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## Affine-TS-Based Fuzzy Tracking Design

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### 1. Introduction

Research in fuzzy modelling and fuzzy control have come of age (1), (2), (3), (4). There are two model-based approaches to theoretically construct the T-S fuzzy system of a nonlinear system. One is from local linear approximation, which generates linear singleton-included rule-consequences (*affine* T-S fuzzy system). The other is via the sector nonlinearity concept (5), (6), (7), which generates linear singleton-free rule-consequences (*linear* T-S fuzzy system). Both are demonstrated to be universal approximations to any smooth nonlinear systems (8), (9), (10). The *linear* type T-S system is popular due to its further intrinsic analysis. A linear matrix inequality (LMI)-based fuzzy controller was used to minimize the upper bound of a performance index (9). Structure-oriented and switching fuzzy controllers were developed for more complicated systems (7), (11), (12). Optimal fuzzy control techniques were proposed to minimize a performance index from local-concept and global-concept approach, respectively (13), (14), (15). Yang and coworkers used an input-free T-S fuzzy system to approximate a uncertain nonlinear state function, and adopted hybrid sliding-mode, adaptive and back-stepping control techniques to control a strick-feedback uncertainty-included nonlinear system (16). Via a fuzzy-static-output-feedback technique, Lo and Lin transformed a robust  $H_\infty$  quadratic tracking problem into a bilinear-matrix inequality (17).

Target tracking is common in the real world. However, it is tough work to construct a system to achieve perfect tracking. We derived local-concept-based tracking technologies for various tracking problems (18). Cuevas and Toledo solved a chaotic-synchronization problem, and found out two Lorenz' attractors (19). Uang and Hung focused on a model-following tracking problem (20). Chen and coauthors reformulated a  $H_\infty$  tracking problem into a LMI problem (21). They also adopted this technique to derive a reference-tracking-control design for an interconnected system (22). Recently, they used a T-S fuzzy model to describe a fuzzy stochastic moving-average system, and derived a minimum-variance (23).

However, it is impractical to *theoretically* convert a mathematical model into a T-S fuzzy model if a nonlinear system is too complex to describe. More and more researchers attempt to identify fuzzy models from input-output data (24), (25). The approach of model-free nonlinear systems to guarantee the proposed fuzzy model under limited modelling error and the corresponding fuzzy control with desirable implementation is still in development. For this model-free approach, an *affine* type fuzzy model will be more preferred than *linear* type on providing one more adjustable parameter during computation-intelligent (neural-fuzzy-evolution) learning process (26). However, no affine-type tracking-controller and few affine-type regulating-controllers were proposed. Hsiao and coworkers proposed

a hybrid-compensation controller (27). E. Kim and coworkers used convex optimization technique to construct a LMI-based affine-type fuzzy controller (28), (29). They recently specialized in an affine T-S fuzzy system with *constant input-matrix* and transformed a regulating problem into a bilinear-matrix inequality (30). P. Bergsten and coworkers tried to derive an affine-type observer by regarding the singleton of an affine rule-consequence as a trivial term. Therefore, the result was, in fact, belong to a typical *linear-type* case (31).

Here, we realize a tracking system as an affine T-S fuzzy systems, and formulate a tracking problem as a fuzzy quadratic-tracking problem. The tracking-control design schemes for an affine TS-based nonlinear system to trace two kinds of targets (moving target and model-following target) are derived in Sections 2 and 3. These two sections describe fuzzy quadratic tracking problems, the derived optimal fuzzy tracking-controllers, and the Lyapunov-based stability analysis. The performance of the proposed affine-based trackers for these two targets is examined in Section 4. Section 5 summarizes the results of our research and suggests areas for further research.

## 2. Moving-target tracking problem

By local-linear approximation or neural fuzzy inference networks, a nonlinear system can be realized as an *affine* T-S fuzzy system,

$$\begin{aligned} R^i : \text{ If } x_1 \text{ is } T_{1i}, \dots, x_n \text{ is } T_{ni}, \text{ then } \dot{X}(t) &= A_i X(t) + B_i u(t) + D_i, \quad i = 1, \dots, r, \\ Y(t) &= C X(t), \end{aligned} \quad (1)$$

where  $R^i$  denotes the  $i$ th rule of the fuzzy model;  $x_1, \dots, x_n$  are system states;  $T_{1i}, \dots, T_{ni}$  are input fuzzy terms in the  $i$ th rule;  $X(t) = [x_1, \dots, x_n]^t \in \mathbb{R}^n$  is state vector,  $Y(t) = [y_1, \dots, y_{n'}]^t \in \mathbb{R}^{n'}$  is system output vector,  $u(t) \in \mathbb{R}^m$  is system input; and  $A_i, B_i, C_i$  and  $D_i$  are  $n \times n, n \times m, n' \times n$  and  $n \times 1$  matrices, respectively. We note that the entire T-S fuzzy system in Eq. (1) can be expressed as

$$\dot{X}(t) = \sum_{i=1}^r h_i(X(t)) (A_i X(t) + B_i u(t) + D_i) \quad (2)$$

with  $X(0) = X_0 \in \mathbb{R}^n$ ;  $h_i(X(t))$  denotes the normalized firing-strength of the  $i$ th rule of the fuzzy system,  $h_i(X(t)) = \alpha_i / \sum_{i=1}^r \alpha_i$  with  $\alpha_i = \prod_{j=1}^n \mu_{T_{ji}}(X(t))$ , where  $\mu_{T_{ji}}(X(t))$  is the membership function of fuzzy term  $T_{ji}$ .

The problem of minimal energy consumption for the moving-target tracking control of a nonlinear system is to control the system such that its output  $Y(t)$  keeps close to the moving-target  $Y^d(t)$  under a minimal energy condition. Therefore, we can formulate it as a fuzzy quadratic tracking problem.

**PROBLEM 1.** Given a rule-based fuzzy tracking system in Eq. (1) with  $X(t_0) = X_0 \in \mathbb{R}^n$  and a rule-based fuzzy tracking-controller,

$$R^i : \text{ If } y_1 \text{ is } S_{1i}, \dots, y_{n'} \text{ is } S_{n'i}, \text{ then } u(t) = r_i(t), \quad i = 1, \dots, \delta, \quad (3)$$

find the individual tracking-law,  $r_i^*(\cdot), i = 1, \dots, \delta$ , such that the composed tracking controller,  $u^*(\cdot)$ , can minimize a quadratic cost function,

$$J(u(\cdot)) = \int_{t_0}^{\infty} [u^t(t) S u(t) + X^t(t) L_1 X(t) + (Y(t) - Y^d(t))^t L_2 (Y(t) - Y^d(t))] dt, \quad (4)$$

where  $L_1 = [I_n - C^t(CC^t)^{-1}C]^t L_3 [I_n - C^t(CC^t)^{-1}C]$ ;  $S$ ,  $L_2$  and  $L_3$  are, respectively,  $m \times m$ ,  $n' \times n'$  and  $n \times n$  positive symmetric matrices;  $X^t(t)L_1X(t)$  is the state-trajectory penalty to produce smooth response,  $u^t(t)Su(t)$  denotes fuel consumption, and the last term in  $J(u(\cdot))$  relates to error cost. Let  $L = L_1 + C^tL_2C$ . And, we define an *artificial desired-trajectory* ( $X^d(t) = C^t[CC^t]^{-1}Y^d(t)$ ) to simplify the cost function as

$$J(u(\cdot)) = \int_{t_0}^{\infty} [u^t(t)Su(t) + (X(t) - X^d(t))^t L (X(t) - X^d(t))] dt. \quad (5)$$

