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An Analytical Solution for Transient Heat and Moisture Diffusion in a Double-Layer Plate

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1. Introduction

In most materials, there exists a coupling effect between heat and moisture during their transient diffusion. In particular, the coupling effect gives a significant change in the distributions of temperature and moisture concentration in some porous materials and resin composites (Chang, et al., 1991). Moreover, it is known that the absorption of moisture by hygroscopic materials under high-temperature environments causes considerable hygrothermal stresses, and meanwhile their mechanical stiffness and strength are degraded a great deal (Komai, et al., 1991). Therefore, it is important to predict accurately the coupled heat and moisture diffusion behaviour within the materials in assessing the life of moisture-conditioning building materials and resin-based structural materials such as CFRP and GFRP in hygrothermal environments.

With regard to the transient heat and moisture diffusion problems, some researchers conducted theoretical analyses using analytical (mathematical) or numerical techniques. For example, Sih et al. presented analytical or numerical solutions for the coupled heat and moisture diffusion and resulting hygrothermal stress problems (Hartranft and Sih, 1980 a) (Hartranft and Sih, 1980 b, Sih, 1983, Sih, et al., 1980) (Sih and Ogawa, 1982) (Hartranft and Sih, 1981) (Sih, 1983, Sih, et al., 1981). Chang et al. used a decoupling technique to obtain analytical solutions for the heat and moisture diffusion occurring in a hollow cylinder (Chang, et al., 1991) and a solid cylinder (Chang, 1994) subjected to hygrothermal loadings. Subsequently, using the same technique, Sugano et al. (Sugano and Chuuman, 1993 a, Sugano and Chuuman, 1993 b) obtained analytical solutions for a hollow cylinder subjected to nonaxisymmetric hygrothermal loadings. All the above-mentioned papers, however, focus on/ target a single material body.

Studies that address the coupled heat and moisture diffusion problem for composite regions (e.g., layered bodies) are limited. Chen et al. (Chen, et al., 1992) analysed the coupled diffusion problem in a double-layered cylinder using the FEM, which leads to time-consuming computation. In order to improve this disadvantage, Chang et al. (Chang and Weng, 1997) later proposed an analytical technique including Hankel and Laplace transforms, and significantly reduced the computational time compared to the FEM analysis. However, the exact continuity of moisture flux was not fulfilled at the layer interface although the coupling terms were included in the governing equations.

In this chapter, under the exact continuity condition the one-dimensional transient coupled heat and moisture diffusion problem is analytically solved for a double-layer plate subjected

to time-varying hygrothermal loadings at the external surfaces, and analytical solutions for the temperature and moisture fields are presented. The solutions are explicitly derived without complicated mathematical procedures such as Laplace transform and its inversion by applying an integral transform technique—Vodicka's method. For simplicity, the diffusion problem treated here is assumed to be a one-way coupled problem, which considers only the effect of heat diffusion on the moisture diffusion, not a fully-coupled problem, in which heat and moisture diffusions affect each other. Since, in some real cases, moisture-induced effect on the heat diffusion (i.e., the latent heat diffusion) is evaluated to be insignificant (Khoshbakht and Lin, 2010, Khoshbakht, et al., 2009), this assumption is reasonable.

Numerical calculations are performed for a double-layer plate composed of distinct resin-based composites that the temperature and moisture concentration are kept constant at the external surfaces (the 1st kind boundary condition). The effects of coupling terms included in the continuity condition of the moisture flux at the layer interface on the transient moisture distribution in the plate are quantitatively evaluated. Numerical results demonstrate that for an accurate prediction of heat and moisture diffusion behaviour, the coupling terms in the continuity condition should be taken into consideration.

