hard cover, 424 pages

Publisher InTech

Published online 28, February, 2011

Published in print edition February, 2011

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How to reference

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Snorre Lindset (2011). Utilizing Monte Carlo Simulation for Valuation: the Case of Barrier Options under Stochastic Interest Rates, Applications of Monte Carlo Methods in Biology, Medicine and Other Fields of Science, Prof. Charles J. Mode (Ed.), ISBN: 978-953-307-427-6, InTech, Available from: <http://www.intechopen.com/books/applications-of-monte-carlo-methods-in-biology-medicine-and-other-fields-of-science/utilizing-monte-carlo-simulation-for-valuation-the-case-of-barrier-options-under-stochastic-interest>

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