According to a dynamic-programming formalism, this quadratic optimal problem can be regarded as a successively on-going dynamic problem with regard to the state from the previous decision. At any time-instant, the energy of the entire fuzzy system is the “fuzzy summation” of the energy of each fuzzy subsystem. Therefore, solving a quadratic optimal tracking-control problem is to find *only one corresponding optimal* solution of the fuzzy tracking-controller for *each* rule of the *affine* fuzzy model. We further introduce an *augmented target* to re-formulate the *affine-type* local quadratic problem into a *linear-type* problem. And then, we can obtain our tracking-control design scheme as follows.

**Theorem 1 (moving-target)** For affine T-S fuzzy system in Eq. (1) and fuzzy tracking-controller in Eq. (3), if  $A_i$  is nonsingular,  $\bar{\pi}_i^{-1}(L + A_i^t \bar{\pi}_i) > 0$ ,  $(A_i, B_i)$  is completely controllable (c.c.), and  $(A_i, C)$  is completely observable (c.o.) for  $i = 1, \dots, r$ , then (1) the local fuzzy tracking-law is

$$r_i^*(t) = -S^{-1}B_i^t \bar{\pi}_i X^*(t) + \bar{r}_i^s + \bar{r}_i^{ext}(t), \quad i = 1, \dots, r; \quad (6)$$

their “blending” global fuzzy tracking-controller ( $u^*(t) = \sum_{i=1}^r h_i(X^*(t))r_i^*(t)$ ) minimizes  $J(u(\cdot))$  in (4), where  $\bar{r}_i^s = -S^{-1}B_i^t(\bar{\pi}_i \bar{X}_i^s + \bar{b}_i^s)$ ,  $\bar{r}_i^{ext}(t) = -S^{-1}B_i^t \bar{b}_i(t)$ ,  $\bar{X}_i^s = A_i^{-1}D_i$ ,  $\bar{b}_i(t)$  is the target-dependent variable for adapting to the target-variation,  $\bar{b}_i^s$  is a fuzzy singleton-related constant,

$$\dot{b}_i(t) = -(A_i - B_i S^{-1} B_i^t \bar{\pi}_i)^t b_i(t) + L X^d(t), \quad b_i(\infty) = \mathbf{0}_{n \times 1}, \quad (7)$$

$$\bar{b}_i^s = -\int_0^{\infty} e^{[A_i - B_i S^{-1} B_i^t \bar{\pi}_i]^t \tau} d\tau \cdot L \bar{X}_i^s, \quad (8)$$

and  $\bar{\pi}_i$  is a unique symmetric positive-definite solution of a Riccati equation,

$$K_i A_i + A_i^t K_i - K_i B_i S^{-1} B_i^t K_i + L = \mathbf{0}; \quad (9)$$

(2) the entire feedback fuzzy tracking system is stable,

$$\dot{X}^*(t) = \sum_{i=1}^r h_i(X^*(t)) [(A_i - B_i S^{-1} B_i^t \bar{\pi}_i) X^*(t) + B_i (\bar{r}_i^s + \bar{r}_i^{ext}(t)) + D_i]. \quad (10)$$

**Proof.** See the Appendix.

### 3. Model-following tracking problem

In this section, we consider another fuzzy tracking problem, whose target comes from the response of a reference model. We shall derive a tracking design scheme to control a nonlinear system such that system-output  $Y(t)$  keeps following the model-response-target  $Y^d(t)$  with minimal energy consumption.

**PROBLEM 2.** Given an affine T-S fuzzy tracking system in Eq. (1) and a fuzzy tracking-controller in Eq. (3) with  $X(t_0) = X_0 \in \mathbb{R}^n$ , find the individual tracking-law,  $r_i^*(\cdot)$ ,  $i = 1, \dots, \delta$ , such that the composed tracking-controller,  $u^*(\cdot)$ , can minimize  $J(u(\cdot))$  in Eq. (4) and follow the target  $Y^d(t)$ , which is the response of a linear model,

$$\begin{aligned} \dot{z}_1(t) &= F_1 z_1(t) + J_1 v(t), \\ Y^d(t) &= E_1 z_1(t) \end{aligned} \quad (11)$$

with  $z_1(t_0) = z_{10}$ ; and input-command  $v(t) \in \mathbb{R}^{m'}$  is the zero-input response of the system:  $\dot{z}_2(t) = F_2 z_2(t)$  and  $v(t) = E_2 z_2(t)$  with  $z_2(t_0) = z_{20}$ , where  $z_1(t) \in \mathbb{R}^h$  and  $z_2(t) \in \mathbb{R}^{h'}$  are system states;  $F_1, J_1, E_1, F_2$  and  $E_2$  are respectively  $h \times h, h \times m', n' \times h, h' \times h'$  and  $m' \times h'$  matrices.

By defining  $Z(t) = [z_1^t(t) \ z_2^t(t)]^t$ , we can rewrite the desired tracked system as

$$\begin{aligned} \dot{Z}(t) &= \begin{bmatrix} F_1 & J_1 E_2 \\ \mathbf{0}_{h' \times h} & F_2 \end{bmatrix} Z(t) = FZ(t), \\ Y^d(t) &= [E_1 \ \mathbf{0}_{n' \times h'}] Z(t) = EZ(t). \end{aligned} \quad (12)$$

We further define  $\tilde{X}(t) = [X^t(t) \ Z^t(t)]^t$  to simply Problem 2 into the following *augmented regulating* problem.

