

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

185,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Inertia-Independent Generalized Dynamic Inversion Control of Spacecraft Attitude Maneuvers

Abdulrahman Bajodah
King Abdulaziz University
Saudi Arabia

1. Introduction

Dynamic inversion (DI) is a transformation from a nonlinear system to an equivalent linear system, performed by means of a change of variables and through feedback. The theory of DI was initially formalized in (Su, 1982) and (Hunt et al., 1983), and its first reported application to spacecraft attitude control problem is due to (Dwyer III, 1984). The methodology is widely accepted among control system practitioners because it substantially facilitates control system design. Additionally, it preserves the nonlinear nature of plant's dynamics and thus it avoids limitations of linearizing approximations.

Classical DI is based on constructing inverse mapping of the controlled plant and augmenting it within the feedback control system. Therefore the linearizing transformation depends heavily on nature of the plant, and it becomes difficult or impossible as complexity of the plant increases. For this reason it may become necessary to introduce simplifying approximations to the plant's mathematical model in order to obtain the DI linearizing transformation, which adversely affects closed loop control system stability and performance characteristics in real implementations of the transformation. Additionally, DI in particular situations must be local in state space, as it is the case for spacecraft attitude dynamics (Dwyer III, 1984).

A paradigm shift was made to DI in (Paielli & Bach, 1993) in the context of spacecraft attitude control. Their approach aims to impose a prescribed dynamics on the errors of spacecraft attitude variables from their desired trajectory values. Rather than inverting the mathematical model of the spacecraft, the desired attitude error dynamics is inverted for the control variables that realize the dynamics. The transformation is global and does not involve deriving inverse equations of motion. It involves simple mathematical inversions of terms that include motion variables and control system design parameters, and therefore it is easier and more systematic than its counterpart.

Nevertheless, a common feature between the above mentioned DI approaches is that the linearizing transformation eliminates nonlinearities from the transformed closed loop system dynamics without distinguishing between types of nonlinearities. For instance, a nonlinearity may cause the spacecraft at a particular time instant to accelerate in a manner that is in favor of the control objective, e.g., in performing a desired maneuver. Yet a needless control effort is made to eliminate that nonlinearity, and an additional control effort is made to satisfy the control objective. This can be extremely disadvantageous as large control signals may cause actuator saturation and control system's failure.

It is therefore desirable to come up with a dynamic inversion control design methodology that provides a global linearizing transformation, alleviates the difficulty of plant's mathematical

model inversion, and requires less control effort to perform the inversion by avoiding blind cancelation of dynamical system's nonlinearity. These features are offered by generalized dynamic inversion (GDI) control. Some basic elements of GDI were introduced in (Bajodah et al., 2005; Bajodah, 2008; 2009), together with particular GDI control designs. Every design exhibits different characteristics in terms of closed loop system stability, performance, and control signal behavior.

The GDI control combines the flexibility of non-square inversion with the simplicity of DI by observing that the inverse system dynamics problem is in general a problem with non-unique solution, i.e., there exist infinite control strategies that realize a prescribed outer system dynamics, and infinite ways by which the system's inner states evolve in time to realize that outer dynamics. Therefore, the original philosophy of dynamic inversion is quite restrictive, and there must exist infinite inverse control laws that realize a servo-constraint dynamics, i.e., the differential equation in system's variables which has its steady state solution satisfies the control design objective.

A GDI spacecraft control design begins by defining a norm measure function of attitude error from desired attitude trajectory. An asymptotically stable linear time varying second-order differential equation in the norm function is prescribed, resembling the desired servo-constraint dynamics. The differential equation is then transformed to a relation that is linear in the control vector by differentiating the norm measure function along the trajectories defined by solution of the spacecraft's state space mathematical model. The Greville formula (Greville, 1959; Ben-Israel & Greville, 2003) is utilized thereafter to invert this relation for the control law required to realize the desired stable linear servo-constraint dynamics.

The Greville formula is capable of modeling solution nonuniqueness to problems where requirements can be satisfied in more than one course of action. For that reason, the formula had remarkable contributions towards advancements in science and engineering. In the arena of robotics, it has been extensively used in analysis and design of kinematically redundant manipulators (Siciliano & Khatib, 2008). Utilization of the formula in the field of analytical dynamics was made by deriving the Udwadia-Kalaba equations of motion for constrained dynamical systems (Udwadia & Kalaba, 1996). Other applications include the evolving subject of pointwise optimal control in the sense of Gauss' principle of least constraints (Gauss, 1829), e.g., (De Sapio et al., 2008; Udwadia, 2008; Peters et al., 2008).

The GDI control law exhibits useful geometrical features of generalized inversion. It consists of auxiliary and particular parts, residing in the nullspace of the inverted matrix and the complementary orthogonal range space of its transpose, respectively. The particular part involves the standard Moore-Penrose generalized inverse (MPGI) (Moore, 1920; Penrose, 1955), and the auxiliary part involves a free null-vector that is projected onto nullspace of the inverted matrix by means of a nullprojection matrix.

Orthogonality of the two parts composing the GDI control law makes it possible for the control system to satisfy multi-design objectives in a noninterfering manner, and makes it possible to merge dynamic inversion with other control design methodologies to enhance closed loop system design features. This is achieved through construction of the null-control vector that appears explicitly in the auxiliary part of the control law. In the present context, the null-control vector provides by its affine parametrization of controls coefficient's nullspace a convenient way to stabilize the inner dynamics of the closed loop control system without affecting servo-constraint realization.

The geometric structure of the GDI control law motivates employing the controls coefficient's

nullprojection matrix to simplify designing the null-control vector. Hence, a positive semidefinite control Lyapunov function that involves the nullprojection matrix is utilized for this purpose. It is shown in Refs. (Iqqidr et al., 1996; Bensoubaya et al., 1999) that a semi-definite Lyapunov function is usable to show stability of a dynamical system if some conditions on system trajectories in the null value of the Lyapunov function are satisfied. Applying Lyapunov direct method (Khalil, 2002) yields a controls coefficient null-projected Lyapunov equation. The equation is solved to obtain a simple control law for global asymptotic stability of inner spacecraft dynamics.

Generalized inversion singularity is a well-known problem in the applications of an MPGI with dynamic matrix elements, and it has been thoroughly investigated in the subject of inverse kinematics, e.g., (Baker & Wampler II, 1988). The reason for MPGI singularity is that a matrix with continuous function elements has discontinuous MPGI function elements. These discontinuities occur whenever the inverted matrix changes rank. Moreover, these discontinuous elements approach infinite values at discontinuities. Accordingly, the corresponding solutions provided by the Greville formula must also be discontinuous and unbounded.

The MPGI singularity forms an obstacle in the way of utilizing the Greville formula in engineering solutions. Several remedies for the problem of generalized inversion instability due to MPGI singularity have been offered in the literature of robotics and control moment gyroscopic devices, in what has become known as the singularity avoidance problem. Remedies are either nullspace parametrization-based, made by proper choices of the null-vector in the auxiliary part of the Greville formula, e.g., (Liegeois, 1977; Mayorga et al., 1995; Yoon & Tsiotras, 2004), or approximation-based, made by modifying the definition of the generalized inverse itself in the particular part of the formula, e.g., (Nakamura & Hanafusa, 1986; Wampler II, 1986; Oh & Vadali, 1991).

A series of solutions to the GDI closed loop instability due to MPGI singularity have been provided in the context of GDI control. One solution is made by deactivating the particular part of the GDI control law in the vicinity of singularity, resulting in discontinuous control laws (Bajodah, 2006). Another solution is presented in (Bajodah, 2008), made by modifying the definition of MPGI by means of a damping factor, resulting in uniformly ultimately bounded attitude trajectory tracking and a tradeoff between generalized inversion stability and closed loop system performance.

The concept of dynamically scaled generalized inversion for GDI singularity avoidance is introduced in (Bajodah, 2010). The concept is based on replacing the MPGI in the particular part of the GDI control law by a growth-controlled dynamically scaled generalized inverse (DSGI), such that the DSGI elements converge uniformly to the standard MPGI elements. The DSGI is constructed by adding a dynamic scaling factor to each denominator of MPGI's elements. The dynamic scaling factor is the p^{th} integer power of a vector p norm of the difference between spacecraft body angular velocity and reference angular velocity. The null-control vector in the auxiliary part of the control law is designed to nullify the dynamic scaling factor such that the DSGI recovers the structure of the MPGI. This causes the particular part of the control law to converge to its projection on the range space of the controls coefficient's MPGI, which drives the attitude variables to satisfy desired servo-constraint stable dynamics, resulting in global attitude trajectory tracking.

This work adopts a generalization of the concept of dynamically scaled generalized inversion, based on augmenting an integrator of the dynamic scaling factor to increase the order of the closed loop dynamics and cause a delay in the scaling factor dynamics. The augmented

stable mode is driven by the spacecraft angular velocity error's norm from reference angular velocity, and is designed to be fast compared to spacecraft dynamics. The dynamic scaling delay caused by the augmented stable mode enhances singularity avoidance performance of dynamic scaling.

The attitude error norm function reduces the order of attitude dynamics from three to one. This feature in addition to skew symmetry of the angular velocity cross product matrix makes the proposed GDI control law totally independent from the inertia matrix. Furthermore, the GDI control methodology does not involve inertia matrix identification in the control design. Two spacecraft attitude maneuvers with different desired asymptotic behaviors are used to illustrate the present GDI control methodology. The first is a rest-to-rest slew maneuver that aims to reorient the spacecraft from an initial attitude to another prescribed attitude. The second is a sinusoidal angular velocity-commanded attitude quaternion trajectory tracking maneuver. Asymptotic tracking is achieved for the first maneuver because the spacecraft angular velocity components asymptotically converge to the reference angular velocity components.

