2

Takagi-Sugeno Fuzzy Control Based on Robust Stability Specifications

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1. Introduction

In the design of modern and classical control systems, the first step is establish a suitable mathematical model to describe the behavior of the controlled plant (Takagi & Sugeno, 1985; Ying et al., 1990). However, in practical situations, such a requirement is not feasible because in practical control systems the plants are always nonlinear systems, which makes this task analytically unfeasible for complex systems (Cetin & Demir, 2008; Dong et al., 2009; Park et al., 2007; Pelladra et al., 2009). This fact has motivated the use of fuzzy logic in the development of fuzzy model based control systems. In this context, The Fuzzy Systems have been widely used due to flexibility of its structure to incorporate linguistic information (knowledge expert) with numerical information (sensors and actuators measurements), as well as its functional efficiency as universal approximator capable of treat adequately uncertainties, parametric variations and nonlinearity of the plant to be controlled (Castro-Sitiriche et al., 2008; Cetin & Demir, 2008; Cheng et al., 2009; Ibrahim, 2003; Mishra et al., 2000; Park et al., 2007; Wen-Xu et al., 2009). Modeling is the task that simplifies a real system or complex reality with the aim of easing its understanding. In this sense, an effective approach to the identification of complex nonlinear systems is to partition the available data into subsets and approximate each subset by simple model. Fuzzy Clustering can be used as a tool to obtain a partitioning of experimental data where the transitions between the subsets are gradual rather than abrupt. The potential of fuzzy clustering algorithms to reveal the underlying structures in data can be exploited, not only for classification and pattern recognition in the available data, but also for the reduction of complexity in modeling and identification. One of the major applications of the model is the design of a controller for the true system. The ultimate goal of a control-system is to build a system that will work in the real environment. Since the real environment may change with time (parametric variations and nonlinearity) or operating conditions may vary (noise and disturbance), the control system must be able to withstand these variations (Petros & Sun, 1996). This fact has motivated, since 1980's, the proposal of new methodologies for design of robust controllers. In this context, fuzzy systems have been widely used in robust controllers design (Barton, 2004; Serra & Boturra, 2006; Silva & Serra, 2009; Tanaka & Sugeno, 1993; Zhan, 2010). In this paper a robust fuzzy control design based on gain and phase margins specifications for nonlinear systems, in the continuous time domain, is proposed. A mathematical formulation based on Takagi-Sugeno fuzzy model structure as well as the PDC strategy is presented. Analytical formulas are deduced for the sub-controllers parameters, in the robust fuzzy controller rules base, according to the fuzzy model parameters of the fuzzy model plant to be controlled. Results for the necessary and sufficient conditions for the fuzzy controller design, from the proposed robust methodology, with one axiom and two theorems are presented. Simulation results, based on robust methodology, for a single link robotic manipulator are presented. The paper is organized as follows: In section II, it is introduced firstly the preliminary concepts fot the proposal methodology; secondly the the robust fuzzy control design and tuning formulas, based on gain and phase margins specifications, as well as the robust stability analysis of the fuzzy controller, are proposed in section III. Finally, Simulation results and conclusions are drawn in sections IV and V, repectively.

2. Preliminary concepts

In this section, some importants concepts to development the proposal methodology are presented.

2.1 Takagi-Sugeno fuzzy inference systems

The TS fuzzy model, originally proposed by Takagi and Sugeno (Takagi & Sugeno, 1985), is composed of a fuzzy **IF-THEN** rule base that partitions a space - usually called the *universe of discourse* - into fuzzy regions described by the *antecedents*. The *consequent* of each rule i is a simple functional expression of model inputs and that all fuzzy terms are monotonic functions. In this case, specifically, the TS fuzzy model can be regarded as a mapping from the antecedent (input) space to a convex region (polytope) in the local sub-models space into the consequent, defined by the variants consequents parameters of the plant to be controlled. This property simplifies the analysis of the TS fuzzy model in a context of robust time-variant and linear system for design of controllers with desired characteristics of the closed loop control system or stability analysis.
The $i|^{[i=1,2,...,l]}$-th TS rule, without loss of generality, the following structure:

$$R^{(i)}: \text{ IF } \tilde{x}_1 \text{ is } F^i_{j|\tilde{x}_1} \text{ AND } \cdots \text{ AND } \tilde{x}_n \text{ is } F^i_{j|\tilde{x}_n} \text{ THEN } \tilde{y}_i = f_i\left(\tilde{\mathbf{x}}\right) \tag{1}$$

where

$$\tilde{\mathbf{x}}^T = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n],$$
$$\tilde{\mathbf{y}}^T = [\tilde{y}_1, \tilde{y}_2, \cdots, \tilde{y}_n],$$

l is the number of fuzzy **IF-THEN** rules. The vector $\tilde{\mathbf{x}} \in \Re^n$ contains the antecedent linguistic variables. Each linguistic variable has its own universe of discourse $U_{\tilde{x}_1}, \cdots, U_{\tilde{x}_n}$ partitioned by fuzzy sets representing the linguistic terms. The variable $\tilde{x}_t|^{[t=1,2,...,n]}$ belongs to the fuzzy set $F^i_{j|\tilde{x}_t}$ with a value $\mu^i_{F_{j|\tilde{x}_t}}$ defined by a membership function $\mu^i_{\tilde{x}_t}: \Re \to [0,1]$, with $\mu^i_{F_{j|\tilde{x}_t}} \in \mu^i_{F_{1|\tilde{x}_t}}, \mu^i_{F_{2|\tilde{x}_t}}, \mu^i_{F_{3|\tilde{x}_t}}, \ldots, \mu^i_{F_{p_{\tilde{x}_t}|\tilde{x}_t}}$, where $p_{\tilde{x}_t}$ is the number of partitions of the universe of discourse associated to the linguistic variable $\tilde{\mathbf{x}}$. The activation degree of h_i for the rule i, is given by:

$$h_i\left(\tilde{\mathbf{x}}\right) = \mu^i_{F_{j|\tilde{x}^*_1}} \otimes \mu^i_{F_{j|\tilde{x}^*_2}} \otimes \cdots \mu^i_{F_{j|\tilde{x}^*_n}} \tag{2}$$

where $\tilde{x}^*_t$ is some point in $U_{\tilde{x}_t}$. The normalized activation degree for the rule i, is given by:

$$\gamma_i(\tilde{\mathbf{x}}) = \frac{h_i(\tilde{\mathbf{x}})}{\sum_{\lambda=1}^{l} h_\lambda(\tilde{\mathbf{x}})} \tag{3}$$

where it is assumed that

$$\sum_{\lambda=1}^{l} h_\lambda(\tilde{\mathbf{x}}) > 0,$$

$$h_\lambda(\tilde{\mathbf{x}}) \geq 0, \quad i = 1, 2, \cdots, l$$

And, this normalization implies that

$$\sum_{i=1}^{l} \gamma_i(\tilde{\mathbf{x}}) = 1 \tag{4}$$

The TS fuzzy model response is a weighted sum of the consequent parameters, i.e., a convex linear combination of the local functions (models) f_i, which reads

$$f_i(\tilde{\mathbf{x}}) = \sum_{i=1}^{l} \gamma_i(\tilde{\mathbf{x}}) f_i(\tilde{\mathbf{x}}) \tag{5}$$

Each linear component $f_i(\tilde{\mathbf{x}})$ is called a *subsystem*. This model can be seen as a Linear Parameters Varying (LPV) System (Balas et al., 1997; Shamma & Athans, 1991). This property simplifies the analysis of the TS fuzzy model in a context of robust time-variant and linear system for design of controllers with desired characteristics of the closed loop control system or stability analysis.

2.2 Fuzzy model based control design steps

The design of a controller that can alter or modify the behavior and response of an unknown plant to meet certain perfomance requirements can be a tedious and challenging problem in many control applications. The plant inputs u are processed to produce several plant outputs y that represent the measured output response of the plant. The control design task is to choose the input u so that the output response $y(t)$ satisfies certain given performance requirements. Because the plant process is usually complex, i.e., it may consist of various mechanical, electronic, hydraulic parts, etc., the appropriate choise of u is in general straightforward. The control design steps often followed by most control engineers in choosing the input u are explained below.