**PROBLEM 2.1.** Given an *augmented* affine T-S fuzzy *regulating* system,

$$\dot{\tilde{X}}(t) = \sum_{i=1}^r h_i(\tilde{X}(t)) [\tilde{A}_i \tilde{X}(t) + \tilde{B}_i u(t) + \tilde{D}_i] \quad (13)$$

with  $\tilde{X}(t_0) = \tilde{X}_0 \in \mathbb{R}^{n+h+h'}$ ,  $h_i(\tilde{X}(t)) = h_i(X(t))$ , and rule-based fuzzy controller in Eq. (3), find the individual regulating law,  $r_i^*(\cdot)$ ,  $i = 1, \dots, \delta$ , to minimize

$$J(r_i(\cdot)) = \int_{t_0}^{\infty} [\tilde{X}^t(t) \tilde{L} \tilde{X}(t) + u^t(t) S u(t)] dt, \quad (14)$$

where parameters

$$\begin{aligned} \tilde{B}_i &= \begin{bmatrix} B_i \\ \mathbf{0}_{(h+h') \times m} \end{bmatrix}, \tilde{D}_i = \begin{bmatrix} D_i \\ \mathbf{0}_{(h+h') \times 1} \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} A_i & \mathbf{0}_{n \times (h+h')} \\ \mathbf{0}_{(h+h') \times n} & F \end{bmatrix}, \\ \text{and} \\ \tilde{L} &= \begin{bmatrix} L & -LC^t[CC^t]^{-1}E \\ -E^t[CC^t]^{-1}CL & E^t[CC^t]^{-1}CLC^t[CC^t]^{-1}E \end{bmatrix}. \end{aligned}$$

To solve this issue, we first regards Problem 2.1 as Problem 1 in case of  $Y^d(t) = \mathbf{0}_{n' \times 1}$ . Then, we introduce two parameters,  $\tilde{K}_i(t) = \begin{bmatrix} K_i(t) & K_i^{21^t}(t) \\ K_i^{21}(t) & K_i^{22}(t) \end{bmatrix}$  and  $\tilde{b}_i(t) = \begin{bmatrix} b_i^s(t) \\ b_{2i}(t) \end{bmatrix}$ . After a series of complicated matrix-manipulations, we derive the following theorem.

**Theorem 2 (model-following)** For affine T-S fuzzy system in Eq. (1) and fuzzy tracking-controller in Eq. (3), let an artificial desired trajectory  $X^d(t)$  be defined as  $Y^d(t) = CX^d(t)$ , where  $Y^d(t)$  is the output of a tracked model in Eq. (11), and

$$Z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}, F = \begin{bmatrix} F_1 & J_1 E_2 \\ \mathbf{0}_{h' \times h} & F_2 \end{bmatrix},$$

$E = [E_1 \quad \mathbf{0}_{n' \times h'}]$ . If  $A_i$  is nonsingular,  $\bar{\pi}_i^{-1}(L + A_i^t \bar{\pi}_i) > 0$ ,  $(A_i, B_i)$  is c.c. and  $(A_i, C)$  is c.o. for  $i = 1, \dots, r$ , then

(1) the local fuzzy tracking-law is

$$r_i^*(t) = -S^{-1}B_i^t[\bar{\pi}_i X^* + \bar{\pi}_i^{21t} Z(t)] + \bar{r}_i^s, i = 1, \dots, r; \quad (15)$$

their "blending" global fuzzy tracking-controller ( $u^*(t) = \sum_{i=1}^r h_i(X^*(t))r_i^*(t)$ ) minimizes  $J(u(\cdot))$  in (4), where  $\bar{r}_i^s = -S^{-1}B_i^t(\bar{\pi}_i \bar{X}_i^s + \bar{b}_i^s)$ ,  $\bar{X}_i^s = A_i^{-1}D_i$ ,  $\bar{b}_i^s$  satisfies Eq. (8),  $\bar{\pi}_i$  is the unique solution of the Riccati equation in Eq. (9), and

$$\bar{\pi}_i^{21} = -\int_0^\infty e^{F^t \tau} \cdot E^t (CC^t)^{-1} CL \cdot e^{(A_i - B_i S^{-1} B_i^t \bar{\pi}_i) \tau} \cdot d\tau; \quad (16)$$

(2) the entire feedback fuzzy tracking system is stable,

$$\dot{X}^*(t) = \sum_{i=1}^r h_i(X^*(t))[(A_i - B_i S^{-1} B_i^t \bar{\pi}_i)X^*(t) - B_i S^{-1} B_i^t \bar{\pi}_i^{21t} Z(t) + B_i \bar{r}_i^s + D_i]. \quad (17)$$

**Proof.** See the Appendix.

#### 4. Numerical simulation

In this section, we use an affine TS-based nonlinear system to examine the performance of our fuzzy tracking-controllers. As we know, via analytical or hybrid-soft-computing technique any nonlinear system can be approximated by an affine T-S fuzzy system. Therefore, we can choose any affine-TS-based nonlinear system as our tracking system. We shall examine the tracking performance for moving-target first, and then for model-following-target. We consider our system as

$$\begin{aligned} R^1: & \quad \text{If } x(t) \text{ is } F_1^1 \text{ and } \dot{x}(t) \text{ is } F_2^1, \text{ then } \dot{X}(t) = A_1 X(t) + B_1 u(t) + D_1, \\ R^2: & \quad \text{If } x(t) \text{ is } F_1^1 \text{ and } \dot{x}(t) \text{ is } F_2^2, \text{ then } \dot{X}(t) = A_2 X(t) + B_2 u(t) + D_2, \\ R^3: & \quad \text{If } x(t) \text{ is } F_1^2 \text{ and } \dot{x}(t) \text{ is } F_2^1, \text{ then } \dot{X}(t) = A_3 X(t) + B_3 u(t) + D_3, \\ R^4: & \quad \text{If } x(t) \text{ is } F_1^2 \text{ and } \dot{x}(t) \text{ is } F_2^2, \text{ then } \dot{X}(t) = A_4 X(t) + B_4 u(t) + D_4 \end{aligned} \quad (18)$$

with system-output  $Y(t) = CX(t)$ , where for each rule  $C = [0 \ 1]$  and

$$\begin{aligned} X(t) &= \begin{bmatrix} \dot{x}(t) \\ x(t) \end{bmatrix}; D_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, D_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, D_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, D_4 = \begin{bmatrix} 0.3 \\ 1 \end{bmatrix}; \\ A_1 &= \begin{bmatrix} 0 & -0.02 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.225 & -0.02 \\ 1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & -1.5275 \\ 1 & 0 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} -0.225 & -1.5275 \\ 1 & 0 \end{bmatrix}; B_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, i = 1, \dots, 4. \end{aligned}$$