However, since the steady state reference trajectories for the second maneuver are time varying, then spacecraft angular velocity components continue to exhibit small errors from reference angular velocity components during steady state phase of closed loop response. Hence, the stable augmented mode continues to get excited, and asymptotic quaternion attitude tracking is lost. Instead, a practical ultimately bounded tracking is achieved.

This chapter reformulates the GDI spacecraft attitude control methodology in terms of multiplicative quaternion attitude errors that accommodate for spacecraft maneuvers with big changes in attitude angles. Time-varying linear attitude deviation servo-constraint is used to reduce the control load at initial stage of closed loop response. Additionally, a new nullprojection control Lyapunov design is made for constructing the null-control vector, and a modified dynamic scaling factoring is used for improved singularity-free GDI quaternion attitude trajectory tracking.

2. Spacecraft mathematical model

The unit quaternion attitude vector \mathbf{q} that represents the attitude of spacecraft body reference frame \mathcal{B} relative to the inertial reference frame \mathcal{I} is defined as (Wertz, 1980)

$$\mathbf{q} = [q^T \quad q_4]^T \quad (1)$$

where $q \in \mathbb{R}^3$ is given by

$$q = [q_1 \quad q_2 \quad q_3]^T \quad (2)$$

and $q_4 \in \mathbb{R}$. The four attitude unit quaternion scalars q_1, \dots, q_4 are constrained such that

$$\mathbf{q}^T \mathbf{q} = 1. \quad (3)$$

The skew-symmetric cross product matrix ζ^\times that corresponds to a vector $\zeta = [\zeta_1 \quad \zeta_2 \quad \zeta_3]^T$ is defined as

$$\zeta^\times = \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix}. \quad (4)$$

The spacecraft attitude dynamics is governed by the following system of kinematical differential equations (Wertz, 1980)

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{q} \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (q^\times + q_4 I_{3 \times 3}) \\ -q^T \end{bmatrix} \omega, \quad \mathbf{q}(0) = \mathbf{q}_0 \quad (5)$$

where $\omega \in \mathbb{R}^{3 \times 1}$ is the vector of angular velocity of \mathcal{B} relative to \mathcal{I} expressed in \mathcal{B} and $I_{3 \times 3}$ is the identity matrix. The spacecraft inner (angular velocity) dynamics is given by the following system of dynamical differential equations

$$\dot{\omega} = -J^{-1} \omega^\times J \omega + \tau, \quad \omega(0) = \omega_0 \quad (6)$$

where $J \in \mathbb{R}^{3 \times 3}$ is the spacecraft's body-fixed moments of inertia matrix, and $\tau := J^{-1} u \in \mathbb{R}^{3 \times 1}$ is the vector of scaled control torques, where $u \in \mathbb{R}^{3 \times 1}$ contains the applied gas jet actuator torque components about the spacecraft's body axes.

3. Attitude deviation dynamics

The orthogonal rotation transformation matrix $\mathbf{R} \in \mathbf{SO}(3)$ will be used to quantify large spacecraft attitude changes. The matrix \mathbf{R} is expressed in terms of the unit quaternion components as (Wertz, 1980)

$$\mathbf{R}(\mathbf{q}) = (q_4^2 - q^T q) I_{3 \times 3} + 2q q^T - 2q_4 q^\times. \quad (7)$$

Let $\mathbf{q}_d(t) = [q_d^T(t) \quad q_{d4}^T(t)]^T$ be a twice continuously differentiable unit quaternion vector trajectory that represents the prescribed attitude of desired spacecraft frame \mathcal{D} relative to the attitude of \mathcal{I} , where $q_d(t)$ and $q_{d4}(t)$ are such that $\mathbf{q}_d^T(t) \mathbf{q}_d(t) = 1$. The corresponding rotation transformation $\mathbf{R}(\mathbf{q}_d)$ is composed of two consecutive rotation transformations; the transformation $\mathbf{R}(\mathbf{q})$ that brings the attitude of \mathcal{I} to the current attitude of \mathcal{B} , followed by the attitude error transformation $\mathbf{R}(\mathbf{q}_e)$ that brings the attitude of \mathcal{B} to that of \mathcal{D} . Therefore, $\mathbf{R}(\mathbf{q}_d)$ is given by

$$\mathbf{R}(\mathbf{q}_d) = \mathbf{R}(\mathbf{q}_e) \mathbf{R}(\mathbf{q}). \quad (8)$$

Solving for $\mathbf{R}(\mathbf{q}_e)$ yields

$$\mathbf{R}(\mathbf{q}_e) = \mathbf{R}(\mathbf{q}_d) (\mathbf{R}(\mathbf{q}))^{-1} = \mathbf{R}(\mathbf{q}_d) \mathbf{R}^T(\mathbf{q}). \quad (9)$$

In terms of quaternion products, the attitude quaternion error vector $\mathbf{q}_e(\mathbf{q}, t)$ is equivalently given by

$$\mathbf{q}_e(\mathbf{q}, t) = \mathbf{q}^{-1} \otimes \mathbf{q}_d(t), \quad \forall t \in [0, \infty) \quad (10)$$

where \mathbf{q}^{-1} is the spacecraft conjugate attitude quaternion given by

$$\mathbf{q}^{-1} = [-q^T \quad q_4]^T. \quad (11)$$

For convenience, the quaternion product given by (10) is written in the compact form (Show & Juang, 2003)

$$\mathbf{q}_e(\mathbf{q}, t) = \begin{bmatrix} q_e \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_{d4}(t)q - q_4 q_d(t) - q_d^\times(t)q \\ q_4 q_{d4}(t) + q^T q_d(t) \end{bmatrix}. \quad (12)$$

It can be easily verified that the above expression satisfies the quaternion constraint (Show & Juang, 2003)

$$\mathbf{q}_e^T(\mathbf{q}, t) \mathbf{q}_e(\mathbf{q}, t) = (q^T q + q_4^2)(q_d^T(t) q_d(t) + q_{d4}(t)^2) = 1. \quad (13)$$

Let $\omega_r(t) : [0, \infty) \rightarrow \mathbb{R}^3$ be the prescribed angular velocity vector of \mathcal{D} relative to \mathcal{I} expressed in \mathcal{B} . The quaternion error kinematical differential equations are given by

$$\dot{\mathbf{q}}_e = \begin{bmatrix} \dot{q}_e \\ \dot{q}_{e4} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_e^\times + q_{e4} I_{3 \times 3} \\ -q_e^T \end{bmatrix} \omega_e \quad (14)$$

where $\omega_e := \omega - \omega_r(t)$. The reference angular velocity vector $\omega_r(t)$ satisfies

$$\dot{\omega}_r(t) = -J^{-1} \omega_r^\times(t) J \omega_r(t). \quad (15)$$

Therefore,

$$\dot{\omega}_e = \dot{\omega} - \dot{\omega}_r(t) \quad (16)$$

$$= -J^{-1} \omega^\times J \omega + J^{-1} \omega_r^\times(t) J \omega_r(t) + \tau. \quad (17)$$

A scalar attitude deviation norm measure function $\phi : [-1, 1] \rightarrow [0, 1]$ is defined as

$$\phi(q_{e4}) = 1 - q_{e4}^2 \quad (18)$$

and the control objective is to enforce the servo-constraint

$$\phi(q_{e4}) \equiv 0. \quad (19)$$

From (13), the same servo-constraint requirement can also be written as

$$q_e \equiv \mathbf{0}_{3 \times 1}. \quad (20)$$

The first two time derivatives of ϕ along the spacecraft error trajectories given by the solutions of (14) and (17) are

$$\dot{\phi} = q_{e4} q_e^T \omega_e \quad (21)$$

and

$$\ddot{\phi} = \frac{1}{2} \omega_e^T (q_{e4}^2 I_{3 \times 3} - q_e q_e^T) \omega_e + q_{e4} q_e^T (-J^{-1} \omega^\times J \omega + J^{-1} \omega_r^\times J \omega_r + \tau). \quad (22)$$

Skew symmetries of the cross product matrices ω^\times and ω_r^\times imply that the corresponding terms in $\ddot{\phi}$ are zeros. Hence, the expression of (22) reduces to

$$\ddot{\phi} = \frac{1}{2} \omega_e^T (q_{e4}^2 I_{3 \times 3} - q_e q_e^T) \omega_e + q_{e4} q_e^T \tau. \quad (23)$$

A desired dynamics of ϕ that leads to asymptotic realization of the servo-constraint given by (19) is described to be stable second-order in the general functional form given by

$$\ddot{\phi} = \mathcal{L}(\phi, \dot{\phi}, t) \quad (24)$$

where \mathcal{L} is continuous in its arguments. A special choice of $\mathcal{L}(\phi, \dot{\phi}, t)$ is

$$\mathcal{L}(\phi, \dot{\phi}, t) = -c_1(t) \dot{\phi} - c_2(t) \phi \quad (25)$$

where $c_1(t)$ and $c_2(t)$ are continuous scalar functions. With this choice of $\mathcal{L}(\phi, \dot{\phi}, t)$, the stable attitude deviation servo-constraint dynamics given by (24) becomes linear in the form

$$\ddot{\phi} + c_1(t)\dot{\phi} + c_2(t)\phi = 0. \quad (26)$$

With ϕ , $\dot{\phi}$, and $\ddot{\phi}$ given by (18), (21), and (23), it is possible to write (26) in the pointwise-linear form

$$\mathcal{A}(\mathbf{q}_e)\tau = \mathcal{B}(\mathbf{q}_e, \omega_e), \quad (27)$$

where the vector valued the controls coefficient function $\mathcal{A}(\mathbf{q}_e)$ is given by

$$\mathcal{A}(\mathbf{q}_e) = q_{e4}q_e^T \quad (28)$$

and the scalar valued controls load function $\mathcal{B}(\mathbf{q}_e, \omega_e)$ is given by

$$\mathcal{B}(\mathbf{q}_e, \omega_e) = -\frac{1}{2}\omega_e^T \left(q_{e4}^2 I_{3 \times 3} - q_e q_e^T \right) \omega_e - c_1(t) q_{e4} q_e^T \omega_e - c_2(t)(1 - q_{e4}^2). \quad (29)$$

4. Generalized dynamic inversion attitude control

The MPGI-based Greville formula is used now to obtain a preliminary form of GDI spacecraft attitude control laws.