Nomenclature

- a : interface location, m
 B : Biot number for heat transfer ($= hl / \lambda_{\text{ref}}$)
 B^* : Biot number for moisture transfer ($= \chi l / \Lambda_{\text{ref}}$)
 c : moisture capacity, $\text{kg}/(\text{kg} \cdot ^\circ\text{M})$
 h : heat transfer coefficient, $\text{W}/(\text{m}^2 \cdot \text{K})$
 l : total thickness, m
 L : Luikov number ($= \eta / \kappa_{\text{ref}}$)
 m : moisture content ($= c \cdot u$), wt. %
 P : Possnov number ($= \varepsilon(T_{\text{ref}} - T_0) / (u_0 - u_{\text{ref}})$)
 t : time, s
 T : temperature, K
 \bar{T} : dimensionless temperature ($= (T - T_0) / (T_{\text{ref}} - T_0)$)
 u : moisture potential, $^\circ\text{M}$
 \bar{u} : dimensionless moisture potential ($= (u_0 - u) / (u_0 - u_{\text{ref}})$)
 x, y, z : coordinates, m
 Z : dimensionless coordinate ($= z / l$)
 χ : moisture transfer coefficient, $\text{kg}/(\text{m}^2 \cdot \text{s} \cdot ^\circ\text{M})$
 δ_{ij} : Kronecker delta
 ε : thermogradient coefficient, $^\circ\text{M}/\text{K}$
 γ : eigenvalue for temperature field
 η : moisture diffusivity, m^2/s
 κ : thermal diffusivity, m^2/s
 $\bar{\kappa}$: dimensionless thermal diffusivity ($= \kappa / \kappa_{\text{ref}}$)
 λ : thermal conductivity, $\text{W}/(\text{m} \cdot \text{K})$
 μ : eigenvalue for moisture field
 τ : Fourier number ($= \kappa_{\text{ref}} t / l^2$)
 Λ : conductivity coefficient of moisture content, $\text{kg}/(\text{m} \cdot \text{s} \cdot ^\circ\text{M})$

Subscripts

- b: bottom surface
- i: layer number
- m: eigenvalue number
- ref: reference value
- t: top surface
- 0: initial
- ∞ : surrounding medium

2. Theoretical analysis

Consider an infinite double-layer plate constructed of hygroscopic materials, which is referred to Cartesian coordinate system as shown in Fig. 1. The total thickness of the plate is represented by l . The quantities with subscript 1 or 2 denote those for the 1st or 2nd layer of the double-layer plate throughout the chapter. The coordinate value a indicates the location of the layer interface. The temperature and moisture content measured by the moisture potential in the plate are assumed to be initially T_0 and u_0 , respectively. We denote the temperatures of the surrounding media by functions $T_{t\infty}(t)$ and $T_{b\infty}(t)$ and the moisture potentials of them by $u_{t\infty}(t)$ and $u_{b\infty}(t)$. The plate is subjected to these hygrothermal loadings via heat and moisture transfer coefficients h_t, h_b, χ_t and χ_b .

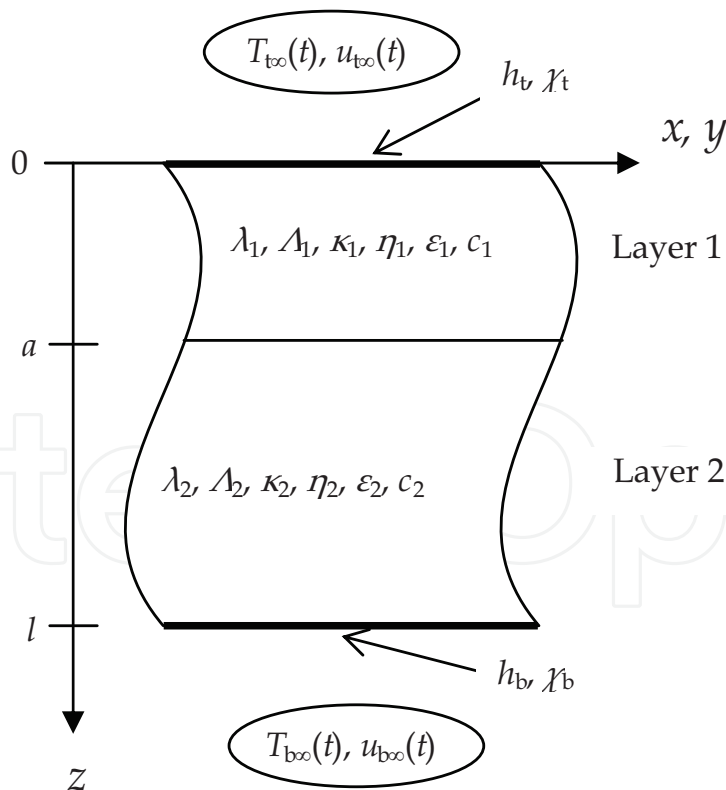


Fig. 1. Physical model and coordinate system

For the one-dimensional case shown in Fig.1, heat and moisture move along the z axis only. When the effect of the moisture content (or potential) gradient in the energy equation is