2.2.1 Modeling

The task os the control engineer in this step is to undestand the processing mechanism of the plant, which takes a given input signal $u(t)$ and produces the output response $y(t)$, to the point that he or she can describe it in the form of some mathematical equations. These equations constitute the mathematical model of the plant. An exact plant model should produce the same output response as the plant, provided the input to the model and initial conditions are exactly the same as those of the plant. The complexity of most physical plants, however, makes the development of such an exact model unwarranted or even impossible. But even if the exact plant model becomes available, its dimension is likely to be infinite, and its description nonlinear or time time varying to the point that its usefulness from the control design viewpoint is minimal or none. This makes the task of modeling even more difficult and challenging, because the control engineer has to came up with a mathematical model that describes accurately the input/output behavior of the plant and yet is simple enough to be used for control design purposes. A simple model usually leads to a simple controller that is easier to understand and implement, and often more reliable for practical purposes. A simple model usually leads to a simple controller that is easier to understand and implement, and often more reliable for practical purposes.

A plant model may be developed by using physical laws or by processing the plant input/output (I/O) data obtained by performing various experiments. Such a model, however, mat still be complicated enough from the control design viewpoint and further simplifications may be necessary. Some of the approaches often used to obtain a simplified model are:

(a) Linearization around operating points;

(b) Model order reduction techniques;

(c) Fuzzy Clustering.

In approach (a) the plant is approximated by a linear model that is valid around a given operating point. Different operating points may lead to several different linear models that are used as plant models. Linearization is achieved by using Taylor's series expansion and approximation, fitting of experimental data to a linear model, etc.

In approach (b) small effects and phenomena outside the frequency range of interest are neglected leading to a lower order and simpler plant model.

In approach (c), used in this work, the fuzzy clustering algorithms are used to construct fuzzy models from experimental data. Among the most popular methods are the following: *Fuzzy C - Means (FCM)*, *Gustafson - Kessel (GK)* and *Fuzzy Maximum Likelihood Estimates (FLME) algorithms*. All these algorithms share the following definitions.

A ***cluster*** is a group of objects that are more similar to another than to members of other clusters (Bezdek, 1981; Jain & Dubes, 1988). The term *"similarity"* should be understood as mathematical similarity, measure in some well-define sense. In metric spaces, similarity is often defined by means of a distance norm. Distance can be measure from a data vector to some cluster prototypical (center). Data can reveal clusters of different geometric shapes, sizes and densities. While clusters can be characterized as linear and nonlinear subspaces of the data space.

The objective of clustering is to partition the data set Z into *c* clusters. Assume that *c* is known, based on priori knowledge. The ***fuzzy partition*** de Z can be defined as a family of subsets $\{A_i | 1 \leq i \leq c\} \subset P(Z)$, with the following properties:

$$\bigcup_{i=1}^{c} A_i = Z \tag{6}$$

$$A_i \cap A_j = 0 \tag{7}$$

$$0 \subset A_i \subset Z_i \tag{8}$$

Equation 6 means that the subsets A_i collectively contain all the data in Z. The subsets must be disjoint, as stated by 7, and none off than is empty nor contains all the data in Z, as stated by 8.

In terms of *membership functions*, μ_{A_i} is the membership function of A_i. To simplifly the notation, in this work we use μ_{ik} instead $\mu_i(z_k)$. The cxN matrix $\mathbf{U} = [\mu_{ik}]$ represents a fuzzy partitioning space if and only if:

$$M_{fc} = \left\{ \mathbf{U} \in \Re^{cxN} | \mu_{ik} \in [0,1], \forall i,k; \sum_{i=1}^{c} \mu_{ik} = 1, \forall k; 0 < \sum_{k=1}^{N} \mu_{ik} < N, \forall i \right\} \tag{9}$$

The i-th row of the fuzzy partition matrix $\mathbf{U}$ contains values of the i-th membership function of the fuzzy subset A_i of Z.
The clustering algorithms optimizes an initial set of centroids by minimizing a ***cost function*** J in an iterative process. Such function is usually formulated as:

$$\mathbf{J}(\mathbf{Z}; \mathbf{U}, \mathbf{V}, \mathbf{A}) = \sum_{i=1}^{c} \sum_{k=1}^{N} \mu_{ik}^{m} D_{ikA_i}^{2} \tag{10}$$

where, $\mathbf{Z} = \{z_1, z_2, \cdots, z_N\}$ is a finite data set. $\mathbf{U} = [\mu_{ik}] \in M_{fc}$ is a fuzzy partition of $\mathbf{Z}$. $\mathbf{V} = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_c\}, \mathbf{v}_i \in \Re^n$, is a vector of cluster prototypes (centers). $\mathbf{A}$ denote a c-tuple of the norm-induting matrices: $\mathbf{A} = (\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_c)$. $D_{ik\mathbf{A}_i}^2$ is a squared inner-product distance norm. $m \in [1, \infty)$ is a weighting exponent which determines the fuzziness of the resulting clusters.
The clustering algorithms differ in the choice of the norm distance. The ***norm metric*** influences the clustering criterion by changing the measure of dissimilarity. The Euclidean norm induces hiperspherical clusters. It's characterizes the ***FCM*** *algorithm*, where norm-inducing matrix $\mathbf{A}_{i_{FCM}}$ is equal to identity matrix ($\mathbf{A}_{i_{FCM}} = \mathbf{I}$), this strictly imposes a circular shape to all clusters. The Euclidean Norm is given by:

$$D_{ik_{FCM}}^2 = (z_k - v_i)^T \mathbf{A}_{i_{FCM}} (z_k - v_i) \tag{11}$$

An adaptative distance norm, in order to detect clusters of different geometrical shapes in one data set, characterizes the ***GK*** *algorithm*:

$$D_{ik_{GK}}^2 = (z_k - v_i)^T \mathbf{A}_{i_{GK}} (z_k - v_i) \tag{12}$$

In this algorithm, each cluster has its own norm-inducing matrix $\mathbf{A}_{i_{GK}}$, where each cluster to adapt the distance norm to the local topological structure of the data set. $\mathbf{A}_{i_{GK}}$ is given by:

$$\mathbf{A}_{i_{GK}} = [\rho_i det(\mathbf{F}_i)]^{1/n} \mathbf{F}_i^{-1}, \tag{13}$$

where ρ_i is cluster volume, usually fixed in one. n is data dimension. $\mathbf{F}_i$ is the *fuzzy covariance matrix* of the i-th cluster defined by:

$$\mathbf{F}_i = \frac{\sum_{k=1}^{N} (\mu_{ik})^m (z_k - v_i)(z_k - v_i)^T}{\sum_{k=1}^{N} (\mu_{ik})^m} \tag{14}$$

The eigenstructure of the cluster covariance matrix provides information about the shape and orientation cluster. The ratio of the hyperellipsoid axes is given by the ratio of the square roots of the eigenvalues of $\mathbf{F}_i$. The directions of the axes are given by the eigenvectores of $\mathbf{F}_i$. The eigenvector corresponding to the smallest eigenvalue determines the normal to the hyperplane, and can be used to compute optimal local linear models from the covariance matrix.

The *fuzzy maximum likelihood estimates* (**FLME**) *algorithms* employs a distance norm based on maximum lekelihood estimates:

$$D_{ik_{FLME}} = \frac{\left[det\mathbf{G}_{i_{FLME}}\right]^{1/2}}{P_i} exp\left[\frac{1}{2}\left(z_k - v_i\right)^T \mathbf{F}_{i_{FLME}}^{-1}\left(z_k - v_i\right)\right] \tag{15}$$

Note that, contrary to the GK algorithm, this distance norm involves an exponential term and thus decreases faster than the inner-product norm. $\mathbf{F}_{i_{FLME}}$ denotes the fuzzy covariance matrix of the i-th cluster, given by equation 14. When m is equal 1, we have a strict algorithm FLME. If m is greater than 1, we have a *extended algorithm FLME*, or *Gath-Geva (GG) algorithm*. P_i is the prior probability of selecting cluster i, given by:

$$\mathbf{P}_i = \frac{1}{N}\sum_{k=1}^{N}\left(\mu_{ik}\right)^m \tag{16}$$

Gath and Geva (Gath & Geva, 1989) reported that the FLME algorithm is able to detect clusters of varying shapes, sizes and densities. This is because the cluster covariance matrix is used in conjuncion with an "exponential" distance, and the clusters are not constrained in volume.
The system identification procedure is illustrated in the Figure 1 below.