Proposition 1 (Linearly parameterized attitude control laws) *The infinite set of all control laws that globally realize the attitude deviation servo-constraint dynamics given by (26) by the spacecraft equations of motion is parameterized by an arbitrarily chosen null-control vector $y \in \mathbb{R}^{3 \times 1}$ as*

$$\tau = \mathcal{A}^+(\mathbf{q}_e)\mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e)y \quad (30)$$

where “ \mathcal{A}^+ ” stands for the MPGI of the controls coefficient (abbreviated as CCGI), and is given by

$$\mathcal{A}^+(\mathbf{q}_e) = \begin{cases} \frac{\mathcal{A}^T(\mathbf{q}_e)}{\mathcal{A}(\mathbf{q}_e)\mathcal{A}^T(\mathbf{q}_e)}, & \mathcal{A}(\mathbf{q}_e) \neq \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{3 \times 1}, & \mathcal{A}(\mathbf{q}_e) = \mathbf{0}_{1 \times 3} \end{cases} \quad (31)$$

and $\mathcal{P}(\mathbf{q}_e)$ is the corresponding controls coefficient nullprojection matrix given by

$$\mathcal{P}(\mathbf{q}_e) = I_{3 \times 3} - \mathcal{A}^+(\mathbf{q}_e)\mathcal{A}(\mathbf{q}_e). \quad (32)$$

Proof 1 *Multiplying both sides of (30) by $\mathcal{A}(\mathbf{q}_e)$ recovers the algebraic system given by (27). Therefore, τ enforces the attitude deviation servo-constraint dynamics given by (26) for all $\mathcal{A}(\mathbf{q}_e) \neq \mathbf{0}_{1 \times 3}$.*

The controls coefficient nullprojector $\mathcal{P}(\mathbf{q}_e)$ projects the null-control vector y onto the nullspace of the controls coefficient $\mathcal{A}(\mathbf{q}_e)$. Therefore, the choice of y does not affect realizability of the linear attitude deviation norm measure dynamics given by (26). Nevertheless, the choice of y substantially affects transient state response and spacecraft inner stability, i.e., stability of the closed loop dynamical subsystem

$$\dot{\omega} = -J^{-1}\omega^\times J\omega + \mathcal{A}^+(\mathbf{q}_e)\mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e)y \quad (33)$$

obtained by substituting (30) in (6).

5. Generalized inverse instability

The expression given by (28) for the controls coefficient implies that if the dynamics given by (26) is realizable by spacecraft equations of motion, then

$$\lim_{\phi \rightarrow 0} \mathcal{A}(\mathbf{q}_e) = \mathbf{0}_{1 \times 3}. \quad (34)$$

Accordingly, the discontinuous expression of $\mathcal{A}^+(\mathbf{q}_e)$ given by (31) implies that for any initial condition $\mathcal{A}(\mathbf{q}_e) \neq \mathbf{0}_{1 \times 3}$, state trajectories of a continuous closed loop control system in the form given by (5) and (33) must evolve such that

$$\lim_{\phi \rightarrow 0} \mathcal{A}^+(\mathbf{q}_e) = \infty_{3 \times 1}. \quad (35)$$

That is, $\mathcal{A}^+(\mathbf{q}_e)$ must go unbounded as the spacecraft dynamics approaches steady state. This is a source of instability for the closed loop system because it causes the control law expression given by (30) to become unbounded. One solution to this problem is made by switching the value of the CCGI according to (31) to $\mathcal{A}^+(\mathbf{q}_e) = \mathbf{0}_{3 \times 1}$ when the controls coefficient $\mathcal{A}(\mathbf{q}_e)$ approaches singularity, which implies deactivating the particular part of the control law as the closed loop system reaches steady state, leading to a discontinuous control law (Bajodah, 2006).

Alternatively, a solution is made by replacing the Moore-Penrose generalized inverse in (30) by a damped generalized inverse (Bajodah, 2008), resulting in uniformly ultimately bounded trajectory tracking errors, and a tradeoff between generalized inversion stability and steady state tracking performance. A solution to this problem that avoids control law discontinuity and improves singularity avoiding trajectory tracking is presented in (Bajodah, 2010), made by replacing the MPGI in (30) by a growth-controlled dynamically scaled generalized inverse. A generalization of the dynamically scaled generalized inverse is presented in the following section.

6. Generalized inverse singularity avoidance by stable mode augmentation

The dynamically scaled generalized inverse provides the necessary generalized inversion singularity avoidance to the GDI control design.

Definition 1 (Dynamically scaled generalized inverse) The DSGI $\mathcal{A}_s^+(\mathbf{q}_e, \nu)$ is given by

$$\mathcal{A}_s^+(\mathbf{q}_e, \nu) = \frac{\mathcal{A}^T(\mathbf{q}_e)}{\mathcal{A}(\mathbf{q}_e)\mathcal{A}^T(\mathbf{q}_e) + \nu} \quad (36)$$

where ν satisfies the asymptotically stable dynamics

$$\dot{\nu} = -a\nu + \|\omega_e\|_p^p, \quad a > 0, \quad p \in \mathbb{Z}^+. \quad (37)$$

The positive integer p is the generalized inversion dynamic scaling index, and $\|\cdot\|_p$ is the vector p norm.

Properties of dynamically scaled generalized inverse

The following properties can be verified by direct evaluation of the CCGI $\mathcal{A}^+(\mathbf{q}_e)$ given by (31) and its dynamic scaling $\mathcal{A}_s^+(\mathbf{q}_e, \nu)$ given by (36).

1. $\mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{A}(\mathbf{q}_e) \mathcal{A}^+(\mathbf{q}_e) = \mathcal{A}_s^+(\mathbf{q}_e, \nu)$
2. $\mathcal{A}^+(\mathbf{q}_e) \mathcal{A}(\mathbf{q}_e) \mathcal{A}_s^+(\mathbf{q}_e, \nu) = \mathcal{A}_s^+(\mathbf{q}_e, \nu)$
3. $(\mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{A}(\mathbf{q}_e))^T = \mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{A}(\mathbf{q}_e)$
4. $\lim_{\|\omega_e\|_p \rightarrow 0} \mathcal{A}_s^+(\mathbf{q}_e, \nu) = \mathcal{A}^+(\mathbf{q}_e)$.

7. Dynamically scaled generalized inverse control

The dynamically scaled generalized inverse control law is obtained by replacing the CCGI in the particular part of the expression given by (30) by the DSGI as

$$\tau_s = \mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e) \mathbf{y} \quad (38)$$

resulting in the following spacecraft closed loop dynamical equations

$$\dot{\omega} = -J^{-1} \omega^\times J \omega + \mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e) \mathbf{y}. \quad (39)$$

Proposition 2 (Asymptotic Attitude Trajectory Tracking) *If the null-control vector \mathbf{y} in the control law expression given by (38) is chosen such*

$$\lim_{t \rightarrow \infty} \omega_e = \mathbf{0}_{3 \times 1} \quad (40)$$

then

$$\lim_{t \rightarrow \infty} q_e = \mathbf{0}_{3 \times 1}. \quad (41)$$

Proof 2 Let ϕ_s be a norm measure function of the attitude deviation obtained by applying the control law given by (38) to the spacecraft equations of motion (5) and (6), and let $\dot{\phi}_s, \ddot{\phi}_s$ be its first two time derivatives. Therefore,

$$\phi_s := \phi_s(\mathbf{q}_e) = \phi(\mathbf{q}_e) \quad (42)$$

$$\dot{\phi}_s := \dot{\phi}_s(\mathbf{q}_e, \omega_e) = \dot{\phi}(\mathbf{q}_e, \omega_e) \quad (43)$$

$$\ddot{\phi}_s := \ddot{\phi}_s(\mathbf{q}_e, \omega_e, \tau_s) = \ddot{\phi}(\mathbf{q}_e, \omega_e, \tau) + \mathcal{A}(\mathbf{q}_e) \tau_s - \mathcal{A}(\mathbf{q}_e) \tau \quad (44)$$

where τ and τ_s are given by (30) and (38), respectively. Adding $c_1(t)\dot{\phi}_s + c_2(t)\phi_s$ to both sides of (44) yields

$$\ddot{\phi}_s + c_1(t)\dot{\phi}_s + c_2(t)\phi_s = \ddot{\phi} + c_1(t)\dot{\phi} + c_2(t)\phi + \mathcal{A}(\mathbf{q}_e) \tau_s - \mathcal{A}(\mathbf{q}_e) \tau \quad (45)$$

$$= \mathcal{A}(\mathbf{q}_e) [\tau_s - \tau]. \quad (46)$$

Therefore, boundedness of the expression of $\mathcal{A}(\mathbf{q}_e)$ given by (28) in addition to satisfaction of (40) imply that

$$\lim_{t \rightarrow \infty} [\ddot{\phi}_s + c_1(t)\dot{\phi}_s + c_2(t)\phi_s] = \lim_{t \rightarrow \infty} [\mathcal{A}(\mathbf{q}_e) [\tau_s - \tau]] = 0 \quad (47)$$

resulting in

$$\lim_{t \rightarrow \infty} \phi_s = 0 \quad (48)$$

and therefore, (41) follows for all permissible initial attitude quaternion vectors $q_0 \in \mathbb{R}^3$. The same conclusion is obtained by multiplying both sides of (38) by $\mathcal{A}(\mathbf{q}_e)$, resulting in

$$\mathcal{A}(\mathbf{q}_e) \tau_s = \mathcal{A}(\mathbf{q}_e) \mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{B}(\mathbf{q}_e, \omega_e) \quad (49)$$

where

$$\mathcal{A}(\mathbf{q}_e)\mathcal{A}_s^+(\mathbf{q}_e, \nu) = \frac{\mathcal{A}(\mathbf{q}_e)\mathcal{A}^T(\mathbf{q}_e)}{\mathcal{A}(\mathbf{q}_e)\mathcal{A}^T(\mathbf{q}_e) + \nu}. \quad (50)$$

