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Resilient Adaptive Control of Uncertain Time-Delay Systems

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1. Introduction

Time delay systems are widely encountered in many real applications, such as chemical processes and communication networks. Hence, the problem of controlling time-delay systems has been investigated by many researchers in the past few decades. It has been found that controlling time-delay systems can be a challenging task, especially in the presence of uncertainties and parameter variations. Several techniques have been studied in the analysis and design of time delay systems with parameter uncertainties. Such techniques include robust control Mahmoud (2000; 2001), H_∞ control Fridman & Shaked (2002); Mahmoud & Zribi (1999); Yang & Wang (2001); Yang et al. (2000), and sliding mode control Choi (2001; 2003); Edwards et al. (2001); Gouaisbaut et al. (2002); Xia & Jia (2003). For time-delay systems with parametric uncertainties Nounou & Mahmoud (2006); Nounou et al. (2007), adaptive control schemes have been developed. The main contribution in Nounou & Mahmoud (2006) is the development of two delay-independent adaptive controllers. The first one is an adaptive state feedback controller when no uncertainties appear in the controller's state feedback gain. This adaptive controller stabilizes the closed-loop system in the sense of uniform ultimate boundedness. The second controller is an adaptive state feedback controller when uncertainties also appear in the controller's state feedback gain. This adaptive controller guarantees asymptotic stabilization of the closed-loop system. In Nounou et al. (2007), the authors focused on the stabilization of the class of time-delay systems with parametric uncertainties and time varying state delay when the states are not assumed to be measurable. For this class of systems, the authors developed two controllers. The first one is a robust output feedback controller when a sliding-mode observer is used to estimate the states of the system, and the second one is an adaptive output feedback controller when a sliding-mode observer is used to estimate the states of the system, such that the uncertainties also appear in the gain of the sliding-mode observer. In the case where uncertain time-delay systems include a nonlinear perturbation, several adaptive control approaches have been introduced Cheres et al. (1989); Wu (1995; 1996; 1997; 1999; 2000). In Cheres et al. (1989); Wu (1996), the authors developed state feedback controllers when the state vector is available for measurement and the upper bound on the delayed state perturbation vector is known. For the case where the upper bound of the nonlinear perturbation is known, more stabilizing controllers with stability conditions have been derived in Wu (1995; 1997). However, in many real control problems, the bounds of the uncertainties are unknown. For such a class of systems, the author in Wu (1999) has developed a continuous time state

feedback adaptive controller to guarantee uniform ultimate boundedness for systems with partially known uncertainties. For a class of systems with multiple uncertain state delays that are assumed to satisfy the matching condition, an adaptive law that guarantees uniform ultimate boundedness has been introduced in Wu (2000). In all of the papers discussed above, the authors investigated delay-independent stabilization and control of time-delay systems. Delay-dependent stabilization and H_∞ control of time-delay systems have been studied in De Souza & Li (1999); Fridman (1998); Fridman & Shaked (2003); He et al. (1998); Lee et al. (2004); Mahmoud (2000); Wang (2004). In Mahmoud (2000), the author discussed stabilization conditions and analyzed passivity of continuous and discrete time-delay systems with time-varying delay and norm-bounded parameter uncertainties. The results in Mahmoud (2000) have been extended in Nounou (2006) to consider designing delay-dependent adaptive controllers for a class of uncertain time-delay systems with time-varying delays in the presence of nonlinear perturbation. In Nounou (2006), the nonlinear perturbation is assumed to be bounded by a weighted norm of the state vector, and for this problem adaptive controllers have been developed for the two cases where the upper bound of the weight is assumed to be known and unknown.

An inherent assumption in the design of all of the above control algorithms is that the controller will be implemented perfectly. Here, the results in Nounou (2006) are extended to investigate the resilient control problem Haddad & Corrado (1997; 1998); Keel & Bhattacharyya (1997), where perturbation in controller state feedback gain is considered.

Here, It is assumed that the nonlinear perturbation is bounded by a weighted norm of the state such that the weight is a positive constant, and the norm of the uncertainty of the state feedback gain is assumed to be bounded by a positive constant. Under these assumptions, adaptive controllers are developed for all combinations when the upper bound of the nonlinear perturbation weight is known and unknown, and when the value of the upper bound of the state feedback gain perturbation is known and unknown. For all these cases, asymptotically stabilizing adaptive controllers are derived.

This chapter is organized as follows. In Section 2, the problem statement is defined. Then, in Section 3, the main stability results are presented. In Section 4, the design schemes are illustrated via a numerical example, and finally in Section 5, some concluding remarks are outlined.

Notations and Facts: In the sequel, the Euclidean norm is used for vectors. We use W^\top , W^{-1} , and $\|W\|$ to denote, respectively, the transpose of, the inverse of, and the induced norm of any square matrix W . We use $W > 0$ ($\geq, <, \leq 0$) to denote a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix W , and I to denote the $n \times n$ identity matrix. The symbol \bullet will be used in some matrix expressions to induce a symmetric structure, that is if the matrices $L = L^\top$ and $R = R^\top$ of appropriate dimensions are given, then

$$\begin{bmatrix} L & N \\ \bullet & R \end{bmatrix} = \begin{bmatrix} L & N \\ N^\top & R \end{bmatrix}.$$

Now, we introduce the following facts that will be used later on to establish the stability results.

Fact 1: Mahmoud (2000) Given matrices Σ_1 and Σ_2 with appropriate dimensions, it follows that

$$\Sigma_1 \Sigma_2 + \Sigma_2^\top \Sigma_1^\top \leq \alpha^{-1} \Sigma_1 \Sigma_1^\top + \alpha \Sigma_2^\top \Sigma_2, \quad \forall \alpha > 0.$$

Fact 2 (Schur Complement): Boukas & Liu (2002); Mahmoud (2000) Given constant matrices Ω_1 , Ω_2 , Ω_3 where $\Omega_1 = \Omega_1^\top$ and $0 < \Omega_2 = \Omega_2^\top$ then $\Omega_1 + \Omega_3^\top \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^\top \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^\top & \Omega_1 \end{bmatrix} < 0.$$

2. Problem statement

Consider the class of dynamical systems with state delay

$$\dot{x}(t) = A_o x(t) + A_d x(t - \tau) + B_o u(t) + E(x(t), t) \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $E(x(t), t) : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ is an unknown continuous vector function that represents a nonlinear perturbation, and τ is some unknown time-varying state delay factor satisfying $0 \leq \tau \leq \tau^+$, where the bound τ^+ is a known constant. The matrices A_o , A_d , and B_o are known real constant matrices of appropriate dimensions. The nonlinear perturbation function is defined to satisfy the following assumption.

Assumption 2.1. The nonlinear perturbation function $E(x(t), t)$ satisfies the following inequality

$$\|E(x(t), t)\| \leq \theta^* \|x(t)\|, \quad (2)$$

where θ^* is some positive constant.

In this chapter, resilient delay-dependent adaptive stabilization results are established for the system (1) when uncertainties appear in the state feedback gain of the following control law:

$$u(t) = (K + \Delta K)x(t) + \mu(t)\mathcal{I}x(t), \quad (3)$$

where $\mathcal{I} \in \mathbb{R}^{m \times n}$ is a matrix whose elements are all ones, $\mu(t) \in \mathbb{R}$ is adapted such that closed-loop asymptotic stabilization is guaranteed, $K \in \mathbb{R}^{m \times n}$ is a state feedback gain, and $\Delta K(t) \in \mathbb{R}^{m \times n}$ is the time varying uncertainty of the state feedback gain that satisfies the following assumption.

Assumption 2.2. The uncertainty of the state feedback gain satisfies the following inequality

$$\|\Delta K(t)\| \leq \rho^*, \quad (4)$$

where ρ^* is some positive constant.

Before we proceed, we start by expressing the delayed state as Mahmoud (2000)

$$\begin{aligned} x(t - \tau) &= x(t) - \int_{-\tau}^0 \dot{x}(t + s) ds \\ &= x(t) - \int_{-\tau}^0 [A_o x(t + s) + A_d x(t - \tau + s) + B_o u(t + s) - E(x(t + s), t + s)] ds \end{aligned} \quad (5)$$

Hence, if we define $A_{od} = A_o + A_d$, then the system (1) can be expressed as

$$\begin{aligned} \dot{x}(t) &= A_{od} x(t) + A_d \eta(t) + B_o u(t) + E(x(t), t), \\ \eta(t) &= - \int_{-\tau}^0 [A_o x(t + s) + A_d x(t - \tau + s) + B_o u(t + s) + E(x(t + s), t + s)] ds. \end{aligned} \quad (6)$$

Here, resilient delay-dependent stabilization results are established for the system (6) considering the following cases:

1. The nonlinear perturbation function satisfies Assumption 2.1 such that θ^* is assumed to be a *known* positive constant, and the uncertainty of the state feedback gain satisfies Assumption 2.2 such that ρ^* is assumed to be a *known* positive constant.
2. The nonlinear perturbation function satisfies Assumption 2.1 such that θ^* is assumed to be a *known* positive constant, and the uncertainty of the state feedback gain satisfies Assumption 2.2 such that ρ^* is assumed to be an *unknown* positive constant.
3. The nonlinear perturbation function satisfies Assumption 2.1 such that θ^* is assumed to be an *unknown* positive constant, and the uncertainty of the state feedback gain satisfies Assumption 2.2 such that ρ^* is assumed to be a *known* positive constant.
4. The nonlinear perturbation function satisfies Assumption 2.1 such that θ^* is assumed to be an *unknown* positive constant, and the uncertainty of the state feedback gain satisfies Assumption 2.2 such that ρ^* is assumed to be an *unknown* positive constant.

3. Main results

In the sequel, the main design results will be presented.

3.1 Adaptive control when both θ^* and ρ^* are known

Here, we wish to stabilize the system (6) considering the control law (3) when both θ^* and ρ^* are known. Let us define $z(t) = \mu(t)x(t)$, and let the Lyapunov-Krasovskii functional for the transformed system (6) be selected as:

$$V_a(x) \triangleq V_1(x) + V_2(x) + V_3(x) + V_4(x) + V_5(x) + V_6(x) + V_7(x) + V_8(x), \quad (7)$$

where

$$V_1(x) = x^\top(t)Px(t), \quad (8)$$

$$V_2(x) = r_1 \int_{-\tau}^0 \int_{t+s}^t x^\top(\alpha) A_o^\top A_o x(\alpha) d\alpha ds, \quad (9)$$

$$V_3(x) = r_2 \int_{-\tau}^0 \int_{t+s-\tau}^t x^\top(\alpha) A_d^\top A_d x(\alpha) d\alpha ds, \quad (10)$$

$$V_4(x) = r_3 \int_{-\tau}^0 \int_{t+s}^t x^\top(\alpha) K^\top B_o^\top B_o K x(\alpha) d\alpha ds, \quad (11)$$

$$V_5(x) = r_4 \int_{-\tau}^0 \int_{t+s}^t x^\top(\alpha) \Delta K^\top(t) B_o^\top B_o \Delta K(t) x(\alpha) d\alpha ds, \quad (12)$$

$$V_6(x) = r_5 \int_{-\tau}^0 \int_{t+s}^t z^\top(\alpha) \mathcal{I}^\top B_o^\top B_o \mathcal{I} z(\alpha) d\alpha ds, \quad (13)$$

$$V_7(x) = r_6 \int_{-\tau}^0 \int_{t+s}^t E^\top(x, \alpha) E(x, \alpha) d\alpha ds, \quad (14)$$

$$V_8(x) = \mu^2(t), \quad (15)$$

where $r_1 > 0, r_2 > 0, r_3 > 0, r_4 > 0, r_5 > 0$ and $r_6 > 0$ are positive scalars, and $P = P^\top \in \mathbb{R}^{n \times n} > 0$. It can be shown that the time derivative of the Lyapunov-Krasovskii functional is

$$\dot{V}_a(x) = \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x) + \dot{V}_4(x) + \dot{V}_5(x) + \dot{V}_6(x) + \dot{V}_7(x) + \dot{V}_8(x), \quad (16)$$

where

$$\dot{V}_1(x) = x^\top(t)P\dot{x}(t) + \dot{x}^\top(t)Px(t), \quad (17)$$

$$\dot{V}_2(x) = \tau r_1 x^\top(t)A_o^\top A_o x(t) - r_1 \int_{-\tau}^0 x^\top(t+s)A_o^\top A_o x(t+s)ds, \quad (18)$$

$$\dot{V}_3(x) = \tau r_2 x^\top(t)A_d^\top A_d x(t) - r_2 \int_{-\tau}^0 x^\top(t+s-\tau)A_d^\top A_d x(t+s-\tau)ds, \quad (19)$$

$$\dot{V}_4(x) = \tau r_3 x^\top(t)K^\top B_o^\top B_o Kx(t) - r_3 \int_{-\tau}^0 x^\top(t+s)K^\top B_o^\top B_o Kx(t+s)ds, \quad (20)$$

$$\begin{aligned} \dot{V}_5(x) = & \tau r_4 x^\top(t)\Delta K(t)^\top B_o^\top B_o \Delta K(t)x(t) \\ & - r_4 \int_{-\tau}^0 x^\top(t+s)\Delta K^\top(t+s)B_o^\top B_o \Delta K(t+s)x(t+s)ds, \end{aligned} \quad (21)$$

$$\dot{V}_6(x) = \tau r_5 z^\top(t)\mathcal{I}^\top B_o^\top B_o \mathcal{I}z(t) - r_5 \int_{-\tau}^0 z^\top(t+s)\mathcal{I}^\top B_o^\top B_o \mathcal{I}z(t+s)ds, \quad (22)$$

$$\dot{V}_7(x) = \tau r_6 E^\top(x,t)E(x,t) - r_6 \int_{-\tau}^0 E^\top(x,t+s)E(x,t+s)ds, \quad (23)$$

$$\dot{V}_8(x) = 2\mu(t)\dot{\mu}(t). \quad (24)$$

The next Theorem provides the main results for this case.

Theorem 1: Consider system (6). If there exist matrices $0 < \mathcal{X} = \mathcal{X}^\top \in \mathbb{R}^{n \times n}$, $\mathcal{Y} \in \mathbb{R}^{m \times n}$, $\mathcal{Z} \in \mathbb{R}^{n \times n}$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > \varepsilon$, $\varepsilon_5 > \varepsilon$ and $\varepsilon_6 > \varepsilon$ (where ε is an arbitrary small positive constant) such that the following LMI

$$\begin{bmatrix} A_{od}\mathcal{X} + \mathcal{X}A_{od} + B_o\mathcal{Y} + \mathcal{Y}^\top B_o^\top & \tau^+ \mathcal{X}A_o^\top & \tau^+ \mathcal{X}A_d^\top & \tau^+ \mathcal{Z} \\ +\tau^+ (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6) A_d A_d^\top & -\tau^+ \varepsilon_1 I & 0 & 0 \\ \bullet & \bullet & -\tau^+ \varepsilon_2 I & 0 \\ \bullet & \bullet & \bullet & -\tau^+ \varepsilon_3 I \end{bmatrix} < 0, \quad (25)$$

has a feasible solution, and $K = \mathcal{Y}\mathcal{X}^{-1}$, and $\mu(t)$ is adapted subject to the adaptive law

$$\dot{\mu}(t) = \text{Proj} \left\{ \alpha_1 \text{sgn}(\mu(t)) \|x(t)\|^2 + \alpha_2 \mu(t) \|x(t)\|^2, \mu(t) \right\}, \quad (26)$$

where $\text{Proj}\{\cdot\}$ Krstic et al. (1995) is applied to ensure that $|\mu(t)| \geq 1$ as follows

$$\mu(t) = \begin{cases} \mu(t) & \text{if } |\mu(t)| \geq 1 \\ 1 & \text{if } 0 \leq \mu(t) < 1 \\ -1 & \text{if } -1 < \mu(t) < 0, \end{cases}$$

and the adaptive law parameters are selected such that

$$\alpha_1 < -\frac{1}{2} \left[\tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| + \tau^+ r_6 (\theta^*)^2 + 2\rho^* \|PB_o\| + 2\|PB_o\| + 2\theta^* \|P\| \right], \quad (27)$$

and

$$\alpha_2 < -\frac{1}{2} \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\|, \quad (28)$$

then the control law (3) will guarantee asymptotic stabilization of the closed-loop system.

Proof As shown in (16), the time derivative of $V_a(x)$ is

$$\begin{aligned}\dot{V}_a(x) &= \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x) + \dot{V}_4(x) + \dot{V}_5(x) + \dot{V}_6(x) + \dot{V}_7(x) + \dot{V}_8(x), \\ &= x^\top(t)P\dot{x}(t) + \dot{x}^\top(t)Px(t) + \dot{V}_2(x) + \dot{V}_3(x) + \dot{V}_4(x) + \dot{V}_5(x) + \dot{V}_6(x) \\ &\quad + \dot{V}_7(x) + \dot{V}_8(x).\end{aligned}\quad (29)$$

Using the system equation defined in (6) and the control law (3), we have

$$\begin{aligned}\dot{V}_a(x) &= x^\top(t) \left[PA_{od} + A_{od}^\top P + PB_oK + K^\top B_o^\top P \right] x(t) \\ &\quad - 2x^\top(t)PA_d \int_{-\tau}^0 A_o x(t+s)ds - 2x^\top(t)PA_d \int_{-\tau}^0 A_d x(t-\tau+s)ds \\ &\quad - 2x^\top(t)PA_d \int_{-\tau}^0 B_o Kx(t+s)ds - 2x^\top(t)PA_d \int_{-\tau}^0 B_o \Delta K(t+s)x(t+s)ds \\ &\quad - 2x^\top(t)PA_d \int_{-\tau}^0 \mu(t+s)B_o \mathcal{I}x(t+s)ds - 2x^\top(t)PA_d \int_{-\tau}^0 E(x, t+s)ds \\ &\quad + 2x^\top(t)PB_o \Delta K(t)x(t) + 2\mu(t)x^\top(t)PB_o \mathcal{I}x(t) + 2x^\top(t)PE(x, t) \\ &\quad + \dot{V}_2(x) + \dot{V}_3(x) + \dot{V}_4(x) + \dot{V}_5(x) + \dot{V}_6(x) + \dot{V}_7(x) + \dot{V}_8(x).\end{aligned}\quad (30)$$

By applying Fact 1, we have

$$\begin{aligned}-2x^\top(t)PA_d \int_{-\tau}^0 A_o x(t+s)ds &\leq r_1^{-1} \int_{-\tau}^0 x^\top(s)PA_d A_d^\top Px(s)ds \\ &\quad + r_1 \int_{-\tau}^0 x^\top(t+s)A_o^\top A_o x(t+s)ds \\ &\leq \tau^+ r_1^{-1} x^\top(t)PA_d A_d^\top Px(t) \\ &\quad + r_1 \int_{-\tau}^0 x^\top(t+s)A_o^\top A_o x(t+s)ds,\end{aligned}\quad (31)$$

where r_1 is a positive scalar. Similarly, if r_2, r_3 and r_4 are positive scalars, we have

$$\begin{aligned}-2x^\top(t)PA_d \int_{-\tau}^0 A_d x(t-\tau+s)ds &\leq \tau^+ r_2^{-1} x^\top(t)PA_d A_d^\top Px(t) \\ &\quad + r_2 \int_{-\tau}^0 x^\top(t-\tau+s)A_d^\top A_d x(t-\tau+s)ds,\end{aligned}\quad (32)$$

$$\begin{aligned}-2x^\top(t)PA_d \int_{-\tau}^0 B_o Kx(t+s)ds &\leq \tau^+ r_3^{-1} x^\top(t)PA_d A_d^\top Px(t) \\ &\quad + r_3 \int_{-\tau}^0 x^\top(t+s)K^\top B_o^\top B_o Kx(t+s)ds,\end{aligned}\quad (33)$$

and

$$\begin{aligned}-2x^\top(t)PA_d \int_{-\tau}^0 B_o \Delta K(t+s)x(t+s)ds &\leq \tau^+ r_4^{-1} x^\top(t)PA_d A_d^\top Px(t) \\ &\quad + r_4 \int_{-\tau}^0 x^\top(t+s)\Delta K^\top(t+s)B_o^\top B_o \Delta K(t+s)x(t+s)ds.\end{aligned}\quad (34)$$

Now, let r_5 be a positive scalar, then using Fact 1 we have

$$\begin{aligned} -2x^\top(t)PA_d \int_{-\tau}^0 \mu(t+s)B_o \mathcal{I}x(t+s)ds &= -2x^\top(t)PA_d \int_{-\tau}^0 B_o \mathcal{I}z(t+s)ds \\ &\leq \tau^+ r_5^{-1} x^\top(t)PA_d A_d^\top Px(t) + r_5 \int_{-\tau}^0 z^\top(t+s)\mathcal{I}^\top B_o^\top B_o \mathcal{I}z(t+s)ds. \end{aligned} \quad (35)$$

Also, if r_6 is a positive scalar, then using Fact 1 we have

$$\begin{aligned} -2x^\top(t)PA_d \int_{-\tau}^0 E(x, t+s)ds &\leq \tau^+ r_6^{-1} x^\top(t)PA_d A_d^\top Px(t) \\ &+ r_6 \int_{-\tau}^0 E^\top(x, t+s)E(x, t+s)ds. \end{aligned} \quad (36)$$

It is known that

$$2\mu(t)x^\top(t)PB_o \mathcal{I}x(t) \leq 2\|PB_o \mathcal{I}\| \|\mu(t)\| \|x(t)\|^2. \quad (37)$$

Also, using Assumption 2.1, it can be shown that

$$2x^\top(t)PE(x, t) \leq 2\|P\| \theta^* \|x(t)\|^2. \quad (38)$$

Using equations (31)- (38) and equations (17)- (24) (with the fact that $0 \leq \tau \leq \tau^+$) in (30), we have

$$\begin{aligned} \dot{V}_a(x) &\leq x^\top(t)\Xi x(t) + \tau^+ r_4 x^\top(t)\Delta K^\top(t)B_o^\top B_o \Delta K(t)x(t) \\ &+ \tau^+ r_5 z^\top(t)\mathcal{I}^\top B_o^\top B_o \mathcal{I}z(t) + \tau^+ r_6 E^\top(x, t)E(x, t) + 2\rho^* \|PB_o\| \|x(t)\|^2 \\ &+ 2\|PB_o \mathcal{I}\| \|\mu(t)\| \|x(t)\|^2 + 2\theta^* \|P\| \|x(t)\|^2 + 2\mu(t) \dot{\mu}(t). \end{aligned} \quad (39)$$

where

$$\begin{aligned} \Xi &= PA_{od} + A_{od}^\top P + PB_o K + K^\top B_o^\top P + \tau^+ r_1 A_o^\top A_o + \tau^+ r_2 A_d^\top A_d + \tau^+ r_3 B_o K K^\top B_o^\top \\ &+ \tau^+ \left(r_1^{-1} + r_2^{-1} + r_3^{-1} + r_4^{-1} + r_5^{-1} + r_6^{-1} \right) PA_d A_d^\top P. \end{aligned} \quad (40)$$

To guarantee that $x^\top(t)\Xi x(t) < 0$, it sufficient to show that $\Xi < 0$. Let us introduce the linearizing terms, $\mathcal{X} = P^{-1}$, $\mathcal{Y} = K\mathcal{X}$, and $\mathcal{Z} = \mathcal{X}B_o K$. Also, let $\varepsilon_1 = r_1^{-1}$, $\varepsilon_2 = r_2^{-1}$, $\varepsilon_3 = r_3^{-1}$, $\varepsilon_4 = r_4^{-1}$, $\varepsilon_5 = r_5^{-1}$ and $\varepsilon_6 = r_6^{-1}$. Now, by pre-multiplying and post-multiplying Ξ by \mathcal{X} and invoking the Schur complement, we arrive at the LMI (25) which guarantees that $\Xi < 0$, and consequently $x^\top(t)\Xi x(t) < 0$. Now, we need to show that the remaining terms of (39) are negative definite. Using the definition of $z(t) = \mu(t)x(t)$, we know that

$$\tau^+ r_5 z^\top(t)\mathcal{I}^\top B_o^\top B_o \mathcal{I}z(t) \leq \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2. \quad (41)$$

Also, using Assumptions 2.1 and 2.2, we have

$$\tau^+ r_6 E^\top(x, t)E(x, t) \leq \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2, \quad (42)$$

and

$$\tau^+ r_4 x^\top(t)\Delta K^\top(t)B_o^\top B_o \Delta K(t)x(t) \leq \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2. \quad (43)$$

Now, using (41)- (43), the adaptive law (26), and the fact that $|\mu(t)| \geq 1$, equation (39) becomes

$$\begin{aligned} \dot{V}_a(x) \leq & x^\top(t) \Xi x(t) + \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2 + \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2 \\ & + \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2 + 2\rho^* \|PB_o\| \|x(t)\|^2 + 2\|PB_o \mathcal{I}\| |\mu(t)| \|x(t)\|^2 \\ & + 2\theta^* \|P\| \|x(t)\|^2 + 2\alpha_1 |\mu(t)| \|x(t)\|^2 + 2\alpha_2 \mu^2(t) \|x(t)\|^2. \end{aligned} \quad (44)$$

It can be easily shown that by selecting α_1 and α_2 as in (27) and (28), we guarantee that

$$\dot{V}_a(x) \leq x^\top(t) \Xi x(t), \quad (45)$$

where $\Xi < 0$. Hence, $\dot{V}_a(x) < 0$ which guarantees asymptotic stabilization of the closed-loop system. ■

3.2 Adaptive control when θ^* is known and ρ^* is unknown

Here, we wish to stabilize the system (6) considering the control law (3) when θ^* is known and ρ^* is unknown. Before we present the stability results for this case, let us define $\tilde{\rho}(t) = \hat{\rho}(t) - \rho^*$, where $\hat{\rho}(t)$ is the estimate of ρ^* , and $\tilde{\rho}(t)$ is error between the estimate and the true value of ρ^* . Let the Lyapunov-Krasovskii functional for the transformed system (6) be selected as:

$$V_b(x) \triangleq V_a(x) + V_9(x), \quad (46)$$

where $V_a(x)$ is defined in equations (7), and $V_9(x)$ is defined as

$$V_9(x) = (1 + \rho^*) [\tilde{\rho}(t)]^2, \quad (47)$$

where its time derivative is

$$\dot{V}_9(x) = 2 (1 + \rho^*) \tilde{\rho}(t) \dot{\tilde{\rho}}(t). \quad (48)$$

Since $\tilde{\rho}(t) = \hat{\rho}(t) - \rho^*$, then $\dot{\tilde{\rho}}(t) = \dot{\hat{\rho}}(t)$. Hence, equation (48) becomes

$$\dot{V}_9(x) = 2 (1 + \rho^*) [\hat{\rho}(t) - \rho^*] \dot{\hat{\rho}}(t). \quad (49)$$

The next Theorem provides the main results for this case.

Theorem 2: Consider system (6). If there exist matrices $0 < \mathcal{X} = \mathcal{X}^\top \in \mathbb{R}^{n \times n}$, $\mathcal{Y} \in \mathbb{R}^{m \times n}$, $\mathcal{Z} \in \mathbb{R}^{n \times n}$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > \varepsilon$, $\varepsilon_5 > \varepsilon$ and $\varepsilon_6 > \varepsilon$ (where ε is an arbitrary small positive constant) such that the LMI (25) has a feasible solution, and $K = \mathcal{Y}\mathcal{X}^{-1}$, and $\mu(t)$ and $\hat{\rho}(t)$ are adapted subject to the adaptive laws

$$\dot{\mu}(t) = \text{Proj} \left\{ [\beta_1 \text{sgn}(\mu(t)) + \beta_2 \mu(t) + \beta_3 \text{sgn}(\mu(t)) \hat{\rho}(t)] \|x(t)\|^2, \mu(t) \right\} \quad (50)$$

$$\dot{\hat{\rho}}(t) = \gamma \|x(t)\|^2, \quad (51)$$

where $\text{Proj}\{\cdot\}$ Krstic et al. (1995) is applied to ensure that $|\mu(t)| \geq 1$ as follows:

$$\mu(t) = \begin{cases} \mu(t) & \text{if } |\mu(t)| \geq 1 \\ 1 & \text{if } 0 \leq \mu(t) < 1 \\ -1 & \text{if } -1 < \mu(t) < 0, \end{cases}$$

and the adaptive law parameters are selected such that $\beta_1 < -\frac{1}{2} \left[\tau^+ r_6 (\theta^*)^2 + 2 \|PB_o \mathcal{I}\| + 2\theta^* \|P\| \right]$, $\beta_2 < -\frac{1}{2} \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\|$, $\gamma > \frac{1}{2} \tau^+ r_4 \|B_o^\top B_o\|$,

$\beta_3 < -\gamma$, and $\hat{\rho}(0) > 1$, then the control law (3) will guarantee asymptotic stabilization of the closed-loop system.

Proof The time derivative of $V_b(x)$ is

$$\dot{V}_b(x) = \dot{V}_a(x) + \dot{V}_9(x). \quad (52)$$

Following the steps used in the proof of Theorem 1 and using equation (49), it can be shown that

$$\begin{aligned} \dot{V}_b(x) \leq & x^\top(t) \Xi x(t) + \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2 + \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2 \\ & + \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2 + 2\rho^* \|PB_o\| \|x(t)\|^2 + 2\|PB_o \mathcal{I}\| |\mu(t)| \|x(t)\|^2 \\ & + 2\theta^* \|P\| \|x(t)\|^2 + 2\mu(t) \dot{\mu}(t) + 2(1 + \rho^*) [\hat{\rho}(t) - \rho^*] \dot{\hat{\rho}}(t), \end{aligned} \quad (53)$$

where Ξ is defined in equation (40). Using the linearization procedure and invoking the Schur complement (as in the proof of Theorem 1), it can be shown that Ξ is guaranteed to be negative definite whenever the LMI (25) has a feasible solution. Using the adaptive laws (50)- (51) in (53) and the fact that $|\mu(t)| \geq 1$, we get

$$\begin{aligned} \dot{V}_b(x) \leq & x^\top(t) \Xi x(t) + \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2 + \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2 \\ & + \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2 + 2\rho^* \|PB_o\| \|x(t)\|^2 \\ & + 2\|PB_o \mathcal{I}\| |\mu(t)| \|x(t)\|^2 + 2\theta^* \|P\| \|x(t)\|^2 \\ & + 2\beta_1 |\mu(t)| \|x(t)\|^2 + 2\beta_2 \mu^2(t) \|x(t)\|^2 + 2\beta_3 \hat{\rho}(t) |\mu(t)| \|x(t)\|^2 + 2\gamma \hat{\rho}(t) \|x(t)\|^2 \\ & - 2\gamma \rho^* \|x(t)\|^2 - 2\gamma \rho^* \hat{\rho}(t) \|x(t)\|^2 - 2\gamma (\rho^*)^2 \|x(t)\|^2. \end{aligned} \quad (54)$$

Using the fact that $|\mu(t)| > 1$ and arranging terms of equation (54), it can be shown that $\dot{V}_b(x) < 0$ if we select $\beta_1 < -\frac{1}{2} \left[\tau^+ r_6 (\theta^*)^2 + 2\|PB_o \mathcal{I}\| + 2\theta^* \|P\| \right]$, $\beta_2 < -\frac{1}{2} \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\|$, and $\beta_3 < -\gamma$, where γ needs to be selected to satisfy the following two conditions:

$$\gamma > \frac{1}{2} \tau^+ r_4 \|B_o^\top B_o\|, \quad (55)$$

and

$$2\|PB_o\| - 2\gamma + 2\gamma \hat{\rho}(t) < 0. \quad (56)$$

Hence, we need to select γ such that

$$\gamma > \max \left\{ \frac{1}{2} \tau^+ r_4 \|B_o^\top B_o\|, \frac{\|PB_o\|}{1 - \hat{\rho}(t)} \right\}. \quad (57)$$

It is clear that when $\hat{\rho}(t) > 1$, we only need to ensure that $\gamma > \frac{1}{2} \tau^+ r_4 \|B_o^\top B_o\|$. Note that from equation (51), $\hat{\rho}(t) > 1$ can be easily ensured by selecting $\hat{\rho}(0) > 1$ and $\gamma > \frac{1}{2} \tau^+ r_4 \|B_o^\top B_o\|$ to guarantee that $\hat{\rho}(t)$ in equation (51) is monotonically increasing. Hence, we guarantee that

$$\dot{V}_b(x) \leq x^\top(t) \Xi x(t), \quad (58)$$

where $\Xi < 0$. Hence, $\dot{V}_b(x) < 0$ which guarantees asymptotic stabilization of the closed-loop system. ■

3.3 Adaptive control when θ^* is unknown and ρ^* is known

Here, we wish to stabilize the system (6) considering the control law (3) when θ^* is unknown and ρ^* is known. Since θ^* is unknown, let us define $\tilde{\theta}(t) = \hat{\theta}(t) - \theta^*$, where $\hat{\theta}(t)$ is the estimate of θ^* , and $\tilde{\theta}(t)$ is error between the estimate and the true value of θ^* . Also, let the Lyapunov-Krasovskii functional for the transformed system (6) be selected as:

$$V_c(x) \stackrel{\Delta}{=} V_a(x) + V_{10}(x), \quad (59)$$

where

$$V_{10}(x) = (1 + \theta^*) [\tilde{\theta}(t)]^2, \quad (60)$$

where its time derivative is

$$\begin{aligned} \dot{V}_{10}(x) &= 2(1 + \theta^*) \tilde{\theta}(t) \dot{\tilde{\theta}}(t), \\ &= 2(1 + \theta^*) [\hat{\theta}(t) - \theta^*] \dot{\hat{\theta}}(t). \end{aligned} \quad (61)$$

The next Theorem provides the main results for this case.

Theorem 3: Consider system (6). If there exist matrices $0 < \mathcal{X} = \mathcal{X}^\top \in \mathbb{R}^{n \times n}$, $\mathcal{Y} \in \mathbb{R}^{m \times n}$, $\mathcal{Z} \in \mathbb{R}^{n \times n}$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > \varepsilon$, $\varepsilon_5 > \varepsilon$ and $\varepsilon_6 > \varepsilon$ (where ε is an arbitrary small positive constant) such that the LMI (25) has a feasible solution, and $K = \mathcal{Y}\mathcal{X}^{-1}$, and $\mu(t)$ is adapted subject to the adaptive laws

$$\begin{aligned} \dot{\mu}(t) &= \text{Proj} \left\{ \delta_1 \text{sgn}(\mu(t)) \|x(t)\|^2 + \delta_2 \mu(t) \|x(t)\|^2 + \delta_3 \text{sgn}(\mu(t)) \hat{\theta}(t) \|x(t)\|^2, \mu(t) \right\} \\ \dot{\hat{\theta}}(t) &= \kappa \|x(t)\|^2, \end{aligned} \quad (62)$$

where $\text{Proj}\{\cdot\}$ Krstic et al. (1995) is applied to ensure that $|\mu(t)| \geq 1$ as follows

$$\mu(t) = \begin{cases} \mu(t) & \text{if } |\mu(t)| \geq 1 \\ 1 & \text{if } 0 \leq \mu(t) < 1 \\ -1 & \text{if } -1 < \mu(t) < 0, \end{cases}$$

and the adaptive law parameters are selected such that $\delta_1 < -\left[\|PB_o\mathcal{I}\| + \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| + \rho^* \|PB_o\|\right]$, $\delta_2 < -\frac{1}{2}\tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\|$, $\delta_3 < -\kappa$, $\kappa > \frac{1}{2}\tau^+ r_6$ and $\hat{\theta}(0) > 1$, then the control law (3) will guarantee asymptotic stabilization of the closed-loop system.

Proof The time derivative of $V_c(x)$ is

$$\dot{V}_c(x) = \dot{V}_a(x) + \dot{V}_{10}(x). \quad (64)$$

Following the steps used in the proof of Theorem 1 and using equation (61), it can be shown that

$$\begin{aligned} \dot{V}_c(x) &\leq x^\top(t) \Xi x(t) + \tau^+ r_4 x^\top(t) \Delta K^\top(t) B_o^\top B_o \Delta K(t) x(t) + \tau^+ r_5 z^\top(t) \mathcal{I}^\top B_o^\top B_o \mathcal{I} z(t) \\ &\quad + \tau^+ r_6 E^\top(x, t) E(x, t) + 2\rho^* \|PB_o\| \|x(t)\|^2 + 2\|PB_o\mathcal{I}\| |\mu(t)| \|x(t)\|^2 \\ &\quad + 2\theta^* \|P\| \|x(t)\|^2 + 2\mu(t) \dot{\mu}(t) + 2(1 + \theta^*) [\hat{\theta}(t) - \theta^*] \dot{\hat{\theta}}(t), \end{aligned} \quad (65)$$

where Ξ is defined in equation (40). Using the linearization procedure and invoking the Schur complement (as in the proof of Theorem 1), it can be shown that Ξ is guaranteed to be negative definite whenever the LMI (25) has a feasible solution. Now, we need to show that the remaining terms of (65) are negative definite. Using the definition of $z(t) = \mu(t)x(t)$, we know that

$$\tau^+ r_5 z^\top(t) \mathcal{I}^\top B_o^\top B_o \mathcal{I} z(t) \leq \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2. \quad (66)$$

Also, using Assumptions 2.1 and 2.2, we have

$$\tau^+ r_6 E^\top(x, t) E(x, t) \leq \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2, \quad (67)$$

and

$$\tau^+ r_4 x^\top(t) \Delta K^\top(t) B_o^\top B_o \Delta K(t) x(t) \leq \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2. \quad (68)$$

Now, using (66)- (68), the adaptive laws (62)- (63), and the fact that $|\mu(t)| \geq 1$, equation (65) becomes

$$\begin{aligned} \dot{V}_c(x) \leq & x^\top(t) \Xi x(t) + \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2 + \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2 \\ & + \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2 + 2\rho^* \|PB_o\| \|x(t)\|^2 + 2\|PB_o \mathcal{I}\| |\mu(t)| \|x(t)\|^2 \\ & 6 + 2\theta^* \|P\| \|x(t)\|^2 + 2\delta_1 |\mu(t)| \|x(t)\|^2 + 2\delta_2 \mu^2(t) \|x(t)\|^2 \\ & + 2\delta_3 |\mu(t)| \hat{\theta}(t) \|x(t)\|^2 + 2\kappa |\mu(t)| \hat{\theta}(t) \|x(t)\|^2 - 2\kappa \theta^* \|x(t)\|^2 \\ & + 2\kappa \theta^* \hat{\theta}(t) \|x(t)\|^2 - 2\kappa (\theta^*)^2 \|x(t)\|^2. \end{aligned} \quad (69)$$

It can be shown that $\dot{V}_c(x) < 0$ if the adaptive law parameters δ_1 , δ_2 , and δ_3 are selected as stated in Theorem 3, and κ is selected to satisfy the following two conditions: $\kappa > \frac{1}{2} \tau^+ r_6$ and $\|P\| - \kappa + \kappa \hat{\theta}(t) < 0$. Hence, we need to select κ such that

$$\kappa > \max \left\{ \frac{1}{2} \tau^+ r_6, \frac{\|P\|}{1 - \hat{\theta}(t)} \right\}. \quad (70)$$

It is clear that when $\hat{\theta}(t) > 1$, we only need to ensure that $\kappa > \frac{1}{2} \tau^+ r_6$. Note that from equation (63), $\hat{\theta}(t) > 1$ can be easily ensured by selecting $\hat{\theta}(0) > 1$ and $\kappa > \frac{1}{2} \tau^+ r_6$ to guarantee that $\hat{\theta}(t)$ in equation (63) is monotonically increasing. Hence, we guarantee that

$$\dot{V}_c(x) \leq x^\top(t) \Xi x(t), \quad (71)$$

where $\Xi < 0$. Hence, $\dot{V}_c(x) < 0$ which guarantees asymptotic stabilization of the closed-loop system. ■

3.4 Adaptive control when both θ^* and ρ^* are unknown

Here, we wish to stabilize the system (6) considering the control law (3) when both θ^* and ρ^* are unknown. Here, the following Lyapunov-Krasovskii functional is used

$$V_d(x) = V_c(x) + V_{11}(x), \quad (72)$$

where $V_c(x)$ is defined in equations (59), and $V_{11}(x)$ is defined as

$$V_{11}(x) = (1 + \rho^*) [\tilde{\rho}(t)]^2, \quad (73)$$

where its time derivative is

$$\dot{V}_{11}(x) = 2 (1 + \rho^*) \tilde{\rho}(t) \dot{\tilde{\rho}}(t). \quad (74)$$

Since $\tilde{\rho}(t) = \hat{\rho}(t) - \rho^*$, then $\dot{\tilde{\rho}}(t) = \dot{\hat{\rho}}(t)$. Hence, equation (74) becomes

$$\dot{V}_{11}(x) = 2 (1 + \rho^*) [\hat{\rho}(t) - \rho^*] \dot{\hat{\rho}}(t). \quad (75)$$

The next Theorem provides the main results for this case.

Theorem 4: Consider system (6). If there exist matrices $0 < \mathcal{X} = \mathcal{X}^\top \in \mathbb{R}^{n \times n}$, $\mathcal{Y} \in \mathbb{R}^{m \times n}$, $\mathcal{Z} \in \mathbb{R}^{n \times n}$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > \varepsilon$, $\varepsilon_5 > \varepsilon$ and $\varepsilon_6 > \varepsilon$ (where ε is an arbitrary small positive constant) such that the LMI (25) has a feasible solution, and $K = \mathcal{Y}\mathcal{X}^{-1}$, and $\mu(t)$ is adapted subject to the adaptive laws

$$\begin{aligned} \dot{\mu}(t) = & \text{Proj} \left\{ \lambda_1 \text{sgn}(\mu(t)) \|x(t)\|^2 + \lambda_2 \mu(t) \|x(t)\|^2 \right. \\ & \left. + \lambda_3 \text{sgn}(\mu(t)) \hat{\theta}(t) \|x(t)\|^2 + \lambda_4 \text{sgn}(\mu(t)) \hat{\rho}(t) \|x(t)\|^2, \mu(t) \right\}, \end{aligned} \quad (76)$$

$$\dot{\hat{\theta}}(t) = \sigma \|x(t)\|^2, \quad (77)$$

$$\dot{\hat{\rho}}(t) = \varsigma \|x(t)\|^2, \quad (78)$$

where $\text{Proj}\{\cdot\}$ Krstic et al. (1995) is applied to ensure that $|\mu(t)| \geq 1$ as follows

$$\mu(t) = \begin{cases} \mu(t) & \text{if } |\mu(t)| \geq 1 \\ 1 & \text{if } 0 \leq \mu(t) < 1 \\ -1 & \text{if } -1 < \mu(t) < 0, \end{cases}$$

and the adaptive law parameters are selected such that $\lambda_1 < -\|PB_o\mathcal{I}\|$, $\lambda_2 < -\frac{1}{2}\tau^+r_5\|\mathcal{I}^\top B_o^\top B_o\mathcal{I}\|$, $\lambda_3 < -\sigma$, $\lambda_4 < -\varsigma$, $\sigma > \frac{1}{2}\tau^+r_6$, $\varsigma > \frac{1}{2}\tau^+r_4\|B_o^\top B_o\|$, $\hat{\theta}(0) > 1$ and $\hat{\rho}(0) > 1$, then the control law (3) will guarantee asymptotic stabilization of the closed-loop system.

Proof The time derivative of $V_d(x)$ is

$$\dot{V}_d(x) = \dot{V}_c(x) + \dot{V}_{11}(x). \quad (79)$$

Following the steps used in the proof of Theorem 3 and using equation (75), it can be shown that

$$\begin{aligned} \dot{V}_d(x) \leq & x^\top(t) \Xi x(t) + \tau^+ r_4 x^\top(t) \Delta K^\top(t) B_o^\top B_o \Delta K(t) x(t) \\ & + \tau^+ r_5 z^\top(t) \mathcal{I}^\top B_o^\top B_o \mathcal{I} z(t) + \tau^+ r_6 E^\top(x, t) E(x, t) + 2\rho^* \|PB_o\| \|x(t)\|^2 \\ & + 2\|PB_o\mathcal{I}\| |\mu(t)| \|x(t)\|^2 + 2\theta^* \|P\| \|x(t)\|^2 + 2\mu(t) \dot{\mu}(t) \\ & + 2(1 + \theta^*) [\hat{\theta}(t) - \theta^*] \dot{\hat{\theta}}(t) + 2(1 + \rho^*) [\hat{\rho}(t) - \rho^*] \dot{\hat{\rho}}(t), \end{aligned} \quad (80)$$

where Ξ is defined in equation (40). Using the linearization procedure and invoking the Schur complement (as in the proof of Theorem 1), it can be shown that Ξ is guaranteed to be negative definite whenever the LMI (25) has a feasible solution. Using the adaptive laws (76)- (78)

in (80) and the fact that $|\mu(t)| \geq 1$, we get

$$\begin{aligned} \dot{V}_b(x) \leq & x^\top(t) \Xi x(t) + \tau^+ r_4 (\rho^*)^2 \|B_o^\top B_o\| \|x(t)\|^2 + \tau^+ r_5 \|\mathcal{I}^\top B_o^\top B_o \mathcal{I}\| \mu^2(t) \|x(t)\|^2 \\ & + \tau^+ r_6 (\theta^*)^2 \|x(t)\|^2 + 2\rho^* \|PB_o\| \|x(t)\|^2 + 2\|PB_o \mathcal{I}\| |\mu(t)| \|x(t)\|^2 \\ & + 2\theta^* \|P\| \|x(t)\|^2 + 2\lambda_1 |\mu(t)| \|x(t)\|^2 + 2\lambda_2 \mu^2(t) \|x(t)\|^2 \\ & + 2\lambda_3 |\mu(t)| \hat{\theta}(t) \|x(t)\|^2 + 2\lambda_4 |\mu(t)| \hat{\rho}(t) \|x(t)\|^2 + 2\sigma |\mu(t)| \hat{\theta}(t) \|x(t)\|^2 \\ & - 2\sigma \theta^* \|x(t)\|^2 + 2\sigma \theta^* \hat{\theta}(t) \|x(t)\|^2 - 2\sigma (\theta^*)^2 \|x(t)\|^2 + 2\varsigma |\mu(t)| \hat{\rho}(t) \|x(t)\|^2 \\ & - 2\varsigma \rho^* \|x(t)\|^2 + 2\varsigma \rho^* \hat{\rho}(t) \|x(t)\|^2 - 2\varsigma (\rho^*)^2 \|x(t)\|^2. \end{aligned} \quad (81)$$

Arranging terms of equation (81), it can be shown that $\dot{V}_d(x) < 0$ if the adaptive law parameters $\lambda_1, \lambda_2, \lambda_3$, and λ_4 are selected as stated in Theorem 4, and σ and ς are selected to satisfy the following conditions: $\sigma > \frac{1}{2}\tau^+ r_6, 2\|P\| - \sigma + \sigma\hat{\theta}(t) < 0, \varsigma > \frac{1}{2}\tau^+ r_4 \|B_o^\top B_o\|$, and $\|PB_o\| - \varsigma + \varsigma\hat{\rho}(t) < 0$. Hence, we need to select σ and ς such that

$$\sigma > \max \left\{ \frac{1}{2}\tau^+ r_6, \frac{\|P\|}{1 - \hat{\theta}(t)} \right\}, \quad (82)$$

$$\varsigma > \max \left\{ \frac{1}{2}\tau^+ r_4 \|B_o^\top B_o\|, \frac{\|PB_o\|}{1 - \hat{\rho}(t)} \right\}. \quad (83)$$

It is clear that when $\hat{\theta}(t) > 1$ and $\hat{\rho}(t) > 1$, we only need to ensure that $\sigma > \frac{1}{2}\tau^+ r_6$ and $\varsigma > \frac{1}{2}\tau^+ r_4 \|B_o^\top B_o\|$. Note that from equations (77)-(78), $\hat{\theta}(t) > 1$ and $\hat{\rho}(t) > 1$ can be easily ensured by selecting $\hat{\theta}(0) > 1$ and $\hat{\rho}(0) > 1$ and σ and ς as stated in Theorem 4 to guarantee that $\hat{\theta}(t)$ and $\hat{\rho}(t)$ are monotonically increasing. Hence, we guarantee that

$$\dot{V}_d(x) \leq x^\top(t) \Xi x(t), \quad (84)$$

where $\Xi < 0$. Hence, $\dot{V}_d(x) < 0$ which guarantees asymptotic stabilization of the closed-loop system. ■

Remarks:

1. The results obtained in all theorems stated above are sufficient stabilization results, that is asymptotic stabilization results are guaranteed only if all of the conditions in the theorems are satisfied.
2. The projection for μ may introduce chattering for μ and control input u Utkin (1992). The chattering phenomenon can be undesirable for some applications since it involves high control activity. It can, however, be reduced for easier implementation of the controller. This can be achieved by smoothing out the control discontinuity using, for example, a low pass filter. This, however, affects the robustness of the proposed controller.

4. Simulation example

Consider the second order system in the form of (1) such that

$$A_o = \begin{bmatrix} 2 & 1.1 \\ 2.2 & -3.3 \end{bmatrix}, \quad B_o = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.5 & 0 \\ 0 & -1.2 \end{bmatrix}, \quad (85)$$

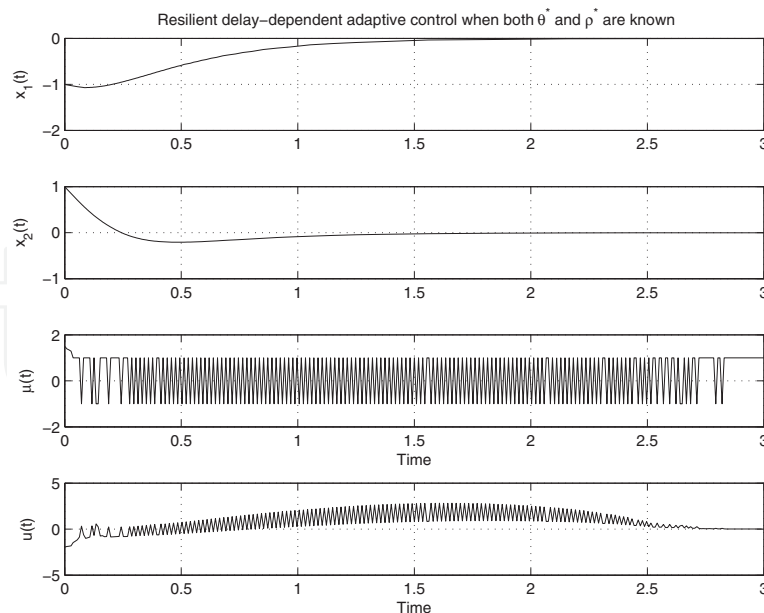


Fig. 1. Closed-loop response when both θ^* and ρ^* are known

and $\tau^* = 0.1$. Using the LMI control toolbox of MATLAB, when the following scalars are selected as $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 1$, the LMI (25) is solved to find the following matrices:

$$\mathcal{X} = \begin{bmatrix} 0.7214 & 0.1639 \\ 0.1639 & 0.2520 \end{bmatrix}, \mathcal{Y} = \begin{bmatrix} -1.7681 & -1.1899 \end{bmatrix}. \quad (86)$$

Using the fact that $K = \mathcal{Y}\mathcal{X}^{-1}$, K is found to be $K = \begin{bmatrix} -1.6173 & -3.6695 \end{bmatrix}$. Here, for simulation purposes, the nonlinear perturbation function is assumed to be $E(x(t)) = \begin{bmatrix} 1.2|x_1(t)| & 1.2|x_2(t)| \end{bmatrix}^\top$, where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^\top$. Based on Assumption 2.1, it can be shown that $\theta^* = 1.2$. Also, the uncertainty of the state feedback gain is assumed to be $\Delta K(t) = \begin{bmatrix} 0.1\sin(t) & 0.1\cos(t) \end{bmatrix}$. Hence, based on Assumption 2.2, it can be shown that $\rho^* = 0.1$.

4.1 Simulation results when both θ^* and ρ^* are Known

For this case, the control law (3) is employed subject to the initial conditions $x(0) = [-1, 1]^\top$ and $\mu(0) = 1.5$. To satisfy the conditions of Theorem 1, the adaptive law parameters are selected as $\alpha_1 = -10$ and $\alpha_2 = -0.5$. The closed-loop response of this case is shown in Fig. 1, where the upper two plots show the response of the two states $x_1(t)$ and $x_2(t)$, and third and fourth plots show the projected signal $\mu(t)$ and the control $u(t)$.

4.2 Simulation results when θ^* is known and ρ^* is unknown

For this case, the control law (3) is employed subject to the initial conditions $x(0) = [-1, 1]^\top$ and $\mu(0) = 1.5$ and $\hat{\rho}(0) = 1.1$. To satisfy the conditions of Theorem 2, the adaptive law parameters are selected as $\beta_1 = -10$, $\beta_2 = -0.5$, $\beta_3 = -0.2$, and $\gamma = 0.1$. For this case, the closed-loop response is shown in Fig. 2, where the upper two plots show the response of the two states $x_1(t)$ and $x_2(t)$, third plot shows the projected signal $\mu(t)$, the fourth plot shows $\hat{\rho}(t)$ and the fifth plot shows the control $u(t)$.

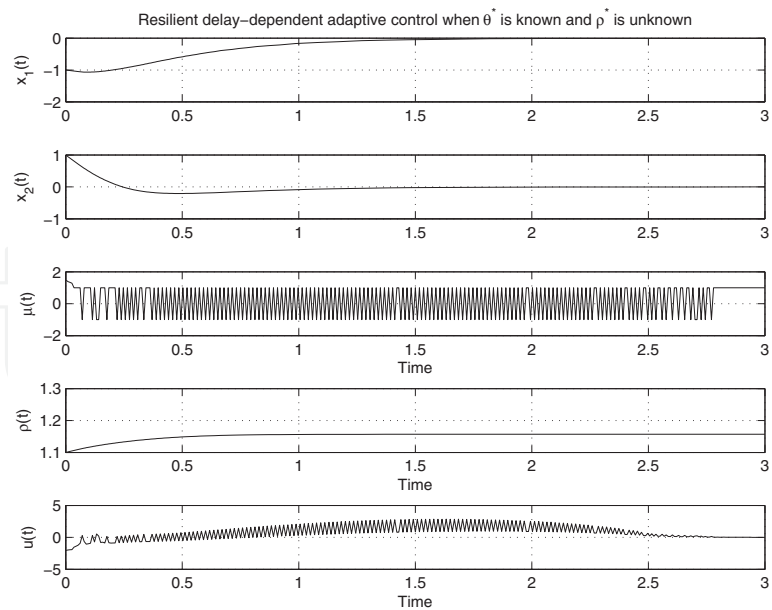


Fig. 2. Closed-loop response when θ^* is known and ρ^* is unknown

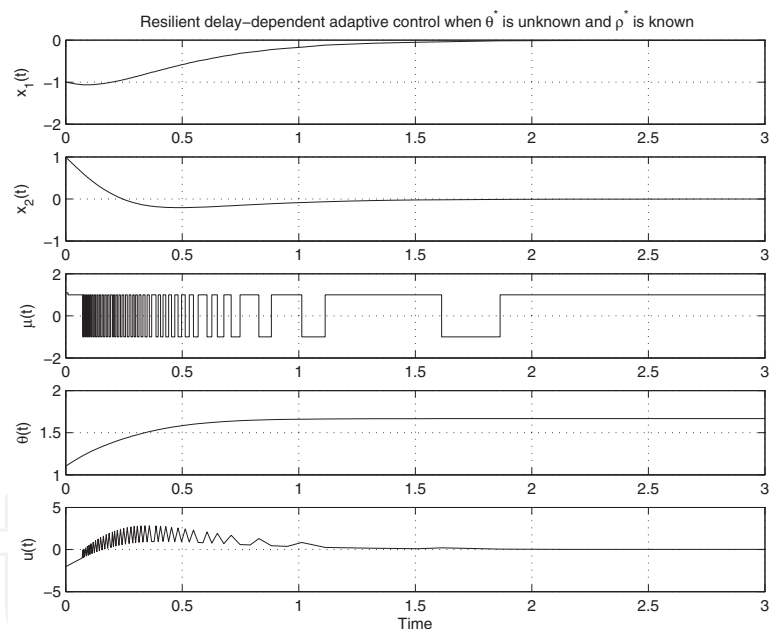


Fig. 3. Closed-loop response when θ^* is unknown and ρ^* is known

4.3 Simulation results when θ^* is unknown and ρ^* is known

For this case, the control law (3) is employed subject to the initial conditions $x(0) = [-1, 1]^T$ and $\mu(0) = 1.1$ and $\hat{\theta}(0) = 1.1$. To satisfy the conditions of Theorem 3, the adaptive law parameters are selected as $\delta_1 = -5$, $\delta_2 = -2$, $\delta_3 = -1.5$ and $\kappa = 1$. For this case, the closed-loop response is shown in Fig. 3, where the upper two plots show the response of the two states $x_1(t)$ and $x_2(t)$, third plot shows the projected signal $\mu(t)$, the fourth plot shows $\hat{\theta}(t)$ and the fifth plot shows the control $u(t)$.

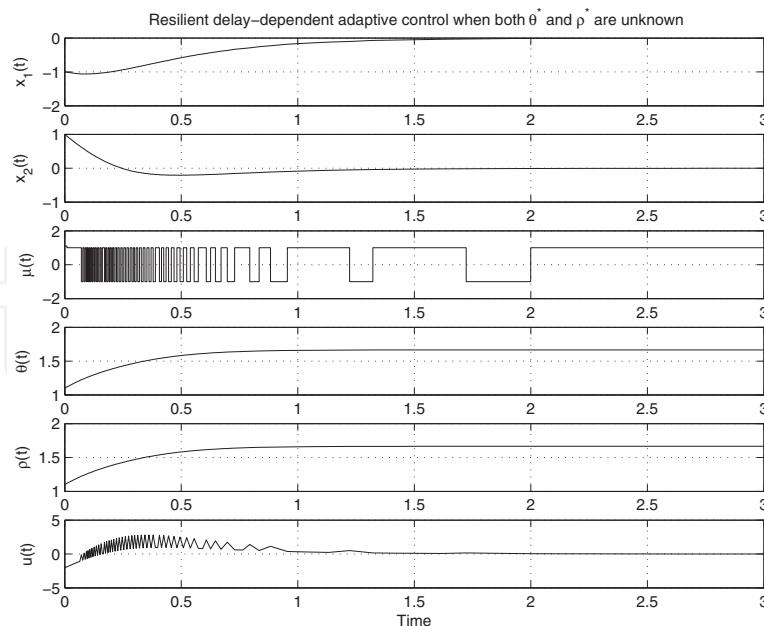


Fig. 4. Closed-loop response when both θ^* and ρ^* are unknown

4.4 Simulation results when both θ^* and ρ^* are unknown

For this case, the control law (3) is employed subject to the initial conditions $x(0) = [-1, 1]^T$ and $\mu(0) = 1.1$, $\hat{\theta}(0) = 1.1$ and $\hat{\rho}(0) = 1.1$. To satisfy the conditions of Theorem 4, the adaptive law parameters are selected as $\lambda_1 = -5$, $\lambda_2 = -1$, $\lambda_3 = -1.5$, $\lambda_4 = -1.5$, $\sigma = 1$, and $\varsigma = 1$. For this case, the closed-loop response is shown in Fig. 4, where the upper two plots show the response of the two states $x_1(t)$ and $x_2(t)$, third plot shows the projected signal $\mu(t)$, the fourth plot shows $\hat{\theta}(t)$, the fifth plot shows $\hat{\rho}(t)$, and the sixth plot shows the control $u(t)$.

5. Conclusion

In this chapter, we investigated the problem of designing resilient delay-dependent adaptive controllers for a class of uncertain time-delay systems with time-varying delays and a nonlinear perturbation when perturbations also appear in the state feedback gain of the controller. It is assumed that the nonlinear perturbation is bounded by a weighted norm of the state vector such that the weight is a positive constant, and the norm of the uncertainty of the state feedback gain is assumed to be bounded by a positive constant. Under these assumptions, adaptive controllers have been developed for all combinations when the upper bound of the nonlinear perturbation weight is known and unknown, and when the value of the upper bound of the state feedback gain perturbation is known and unknown. For all these cases, asymptotically stabilizing adaptive controllers have been derived. Also, a numerical simulation example, that illustrates the design approaches, is presented.

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Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robustness stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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