neglected, the transient heat and moisture diffusion equations for the i th layer ($i = 1, 2$) are written in dimensionless form as follows (Lykov and Mikhailov, 1965):

$$\frac{1}{\bar{\kappa}_i} \frac{\partial \bar{T}_i(Z, \tau)}{\partial \tau} = \frac{\partial^2 \bar{T}_i(Z, \tau)}{\partial Z^2} \quad \tau > 0, \quad i = 1, 2, \quad (1a)$$

$$\frac{1}{L_i} \frac{\partial \bar{u}_i(Z, \tau)}{\partial \tau} = \frac{\partial^2 \bar{u}_i(Z, \tau)}{\partial Z^2} - P_i \frac{\partial^2 \bar{T}_i(Z, \tau)}{\partial Z^2} \quad \tau > 0, \quad i = 1, 2. \quad (1b)$$

The one-way coupled system of equations given by Eqs. (1) is equivalent to the constant properties model presented by Fudym et al. (Fudym, et al., 2004). For constant moisture capacity, the moisture potential u_i and moisture content m_i are related by $m_i = c_i \cdot u_i$.

The initial conditions are defined as:

$$\bar{T}_i(Z, 0) = 0; \quad \bar{u}_i(Z, 0) = 0 \quad i = 1, 2. \quad (2a,b)$$

At the two sides of the plate ($Z = 0$ and $Z = 1$), the mass diffusion caused by the temperature and moisture gradients affects the mass balance (Chang and Weng, 2000 a). At the interface between two constitutive materials, the distributions of temperature and moisture potential are continuous and the moisture flux depending on both temperature and moisture potential gradients must be also continuous as well as the heat flux, provided that interfacial contact resistance is negligible. Therefore, the boundary and continuity conditions can be given as follows:

$$\frac{\partial \bar{T}_1(0, \tau)}{\partial Z} + B_t [\bar{T}_\infty(\tau) - \bar{T}_1(0, \tau)] = 0, \quad (3a)$$

$$-\frac{\partial \bar{u}_1(0, \tau)}{\partial Z} + P_1 \frac{\partial \bar{T}_1(0, \tau)}{\partial Z} - B_t^* [\bar{u}_\infty(\tau) - \bar{u}_1(0, \tau)] = 0, \quad (3b)$$

$$\frac{\partial \bar{T}_2(1, \tau)}{\partial Z} - B_b [\bar{T}_\infty(\tau) - \bar{T}_2(1, \tau)] = 0, \quad (3c)$$

$$-\frac{\partial \bar{u}_2(1, \tau)}{\partial Z} + P_2 \frac{\partial \bar{T}_2(1, \tau)}{\partial Z} + B_b^* [\bar{u}_\infty(\tau) - \bar{u}_2(1, \tau)] = 0, \quad (3d)$$

$$\bar{T}_1(Z_1, \tau) = \bar{T}_2(Z_1, \tau); \quad \bar{u}_1(Z_1, \tau) = \bar{u}_2(Z_1, \tau), \quad (4a,b)$$

$$\bar{\lambda}_1 \frac{\partial \bar{T}_1(Z_1, \tau)}{\partial Z} = \bar{\lambda}_2 \frac{\partial \bar{T}_2(Z_1, \tau)}{\partial Z}, \quad (4c)$$

$$-\bar{\Lambda}_1 \frac{\partial \bar{u}_1(Z_1, \tau)}{\partial Z} + \bar{\Lambda}_1 P_1 \frac{\partial \bar{T}_1(Z_1, \tau)}{\partial Z} = -\bar{\Lambda}_2 \frac{\partial \bar{u}_2(Z_1, \tau)}{\partial Z} + \bar{\Lambda}_2 P_2 \frac{\partial \bar{T}_2(Z_1, \tau)}{\partial Z}, \quad (4d)$$

where $Z_1 = a/l$, $\bar{\lambda}_i = \lambda_i / \lambda_{\text{ref}}$ and $\bar{\Lambda}_i = \Lambda_i / \Lambda_{\text{ref}}$. In existing analytical studies, the second terms of both sides of Eq. (4d) were omitted because of mathematical difficulties.