Therefore,

$$0 < \mathcal{A}(\mathbf{q}_e)\mathcal{A}_s^+(\mathbf{q}_e, \nu) \leq 1 \quad (51)$$

and

$$\lim_{\omega_e \rightarrow \mathbf{0}_{3 \times 1}} \mathcal{A}(\mathbf{q}_e)\mathcal{A}_s^+(\mathbf{q}_e, \nu) = 1. \quad (52)$$

Dividing both sides of (49) by $\mathcal{A}(\mathbf{q}_e)\mathcal{A}_s^+(\mathbf{q}_e, \nu)$ yields

$$\mathcal{A}(\mathbf{q}_e)\bar{\tau} = \mathcal{B}(\mathbf{q}_e, \omega_e) \quad (53)$$

where $\mathcal{A}(\mathbf{q}_e)$ and $\mathcal{B}(\mathbf{q}_e, \omega_e)$ are the same controls coefficient and controls load in (27), and

$$\bar{\tau} = \frac{\tau_s}{\mathcal{A}(\mathbf{q}_e)\mathcal{A}_s^+(\mathbf{q}_e, \nu)}. \quad (54)$$

Furthermore, (52) implies that

$$\lim_{\omega_e \rightarrow \mathbf{0}_{3 \times 1}} \bar{\tau} = \lim_{\omega_e \rightarrow \mathbf{0}_{3 \times 1}} \tau_s = \tau. \quad (55)$$

Therefore, $\bar{\tau}$ in the algebraic system given by (53) asymptotically converges to τ , recovering the algebraic system given by (27), and resulting in asymptotic convergence of $\phi_s(t)$ to $\phi_s = \phi = 0$, and q to $q_d(t)$.

Proposition 2 states that using the DSGI $\mathcal{A}_s^+(\mathbf{q}_e, \nu)$ in the attitude control law yields the same attitude convergence property that is obtained by using the CCGI $\mathcal{A}^+(\mathbf{q}_e)$, provided that the condition given by (40) is satisfied. A design of the null-control vector y is made in the next section to guarantee global satisfaction of the condition given by (40).

Remark 1 It is well-known that topological obstruction of the attitude rotation matrix precludes the existence of globally stable equilibria for the attitude dynamics (Bhat & Bernstein, 2000). Therefore, although the servo-constraint attitude deviation dynamics given by (26) is globally realizable, there exists no null-control that renders the spacecraft attitude dynamics globally stable. In particular, if $q_d(t) \equiv \mathbf{0}_{3 \times 1}$ then for any null-control vector y there exists an attitude vector q_0 such that the closed loop system given by (5) and (39) is unstable in the sense of Lyapunov.

8. Nullprojection Lyapunov control design

A Lyapunov-based design of null-control vector y is introduced in this section to enforce spacecraft inner stability. Let y be chosen as

$$y = K\omega_e(t) \quad (56)$$

where $K \in \mathbb{R}^{3 \times 3}$ is a matrix gain that is to be determined. Hence, a class of control laws that realize the attitude deviation norm measure dynamics given by (26) is obtained by substituting this choice of y in (38) such that

$$\tau_s = \mathcal{A}_s^+(\mathbf{q}_e, \nu)\mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e)K\omega_e(t). \quad (57)$$

Consequently, a class of spacecraft closed loop dynamical subsystems that realize the servo-constraint dynamics given by (26) is obtained by substituting the control law given by (57) in (6), and it takes the form

$$\dot{\omega} = -J^{-1}\omega^\times J\omega + \mathcal{A}_s^+(\mathbf{q}_e, \nu)\mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e)K\omega_e(t) \quad (58)$$

and the closed loop error dynamics $\dot{\omega}_e$ is obtained from (17) as

$$\dot{\omega}_e = -J^{-1}\omega^\times J\omega + J^{-1}\omega_r^\times J\omega_r + \mathcal{A}_s^+(\mathbf{q}_e, \nu)\mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}(\mathbf{q}_e)K\omega_e. \quad (59)$$

The matrix gain K is synthesized by utilizing the positive-semidefinite control Lyapunov function

$$V(\mathbf{q}_e, \omega_e) = \omega_e^T \mathcal{P}(\mathbf{q}_e) \omega_e. \quad (60)$$

Evaluating the time derivative of $V(\mathbf{q}_e, \omega_e)$ along solution trajectories of the error dynamics given by (59) yields

$$\begin{aligned} \dot{V}(\mathbf{q}_e, \omega_e) = 2\omega_e^T \mathcal{P}(\mathbf{q}_e) \Big[& -J^{-1}\omega^\times J\omega + J^{-1}\omega_r^\times(t)J\omega_r(t) \\ & + \mathcal{A}_s^+(\mathbf{q}_e, \nu)\mathcal{B}(\mathbf{q}_e, \omega_e) \Big] + 2\omega_e^T \mathcal{P}(\mathbf{q}_e)K\omega_e + \omega_e^T \dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)\omega_e \end{aligned} \quad (61)$$

where $\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)$ is obtained by differentiating the elements of $\mathcal{P}(\mathbf{q}_e)$ along attitude trajectory solutions of the closed loop kinematical subsystem given by (14). Skew symmetry of the cross product matrix $[\cdot]^\times$, the nullprojection property of $\mathcal{P}(\mathbf{q}_e)$, and the second property of $\mathcal{A}_s^+(\mathbf{q}_e, \omega_e)$ imply that the first term in the above equation is zero. Therefore,

$$\dot{V}(\mathbf{q}_e, \omega_e) = 2\omega_e^T \mathcal{P}(\mathbf{q}_e)K\omega_e + \omega_e^T \dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)\omega_e. \quad (62)$$

Because $V(\mathbf{q}_e, \omega_e)$ is only positive semidefinite, it is impossible to design a matrix gain K that renders $\dot{V}(\mathbf{q}_e, \omega_e)$ negative definite. Nevertheless, a matrix gain K that renders $\dot{V}(\mathbf{q}_e, \omega_e)$ negative semidefinite guarantees Lyapunov stability of $\omega_e = \mathbf{0}_{3 \times 1}$ if it asymptotically stabilizes $\omega_e = \mathbf{0}_{3 \times 1}$ over the invariant set of \mathbf{q}_e and ω_e values on which $V(\mathbf{q}_e, \omega_e) = 0$. Moreover, the same gain matrix asymptotically stabilizes $\omega_e = \mathbf{0}_{3 \times 1}$ if and only if it asymptotically stabilizes $\omega_e = \mathbf{0}_{3 \times 1}$ over the largest invariant set of \mathbf{q}_e and ω_e values on which $\dot{V}(\mathbf{q}_e, \omega_e) = 0$ (Iqdr et al., 1996).

Proposition 3 Let $K = K(\mathbf{q}_e, \omega_e)$ be a full-rank normal matrix gain, i.e., $KK^T = K^TK$ for all $t \geq 0$. Then the equilibrium point $\omega_e = \mathbf{0}_{3 \times 1}$ of the closed loop error dynamics given by (59) is asymptotically stable over the invariant set of \mathbf{q}_e , and ω_e values on which $V(\mathbf{q}_e, \omega_e) = 0$.

Proof 3 Since the matrix $\mathcal{P}(\mathbf{q}_e)$ is idempotent, the function $V(\mathbf{q}_e, \omega_e)$ can be rewritten as

$$V(\mathbf{q}_e, \omega_e) = \omega_e^T \mathcal{P}(\mathbf{q}_e) \omega_e = \omega_e^T \mathcal{P}(\mathbf{q}_e) \mathcal{P}(\mathbf{q}_e) \omega_e \quad (63)$$

which implies that

$$V(\mathbf{q}_e, \omega_e) = 0 \Leftrightarrow \mathcal{P}(\mathbf{q}_e)\omega_e = \mathbf{0}_{3 \times 1}. \quad (64)$$

Therefore,

$$V(\mathbf{q}_e, \omega_e) = 0 \Leftrightarrow \omega_e \in \mathcal{N}(\mathcal{P}(\mathbf{q}_e)) \quad (65)$$

where $\mathcal{N}(\cdot)$ refers to matrix nullspace. Since the matrix $K(\mathbf{q}_e, \omega_e)$ is normal and of full-rank, it preserves matrix range space and nullspace under multiplication. Accordingly,

$$\mathcal{N}(\mathcal{P}(\mathbf{q}_e)) = \mathcal{N}(\mathcal{P}(\mathbf{q}_e)K(\mathbf{q}_e, \omega_e)) \quad (66)$$

which implies from (64) that

$$V(\mathbf{q}_e, \omega_e) = 0 \Leftrightarrow \mathcal{P}(\mathbf{q}_e)K(\mathbf{q}_e, \omega_e)\omega_e = \mathbf{0}_{3 \times 1}. \quad (67)$$

Therefore, the last term in the closed loop error dynamics given by (59) is the zero vector, and the closed loop error dynamics becomes

$$\dot{\omega}_e = -J^{-1}\omega^\times J\omega + J^{-1}\omega_r^\times(t)J\omega_r(t) + \mathcal{A}_s^+(\mathbf{q}_e, \nu)\mathcal{B}(\mathbf{q}_e, \omega_e). \quad (68)$$

On the other hand, since

$$\mathcal{N}(\mathcal{P}(\mathbf{q}_e)) = \mathcal{R}(\mathcal{A}^T(\mathbf{q}_e)) \quad (69)$$

it follows from (65) that

$$V(\mathbf{q}_e, \omega_e) = 0 \Leftrightarrow \omega_e \in \mathcal{R}(\mathcal{A}^T(\mathbf{q}_e)). \quad (70)$$

Accordingly, $V(\mathbf{q}_e, \omega_e) = 0$ if and only if there exists a continuous scalar function $a(t)$, $t \geq 0$, satisfying

$$0 < |a(t)| < \infty \quad (71)$$

such that

$$\omega_e = a(t)\mathcal{A}^T(\mathbf{q}_e). \quad (72)$$

Since the expression of $\mathcal{A}(\mathbf{q}_e)$ given by (28) is bounded for all values of \mathbf{q}_e , it follows from (72) that ω_e is also bounded. Therefore, the trajectory of ω_e must remain in a finite region, and it follows from the Poincare-Bendixon theorem (Slotine & Li, 1991) that the trajectory goes to the equilibrium point $\omega_e = \mathbf{0}_{3 \times 1}$.

Theorem 1 (CCNP Lyapunov control design) Let the nullprojection gain matrix $K(\mathbf{q}_e, \omega_e)$ be

$$K(\mathbf{q}_e, \omega_e) = -\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e) - \sigma_{\max}(\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e))I_{3 \times 3} - Q \quad (73)$$

where $\sigma_{\max}(\cdot)$ denotes the maximum singular value, and $Q \in \mathbb{R}^{3 \times 3}$ is arbitrary positive definite. Then the equilibrium point $\omega_e = \mathbf{0}_{3 \times 1}$ of the closed loop error dynamics given by (59) is globally asymptotically stable, and

$$\lim_{t \rightarrow \infty} q_e = \mathbf{0}_{3 \times 1}. \quad (74)$$

Proof 4 Let $\mathcal{Q}(\mathbf{q}_e, \omega) : \mathbb{R}^{4 \times 1} \times \mathbb{R}^{3 \times 1} \rightarrow \mathbb{R}^{3 \times 3}$ be a positive semidefinite matrix function. Then, a matrix gain K that enforces negative semidefiniteness of $\dot{V}(\mathbf{q}_e, \omega_e)$ is obtained by setting

$$\dot{V}(\mathbf{q}_e, \omega_e) = 2\omega_e^T \mathcal{P}(\mathbf{q}_e)K\omega_e + \omega_e^T \dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)\omega_e = -2\omega_e^T \mathcal{Q}(\mathbf{q}_e, \omega_e)\omega_e. \quad (75)$$

Hence, K satisfies the following Lyapunov equation

$$2\mathcal{P}(\mathbf{q}_e)K + \dot{\mathcal{P}}(\mathbf{q}_e, \omega_e) + 2\mathcal{Q}(\mathbf{q}_e, \omega_e) = \mathbf{0}_{3 \times 3}. \quad (76)$$

Consistency of the above-written nullprojection equation implies that every term maps into $\mathcal{P}(\mathbf{q}_e)$. The range space of $\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)$ is a subset of the range space of $\mathcal{P}(\mathbf{q}_e)$. This is shown by writing

$$\mathcal{P}(\mathbf{q}_e) = \mathcal{P}(\mathbf{q}_e)\mathcal{P}(\mathbf{q}_e) \Rightarrow \dot{\mathcal{P}}(\mathbf{q}_e, \omega_e) = 2\mathcal{P}(\mathbf{q}_e)\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e) \quad (77)$$

so that

$$\mathcal{R}[\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)] = \mathcal{R}[\mathcal{P}(\mathbf{q}_e)\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)] \subseteq \mathcal{R}[\mathcal{P}(\mathbf{q}_e)] \quad (78)$$

where $\mathcal{R}(\cdot)$ refers to matrix range space. Moreover, for $\mathcal{Q}(\mathbf{q}_e, \omega_e)$ to map into the range space of $\mathcal{P}(\mathbf{q}_e)$, then there must exist a positive definite matrix function $\bar{\mathcal{Q}}(\mathbf{q}_e, \omega_e) : \mathbb{R}^{4 \times 1} \times \mathbb{R}^{3 \times 1} \rightarrow \mathbb{R}^{3 \times 3}$ such that a polar decomposition of $\mathcal{Q}(\mathbf{q}_e, \omega_e)$ is given by

$$\mathcal{Q}(\mathbf{q}_e, \omega_e) = \mathcal{P}(\mathbf{q}_e)\bar{\mathcal{Q}}(\mathbf{q}_e, \omega_e). \quad (79)$$

By substituting the expressions of $\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e)$ and $\mathcal{Q}(\mathbf{q}_e, \omega_e)$ given by (77) and (79) in (76), a solution for K that renders $\dot{V}(\mathbf{q}_e, \omega_e)$ negative semidefinite is obtained as

$$K(\mathbf{q}_e, \omega_e) = -\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e) - \bar{\mathcal{Q}}(\mathbf{q}_e, \omega_e). \quad (80)$$

Furthermore, it follows from Proposition 3 that K guarantees asymptotic stability of $\omega_e = \mathbf{0}_{3 \times 1}$ over the invariant set of \mathbf{q}_e , and ω_e values on which $V(\mathbf{q}_e, \omega_e) = 0$ if K remains nonsingular for all $t \geq 0$. This is achieved by choosing $\bar{\mathcal{Q}}(\mathbf{q}_e, \omega_e)$ as

$$\bar{\mathcal{Q}}(\mathbf{q}_e, \omega_e) = \sigma_{\max}(\dot{\mathcal{P}}(\mathbf{q}_e, \omega_e))I_{3 \times 3} + \mathcal{Q} \quad (81)$$

so that $K(\mathbf{q}_e, \omega_e)$ remains negative definite. Substituting the above written expression for $\bar{\mathcal{Q}}(\mathbf{q}_e, \omega_e)$ in (80) results in the expression of $K(\mathbf{q}_e, \omega_e)$ given by (73). Therefore, in addition to rendering $\dot{V}(\mathbf{q}_e, \omega_e)$ negative semidefinite, $K(\mathbf{q}_e, \omega_e)$ guarantees asymptotic stability of $\omega_e = \mathbf{0}_{3 \times 1}$ over the invariant set of \mathbf{q}_e and ω_e values on which $V(\mathbf{q}_e, \omega_e) = 0$, and Lyapunov stability of $\omega_e = \mathbf{0}_{3 \times 1}$ follows (Iqqidr et al., 1996). Since $V(\mathbf{q}_e, \omega_e)$ is radially unbounded with respect to ω_e , Lyapunov stability of $\omega_e = \mathbf{0}_{3 \times 1}$ is global. Moreover, it is noticed from the expression of $\dot{V}(\mathbf{q}_e, \omega_e)$ given by (61) and from (78) that the largest invariant set of \mathbf{q}_e and ω_e on which $\dot{V}(\mathbf{q}_e, \omega_e) = 0$ is the same invariant set on which $V(\mathbf{q}_e, \omega_e) = 0$, implying global asymptotic stability of the equilibrium point $\omega_e = \mathbf{0}_{3 \times 1}$ (Iqqidr et al., 1996). Global asymptotic convergence of the attitude vector q to the desired attitude vector $q_d(t)$ follows from Proposition 2.

9. Damped controls coefficient nullprojector

Although the CCNP $\mathcal{P}(q_e)$ has bounded elements, dependency of CCNP on the unbounded vector $\mathcal{A}^+(\mathbf{q}_e)$ may cause undesirable behavior of the auxiliary part in the control law τ_s during steady state tracking response of time varying trajectories. For this reason, a damped controls coefficient nullprojector (DCCN) $\mathcal{P}_d(\mathbf{q}_e, \epsilon)$ is used in place of $\mathcal{P}(\mathbf{q}_e)$ in (57). The DCCN is defined as

$$\mathcal{P}_d(\mathbf{q}_e, \epsilon) := I_{3 \times 3} - \mathcal{A}_d^+(\mathbf{q}_e, \epsilon)\mathcal{A}(\mathbf{q}_e) \quad (82)$$

where ϵ is a small positive number, and $\mathcal{A}_d^+(\mathbf{q}_e, \epsilon)$ is given by

$$\mathcal{A}_d^+(\mathbf{q}_e, \epsilon) := \frac{\mathcal{A}^T(\mathbf{q}_e)}{\mathcal{A}(\mathbf{q}_e)\mathcal{A}^T(\mathbf{q}_e) + \epsilon}. \quad (83)$$

Therefore,

$$\lim_{\epsilon \rightarrow 0} \mathcal{A}_d^+(\mathbf{q}_e, \epsilon) = \mathbf{0}_{3 \times 1} \quad (84)$$

and consequently,

$$\lim_{\epsilon \rightarrow 0} \mathcal{P}_d(\mathbf{q}_e, \epsilon) = I_{3 \times 3}. \quad (85)$$

Hence, the DCCN maps the null-control vector to itself in steady state phase of response, during which the auxiliary part of the control law converges to the null-control vector.

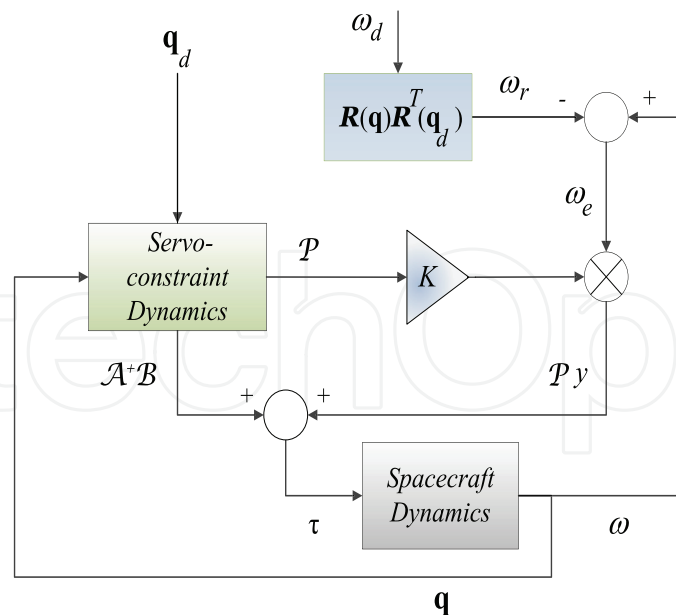


Fig. 1. Schematic of GDI spacecraft attitude control system

Independency of nullprojection on the attitude state of the spacecraft substantially eliminates unnecessary abrupt behavior of the control vector. Replacing $\mathcal{P}(\mathbf{q}_e)$ by $\mathcal{P}_d(\mathbf{q}_e, \epsilon)$ in the control expression given by (57) yields the following form of the GDI control law

$$\tau_{sd} = \mathcal{A}_s^+(\mathbf{q}_e, \nu) \mathcal{B}(\mathbf{q}_e, \omega_e) + \mathcal{P}_d(\mathbf{q}_e, \epsilon) K \omega_e(t). \quad (86)$$

A schematic of the GDI spacecraft attitude control system is shown in Fig. 1.

10. Tuning the GDI control design parameters

When the second-order deviation dynamics given by (26) is chosen to be time invariant, then increasing the value of the constant c_1 increases the damping ratio of closed loop spacecraft dynamics. Additionally, increasing the value of c_2 improves steady state trajectory tracking accuracy. Nevertheless, excessively large values of c_1 and c_2 require large control torque inputs and cause large amplitude oscillations of spacecraft body angular velocity components, particularly during the initial phase of response when the state deviation variable ϕ and its time derivative $\dot{\phi}$ are at their biggest magnitudes, i.e., when the controls load $\mathcal{B}(\mathbf{q}_e, \omega_e)$ has a large value. Accordingly, to increase damping and to improve steady state tracking with simultaneous avoidance of these drawbacks, the coefficients $c_1(t)$ and $c_2(t)$ are chosen to be of the form $c_1(t) = C_1(1 - e^{-\alpha_1 t})$ and $c_2(t) = C_2(1 - e^{-\alpha_2 t})$, where C_1 , C_2 , α_1 , and α_2 are positive constants. Hence, $c_1(0) = 0$ and $c_2(0) = 0$, which substantially decreases the magnitude of $\mathcal{B}(\mathbf{q}_e, \omega_e)$.

11. Numerical simulations

The spacecraft model has inertia scalars $I_{11} = 200 \text{ Kg.m}^2$, $I_{22} = 150 \text{ Kg.m}^2$, $I_{33} = 175 \text{ Kg.m}^2$, $I_{12} = -100 \text{ Kg.m}^2$, $I_{13} = I_{23} = 0 \text{ Kg.m}^2$. The first maneuver considered is a rest-to-rest slew maneuver, aiming to reorient the spacecraft at the initial attitude given by $\mathbf{q}(0) = \mathbf{q}_0$ to a different attitude given by $\mathbf{q}_d(T)$, where T is duration of the maneuver. It is required that the spacecraft quaternion attitude variables follow the trajectories given by the following

transition functions (McInnes, 1998)

$$q_d(t) = q_d(0) + \left[10 \left(\frac{t}{T} \right)^3 - 15 \left(\frac{t}{T} \right)^4 + 6 \left(\frac{t}{T} \right)^5 \right] [q_d(T) - q_d(0)] \quad (87)$$

$$q_{4d}(t) = \sqrt{1 - q_d^T(t)q_d(t)}. \quad (88)$$

The desired quaternion attitude variables and their derivatives satisfy the differential equations

$$\dot{\mathbf{q}}_d(t) = \begin{bmatrix} \dot{q}_d(t) \\ \dot{q}_{4d}(t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (q_d^\times(t) + q_{4d}(t)I_{3 \times 3}) \\ -q_d^T(t) \end{bmatrix} \omega_d(t) \quad (89)$$

where $\omega_d(t)$ is the angular velocity of \mathcal{D} relative to \mathcal{I} expressed in \mathcal{D} . Equations (89) can be inverted to calculate $\omega_d(t)$ as (Behal et al., 2002)

$$\omega_d(t) = 2(q_{4d}(t)\dot{q}_d(t) - q_d(t)\dot{q}_{4d}(t)) - 2q_d^\times(t)\dot{q}_d(t). \quad (90)$$

Accordingly, $\omega_r(t)$ is obtained as

$$\omega_r(t) = \mathbf{R}(\mathbf{q})\mathbf{R}^T(\mathbf{q}_d)\omega_d(t) \quad (91)$$

and is used in the control expression τ_{sd} given by (86). Values of second-order attitude deviation dynamics functions are chosen to be $c_1(t) = 20(1 - e^{-0.07t})$ and $c_2(t) = 10(1 - e^{-0.07t})$. With $q_d(0) = [0.7 \ -0.4 \ 0.5]^T$, $q_d(T) = [0 \ 0 \ 0]^T$, $T = 60$ sec., $Q = 0.1 \times I_{3 \times 3}$, $a = 100$, $p = 2$, $\epsilon = 10^{-4}$ and an arbitrary initial attitude, Fig. 2 shows the excellent asymptotic tracking of attitude quaternion variables q_1, \dots, q_4 trajectories. Figs. 3 and 4 show the corresponding time histories of spacecraft's angular velocity components $\omega_1, \omega_2, \omega_3$ and the GDI control variables u_1, u_2, u_3 .

The second maneuver considered is a trajectory tracking maneuver. The reference trajectory is determined via a sinusoidal trajectory generator at the angular velocity level that is given by

$$\omega_d(t) = [\cos(0.1t) \ -\cos(0.2t) \ \sin(0.3t)]^T. \quad (92)$$

Values of second-order attitude deviation dynamics functions are chosen to be $c_1(t) = 45(1 - e^{-0.40t})$ and $c_2(t) = 40(1 - e^{-0.02t})$. With $Q = 0.1 \times I_{3 \times 3}$, $a = 200$, $p = 2$, $\epsilon = 10^{-4}$ and arbitrary initial conditions, Fig. 5 shows the attitude quaternion error variables q_{e1}, \dots, q_{e4} trajectories. Figs. 6 and 7 show the corresponding time histories of spacecraft's angular velocity components $\omega_1, \omega_2, \omega_3$ and the GDI control variables u_1, u_2, u_3 .

12. Conclusion

Despite that the attitude parametrization provided by quaternion attitude variables is nonminimal, quaternion algebraic properties and multiplicative attitude quaternion error dynamics simplify the expressions of controls coefficient and controls load functions, and therefore simplify the GDI control law. Lyapunov control system design is well-known to consume less energy than classical DI design. The geometric properties of the GDI control law makes it possible to combine DI with Lyapunov control to reduce the control energy required to perform DI.

The choice of desired stable servo-constraint dynamics has its tangible effect on closed loop system response. For instance, choosing the linear servo-constraint dynamics coefficients to

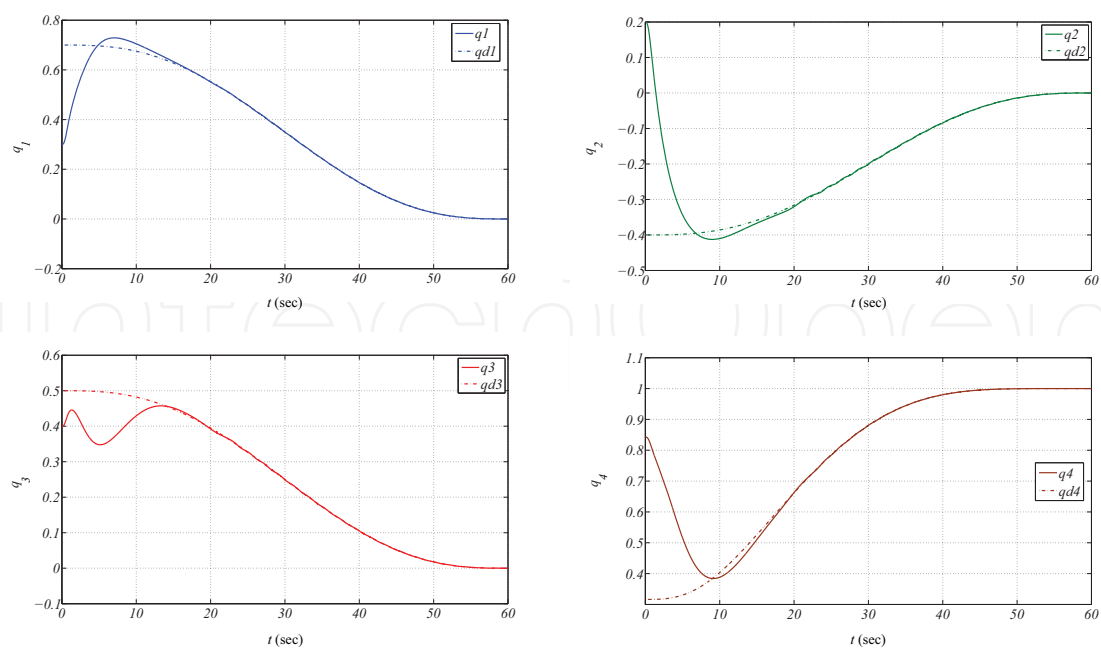


Fig. 2. Quaternion attitude parameters vs. Time: rest-to-rest slew maneuver

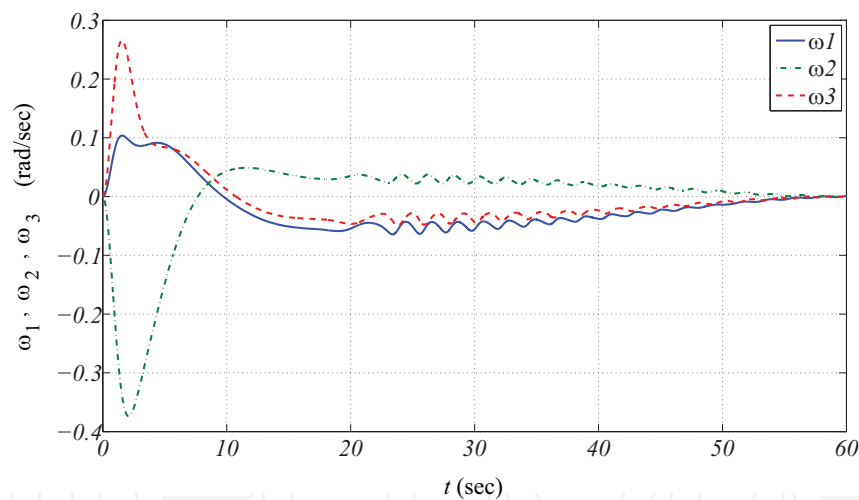


Fig. 3. Angular velocity components vs. Time: rest-to-rest slew maneuver

be time varying with vanishing values at initial time substantially reduces the magnitude of controls load function, and hence substantially reduces initial control signal magnitude. The null-control vector in the auxiliary part of the control law is designed to be linear in angular velocity's error vector. A novel construction of the state dependent linearity gain matrix is made by means of positive semidefinite control Lyapunov function and nullprojected control Lyapunov equation that utilize geometric features of the GDI control law's structure. The generalized inversion stable mode augmentation generalizes the concept of dynamic scaling, and it effectively overcomes controls coefficient generalized inversion singularity. If the augmented mode is designed to be very fast, then the delayed DSGI closely approximates the instantaneous DSGI. For problems involving time invariant steady state trajectory

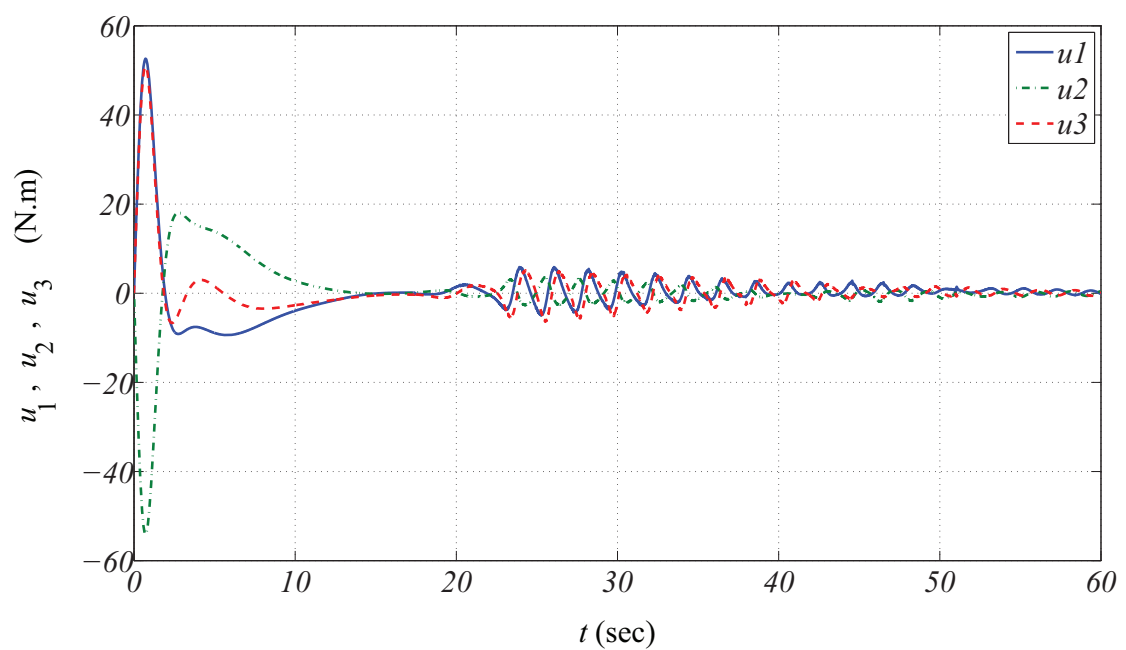


Fig. 4. Control variables vs. Time: rest-to-rest slew maneuver

tracking, the particular part of the control law asymptotically converges to its projection on the range space of the controls coefficient’s MPGI, leading to asymptotic realization of desired servo-constraint stable dynamics. Practically stable trajectory tracking control is achieved otherwise.

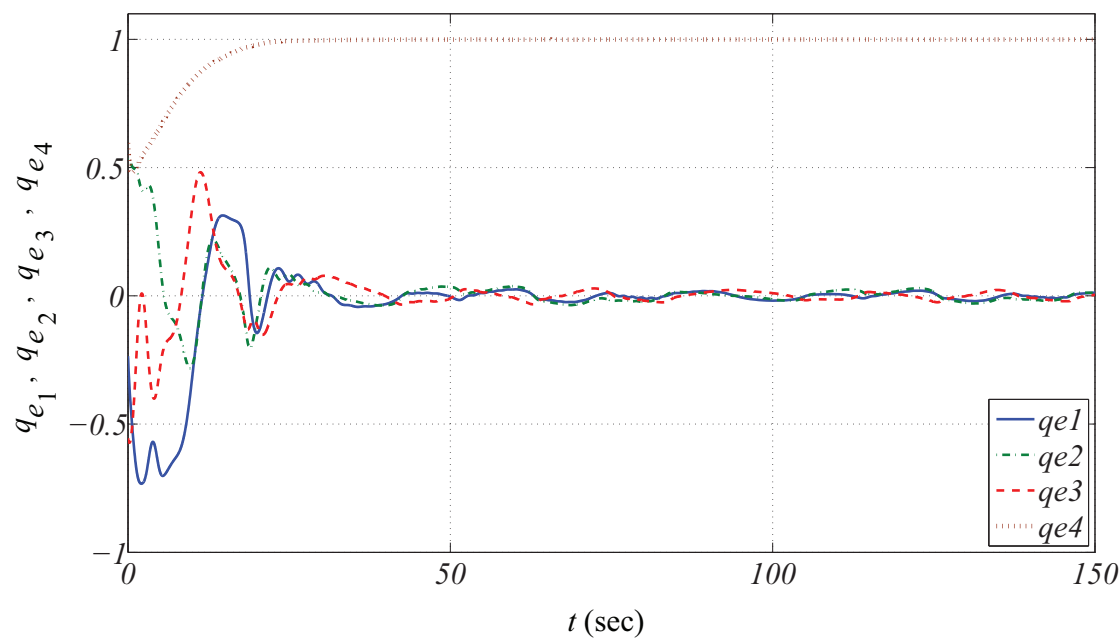


Fig. 5. Quaternion attitude parameters errors vs. Time: trajectory tracking maneuver

