We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



8

Chaotic Clustering: Fragmentary Synchronization of Fractal Waves

Elena N. Benderskaya[1] and Sofya V. Zhukova[2]
[1]*St.Petersburg State Politechnical University*
[2]*Graduate School of Management, SPbSU*
Russia

1. Introduction

Chaotic neural networks seized attention of scientists from various points of view due to the amazing effects they produce. Phenomenology of structure formation in nature inspired scholars to mimic complex and with the same time quasi-optimal solutions to generate artificial systems with similar capabilities. One of the dominant ways to provide collective dynamics of previously unordered elements is self-synchronization that happens without any outside enforcement.

In theory of dynamic chaos and chaotic synchronization were discovered different types of synchronous behaviour of coupled dynamic systems: complete, lag, generalized, phase, time-scale, frequency, partial synchronizations (Pikovsky & Maistrenko, 2008; Anishchenko & et. al., 2007; Koronovskii & Maistrenko, 2009; Fujisaka & Shimada, 1997, Kapitaniak & Maistrenko 1996; Kurths & et. al., 1997). Most of the results were obtained by computer modelling and visualization techniques. Unpredictable and instable long time period behaviour stems from peculiarities of elements self-dynamics governed by deterministic function that predetermine chaotic behaviour (Lyapunov coefficients are positive). Research subject in numerous articles is collective dynamics of chaotic elements that somehow happens to be stable in terms of group oscillations, but vulnerable when individual trajectory is considered (Politi & Torcini, 2010; Liu & et. al., 2010 ; Pikovsky & et. al., 2003; Manrubia & Mikhailov, 1999). Strange attractors correspond to synchronous clusters (Benderskaya & Zhukova, 2008; Chen & et. al., 2006; Anishenko & et. al., 2002).

Extreme complexity of elements individual dynamics constrains the type of interconnection strength to be homogeneous in overwhelming majority of articles. Most of them consider dependence of synchronous regimes from strength of coupling equal for all elements. This limitation prevents from further generalization of results and makes them to be partial solutions. In most recent researches are considered heterogeneous couplings strength. Heterogeneous field of links between elements extends greatly the analysis complexity (especially mathematical analysis) but allows to reveal interesting effects (Inoue & Kaneko, 2010; Li & et. al., 2004; Popovych & et. al., 2000).

In this paper we consider the analysis of fragmentary synchronization of fractal waves generated by large dimension inhomogeneous chaotic neural network. In our previous papers fragmentary synchronization phenomenon was discovered and applied. Further research of new synchronization type shows that we can speak about fractal structure of

synchronous clusters. We propose that the fact of fractal structure existence can settle the problem of finding out the end of transition period in order to reduce computation complexity of clustering method introduced in our previous papers (Benderskaya & Zhukova, 2008, 2009).

2. Chaotic oscillators

During the last decades, the emergence of collective dynamics in large networks of coupled units has been investigated in disciplines such as physics, chemistry, biology, and ecology (Yamapi et. al., 2010; Wang et. al., 2010; Vasconcelos et. al., 2004). In particular, the effect of synchronization in systems of coupled oscillators nowadays provides a unifying framework for different phenomena observed in nature. Complex networks have recently provided a challenging framework for synchronization studies on dynamic objects with main focus on the interplay between topology complexity and local dynamic properties of coupled elements. A key problem is to define conditions that guarantee the stability of neurons synchronous behavior for this or that network topology.

The complexity of interconnected chaotic systems comes from different directions:

a. nonlinear dynamics of elements;
b. exponential dependence on initial conditions;
c. unpredictable result of the influence on nonlinear vulnerable system from other ones;
d. insufficient mathematical apparatus that help to describe multidimensional nonlinear systems;
e. computer modelling investigation methodology (the calculations precision starts to be critical in terms of forecasting the long term behaviour of nonlinear systems);

All this factors predetermines the research focus mainly on the analysis of interdependent pair of chaotic oscillators, or on the collective dynamics of oscillators ensemble, but with homogeneous type of linkage (Li et. al., 2004; Popovych et. al., 2000).

As it is hard to find formal mathematical solution for a system of multidimensional difference equations we can try to obtain solution by means of computer programming and visualizing of the results. Rapid development of computer technology extended the abilities of scientist to find answers with computer modeling techniques. We use different visualization techniques.

2.1 Synchronization phenomenon

Synchronization as a universal concept is thoroughly discussed in literature (Pykovski & et. al., 2003). One of the most important generalization of inner synchronization effects are the conditions that cause inner synchronous motions among groups of nonlinear elements:

a. large amount of globally coupled nonlinear elements;
b. weak coupling strength to exclude the possibility of several elements to suppress
c. individual dynamics of all others;
d. instability dynamics of each nonlinear element;
e. feedbacks to provide own element's dynamics tuning to the neighbors' fluctuations.

Primary results on modeling high dimensional chaotic map lattices were published by K. Kaneko (Kaneko, 1987, 1989). His works showed up the fact that globally coupled chaotic map lattices exhibit formation of ensembles synchronously oscillating elements. These ensembles were called clusters serving as system's attractors. If there appear to be several clusters then the system is characterized by multistability, when several attractors coexist in

the phase space at the same parameters values. The main focus of research in terms of synchronization is on the combination of systems parameters that predetermine the appearance of different synchronization types corresponding to functioning regimes.

2.2 Clustering

The fundamental research provides the basis for various applications of chaotic networks (Fang et. al., 2009; Herrera Martín, 2009; Hammami et. al., 2009, Mosekilde et. al., 2002). In this paper we apply chaotic neural network to 2D and 3D clustering problem. L. Angelini introduced to correspond dataset groups to dynamical oscillatory clusters by means of neural network parametrization (Angelini & et. al., 2000, 2001, 2003). In our previous papers we introduced modifications of chaotic neural network (CNN) clustering method, proposed by Angelini, in order to ensure better clusterization quality (Benderskaya & Zhukova, 2008, 2009). In this paper we give the description of modified CNN model.

3. Chaotic neural network model

Let us consider chaotic neural network model simultaneously from several angels mentioned above as its complexity has many dimensions.

3.1 Structure complexity

CNN does not have classical inputs – it is recurrent neural network with one layer of N neurons. Each neuron is responsible for one object in the dataset, but the image itself is not given to inputs. Instead input dataset is given to logistic map network by means of inhomogeneous weights assignment.

$$W = \{w_{ij}\} = \exp\left(-\frac{{d_{ij}}^2}{2a}\right),\ d_{ij} = |\ x_i - x_j\ |,\ \ i,j = \overline{1,N}, \tag{1}$$

where N – number of elements, w_{ij} - strength of link between elements i and j, d_{ij} - Euclidean distance between neurons i and j, a – local scale, depending on k-nearest neighbors. Influence of linkage mean field on the dynamics of CNN is demonstrated on Fig. 1.

The value of a is fixed as the average distance of k-nearest neighbor pairs of points in the whole system. To define nearest neighbors taking into account image topology we use Delaunay triangulation. Delaunay triangulation (Preparata & Shamos, 1993) gives us all the nearest neighbors of each point from all directions. The value of a is now fixed as the average distance of Delaunay-nearest neighbor pairs of points in the whole system. Thus we form the proper mean field that contributes greatly to convergence of CNN dynamics to macroscopic attractor.

Evolution of each neuron is governed by

$$f(y(t) = 1 - \lambda y^2(t) \tag{2}$$

$$y_i(t+1) = \frac{1}{C_i}\sum_{i \neq j}^{N} w_{ij} f(y_i(t)),\ \ t = 1...T, \tag{3}$$

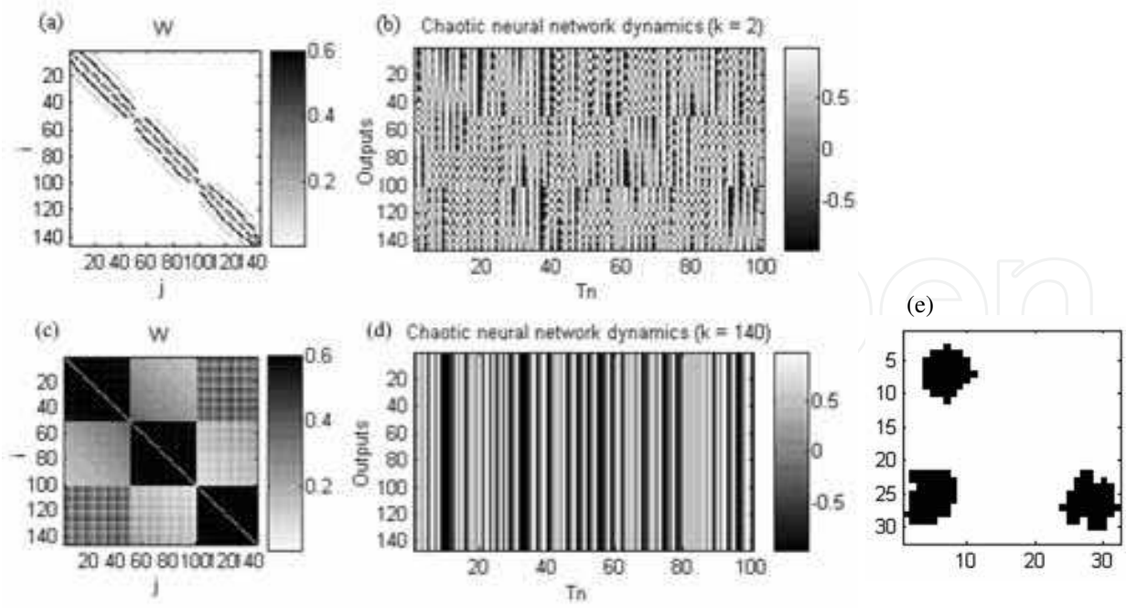


Fig. 1. Example of improper field of weight coefficients and the corresponding dynamics of the CNN for the image (e) that is clustered: (a, b)—the number of nearest neighbors k = 2 (cluster synchronization is absent); (c, d)—the number of nearest neighbors k = 140 - all the neurons oscillate synchronously and division into clusters is impossible. The values of the weight coefficients are represented by concrete colours in correspondence with the nearby given colorbars).

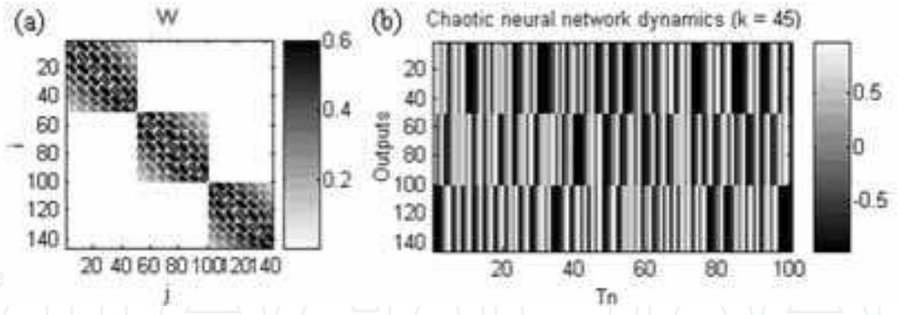


Fig. 2. The inhomogeneous field of weight coefficients within one cluster (distinctly pronounced oscillation clusters when a = 2.2, calculated on the average distance of Delaunay neighbours: (a)—visual map of the weight coefficient matrix; (b)—the change of the CNN output values in time.

where $C_i = \sum_{i \neq j} w_{ij}$, $i, j = \overline{1, N}$, T – time interval, N – number of elements, λ - logistic map parameter. Neurons state is dependent on the state of all other elements. The logistic map with parameter λ =2 predetermines the chaotic behaviour of each neuron.

The structure of chaotic neural network (number of neurons, field of weight coefficients) and dynamics depends on the image size and topology. It happens to be very difficult to find out formal measures to forecast oscillators behaviour especially because of chaotic nature of each neuron. Thus we deal with N-dimensional inhomogeneous system of chaotic

oscillators that evolves in discrete time and generates continuous outputs. The main research method: computer modelling.

3.2 Extreme instability of each neuron

One of the basic models of chaotic nonlinear systems is logistic map (Peitgen, 2004). If the parameter is set as λ =2, then strange attractor can be detected in the phase space (Fig. 3).

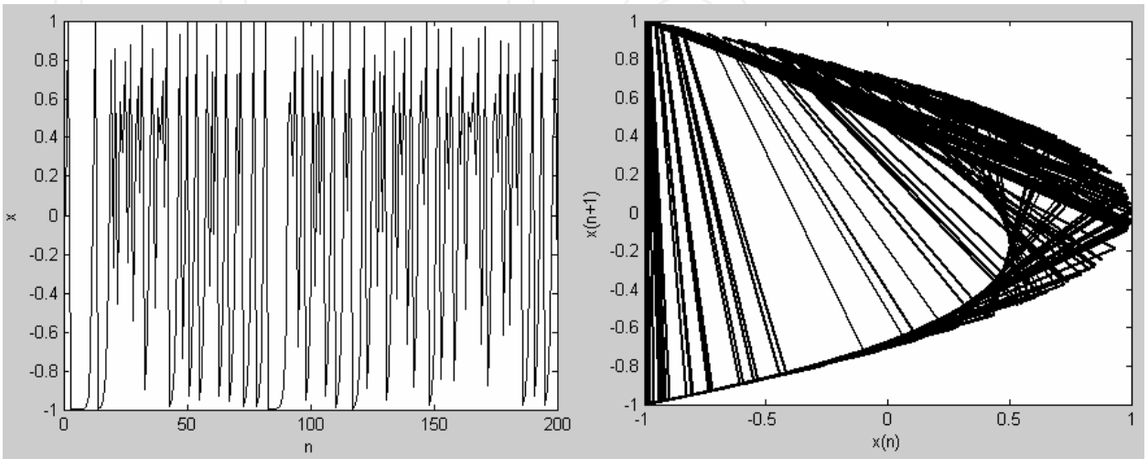


Fig. 3. One neuron dynamics analysis: strange attractor is constructed when λ =2.

One of the main characteristic is exponential instability to initial conditions. The logistic map fully demonstrates this quality (Fig. 4). As we can see even small delta of 0.0000001 leads to the serious trajectories' changes after 24 iterations.