The membership functions are chosen as  $\mu_{F_1^1}(x(t)) = 1 - \frac{x^2(t)}{2.25}$ ,  $\mu_{F_1^2}(x(t)) = \frac{x^2(t)}{2.25}$ ,  $\mu_{F_2^1}(\dot{x}(t)) = 1 - \frac{\dot{x}^2(t)}{2.25}$ , and  $\mu_{F_2^2}(\dot{x}(t)) = \frac{\dot{x}^2(t)}{2.25}$ . We shall design a fuzzy-rule-based tracking-controller,

$$\begin{aligned} R^1: & \quad \text{If } x(t) \text{ is } F_1^1 \text{ and } \dot{x}(t) \text{ is } F_2^1, \text{ then } u(t) = r_1(t), \\ R^2: & \quad \text{If } x(t) \text{ is } F_1^1 \text{ and } \dot{x}(t) \text{ is } F_2^2, \text{ then } u(t) = r_2(t) \end{aligned} \quad (19)$$

$$\begin{aligned} R^3: & \quad \text{If } x(t) \text{ is } F_1^2 \text{ and } \dot{x}(t) \text{ is } F_2^1, \text{ then } u(t) = r_3(t), \\ R^4: & \quad \text{If } x(t) \text{ is } F_1^2 \text{ and } \dot{x}(t) \text{ is } F_2^2, \text{ then } u(t) = r_4(t), \end{aligned} \quad (20)$$

such that the designed affine-type closed-loop fuzzy system can keep tracing the moving-target with minimal cost consumption,

$$J(u(\cdot)) = \int_0^\infty [u^t(t)Su(t) + X^t(t)L_1(t)X(t) + e_y(t)^tL_2e_y(t)]dt \quad (21)$$

with  $e_y(t) = Y(t) - Y^d(t)$  and  $L_1 = [I_2 - C^t(CC^t)^{-1}C]^tL_3[I_2 - C^t(CC^t)^{-1}C]$ . We now set penalty-parameters as  $S = 0.001$ ,  $L_3 = I_2$ , and  $L_2 = I_1$ . (These parameters can be regarded as weighting factors for cost, trajectory-smoothness and output-error. Therefore, they can be chosen optionally.) Since each fuzzy subsystem is well-behaved ( $\text{rank}[A_i \ A_iB_i] = 2$  and  $\text{rank}[C^t \ A_i^tC^t]^t = 2$  for  $i = 1, \dots, 4$ ), we have the unique symmetric positive-definite solution of the algebraic Riccati equation,

$$\begin{aligned} \bar{\pi}_1 &= \begin{bmatrix} 0.0326 & 0.0316 \\ 0.0316 & 1.0311 \end{bmatrix}, \\ \bar{\pi}_2 &= \begin{bmatrix} 0.0324 & 0.0316 \\ 0.0316 & 1.0311 \end{bmatrix}, \\ \bar{\pi}_3 &= \begin{bmatrix} 0.0326 & 0.0301 \\ 0.0301 & 1.0309 \end{bmatrix}, \bar{\pi}_4 = \begin{bmatrix} 0.0323 & 0.0301 \\ 0.0301 & 1.0306 \end{bmatrix}. \end{aligned}$$

And, we have  $\bar{\pi}_i^{-1}(L + A_i^t\bar{\pi}_i) > 0$  for each rule.

So, the tracking-controller is  $u^*(t) = \sum_{i=1}^r h_i(X^*(t))[-S^{-1}B_i^t\bar{\pi}_iX^*(t) + \bar{r}_i^s + \bar{r}_i^{ext}(t)]$ , where  $\bar{r}_i^s = -S^{-1}B_i^t(\bar{\pi}_i\bar{X}_i^s + \bar{b}_i^s)$ ,  $\bar{r}_i^{ext}(t) = -S^{-1}B_i^t\bar{b}_i(t)$ ,  $\bar{X}_i^s = A_i^{-1}D_i$ . Figure 1 shows the tracking trajectories for various moving targets ( $Y^d$  is a step'wise function,  $Y^d(t) = 3 + 2\sin 0.2t$ ,  $Y^d(t) = 3 + 4e^{-0.3t}$ , and  $Y^d(t) = 0.5 + 2\log(3 + t)$ ).

This affine TS fuzzy system is also used for model-following-target tracking. Our tracked target is from a model-response  $Y^d(t)$  in Eq. (11) with initial-condition  $z_{10} = 10$ , where parameters are set at  $(F_1, J_1, E_1) = (-1, 1, 1)$ ; and input-command  $v$  is from a zero-input linear system with initial-condition  $z_{20} = 10$  and parameters  $E_2 = 1$ ,  $F_2 = -0.2$ . That is, our augmented tracked system  $Z(t)$  is in Eq. (12) with initial condition,

$$Z(0) = \begin{bmatrix} z_{10} \\ z_{20} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix},$$

and parameters,

$$F = \begin{bmatrix} F_1 & J_1E_2 \\ \mathbf{0} & F_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -0.2 \end{bmatrix},$$

$E = [E_1 \ \mathbf{0}] = [1 \ 0]$ . Based on Theorem 2, we have the model-following controller,  $u^*(t) = \sum_{i=1}^r h_i(X^*(t))[-S^{-1}B_i^t(\bar{\pi}_iX^* + \bar{\pi}_i^{21}Z(t)) + \bar{r}_i^s]$ , where  $\bar{\pi}_i^{21} = -\int_0^\infty e^{F^t\tau} \cdot E^t(CC^t)^{-1}CL \cdot e^{(A_i - B_iS^{-1}B_i^t\bar{\pi}_i)\tau} \cdot d\tau$ . Figure 2 shows the tracking-performance also for  $(F_1, F_2) = (-0.2, -1)$ ,  $(-5, -\frac{1}{30})$  and  $(-\frac{1}{30}, -5)$ . Simulation results show that the designed affine-type optimal fuzzy tracking-controllers can efficiently push the tracking system to trace various moving-targets (Fig. 1) and to follow various parameter-variation tracked model (Fig. 2) in a short time.

## 5. Conclusions

Although a *linear* type T-S fuzzy system is very popular, and has been successfully applied to various fields, the *affine* type system is more preferred for computation-intelligent (neural-fuzzy-evolution) modelling as a system is too complex to be described. In this paper, we derive an affine-type fuzzy tracking-controller for tracking various moving-targets and another for keeping following a linear system whose input-command is generated from another zero-input system. Both designed closed-loop tracking systems are demonstrated to be globally stable. We use an affine-based nonlinear system to demonstrate that the proposed tracking-controllers can quickly reach a perfect tracking-effect. In the future, we shall discuss the robustness of the designed systems.

## 6. Appendix

### Proof of Theorem 1.

(1) From the essence of the dynamic programming formalism, the operation of minimizing  $J(u(\cdot))$  in Eq. (??) can be decomposed as follows:

$$\min_{u_{[t_0, \infty]}} J(u(\cdot)) = \min_{u_{[t, \infty]}} \left\{ \int_t^\infty (e_{x_l}^t L e_{x_l} + u_l^t S u_l) dl + \min_{u_{[t_0, t]}} \int_{t_0}^t (e_{x_l}^t L e_{x_l} + u_l^t S u_l) dl \right\}, \quad (22)$$

where state-error  $e_{x_l} = X_l - X_l^d$  and the lower index is used to denote time-dependence for notation-simplification ( $X_l$  for  $X(l)$ ). Therefore, the quadratic optimization problem is, in fact, a successively on-going dynamic problem with regard to the system-state resulting from the previous decision, i.e., the initial system-state at time  $t$  is  $X_{0_t} = X_t^*$ . So, the objective of Problem 1 is to successively find the *optimal global decision* (*global optimal fuzzy controller*)  $u_t^*$  for minimizing the cost function,

$$J_t(u_t) = \int_t^\infty (e_{x_l}^t L e_{x_l} + u_l^t S u_l) dl, \quad t \in [t_0, \infty], \quad (23)$$

and for estimating  $X_{t^+}^*$  with regard to initial-state  $X_t^*$ , where  $t^+$  denotes the time-instant slightly later than time  $t$ ; and then, with the new initial-state,  $X_{t^+}^*$ , resolving  $u_{t^+}^*$  to minimize  $J_{t^+}(u_{t^+})$ . At any time-instant  $t$ , the *optimal local decision* (*local optimal fuzzy tracking-law*) derives from minimizing  $J_t(u_t)$  in Eq. (23) with regard to the *fuzzy subsystem*,

$$\dot{X}_l = A_i X_l + B_i u_l + D_i, \quad l \in [t, \infty], \quad i = 1, \dots, r; \quad (24)$$

and the *optimal global decision* derives from minimizing  $J_t(u_t)$  with regard to the *entire fuzzy system*,

$$\dot{X}_l = \sum_{i=1}^r h_i(X_l)(A_i X_l + B_i u_l + D_i), \quad l \in [t, \infty]. \quad (25)$$

Since  $u_t^*$  is just a variable to be solved either for the aforementioned local optimization problem or for the global optimization issue, we can use  $r_{i_t}^*$  to denote the optimal local decision of the  $i$ -th fuzzy subsystem.

(2) We shall further demonstrated that the global optimal decision in Problem 1 can be obtained by fuzzily blending those optimal local decisions. Now, let  $\zeta_l(X_l, u_l)$  and  $\zeta_{i_l}(X_l, r_{i_l})$ ,  $i = 1, \dots, r$ , denote, respectively, the entire energy and local energy at any time-instant  $l$ ,  $l \in [t, \infty]$ . Then,  $J_t(u_t) = \int_t^\infty \zeta_l(X_l, u_l) dl$  and  $J_t(r_{i_t}) = \int_t^\infty \zeta_{i_l}(X_l, r_{i_l}) dl$ . At any time-instant, the

energy of the entire fuzzy system is *some kinds of nonlinear combination* (fuzzy summation) of the energy of fuzzy subsystems. In other words, the global energy can be expressed in terms of rule-based local energy. We note that this combination is only state-dependent, which is nothing to do with system-input. Therefore, no matter that system behavior is nonlinear with regard to input or not, the input-term cannot be included into fuzzy-precondition for physical realizable consideration, even it is reasonable in mathematical concept. And, this nonlinear combination is not necessary to be the same type as that for blending fuzzy subsystems into an entire system. Therefore, we use  $h'(X(t))$  to denote that (a) nonlinear summation is only state dependent; (b) the energy-relationship between entire system and subsystems could be totally different from the behavior-relationship, which is denoted by normalized membership function  $h(X(t))$ . Therefore, we write  $\zeta_l(X_l, u_l) = \sum_{i=1}^r h'_i(X_l) \zeta_{i_l}(X_l, r_{i_l})$ . At time-instant  $t$  with initial-condition  $X_t^*$ , let  $r_{i_t}^*$  denote the optimal local decision to minimize  $J_t(r_{i_t})$  for all  $i = 1, \dots, r$ , i.e.,

$$\begin{aligned} \frac{\partial J_t(r_{i_t})}{\partial r_{i_t}} \Big|_{r_{i_t}^*} &= \frac{\partial}{\partial r_{i_t}} \int_t^\infty \zeta_{i_l}(X_l, r_{i_l}) dl \Big|_{r_{i_t}^*} = \frac{\partial \zeta_{i_t}(X_t^*, r_{i_t})}{\partial r_{i_t}} \Big|_{r_{i_t}^*} = 0, \\ \frac{\partial^2 J_t(r_{i_t})}{\partial r_{i_t}^2} \Big|_{r_{i_t}^*} &= \frac{\partial^2 \zeta_{i_t}(X_t^*, r_{i_t})}{\partial r_{i_t}^2} \Big|_{r_{i_t}^*} > 0 \end{aligned}$$

Their corresponding global decision  $\check{u}_t = \sum_{i=1}^r h_i(X_t^*) r_{i_t}^* \Big|_{r_{i_t}^*}$  can satisfy

$$\begin{aligned} \frac{\partial J_t(u_t)}{\partial u_t} \Big|_{\check{u}_t} &= \frac{\partial}{\partial u_t} \int_t^\infty \zeta_l(X_l, u_l) dl \Big|_{\check{u}_t} \\ &= \frac{\partial}{\partial u_t} \int_t^\infty \sum_{i=1}^r h'_i(X_l) \zeta_{i_l}(X_l, r_{i_l}) dl \Big|_{\check{u}_t} \\ &= \sum_{i=1}^r h'_i(X_t^*) \frac{\partial \zeta_{i_t}(X_t^*, r_{i_t})}{\partial u_t} \Big|_{\check{u}_t} \\ &= \sum_{i=1}^r h'_i(X_t^*) \frac{\partial \zeta_{i_t}(X_t^*, r_{i_t})}{\partial r_{i_t}} \cdot \frac{\partial r_{i_t}}{\partial u_t} \Big|_{\check{u}_t} = 0, \\ \frac{\partial^2 J_t(u_t)}{\partial u_t^2} \Big|_{\check{u}_t} &= \sum_{i=1}^r h'_i(X_t^*) \frac{\partial^2 \zeta_{i_t}(X_t^*, r_{i_t})}{\partial r_{i_t}^2} \cdot \left( \frac{\partial r_{i_t}}{\partial u_t} \right)^2 \Big|_{\check{u}_t} \\ &\quad + \sum_{i=1}^r h'_i(X_t^*) \frac{\partial \zeta_{i_t}(X_t^*, r_{i_t})}{\partial r_{i_t}} \cdot \frac{\partial^2 r_{i_t}}{\partial u_t^2} \Big|_{\check{u}_t} > 0, \end{aligned}$$

i.e.,  $\check{u}_t = u_t^*$ . Therefore, at any time instant  $t$  if we can find  $r_{i_t}^*$  to minimize  $J_t(r_{i_t})$ , then it follows that their composed global decision  $u_t^*$  can be the global minimizer of the total cost  $J_t(u_t)$ .

(3) We assume  $A_i$  to be nonsingular for  $i = 1, \dots, r$ , and let  $\hat{X}(t) = X(t) + \bar{X}_i^s$ , where  $\bar{X}_i^s = A_i^{-1} D_i$ . An affine-type local tracking problem can be rewritten as a linear-type tracking issue with augmented target  $\hat{X}_i^d(t) = \bar{X}_i^s + X^d(t)$ . In other words, our problem is reformulated as: Given a fuzzy subsystem,  $\dot{\hat{X}}_l = A_i \hat{X}_l + B_i r_{i_l}$ ,  $l \in [t, \infty]$ ,  $i = 1, \dots, r$  with  $\hat{X}_{0_i} = \hat{X}_i^*$ , find the local decision at time-instant  $t$ ,  $r_{i_t}^*$ , for minimizing a cost function,

$J_t(r_{i_t}) = \int_t^\infty ((\hat{X}_l - \hat{X}_i^d(t))^t L (\hat{X}_l - \hat{X}_i^d(t)) + r_{i_t}^t S r_{i_t}) dl$ . Then, we obtain the tracking law (18),  $r_{i_t}^*(t) = -S^{-1} B_i^t [\bar{\pi}_i \hat{X}^*(t) + \hat{b}_i(t)]$ , the fuzzy subsystem,

$\dot{X}^*(t) = (A_i - B_i S^{-1} B_i^t \bar{\pi}_i) \hat{X}^*(t) - B_i S^{-1} B_i^t \hat{b}_i(t)$ , where  $\bar{\pi}_i$  satisfies Eq. (9) and  $\hat{b}_i(t)$  satisfies

$$\dot{\hat{b}}_i(t) = -(A_i - B_i S^{-1} B_i^t \bar{\pi}_i) \hat{b}_i(t) + L \hat{X}_i^d(t), \quad \hat{b}_i(\infty) = \mathbf{0}_{n \times 1}. \quad (26)$$

According to the linearity property, we have  $\hat{b}_i(t) = b_i^s(t) + b_i(t)$ , where  $b_i(t) = \bar{b}_i(t)$  in Eq. (7) is to adaptively trace the target and

$$\dot{b}_i^s(t) = -(A_i - B_i S^{-1} B_i^t \bar{\pi}_i) b_i^s(t) + L \bar{X}_i^s, \quad b_i^s(\infty) = \mathbf{0}_{n \times 1}. \quad (27)$$

Further,  $(A_i, B_i)$  is c.c. and  $(A_i, C)$  is c.o. So, we have  $b_i^s(t) = \bar{b}_i^s$  in Eq. (8) to response to fuzzy consequence-singleton  $D_i$ . By using  $\bar{r}_i^s = -S^{-1} B_i^t (\bar{\pi}_i \bar{X}_i^s + \bar{b}_i^s)$  to denote the local singleton-related law, and  $\bar{r}_i^{ext}(t) = -S^{-1} B_i^t \bar{b}_i(t)$  to denote local target-related law, we obtain the tracking law  $r_i^*(t)$  in Eq. (6) and the fuzzy system  $X_i^*(t)$  in Eq. (10).

(4) Stability analysis: We use  $\bar{U}_i^{aug}$  to denote an augmented-target-associated input,  $\bar{U}_i^{aug} = B_i(\bar{r}_i^s + \bar{r}_i^{ext}) + D_i$ . Then, the designed feedback system in Eq. (10) can be rewritten as Eq. (28). Its stability concurs with the zero-input system in Eq. (29).

$$\dot{X}^*(t) = \sum_{i=1}^r h_i(X^*(t)) [A_i - B_i S^{-1} B_i^t \bar{\pi}_i] X^*(t) + \bar{U}_i^{aug}, \quad (28)$$

$$\dot{X}^*(t) = \sum_{i=1}^r h_i(X^*(t)) [A_i - B_i S^{-1} B_i^t \bar{\pi}_i] X^*(t). \quad (29)$$

We define a Lyapunov function  $V(X) = X^t P X$ , where  $P$  is a symmetric positive-definite matrix. According to Eq. (9), we obtain

$$A_i - B_i S^{-1} B_i^t \bar{\pi}_i = -\bar{\pi}_i^{-1} (L + A_i^t \bar{\pi}_i),$$

and

$$\begin{aligned} \dot{V}(X) &= \dot{X}^t P X + X^t P \dot{X} \\ &= \left[ \sum_{i=1}^r h_i(X(t)) X^t (A_i - B_i S^{-1} B_i^t \bar{\pi}_i)^t \right] P X \\ &\quad + X^t P \left[ \sum_{i=1}^r h_i(X(t)) (A_i - B_i S^{-1} B_i^t \bar{\pi}_i) X \right] \\ &= - \sum_{i=1}^r h_i(X(t)) \{ X^t [(L + A_i^t \bar{\pi}_i)^t \bar{\pi}_i^{-1} P \\ &\quad + P \bar{\pi}_i^{-1} (L + A_i^t \bar{\pi}_i)] X \} \\ &= -2 \sum_{i=1}^r h_i(X(t)) [X^t P \bar{\pi}_i^{-1} (L + A_i^t \bar{\pi}_i) X] \\ &= -2 \sum_{i=1}^r h_i(X(t)) [X^t \bar{\pi}_i^{-1} (L + A_i^t \bar{\pi}_i) X] < 0 \end{aligned}$$

for  $P = I$  and  $\bar{\pi}_i^{-1} (L + A_i^t \bar{\pi}_i) > 0$  since  $h_i(X(t))$  is a positive number always.

#### **Proof of Theorem 2.**

(1) An affine-type fuzzy regulating issue in Problem 2.1 is equivalent to Problem 1 in the case of  $Y^d(t) = \mathbf{0}_{n' \times 1}$ . Therefore, based on Theorem 1 and its proof, we have

$$\begin{aligned}
r_i^*(t) &= -S^{-1}\tilde{B}_i^t \tilde{\pi}_i(t, t_1) \tilde{X}^*(t) + \tilde{r}_i^s(t), \\
\dot{\tilde{X}}^*(t) &= [\tilde{A}_i - \tilde{B}_i S^{-1} \tilde{B}_i^t \tilde{\pi}_i(t, t_1)] \tilde{X}^*(t) + \tilde{B}_i \tilde{r}_i^s(t) + \tilde{D}_i, \\
\dot{\tilde{b}}_i^s(t) &= -[\tilde{A}_i - \tilde{B}_i S^{-1} \tilde{B}_i^t \tilde{\pi}_i(t, t_1)]^t \tilde{b}_i^s(t) + \tilde{L} \tilde{X}_i^s, \quad \tilde{b}_i^s(t_1) = \mathbf{0}_{(n+h+h') \times 1},
\end{aligned}$$

where

$\tilde{r}_i^s(t) = -S^{-1}\tilde{B}_i^t (\tilde{\pi}_i(t, t_1) \tilde{X}_i^s + \tilde{b}_i^s(t))$ ,  $\tilde{X}_i^s = \tilde{A}_i^{-1} \tilde{D}_i = \begin{bmatrix} \tilde{X}_i^s \\ \mathbf{0}_{h \times 1} \end{bmatrix}$ , and  $\tilde{\pi}_i(t, t_1)$  is the symmetric positive-definite solution of a Riccati equation,

$$-\dot{\tilde{K}}_i(t) = \tilde{K}_i(t) \tilde{A}_i + \tilde{A}_i^t \tilde{K}_i(t) - \tilde{K}_i(t) \tilde{B}_i S^{-1} \tilde{B}_i^t \tilde{K}_i(t) + \tilde{L}, \quad \tilde{K}_i(t_1) = \mathbf{0}_{(n+h) \times (n+h)}.$$

We here replace  $\tilde{b}_i^s(\infty)$  and  $\tilde{\pi}_i$  by  $\tilde{b}_i^s(t_1)$  and  $\tilde{\pi}(t, t_1)$  for further derivation; in fact,  $\tilde{\pi}_i = \lim_{t_1 \rightarrow \infty} \tilde{\pi}(t, t_1)$ .

Now, let

$$\tilde{K}_i(t) = \begin{bmatrix} K_i(t) & K_i^{21}(t) \\ K_i^{21}(t) & K_i^{22}(t) \end{bmatrix},$$

$$\tilde{b}_i(t) = \begin{bmatrix} b_i^s(t) \\ b_{2i}(t) \end{bmatrix},$$

and

$$\tilde{X}(t) = \begin{bmatrix} X(t) \\ Z(t) \end{bmatrix}.$$

For infinite-horizon case ( $t_1 = \infty$ ), after a series of complicated matrix-manipulations we obtain  $r_i^*(t)$  in Eq. (15) and  $X^*(t)$  in Eq. (17), where  $\tilde{r}_i^s = -S^{-1}\tilde{B}_i^t (\tilde{\pi}_i \tilde{X}_i^s + \tilde{b}_i^s)$ ;  $\tilde{b}_i^s$  is in Eq. (8) as  $(A_i, B_i)$  is c.c. and  $(A_i, C)$  is c.o.;  $\tilde{\pi}_i = \lim_{t_1 \rightarrow \infty} \pi(t, t_1)$  is the unique solution of the Riccati equation in Eq. (9);  $b_{2i}(t)$ ,  $K_i^{21}$  and  $K_i^{22}(t)$  satisfy

$$\dot{b}_{2i}(t) = K_{21}(t) B_i S^{-1} B_i^t b_i^s(t) + F^t b_i(t) - E^t (CC^t)^{-1} CL \tilde{X}_i^s, \quad (30)$$

$$-\dot{K}_{21}(t) = F^t K_{21}(t) + K_{21}(t) A_i - K_{21}(t) B_i S^{-1} B_i^t \tilde{\pi}_i - E^t (CC^t)^{-1} CL, \quad (31)$$

$$-\dot{K}_{22}(t) = E^t (CC^t)^{-1} CL C^t (CC^t)^{-1} E + F^t K_{22}(t) - K_{21}(t) B_i S^{-1} B_i^t K_{21}^t(t) + K_{22}(t) F \quad (32)$$

with  $b_{2i}(\infty) = \mathbf{0}_{h \times 1}$ ,  $K_{21}(\infty) = \mathbf{0}_{h \times n}$ , and  $K_{22}(\infty) = \mathbf{0}_{h \times h}$ .

(2) We will derive the solution of Eq. (31) is  $\tilde{\pi}_i^{21}$  in Eq. (16). Now, we rewrite Eq. (31) as

$$-\dot{K}_{21}(t) = F^t K_{21}(t) + K_{21}(t) A_{ci} - L_{21}, \quad (33)$$

where  $A_{ci} = A_i - B_i S^{-1} B_i^t \tilde{\pi}_i$  and  $L_{21} = E^t (CC^t)^{-1} CL$ . We then obtain

$$K_{21}(t) = \phi_1(t, t_0) K_{21}(t_0) \phi_2^t(t, t_0) + \int_{t_0}^t \phi_1(t, \tau) L_{21} \phi_2^t(t, \tau) d\tau, \quad (34)$$

where  $\phi_1(t, t_0)$  and  $\phi_2(t, t_0)$  are state-transition matrices of  $\dot{X}(t) = -F^t X(t)$  and  $\dot{X}(t) = -A_{ci}^t X(t)$ , respectively. Therefore, we have

$$K_{21}(t_1) = \phi_1(t_1, t_0) K_{21}(t_0) \phi_2^t(t_1, t_0) + \int_{t_0}^{t_1} \phi_1(t_1, \tau) L_{21} \phi_2^t(t_1, \tau) d\tau,$$

$$K_{21}(t_0) = \phi_1(t_0, t_1) K_{21}(t_1) \phi_2^t(t_0, t_1) - \int_{t_0}^{t_1} \phi_1(t_0, \tau) L_{21} \phi_2^t(t_0, \tau) d\tau.$$

Substituting  $K_{21}(t_0)$  into Eq. (34), we obtain

$$K_{21}(t) = \phi_1(t,t_1)K_{21}(t_1)\phi_2^t(t,t_1) - \int_t^{t_1} \phi_1(t,\tau)L_{21}\phi_2^t(t,\tau)d\tau.$$

Since  $\lim_{t_1 \rightarrow \infty} K_{21}(t_1) = \mathbf{0}_{h \times n}$ , we have the solution of  $\bar{K}_{21}(t)$  in Eq. (31),

$$\bar{K}_{21}(t) = - \int_t^\infty e^{-F^t(t-\tau)} \cdot L_{21} \cdot e^{-A_{ci}^t(t-\tau)} d\tau = - \int_0^\infty e^{F^t\tau} \cdot L_{21} \cdot e^{A_{ci}^t\tau} d\tau. \tag{35}$$

Therefore, we have  $\bar{\pi}_i^{21}$  in Eq. (16) satisfies Eq. (35) and also Eq. (31).

(3) Stability analysis:  $S^{-1}B_i^t\bar{\pi}_i^{21t}Z(t)$  in Eq. (17) is associated only with the target.  $B_i\bar{r}_i^s + D_i$  in Eq. (17) relates to the fuzzy-singleton. So, we lump these two together into an augmented artificial-target,  $\bar{U}_i^{art}$ . Then, we can rewrite the proposed closed-loop tracking-control system in Eq. (17) as

$$\dot{X}^*(t) = \sum_{i=1}^r h_i(X^*(t))[(A_i - B_iS^{-1}B_i^t\bar{\pi}_i)X^*(t) + \bar{U}_i^{art}], \tag{36}$$

where  $\bar{U}_i^{art} = S^{-1}B_i^t\bar{\pi}_i^{21t}Z(t) + B_i\bar{r}_i^s + D_i$ . As we know, the stability of nonlinear tracking fuzzy system in Eq. (36) is coincident with that of zero-input fuzzy system in Eq. (29). In Proof of Theorem 1, we have demonstrated that the system in Eq. (29) is globally stable if each fuzzy subsystem satisfies  $(A_i, B_i)$  c.c. and  $(A_i, C)$  c.o. and  $\bar{\pi}_i^{-1}(L + A_i^t\bar{\pi}_i) > 0$ .

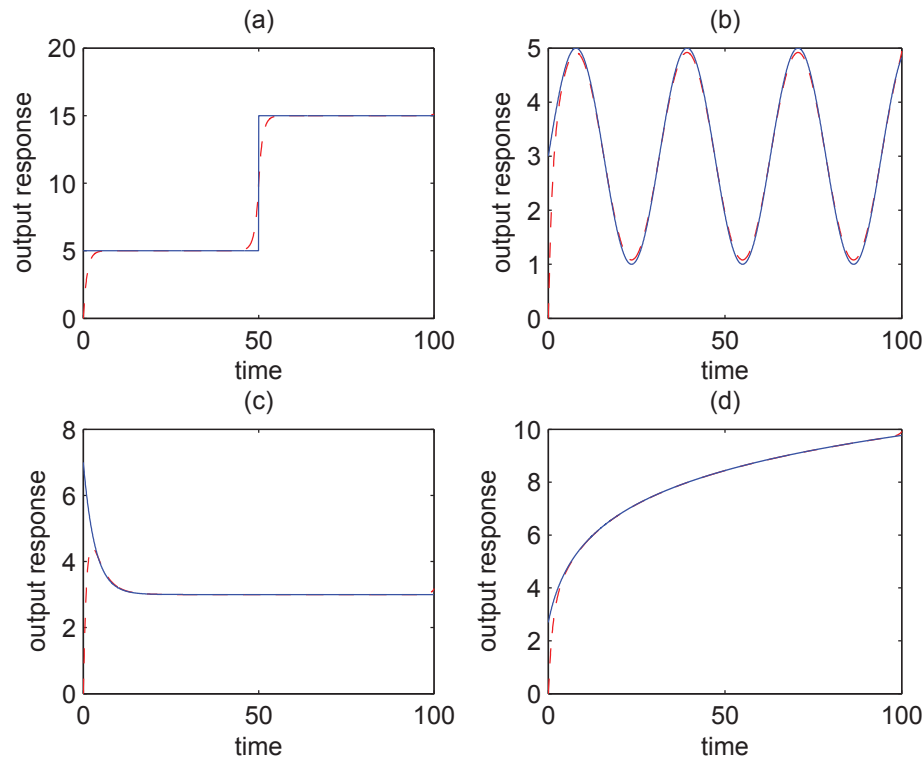


Fig. 1. Output responses (denoted by dashed line) of the affine T-S fuzzy tracking system with the designed affine-type fuzzy *moving-target* tracking controllers in Section 2 for various moving targets (denoted by solid line), where (a)  $Y^d(t)$  being a step'wise target, (b)  $Y^d(t) = 3 + 2\sin 0.2t$ , (c)  $Y^d(t) = 3 + 4e^{-0.3t}$  and (d)  $Y^d(t) = 0.5 + 2\log(3 + t)$ .

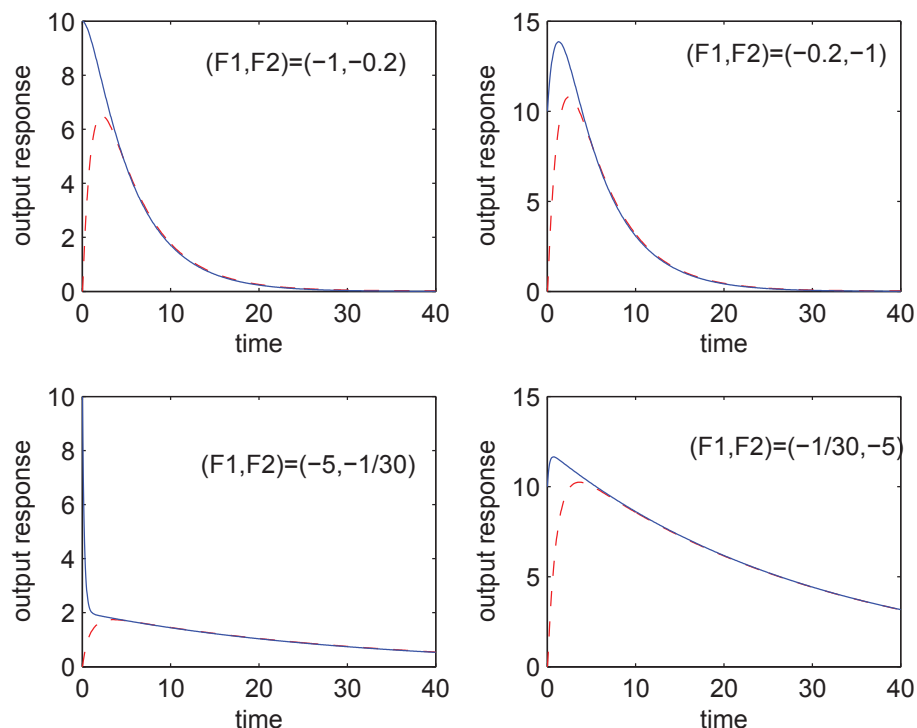


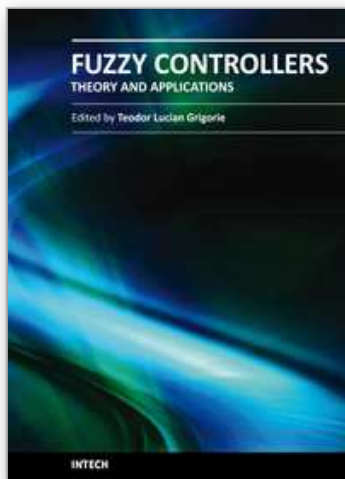
Fig. 2. Output responses (denoted by dashed line) of the affine T-S fuzzy tracking system with the designed affine-type fuzzy *model-following* tracking controllers in Section 3 for the targets (denoted by solid line) from the *tracked model* in Eq. (11) with  $E_1 = 1$ ,  $E_2 = 1$  and four different sets of parameters  $(F_1, F_2) = (-1, -0.2)$ ,  $(-0.2, -1)$ ,  $(-5, -1/30)$  and  $(-1/30, -5)$ .

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## **Fuzzy Controllers, Theory and Applications**

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Trying to meet the requirements in the field, present book treats different fuzzy control architectures both in terms of the theoretical design and in terms of comparative validation studies in various applications, numerically simulated or experimentally developed. Through the subject matter and through the inter and multidisciplinary content, this book is addressed mainly to the researchers, doctoral students and students interested in developing new applications of intelligent control, but also to the people who want to become familiar with the control concepts based on fuzzy techniques. Bibliographic resources used to perform the work includes books and articles of present interest in the field, published in prestigious journals and publishing houses, and websites dedicated to various applications of fuzzy control. Its structure and the presented studies include the book in the category of those who make a direct connection between theoretical developments and practical applications, thereby constituting a real support for the specialists in artificial intelligence, modelling and control fields.

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