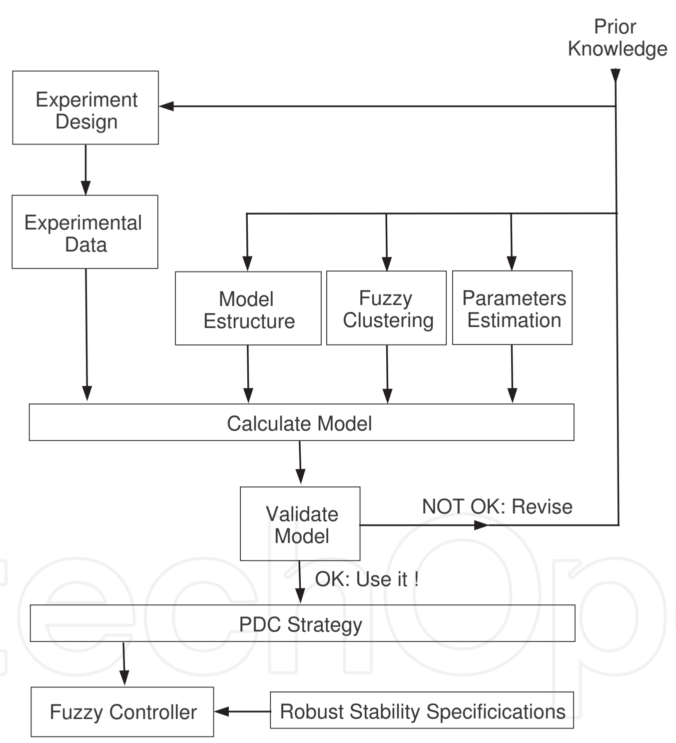


Fig. 1. The control system diagram

The fuzzy clustering algorithms can be used to approximate a set of experimental data by local linear models. Each of these models is represented by a fuzzy subset in the data set available for identification. In order to obtain a model useful for controller design, an additional step must be applied to generate a model independent of the identification data. Such a model can be represented either as a rule base. Each cluster obtained by clustering algorithms of the identification data set can be regarded as a local linear approximation of the regression hypersurface. The global model can be conveniently represented as a set affine Takagi-Sugeno

(TS) rules, can decribed in equation 1. The antecedent fuzzy sets can be computed analitically in the antecedent product space, or can be extracted from the fuzzy partition matriz by projections.
The consequent parameters are estimated from the data using the weighted least-squares method. Where, the identification data and the membership degrees of the fuzzy partition are arranged in the following matrices:

$$X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_N^T \end{bmatrix}, \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \qquad \Omega_i = \begin{bmatrix} \mu_{i1} & 0 & \cdots & 0 \\ 0 & \mu_{i1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu_{iN} \end{bmatrix} \tag{17}$$

The consequent parameters of the rule belonging to the i-th cluster, depending of the model identification structure, are concatenated into a single parameter vector, θ_i, for example:

$$\theta_i = \left[a_i^T, b_i^T\right] \tag{18}$$

X_{reg} gives the extended regressor matrix, depending too of the model identification structure. Assuming that each cluster represents a local linear model of the system, the consequent parameter vectors θ_i, $i = 1, 2, \cdots, c$, can be estimated independently by the weighted least-squares method. The membership degrees μ_{ik} of the fuzzy partition serve as the weights expressing the relevance of the data pair (x_k, y_k) to that local model. If the columns of X_{reg} are linearly independent and $\mu_{ik} > 0$ for $1 \leq k \leq N$, then

$$\theta_i = \left[X_{reg}^T \Omega_i X_{reg}\right]^{-1} X_{reg}^T \Omega_i y \tag{19}$$

Since that,

$$\tilde{y}_k = f_i\left(x_k; \theta_i\right) \tag{20}$$

where the functions f_i are parameterized by $\theta_i \in \Re^{p_i}$. We have,

$$\tilde{y}\left(\tilde{\mathbf{x}}\right) = \frac{\sum_{i=1}^{l} h_i\left(\tilde{\mathbf{x}}\right) \tilde{y}_i}{\sum_{i=1}^{l} h_i\left(\tilde{\mathbf{x}}\right)} \tag{21}$$

2.2.2 Controller design

Once a model of the plant is available, one can proceed with the controller design. The controller is designed to meet the performance requirements for the plant model. If the model is a good approximation of the plant, one would hope that the controller performance for the plant model to be close to that achieved when the same controller is applied to the plant. In this sence, the robust stability control problem is to find a control law which maintains system response and error signals within prescribed tolerances despite the effects of parametric variations on the plant. In this paper a robust fuzzy control design based on gain and phase margins specifications for nonlinear systems, in the continuous time domain, is proposed. A mathematical formulation based on Takagi-Sugeno fuzzy model structure as well as the PDC strategy is presented. Analytical formulas are deduced for the sub-controllers parameters, in the robust fuzzy controller rules base, according to the fuzzy model parameters of the fuzzy model plant to be controlled.

2.2.3 Implementation

In this step, a controller designed in previous step, which is shown to meet performance requirements for the plant model and is robust with respect possible plant model disturbances, is ready to be applied to the unknow plant. The implementation can be done using a digital computer, all though in some applications analog computers may be used too. Issue such as the type of computer available, the type of inference devices between the computer and the plant, software tools, etc., need to be considered priori. Computer speed and accuracy limitations may put constraints complexity of the complexity of the controller that may force the control engineer to go back to previous step or even first step to come up with a simpler controller without violating the performance requirements.
Another important aspect of implementation is the final adjustment or as often called the tuning, of the controller to improve performance be compensating for the plant model disturbances that are not accounted for during the design process. Tuning is often done by trial and error, and depends very much on the experience and intuition of control engineer. In this work, the adjusts are done based on gain and phase margin especifications.

2.3 Gain and phase margins especifications

A successfully designed control system should be always able to maintain stability and performance level in spite of disturbances in system dynamics and/or in the working environment to a certain degree. Gain margin and phase margin have always served as important measures of robustness. It is also known from classical control that phase margin is related to the damping of the system, and can therefore also serve as a performance measure (Franklin et al., 1986). Controller designs to satisfy gain margin and phase margin (GPM) criteria are not new (Franklin et al., 1986; Ogata, 2002).
The **Phase Margin** is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability. The gain crossover frequency is the frequency at which $|G(j\omega)|$, the magnitude of the open-loop transfer function, is unity. The phase margin ϕ_m is 180° plus the phase angle $\angle G(j\omega)$ of the open-loop transfer function at the gain crossover frequency, or:

$$\phi_m = \angle G(j\omega) + \pi \tag{22}$$

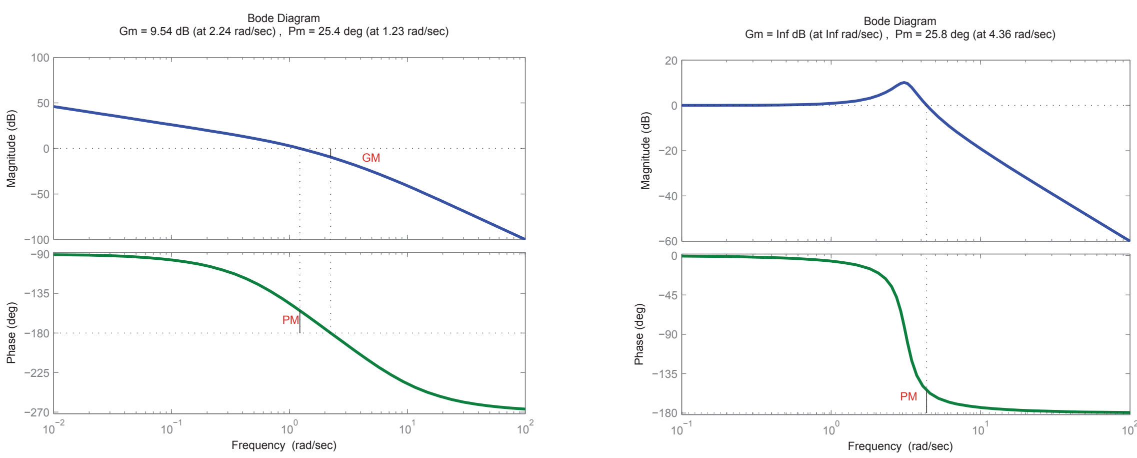
The phase margin is positive for $\phi_m > 0$ and negative for $\phi_m < 0$. For a minimum-phase system[1] to be stable, the phase margin must be positive.
The **Gain Margin** is the reciprocal of the magnitude $|G(j\omega)|$ at the frequency at which the phase angle is $-180°$. Defining the phase crossover frequency ω_p, to be the frequency at which the phase angle of the open-loop transfer function equals $-180°$ gives the gain margin A_m:

$$A_m = \frac{1}{|G(j\omega_g)|} \tag{23}$$

The gain margin expressed in decibels is positive if A_m, is greater than unity and negative if A_m is smaller than unity. Thus, a positive gain margin (in decibels) means that the system is stable, and a negative gain margin (in decibels) means that the system is unstable. For a stable minimum-phase system, the gain margin indicates how much the gain can be increased

[1] Transfer functions having neither poles nor zeros in the right-half s plane are *minimum-phase* transfer functions, whereas those having poles and/or zeros in the right-half s plane are *nonminimum-phase* transfer functions

before the system becomes unstable. For an unstable system, the gain margin is indicative of how much the gain must be decreased to make the system stable. For a minimum-phase system, both the phase and gain margins must be positive for the system to be stable. Negative margins indicate instability. Proper phase and gain margins ensure us against variations in the system components and are specified for definite positive values. The two values bound the behavior of the closed-loop system near the resonant frequency. For satisfactory performance, the phase margin should be between 30° and 60°, and the gain margin should be greater than 6 dB. With these values, a minimum-phase system has guaranteed stability, even if the openloop gain and time constants of the components vary to a certain extent. Although the phase and gain margins give only rough estimates of the effective damping ratio of the closed-loop system, they do offer a convenient means for designing control systems or adjusting the gain constants of systems. the Figure 2 shows the gain and phase margins for two different systems.



(a) $G_1 = 10/s(s+1)(s+5)$ (b) $G_2 = 10/(s^2+s+10)$

Fig. 2. The phase (9.54dB) and gain (25.4dB) margins of the system $10/s(s+1)(s+5)$ is showed in (a). The phase (25.8dB) and gain (inf.) margins of the system $10/(s^2+s+10)$ is showed in (b). Note that the gain margin of a first or second-order system is infinite since the polar plots for such systems do not cross the negative real axis.

For minimum-phase systems, the magnitude and phase characteristics of the openloop transfer function are definitely related. The requirement that the phase margin be between 30° and 60° means that in a Bode diagram the slope of the log-magnitude curve at the gain crossover frequency should be more gradual than -40 dB/decade. In most practical cases, a slope of -20 dB/decade is desirable at the gain crossover frequency for stability. If it is -40 dB/decade, the systems could be either stable or unstable. (Even if the system is stable, however, the phase margin is small.) If the slope at the gain crossover frequency is -60 dB/decade or steeper, the system is most likely unstable.
Denote the process and the controller transfer function by $G_p(s)$ and $G_c(s)$, and the specified gain and phase margins by A_m and ϕ_m, respectively. The formulas for gain margin and phase margin are as follows:

$$arg\left[G_c(j\omega_p)G_p(j\omega_p)\right] = -\pi \tag{24}$$

$$A_m = \frac{1}{\left|G_c(j\omega_p)G_p(j\omega_p)\right|} \tag{25}$$

$$\left|G_c(j\omega_g)G_p(j\omega_g)\right| = 1 \tag{26}$$

$$\phi_m = arg\left[G_c(j\omega_g)G_p(j\omega_g)\right] + \pi \tag{27}$$

where the gain margin is defined by Eqs. 24 and 25, and the phase margin by Eqs. 26 and 27. The frequency ω_p at which the Nyquist curve has a phase of $-\pi$ is known in classical terminology as the phase crossover frequency, and the frequency ω_g at which the Nyquist curve has an amplitude of 1 as the gain crossover frequency.

2.4 Parallel Distributed Compesation (PDC) strategy

The history of the so-called parallel distributed compensation (PDC) began with a model-based design procedure proposed by Wang (Wang et al., 1995). The PDC offers a procedure to design a fuzzy controller from a given T-S fuzzy model. To realize the PDC, a controlled plant is first represented by a T-S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. The Figure 3 shows the concept of PDC design.

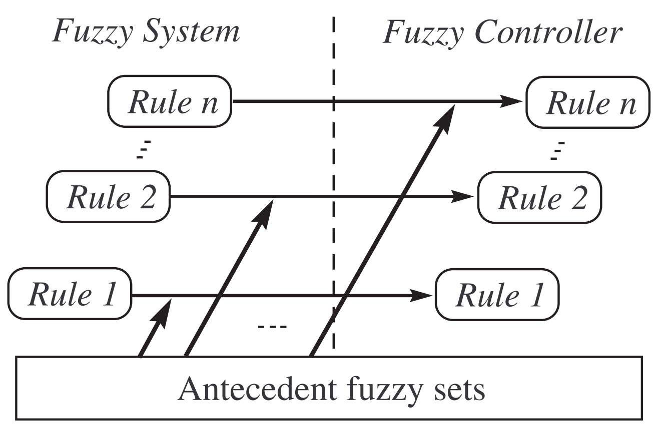


Fig. 3. In the PDC strategy, the fuzzy controller shares the same fuzzy sets with the fuzzy system.

In this paper is presented an fuzzy robust model based control scheme from the TS fuzzy model structure, the PDC strategy and gain and phase margins robust specifications. In the proposed methodology, the fuzzy controller parameters, with TS structure, are obtained through analytical formulas from the definition of gain and phase margins specifications. The robust fuzzy controller designed and the TS fuzzy model of the plant model to be controlled shares the same fuzzy sets, in the antecedents. In the fuzzy inference engine the sub-controller is selected based on the plant dynamic behavior and the gain and phase margins robust specifications. The dynamic system class under analysis for the fuzzy control design structure of the robust control is proposed with the objective to obtain the above robustness characteristics, from generalized analytical formulas.

3. Robust fuzzy control based on gain and phase margins especifications

In this section, the robust fuzzy control methodology based on gain and phase margins especifications are presented.

3.1 TS fuzzy dynamic model

The TS fuzzy inference system for a second-order plant, $G_p(s)$,presents in the $i|^{[i=1,2,\ldots,l]}$-th rule, without loss of generality, the following structure:

$$R^{(i)}: \quad IF \quad \tilde{\tau} \quad \text{is} \quad F^i_{k|\tilde{\tau}} \quad AND \quad \tilde{\tau}' \quad \text{is} \quad G^i_{k|\tau'} \quad AND \quad \tilde{K}_p \quad \text{is} \quad H^i_{k|\tilde{K}_p}$$

$$\text{THEN} \quad G^i_p(s) = \frac{K^i_p}{(1+s\tau^i)(1+s\tau'^i)}e^{-sL} \tag{28}$$

The time constants $\tilde{\tau}$ and $\tilde{\tau'}$, where $\tilde{\tau} \geq \tilde{\tau}^{'}$, and the gain $\tilde{K}_p$, represent the linguistic variables of the antecedent. The activation degree of h_i for the rule i, is given by:

$$h_i\left(\tilde{\tau}, \tilde{\tau'}, \tilde{K}_p\right) = \mu^i_{F_{k|\tilde{\tau}^*}} \otimes \mu^i_{G_{k|\tilde{\tau}'^*}} \otimes \mu^i_{H_{k|\tilde{K}^*_p}} \tag{29}$$

The normalized activation degree for the rule i, is given by:

$$\gamma_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right) = \frac{h_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)}{\sum_{\lambda=1}^{l} h_\lambda\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)} \tag{30}$$

And, this normalization implies

$$\sum_{i=1}^{l} \gamma_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right) = 1 \tag{31}$$

Therefore, the TS fuzzy model, $G^i_p(s)$, of the plant is a weighted sum of second order linear sub-models, as follow:

$$G_p\left(s, \tilde{\tau}, \tilde{\tau'}, \tilde{K}_p\right) = \sum_{i=1}^{l} \gamma_i\left(\tilde{\tau}, \tilde{\tau'}, \tilde{K}_p\right) \frac{K^i_p}{(1+s\tau^i)(1+s\tau'^i)}e^{-sL} \tag{32}$$

3.2 TS robust fuzzy controller

The TS fuzzy inference system proposed for the fuzzy controller, $G_c\,(s)$, whereas the definition of parallel distributed compensation, presents in the $j|^{[j=1,2,\ldots,l]}$ -th rule, without loss of generality, is given by:

$$R^{(j)}: \quad IF \quad \tilde{\tau} \quad \text{is} \quad F^j_{k|\tilde{\tau}} \quad AND \quad \tilde{\tau}^{'} \quad \text{is} \quad G^j_{k|\tau'} \quad AND \quad \tilde{K}_p \quad \text{is} \quad H^j_{k|\tilde{K}_p}$$

$$\text{THEN} \quad G^j_c(s) = \frac{K^j_c\left(1+sT^j_I\right)\left(1+sT^j_D\right)}{sT^j_I} \tag{33}$$

The activation degree h_j for the rule j, is given by:

$$h_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) = \mu^j_{F_{k|\tilde{\tau}^*}} \otimes \mu^j_{G_{k|\tilde{\tau}'^*}} \otimes \mu^j_{H_{k|\tilde{K}_p^*}} \tag{34}$$

where $\tilde{\tau}^*$, $\tilde{\tau}'^*$ and $\tilde{K}_p^*$ are some point in $U_{\tilde{\tau}}, U_{\tilde{\tau}'}$ and $U_{\tilde{K}_p}$, respectively. The normalized activation degree for the rule j, is given by:

$$\gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) = \frac{h_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)}{\sum\limits_{\lambda=1}^{l} h_\lambda\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)} \tag{35}$$

And, this normalization implies

$$\sum_{j=1}^{l} \gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) = 1 \tag{36}$$

Therefore, the TS fuzzy model for the fuzzy controller, $G_c\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p,s\right)$, is a weighted sum of the local fuzzy sub-controllers, as follows:

$$G_c\left(s,\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) = \sum_{i=1}^{l} \gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) \frac{K_c^j\left(1+sT_I^j\right)\left(1+sT_D^j\right)}{sT_I^j} \tag{37}$$

The compensated open-loop fuzzy model (Figure 4), according to the PDC strategy, with the controller and the plant, from the equations 32 and 37, respectively, is

$$G_p(s)G_c(s) = \sum_{j=1}^{l}\sum_{i=1}^{l} \gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) \times \tag{38}$$

$$\times \frac{K_c^j K_p^i\left(1+sT_I^j\right)\left(1+sT_D^j\right)}{sT_I^j\left(1+s\tau^i\right)\left(1+s\tau'^i\right)} e^{-sL}$$

3.3 Robust stability based on gain and phase margins

Denote the process and the controller transfer function by $G_p(s)$ and $G_c(s)$, and the specified gain and phase margins by A_m and ϕ_m, respectively, as defined previously in the Section 2.3. The formulas for gain margin and phase margin, in the fuzzy context, are as follows::

$$arg\left[G_c(\tilde{\tau},\tilde{\tau}',\tilde{K}_p,j\omega_p)G_p(\tilde{\tau},\tilde{K}_p,j\omega_p)\right] = -\pi \tag{39}$$

$$A_m = \frac{1}{\left|G_c(\tilde{\tau},\tilde{\tau}',\tilde{K}_p,j\omega_p)G_p(\tilde{\tau},\tilde{K}_p,j\omega_p)\right|} \tag{40}$$

$$\left|G_c(\tilde{\tau},\tilde{K}_p,j\omega_g)G_p(\tilde{\tau},\tilde{\tau}',\tilde{K}_p,j\omega_g)\right| = 1 \tag{41}$$

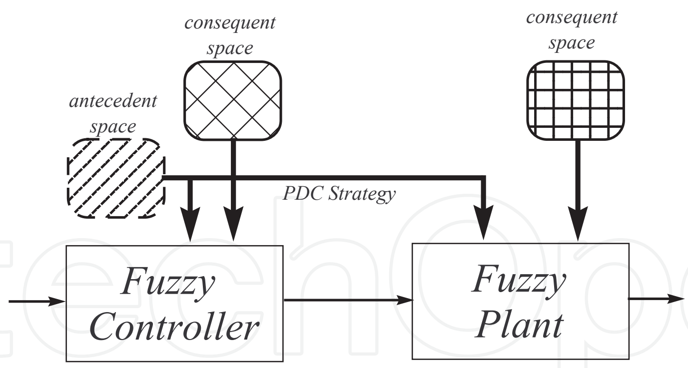


Fig. 4. Controller and plant fuzzy model in open-loop share the same fuzzy sets in the antecedent space.

$$\phi_m = arg\left[G_c(\tilde{\tau}, \tilde{K}_p, j\omega_g) G_p(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p, j\omega_g)\right] + \pi \tag{42}$$

Replacing the equation 38 in 39-42, it has:

$$l\left[\sum_{i=1}^{l}\left(\arctan\left(\omega_p T_I^i\right) - \arctan\left(\omega_p \tau^i\right)\right) - \frac{\pi}{2} - \omega_g L\ \right] = -\pi \tag{43}$$

$$A_m = \frac{1}{\sum_{j=1}^{l}\sum_{i=1}^{l}\gamma_j\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\gamma_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\frac{K_c^j K_p^i}{\omega_p T_I^j}\right)\left(\sqrt{\frac{\left(\omega_p T_I^j\right)^2 + 1}{\left(\omega_p \tau^i\right)^2 + 1}}\right)} \tag{44}$$

$$\sum_{j=1}^{l}\sum_{i=1}^{l}\gamma_j\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\gamma_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\frac{K_c^j K_p^i}{\omega_g T_I^j}\right)\left(\sqrt{\frac{\left(\omega_g T_I^j\right)^2 + 1}{\left(\omega_g \tau^i\right)^2 + 1}}\right) = 1 \tag{45}$$

$$\phi_m = l\left[\sum_{i=1}^{l}\left(\arctan\left(\omega_g T_I^i\right) - \arctan\left(\omega_g \tau^i\right)\right) - \frac{\pi}{2} - \omega_p L\right] + \pi \tag{46}$$

For a given linear sub-model, $G^i(s, \tilde{K}_p^i, \tilde{\tau}^i, \tilde{\tau}'^i)$, and gain and phase margins specifications (A_m, ϕ_m), the Equations 43-46 can be used to determine the parameters of the PID sub-controllers, $G_c^j(s, K_c^j, T_I^j, T_D^j)$. Therefore, using the approximation of arctan function in the case $|x| > 1$, the Equations 44 and 45 are given by:

$$\sum_{j=1}^{l}\sum_{i=1}^{l}\gamma_j\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\gamma_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\frac{A_m}{\omega_p}\left(\frac{K_c^j K_p^i}{\tau^i}\right) = 1 \tag{47}$$

$$\sum_{j=1}^{l}\sum_{i=1}^{l}\gamma_j\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\gamma_i\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\frac{K_c^j K_p^i}{\omega_g \tau^i}\right) = 1 \tag{48}$$

respectively. Using the same approach, the Equations 43 and 46 are given by:

$$l\left[\sum_{i=1}^{l}\left(\frac{\pi}{4\omega_p\tau^i}-\frac{\pi}{\omega_p T_I^i}-\frac{\pi}{2}-\omega_p L\right)\right]=-\pi \tag{49}$$

$$\phi_m=l\left[\sum_{i=1}^{l}\left(\frac{\pi}{4\omega_g\tau^i}-\frac{\pi}{\omega_g T_I^i}-\frac{\pi}{2}-\omega_g L\right)\right]+\pi \tag{50}$$

respectively. Therefore, the analytical solution for the tuning of the PID sub-controllers parameters, $G_c^j(s)\Big|^{[i=1,2,\ldots,l]}$, according to Equations 47-50, is given by

$$T_D^j=\tau'^i \tag{51}$$

$$\begin{bmatrix}\sum_{i=1}^{l}\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left(\frac{K_p^i}{\tau^i}\right) & \cdots & \sum_{i=1}^{l}\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left(\frac{K_p^i}{\tau^i}\right)\\ \sum_{i=1}^{l}\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left(\frac{K_p^i}{\tau^i}\right) & \cdots & \sum_{i=1}^{l}\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left(\frac{K_p^i}{\tau^i}\right)\end{bmatrix}\times$$

$$\times\begin{bmatrix}\gamma_1\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \gamma_l\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\end{bmatrix}\begin{bmatrix}K_c^1\\ \vdots\\ K_c^l\end{bmatrix}=\begin{bmatrix}\frac{\omega_p}{A_m}\\ \omega_g\end{bmatrix} \tag{52}$$

and

$$\begin{bmatrix}l\frac{\pi}{\omega_p} & \cdots & l\frac{\pi}{\omega_p}\\ l\frac{\pi}{\omega_g} & \cdots & l\frac{\pi}{\omega_g}\end{bmatrix}\begin{bmatrix}\left(T_I^1\right)^{-1}\\ \vdots\\ \left(T_I^l\right)^{-1}\end{bmatrix}=\begin{bmatrix}l\left\{\sum_{i=1}^{l}\left(\frac{\pi}{4\omega_p\tau^i}\right)-\frac{\pi}{2}-\omega_p L\right\}+\pi\\ l\left\{\sum_{i=1}^{l}\left(\frac{\pi}{4\omega_g\tau^i}\right)-\frac{\pi}{2}-\omega_g L\right\}-\phi_m+\pi\end{bmatrix} \tag{53}$$

where ω_p is given by:

$$\omega_p=\frac{A_m\phi_m+\frac{1}{2}\pi A_m(A_m-1)}{(A_m^2-1)L} \tag{54}$$

3.3.1 Robust stability analysis

For the design of robust fuzzy PID controller, from Equations 51-53, respectively, based on the gain and phase margins specifications, the following Axiom and Theorems are proposed:

Axiom: *The linear sub-models, $G_p^i(s)\,\Big|^{[i=1,2,\ldots,l]}$, of the plant, are necessarily of minimum phase, i.e., all poles of the characteristic equation are placed in the left half-plane of the complex plane.*

Theorem 1: Each robust PID sub-controller, $G_c^j(s)\,\Big|^{[j=1,2,\ldots,l]}$, guarantee the gain and phase margins specifications for the linear sub-model, $G_p^i(s)\,\Big|^{[i=1,2,\ldots,l]}$ with $i = j$, of the plant to be controlled.

Proof: The normalized activation degree, in a given operating point, on the rules base of the robust PID fuzzy controller, satisfies the following condition:

$$\sum_{i=1}^{l} \gamma_j\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right) = 1 \tag{55}$$

The total normalized activation degree, for a simple p-th rule activated, as defined in the equation 4, is given by

$$\gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right) = 1 \tag{56}$$

Based on the Parallel Distributed Compensation strategy, it has

$$\begin{bmatrix} \gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\dfrac{K_p^p}{\tau^p}\right) \ldots \gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\dfrac{K_p^p}{\tau^p}\right) \\ \\ \gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\dfrac{K_p^p}{\tau^p}\right) \ldots \gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\dfrac{K_p^p}{\tau^p}\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 0 & \gamma_1\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right) & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix} \begin{bmatrix} 0 \\ K_c^p \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\omega_p}{A_m} \\ \\ \omega_g \end{bmatrix} \tag{57}$$

Solving the Equation 57 for K_c, it has

$$\gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\frac{K_p^p}{\tau^p}\right)\gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(K_c^p\right) = \frac{\omega_p}{A_m} \tag{58}$$

and

$$\gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(\frac{K_p^p}{\tau^p}\right)\gamma_p\left(\tilde{\tau}, \tilde{\tau}', \tilde{K}_p\right)\left(K_c^p\right) = \omega_g \tag{59}$$

Isolating K_c^p, the Equation 58, is given by:

$$K_c^p = \left(\frac{\tau^p}{K_p^p}\right)\left(\frac{\omega_p}{A_m}\right)\left(\frac{1}{\gamma_p\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)^2}\right) \tag{60}$$

To obtain the parameter T_I^p, in a given time, as defined previously, it has:

$$\begin{bmatrix} l\dfrac{\pi}{\omega_p} \ldots l\dfrac{\pi}{\omega_p} \\ \\ l\dfrac{\pi}{\omega_g} \ldots l\dfrac{\pi}{\omega_g} \end{bmatrix} \begin{bmatrix} 0 \\ \left(T_I^p\right)^{-1} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} l\left(\dfrac{\pi}{4\omega_p\tau^p} - \dfrac{\pi}{2} - \omega_p L\right) + \pi \\ \\ l\left(\dfrac{\pi}{4\omega_g\tau^p} - \dfrac{\pi}{2} - \omega_g L\right) + \pi - \phi_m \end{bmatrix} \tag{61}$$

which results in

$$\left(\frac{\pi}{4\omega_p\tau^p} - \frac{\pi}{2} - \omega_p L\right) + \pi \tag{62}$$

and

$$l\frac{\pi}{\omega_g}\frac{1}{T_I^p} = l\left(\frac{\pi}{4\omega_g\tau^p} - \frac{\pi}{2} - \omega_g L\right) + \pi - \phi_m \tag{63}$$

Isolating ϕ_m, the Equation 63, is given by:

$$\phi_m = l\left(\frac{\pi}{4\omega_g\tau^p} - \frac{\pi}{\omega_g}\frac{1}{T_I^p} - \frac{\pi}{2} - \omega_g L\right) + \pi \tag{64}$$

and,

$$\gamma_p\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\gamma_p\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left(\frac{K_p^p A_m}{\tau^p\omega_p}\right)\times$$

$$\left(\frac{\tau^p\omega_p}{K_p^p A_m}\right)\left(\frac{1}{\gamma_p\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\gamma_p\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)}\right) = 1 \tag{65}$$

and

$$A_m = A_m \tag{66}$$

Assuming, in a given time, the total activation of a simple rule p, as defined previously, in Equation 35, we have:

$$\phi_m = l\left(\frac{\pi}{4\omega_g\tau^p} - \frac{\pi}{\omega_g T_I^p} - \frac{\pi}{2} - \omega_g L\right) + \pi \tag{67}$$

Comparing the Equation 67 with 64, it has

$$\phi_m = \phi_m \tag{68}$$

From those analysis, the robust fuzzy PID controller guarantee the gain and phase margins specifications for the plant to be controlled.

Theorem 2: Each robust PID sub-controller, $G_c^j(s)\Big|^{[j=1,2,\ldots,l]}$, guarantee the stability for all linear sub-models, $G_p^i(s)\Big|^{[i=1,2,\ldots,l]}$, of the non-linear plant to be controlled.

Proof: The closed-loop transfer function is given by:

$$G_{MF}\left(s,\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right) = \sum_{j=1}^{l}\sum_{i=1}^{l}\gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\frac{K_c^jK_p^i\left(1+sT_I^j\right)e^{-sL}}{\left[sT_I^j\left(1+s\tau^i\right)+K_c^jK_p^i\left(1+sT_I^j\right)\right]} \tag{69}$$

For the stability condition, the characteristic equation of the closed-loop transfer function, given in Equation 69, must have roots (poles) in the left half-plane of the complex plane (negative real part). Therefore, it has

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left[sT_I^j\left(1+s\tau^i\right)+\;+K_p^iK_c^j\left(1+sT_I^j\right)\right]=0 \tag{70}$$

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\gamma_i\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\gamma_j\left(\tilde{\tau},\tilde{\tau}',\tilde{K}_p\right)\left[\tau^iT_I^js^2+\;+\left(T_I^j+K_p^iK_c^jT_I^j\right)s+\left(K_p^iK_c^j\right)\right]=0$$

By application of the Routh Stability Criterion Franklin et al. (1986) in 69, it has

$$\begin{array}{c|cc} s^2 & \tau^iT_I^j & K_p^iK_c^j \\ s^1 & (T_I^j+K_p^iK_c^jT_I^j) & 0 \\ s^0 & K_p^iK_c^j & \end{array} \tag{71}$$

And, it is necessary that all terms of the first column are positive:

$$\tau^iT_I^j > 0 \tag{72}$$

$$(T_I^j+K_p^iK_c^jT_I^j) > 0 \tag{73}$$

$$K_p^iK_c^j > 0 \tag{74}$$

Since the parameters of the stable sub-models of the plant to be controlled (τ^i, τ'^i e K_p^i), according to **Axiom**, are positive as well as the gain and phase margins specifications (A_m e ϕ_m), from Equations 51-53, the values of the robust fuzzy PID controller parameters (K_c^j, T_I^j, T_D^j) are positive. Therefore, the inequalities, in Equations 72-74, are satisfied, and each robust PID sub-controller guarantee the stability for all sub-models of the plant to be controlled.

4. Computational results

This section describes the experimental results of the robust fuzzy control method in this paper.

4.1 Dynamic system description

To illustrate the proposed robust fuzzy control method in this paper, a simulation example is carried out for a one-link robotic manipulator showed in Figure 5. The dinamic equation of the one-link robotic manipulator is given by:

$$ml^2\ddot{\theta} + d\dot{\theta} + mglsin(\theta) = u \quad (75)$$

with,

$m = 1kg$, payload,
$l = 1m$, length of link,
$g = 9.81m/s^2$, gravitational constant,
$d = 1kgm^2/s$, damping factor,
u =control variable (kgm^2/s^2).

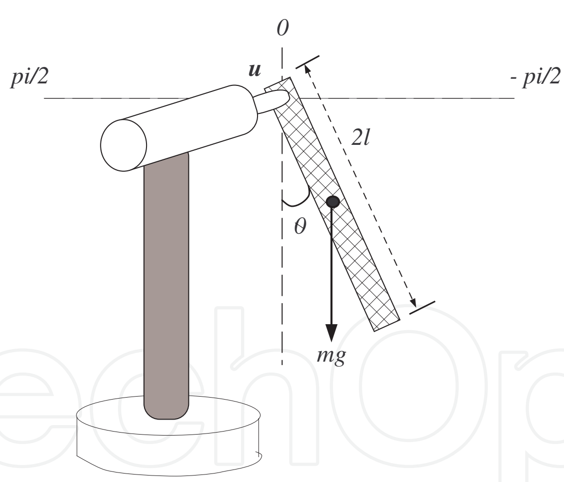


Fig. 5. One-link robotic manipulator.

This process has as input the torque, and as output the robotic manipulator angular position, denoted by θ.

4.2 Data collection

Several simulations were performed to collect suitable identification and validation data. The input of the system were excited with chirp signal. The left plots in Figure 6 show the input signal and the right plot shows the corresponding output.

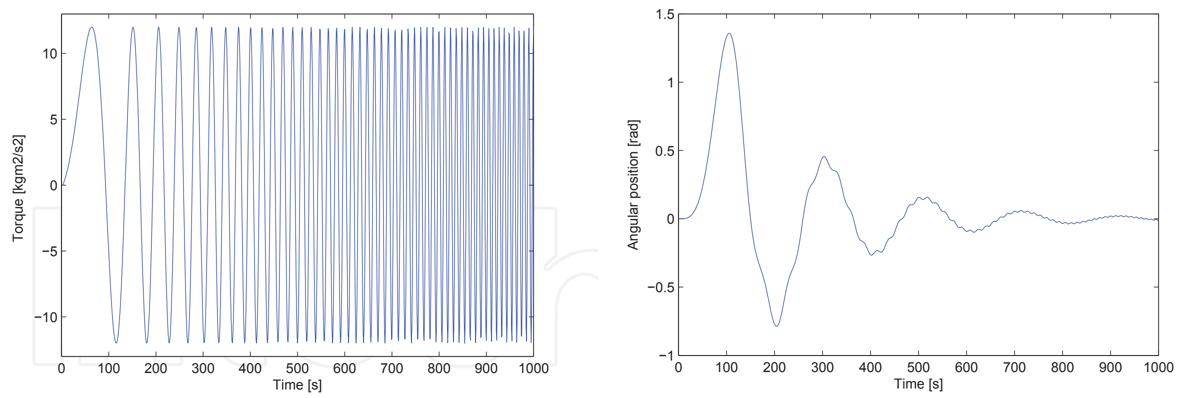


Fig. 6. Identification data set

4.3 Takagi-Sugeno fuzzy model

Based on the prior knowledge about the process, a second-order structure in transfer function terms was selected, resulting in TS rules of the following form:

$$\textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^i_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^i_p(s) = \frac{b^i}{s^2 + a^i_1 s + a^i_2} e^{-sL} \tag{76}$$

where $\tilde{\theta}(t)$ is the angular position at the time t. The membership functions of the antecedent linguistic term $A^i_{k|\tilde{\theta}}$, as well as the consequent parameters b^i, a^i_1 and a^i_2 were estimated from the data by fuzzy clustering, as described in subsection 2.2.1.
First the data matrix $\mathbf{Z}$ is formed, which contains the regressors $u(t), \dot{\theta}(t)$ and $\ddot{\theta}(t)$:

$$\mathbf{Z} = \begin{bmatrix} u(1) & \dot{\theta}(1) & \ddot{\theta}(1) \\ u(2) & \dot{\theta}(2) & \ddot{\theta}(2) \\ \vdots & \vdots & \vdots \\ u(t-1) & \dot{\theta}(t-1) & \ddot{\theta}(t-1) \end{bmatrix} \tag{77}$$

All the clustering algorithms, described in this paper, was applied to the data, but the GK algorithm has selected. We choose the fuzzification factor $m = 2$ and the termination criterion $\epsilon = 0.001$. The clusters number varied from 2 to 5. Due the lower mean square error or MSE obtained for five clusters, as show in Figure 8, the following data classification and clustering, as show in Figure 7, is obtained.
Each obtained cluster corresponds to one rule of the TS fuzzy model. The antecedent membership degrees are directly obtained in the product space of the antecedent variable, and the consequent parameters are estimated by weighted least-squares method. Using the identification method based on fuzzy clustering, the following five TS rules, to plant model, were extracted from identification data:

$$\textit{Rule } 1: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^1_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^1_p(s) = \frac{1.001}{s^2 + 1.014s + 9.464} e^{-0.1s}$$

$$\textit{Rule } 2: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^2_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^2_p(s) = \frac{0.998}{s^2 + 1.002s + 9.125} e^{-0.1s}$$

$$Rule\ 3: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^3_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^3_p(s) = \frac{0.892}{s^2 + 0.706s + 7.828} e^{-0.1s}$$

$$Rule\ 4: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^4_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^4_p(s) = \frac{0.999}{s^2 + 1.023s + 9.389} e^{-0.1s}$$

$$Rule\ 5: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^5_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^5_p(s) = \frac{0.998}{s^2 + 0.991s + 9.342} e^{-0.1s}$$

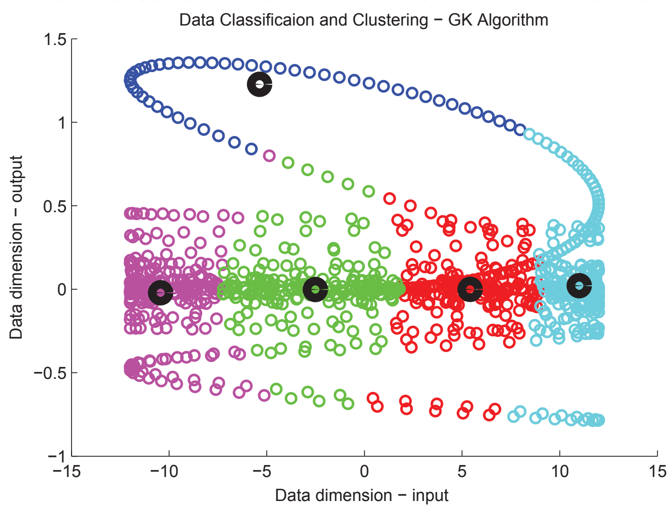


Fig. 7. The dark dots represents the obtained clusters and data classification. Each cluster represents the estimated local sub-models.

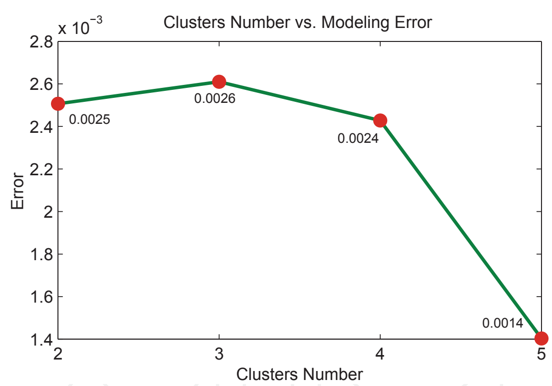


Fig. 8. Modeling error vs. clusters number.

Validation was performed on a different data set than the one used for identification. From Figure 9 one can see that the TS model follows the process output with a reasonable accuracy.

4.4 Robust fuzzy control based on gain and phase margins

Based on the PDC strategy, each control rule in the robust fuzzy controller rules base is designed from the corresponding rule of the TS fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. The robust fuzzy controller rule base is:

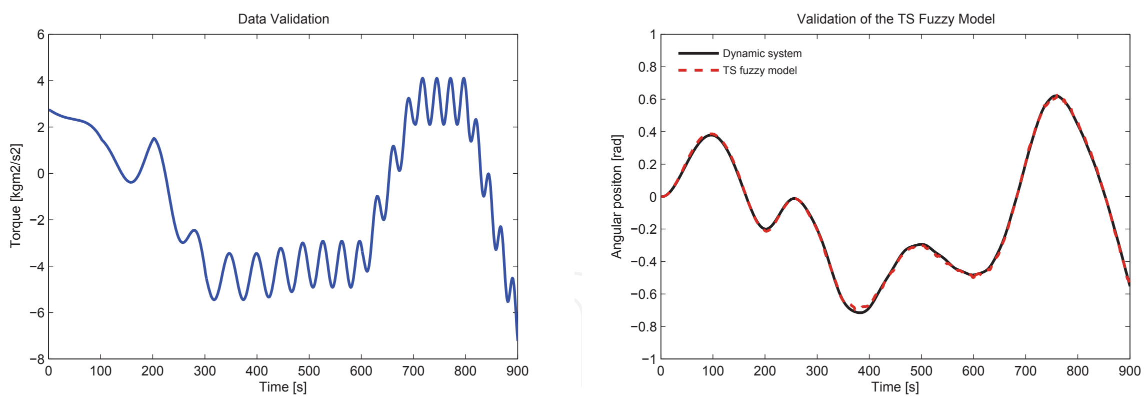


Fig. 9. Validation of the TS fuzzy model. Solid line: dynamic system, dashed line: TS fuzzy model.

$$Rule\ 1: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^1_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^1_c(s) = \frac{0.647s^2 + 3.292s + 3.979}{0.507s}$$

$$Rule\ 2: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^2_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^2_c(s) = \frac{0.643s^2 + 3.258s + 3.939}{0.500s}$$

$$Rule\ 3: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^3_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^3_c(s) = \frac{0.389s^2 + 2.199s + 3.107}{0.353s}$$

$$Rule\ 4: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^4_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^4_c(s) = \frac{0.662s^2 + 3.349s + 4.019}{0.511s}$$

$$Rule\ 5: \quad \textbf{IF} \quad \tilde{\theta}(t) \quad \textbf{is} \quad A^5_{k|\tilde{\theta}} \quad \textbf{THEN} \quad G^5_c(s) = \frac{0.624s^2 + 3.190s + 3.898}{0.495s}$$

For robust fuzzy controller design, different gain margins and phase margins are specified for the model of robotic manipulator plus dead-time in Table 1. Observed that among the gain and phase margins specifications obtained (marked by *), to $A_m = 2$ and $\phi_m = 45$, and $A_m = 3$ and $\phi_m = 60$ the phase margin is quite close to the specified ones. The largest error occurred for gain margin. The dead-time process is 0.1s and the Padé approximation order is 2.

Specified		**Resultant**	
A_m	ϕ_m	A^*_m	ϕ^*_m
6.02	45	12.62	48.86
9.54	45	12.9	20.23
13.98	45	13.8	36.58
9.54	60	13.7	60.28
13.98	60	14.1	66.30

Table 1. Gain and phase margins obtained from the especifications.

The Figure 10 shows the results obtained with the fuzzy robust controller based on gain and phase margins specifications plus the TS fuzzy model. As well as the gain and phase margins resulting.

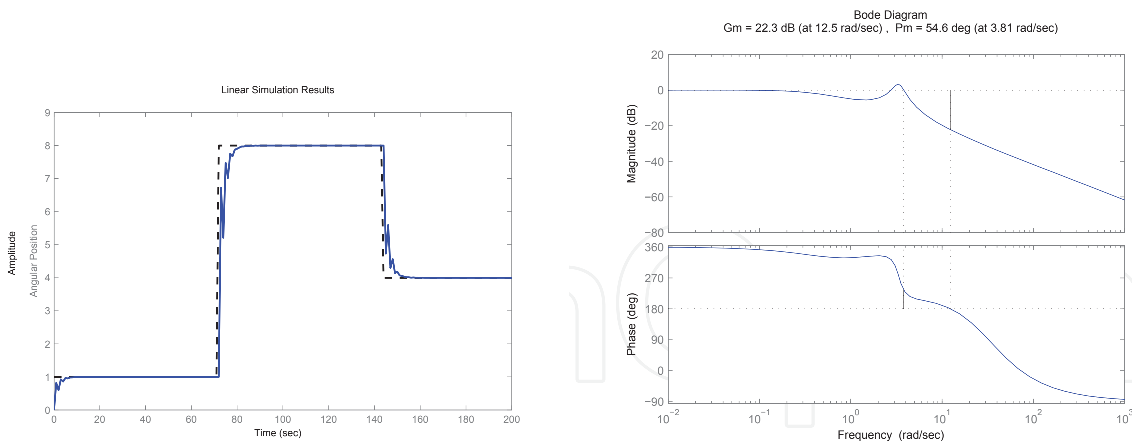


Fig. 10. Performance of the robust fuzzy controller based on the gain and phase margins specifications. The dashed line is the reference and the solid line is the robotic manipulator model with the robust fuzzy controller.

5. Conclusion

This paper presented a proposal for analysis and design of robust fuzzy control, for non-linear systems based on gain and phase margins specifications. From the proposed analysis and design, it has the following final remarks:

- The TS fuzzy model, due to the flexibility to incorporate in its structure the linear sub-models of the non-linear plant made possible, via PDC strategy, the design of robust fuzzy sub-controllers;
- The proposed Axiom and Theorems guaranteed the robust stability, since all formulation and analysis were made in the frequency domain, based on gain and phase margins specifications;
- As noted, the identification method based on fuzzy clustering is effective for modeling the robotic manipulator;
- The proposed robust fuzzy controller, based on gain and phase margins specifications, guarantees the stability of the obtained model as observed.

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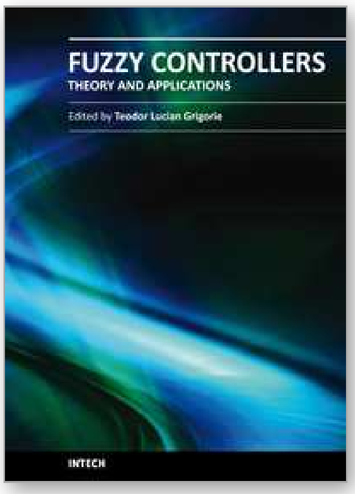

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Trying to meet the requirements in the field, present book treats different fuzzy control architectures both in terms of the theoretical design and in terms of comparative validation studies in various applications, numerically simulated or experimentally developed. Through the subject matter and through the inter and multidisciplinary content, this book is addressed mainly to the researchers, doctoral students and students interested in developing new applications of intelligent control, but also to the people who want to become familiar with the control concepts based on fuzzy techniques. Bibliographic resources used to perform the work includes books and articles of present interest in the field, published in prestigious journals and publishing houses, and websites dedicated to various applications of fuzzy control. Its structure and the presented studies include the book in the category of those who make a direct connection between theoretical developments and practical applications, thereby constituting a real support for the specialists in artificial intelligence, modelling and control fields.