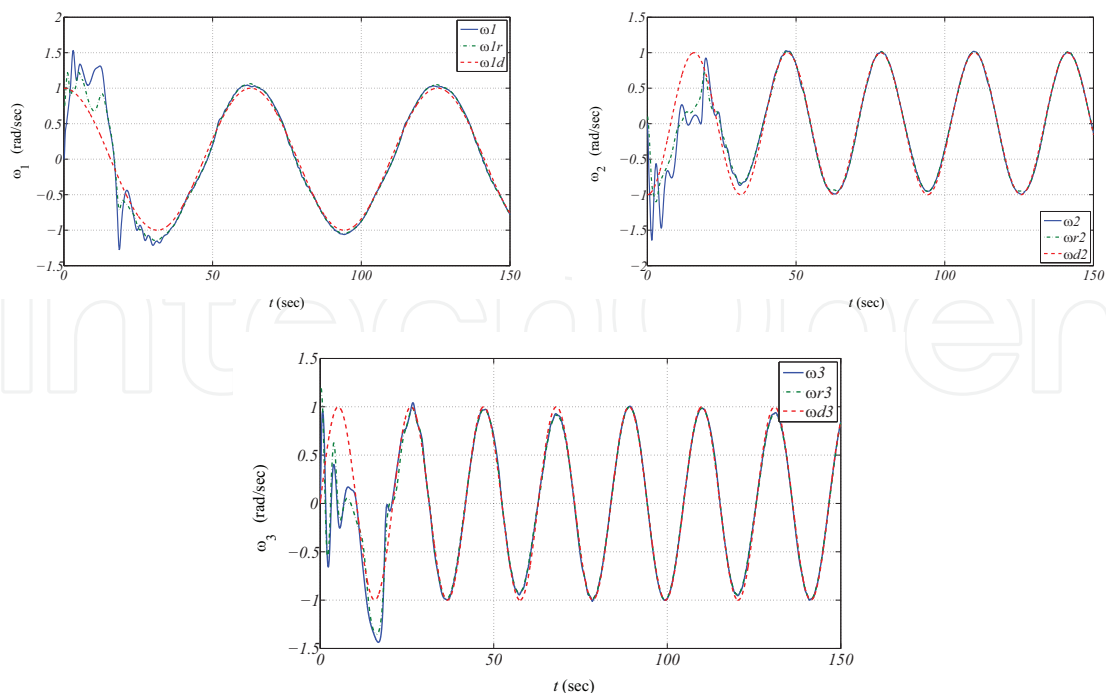


Fig. 6. Angular velocity components vs. Time: trajectory tracking maneuver

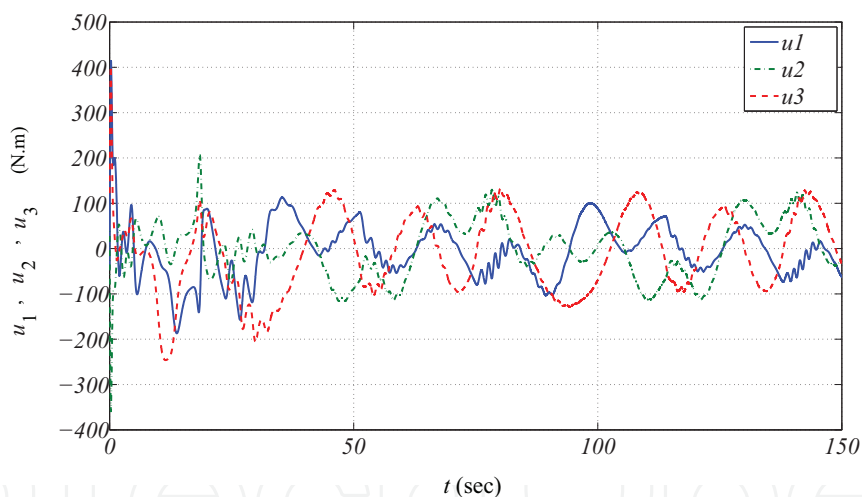


Fig. 7. Control variables vs. Time: trajectory tracking maneuver

13. References

- Bajodah, A. H. (2006). Stabilization of underactuated spacecraft dynamics via singularly perturbed feedback linearization, *Journal of King Abdulaziz University: Engineering Sciences* 16(2): 35–53.
- Bajodah, A. H. (2008). Singularly perturbed feedback linearization with linear attitude deviation dynamics realization, *Nonlinear Dynamics* 53(4): 321–343.
- Bajodah, A. H. (2009). Generalized dynamic inversion spacecraft control design methodologies, *IET Control Theory & Applications* 3(2): 239–251.
- Bajodah, A. H. (2010). Asymptotic generalized inversion attitude control, *IET Control Theory*

- & Applications* 4(5): 827–840.
- Bajodah, A. H., Hodges, D. H. & Chen, Y. H. (2005). Inverse dynamics of servo-constraints based on the generalized inverse, *Nonlinear Dynamics* 39(1-2): 179–196.
- Baker, D. R. & Wampler II, C. W. (1988). On inverse kinematics of redundant manipulators, *International Journal of Robotics Research* 7(2): 3–21.
- Behal, A., Dawson, D., Zergeroglu, E. & Fang, Y. (2002). Nonlinear tracking control of an underactuated spacecraft, *Journal of Guidance, Control, and Dynamics* 25(5): 979–985.
- Ben-Israel, A. & Greville, T. N. E. (2003). *Generalized Inverses, Theory and Applications*, second edn, Springer-Verlag New York, Inc.
- Bensoubaya, M., Ferfera, A. & Iqqidr, A. (1999). Stabilization of nonlinear systems by use of semidefinite Lyapunov functions, *Applied Mathematics Letters* 12(7): 11–17.
- Bhat, S. & Bernstein, D. (2000). A topological obstruction to continuous global stabilization of rotational motion and the unwinding phenomenon, *Systems & Control Letters* 39(1): 63–70.
- De Sapio, V., Khatib, O. & Delp, S. (2008). Least action principles and their application to constrained and task-level problems in robotics and biomechanics, *Multibody System Dynamics* 19(3): 303–322.
- Dwyer III, T. (1984). Exact nonlinear control of large angle rotational maneuvers, *IEEE Transactions on Automatic Control* 29(9): 769–774.
- Gauss, C. F. (1829). Ueber ein neues allgemeines grundgesetz der mechanik, *Zeitschrift Fuer die reine und angewandte Mathematik* 4: 232–235.
- Greville, T. N. E. (1959). The pseudoinverse of a rectangular or singular matrix and its applications to the solutions of systems of linear equations, *SIAM Review* 1(1): 38–43.
- Hunt, L., Su, R. & Meyer, G. (1983). Global transformations of nonlinear systems, *IEEE Transactions on Automatic Control* AC-28: 24–31.
- Iqqidr, A., Kalitine, B. & Outbib, R. (1996). Semidefinite Lyapunov functions stability and stabilization, *Mathematics of Control, Signals and Systems* 9(2): 95–106.
- Khalil, H. K. (2002). *Nonlinear Systems*, 3rd edn, Prentice-Hall, Inc.
- Liegeois, A. (1977). Automatic supervisory control of the configuration and behavior of multi-body mechanisms, *IEEE Transactions on Systems, Man, and Cybernetics* 7(12): 868–871.
- Mayorga, R. V., Janabi-Sharifi, F. & Wong, A. K. C. (1995). A fast approach for the robust trajectory planning of redundant robot manipulators, *Journal of Robotic Systems* 12(2): 147–161.
- McInnes, C. R. (1998). Satellite attitude slew manoeuvres using inverse control, *Aeronautical Journal* 102: 259–265.
- Moore, E. H. (1920). On the reciprocal of the general algebraic matrix, *Bulletin of the American Mathematical Society* 26: 394–395.
- Nakamura, Y. & Hanafusa, H. (1986). Inverse kinematic solutions with singularity robustness for robot manipulator control, *Journal of Dynamic Systems, Measurements, and Control* 10(8): 163–171.
- Oh, H. S. & Vadali, S. R. (1991). Feedback control and steering laws for spacecraft using single gimbal control moment gyros, *Journal of the Astronautical Sciences* 39(2): 183–203.
- Paielli, R. A. & Bach, R. E. (1993). Attitude control with realization of linear error dynamics, *Journal of Guidance, Control, and Dynamics* 16(1): 182–189.
- Penrose, R. (1955). A generalized inverse for matrices, *Proceedings of the Cambridge Philosophical Society* 51: 406–413.

- Peters, J., Mistry, M., Udwadia, F., Nakanishi, J. & Schaal, S. (2008). A unifying framework for robot control with redundant DOFs, *Autonomous Robots* 24(1): 1–12.
- Show, L.-L. & Juang, J.-C. (2003). Satellite large angle tracking control design: Thruster control approach, *Proceedings of the American Control Conference*, Denver, Colorado, pp. 1098–1103.
- Siciliano, B. & Khatib, O. (2008). *Springer Handbook of Robotics*, Springer.
- Slotine, J. E. & Li, W. (1991). *Applied Nonlinear Control*, Prentice-Hall.
- Su, R. (1982). On the linear equivalents of nonlinear systems, *Systems and Control Letters* 2: 48–52.
- Udwadia, F. E. (2008). Optimal tracking control of nonlinear dynamical systems, *Proceedings of the Royal Society of London. Series A* 464: 2341–2363.
- Udwadia, F. E. & Kalaba, R. E. (1996). *Analytical Dynamics, A New Approach*, Cambridge University Press, New York, NY.
- Wampler II, C. W. (1986). Manipulator inverse kinematic solutions based on vector formulations and damped least-squares methods, *IEEE Transactions on Systems, Man, and Cybernetics* SMC-16: 93–101.
- Wertz, J. W. (1980). *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Co., Dordrecht, Holland.
- Yoon, H. & Tsiotras, P. (2004). Singularity analysis of variable-speed control moment gyros, *Journal of Guidance, Control, and Dynamics* 27(3): 374–386.

IntechOpen



Advances in Spacecraft Technologies

Edited by Dr Jason Hall

ISBN 978-953-307-551-8

Hard cover, 596 pages

Publisher InTech

Published online 14, February, 2011

Published in print edition February, 2011

The development and launch of the first artificial satellite Sputnik more than five decades ago propelled both the scientific and engineering communities to new heights as they worked together to develop novel solutions to the challenges of spacecraft system design. This symbiotic relationship has brought significant technological advances that have enabled the design of systems that can withstand the rigors of space while providing valuable space-based services. With its 26 chapters divided into three sections, this book brings together critical contributions from renowned international researchers to provide an outstanding survey of recent advances in spacecraft technologies. The first section includes nine chapters that focus on innovative hardware technologies while the next section is comprised of seven chapters that center on cutting-edge state estimation techniques. The final section contains eleven chapters that present a series of novel control methods for spacecraft orbit and attitude control.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Abdulrahman Bajodah (2011). Inertia-Independent Generalized Dynamic Inversion Control of Spacecraft Attitude Maneuvers, *Advances in Spacecraft Technologies*, Dr Jason Hall (Ed.), ISBN: 978-953-307-551-8, InTech, Available from: <http://www.intechopen.com/books/advances-in-spacecraft-technologies/inertia-independent-generalized-dynamic-inversion-control-of-spacecraft-attitude-maneuvers>

INTECH
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen