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Extended Kalman Filter for the Estimation and Fuzzy Optimal Control of Takagi-Sugeno Model

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1. Introduction

This chapter is aimed at improving the local and global approximation and modelling capability of Takagi-Sugeno (T-S) fuzzy model and the design of an optimal fuzzy controller. The main aim is obtaining high function approximation accuracy and fast convergence. The approach developed here can be considered as a generalized version of T-S fuzzy identification method with optimized performance in estimating nonlinear functions. We propose an iterative method by applying the extended Kalman filter. We show that the Kalman filter is an effective tool in the estimation of T-S fuzzy model. It is a powerful mathematical tool for stochastic estimation from noisy environment. It has various applications in optimizing fuzzy systems. For example, it has been used to extract fuzzy rules from a given rule base (Wang, L. & Yen, J. (1998)) and to optimize the output function parameters of T-S fuzzy systems (Ramaswamy, P.; Edwards, R. R. & Lee, K. (1993)). For linear systems with white noise and measurement noise, the Kalman filter is known to be an optimal estimator. For nonlinear dynamic systems with coloured noise, the Kalman filter can be extended by linearizing the system around the current parameter estimates. This algorithm updates parameters in a way that is consistent with all previously measured data and generally converges in a few iterations. In this chapter, we describe how the extended Kalman filter can be applied to fuzzy system optimization. Fuzzy logic has been used to compute the gains of a bank of parallel Kalman filters in order to combine their outputs (Hsiao, C. (1999)). Fuzzy logic has also been used to tune the parameters of a Kalman filter (Kobayashi, K.; Cheok, K. & Watanabe, K. (1995)).

A fuzzy controller (FC) based Linear Quadratic Regulator (LQR) is then proposed in order to show the effectiveness of the estimation method developed here in control applications. An illustrative example of a highly nonlinear system is chosen to evaluate the robustness and remarkable performance of the proposed method. The main idea is to design a supervisory fuzzy controller capable to adjust the controller parameters in order to obtain the desired response. The reason behind this scheme is to combine the best features of fuzzy control and those of the optimal LQR.

In control design, it is often of interest to design a controller to fulfil, in an optimal form, certain performance criteria and constraints in addition to stability. The theme of optimal control addresses this aspect of control system design. For linear systems, the problem of designing optimal controllers reduces to solving algebraic Riccati equations, which are usually easy to solve and detailed literature of their solutions can be found in many

references. Nevertheless, for nonlinear systems, the optimization problem reduces to the so-called Hamilton-Jacobi (HJ) equations, which are nonlinear partial differential equations. Different from their counterparts for linear systems, HJ equations are usually difficult to solve both numerically and analytically. Improvements have also been carried out on the numerical solution of the approximated solution of HJ equations. But few results so far can provide an effective way of designing optimal controllers for general nonlinear systems.

In the past, the design of controllers based on a linearized model of real control systems. In many cases a good response of complex and highly non-linear real process is difficult to obtain by applying conventional control techniques which often employ linear mathematical models of the process. One reason for this lack of a satisfactory performance is the fact that linearization of a non-linear system might be valid only as an approximation to the real system around a determined operating point.

However, fuzzy controllers are basically non-linear, and effective enough to provide the desired non-linear control actions by carefully adjusting their parameters.

In this chapter, we propose an effective method to nonlinear optimal control based on fuzzy control. The optimal fuzzy control methodology presented in this chapter is based on a quadratic performance index. The optimal fuzzy controller is designed by solving a minimization problem that minimizes a given quadratic performance function.

Both the controlled system and the fuzzy controller are represented by the affine T-S fuzzy model taking into consideration the effect of the constant term. Most of the research works analyzed the T-S model assuming that the non-linear system is linearized with respect to the origin in each IF-THEN rule, which means that the consequent part of each rule is a linear function with zero constant term. This will in turn reduce the accuracy of approximating non-linear systems. Moreover, in linear control theory, the independent term does not affect the dynamics of the system rather the input to it. In the case of fuzzy control, the blending of the independent term of each rule will no longer be a constant but a function of the variables of the system and thus affects the dynamics of the resultant system. A necessary condition has been added to deal with the independent term. The control is carried out based on the fuzzy model via the so-called parallel distributed compensation scheme. The idea is that for each local affine model, an affine linear feedback control is designed. The resulting overall controller, which is also a non-linear one, is again a blending of each individual affine linear controller.

LQR is used to determine best values for parameters in fuzzy control rules in which the robustness is inherent in the LQR thereby robustness in fuzzy control can be improved. With the aid of LQR, it provides an effective design method of fuzzy control to achieve rapid, robust and accurate tracking control of a class of nonlinear systems.

In this chapter, we will also show how the LQR, the structure of which is based on mathematical analysis, can be made more appropriate for actual implementation by introduction of fuzzy rules.

The results obtained show a robust and stable behaviour when the system is subjected to various initial conditions, moment of inertia and to disturbances.

The content of this chapter is organized as follows. In section 2, an overview of Kalman Filter's Estimation and Optimal Control Techniques for Fuzzy Systems are presented. Section 3 presents the identification of T-S model. Section 4 demonstrates the iterative parameters' identification using extended Kalman filter. In section 5, a design of a fuzzy optimal controller is developed. Section 6 entails the application of the proposed FC-LQR on nonlinear model to demonstrate the validity of the proposed approach. This example

shows that the proposed approach gives a stable and well damped response in front of various initial conditions, moment of inertia and a robust behaviour in the presence of disturbances. The conclusion of the effectiveness and validity of the proposed approach is explained in section 7.

2. Overview of identification and estimation of fuzzy systems

Nonlinear control systems based on the T-S fuzzy model (Sugeno, M. & Tanaka, K. (1991).), (Sugeno, M. & Kang, G. (1988)) and (Takagi T. & Sugeno, M. (1985)) have attracted lots of attention during the last twenty years (e.g., see (Baranyi, P. (2003), (Tanaka, K.; Wang, H. O. (2001).), (Hou, Y.; Zurada, J. M.; Karwowski, W.; Marras, W. S. & Davis, K. (2007)), (Xian-Tu, P. (1990)), (Gang, F. (2006)), (Lian, K.-Y.; Su, C.-H. & Huang, C.-S. (2006)), (Hseng, T.; Li, S. & Tsai, S.-H. (2007)) and (Kim, J.-H.; Hyun, C.-H.; Kim, E. & Park, M. (2007).). It provides a powerful solution for development of function approximation, systematic techniques to stability analysis and controller design of fuzzy control systems in view of fruitful conventional control theory and techniques. They also allow relatively easy application of powerful learning techniques for their identification from data.

This model is formed by using a set of fuzzy rules to represent a nonlinear system as a set of local affine models which are connected by fuzzy membership functions (Cao, S. G.; Rees, N. W. & and Feng, G. (1996).).

This fuzzy modelling method presents an alternative technique to represent complex nonlinear systems (Fantuzzi, C. & Rovatti, R. (1996).), (Ying, H. (1998)) and (Zeng, K.; Zhang, N. Y. & Xu, W. L. (2000)) and reduces the number of rules in modelling higher order nonlinear systems (Takagi T. & Sugeno, M. (1985)) and (Gang, F. (2006)).

T-S fuzzy models are proved to be universal function approximators as they are able to approximate any smooth nonlinear functions to any degree of accuracy in any convex compact region (Fantuzzi, C. & Rovatti, R. (1996).), (Johansen, T. A.; Shorten, R. & Murray-Smith, R. (2000)), (Ying, H. (1998)) and (Zeng, K.; Zhang, N. Y. & Xu, W. L. (2000)). This result provides a theoretical foundation for applying T-S fuzzy models to represent complex nonlinear systems approximately.

Great attention has been paid to the identification of T-S fuzzy models and several results have been obtained (Cao, S. G.; Rees, N. W. & and Feng, G. (1997)), (Teixeira, M. C. M.; Assunção, E. & Avellar, R. G. (2003)) and (Yu, W. & Li, X. O. (2004)). They are based upon two kinds of approaches, one is to linearize the original nonlinear system in various operating points when the model of the system is known, and the other is based on the input-output data collected from the original nonlinear system when its model is unknown. The authors in (Cao, S. G.; Rees, N. W. & and Feng, G. (1997) use a fuzzy clustering method to identify T-S fuzzy models, including identification of the number of fuzzy rules and parameters of fuzzy membership functions, and identification of parameters of local linear models by using a least squares method (Skrjanc, I.; Blazic, S. & Agamennoni, O. (2005)) and (Wang, L. X. & Mendel, J. M. (1992).). The goal is to minimize the error between T-S fuzzy models and the corresponding original nonlinear systems. In (Klawonn, F. & Kruse, R. (1997)), Klawonn *et al.* explained how fuzzy clustering techniques could be applied to design a fuzzy controller from the training data. In (Hong T. P. & Lee, C. Y. (1996)), Hong and Lee have analyzed that the disadvantages of most fuzzy systems are that the membership functions and fuzzy rules should be predefined to map numerical data into linguistic terms and to make fuzzy reasoning work. They suggested a method based on the fuzzy clustering

technique and the decision tables to derive membership functions and fuzzy rules from numerical data. However, Hong and Lee's algorithm presented in (Hong T. P. & Lee, C. Y. (1996)) needs to predefine the membership functions of the input linguistic variables and it simplifies fuzzy rules by a series of merge operations. As the number of variables becomes larger, the decision table will grow tremendously and the process of the rule simplification based on the decision tables becomes more complicated.

In (Matía, F.; Jiménez, A. & Al-Hadithi, B. M. (2008)), the authors proposed to obtain the best features of Mamdani and Takagi-Sugeno models by using an affine global model with function approximation capabilities which maintains local interpretation. The suggested model is composed of variant coefficients which are independently governed by a zeroth order fuzzy inference system. This model may be interpreted as a generalization of T-S model in which dynamics coefficients have been decoupled. They have shown that Mamdani and Takagi-Sugeno models can be combined so that local and global interpretations are preserved.

The authors in (Johansen, T. A.; Shorten, R. & Murray-Smith, R. (2000)) suggest a method to identify T-S fuzzy models. Their method aims at improving the local and global approximation of T-S model. However, this complicates the approximation in order to obtain both targets. It has been shown that constrained and regularized identification methods may improve interpretability of constituent local models as local linearizations, and locally weighted least squares method may explicitly address the trade-off between the local and global accuracy of T-S fuzzy models.

In (Skrjanc, I.; Blazic, S. & Agamennoni, O. (2005)) a new method of interval fuzzy model identification was developed. The method combines a fuzzy identification methodology with some ideas from linear programming theory. The idea is then extended to modelling the optimal lower and upper bound functions that define the band which contains all the measurement values. This results in lower and upper fuzzy models or a fuzzy model with a set of lower and upper parameters. This approach can also be used to compress information in the case of large amount of data and in the case of robust system identification. The method can be efficiently used in the case of the approximation of the nonlinear functions family. The paper focuses on the development of an interval L_∞ -norm function approximation methodology problem using the LP technique and the TS fuzzy logic approach. This results in lower and upper fuzzy models or a fuzzy model with lower and upper parameters.

In (Mencattini, A.; Salmeri M. & Salsano, A. (2005)) a constructive method to synthesize a MISO TS fuzzy logic system imposing the requested derivative constraints on the function representing its behaviour is presented. The values of that function and its partial derivatives on the grid points of the input space permit to define a suitable interpolator of the function itself.

In (Kim, J.; Suga, Y. & Won, S. (2006).), a new approach to fuzzy modelling using the relevance vector learning mechanism (RVM) based on a kernel-based Bayesian estimation is introduced. The main concern is to find the best structure of the T-S fuzzy model for modelling nonlinear dynamic systems with measurement error. The number of rules and the parameter values of membership functions can be found as optimizing the marginal likelihood of the RVM in the proposed FIS. Because the RVM is not necessary to satisfy Mercer's condition, selection of kernel function is beyond the limit of the positive definite continuous symmetric function of SVM. The relaxed condition of kernel function can satisfy various types of membership functions in fuzzy model. The RVM which was compared

with support vector learning mechanism in examples had the small model capacity and described good generalization. Simulated results showed the effectiveness of the proposed FIS for modelling of nonlinear dynamic systems with noise.

In (Takagi T. & Sugeno, M. (1985)), the authors develop an interesting method to identify nonlinear systems using input-output data. They divide the identification process in three steps; premise variables, membership functions and consequent parameters. With respect to membership functions, they apply nonlinear programming technique using the complex method for the minimization of the performance index.

In 1991, Wang and Mendel developed a method for generating fuzzy rules by learning from examples (Wang, L. X. & Mendel, J. M. (1992).) and proved that a fuzzy inference system is a universal approximator by the Stone-Weierstrass theorem (Wang, L. (1994)).

In 1995, Wang proposed a new state-space analytical approach to fuzzy identification of nonlinear dynamical systems (Wang, L. X. & Mendel, J. M. (1995)). In 1996, Langari and Wang proposed achieving structure identification of a T-S fuzzy model by using a combination of fuzzy c-means clustering technique and a fuzzy discretization technique (Langari R. & Wang, L. (1996).

In (Nozaki, K.; Ishibuchi, H. & Tanaka, H. (1997), Nozaki *et al.* presented a heuristic method for generating T-S fuzzy rules from numerical data, and then converted the consequent parts of T-S fuzzy rules into linguistic representation.

In (Kumar et al., 2006), a study has outlined a new min-max approach to the fuzzy clustering, estimation, and identification with uncertain data. The proposed approach minimizes the worst-case effect of data uncertainties and modelling errors on estimation performance without making any statistical assumption and requiring a priori knowledge of uncertainties. Simulation studies have been provided to show the better performance of the proposed method in comparison to the standard techniques. The developed fuzzy estimation theory was applied to a real world application of physical fitness classification and modelling.

A new fuzzy system containing a dynamic rule base is proposed in (Chen, W. & Saif, M. (2005).). The characteristic of the proposed system is in the dynamic nature of its rule base which has a fixed number of rules and allows the fuzzy sets to dynamically change or move with the inputs. The number of the rules in the proposed system can be small, and chosen by the designer. The focus of article is mainly on the approximation capability of this fuzzy system. The proposed system is capable of approximating any continuous function on an arbitrarily large compact domain. Moreover, it can even approximate any uniformly continuous function on infinite domains. This paper addresses existence conditions, and as well provides constructive sufficient conditions so that the new fuzzy system can approximate any continuous function with bounded partial derivatives.

Fuzzy system optimization problem is studied in several works. Some articles focused on choosing proper rules in the inference engine (Cordon, O.; Herrera; Magdalena; F. L. & Villar P. (2001)) and (Xian-Tu, P. (1990)). Also tuning of the input and output scale factors are proposed (Cordon, O.; Herrera; Magdalena; F. L. & Villar P. (2001)), (Gudwin, R.; Gomide, F. & Pedrycz, W. (1998)) and (Pedrycz, W.; Gudwin, R. & Gomide, F. (1997)). Because of the importance of selecting proper MF's for fuzzy systems (Cordon, O.; Herrera. & Villar P. (2000)), several methods are used to deal with the problem of optimizing membership functions, which are either derivative-based or derivative-free methods. The derivative free approaches do not use the derivative of the performance of the system with

respect to the MF's parameters, and they are desirable because they are more robust than derivative-based methods with respect to finding global minimum and with respect to a wide range of objective function and MFs types. The main drawback is that they converge more slowly than derivative-based techniques (Tao, C. & Taur, J. (1999).

On the other hand, derivative-based methods have the advantage of fast convergence and fine tuning in finding the optimum functions, but they tend to converge to local minimums. In addition, due to their dependence on analytical derivatives, they are limited to specific objective functions and types of inference and MFs. The most common approaches are: gradient descent (Simon, D. (2000).), least squares (Skrjanc, I.; Blazic, S. & Agamennoni, O. (2005)), back propagation (Wang, L. & Mendel, J. (1992).) and Kalman filtering (Simon, D. (2002).), (Simon, 2002).

The use of Kalman filter training to optimize the MFs of a fuzzy system was introduced by Simon (Simon, D. (2002).) for motor winding current estimation. The used MFs were assumed as symmetric triangular forms. The Kalman filter training was extended to asymmetric triangles in (Simon, 2002), and a matrix was defined relating the parameters of the MFs together based on the sum-normality conditions, then projecting this matrix in each iteration of optimization to constrain the MFs to sum normal types.

Since the derivatives of the functions are used in Kalman filtering, it is limited to special type of MFs because of complicated and time consuming calculations. So far only triangular types are optimized for both inputs and outputs of a FLC Simon, D. (2002).), (Simon, 2002).

3. Identification of T-S model

An interesting method of identification is presented in (Takagi T. & Sugeno, M. (1985)). The idea is based on estimating the nonlinear system parameters minimizing a quadratic performance index. The method is based on the identification of functions of the following form:

$$\begin{aligned} f: \mathcal{R}^n &\rightarrow \mathcal{R} \\ y &= f(x_1, x_2, \dots, x_n) \end{aligned} \quad (1)$$

Each IF-THEN rule $R_i^{1 \dots i_n}$, for an n^{th} order system can be rewritten as follows:

$$\begin{aligned} S^{(i_1 \dots i_n)} : & \text{if } x_1 \text{ is } M_1^{i_1} \text{ and } x_2 \text{ is } M_2^{i_2} \text{ and } \dots x_n \text{ is } M_n^{i_n} \text{ then} \\ \hat{y} &= p_0^{(i_1 \dots i_n)} + p_1^{(i_1 \dots i_n)} x_1 + p_2^{(i_1 \dots i_n)} x_2 + \dots + p_n^{(i_1 \dots i_n)} x_n \end{aligned} \quad (2)$$

Where the fuzzy estimation of the output is:

$$\hat{y} = \frac{\sum_{i_1=1}^{r_1} \dots \sum_{i_n=1}^{r_n} w^{(i_1 \dots i_n)}(x) \left[p_0^{(i_1 \dots i_n)} + p_1^{(i_1 \dots i_n)} x_1 + \dots + p_n^{(i_1 \dots i_n)} x_n \right]}{\sum_{i_1=1}^{r_1} \dots \sum_{i_n=1}^{r_n} w^{(i_1 \dots i_n)}(x)} \quad (3)$$

Let m be a set of input/output system samples $\{x_{1k}, x_{2k}, \dots, x_{nk}, y_k\}$. The parameters of the fuzzy system can be calculated as a result of minimizing a quadratic performance index:

$$J = \sum_{k=1}^m (y_k - \hat{y}_k)^2 = \|Y - XP\|^2 \quad (4)$$

where

$$Y = [y_1 \ y_2 \ \dots \ y_m]^t \quad (5)$$

$$P = \begin{bmatrix} p_0^{(1\dots 1)} & p_1^{(1\dots 1)} & p_2^{(1\dots 1)} & \dots & p_n^{(1\dots 1)} & \dots & p_0^{(r_1\dots r_n)} & \dots & p_n^{(r_1\dots r_n)} \end{bmatrix}^t \quad (6)$$

$$X = \begin{bmatrix} \beta_1^{(1\dots 1)} & \beta_1^{(1\dots 1)}x_{11} & \dots & \beta_1^{(1\dots 1)}x_{n1} & \dots & \beta_1^{(r_1\dots r_n)} & \dots & \beta_1^{(r_1\dots r_n)}x_{n1} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \beta_m^{(1\dots 1)} & \beta_m^{(1\dots 1)}x_{1m} & \dots & \beta_m^{(1\dots 1)}x_{nm} & \dots & \beta_m^{(r_1\dots r_n)} & \dots & \beta_m^{(r_1\dots r_n)}x_{nm} \end{bmatrix}$$

and

$$\beta_k^{(i_1\dots i_n)} = \frac{w^{(i_1\dots i_n)}(x_k)}{\sum_{i_1=1}^{r_1} \dots \sum_{i_n=1}^{r_n} w^{(i_1\dots i_n)}(x_k)} \quad (7)$$

If X is a matrix of complete rank, the solution is obtained as follows:

$$\begin{aligned} J &= \|Y - XP\|^2 = (Y - XP)^t (Y - XP) \\ \nabla J &= X^t (Y - XP) = X^t Y - X^t X P = 0 \\ P &= (X^t X)^{-1} X^t Y \end{aligned} \quad (8)$$

4. Iterative parameters' identification

The inconvenient feature of the non iterative methods is the amplification of the matrix X throughout the time, so that they become inappropriate to be used in real time application as adaptive control for example (Jiménez, A.; Al-Hadithi, B. M. & Matía, F. (2008)). The solution is finding an iterative method so that the dimension of the calculation will not be augmented for each sample. In this work, we use an iterative method based on the extended Kalman.

4.1 Kalman filter

Kalman filter is widely used for state estimation. It was developed by Rudolph E. Kalman (Kalman, R. E. (1960). Kalman filter is known to be optimum for linear systems (Maybeck, P. S. (1979)) with white process and measurement noises. It is assumed that the system is described by the following sampled model:

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) + v(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \quad (9)$$

$$x(k), x(k+1), v(k) \in \mathbb{R}^n \quad u(k) \in \mathbb{R}^m \quad y(k), e(k) \in \mathbb{R}^p$$

where $x(k)$ represents the state of the dynamic system, $u(k)$ is the input vector and $y(k)$ is the output vector. The vector $v(k)$ represents the Gaussian-white noise of the system and

$e(k)$ is Gaussian-white noise of the output measure. Both of them are independent from each other with zero mean. The objective of the Kalman filter is to obtain an optimum estimation $\hat{x}(k)$ of the state $x(k)$ from measurements of the input / output vectors. The covariance matrices are supposed to be known and are given as:

$$\begin{aligned} R_1 &= E(v(k) \cdot v^t(k)) \\ R_{12} &= E(v(k) \cdot e^t(k)) \\ R_2 &= E(e(k) \cdot e^t(k)) \end{aligned} \quad (10)$$

where $E(\cdot)$ is the expectation operator. It is also assumed that the initial condition $x(0)$ is Gaussian distributed with

$$\begin{aligned} m_0 &= E(x(0)) \\ R_0 &= E((x(0) - m_0) \cdot (x(0) - m_0)^t) \end{aligned} \quad (11)$$

It is supposed that $\hat{x}(k/k-1)$, $u(k)$ and $y(k)$ are known and the objective is to estimate $\hat{x}(k+1/k)$. The prediction problem can be improved by introducing the difference between the measured and estimated outputs, $(y(k) - C\hat{x}(k/k-1))$ as a feedback gain:

$$\hat{x}(k+1/k) = \Phi\hat{x}(k/k-1) + \Gamma u(k) + K(k)(y(k) - C\hat{x}(k/k-1)) \quad (12)$$

The resultant prediction error is the difference between the state of the true system and the estimated one:

$$\varepsilon(k+1) = x(k+1) - \hat{x}(k+1/k) \quad (13)$$

It should be observed that as above mentioned Gaussian errors $v(k)$ and $e(k)$ are with zero mean, it can be verified that:

$$\bar{\varepsilon}(k+1) = (\Phi - K(k)C)\bar{\varepsilon}(k) \quad (14)$$

Thus,

$$\text{If } \bar{\varepsilon}(0) = 0 (\Rightarrow \hat{x}(0) = m_0) \Rightarrow \forall k > 0 \varepsilon(k) = 0 (\hat{x}(k) = m_k) \quad (15)$$

And if the dynamics of (3) is stable, then:

$$\forall x(0) \lim_{k \rightarrow \infty} \bar{\varepsilon}(k) = 0 \Rightarrow \lim_{k \rightarrow \infty} \hat{x}(k) = m_k \quad (16)$$

The secondary objective is to minimize the covariance matrix which denoted as $P(k)$,

$$P(k) = E((\varepsilon - \bar{\varepsilon}) \cdot (\varepsilon - \bar{\varepsilon})^t) \quad (17)$$

in the sense that it approaches its minimum for:

$$\min(\alpha^t P(k) \alpha) \quad \forall \alpha \in \Re^n \quad (18)$$

The algorithm of Kalman filter can be summarized by the following iterative process:

$$\begin{aligned} K(k) &= \left(\Phi P(k/k-1)C^t + R_{12} \right) \left(CP(k/k-1)C^t + R_2 \right)^{-1} \\ \hat{x}(k+1/k) &= \Phi \hat{x}(k/k-1) + \Gamma u(k) + K(k) \left(y(k) - C\hat{x}(k/k-1) \right) \\ P(k+1/k) &= \Phi P(k/k-1)\Phi^t + R_1 - K(k) \left(CP(k/k-1)\Phi^t + R_{12}^t \right) \end{aligned} \quad (19)$$

This process is initialized with $\hat{x}(0) = m_0$ and $P(0) = R_0$ which have been initially estimated. The classic formulation of Kalman filter can be complemented with an additional useful filtering process for certain applications.

4.2 Extended Kalman filter

Kalman filter can also be used for state estimation of nonlinear systems. For nonlinear systems, e.g. fuzzy systems, the Kalman filter can not be applied directly; but if the nonlinearity of the system be sufficiently smooth, then we can linearize it about the current mean and covariance of the state estimation. This is called Extended Kalman Filter (EKF) with white process and measurement noises. Derivations of the extended Kalman filter are widely available in the literature (Gelb, A. (1974)). In this section, we briefly outline the algorithm and show how it can be applied to fuzzy system optimization.

Consider a nonlinear discrete time system of the form:

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) + v(k) \\ y(k) &= g(x(k)) + e(k) \end{aligned} \quad (20)$$

In this case, Jacobian matrices are those which represent the nonlinear systems:

$$\begin{aligned} \Phi(x(k), u(k)) &= \left. \frac{\partial f}{\partial x} \right|_{x=x(k), u=u(k)} \\ \Gamma(x(k), u(k)) &= \left. \frac{\partial f}{\partial u} \right|_{x=x(k), u=u(k)} \\ C(x(k)) &= \left. \frac{\partial g}{\partial x} \right|_{x=x(k)} \end{aligned} \quad (21)$$

Moreover, the prediction formula for the nonlinear case is the following:

$$\hat{x}(k+1/k) = \Phi \hat{x}(k/k-1) + \Gamma u(k) + K(k) \left(y(k) - g(\hat{x}(k/k-1), u(k)) \right) \quad (22)$$

It must be noted that in this case, the system matrices in this depend on both the state and input of the system in each instant. Thus, it becomes necessary the calculation of these matrices in each iteration of the algorithm.

4.3 Kalman filter for parameters' identification

One of the applications of Kalman filter is the identification of parameters. Let us suppose that a function depends on q parameters p_1, p_2, \dots, p_q

$$\begin{aligned} f: \mathfrak{R}^n &\rightarrow \mathfrak{R}^m \\ y &= f(x_1, x_2, \dots, x_n, p_1, p_2, \dots, p_q) = f(x, p) \end{aligned} \quad (23)$$

The problem of identification of parameters can be explained as a problem of estimation of systems' states.

$$\begin{aligned} p(k+1) &= p(k) \\ y(k) &= f(p(k)) + e(k) \end{aligned} \quad (24)$$

Then, if we have a set of m samples $\{x_{1k}, x_{2k}, \dots, x_{nk}, y_k\}$ of the function to be identified, Kalman filter can be used with the following particularities. The matrix Φ will be an identity matrix in this case. It is assumed a free system without an external input so the matrix Γ is null and the matrix C can be calculated as follows:

$$C(p(k)) = \left. \frac{\partial f}{\partial p} \right|_{p=\hat{p}(k)} \quad (25)$$

The matrices R_1 y R_{12} become null, while R_2 is selected based on trial and error. If $y \in \mathfrak{R}$ and we suppose that $R_2 = I$, which would correspond to Gaussians error functions $N(0,1)$, and the function is a linear one, the algorithm becomes equivalent to the recursive minimum square one.

The initial covariance state matrix is supposed to be $P(0) = cI$ where C is a number relatively large with respect to the data of the problem. The algorithm becomes:

$$\begin{aligned} K(k) &= \left(P(k/k-1)C^t \right) \left(CP(k/k-1)C^t + R_2 \right)^{-1} \hat{p}(k+1/k) = \\ &\hat{p}(k/k-1) + K(k)(y_k - f(x_{1k}, x_{2k}, \dots, x_{nk}, \hat{p}(k/k-1))) \\ P(k+1/k) &= P(k/k-1) - K(k)CP(k/k-1) \end{aligned} \quad (26)$$

4.4 Application of Kalman filter for T-S fuzzy model

Motivated by the successful use of the Kalman filter for training neural networks (Puskorius, G. & Feldkamp, L. (1994).) and for defuzzification strategies, we can apply a similar method to the training of fuzzy systems. In general, we can view the identification of fuzzy systems a weighted least-squares minimization problem, where the error vector is the difference between the fuzzy model outputs and the target values for those outputs. The proposed solution for its application is to combine it with a minimization of a weighting of the norm of the vector of parameters p . Let a function be represented as:

$$\begin{aligned} f: \mathfrak{R}^n &\rightarrow \mathfrak{R} \\ y &= f(x_1, x_2, \dots, x_n) \end{aligned} \quad (27)$$

The optimum approximation of the function is searched by describing the function as a fuzzy system represented in the following form:

$$\begin{aligned} S^{(i_1 \dots i_n)} : &\text{if } x_1 \text{ is } M_1^{i_1} \text{ and } x_2 \text{ is } M_2^{i_2} \text{ and } \dots x_n \text{ is } M_n^{i_n} \\ \text{then } \hat{y} &= p_0^{(i_1 \dots i_n)} + p_1^{(i_1 \dots i_n)} x_1 + p_2^{(i_1 \dots i_n)} x_2 + \dots + p_n^{(i_1 \dots i_n)} x_n \end{aligned} \quad (28)$$

In order to cast the fuzzy system identification problem in a form suitable for Kalman filtering, we let the parameters of the rules constitute the state of a nonlinear system, and we consider the output of the fuzzy system as the output of the nonlinear system to which the Kalman filter is applied.

$$\begin{aligned} p(k+1) &= p(k) \\ y_k &= f(x_{1k}, x_{2k}, \dots, x_{nk}, p(k)) + e(k) \end{aligned} \quad (29)$$

It should be noticed that the function f is a linear one with respect to the parameters and therefore it can be calculated in a direct form as follows:

$$\left. \frac{\partial f}{\partial p_0^{(i_1 \dots i_n)}} \right|_{p=\hat{p}(k)} = \frac{w^{(i_1 \dots i_n)}(x_k)}{\sum_{i_1=1}^{r_1} \dots \sum_{i_n=1}^{r_n} w^{(i_1 \dots i_n)}(x_k)} \quad (30)$$

and

$$\left. \frac{\partial f}{\partial p_j^{(i_1 \dots i_n)}} \right|_{p=\hat{p}(k)} = \frac{w^{(i_1 \dots i_n)}(x_k)}{\sum_{i_1=1}^{r_1} \dots \sum_{i_n=1}^{r_n} w^{(i_1 \dots i_n)}(x_k)} x_{jk} \quad \text{for } j = 1 \dots n \quad (31)$$

Therefore, the Jacobian coincides with the row k of the matrix X defined in section II.

$$C(p(k)) = \left. \frac{\partial f}{\partial p} \right|_{p=\hat{p}(k)} = X_k = \left[\beta_k^{(1 \dots 1)} \beta_k^{(1 \dots 1)} x_{1k} \dots \beta_k^{(1 \dots 1)} x_{nk} \dots \beta_k^{(r_1 \dots r_n)} \dots \beta_k^{(r_1 \dots r_n)} x_{nk} \right] \quad (32)$$

And thus, the problem can be formulated as an estimation of the state of the linear system

$$\begin{aligned} p(k+1) &= p(k) \\ y_k &= C(\hat{p}(k)) \cdot p(k) + e(k) \end{aligned} \quad (33)$$

The prediction formula in this case becomes:

$$\hat{p}(k+1/k) = \hat{p}(k/k-1) + K(k)(y_k - C(\hat{p}(k)) \cdot \hat{p}(k)) \quad (34)$$

5. Design of optimal fuzzy controller

In order to show the effectiveness of the proposed estimation methods, a design of an optimal controller is carried out for a dynamic system whose model is of the following form:

$$x^{(n)} = f(x, x', \dots, x^{(n-1)}, u) \quad (35)$$

Applying the proposed estimation method, the T-S model can be adjusted as follows:

$$\begin{aligned} S^{(i_1 \dots i_n)} : & \text{if } x \text{ is } M_1^{i_1} \text{ and } x' \text{ is } M_2^{i_2} \text{ and } \dots x^{(n-1)} \text{ is } M_n^{i_n} \text{ then} \\ x^{(n)} &= a_0^{(i_1 \dots i_n)} + a_1^{(i_1 \dots i_n)} x + a_2^{(i_1 \dots i_n)} x' + \dots + a_n^{(i_1 \dots i_n)} x^{(n-1)} + b^{(i_1 \dots i_n)} u \end{aligned} \quad (36)$$

The controller fuzzy rule is represented in a similar form:

$$C^{(i_1 \dots i_n)} : \text{if } x \text{ is } M_1^{i_1} \text{ and } x' \text{ is } M_2^{i_2} \text{ and } \dots x^{(n-1)} \text{ is } M_n^{i_n} \text{ then} \\ u = r - \left(k_0^{(i_1 \dots i_n)} + k_1^{(i_1 \dots i_n)} x + k_1^{(i_1 \dots i_n)} x' + \dots + k_1^{(i_1 \dots i_n)} x^{(n-1)} \right) \quad (37)$$

5.1 Calculation of the affine term

The proposed methodology of design is based on the possibility of formulate the feedback system as shown previously

$$SC^{(i_1 \dots i_n)} : \text{if } x \text{ is } M_1^{i_1} \text{ and } x' \text{ is } M_2^{i_2} \text{ and } \dots x^{(n-1)} \text{ is } M_n^{i_n} \text{ then} \\ x^{(n)} = a_0^{(i_1 \dots i_n)} + a_1^{(i_1 \dots i_n)} x + \dots + a_n^{(i_1 \dots i_n)} x^{(n-1)} + b^{(i_1 \dots i_n)} \left[r - \left(k_0^{(i_1 \dots i_n)} + k_1^{(i_1 \dots i_n)} x + \dots + k_1^{(i_1 \dots i_n)} x^{(n-1)} \right) \right] \quad (38)$$

Firstly, the affine term of the control action is used to eliminate the affine term of the system:

$$a_0^{(i_1 \dots i_n)} + b^{(i_1 \dots i_n)} k_0^{(i_1 \dots i_n)} = 0 \Rightarrow k_0^{(i_1 \dots i_n)} = -\frac{a_0^{(i_1 \dots i_n)}}{b^{(i_1 \dots i_n)}} \quad (39)$$

and the feedback system is rewritten as follows:

$$SC^{(i_1 \dots i_n)} : \text{if } x \text{ is } M_1^{i_1} \text{ and } x' \text{ is } M_2^{i_2} \text{ and } \dots x^{(n-1)} \text{ is } M_n^{i_n} \text{ then} \\ x^{(n)} = a_1^{(i_1 \dots i_n)} x + \dots + a_n^{(i_1 \dots i_n)} x^{(n-1)} + b^{(i_1 \dots i_n)} \left[r - \left(k_1^{(i_1 \dots i_n)} x + \dots + k_1^{(i_1 \dots i_n)} x^{(n-1)} \right) \right] \quad (40)$$

5.2 State space feedback control based on the proposed estimation approach

Any control methodology by state feedback design can be applied to calculate the rest of control coefficients. Together with the proposed estimation method, the well known LQR method might be an appropriate choice (Aström, K. J.; Wittenmark, B1985). The system can be represented in state space form:

$$x' = Ax + Bu, \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m} \quad (41)$$

The objective is to find the control action $u(t)$ to transfer the system from any initial state $x(t_0)$ to some final state $x(\infty) = 0$ in an infinite time interval, minimizing a quadratic performance index of the form:

$$J = \int_{t_0}^{\infty} (x^t Q x + u^t R u) dt \quad (42)$$

where $Q \in \mathbb{R}^{n \times n}$ is a symmetric matrix, at least positive a semidefinite one and $R \in \mathbb{R}^{m \times m}$ is also a symmetric positive definite matrix. The optimal control law is computed as follows:

$$u(t) = -Kx(t) = -R^{-1}B^t Lx(t) \quad (43)$$

where the matrix $L \in \Re^{n \times n}$ is a solution of the Riccati equation:

$$0 = -Q + LBR^{-1}B^tL - LA - A^tL \quad (44)$$

The design algorithm includes firstly the cancellation of the affine term in each subsystem of the form:

$$x^{(n)} = a_0^{(i_1 \dots i_n)} + a_1^{(i_1 \dots i_n)}x + a_2^{(i_1 \dots i_n)}x' + \dots + a_n^{(i_1 \dots i_n)}x^{(n-1)} + b^{(i_1 \dots i_n)}u \quad (45)$$

The system is then represented in state space form as:

$$x = \begin{bmatrix} x & x' & \dots & x^{(n-1)} \end{bmatrix}^t \quad (46)$$

$$A^{(i_1 \dots i_n)} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a_1^{(i_1 \dots i_n)} & a_2^{(i_1 \dots i_n)} & a_3^{(i_1 \dots i_n)} & \dots & a_n^{(i_1 \dots i_n)} \end{bmatrix}, \quad B^{(i_1 \dots i_n)} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b^{(i_1 \dots i_n)} \end{bmatrix} \quad (47)$$

Secondly, the LQR methodology is applied for each subsystem using a common state weighting matrix Q and input matrix R for all the rules. Thus, Riccati equation is solved for each subsystem as follows:

$$0 = -Q + L^{(i_1 \dots i_n)}B^{(i_1 \dots i_n)}R^{-1}B^{(i_1 \dots i_n)t}L^{(i_1 \dots i_n)} - L^{(i_1 \dots i_n)}A^{(i_1 \dots i_n)} - A^{(i_1 \dots i_n)t}L^{(i_1 \dots i_n)} \quad (48)$$

Then the state feedback gain vector can be obtained as follows:

$$K^{(i_1 \dots i_n)} = \begin{bmatrix} k_1^{(i_1 \dots i_n)} & k_2^{(i_1 \dots i_n)} & \dots & k_n^{(i_1 \dots i_n)} \end{bmatrix} = R^{-1}B^{(i_1 \dots i_n)t}L^{(i_1 \dots i_n)} \quad (49)$$

6. Example

In this section the proposed estimation method and its application to control design of FC-LQR is illustrated by an example of an inverted pendulum.

Consider the problem of stabilizing and balancing of swing up of an inverted pendulum (see figure 1). The control of this system is a widely used performance measure of a controller, since this system is unstable and highly nonlinear. The objective is to maintain the inverted pendulum upright with θ despite small disturbances due to wind or system noises. The inverted pendulum can be represented as follows:

$$\ddot{\varphi} = \frac{g \sin \varphi - \cos \varphi \left(\frac{u + ml\dot{\varphi}^2 \sin \varphi}{M + m} \right)}{l \left(\frac{4}{3} - \frac{m \cos^2 \varphi}{M + m} \right)} \quad (50)$$

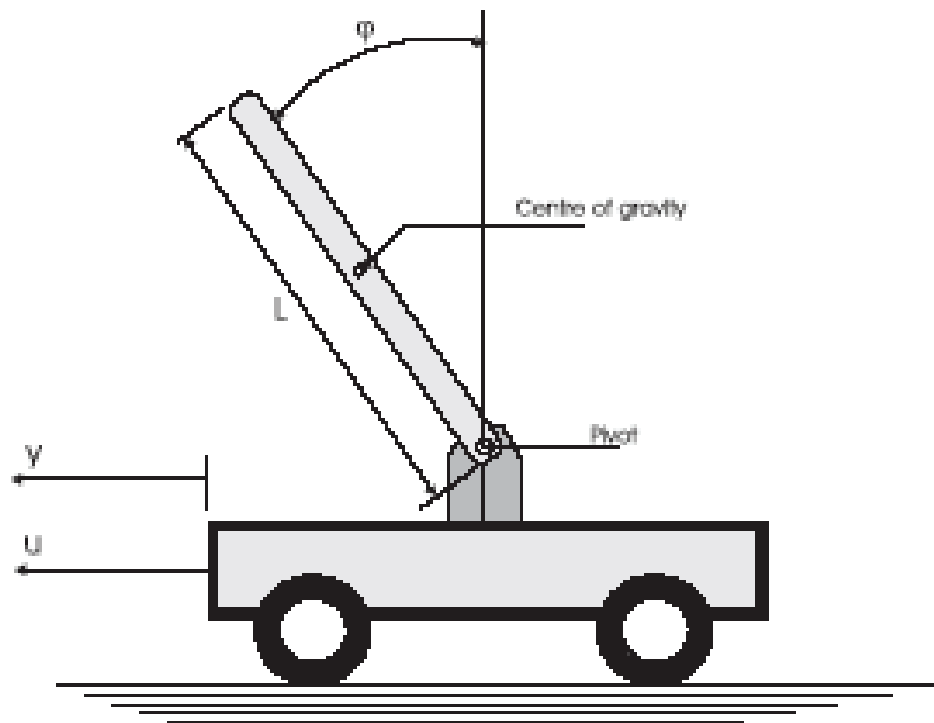


Fig. 1. Inverted pendulum

where θ denotes the angular position (in radians) deviated from the equilibrium position (vertical axis) of the pendulum and $\dot{\theta}$ is the angular velocity, g (gravity acceleration)=9.8 m/sec², M (mass) of the cart=1 kg, m (mass) of the pole=0.1 kg, l is the distance from the center of the mass (m) of the pole to the cart=0.5 m. Assuming that $x_1=\theta$ and $x_2=\dot{\theta}$, then the inverted pendulum model can be rewritten in state space form as follows:

$$\begin{aligned} x_1 &= \varphi \\ x_1' &= x_2 \\ x_2 &= \dot{\varphi} \\ x_2' &= \frac{g \sin x_1 - \cos x_1 \left(\frac{u + m l x_2^2 \sin x_1}{M + m} \right)}{l \left(\frac{4}{3} - \frac{m \cos^2 x_1}{M + m} \right)} \end{aligned} \quad (51)$$

The aim is to move the pendulum to its instable equilibrium position, i.e., $x_1 = x_2 = u = 0$. The membership functions are as shown in figures 2 and 3:

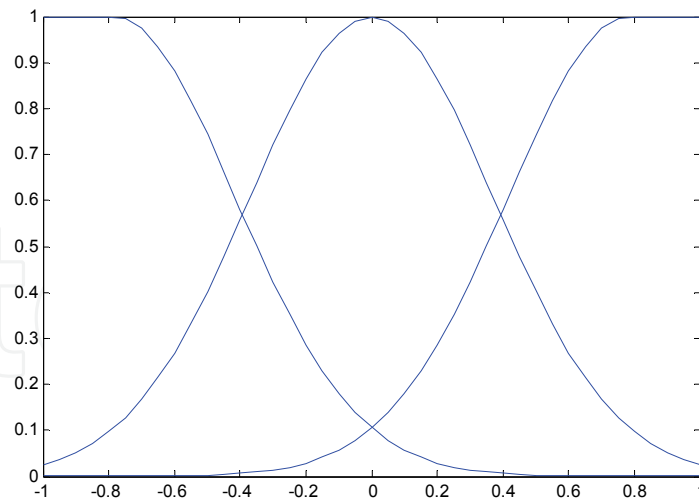


Fig. 2. Membership functions for the angle position

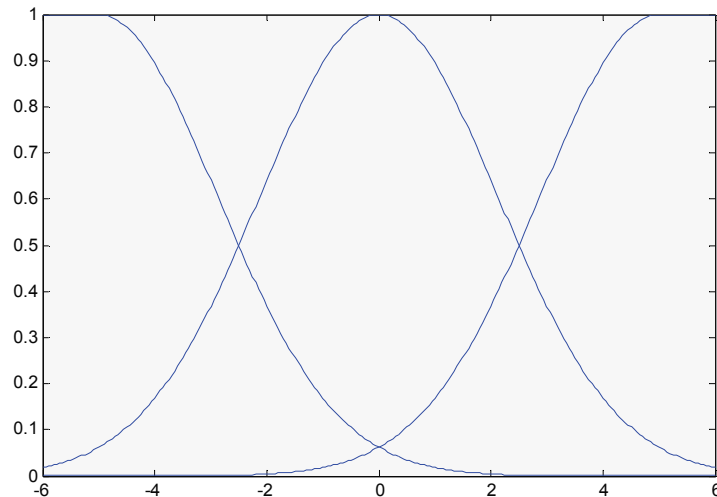


Fig. 3. Membership functions for the angular velocity

Using the iterative method mentioned above, the inverted pendulum fuzzy model can be represented as follows:

$$\begin{aligned}
 S^{11} : & \text{if } x_1 \text{ is } M_1^1 \text{ and } x_2 \text{ is } M_2^1 \text{ then } x'_2 = -2.9699 + 10.3602x_1 - 0.2535x_2 - 1.0001u \\
 S^{12} : & \text{if } x_1 \text{ is } M_1^1 \text{ and } x_2 \text{ is } M_2^2 \text{ then } x'_2 = -2.4941 + 10.4319x_1 - 0.0000x_2 - 1.0001u \\
 S^{13} : & \text{if } x_1 \text{ is } M_1^1 \text{ and } x_2 \text{ is } M_2^3 \text{ then } x'_2 = -2.9699 + 10.3602x_1 + 0.2535x_2 - 1.0001u \\
 S^{21} : & \text{if } x_1 \text{ is } M_1^2 \text{ and } x_2 \text{ is } M_2^1 \text{ then } x'_2 = -0.0066 + 12.7376x_1 - 0.0079x_2 - 1.5883u \\
 S^{22} : & \text{if } x_1 \text{ is } M_1^2 \text{ and } x_2 \text{ is } M_2^2 \text{ then } x'_2 = 13.9336x_1 - 0.0000x_2 - 1.5883u \\
 S^{23} : & \text{if } x_1 \text{ is } M_1^2 \text{ and } x_2 \text{ is } M_2^3 \text{ then } x'_2 = -0.0066 + 12.7376x_1 + 0.0079x_2 - 1.5883u \\
 S^{31} : & \text{if } x_1 \text{ is } M_1^3 \text{ and } x_2 \text{ is } M_2^1 \text{ then } x'_2 = 2.8833 + 10.7086x_1 + 0.2823x_2 - 1.0567u \\
 S^{32} : & \text{if } x_1 \text{ is } M_1^3 \text{ and } x_2 \text{ is } M_2^2 \text{ then } x'_2 = 2.1717 + 10.9258x_1 + 0.0000x_2 - 1.0567u \\
 S^{33} : & \text{if } x_1 \text{ is } M_1^3 \text{ and } x_2 \text{ is } M_2^3 \text{ then } x'_2 = 2.8833 + 10.7086x_1 - 0.2823x_2 - 1.0567u
 \end{aligned} \tag{52}$$

The resultant mean square error from this approximation is 0.0014. For each one of these subsystems, a feedback state LQR controller has been designed with the affine term. Firstly, the affine term of the controller is used to eliminate the affine term of the system. The other terms are calculated by the LQR minimizing the following performance index:

$$J = \int_{t_0}^{\infty} (100x_1^2 + 10x_2^2 + u^2) dt \quad (53)$$

Thus, the resultant fuzzy optimal LQR is:

$$\begin{aligned} R^{11} : & \text{if } x_1 \text{ is } M_1^1 \text{ and } x_2 \text{ is } M_2^1 \text{ then } u = -2.9695 + 24.7569x_1 + 7.4648x_2 \\ R^{12} : & \text{if } x_1 \text{ is } M_1^1 \text{ and } x_2 \text{ is } M_2^2 \text{ then } u = -2.4937 + 24.8803x_1 + 7.7301x_2 \\ R^{13} : & \text{if } x_1 \text{ is } M_1^1 \text{ and } x_2 \text{ is } M_2^3 \text{ then } u = -2.9695 + 24.7569x_1 + 7.9717x_2 \\ R^{21} : & \text{if } x_1 \text{ is } M_1^2 \text{ and } x_2 \text{ is } M_2^1 \text{ then } u = -0.0042 + 20.8385x_1 + 6.0150x_2 \\ R^{22} : & \text{if } x_1 \text{ is } M_1^2 \text{ and } x_2 \text{ is } M_2^2 \text{ then } u = 22.0756x_1 + 6.1480x_2 \\ R^{23} : & \text{if } x_1 \text{ is } M_1^2 \text{ and } x_2 \text{ is } M_2^3 \text{ then } u = -0.0042 + 20.8385x_1 + 6.0250x_2 \\ R^{31} : & \text{if } x_1 \text{ is } M_1^3 \text{ and } x_2 \text{ is } M_2^1 \text{ then } u = 2.7286 + 24.3712x_1 + 7.7638x_2 \\ R^{32} : & \text{if } x_1 \text{ is } M_1^3 \text{ and } x_2 \text{ is } M_2^2 \text{ then } u = 2.0551 + 24.7239x_1 + 7.5362x_2 \\ R^{33} : & \text{if } x_1 \text{ is } M_1^3 \text{ and } x_2 \text{ is } M_2^3 \text{ then } u = 2.7286 + 24.3712x_1 + 7.2294x_2 \end{aligned} \quad (54)$$

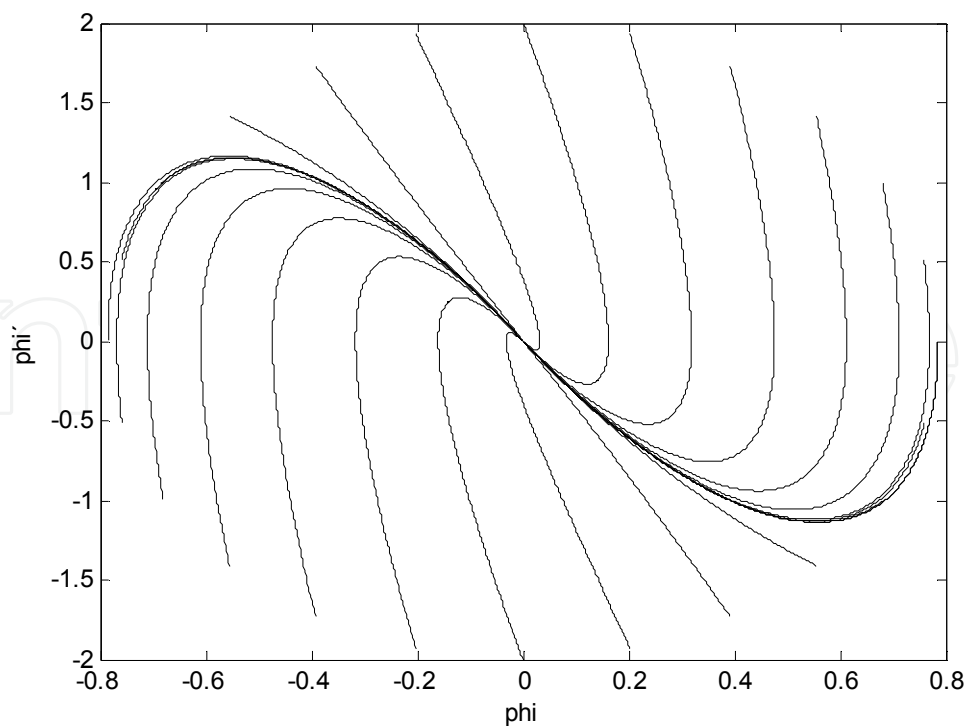


Fig. 4. Several trajectories in state space form of the system for several initial conditions

7. Conclusions

New efficient approach has been presented to improve the local and global estimation of T-S fuzzy model. The approach developed here can be considered as a generalized version of T-S method with optimized performance in approximating nonlinear functions. A simple and less computational method, based on the extended Kalman filter has been developed. A FC based LQR has been proposed in order to show the effectiveness of the estimation method developed here in control applications. An Illustrative example of an inverted pendulum has been chosen to evaluate the robustness and remarkable performance of the proposed method and the high accuracy obtained in approximating nonlinear and unstable systems locally and globally in comparison with the original T-S model. Simulation results have shown the potential, simplicity and generality of the algorithm.

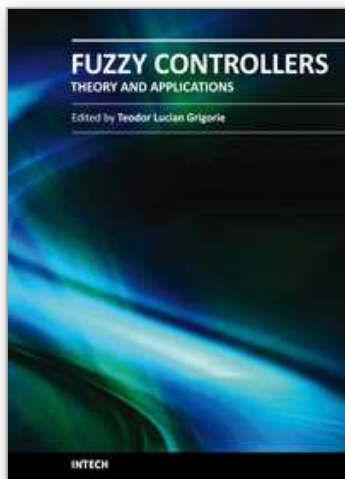
8. References

- Aström, K. J.; Wittenmark, B. (1997). *Computer-Controlled Systems*", Prentice Hall, USA.
- Cordon, O.; Herrera; Magdalena; F. L. & Villar P. (2001). A Genetic Learning Process for the Scaling Factors, Granularity and Contexts of the Fuzzy Rule-Based System Data Base, *Information Sciences*, 136, 85- 107.
- Cordon, O.; Herrera. & Villar P. (2000). "Analysis and Guidelines to Obtain a Good Uniform Fuzzy Partition Granularity for Fuzzy Rule-Based Systems Using Simulated Annealing, *International Journal of Approximate Reasoning*, 25, 187-216.
- Gelb, A. (1974). *Applied Optimal Estimation*, MITPress, Cambridge, MA.
- Hsiao, C. (1999). Modified gain fuzzy Kalman filtering algorithm, *JSME Internat. J. Ser. C* 42 363-368.
- Jiménez, A.; Al-Hadithi, B. M. & Matía, F. (2008). "An Optimal T-S Model for the Estimation and Identification of Nonlinear Functions", *WSEAS Transactions on Systems and Control*, Vol. 3, Issue 10, pp. 897-906.
- Kalman, R. E. (1960). "A New Approach to Linear Filtering and Prediction Problems," *Transactions of the ASME-Journal of Basic Engineering*, Vol. 82(series D), pp.35-45.
- Kobayashi, K.; Cheok, K. & Watanabe, K.(1995). Estimation of the absolute vehicle speedusing fuzzy logic rule-based Kalman filter, *American Control Conf.*, Seattle, pp. 3086-3090.
- Maybeck, P. S. (1979). *Stochastic models, estimation, and control*, Vol. 1, Academic Press.
- Puskorius, G. & Feldkamp, L. (1994). Neurocontrol of nonlinear dynamical systems with Kalman filter trained recurrent networks, *IEEE Trans. Neural Networks* 5, pp. 279-297.
- Baranyi, P. (2003). SVD-based reduction to MISO TS models, *IEEE Trans. Ind. Electron.*, vol. 51, no. 1, pp. 232-242.
- Cao, S. G.; Rees, N. W. & and Feng, G. (1996). Stability analysis of fuzzy control systems, *IEEE Trans. Syst., Man, Cybern., B, Cybern.*, vol. 26, no. 1, pp. 201-204.
- Cao, S. G.; Rees, N. W. & and Feng, G. (1997). Analysis and design for a class of complex control systems— Part I: Fuzzy modeling and identification," *Automatica*, vol. 33, pp. 1017-1028.

- Chen, W. & Saif, M. (2005). A Novel Fuzzy System With Dynamic Rule Base, *IEEE Trans. Fuzzy Syst.* 13(5), pp. 569-582.
- Fantuzzi, C. & Rovatti, R. (1996). On the approximation capabilities of the homogeneous Takagi-Sugeno model, in *Proc. 5th IEEE Int. Conf. Fuzzy Systems*, New Orleans, LA, pp. 1067-1072.
- Gang, F. (2006). A survey on analysis and design of model-based fuzzy control systems. *IEEE Trans. Fuzzy Syst.* 14(5), 676-697.
- Gudwin, R.; Gomide, F. & Pedrycz, W. (1998). Context Adaptation in Fuzzy Processing and Genetic Algorithms, *International Journal of Intelligent Systems*, 13, 929-948.
- Hong T. P. & Lee, C. Y. (1996). Induction of fuzzy rules and membership functions from training examples, *Fuzzy Sets Syst.*, vol. 84, no. 1, pp. 33-47.
- Hou, Y.; Zurada, J. M.; Karwowski, W.; Marras, W. S. & Davis, K. (2007). Identification of Key Variables Using Fuzzy Average With Fuzzy Cluster Distribution, *IEEE Trans. Fuzzy Syst.* 15(4), 673-685.
- Hseng, T.; Li, S. & Tsai, S.-H. (2007). T-S Fuzzy Bilinear Model and Fuzzy Controller Design for a Class of Nonlinear Systems, *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 3, pp. 494-506.
- Jiménez, A.; Al-Hadithi, B. M. & Matía, F. (2008). An Optimal T-S Model for the Estimation and Identification of Nonlinear Functions", *WSEAS Transactions on Systems and Control*, Vol. 3, Issue 10, pp. 897-906.
- Johansen, T. A.; Shorten, R. & Murray-Smith, R. (2000) On the interpretation and identification of dynamic Takagi-Sugeno models, *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 3, pp. 297-313.
- Kim, J.-H.; Hyun, C.-H.; Kim, E. & Park, M. (2007). New Adaptive Synchronization of Uncertain Chaotic Systems Based on T-S Fuzzy Model, *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 3, pp. 359-369.
- Kim, J.; Suga, Y. & Won, S. (2006). A New Approach to Fuzzy Modeling of Nonlinear Dynamic Systems With Noise: Relevance Vector Learning Mechanism", *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 2, pp. 222-231.
- Klawonn, F. & Kruse, R. (1997). Constructing a fuzzy controller from data, *Fuzzy Sets Syst.*, vol. 85, no. 2, pp. 177-193.
- Mencattini, A.; Salmeri M. & Salsano, A. (2005). Sufficient Conditions to Impose Derivative Constraints on MISO Takagi-Sugeno Fuzzy Logic Systems", *IEEE Trans. Fuzzy Syst.*, Vol. 13, No. 4.
- Kumar, M.; Stoll, R. & Stoll, N. (2006). A Min-Max Approach to Fuzzy Clustering, Estimation, and Identification, *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 2, pp. 248-262
- Langari R. & Wang, L. (1996). Complex systems modeling via fuzzy logic," *IEEE Trans. Syst., Man, Cybern. B*, vol. 26, pp. 100-106.
- Lian, K.-Y.; Su, C.-H. & Huang, C.-S. (2006). Performance Enhancement for T-S Fuzzy Control Using Neural Networks, *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 619-627.
- Matía, F.; Jiménez, A. & Al-Hadithi, B. M. (2008). An Affine model with Decoupled dynamics, *Proceedings of IPMU'08*, pp. 713-720, 22-27.
- Mollov, S.; Babuska, R.; Abonyi, J. & Verbruggen, H. B. (2004). Effective optimization for fuzzy model predictive control, *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 5, pp. 661-675.

- Nozaki, K.; Ishibuchi, H. & Tanaka, H. (1997). A simple but powerful heuristic method for generating fuzzy rules from numerical data, *Fuzzy Sets Syst.*, vol. 86, no. 3, pp. 251–270.
- Pedrycz, W.; Gudwin, R. & Gomide, F. (1997). Nonlinear Context Adaptation in the calibration of Fuzzy Sets, *Fuzzy Sets and Systems*, 88, pp. 91-97.
- Ramaswamy, P.; Edwards, R. R. & Lee, K. (1993). Two approaches for automating the tuning process of fuzzy logic controllers, *IEEE Conf. on Decision and Control*, San Antonio, TX, pp.1753-1758.
- Simon, D. (2000). Design and rule base reduction of a fuzzy filter for the estimation of motor currents, *International Journal of Approximate Reasoning*, 25, pp. 145-167.
- Simon, D. (2002). Training Fuzzy Systems with the Extended Kalman Filter, *Fuzzy Sets and Systems*, Vol. 132, 189-199.
- Simon, D. (2002). Sum normal optimization of fuzzy membership functions, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, Vol. 10, No. 4, pp.363-384.
- Sugeno, M. & Tanaka, K. (1991). Successive identification of a fuzzy model and its applications to prediction of a complex system, *Fuzzy Sets and Systems* 42, pp. 315–334.
- Sugeno, M. & Kang, G. (1988). Structure identification of fuzzy model, *Fuzzy Sets Syst.*, vol. 28, pp.15–33.
- Skrjanc, I.; Blazic, S. & Agamennoni, O. (2005). Interval Fuzzy Model identification Using l_∞ -Norm, *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 5.
- Takagi T. & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132.
- Tanaka, K.; Wang, H. O. (2001). *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York: Wiley.
- Tao, C. & Taur, J. (1999). Design of Fuzzy Controllers with Adaptive Rule Insertion, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 29, pp. 389- 397.
- Teixeira, M. C. M.; Assunção, E. & Avellar, R. G. (2003). On relaxed LMI based designs for fuzzy regulators and fuzzy observers, *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 613–623.
- Wang, L. & Mendel, J. (1992). Back-propagation of Fuzzy Systems as Nonlinear Dynamic System Identifiers, *IEEE International Conference on Fuzzy Systems*, San Diego, California, pp. 1409-1418.
- Wang, L. (1994). *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Prentice-Hall, Englewood Cliffs, NJ, pp. 27-28.
- Wang, L. X. & Mendel, J. M. (1995). Design and analysis of fuzzy identifiers of nonlinear dynamic systems," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 11–23.
- Wang, L. X. & Mendel, J. M. (1992). Generating fuzzy rules by learning from examples, *IEEE Trans. Syst., Man, Cybern.*, vol. 22, pp. 1414–1427.
- Wang, L. & Yen, J. (1998). Extracting fuzzy rules for system modelling using a hybrid of genetic algorithms and Kalman filter, *Fuzzy Sets and Systems* 101, pp. 353–362.

- Xian-Tu, P. (1990). Generating Rules for Fuzzy Logic Controllers by Functions," *Fuzzy Sets and Systems* 36, pp. 83-89.
- Ying, H. (1998). General SISO Takagi-Sugeno fuzzy systems with linear rule consequent are universal approximators, *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 4, pp. 582-587.
- Yu, W. & Li, X. O. (2004). Fuzzy identification using fuzzy neural networks with stable learning algorithms, *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 3, pp. 411-420.
- Zeng, K.; Zhang, N. Y. & Xu, W. L. (2000). A comparative study on sufficient conditions for Takagi-Sugeno fuzzy systems as universal approximators, *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 6, pp. 773-780.



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Trying to meet the requirements in the field, present book treats different fuzzy control architectures both in terms of the theoretical design and in terms of comparative validation studies in various applications, numerically simulated or experimentally developed. Through the subject matter and through the inter and multidisciplinary content, this book is addressed mainly to the researchers, doctoral students and students interested in developing new applications of intelligent control, but also to the people who want to become familiar with the control concepts based on fuzzy techniques. Bibliographic resources used to perform the work includes books and articles of present interest in the field, published in prestigious journals and publishing houses, and websites dedicated to various applications of fuzzy control. Its structure and the presented studies include the book in the category of those who make a direct connection between theoretical developments and practical applications, thereby constituting a real support for the specialists in artificial intelligence, modelling and control fields.

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