An analytical solution to the transient heat conduction problem expressed by Eqs. (1a), (2a), (3a), (3c), (4a) and (4c) has already been derived by Sugano et al. (Sugano, et al., 1993) as follows:

$$\bar{T}_i(Z, \tau) = \sum_{m=1}^{\infty} \phi_m(\tau) \left[A_{im} \cos\left(\frac{\gamma_m Z}{\sqrt{\bar{\kappa}_i}}\right) + B_{im} \sin\left(\frac{\gamma_m Z}{\sqrt{\bar{\kappa}_i}}\right) \right] + \sum_{j=1}^2 (C_{ij}Z + D_{ij})V_j(\tau) \quad \text{for } i = 1, 2, \quad (5)$$

where

$$V_1(\tau) = -\bar{T}_{t\infty}(\tau); \quad V_2(\tau) = \bar{T}_{b\infty}(\tau), \quad (6a, b)$$

$$\phi_m(\tau) = \exp(-\gamma_m^2 \tau) \left[g_m - \int_0^\tau \exp(\gamma_m^2 t) \sum_{j=1}^2 f_{mj} \frac{dV_j(t)}{dt} dt \right]. \quad (7)$$

The procedure for determining the eigenvalues γ_m ($m = 1, 2, \dots$) and the expansion coefficients g_m and f_{mj} ($j = 1, 2$) can be found in (Sugano, et al., 1993). The constants A_{im} , B_{im} , C_{ij} and D_{ij} in Eq. (5) are determined from the boundary and continuity conditions, Eqs. (3a), (3c), (4a) and (4c).

Rewriting Eq. (1b) with Eq. (5) yields

$$\frac{1}{L_i} \frac{\partial \bar{u}_i(Z, \tau)}{\partial \tau} = \frac{\partial^2 \bar{u}_i(Z, \tau)}{\partial Z^2} + Q_i(Z, \tau) \quad \text{for } i = 1, 2, \quad (8)$$

where

$$Q_i(Z, \tau) = \frac{P_i}{\bar{\kappa}_i} \sum_{m=1}^{\infty} \gamma_m^2 \phi_m(\tau) \left[A_{im} \cos\left(\frac{\gamma_m Z}{\sqrt{\bar{\kappa}_i}}\right) + B_{im} \sin\left(\frac{\gamma_m Z}{\sqrt{\bar{\kappa}_i}}\right) \right]. \quad (9)$$

The transient moisture diffusion problem for a composite medium represented by Eqs. (2b), (3b), (3d), (4b), (4d) and (8) is analysed by extending the ideas in an integral transform technique—Vodicka's method (Vodicka, 1955). Using this method, the solution to the moisture diffusion problem is obtained as

$$\bar{u}_i(Z, \tau) = \sum_{m=1}^{\infty} \psi_m(\tau) R_{im}(Z) + \sum_{j=1}^3 F_{ij}(Z) W_j(\tau), \quad i = 1, 2, \quad (10)$$

with the following functions:

$$F_{ij}(Z) = C_{ij}^* Z + D_{ij}^*, \quad j = 1, 2, 3, \quad (11)$$

$$W_1(\tau) = \frac{P_1}{B_t^*} \frac{\partial \bar{T}_1(0, \tau)}{\partial Z} - \bar{u}_{t\infty}(\tau) \quad \text{for } B_t^* \neq 0, \quad W_1(\tau) = 0 \quad \text{for } B_t^* = 0, \quad (12a)$$

$$W_3(\tau) = \frac{P_2}{B_b^*} \frac{\partial \bar{T}_2(1, \tau)}{\partial Z} + \bar{u}_{b\infty}(\tau) \quad \text{for } B_b^* \neq 0, \quad W_3(\tau) = 0 \quad \text{for } B_b^* = 0, \quad (12b)$$

$$W_2(\tau) = \bar{\Lambda}_1 P_1 \frac{\partial \bar{T}_1(Z_1, \tau)}{\partial Z} - \bar{\Lambda}_2 P_2 \frac{\partial \bar{T}_2(Z_1, \tau)}{\partial Z}. \quad (12c)$$

The constants C_{ij}^* and D_{ij}^* are determined from the following relationships:

$$F_{1j}(Z_1) - F_{2j}(Z_1) = 0, \quad j = 1, 2, 3, \quad (13a)$$

$$\bar{\Lambda}_1 \frac{dF_{1j}(Z_1)}{dZ} - \bar{\Lambda}_2 \frac{dF_{2j}(Z_1)}{dZ} = \delta_{2,j} \quad j = 1, 2, 3, \quad (13b)$$

$$\frac{dF_{1j}(0)}{dZ} - B_t^* F_{1j}(0) = B_t^* \delta_{1,j}, \quad j = 1, 2, 3, \quad (13c)$$

$$\frac{dF_{2j}(1)}{dZ} + B_b^* F_{2j}(1) = B_b^* \delta_{3,j}, \quad j = 1, 2, 3. \quad (13d)$$

$R_{im}(Z)$ is the solution to the eigenvalue problem corresponding to Eqs. (3b), (3d), (4b), (4d) and (8) and is given as follows:

$$R_{im}(Z) = A_{im}^* \cos\left(\frac{\mu_m Z}{\sqrt{L_i}}\right) + B_{im}^* \sin\left(\frac{\mu_m Z}{\sqrt{L_i}}\right), \quad (14)$$

with μ_m being an eigenvalue. The conditions necessary to determine the unknown constants A_{im}^* and B_{im}^* can be obtained by substituting Eqs. (10)–(14) into Eqs. (3b), (3d), (4b) and (4d) as follows:

$$\frac{dR_{1m}(0)}{dZ} - B_t^* R_{1m}(0) = 0, \quad (15a)$$

$$\frac{dR_{2m}(1)}{dZ} + B_b^* R_{2m}(1) = 0, \quad (15b)$$

$$R_{1m}(Z_1) = R_{2m}(Z_1), \quad (15c)$$

$$\bar{\Lambda}_1 \frac{dR_{1m}(Z_1)}{dZ} = \bar{\Lambda}_2 \frac{dR_{2m}(Z_1)}{dZ}. \quad (15d)$$

The eigenvalues μ_m ($m = 1, 2, \dots$) are obtained from the condition under which all the A_{im}^* and B_{im}^* values are nonzero and are, therefore, positive roots of the following transcendental equation:

$$\mathbf{G} \cdot \mathbf{E} \cdot \mathbf{a} = 0, \quad (16)$$

where

$$\mathbf{G} = \begin{bmatrix} -B_i^* & \frac{\mu_m}{\sqrt{L_1}} \end{bmatrix}; \quad \mathbf{E} = \mathbf{C}^{-1} \cdot \mathbf{D}, \quad (17a,b)$$

$$\mathbf{C} = \begin{bmatrix} \cos\left(\frac{\mu_m Z_1}{\sqrt{L_1}}\right) & \sin\left(\frac{\mu_m Z_1}{\sqrt{L_1}}\right) \\ -\bar{\Lambda}_1 \frac{\mu_m}{\sqrt{L_1}} \sin\left(\frac{\mu_m Z_1}{\sqrt{L_1}}\right) & \bar{\Lambda}_1 \frac{\mu_m}{\sqrt{L_1}} \cos\left(\frac{\mu_m Z_1}{\sqrt{L_1}}\right) \end{bmatrix}, \quad (17c)$$

$$\mathbf{D} = \begin{bmatrix} \cos\left(\frac{\mu_m Z_1}{\sqrt{L_2}}\right) & \sin\left(\frac{\mu_m Z_1}{\sqrt{L_2}}\right) \\ -\bar{\Lambda}_2 \frac{\mu_m}{\sqrt{L_2}} \sin\left(\frac{\mu_m Z_1}{\sqrt{L_2}}\right) & \bar{\Lambda}_2 \frac{\mu_m}{\sqrt{L_2}} \cos\left(\frac{\mu_m Z_1}{\sqrt{L_2}}\right) \end{bmatrix}, \quad (17d)$$

$$\mathbf{a} = \begin{bmatrix} \frac{\mu_m}{\sqrt{L_2}} \cos\left(\frac{\mu_m}{\sqrt{L_2}}\right) + B_b^* \sin\left(\frac{\mu_m}{\sqrt{L_2}}\right) \\ \frac{\mu_m}{\sqrt{L_2}} \sin\left(\frac{\mu_m}{\sqrt{L_2}}\right) - B_b^* \cos\left(\frac{\mu_m}{\sqrt{L_2}}\right) \end{bmatrix}. \quad (17e)$$

The time function $\psi_m(\tau)$ is expressed as

$$\psi_m(\tau) = \exp(-\mu_m^2 \tau) \left\{ g_m^* + \int_0^\tau \exp(\mu_m^2 t) \left[q_m(t) - \sum_{j=1}^3 f_{mj}^* \frac{dW_j(t)}{dt} \right] dt \right\}, \quad (18)$$

where the expansion coefficients g_m^* , $q_m(\tau)$ and f_{mj}^* are given with $Z_0 = 0$ and $Z_2 = 1$ by

$$g_m^* = - \frac{\sum_{i=1}^2 \frac{\bar{\Lambda}_i}{L_i} \int_{Z_{i-1}}^{Z_i} \left[\sum_{j=1}^3 F_{ij}(z) W_j(0) \right] R_{im}(z) dz}{\sum_{i=1}^2 \frac{\bar{\Lambda}_i}{L_i} \int_{Z_{i-1}}^{Z_i} [R_{im}(z)]^2 dz}, \quad (19a)$$

$$q_m(\tau) = \frac{\sum_{i=1}^2 \bar{\Lambda}_i \int_{Z_{i-1}}^{Z_i} Q_i(z, \tau) R_{im}(z) dz}{\sum_{i=1}^2 \frac{\bar{\Lambda}_i}{L_i} \int_{Z_{i-1}}^{Z_i} [R_{im}(z)]^2 dz}, \quad (19b)$$

$$f_{mj}^* = \frac{\sum_{i=1}^2 \frac{\bar{\Lambda}_i}{L_i} \int_{Z_{i-1}}^{Z_i} F_{ij}(z) R_{im}(z) dz}{\sum_{i=1}^2 \frac{\bar{\Lambda}_i}{L_i} \int_{Z_{i-1}}^{Z_i} [R_{im}(z)]^2 dz}. \quad (19c)$$

In order to obtain Eqs. (18) and (19), the following orthogonal relationship with discontinuous weight functions in terms of the eigenfunction $R_{im}(z)$ was used:

$$\sum_{i=1}^2 \frac{\bar{\Lambda}_i}{L_i} \int_{Z_{i-1}}^{Z_i} R_{im}(z) R_{ik}(z) dz = \begin{cases} \text{const.} & (m = k) \\ 0 & (m \neq k) \end{cases} \tag{20}$$

3. Numerical results and discussion

Numerical calculations are performed for a double-layer plate composed of a T300/5208 composite (layer 1) and another hygroscopic material (layer 2). The thickness of both layers is assumed identical, that is, $Z_1 = 0.5$. The material properties are listed as follows (Chang and Weng, 2000 b, Chen, et al., 1992, Sih, et al., 1986, Yang, et al., 2006):

$$\begin{aligned} \kappa_1 &= 5\kappa_2 = 2.16 \times 10^{-5} \text{ m}^2/\text{s}, \quad \eta_1 = \eta_2 = 2.16 \times 10^{-6} \text{ m}^2/\text{s}, \\ c_1 &= 5c_2 = 0.01 \text{ kg}/(\text{kg} \cdot ^\circ\text{M}), \quad \lambda_1 = 5\lambda_2 = 0.65 \text{ W}/(\text{m} \cdot \text{K}), \\ \varepsilon_1 &= \varepsilon_2 = 2.0 \text{ } ^\circ\text{M}/\text{K}, \quad A_1 = A_2 = 2.2 \times 10^{-8} \text{ kg}/(\text{m} \cdot \text{s} \cdot ^\circ\text{M}). \end{aligned}$$

The material properties in the layer 1 are used as reference for the dimensionless quantities. We consider the case in which the same temperature and moisture potential values are prescribed at both external surfaces of the double-layer plate, i.e., $B_t = B_b = B_t^* = B_b^* = \infty$, $\bar{T}_{t\infty} = \bar{T}_{b\infty} = 1$ and $\bar{u}_{t\infty} = \bar{u}_{b\infty} = 1$ for $(u_0 - u_{\text{ref}})/(T_{\text{ref}} - T_0) = 0.82 \text{ } ^\circ\text{M}/\text{K}$ (Yang, et al., 2006). The number of terms in the infinite series in Eqs. (5) and (10) is taken as 100, which is used for the verification of sufficient convergence of the numerical results. Table 1 lists the specific values of up to the fifth eigenvalue for this situation.

| γ_1 | γ_2 | γ_3 | γ_4 | γ_5 |
|------------|------------|------------|------------|------------|
| 2.1774 | 3.7028 | 5.7323 | 8.0017 | 9.6311 |
| μ_1 | μ_2 | μ_3 | μ_4 | μ_5 |
| 0.99344 | 1.9869 | 2.9803 | 3.9738 | 4.9672 |

Table 1. The first five eigenvalues for temperature and moisture fields

Figure 2 illustrates the transient behaviour of temperature and moisture potential distributions. In the moisture potential distribution, numerical results based on the simple continuity condition in which the second terms of both sides of Eq. (4d) are omitted are also shown in order to investigate the effects of the continuity condition of moisture flux. It is observed from Fig. 2(a) that the temperature distribution increases monotonously with time. The difference between temperature gradients at the layer interface in different layers increases with time in the beginning, then starting to decline at a time to vanish ultimately. On the other hand, the variation in the moisture potential is not monotonous owing to the effect of the second derivative of temperature (see Eq. (1b)). Since the second derivative is positive throughout the plate thickness, as shown in Fig. 2(a), the temperature acts as moisture sink in the moisture potential field. Therefore, the values of moisture potential decrease from the initial state ($\bar{u} = 0$) near the thickness centre, where rapid supply of

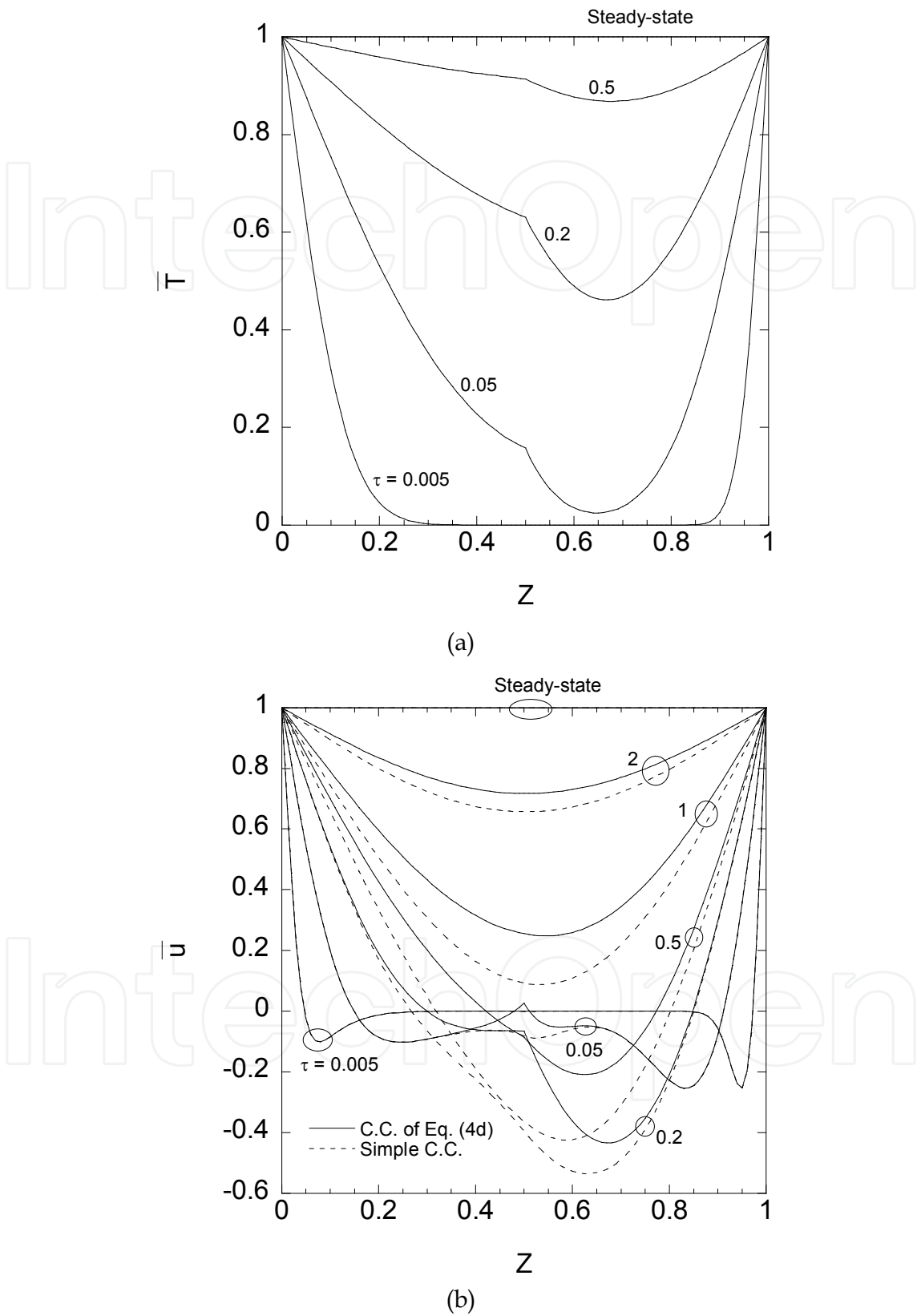


Fig. 2. Transient behaviour of (a) temperature and (b) moisture potential in a double-layer plate

moisture flux from both external surfaces is not expected. Subsequently, the strength of the moisture sink weakens as the temperature distribution approaches the steady-state one and the effect of its second derivative wanes. As a result, the moisture potential increases simply to the steady state ($\bar{u} = 1$).

The effects of the continuity condition on the moisture potential distributions are pronounced. In the case of Eq. (4d), which stipulates the exact continuity of moisture flux at the interface, the values of the moisture potential are higher than the counterparts at all the time instants. Of special note is the singular distribution adjacent to the layer interface, as observed for $\tau = 0.05$. This phenomenon, in which the sign of moisture potential gradient changes at the layer interface, can also be found by finite element analyses (Khoshbakht and Lin, 2010, Khoshbakht, et al., 2009). Figure 2(b) demonstrates that the usage of the simplified continuity condition precludes prediction of that phenomenon. It is obvious that expansion/shrinkage stresses computed based on both moisture potential distributions are substantially different from each other. Hence, for an accurate prediction of heat and moisture diffusion behaviour and resulting hygrothermal stresses, it is of great importance to take into account the coupling term in the continuity condition as well as that in the governing equation.

4. Conclusions

In this chapter, analytical solutions have been presented for the transient temperature and moisture fields in a double-layer plate subjected to time-varying hygrothermal loadings at the external surfaces. The effects of the coupling included in the continuity condition of moisture flux at the layer interface on the distribution of moisture potential (or moisture content) in the plate have been quantitatively evaluated. Numerical results have shown that, in predicting transient heat and moisture diffusion behavior in a layered body, neglecting the moisture flux component due to temperature gradient in the continuity conditions of moisture flux at layer interfaces may result in a significant error.

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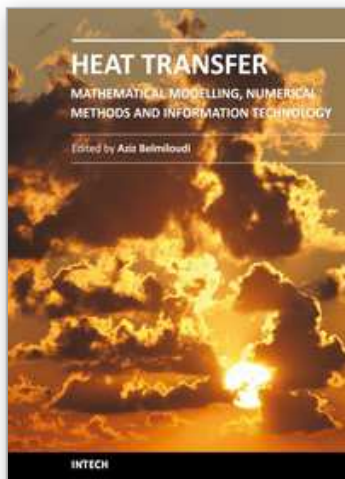
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Over the past few decades there has been a prolific increase in research and development in area of heat transfer, heat exchangers and their associated technologies. This book is a collection of current research in the above mentioned areas and describes modelling, numerical methods, simulation and information technology with modern ideas and methods to analyse and enhance heat transfer for single and multiphase systems. The topics considered include various basic concepts of heat transfer, the fundamental modes of heat transfer (namely conduction, convection and radiation), thermophysical properties, computational methodologies, control, stabilization and optimization problems, condensation, boiling and freezing, with many real-world problems and important modern applications. The book is divided in four sections : "Inverse, Stabilization and Optimization Problems", "Numerical Methods and Calculations", "Heat Transfer in Mini/Micro Systems", "Energy Transfer and Solid Materials", and each section discusses various issues, methods and applications in accordance with the subjects. The combination of fundamental approach with many important practical applications of current interest will make this book of interest to researchers, scientists, engineers and graduate students in many disciplines, who make use of mathematical modelling, inverse problems, implementation of recently developed numerical methods in this multidisciplinary field as well as to experimental and theoretical researchers in the field of heat and mass transfer.

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