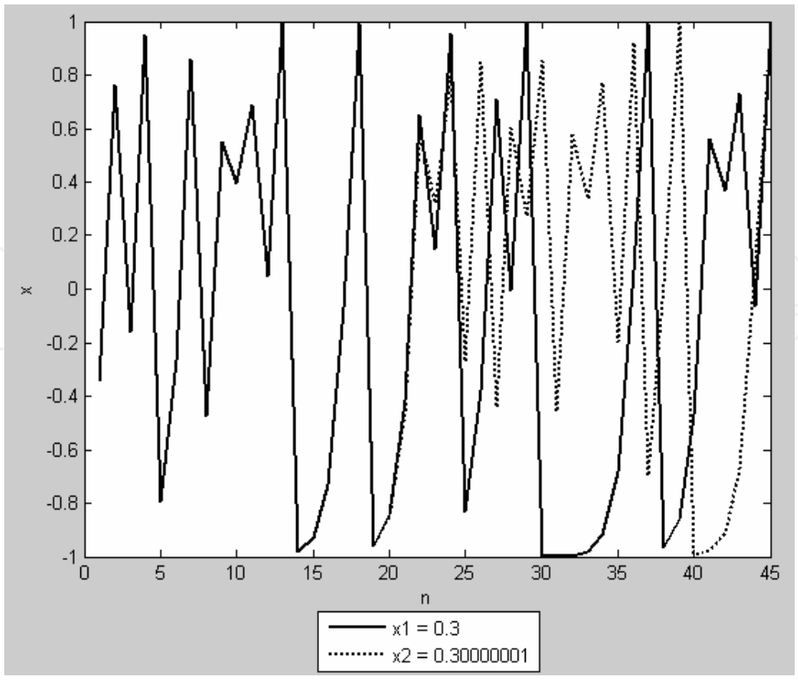


Fig. 4. Trajectories, starting from very close initial conditions with difference of 0.0000001.

Clusterization phenomenon stems from the chaotic oscillations of each neuron. The logistic map parameter λ=2 guarantees chaotic dynamics of each neuron, as the eldest Lyapunov indicator is positive.

3.3 Dynamical clusters and synchronization types

In accordance with (Pykovski et al., 2003; Peitgen et al. 2004) in the ensembles of poorly connected identical neurons emerge synchronization of various types, depending on the system's parameter combination. We introduce these types on the example of CNN:

a. complete synchronization (Fig. 5.a);
b. imphase synchronization (Fig. 5.b);
c. phase synchronization (Fig. 5.c, Fig. 5.d);
d. lag synchronization (time series coincide but with some delay in time);
e. generalized synchronization (there is some functional dependence between time series).

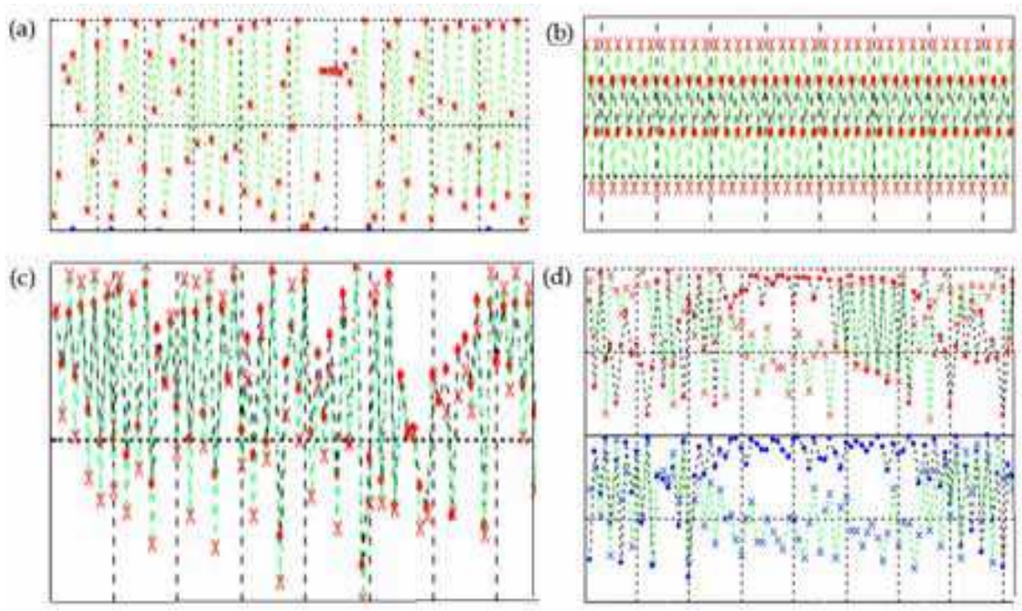


Fig. 5. Synchronization types of chaotic time series: (a) – complete; (b)-imphase; (c) – phase synchronization with slight amplitude deviations; (d) – phase synchronization of neurons pairs within two clusters reveal the outputs changes in the same direction but with significantly different amplitudes.

Besides these well-known synchronization types we found out CNN to produce new synchronization type – we named it fragmentary synchronization. It is characterized by different oscillatory melodies-fragments (Fig 6.e, 6.f). Synchronization is no more about comparing separate trajectories, but about integrative consideration of cluster's music of fragments.

3.4 Macroscopic attractors in oscillations

The dynamics of a separate neuron output highly depends on initial conditions, but the most fruitful about CNN is its ability to form stable (independent of initial conditions) synchronous clusters in terms of joint dynamics of neurons. Stable mutual synchronization

of neurons (points) within each cluster in terms of CNN corresponds to the macroscopic attractor, when we receive indifferent to initial conditions oscillatory clusters, though instant outputs of neurons differ greatly (Fig. 6). The complexity of mutual oscillations depends on the complexity of input image. Simple image comprised by 146 points (Fig 5.a) organized in compact groups located far from each other predetermines almost complete synchronization of oscillations within clusters (Fig. 6.b, 6.c). But if the image of 158 points with less compact topology and inter cluster distance more complex synchronization take place – fragmentary synchronization (Fig. 6.e, 6.f) .

The system is stable in terms of mutual synchronous dynamics of outputs within time but not in terms of instant values of separate neurons.

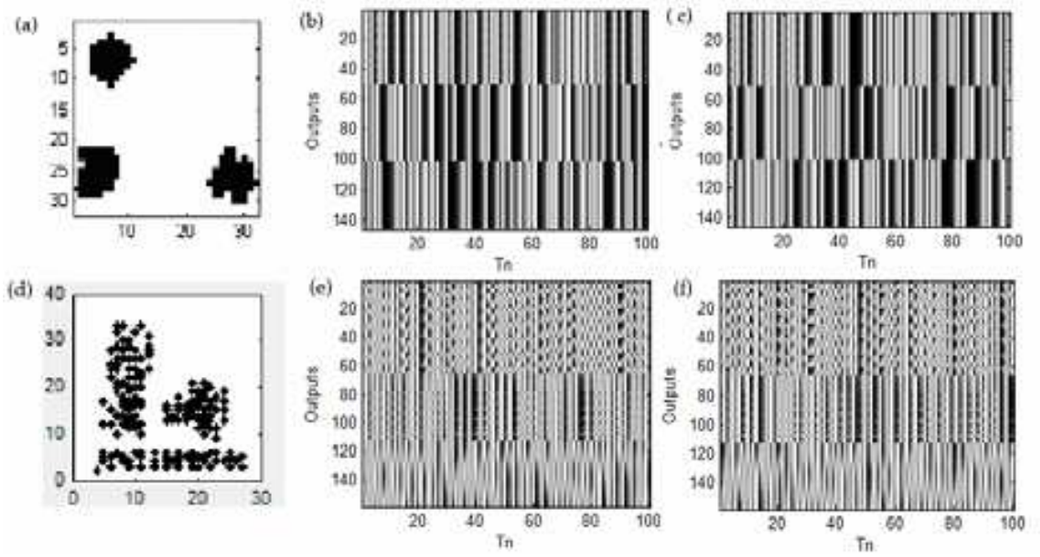


Fig. 6. Visualization of CNN outputs: in stationary regime trajectories being chaotic form the three different oscillatory clusters from absolutely different initial conditions: (a) – simple input dataset to be clustered ; (b), (c) – 146 outputs of CNN completely synchronous within clusters evolving during observation period Tn=100 from different initial conditions; (d) – complex input dataset to be clustered; (e), (f) – 158 fragmentary synchronized outputs of CNN evolving during observation period Tn=100 from different initial conditions.

3.5 Clustering technique drawback

All the figures above demonstrate CNN dynamics statistics gathered after some transition period. One of the unsolved problems at the moment is finding out some formal way to state that transition period is over and it is time of macroscopic attractor to govern trajectories. We introduced some indirect approach that consists in CNN output statistics processing. At different level of resolution due to the theory of hierarchical data mining (Han, 2005) are generated dozens of clusterizations, then they are compared and the variant that repeats more often wins (is considered to be the answer). After that we repeat the procedure all over again to be sure that with the cause of time mutual synchronization remains to be the same. This approach is desperately resource consuming and takes dozens more time than generating CNN oscillatory clusters. Thus it prevents from wide application of CNN clustering technique especially with the growth of objects number in the input

image (it does not matter 2D, 3D or N-dimensional), though it has a wide set of advantages in compare to other clustering methods.
The novelty of this paper consists in discovering fractal structures in fragmentary synchronized outputs. This phenomenon encourages us to shift our focus on the direct analysis of outputs value and not on their desensitization with the main aim to reduce clustering method complexity.

4. In pursuit of strange attractor

The captivating interplay of oscillations within dynamical clusters that we call fragmentary synchronization could hardly be interpreted somehow in a numerical way. Other problem that seemed to have no answer is that the dependence between clustering quality and the size of outputs statistics is not obvious. The extensive growth of CNN states to be analysed sometimes was not successful in terms of clustering problem and predetermined even worse results than those obtained on a smaller dataset. Such observations forced us to focus mainly on synchronization of time-series once more in order to reveal some order in the macroscopic attractor, comprised by temporal sequences. The indication of macroscopic attractor existence is the coincidence of clustering results (synchronous dynamical clusters) obtained for different initial conditions.

4.1 Fractal waves

Under the notion of fractal coexists a wide set of structures, both of spatial and temporal nature that demonstrate self-similarity. The very word fractal is formed from latin fractus which means to consist of fragments. Broad definition tells that fractal is the structure consisted of the parts which are similar the whole (Mandelbrot, 1983). In the case of CNN it is more applicable to say that fractals are signals that display scale-invariant or self-similar behaviour. Fractals reflect nature as inherently complex and nonlinear according to Dardik (Dardic, 1995). Smaller rhythms are imbedded within larger rhythms, and those within larger still. The short biochemical cycles of cells of the human heart waves are embedded within the circadian rhythm of the whole body, and they are all embedded within the larger waves of weeks, months and years. Fractal superwaves spiral in all directions as an inherent continuum of waves nested within other waves. Thus everything is affecting everything else simultaneously and casually, while everything is changing, throughout all scales. In terms of recurrent behaviour of CNN outputs we consider the joint dynamics of neurons as waves of complex form. What does it mean – self-similarity in CNN?
We started with careful consideration of fragmentary synchronized neurons dynamics (Fig. 7). The dynamics statistics was gathered during Tn = 2000 (Fig. 7.a). Then we focused on first counts (states of CNN represented on the figure by vertical lines) and visualized them in a more and more detailed way (Fig. 7.b-e).
After careful consideration we noticed that there exist quasi similar fragments not only in terms of horizontal lines that comprise melodies, but repeating waves in the overall chaotic neural network (Fig. 7b., Fig. 7c). This temporal similarity leads us to the hypothesis of oscillations fractal structure.

4.2 CNN fractals

Temporal fractals as well as space fractals are characterized by self-similarity at different scales of consideration. But in case of CNN we deal not with geometric object, but with

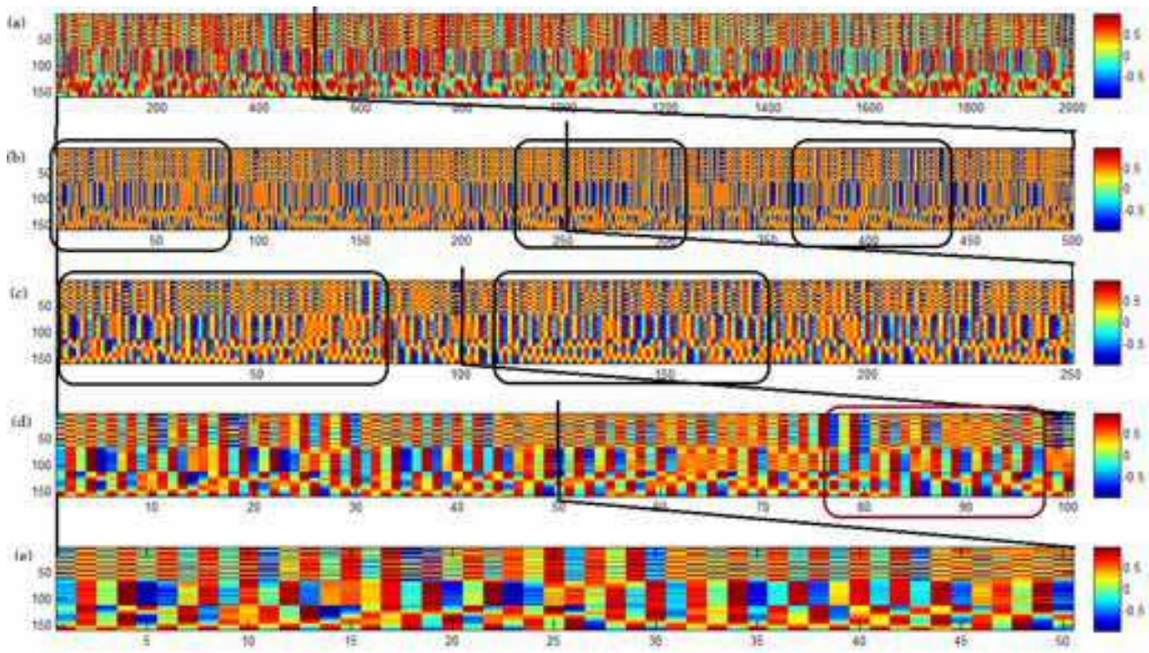


Fig. 7. Fragmentary synchronization with detailed consideration of first counts: (a) – CNN outputs dynamics during 2000 iterations; (b) – bunch of 500 first counts; (c) – bunch of 250 first counts; (d) – bunch of 100 first counts; (e) – bunch of 50 first counts.

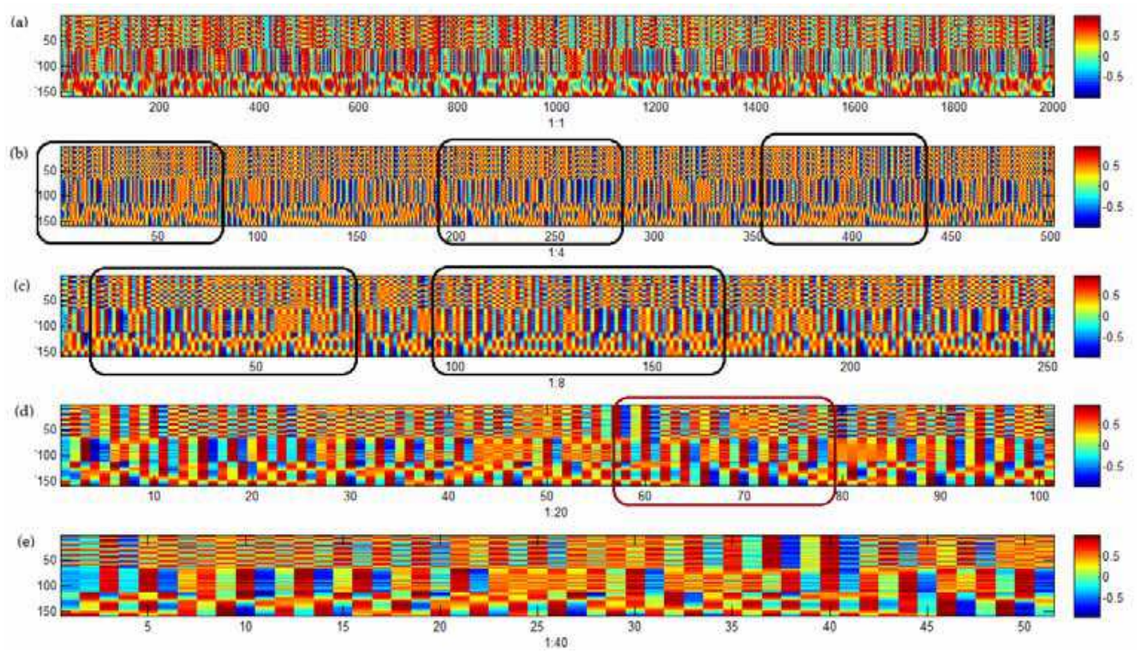


Fig. 8. Scaling of CNN dynamics by means of decimation: (a) – original dynamics; (b) – scale 1:4 (every 4th count out of 2000 original counts); (c) – scale 1:8 (every 8th count out of 2000 original ones); (d) – scale 1:20 (every 20th count out of 2000 original counts); (e) – scale 1:40 (every 20th count out of 2000 original counts).

fractal structure of multidimensional time-series, comprised by CNN counts. The scaling is done by means of time-series decimation (bolting) with different coefficient in order to observe CNN dynamics at various detail levels (this is similar to CNN modeling with different discretization time).
Representation of CNN time-series at different scales is shown on Fig. 8. Interested in less detailed temporal picture we gradually decrease the scale.
Amazing thing about the scaling on Fig. 8 is that it looks like almost the same in comparison to Fig. 7. On Fig. 7.b and Fig. 8.b as well as on Fig. 7.c and Fig. 8.c one can see the same fragments though they are viewed absolutely from different perspectives. More over the similarity is observed not only on small scales (1:4, 1:8) but on rather huge ones (1:20, 1:40) with several counts lag precision (Fig. 7.d and Fig. 8.d).

4.3 Fractal visualization

One of the common ways to reveal fractals is construction of phase portraits. In case of CNN we observe almost the same phase portrait like on Fig. 3, as individual trajectory is forced by logistic map to be chaotic. To investigate recurrent behaviour of complex multidimensional time-series recurrent analysis is applied. In case of CNN we propose to visualize trajectories by means of recurrence plots (RP) introduced by J.P. Eckmann (Eckmann et al., 1987, Romano et al., 2005). Recurrence plots visualize the recurrence of states in a phase space. Usually, a phase space does not have enough dimension (two or three) to be pictured. Higher-dimensional phase spaces can only be visualized by projection into the two or three-dimensional sub-spaces. However, Eckmann's tool enables us to investigate the *m*-dimensional phase space trajectories through a two-dimensional representation of its recurrences. Such recurrence of a state at time *i* at a different time *j* is pictured within a two-dimensional squared matrix with black and white dots, where black dots mark a recurrence, and both axes are time axes. This representation is called recurrence plot. Such an RP can be mathematically expressed as

$$R(i, j) = Q(Eps - ||x(i)-x(j)||), i, j = 1, ... ,N \quad (4)$$

where *N* is the number of states *x(i)* (counts of CNN dynamics) , *Eps* is a threshold distance, ||*|| - a Euclidean norm and *Q* the Heaviside step function. Recurrence plot contains typical small-scale structures, as single dots, diagonal lines and vertical/horizontal lines (or a mixture of those). The large-scale structure, also called texture, can be visually characterised by homogenous, periodic, drift or disrupted. The visual appearance of an RP gives hints about the dynamics of the system.
If recurrence behaviour occurs in two different time-series then synchronization takes place. If self-recurrent pieces are detected or similar dynamics is revealed between original time-series and their scaled copy then we can speak about application of RP to the analysis of CNN fractal temporal structure (self-similarity).
Recurrence plots of CNN dynamics temporal structure is introduced on Fig. 9 in two ways: self-reflection of first counts bunches 500, 250, 100, 50 correspondingly represented on Fig. 9.a, Fig. 9.c, Fig. 9.e, Fig. 9.h and self-reflection of scaled bunches 1:4, 1:8, 1:20, 1:40 represented on Fig. 9.b, Fig. 9.d, Fig. 9.f, Fig. 9.h. We can see that recurrence plots of first bunches and scaled bunches have much in common (partly just the copy of each other). It happens to have no difference whether to observe first 500 counts or picture, comprised by decimated counts with four times magnification (every 4th count out of 2000). This is the evidence for fractal structure of time-series, generated by CNN. The indicator of oscillations

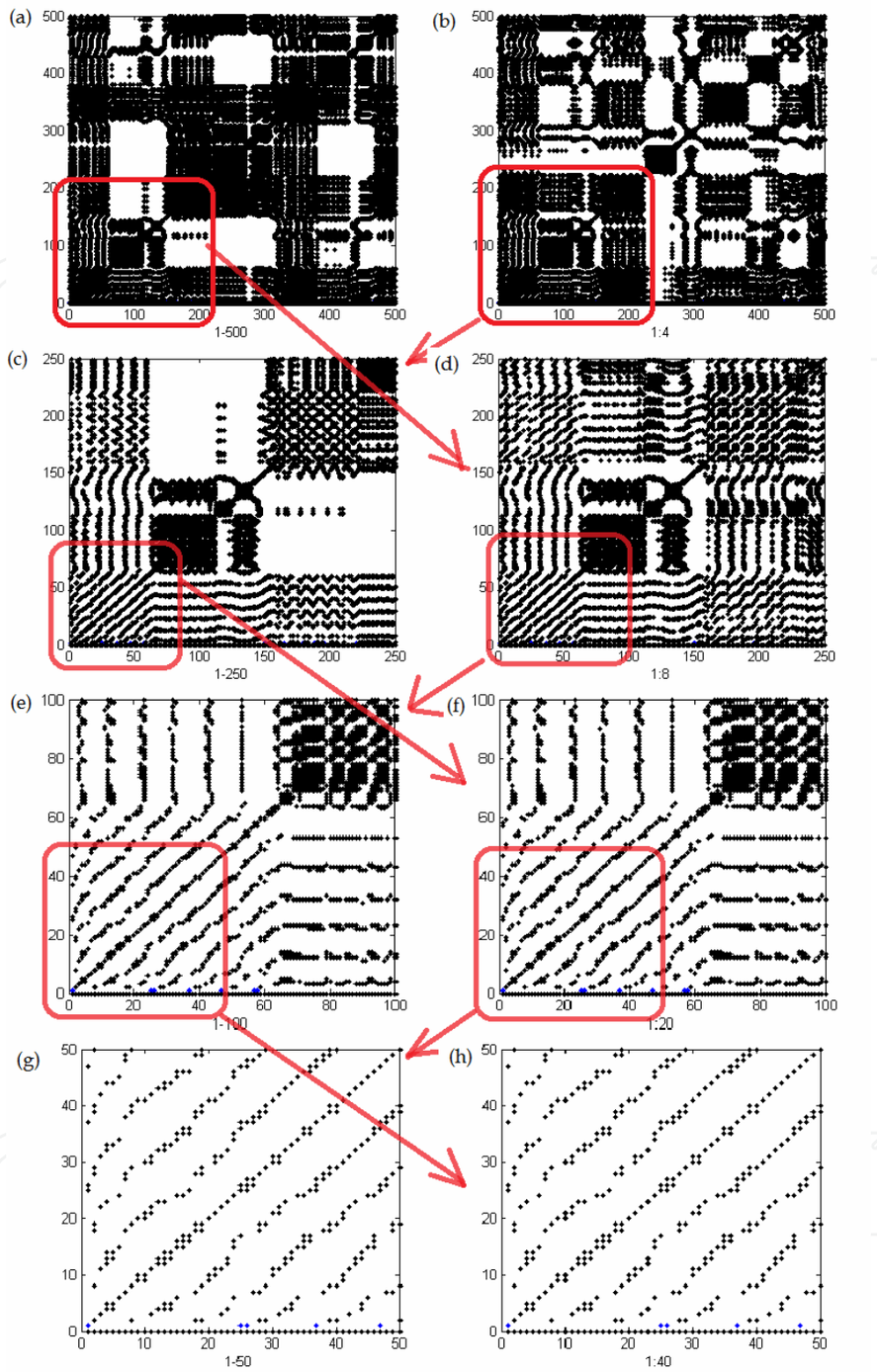


Fig. 9. Recurrence plots of CNN dynamics: (a) - self-reflection of 500 first counts bunch; (b) – self-reflection of scaled dynamics with 1:4 magnification level; (c) - self-reflection of 250 first counts bunch ; (d) – self-reflection of scaled dynamics with 1:8 magnification level; (e) - self-reflection of 100 first counts bunch ; (f) – self-reflection of scaled dynamics with 1:20 magnification level; (g) - self-reflection of 50 first counts bunch ; (h) – self-reflection of scaled dynamics with 1:8 magnification level.

periodical dynamics is the presence of diagonal lines and patterns arranged in staggered order. Irregular fragments speak about chaotic synchronization and quasi periodical character of oscillations. Dissimilar pieces of recurrence plots can also be the consequence of coincidence precision demands, predetermined by (4) (it is well-known practise to use tolerance discrepancy when analysis of multidimensional nonlinear systems with chaotic dynamics is conducted). By means of arrows we note the nesting similarity of different recurrent plots (both first and scale bunches) – evidence for fractal structure.

Further investigation leads us to cross-recurrence analysis. The comparison of first count bunches and scaled count bunches is provided (Fig. 10). And again one can see self-similarity of results: the same as in case of recurrence plots the mutual correspondence among plots is detected in spite of different scaling of original CNN statistics: Fig. 10.a is similar to Fig. 9.b; Fig. 10.c is similar to Fig. 9.d; Fig. 10.b is similar to Fig. 9.f; Fig. 10.d is similar to Fig. 9.h.

To be careful with conclusions we checked the existence of fractal nesting depth and compared the reflection of scaled bunch of counts into each other (Fig. 11). As for recurrence plots on Fig. 9 and Fig. 10 on Fig. 11 the same structure is observed accurate within small fragments. This verifies the importance of discrepancy thresholds not only for absolute values of CNN outputs coincidence but also for comparison of fragments when fragmentary synchronization is analyzed.

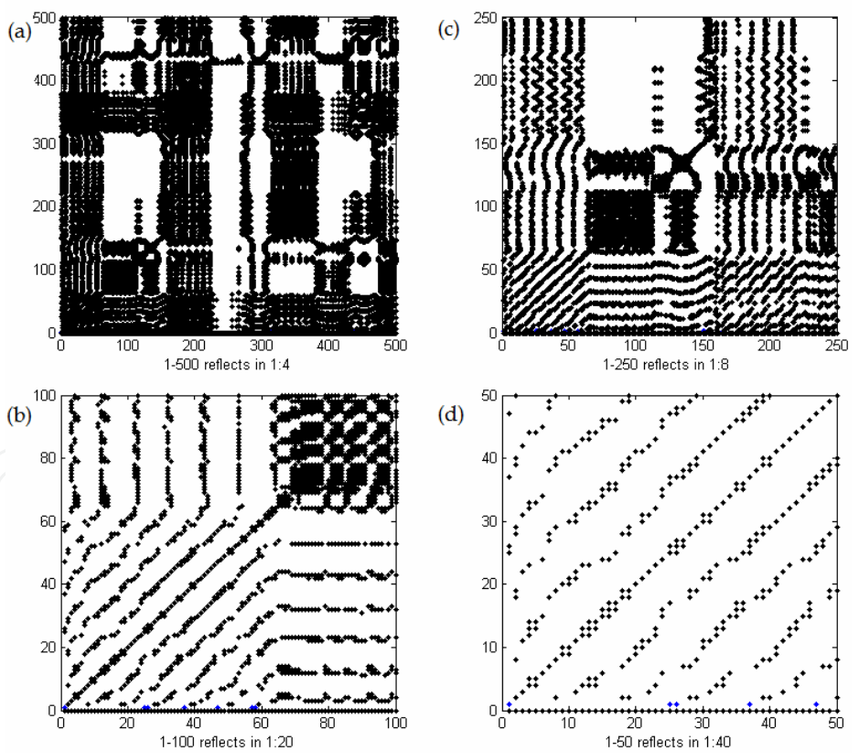


Fig. 10. Cross recurrence plots of CNN dynamics: (a) - reflection of 500 first counts bunch into scaled 1:4 bunch of counts; (b) – reflection of 250 first counts bunch into scaled 1:8 bunch of counts; (c) - reflection of 100 first counts bunch into scaled 1:20 bunch of counts; (d) – reflection of 100 first counts bunch into scaled 1:40 bunch of counts.

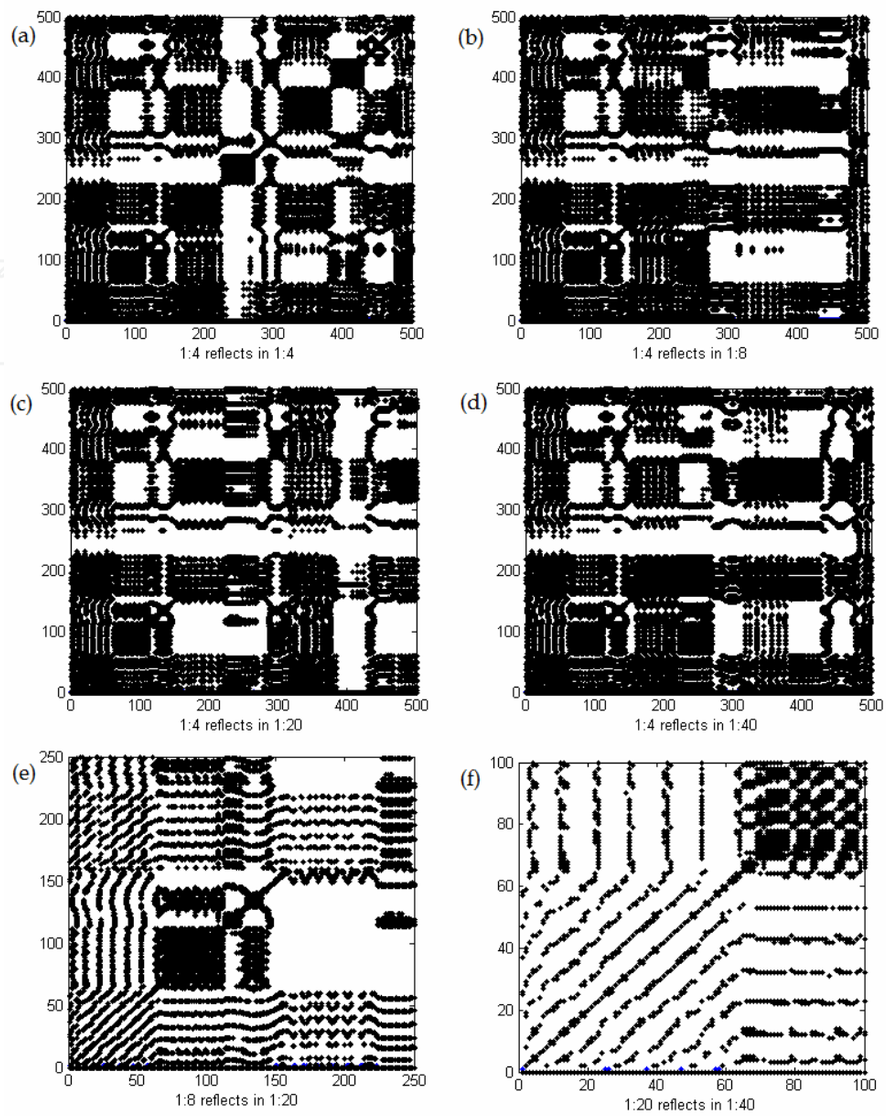


Fig. 11. Cross-recurrent plots of scaled count bunches: (a) – self-reflection of 1:4 dynamics scale; (b) – reflection of 1:4 scaled bunch into 1:8 scaled bunch; (c) – reflection of 1:4 scaled bunch into 1:20 scaled bunch; (d) – reflection of 1:4 scaled bunch into 1:40 scaled bunch; (e) – reflection of 1:8 scaled bunch into 1:20 scaled bunch; (f) – reflection of 1:20 scaled bunch into 1:40 scaled bunch.

5. Conclusion

In this paper fractal structure of fragmentary synchronization is discovered. The structure of fragment's and overall dynamics of CNN was investigated by means of recurrence and cross-recurrence plots visualization techniques. Understanding the mechanism of fragments interplay (periodical vertical similarity) along with oscillatory clusters interplay (horizontal dissimilarity of cluster's melodies) is vital for discovering the low resource consuming algorithm of CNN outputs processing in order to translate nonlinear language of oscillation

into the language of images in data mining field (important to solve general clustering problem).
Analogous fractal effects were obtained for a wide set of well-known clustering datasets. Further research will follow in the direction of fractal structures measurement. This is important to formalize the analysis of inner phase space patterns by means of automatic techniques for recurrence plots analysis.

6. References

Angelini, L. (2003). Antiferromagnetic effects in chaotic map lattices with a conservation law, *Physics Letters A* 307(1), pp.41–49

Angelini, L.; Carlo, F.; Marangi, C.; Pellicoro, M.; Nardullia, M. & Stramaglia, S. (2001). Clustering by inhomogeneous chaotic maps in landmine detection, *Phys. Rev. Lett.* 86, pp.89–132

Angelini, L.; Carlo, F.; Marangi, C.; Pellicoro, M.; Nardullia, M. & Stramaglia, S. (2000). Clustering data by inhomogeneous chaotic map lattices, *Phys. Rev. Lett.* 85, pp.78–102

Anishchenko, V.; Astakhov, V.; Neiman, A.; Vadivasova, T. & Schimansky-Geier, L. (2007). *Nonlinear Dynamics of Chaotic and Stochastic Systems: Tutorial and Modern Developments*, Springer-Verlag, New York, ISBN 3540381643

Anishchenko, V. S.; Sosnovtseva, O. V. ; Kopejkin, A. S.; Matujshkin, D. D. & Klimshin, A. V. (2002). Synchronization effects in networks of stochastic bistable oscillators, *Mathematics and Computers in Simulation*, Volume 58, Issue 4-6 (Mar. 2002), pp. 469-476, Elsevier Science Publishers B. V., Amsterdam

Benderskaya, Elena N. & Zhukova, Sofya V. (2008). Clustering by chaotic neural networks with mean field calculated via Delaunay triangulation, *Proceedings of Third international workshop on Hybrid artificial intelligence systems*, pp. 408-416, ISBN 978-3-540-87655-7, Burgos, Spain, September 24-26, 2008, Lecture Notes in Computer Science, vol. 5271, Springer-Verlag, Berlin-Heidelberg

Benderskaya, Elena N. & Zhukova, Sofya V. (2009). Fragmentary synchronization in chaotic neural network and data mining, *Proceedings of 4th international conference on Hybrid artificial intelligence systems*, pp. 319-326, ISBN 978-3-642-02318-7, Salamanca, Spain, June 10-12, 2009, Lecture Notes in Computer Science, vol. 5572, Springer-Verlag, Berlin-Heidelberg

Chen, Y. ; Rangarajan, G. & Ding, M. (2006). Stability of synchronized dynamics and pattern formation in coupled systems:Review of some recent results, *Communications in Nonlinear Science and Numerical Simulation,* 11 (2006) pp.934–960

Dardik, I. (1995). The law of waves, *Cycles*, Vo. 45, pp. 49-60

Eckman, J.-P.; Kamphorst S.O. & Ruelle, D. (1987). Recurrence Plots of Dynamical Systems, *Europhys. Lett*., 4 (9), pp. 973-977

Fang, X.; Yu, H. & Jiang, Z.(2009). Chaotic synchronization of nearest-neighbor divusive coupling Hindmarsh–Roseneural networks in noisy environments, *Chaos, Solitonsand Fractals* 39 (2009) pp. 2426–2441

Fujisaka H. & Shimada T. (1997). Phase synchronization and nonlinearity decision in the network of chaotic flows, *Phys. Rev. E*, 55(3), pp. 2426-2433, 1997.

Hammami, S.; BenSaad, K. & Benrejeb, M. (2009). On the synchronization of identical and non-identical 4-D chaotic systems using arrow form matrix, *Chaos, Solitons and Fractals,* 42 (2009), pp. 101–112

Han, J.; Kamber, M. & Pei, J. (2005). *Data Mining: Concepts and Techniques*, The Morgan Kaufmann Series in Data Management Systems

Herrera, D. & Martín, J. (2009). Ananalytical study in coupled map lattices of synchronized states and traveling waves, and of their period-doubling cascades, *Chaos, Solitonsand Fractals* 42 (2009) pp. 901–910

Inoue, M. & Kaneko, K. (2010). Dynamics of Coupled Adaptive Elements: Bursting and Intermitteny Oscillations Generated by Frustration in the Network, *Phys. Rev. E,* 81, pp. 126-203

Kaneko, K. (1987). Phenomenology of spatio-temporal chaos, In *Directions in chaos.* - World Scientific Publishing Co., pp. 272-353, Singapore

Kaneko, K. (1989). Chaotic but regular posi-nega switch among coded attractors by cluster-size variations, *Phys. Rev. Lett.,* 63(14), pp. 219-223

Kapitaniak, T. & Maistrenko, Y. (1996). Different types of chaos synchronization in coupled chaotic oscillators, *Phys. Rev. E,* 54(4) pp. 3285-3292

Koronovskii, A. A.; Moskalenko, O. I. & Hramov, A. E. (2009). On the use of chaotic synchronization for secure communication, *Phys. Usp.* 52, pp.1213–1238

Kurths, J.; Pikovsky, A. & Rosenblum M. (1997). From phase to lag synchronization in coupled chaotic oscillators, *Phys. Rev. Lett.* 78, pp. 4193-4196

Li, S.; Liao, X. & Yu, J. (2004). Synchronization in coupled map lattices with small-world delayed interactions, *Physica A* 335, pp. 365–370

Liu, H.; Chen, J.; Lu, J. & Ming Cao (2010). Generalized synchronization in complex dynamical networks via adaptive couplings, *Physica A: Statistical Mechanics and its Applications*, Vol. 389, Issue 8, April 2010, pp. 1759-1770

Mandelbrot, B. (1983). *The Fractal Geometry of Nature*, W. H. Freeman and Co., ISBN 0-7167-1186-9, New York

Manrubia, S. & Mikhailov, A. (1999). Mutual synchronization and clustering in randomly coupled chaotic dynamical networks, *Phys. Rev. E* 60, pp. 1579–1589

Mosekilde, E.; Maistrenko, Yu. & Postnov, D. (2002). *Chaotic synchronization*, World Scientific Series on Nonlinear Science, Series A, Vol. 42

Marwan, N.; Romano, M. C.; Thiel, M. & Kurths, J. (2007). Recurrence Plots for the Analysis of Complex Systems, *Physics Reports*, 438(5-6), pp. 237-329

Peitgen, H.; Jürgens, H. & Dietmar S. (2004). *Chaos and Fractals. New Frontiers of Science*, Springer-Verlag, ISBN 978-0-387-20229-7, New York

Pikovsky, A. & Maistrenko, Y. (2008). *Synchronization: Theory and Application*, NATO Science Series II: Mathematics, Physics and Chemistry, Springer, ISBN 9781402014178

Pikovsky, A.; Rosenblum, M. & Kurths, J. (2003). *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge Nonlinear Science Series) ISBN 978-0521533522, Cambridge University Press

Politi, A. & Torcini, A. (2010). Stable chaos in Nonlinear Dynamics and Chaos: Advances and Perspectives, *In Understanding Complex Systems*, 2010, pp. 103-129, DOI: 10.1007/978-3-642-04629-2_6

Popovych, O.; Maistrenko, Ya.; Mosekildec, E.; Pikovsky, A. & Kurths, J. (2000). Transcritical loss of synchronization in coupled chaotic systems, *Physics Letters A* 275, pp.401–406

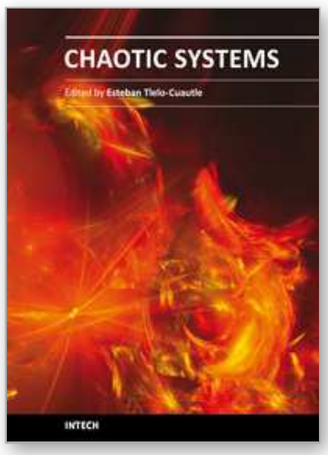
Preparata, F. R. & Shamos, M. I. (1993). *Computational Geometry.* An Introduction, Monographs in Computer Science, Springer

Romano, M.C.; Thiel, M.; Kurths, J. ; Kiss, I.Z. & Hudson, J.L.(2005) . *Europhys. Lett.* 2005. V. 71 (3). P. 466.

Vasconcelos, D .; Viana, R .; Lopes, S.; Batista, A. & Pinto, S. (2004) Spatial correlations and synchronization in coupled map lattices with long-rangeinteractions, *Physica A* 343 (2004) pp.201–218

Wang, C .; Li, S.; Ma, J. & Jin, W. (2010). Synchronization transition in degenerate optical parametric oscillators induced by nonlinear coupling, *Applied Mathematics and Computation* 216 (2010), pp. 647–654

Yamapi, R ; Kadji, E. ; Filatrella G. (2010). Stability of the synchronization manifold in nearest neighbor nonidentical van der Pollike oscillators, *Nonlinear Dynamics,* Volume 61, Numbers 1-2, pp. 275-294



Chaotic Systems

Edited by Prof. Esteban Tlelo-Cuautle

ISBN 978-953-307-564-8
Hard cover, 310 pages
Publisher InTech
Published online 14, February, 2011
Published in print edition February, 2011

This book presents a collection of major developments in chaos systems covering aspects on chaotic behavioral modeling and simulation, control and synchronization of chaos systems, and applications like secure communications. It is a good source to acquire recent knowledge and ideas for future research on chaos systems and to develop experiments applied to real life problems. That way, this book is very interesting for students, academia and industry since the collected chapters provide a rich cocktail while balancing theory and applications.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Elena N. Benderskaya and Sofya V. Zhukova (2011). Chaotic Clustering: Fragmentary Synchronization of Fractal Waves, Chaotic Systems, Prof. Esteban Tlelo-Cuautle (Ed.), ISBN: 978-953-307-564-8, InTech, Available from: http://www.intechopen.com/books/chaotic-systems/chaotic-clustering-fragmentary-synchronization-of-fractal-waves

INTECH
open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License, which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen