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Harmonic Potential Fields: An Effective Tool for Generating a Self-organizing Behavior

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1. Introduction

This work provide a proof-by-example of the ability of harmonic potential fields (HPF) to exhibit a self-organizing behavior that can be utilized in building decentralized, evolutionary, multi-agent systems. It is shown that the strong relation the single agent HPF approach has to the evolutionary artificial life (AL) approach may be utilized at the multi-agent level to synthesize decentralized controllers that can be applied to a large variety of practical problems. We first provide a background of the single agent HPF approach along with its relation to the AL approach. Different multi-agent, HPF-based methods are presented along with simulation examples to demonstrate the utility of these techniques.

Humans have long attempted to bridge the gap between actions under their direct command (control variables) and directly inaccessible desired aspects of the environment they want to influence. This is carried-out by constructing a chain of causality linking the two together; hence making those directly inaccessible aspects indirectly accessible to the human operator. The process that realizes this chain of causality is called a servo-process. There are more than on type of problems that a servo-process have to rectify in order to enable causality to flow from the control side to the desired outcome side. The failure could be caused by insufficient quantity of effort that is being exerted at the control variable side. It may be the result of incompatibility of the control effort with the aspects of the environment that is to be influenced. The lack of organization in terms of the proper spatial-temporal distribution of the assets comprising the servo-process is a serious and difficult to detect source of failure. The sufficiency of the level of information available to constructor of the servo-process is also a fundamental cause of failure. Attention in this chapter is paid to the third type of failure concerning the faulty organization of the servo-process resources. Any servo-process must, among other things, regulate the interaction among its sensory, processing, communication and actuation components. There are a number of distinct modalities in which these components are governed each suited to tackle a certain situation. Each one of these modalities gives rise to a family of planners. A planner is an intelligent, goal-oriented, context-sensitive controller that instructs the servo-process on how to deploy its actuators of motion so that a target situation may be reached in a constrained manner. Probably the most common modality used by a servo-process is the: know-plan-Act modality which is commonly called the: model-based approach (figure 1). Here, the servo-process uses its sensors to collect data about the situation it has to deal with. This data is converted into a representation. The representation is processed to generate a plan or sub-

tasks that have to be sequentially executed. These tasks are in turn fed to an actuation stage. Despite the popularity of this approach, it was found to suffer from problems such as: slow reaction to changes in the environment, aging of information and outdated plans and issues in converting the generated plan into successful actions.

Fig. 1. The model-based approach

To solve the problems of slow reaction to changes in the environment and outdated plans, the sense-act modality was suggested (figure 2). This reactive modality proved to be highly practical; however, its applicability is limited to simple tasks.
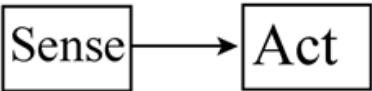
Fig. 2. The reactive approach

With limited success, attempts to improve the robustness of the model-based approach were carried-out by hybridizing it with the reactive approach (figure 3).
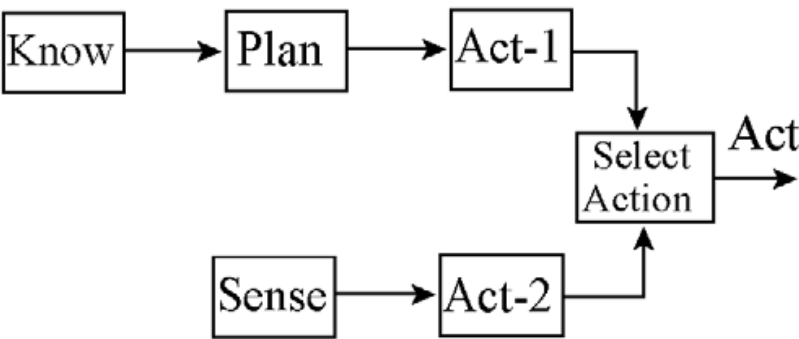
Fig. 3. The hybrid reactive – Model-based approach

Hardwiring sensors to an algorithm that directly feeds an actuation stage was found to be quite an effective modality for behavior generation (figure 4). To best of this author's knowledge, planning techniques based on this modality are difficult to adapt in situational spaces that have dimensionality more than two.
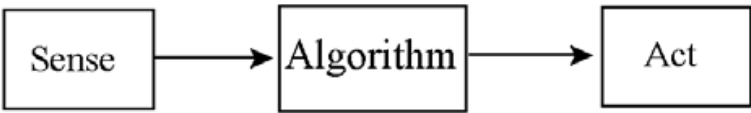
Fig. 4. The algorithmic, sensor-based approach

The subsumbtion architecture (figure 5) proved to be an effective modality for building servo-processes that have high chance of success operating in a realistic environment. This modality relies on direct sensory feed from the environment to a group of nested behaviors which the servo-process can project. These hierarchical behaviors can override each other

when the situation makes necessary to do so. The behavior of the resulting servo-process is situated, embodied, intelligent and emergent.
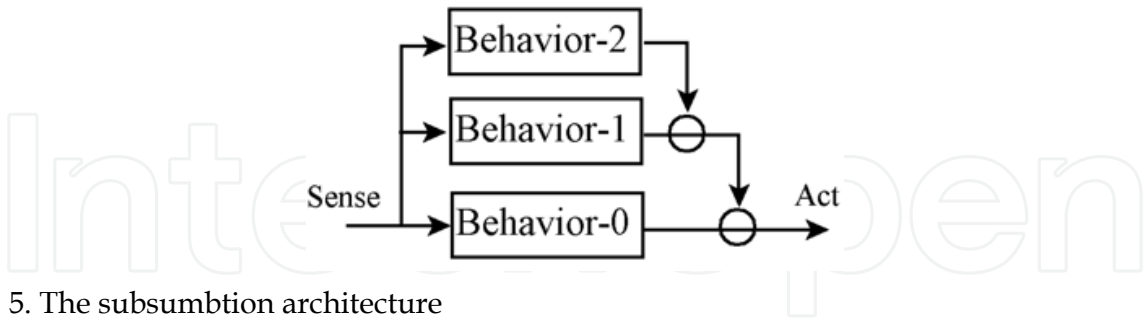

Fig. 5. The subsumbtion architecture

A belief-based approach (figure 6) is a powerful modality that allows a servo-process to act without the need for a common globally-agreed-on representation of the environment. Instead, the process forms its personal representation by continuously shaping its belief based on its experience of the environment.
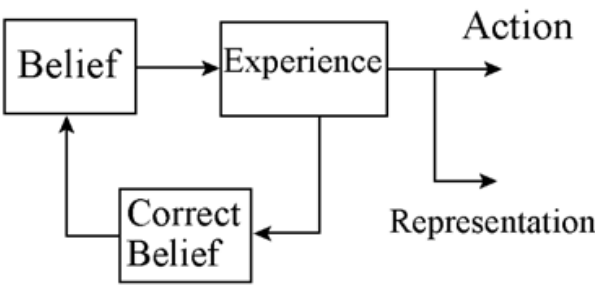

Fig. 6. Belief-based structures

Servo-processes may be built for any task and in any environment physical and nonphysical. A modality that suits the nature of the task is crucial for success. Modern technology has made great advances in miniaturizing and replicating devices and processes. This has strongly brought forward the possibility of building a distributed systems that are actualized by a group of, usually, identical agents to collectively perform a task. There are many modes in which this group may function. A highly sought-after mode requires that the group be able to function without a central, omni-aware supervisor. The group must have distributed asynchronous processing, perception and actuation. Communication among the agents is also limited in terms of reach and connectivity. In other words, the group has to self-organize in order to carry-out the task. The belief-based modality which can only exist if the group is decentralized seems to best fit such situation.

2. Centralized versus decentralized systems

In the following, general properties of centralized and decentralized systems are discussed. A definition equating decentralization to self-organization in a collective of agents is proposed. The artificial life (AL) paradigm and the harmonic potential field approach are suggested to realize a planner that is based on this definition.

Whether it is one or more agents, successful, context-sensitive, purposive behavior requires the presence of a process for generating a regulating control action. This process receives

data from the environment, the agent/s, the target/s, and the constraints on behavior, and converts them into a control action that will successfully propel the agents, in a constrained manner, towards their goals. There are two ways for generating such a regulating action: a centralized approach, and a decentralized approach.
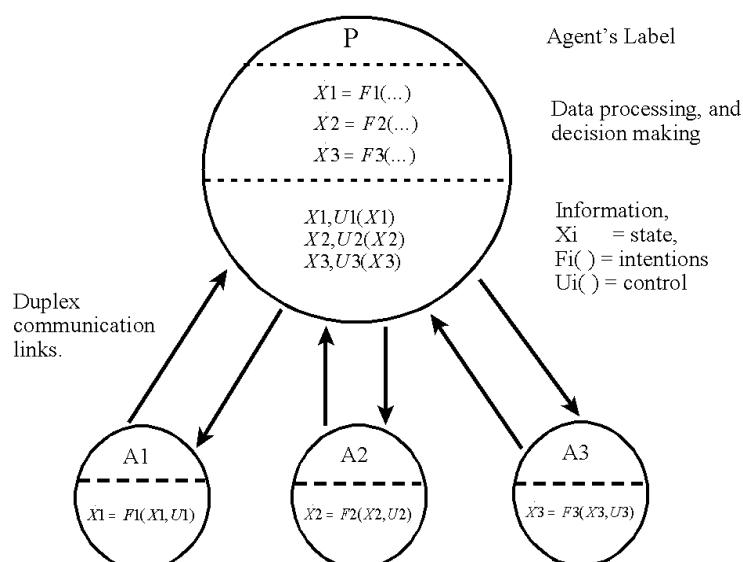

Fig. 7. Centralized approach to control

The centralized approach (figure 7) has a holistic-in-nature, top-down view to the behavior synthesis process. Here, a central agent that has a duplex communication link to each member of the group simultaneously observes the states of the agents and the environment, and processes the database in a manner that is in accordance with the aim of the group and the constraints on behavior. It then generates synchronized sequences of action instructions for each member of the group. The instructions are then communicated to the respective agents for them to progressively modify their trajectories and safely reach their destinations. In this mode of behavior, the generation of the constraint-satisfying, goal-fulfilling, conflict-free solution (i.e. sequence of state-control pair) begins by constructing the hyper action space (HAS) of the group. HAS houses the space of all admissible point actions which the agents may attempt to project. The HAS is then searched for a solution that is in turn communicated to the agents. The agents "blindly and mindlessly" execute the solution with a rigidity that is based on a trust that their actions will lead to the desired conclusion. It is a well-known fact that, in real life, any solution generated by a centralized mechanism is short lived. The dynamic nature of real environments will cause a mismatch between the conditions assumed at the time the controller begins generating the solution, and the actual conditions at the time the solution is handed to the agents for execution. Despite the attempt to alleviate this problem by equipping the agents with local sensory and decision making capabilities, centralized systems still suffer serious problems some of which are stated below:

1. Almost all centralized planning and control problems are known to be PSPACE-complete with a worst case complexity that grows exponentially with the number of agents. The large number of agents a realistic system contains will prevent the central controller from responding to environmental changes in a timely manner, if not cripple the control process altogether.

2. Centralized systems are inflexible in the sense that any changes to the characteristics of one or more agents may translate into a change in the whole HAS. This makes it necessary to repeat the expensive search for a solution. In turn, the desirable property that the size of the effort needed to adjust the control should be commensurate with the size of changes in the setting is not satisfied.

3. Centralized systems are prone to problems in communication and action synchronization. This makes it difficult to reliably operate the system even if the central planner has the assets needed to meet the demands of a realistic environment.

4. Centralized systems are not robust in the sense that the failure of one agent to fulfill its commitment towards the group could lead to the failure of the whole group.

In real life, no agent, no matter how sophisticated it is, has omniscient awareness of its surroundings, let alone infinite resources to instantly store and process data. Sometimes, even reliable communication links between the central agent and the others is difficult to establish. It may even be impossible due to the lack of universally accepted technical language, even vocabularies, for communication. The above are a few reasons why central planning strategies may not succeed in real life. Ruling out the feasibility of a central planning agent leaves only the option of the regulating control action arising from the agents themselves. The fact that the agents possess only local sensing, reasoning, and action capabilities makes it impossible to capture a complete spatial and/or temporal representation of the process. This, in turn, makes it impossible to build an HAS. As a result, the traditional way of control generation that first starts with a representation module followed by reasoning and control action generation can no longer be applied.

A major departure from such a linear, traditional way of thinking is needed. Since any finite sensory data an agent acquires does not reflect the actual content of the environment, the representation based on such data can only be classified as a belief. Under the above described conditions, an agent starts from a "seed" belief about its environment. This belief is coupled to an experiential stage that requires a sensory action continuously engaging the environment. Feedback is then applied to condition belief by experience. The control action is the outcome of this process.

Unlike the traditional approach where representation is an *a priori* that is needed to generate the control action, the suggested approach has representation as a *posteriori*, a byproduct of the action generation process. The local control synthesis modules based on the above approach are set to interact within the confines of their environment. From such nuclear activities of properly designed modules, a global regulating control action will emerge, and the group will "self-organize".

Obviously it is not feasible for agents in a large group with distinct goals to be *a priori* aware of each other's presence, to communicate with each other or with a central agent regarding advice on what action to take. As mentioned above, the only remaining option is for each agent to make its own decision on how to act based on the sensory data which the agent dynamically extracts from its local surroundings (Figure 8). Knowing that there is more than one interpretation of decentralization, the author considers a multi-agent system decentralized if each agent in the group is independent from the others in sensory data acquisition, data processing, and action projection. In a decentralized system, these faculties are configured in a mode that would give rise to coordination in the group without a coordinator. In other words, the group is capable of self-organization. Unlike centralized, top-down approaches, self-organization is a bottom-up approach to behavior synthesis where the system designer is only required to supply the individual agents with basic, "self-

control" capabilities. The overall control action that shapes the behavior of the agents evolves in space and time as a result of the interaction of the agents among themselves and with their environment.
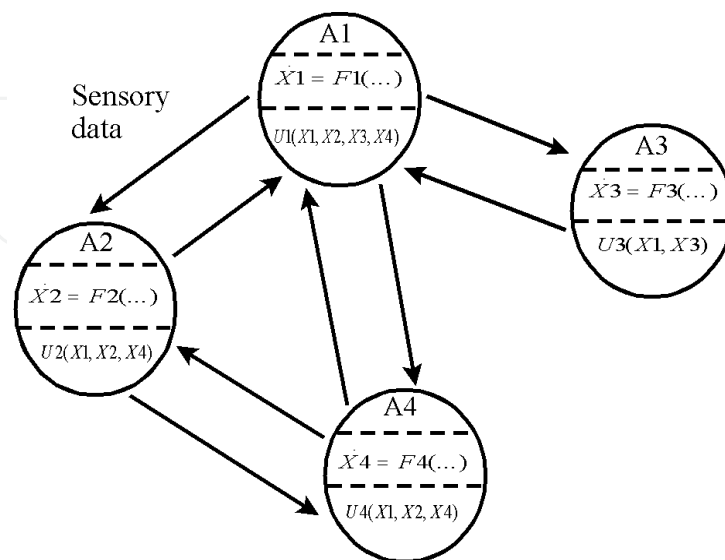

Fig. 8. Decentralized approach to control

Some properties of decentralized systems that conform to the above definition are:
1. There is no need to search or, for that matter, construct the HAS of the group in order to generate a solution. For a decentralized system, the solution emerges as a result of the agents interacting among themselves and with their environment.
2. No inter-agent communication, or communication with a supervisory agent is required. All that an agent is required to do is to observe (not communicate with) other agents in its local neighborhood. No preexisting awareness of the whole group, or the whole environment is required.
3. Synchronous behavior is an emergent phenomenon (instead of an imposed one) that results from asynchronous interaction.
4. The complexity of control in the group grows linearly with the number of agents.
5. Decentralized systems, where every one of its member agents independently sense its environment, process data, and actuate motion, form open systems that enable any agent to join or leave the group without the others having to adjust the manner in which they process information or project action.
6. Unlike centralized systems which are informationally-closed, and organizationally-open, decentralized systems are informationally-open and organizationally-closed.

The difference between centralized and decentralized systems goes far beyond the manner in which the behavior generation faculties are related to the agents. They reach as deep as the process enabling the system to generate the information needed for behavior synthesis. Centralized systems use reasoning coupled with search as the driver of the action selection process (it ought to be mentioned that function/al minimization is a form of search). The search of the system's space of possible actions for a feasible solution may be carried out in a brute force manner, or in an intelligent manner that utilizes heuristics and side information for speed. No matter what form the search assumes or how it is applied, systems relying on search have very serious problems, some of which are mentioned above, if they operate in a

dynamic environment. On the other hand, the action selection driver in decentralized systems that satisfies the above requirements is a synergy-driven evolution. In this mode of behavior information synthesis is the result of the synergetic interaction of the agents among themselves under the influence of their environment. The information that is *a priori* encoded into each agent in the form of self-capabilities to project actions is usually simple and not adequate, on its own, to handle the usually complex planning task which faces the group. It is synergetic interaction within the context of the environment that augment the level of information which the group has to a level that is sufficient for the members to carry out the task at hand (an act of knowledge amplification).

3. The artificial life approach

Despite the abundance of evolutionary techniques (e.g. neural nets, genetic algorithms, reinforcement learning, Bayesian belief networks, etc.) a relatively new area in evolutionary behavior synthesis that is called artificial life (AL) (Langton, 1988) seems to provide a powerful paradigm for explaining the behavior of decentralized systems. It also provides constructive guidelines for their synthesis. In an AL system, the members of the group are equipped with the proper elementary, *a priori* known capabilities for self-control which are called the Geno-type of behavior (G-type). On the other hand, the overall control action that actually governs the behavior of the whole group evolves in space and time as a result of the interpretation of the G-type in the context of a particular environment. The whole control action is called the Pheno-type (P-type) of behavior. This behavior cannot be, exactly, *a priori* predicted, only certain aspects of it can be *a priori* known. It is very flexible, highly adaptive, and far exceeds in complexity and informational content the G-type control. There are two requirements for constructing a proper G-type control action:
1.   Each agent must individually develop a control action to drive it toward its goal. Such a control need not take into consideration the control actions generated by the other agents of the group.
2.   Each agent must have the ability to generate a control that can resolve conflict with other agents through bilateral interaction.

Despite being an inherently multi-agent approach, the AL approach is applicable to the case of an isolated agent trying to synthesize a regulating control action utilizing only finite sensory and data processing resources. This is possible despite the fact that more than one agent is needed for synergetic interaction to take place. The agents needed to trigger synergy need not be physical, they can be a virtual construction of the agent concerned. For this case, the agent starts by densely spreading micro-agents, in its own image, all over the situational space the agent has the potential of occupying (Figure 9).

The only difference between the "mother-agent" and a micro-agent is that the state of the former agent evolves in space and time, while the state of a micro-agent is stagnant and immobilized to only one *a priori* known point in state space. Only the control action associated with each micro-agent is allowed to evolve. The micro-agent concept is used to construct a control action group for the agent by first covering the state space with a manifold that has locally (point-wise) extractable vector features which homogeneously cover the domain on which the control is defined. The vector features are determined by the vector partial differential operator that is used to operate on the manifold to induce a vector field that may be used to describe the action structure of the micro-agent group, therefore generating the action field of the agent. The second step is to provide each micro-agent with

the ability to generate a proper differential behavior. Differential behavior is a self-behavior where a micro-agent does not attempt to influence the other micro-agents with which it is directly interacting. Instead, it forms a soft informational coupling with them where it only observes their behavior and uses this information to derive a self-action that governs its and only its behavior in state space (figure 10). This may be achieved by constraining the vector partial differential operator that is used to emulate the actions of the micro-agents using another partial differential operator. This operator encodes how a micro-agent is going to constrain its behavior with respect to the behavior of the other micro-agents it is interacting with. The third step is to induce a proper action structure over the micro-agent group. In centralized approaches, each micro-agent has to search for the "correct" action in order to generate a group structure that unifies all the micro-agents in one goal-oriented unit. In the proposed approach, a micro-agent is only required not to exert the "wrong" actions that could result in the failure of the agent to reach the goal.
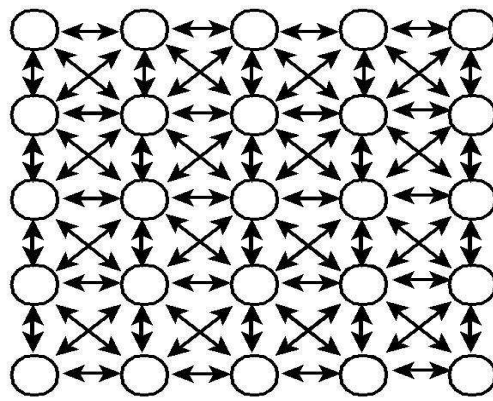

Fig. 9. A dense matrix of interacting virtual agents

Not selecting the wrong actions is not enough, on its own, for each micro-agent to restrict itself to one and only one admissible action that would constitute a proper building block of the global structure needed to turn the micro-agent group into a functioning unit. The additional effort needed to induce the global structure on the micro-agents is a result of the evolution of the behavior of the micro-agent group in time and space under the guidance of the environment (i.e. morphogenesis,(Thom, 1975)). This guidance is what eventually limits each micro-agent to one and only one action that is also a proper component in a functioning group structure. The environment guidance may be factored into the behavior generation process as state boundary conditions which play the role of self-preserving actions that the agent is *a priori* equipped with. The behavior of a micro-agent at a location which the agent believes to be harmful is constrained to an *a priori* known survival action that would drive motion away from it and towards a safe region. The environment guidance could also be in the form of instructions restricting the behavior of the agent at certain region in the admissible space (e.g. constraints on direction along which motion should proceed). The above approach was used to derive a new class of intelligent, emergent, situated, end embodied class of controller called evolutionary, hybrid, pde-ode controllers (EHPCs) that are suitable for constructing the self-control component (G-Type control) of a multi-agent system. the evolutionary, hybrid, PDE-ODE control (EHPC), Figure 11. An EHPC consists of two parts:

1. a discrete time-continuous time system to couple the discrete-in-nature data acquisition process to the continuous-in-nature control action release process;

2. a hybrid, PDE-ODE controller (HPC) to convert the acquired data into in-formation that is encoded in the structure of the micro-control action group. For more details about this type of control see (Masoud & Masoud, 2000a; Masoud & Masoud,1998; Masoud & Masoud, 1997; Masoud & Masoud,1994; Masoud & Masoud, 2000b; Masoud & Masoud, 2002).
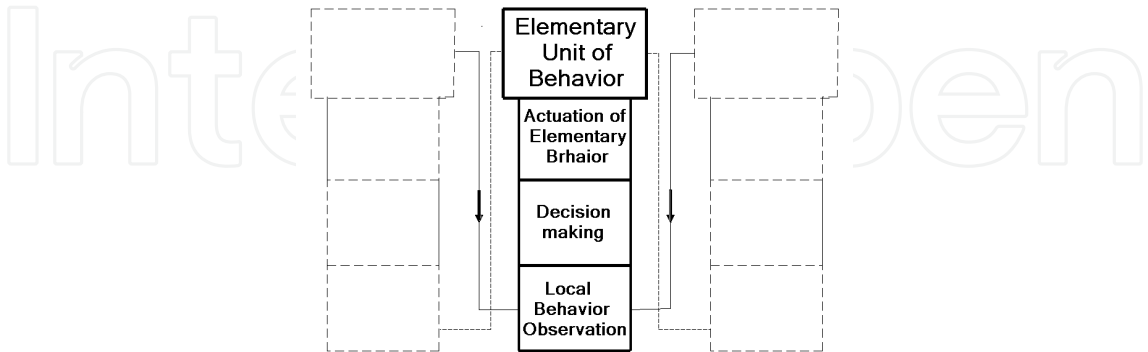

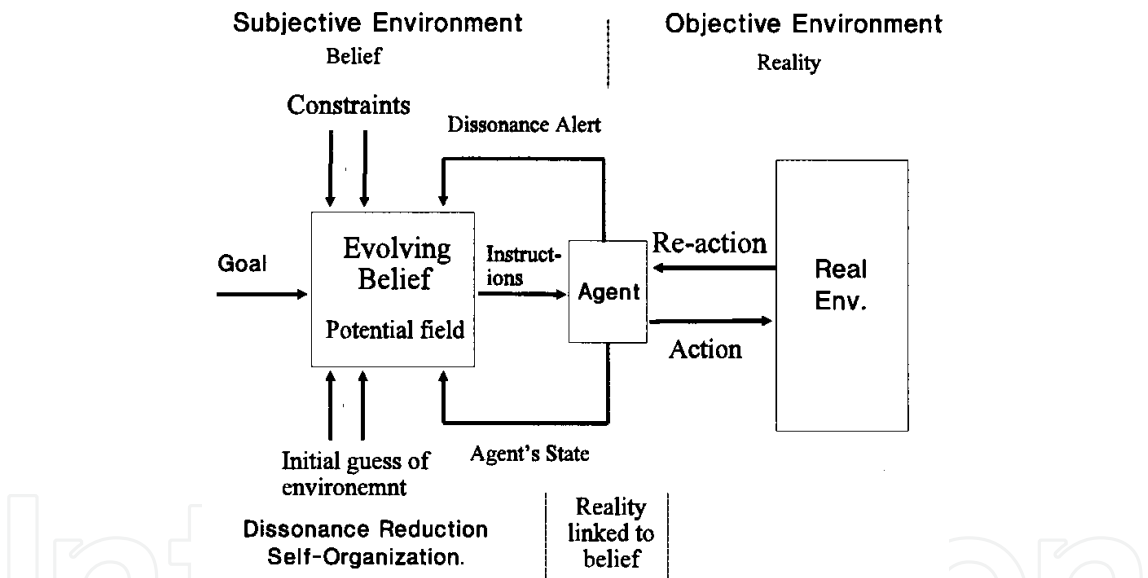Fig. 10. Layers of functions in an interactive micro-agent

Fig. 11. A structure for an EHPC

4. The harmonic potential field approach: a background

The HPF approach is a realization of EHPCs. It mathematically captures the behavior of an AL system by first emulating the dense collective of micro-agents using a situation space cover in the form of a potential field (V) that is acted on by a differential operator (usually the gradient operator, ∇V). The vector differential elements, which may be perceived as the micro-agents, are sensitized to each other using a vector differential relation that locally imposes dependence on the behavior of these vectors. The form to which the structure of the vector differential elements converge to (i.e the interpretation of the G-type action in the context of the environment) may be controlled by boundary conditions or by making G-type

action environment-dependant. Figure 12 shows an HPF-based group action evolving in space and time until it finally reaches a form that allows it to function as desired.
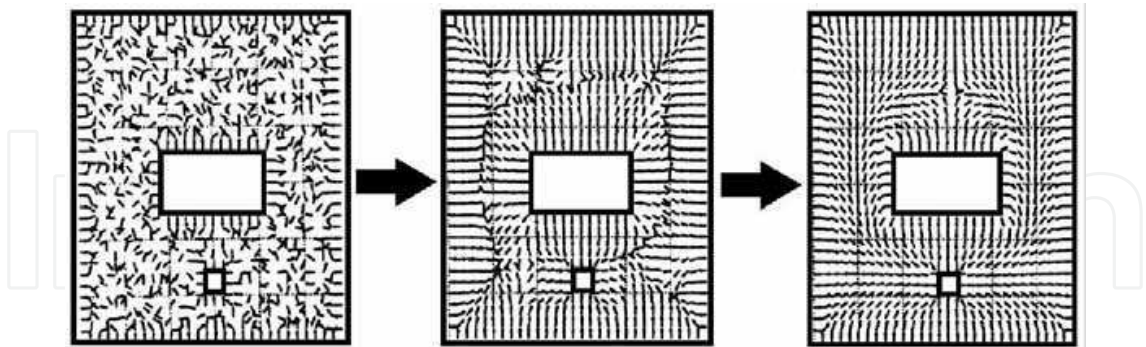

Fig. 12. Evolution of the control action in a PRF component

The harmonic potential field approach is a powerful, versatile and provably-correct means of guiding motion in an N-dimensional abstract space to a goal state subject to a set of constraints. The approach works by converting the goal, representation of the environment and constraints into a reference velocity vector field using the mechanism described above (figure 13). This reference field is usually generated from a properly conditioned negative gradient of an underlying potential field.
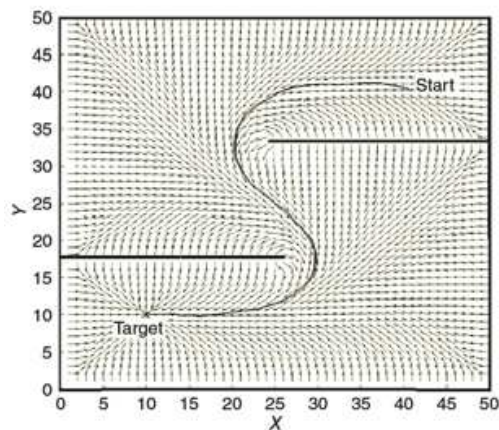
Fig. 13. The velocity field from an HPF along with the resulting trajectory

A basic setting of the HPF approach (1) is:

solve:
$$\nabla^2 V(X) \equiv 0 \qquad X \in \Omega \tag{1}$$

subject to: $V(X) = 1$ at $X = \Gamma$ and $V(X_T) = 0$,
A provably-correct path may be generated using the gradient dynamical system (2):

$$\dot{X} = -\nabla V(X) \tag{2}$$

where X is a point in an abstract N-dimensional space (usually N=3), Ω is the workspace, Γ is its boundary and X_T is the target point.
Many variants of the above setting were later proposed to extend the capabilities of the HPF approach. For example, it is demonstrated that the approach can be used for planning in
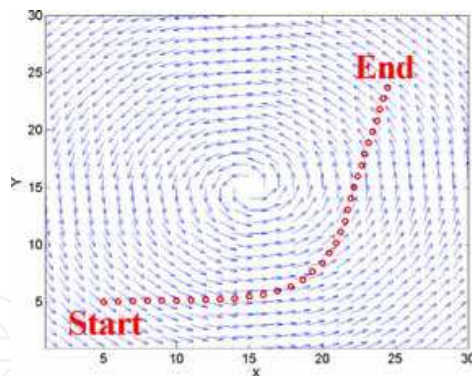
complex unknown environment (Masoud & Masoud, 1997) relying on local sensing only (figure 14),

Fig. 14. HPF-based planning in unknown environments

The HPF approach can also incorporate directional constraints along with regional avoidance constraints (Masoud & Masoud, 2002) in a provably-correct manner to plan a path to a target point (figure 15),

Fig. 15. HPF-based planning with directional and regional avoidance constraints

The HPF approach may also be modified to deal with inherent ambiguity (Masoud, 2009a) that prevents the partitioning of an environment into admissible and forbidden regions (figure 16),

Fig. 16. HPF-based planning in non-divisible environments

It can also be adapted to deal with environments containing obstacles and a drift field which suits planning for energy exhaustive missions (figure 17).

Fig. 17. HPF-based planning in the presence of drift fields

It was demonstrated in (Gupta et al, 2006) that the HPF approach can work with integrated navigation systems that can efficiently function in a real-life situation (figure 18).
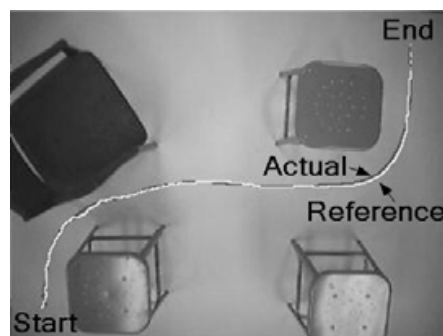
Fig. 18. HPF-based integrated navigation system
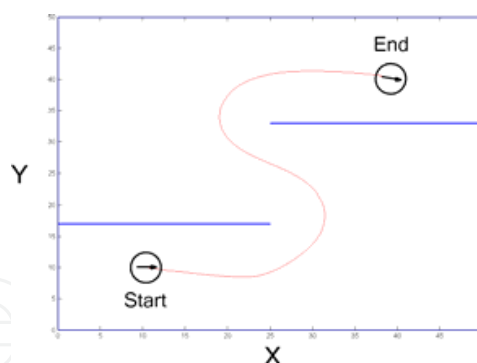
Fig. 19. HPF-based, nonholonomic, dynamical navigation

Work on extending the HPF approach to work with dynamical and nonholonomic systems (figure 19) may be found in (Masoud, 2009b; Masoud, 2010).

5. Decentralized multi-agent HPF planners

The single-agent HPF approach has amassed and is still gaining a wide array of capabilities that makes it applicable to a large number of practical and challenging problems in planning. In the remainder of this chapter, it is demonstrated by examples that the HPF approach does extend to the multi-agent case while maintaining an adaptive, evolutionary, decentralized, self-organizing nature that is compliant with the AL paradigm to behavior

synthesis. It is shown that a single HPF planner by itself or slightly modified does play the role of the G-Type controller (self-control or control protocol) in a multi-agent system.

Formally a multi-agent planner must maximize both the minimum inter-agent distance as well as the minimum distance between the agents and the clutter populating the environment while guaranteeing that each member reaches its destination in the desired manner. Unfortunately formulating the problem in this manner for a large group of agents leads to an intractable situation. Self-organizing optimization methods (Wu & Chow, 2007; Kohonen, 1997; Lampinen & Storn, 2004; Hao et al, 2007) may be used for such a purpose. They are known for their ability to handle nonlinear functions having large degrees of freedom. Neglecting the fact that these methods are not provably correct and cannot guarantee that a solution can be found if one exist, they do not provide acceptable transient behavior that allows them to serve online as trajectory controllers. Instead of seeking a formal and optimal solution to the problem, a practical solution with acceptable properties is suggested using the HPF approach. The solution sought is built around a decentralized paradigm that employs local interaction and sensing among agents (figure 20) in regulating the group's motion. The artificial life paradigm to behavior synthesis does support this mode of operation. Therefore designing a controller for the collective reduces to designing the proper G-Type controller which each agent must use. The controller should be designed such that for the overall system conflict is eliminated and goal for each member is attained while enforcing additional constraints on the individual trajectories if needed.
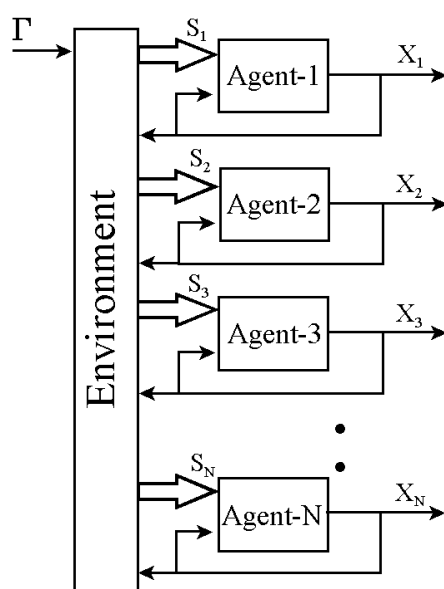

Fig. 20. Overall, decentralized system

5.1 The vector-harmonic multi-agent potential field approach

This decentralized, self-organizing, multi-agent, HPF-based method relies on local information in de-conflicting the workspace. Each agent, independently, uses an HPF method to guide it to its target in the desired manner. The control protocol individually used by the agents to reach their goals is constructed by augmenting the HPF controller with sensor-actuated, local vector potential fields. The HPF component of the protocol is called the purpose field (PRF) and vector potential component is called the conflict resolving field (CRF).

5.1.1 Formulation

In this section the problem of decentralized, multi-agent motion planning in the face of incomplete information is formulated. An agent ($D_i(x)$) is assumed to be massless, and occupy a multi-dimensional, hyper sphere ($x \in R^M$) with a radius ρ_i and a center x_i:

$$D_i(x) = \left\{ x : |x - x_i| \leq \rho_i \right\} \quad i=1,...,L, \tag{3}$$
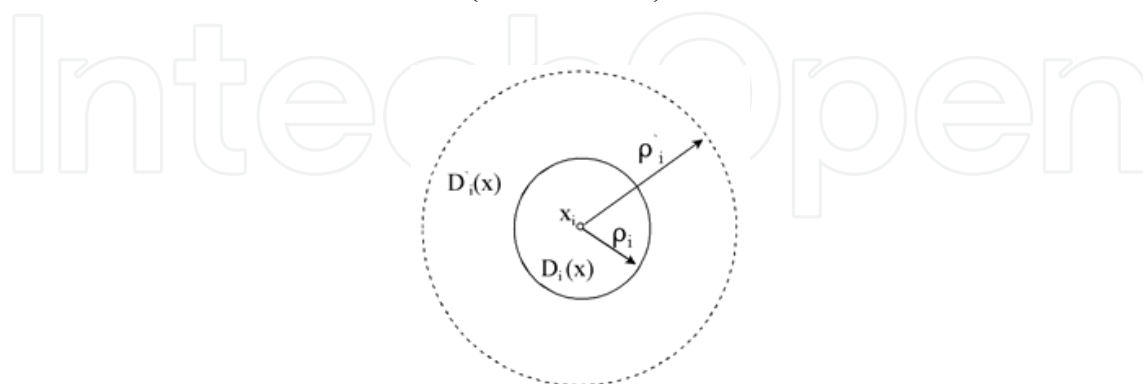
Fig. 21. Zones related to D_i

where L is the number of agents occupying the workspace (figure 21). An enlarged circular region ($D'_i(x)$) with radius ρ'_i ($\rho'_i > \rho_i$) and center x_i is assumed to be surrounding $D_i(x)$:

$$D_i(x) = \left\{ x : |x - x_i| \leq \rho_i \right\} \quad i=1,...,L,$$
$$D_i(x) \subset D'_i(x) \tag{4}$$

The ring $S_i(x)$ ($S_i(x) = D'_i(x) - D_i(x)$) surrounding $D_i(x)$ marks the region illuminated by the sensors of the i'th agent. The time between an agent sensing an event and releasing a control action (data processing and action release delay) is assumed to be small enough to be neglected in practice. Therefore, this region is a dual sensory and action zone. Besides the agents, the environment is assumed to contain static, forbidden regions (O) which the agents must not occupy at any time ($O \cap D_i \equiv \varphi, \forall t, i=1,...,L$). The agents are only allowed to exist in the workspace Ω ($\Omega = R^M - O$). The boundary of the forbidden regions is referred to as Γ ($\Gamma = \partial O$). The destination of the i'th agent is surrounded by the spherical region $T_i(x)$ with a center C_i (figure 22). T_i's are chosen so that:

$$\begin{array}{ll} D'_i(x) \subset T_i(x) & x_i = C_i \\ T_i(x) \cap T_j(x) \equiv \phi & i \neq j \\ O \cap T_i(x) \equiv \phi & i=1,...,L. \end{array} \tag{5}$$

The last two conditions, respectively, mean that the goals of the different agents should not be conflicting, and should be attainable (i.e. lie inside Ω). The partial knowledge the i'th agent has about its stationary environment is represented by Γ_i ($\Gamma \supseteq \Gamma_i \supseteq \varphi$, i=1,..,L). The binary variable Q_i ($Q_i \in \{0,1\}$) marks the event of an agent discovering part of the forbidden regions not previously known to it, i.e.

$$\begin{array}{l} S_i(x) \cap \Gamma \neq \phi, \\ \left(S_i(x) \cap \Gamma \right) \cap \Gamma'_i \equiv \phi \qquad i=1,...,L. \end{array} \tag{6}$$

If at any instant in time (t_n), this condition becomes true, the content of Γ_i is adjusted so that:

$$\Gamma_i^{'}(t_n) = \Gamma_i^{'}(t_n - dt) \cup (S_i(x) \cap \Gamma). \tag{7}$$

If such a situation transpires, Q_i is set to 1, otherwise, its value is set to zero. The i'th agent also actively monitors its immediate neighborhood for the presence of other agents. It forms the set:

$$\chi_i(t) = \{x_j : S_i(x) \cap D_j(x) \neq \phi, j = 1,.., K_i(t), i \neq j\}, \tag{8}$$

where K_i is the number of agents lying in the proximity of the i'th agent at time t.
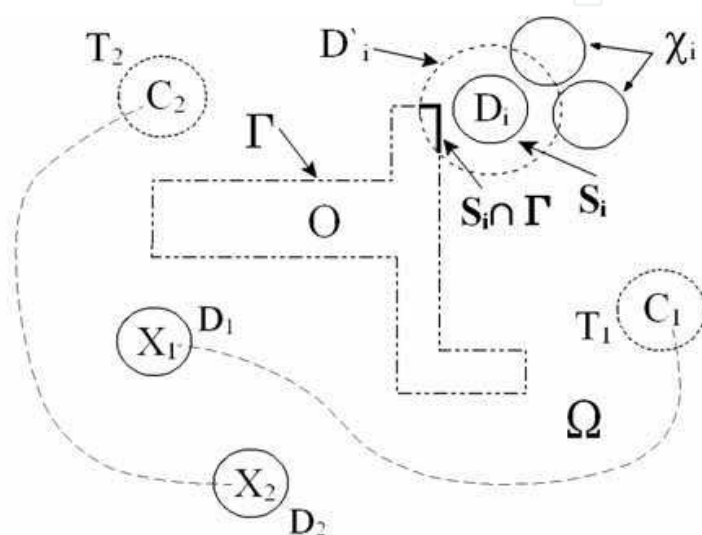
Fig. 22. Goal oriented agents in a cluttered environment

Designing the multi-agent controller requires the synthesis of the dynamical systems:

$$\dot{x}_i = h_i(x_i, C_i, Q_i, \chi_i, \Gamma_i^{'}) \qquad \qquad i=1,..,L, \tag{9}$$

such that the following constraints are satisfied for the overall system:

$$\dot{X} = \mathbf{H}(X, C, Q, \Gamma^{'}),$$

$$\underset{t \to \infty}{Lim}\, x_i(t) \to C_i \qquad \qquad i=1,...,L$$

$$D_i(x) \cap D_j(x) \equiv \phi \qquad \qquad \forall t,\, i \neq j \tag{10}$$

$$O \cap D_i(x) \equiv \phi,$$

where $x_i \in R^M$, X=$[x_1 ... x_L]^t$, C= $[C_1 ... C_L]^t$, Q=$[Q_1 ... Q_L]^t$, $\Gamma^{'}$=$[\Gamma_1^{'}...\Gamma_L^{'}]^t$, $\mathbf{H}$=$[h_1 ... h_L]^t$.

5.1.2 Controller design
As discussed earlier an AL approach to behavior synthesis reduces the job of the designer to only constructing the self-controllers (G-type control) of the agents as individuals. The

overall control action that regulates the behavior of the agents as a group operating in the context of some environment (P-type control) evolves as a result of the constrained synergetic interaction among the agents. The designer is required to synthesize controls for the systems:

$$\dot{x}_i = \mathbf{u_i} = h_i(x_i, C_i, Q_i, \chi_i, \Gamma_i^{`}) \qquad i=1,..,L .$$ (11)

The i'th self-control is divided into the following three components:

$$\mathbf{u_i} = \mathbf{ug_i}(x_i, C_i, Q_i, \Gamma_i^{`}) + \mathbf{uc_i}(x_i, \chi_i) + \mathbf{uo_i}(x_i, \Gamma_i^{`}),$$ (12)

where **ug$_i$** is the PRF component of the i'th self-control, **uc$_i$** is the CRF component, and **uo$_i$** is an optional control component that is included as an extra precaution against collision with stationary obstacles. It ought to be mentioned that **ug$_i$** includes, among other things, the ability to avoid collision. Details about how to construct **uo$_i$** may be found in (Khatib, 1985).

5.1.2.1 The PRF control

The PRF component of the multi-agent controller is required to guide a single agent in a stationary, cluttered environment assuming other agents are not present. In its simplest form, a PRF control assumes the form of simple vector fields that play the role of vocabularies for a language of behavior. For examples, the vector fields in figure 23 denote the behavioral vocabularies: "go to the center point", "move right", and "move right along a straight line" respectively.
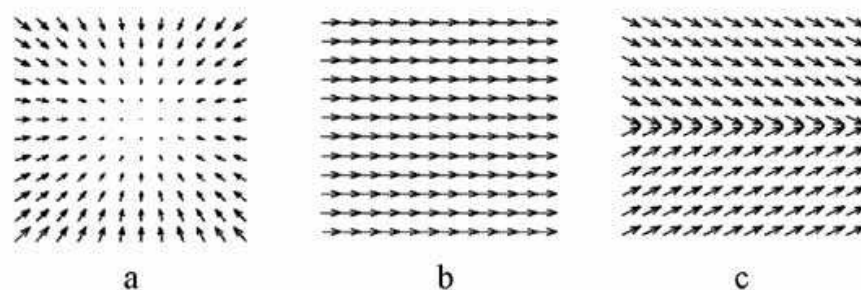

Fig. 23. Vector field-based behavioral primitives for a single agent

Such fields can be useful in simple situations where the agent is operating in a lightly cluttered environment and there are few constraints on behavior. In a realistic situation the environment may consist of heavy, irregular clutter that is not *a priori* known to the agent. The agent may also be required to constrain its behavior in the vicinity of the forbidden regions and inside the workspace. In such situations the approach of using behavioral primitives spatially and/or temporally foliated using a syntax that is determined by an algorithm or a human operator may lead to an undesired outcome. What is needed in such situations is the design of a goal-oriented, context sensitive, intelligent control action that can semantically embed the agent in the context of an environment that need not be *a priori* known. The approach adopted for PRF control synthesis is similar to the one described at the end of section 3. It assumes the lucidity of the control action. In this approach, state space is assumed to be covered with a dense set of freely-configurable control vectors. The structure that converts the individual micro-control actions into a group that can project the desired macro-control is induced on the substrate of micro-control actions using a decentralized, AL-based method (Figure 24).
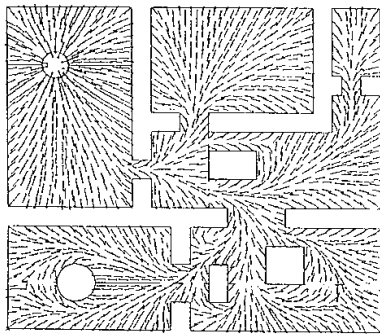
Fig. 24. A PRF field generated for a geometrically complex environment

The EHPC representing the i'th PRF control component is:

$$\mathbf{ug}_i = -\nabla V_i(x_i, C_i, Q_i(t_n), \Gamma_i^{'}(t_n)) \tag{13}$$

so that for the gradient dynamical system:

$$\dot{x}_i = -\nabla V_i(x_i, C_i, Q_i(t_n), \Gamma_i^{'}(t_n)) \ ,$$

$$\underset{\substack{n \to Z \\ t \to \infty}}{\text{Lim}} x_i(t) \to C_i \qquad\qquad \text{i=1,..,L, n=1,...,Z} \tag{14}$$

$$D_i(t) \cap O \equiv \phi \qquad\qquad \forall t,$$

where n represents the n'th instant at which condition (4) becomes valid (t_n) , Z is a finite, positive integer, and ∇ is the gradient operator. At t_n, which marks the transition of $Q_i(t_n)$ from 0 to 1, first the contents of $\Gamma_i^{'}$ are adjusted according to (5). The boundary value problem (BVP) below is then used for synthesizing V_i:

$$\nabla^2 V_i(x) \equiv 0 \qquad\qquad x \in R^N - \Gamma_i^{'} - C_i$$

$$V_i = 0|_{x=C_i} \ \& \ V_i = 1|_{x \in \Gamma_i^{'}} \ . \tag{15}$$

It ought to be mentioned that the above BVP is not the only one that can be used for generating the gradient field. Many other BVPs such as the ones reported in section 4 may be used for such a purpose.

5.1.2.2 The CRF control

There are only two ways conflict could arise in a workspace occupied by more than one purposive, mobile agents, each of which is capable of reaching its target in the absence of other agents:

1. Two or more agents may attempt to occupy the same space at the same time.
2. Two or more agents may block each other's way preventing the movement towards the targets.

A conflict resolving control (**uc$_i$**) that can prevent the above two events from happening will enable the utilizing agent to reach its target.

It is obvious that an agent can prevent another from moving towards it, hence occupying the same space it is using, by exerting a force that is radial (**ucr$_i$**) to its boundary (i.e. pushing the other agent away from it, figure 25). On the other hand, an agent can prevent others

from blocking its path by exerting a force that is tangential (**uct_i**) to its boundary (i.e. moving out of the way, figure 25)
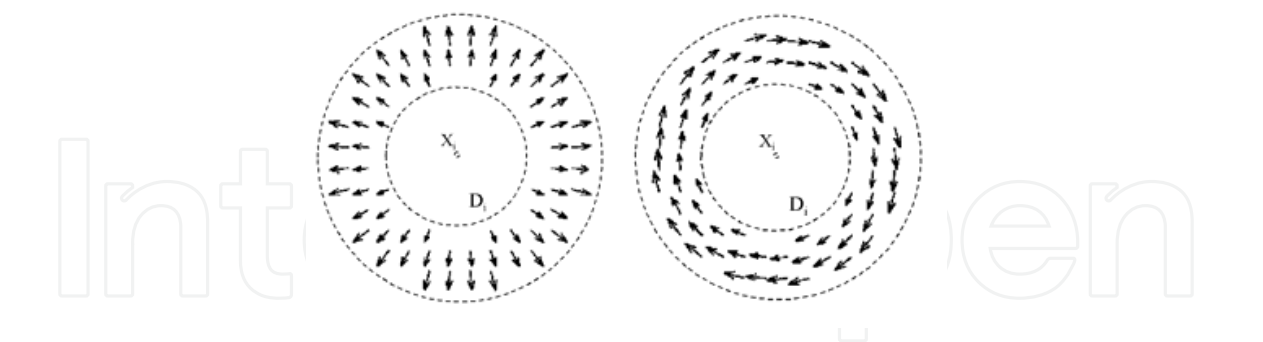

Fig. 25. Radial and Tangential components of the CRF

The CRF component is the sum of the above two actions:

$$\mathbf{uc_i} = \mathbf{ucr_i} + \mathbf{uct_i} . \qquad (16)$$

The radial component of the control (**ucr_i**) may be constructed as:

$$\mathbf{ucr}_i = \sigma(|x - x_i|) \frac{\nabla Vr_i(|x - x_i|)}{|\nabla Vr_i(|x - x_i|)|} , \qquad (17)$$

where both the weighting function σ, and the scalar potentials Vr's are positive, spherically symmetric, monotonically decreasing functions whose values are zero for $|x\text{-}x_i| \geq \rho_i'$. As for **uct_i**, it may be constructed as :

$$\mathbf{uct}_i = \sigma(|x - x_i|) \frac{\nabla \times \mathbf{A}_i(x - x_i)}{|\nabla \times \mathbf{A}_i(x - x_i)|} \qquad \nabla \cdot \mathbf{A_i} \equiv 0, \qquad (18)$$

where $\nabla \cdot$ is the divergence operator, and **A_i** is a vector potential field (Masoud & Masoud, 2000b) selected so that:

$$\nabla Vr_i(|x - x_i|)^t \nabla \times \mathbf{A}_i(x - x_i) \equiv 0 \cdot \qquad (19)$$

For the local tangent fields to form a continuous, global tangential action that has the potential to push the interacting agents out of each other's way and prevent deadlock, all the individual tangent fields must circulate along the same direction (figure 26).
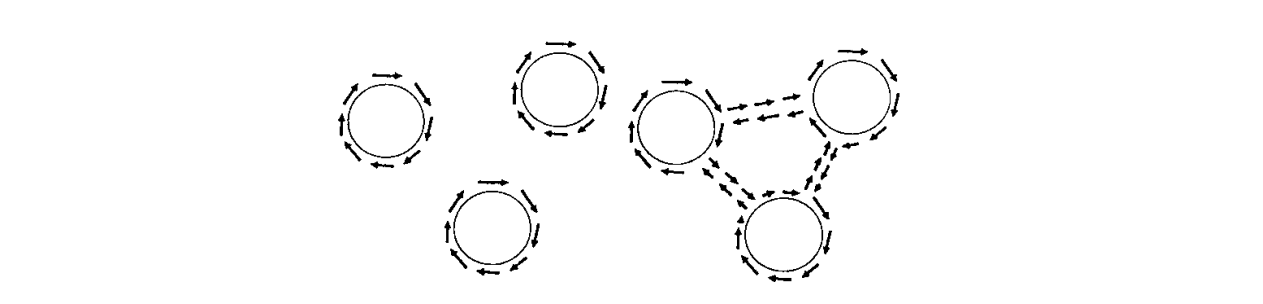

Fig. 26. Same circulations guarantees a larger one circulation along the same direction

The overall controller governing the i'th agent is described by the dynamical system:

$$\dot{x}_i = \mathbf{ug}_i + [\mathbf{ucr}_i + \mathbf{uct}_i] + \mathbf{uo}_i$$
$$= -\nabla V_i(x_i, C_i, Q_i(t_n), \Gamma_i^{'}(t_n)) +$$
$$\sum_{j=1}^{K_i(t)} \sigma(|x_i - x_j|) \left[\frac{\nabla Vr_i(|x_i - x_j|)}{|\nabla Vr_i(|x_i - x_j|)|} + \frac{\nabla \times \mathbf{A}_i(x_i - x_j)}{|\nabla \times \mathbf{A}_i(x_i - x_j)|} \right] + \tag{20}$$
$$\nabla Vo_i(x_i, \Gamma_i^{'}),$$

where $\mathbf{uo_i} = \nabla Vo_i(x_i, \Gamma_i)$, and Vo_i is a scalar, repelling potential field that is strictly localized to the vicinity of the obstacles. The dynamical equation governing the behavior of the collective is:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} \mathbf{ug_1}(x_1, C_1, Q_1(t_n), \Gamma_1^{'}(t_n)) \\ \mathbf{ug_2}(x_2, C_2, Q_2(t_n), \Gamma_2^{'}(t_n)) \\ \vdots \\ \mathbf{ug_L}(x_L, C_L, Q_L(t_n), \Gamma_L^{'}(t_n)) \end{bmatrix} + \begin{bmatrix} \mathbf{uo_1}(x_1, \Gamma_1^{'}(t_n)) \\ \mathbf{uo_2}(x_2, \Gamma_2^{'}(t_n)) \\ \vdots \\ \mathbf{uo_L}(x_L, \Gamma_L^{'}(t_n)) \end{bmatrix} + \begin{bmatrix} \sum_{\substack{j=2, \\ j \neq 1}}^{K_1(t)} \mathbf{uc_1}(x_i - x_j) \\ \sum_{\substack{j=1, \\ j \neq 2}}^{K_2(t)} \mathbf{uc_2}(x_i - x_j) \\ \vdots \\ \sum_{\substack{j=1, \\ j \neq L}}^{K_L(t)} \mathbf{uc_L}(x_i - x_j) \end{bmatrix} \tag{21}
$$

5.1.3 Motion analysis

A detailed proof of the ability of the agents, individually, to reach their respective destinations in an unknown cluttered environment may be found in (Masoud & Masoud, 2002; Masoud & Masoud, 2000a). While it is not hard to see that the ability of the robots to avoid collision with each other and with the obstacles can be guaranteed by making the barrier controls ($\mathbf{uo_i}$, $\mathbf{ucr_i}$) strong enough (some techniques set the strength of the control to infinity at the inner boundary of the robots (Khatib, 1985), their ability to converge to their respective destinations, as a group, needs careful examination. In the following it is shown that the first order dynamical systems in (20) are potentially capable of driving the robots from anywhere in the workspace to their respective destinations provided that the narrowest passage in the workspace is wide enough to allow the largest two robots to pass at all times.

Here, it is shown that under certain conditions the solution of the system in (21) is globally, asymptotically stable. The proof is dependent on a theorem by LaSalle (Theorem-3, (LaSalle, 1960), pp. 524]. The theorem is restated below with minor changes to the notations.

Theorem: Let $\Xi(X)$ be a scalar function with continuous first partials with respect to X. Assume that:

$$\begin{array}{lll} 1- & \Xi(X) > 0 & \forall X \neq C, \\ 2- & \dot{\Xi}(X) \leq 0 & \forall X . \end{array} \tag{22}$$

Let E be the set of all points where $\dot{\Xi} = 0$, and M be the largest invariant set in E. Then every solution of the system:

$$\dot{X} = \mathbf{H}(X, C, Q, \Gamma^{\backprime}) \tag{23}$$

bounded for $t \geq 0$ approaches M as $t \to \infty$.

Proposition-1: For the system in (21), $\exists$ a set of **uct**'s that can guarantee

$$\lim_{t \to \infty} X(t) \to C, \tag{24}$$

provided that: 1- for the gradient dynamical systems:

$$\dot{x}_i = -\nabla V_i(x_i, C_i, Q_i(t_n), \Gamma_i^{'}(t_n))'$$

1- $\lim_{t \to \infty} x_i(t) \to C_i$ i=1,...,L

2- $D_i \cap D_j \equiv \phi,$ $i \neq j$

         $D_i \cap O \equiv \phi,$ (25)

3- $\forall x^{'} \in \Omega \quad \exists x_c, \ni$

         $x^{'} \in \{ x : | x - x_c | \leq \xi \} \subset \Omega \cdot$

where $\xi = \rho^{\backprime}_1 + \rho^{\backprime}_2$, where $\rho^{\backprime}_1$ and $\rho^{\backprime}_2$ are the expanded radii of the two largest robots in the group. The third condition guarantees that nowhere in Ω will the geometry of the environment prevent the agents from resolving the conflict, and instead forces them to project motion along environmentally-determined degrees of freedom (Figure 27) that may not lend themselves to the resolution of the conflict (a restrictive environment).
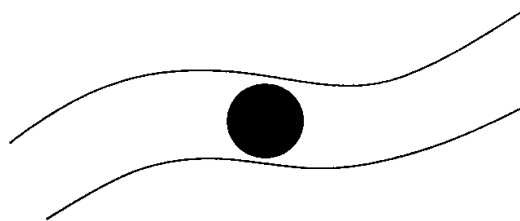

Fig. 27. Restrictive environments force *a priori* determined spatial movement patterns

By guaranteeing that there always exists a local, simply-connected region that is large enough to enable any two robots to interact, guarantees that whatever pattern of motion which the agents arrive at to resolve the conflict can be realized.

Proof: consider the following Liapunov function candidate (LFC):

$$\Xi(X) = \sum_{i=1}^{L} V_i(x_i) , \tag{26}$$

where $V_i(x_i)$ is used to refer to $V_i(X_i, C_i, Q_i(t_n), \Gamma_i^{'}(t_n))$, and $Vo_i(x_i)$ refers to $Vo_i(x_i, \Gamma_i)$. It was shown in (Masoud & Masoud, 2002; Masoud & Masoud, 2000a) that harmonic potential fields are LFCs, i.e. $V_i(x_i)=0$ for $x_i = C_i$, and $V_i(x_i) > 0$ for $x_i \neq C_i$. Therefore the above sum is a

valid LFC, i.e. $\Xi(X) = 0$ for $X = C$, and $\Xi(X) > 0$ for $X \neq C$. The time derivative of Ξ may be computed as:

$$\frac{d}{dt}\Xi = \sum_{i=1}^{L} \nabla V_i(x_i)^t \frac{d}{dt} x_i$$

$$= \sum_{i=1}^{L} \nabla V_i(x_i)^t [-\nabla V_i(x_i) +$$

$$\sum_{\substack{j=1 \\ i \neq j}}^{K_i(t)} \sigma(|x_i - x_j|) \left[\frac{\nabla V r_i(|x_i - x_j|)}{|\nabla V r_i(|x_i - x_j|)|} + \frac{\nabla \times A_i(x_i - x_j)}{|\nabla \times A_i(x_i - x_j)|} \right] +$$

$$\nabla V o_i(x_i)]. \tag{27}$$

The above expression is examined term by term to determine the nature of the time derivative of Ξ. It is obvious that the term:

$$\sum_{i=1}^{L} -\nabla V_i(x_i)^t \nabla V_i(x_i) \tag{28}$$

is negative definite with a zero value (stable global equilibrium) at and only at $x_i = C_i$, $i=1,..,L$, $(X=C)$. As for the term:

$$\sum_{i=1}^{L} \nabla V_i(x_i)^t \nabla V o_i(x_i), \tag{29}$$

One must first notice that $\nabla V o$ is a local field that is strictly limited to a thin narrow region surrounding Γ. Its value is zero everywhere else in Ω. By construction, the field lines of $\nabla V o_i$ emanate normal to Γ (in order to drive the robot away from the obstacles):

$$\nabla V o(x_i) = \begin{bmatrix} \alpha(x_i)\mathbf{n} & x_i \in \Gamma_i^{'} \\ 0 & \text{elsewhere} \end{bmatrix} \tag{30}$$

where $\mathbf{n}$ is a unit vector that is normal to Γ, and α is a smooth, positive, monotonically decreasing scalar function with a value set to zero a small distance (ε) away from the boundary of the obstacles ($x_i^t \mathbf{n}$), i.e. $\alpha(x_i)=0$ for $x_i^t \mathbf{n} > \varepsilon$. The BVPs used for constructing the potential field associated with the PRF control (V_i) admits only two types of basic boundary conditions (BCs):

1- homogeneous Neumann BCs:

$$\frac{\partial}{\partial \mathbf{n}} V_i(x_i) = \nabla V_i(x_i)^t \mathbf{n} \equiv 0, \qquad x_i = \Gamma_i^{'} \tag{31}$$

2-homogeneous Dirichlet BCs: $V_i(x_i) = 1$

which in turn makes: $\frac{\partial}{\partial \mathbf{n}} V_i(x_i) = \nabla V_i(x_i)^t \mathbf{n} < 0,$ \hfill (32)

(i.e. the maximum of V_i is achieved at $x_i = \Gamma_i$ and its value decreases with motion away from $x_i = \Gamma_i$). As a result the above term is in one of the two forms in (33):

$$
\begin{aligned}
&\sum_{i=1}^{L} \nabla V_i(x_i)^t \nabla V_{0i}(x_i) \equiv 0, \\
&\sum_{i=1}^{L} \nabla V_i(x_i)^t \nabla V_{0i}(x_i) < 0, \qquad x_i = \Gamma_i.
\end{aligned}
\tag{33}
$$

As for the second term of (27), it ought to be mentioned that forces surrounding the mobile agents (CRFs) have a local, reactive passive nature. In view of the above, this guarantees that no unbounded growth in the magnitude of the x_i's can occur. The worst case is for those forces to cause a deadlock in motion (i.e, X - C = constant, $t \rightarrow \infty$). Since in the worst case scenario, motion will be brought to a halt (i.e, $\dot{\Xi} = 0$), also taking into consideration the negative definiteness of the other terms, the time derivative of Ξ is always less than or equal to zero:

$$
\dot{\Xi} \leq 0.
\tag{34}
$$

If the i'th robot is in static equilibrium (assuming that the target was not reached), the following identity must hold:

$$
\sum_{j=1}^{K_i(t)} \sigma\left(\left|x_i - x_j\right|\right) \left[\frac{\nabla Vr_i\left(\left|x_i - x_j\right|\right)}{\left|\nabla Vr_i\left(\left|x_i - x_j\right|\right)\right|} + \frac{\nabla \times \mathbf{A}_i(x_i - x_j)}{\left|\nabla \times \mathbf{A}_i(x_i - x_j)\right|} \right] = \nabla V_{0i}(x_i) - \nabla V_i(x_i).
\tag{35}
$$

Therefore, the set E is equal to:

$$
\begin{aligned}
&E = E1 \cup E2 = \{x_i : \frac{d}{dt}\Xi = 0\}, \\
&E1 = \bigcup_i E1_i, \quad E1_i = \{x_i : x_i = C_i\} \qquad i=1,\ldots,L \\
&E2 = \bigcup_i E2_i,
\end{aligned}
\tag{36}
$$

$$
E2_i = \{x_i : \nabla V_{0i}(x_i) - \nabla V_i(x_i) + \sum_{\substack{j=1 \\ i \neq j}}^{K_i(t)} \sigma\left(\left|x_i - x_j\right|\right) \left[\frac{\nabla V_i\left(\left|x_i - x_j\right|\right)}{\left|\nabla V_i\left(\left|x_i - x_j\right|\right)\right|} + \frac{\nabla \times \mathbf{A}_i(x_i - x_j)}{\left|\nabla \times \mathbf{A}_i(x_i - x_j)\right|} \right] = 0, \quad x_i \neq C_i\}.
\tag{37}
$$

The largest invariant set $M \subset E$ is the subset of E that satisfies the equilibrium condition on (21). Before computing M, let us first examine if E2 is an equilibrium set of system (21). For this case the system forces may be computed using the equation:

$$
h_i = \nabla V_{0i}(x_i) - \nabla V_i(x_i) + \sum_{\substack{j=1 \\ i \neq j}}^{K_i(t)} \sigma\left(\left|x_i - x_j\right|\right) \left[\frac{\nabla Vr_i\left(\left|x_i - x_j\right|\right)}{\left|\nabla Vr_i\left(\left|x_i - x_j\right|\right)\right|} + \frac{\nabla \times \mathbf{A}_i(x_i - x_j)}{\left|\nabla \times \mathbf{A}_i(x_i - x_j)\right|} \right]. \quad i=1,\ldots,L.
\tag{38}
$$

It should be noticed that if the second condition of (23) holds, the magnitude of the radial reaction forces (∇V_{0i}, and ∇Vr_i) is determined by the self-forces (∇V_i), and the geometric

configuration the robots assume during deadlock. On the other hand, the magnitude of the circulating forces ($\nabla \times \mathbf{A_i}$) is totally independent of the self-forces. Since the individual circulating forces are made to rotate in the same direction, such fields contain no singularities (Figure 28). In other words, the circulating forces never vanish, always guaranteeing that relative motion among the agents can be actuated. Hence, their strength can be independently set by the designer anywhere in the workspace. Since the goal is to eliminate E2 from M, this freedom is used to guarantee that $h_i \neq 0 \quad x_i \neq C_i$, i=1,..L.

Rotation in the same direction produces
a singularity-free rotating golbal field

Opposite circulation produces singularities
in the circulating field
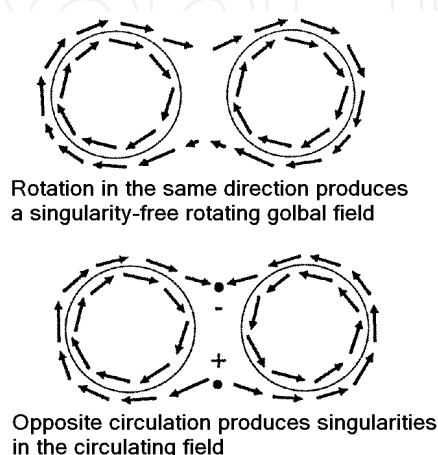
Fig. 28. A global tangent field constructed by local tangent fields circulating in the same direction is free of singularities

Since the self-forces are generated from the gradient flow of a harmonic potential, their magnitude in Ω is bounded:

$$\left| \nabla V_i(x_i) \right| \leq B_i, \quad x_i \neq C_i \qquad \text{i=1,...,L,} \tag{39}$$

where B_i is a positive, and finite constant. Also notice that it is not possible for the magnitude of the passive reaction forces to exceed that of the self-forces. Therefore, a conservative choice of the magnitude of the circulating field that would guarantee that E2 is not an equilibrium set of (23) is

$$\left| \nabla \times \mathbf{A_i}(x_i - x_i) \right| \geq \sum_{i=1}^{L} B_i. \tag{40}$$

It should also be noticed that if the third condition of (25) is not satisfied (i.e. there is not enough free space for the largest two robots to move at all times) and the circulating fields have to push against a static obstacle (a static obstacle can exert infinite reaction force), no realizable choice of B_i's would exist to satisfy condition (38). The above treatment amounts to the simple physical fact that whenever the radial reaction forces of one or more robots are in equilibrium the circulating forces intervene to pull the system out of deadlock. If the above condition is satisfied, E2 is eliminated from M. Also, since the robots have convex geometry, no equilibrium paths can form trapping one or more robots in a limit cycle. As for E1, the fact that the T_i's are taken so that $D`_i \subset T_i$, guarantees that once the robots reach their respective destinations, no interactions among their fields can happen (i.e $\mathbf{uc_i} = 0$, and $\nabla Vo_i = 0$, i=1,..,L). Also since:

$$\nabla \mathbf{V_i}(\mathbf{x_i}) = 0, \quad \mathbf{x_i} = \mathbf{C_i}, \tag{41}$$

system (21) reduces to:

$$\overset{\bullet}{\mathbf{x}}_i = 0, \quad \mathbf{x}_i = C_i \tag{42}$$

making the largest invariant set equal to:

$$M = \underset{i}{\cup}\{\mathbf{x}_i : \mathbf{x}_i = C_i\} \qquad\qquad i=1,..,L. \tag{43}$$

Therefore, according to LaSalle's theorem, the robots will globally, asymptotically converge to their respective destinations, i.e.

$$\underset{t\to\infty}{\text{Lim}}\,\mathbf{x}_i \to C_i \qquad\qquad i=1,..,L \tag{44}$$

As mentioned earlier, the suggested planner is complete provided that conditions (25) and (40) hold. To examine why imposing the third condition of (25) is necessary for the suggested planner to guarantee completeness, begin by noting that behavior, in general, has two components: a spatial one that consists of a vector field that assigns to each point in the workspace a direction along which motion should proceed, and a temporal one which consists of a scalar field that assigns a speed to each point in the workspace. Therefore, completeness for a general class of workspaces implies the existence of a spatio-temporal pattern of behavior which, if executed by the agents, leads to the satisfaction of the goal. In general, tractable environments, where a solution exists provided that behavior be spatially and temporally manipulated, the environment may at one stage deprive the planner from the ability to fully manipulate spatial behavior by forcing one agent or more to follow predetermined spatial behavioral patterns that are set by the geometry of the workspace (figure 27). If such a situation occurs, the planner can only resolve the conflict by manipulating the temporal component of behavior (i.e speed up or slow down the movements of the agents, as well as halt motion or reverse it). Since the suggested planner is totally reliant on manipulating spatial behavior only, it may fail if it encounters situations where both spatial and temporal behavior are to be manipulated. The third condition of (25) guarantees that the environment will never be able to prevent the planner from spatially manipulating behavior in order to resolve a conflict. In a recent work by the author (Masoud & Masoud, 2002; Masoud & Masoud, 2000a), a method for synthesizing a PRF control component that can jointly enforce regional avoidance, and directional constraints, may be used to guarantee that deadlock will not happen in environments with tight passages (see the last example, figures-[39-44]). Unfortunately, this approach for avoiding deadlock may reduce the set of potential solutions to the multi-agent planning problem. In other words, the controller will no longer be complete.

5.1.4 Results
Several simulation experiments were conducted to explore the behavior of the suggested method. Each example is presented as a sequence of frames with each frame depicting the state of the robots at different instants of the solution. The notation used is the same as that in the theoretical development (i.e. D_i represents the i'th robot, x_i its center, and C_i the center of the target zone).

5.1.4.1 A basic example

In figure 29 two robots sharing the same obstacle-free workspace are required to exchange positions. In doing so, each robot makes the simple, but naive, decision of moving along a straight line to the target. Despite the apparent conflict which each is heading towards, each robot proceeds with its plan as if the selected action is conflict-free. Once the conflict is in a phase that is detectable by the local sensors each robot has, corrective actions are taken by each to modify their behavior in order to resolve the conflict (i.e. the CRF control component is activated). As mentioned before, the "seed" CRF activities consist of a component to prevent collision, and another to move the agents out of each other's way. Once the conflict is resolved, the behavior modification activities dissipate and guidance is fully restored to the PRFs (figure 30).
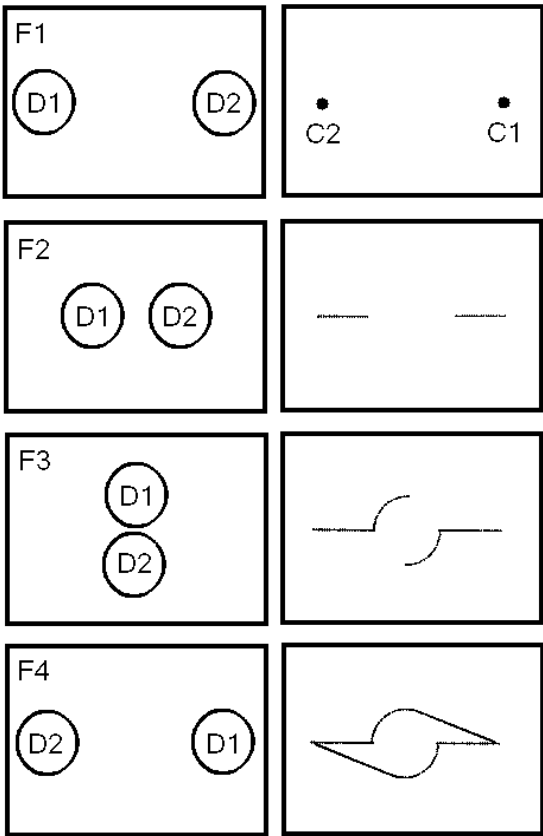
Fig. 29. Two robots exchanging positions

In figure 31, three robots operating in an obstacle-free space, and initially positioned on the vertices of an equilateral triangle are required to proceed towards their symmetric targets. As in the two-robot example, each robot chooses to proceed along a straight line to its target ignoring the apparent conflict to which this choice leads. For this case the response of the robots, once a conflict is detected, exhibits an interesting emergent nature. By reducing the degrees of freedom of the system from six to one, the three robots act as one rotating body to position themselves where each can proceed unimpeded towards its target. It is interesting to note that without being *a priori* programmed to do so, the robots choose to cooperate in order to resolve the conflict. This cooperation is manifested as a reduction in the degrees of freedom of the system during the period of the conflict.
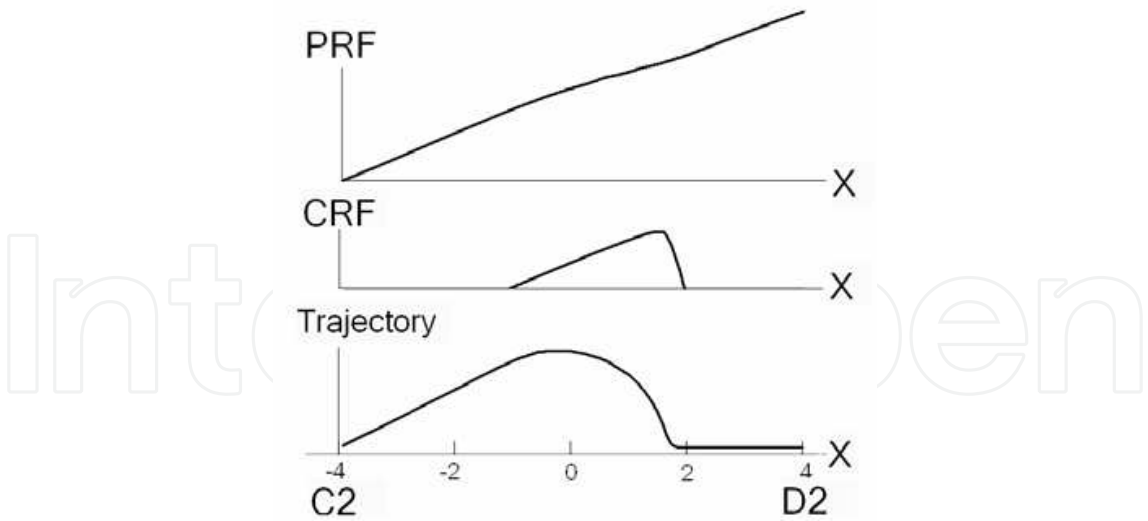
Fig. 30. CRF activities dissipate after conflict is resolved (trajectory of D2)

5.1.4.2 Fault tolerance
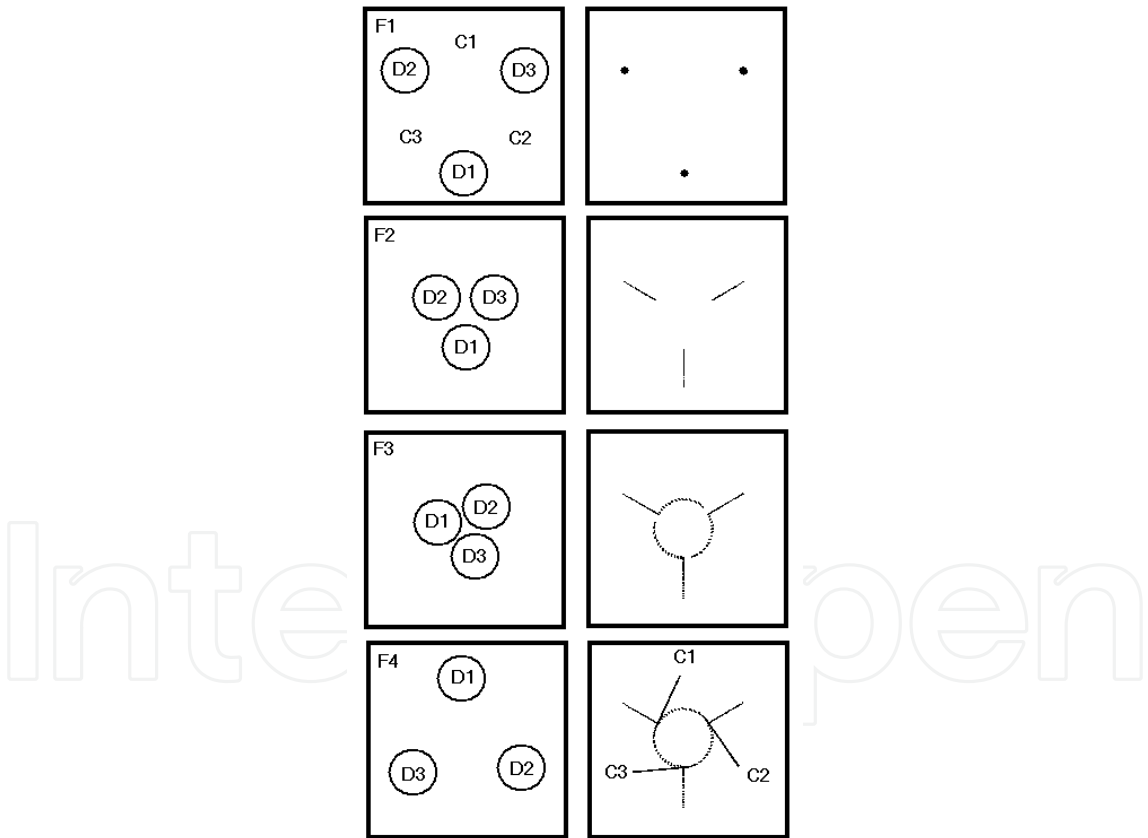
Fig. 31. Three robots moving to their goals, all functioning

In a centralized system the supervisory control assigns each agent the duties it has to fulfill for the whole group to avoid conflict. If one agent fails to fulfill its obligation towards the group, the whole group may be affected. In decentralized systems, conflict evasion has a lucid nature where conflict evasion activities dynamically shift from the unable, or

unwilling agents, to the remaining functional agents. Here, an agent's role keeps adapting to the situation in a manner that would, to the best of the agent's ability, enable all the agents (this includes the offending agents) to reach their targets. The following example examines this intriguing property of decentralized systems. In figure 32, a setting similar to the one in figure 31 is used. The only difference is that D2 refuses to participate in conflict resolution and, instead, follows the plan encoded by its PRF requiring it to move along a straight line to its target. As can be seen, the remaining two agents adjust their behavior to compensate for the intransigence of D2 in such a manner that allows all the agents to reach their destinations.
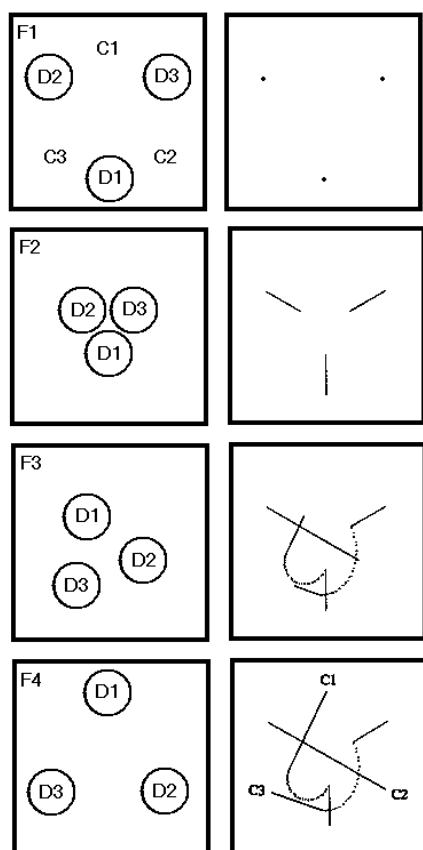
Fig. 32. Three robots moving to their goals, D2 malfunction

5.1.4.3 Morphogenesis of CRF activities

The action of the controller may be mistaken with that of a controller equipped with simple reflexive capabilities. One should keep in mind that the simple actions of collision avoidance and tangent motion are nothing but the G-type of the control. The G-type control should only be viewed as the kernel of the global control action (P-type) which is the one actually controlling the systems. The, more complex, P-type control action emerges from the seed G-type control in a flexible, situation-responsive manner. Figure 33 shows the tangent circulating fields of ten agents approaching each other. As can be seen, once the agents start to get close to each other, the fields begin to interact and their structure begins to gradually mutate until it finally assumes a global form that is very different from the form of the individual fields.
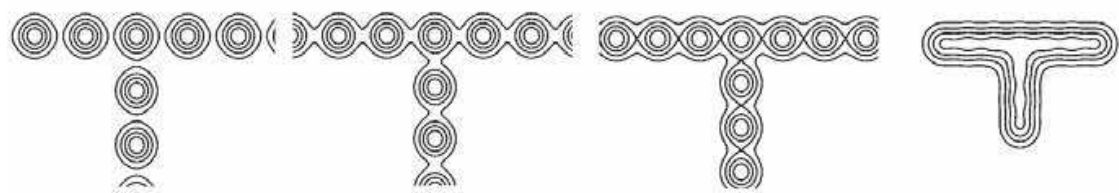
Fig. 33. Context-sensitive evolution of CRF activities

5.1.4.4: Self-organization

In the following two examples the evolutionary, cooperative, self-organizing nature of the controller is clearly demonstrated. In Figure 34 two groups of four robots each are moving in opposite directions along a road with side rails blocking each other's way. The goal is for the left group to move to the right side, and right group to move to the left side. The groups collectively solve the problem by forming right and left lanes and confining the motion of each group to one of the lanes. It should be noted that lane formation is a high-level, holistic organizational activity that fundamentally differs from the local capabilities with which each robot is originally equipped.
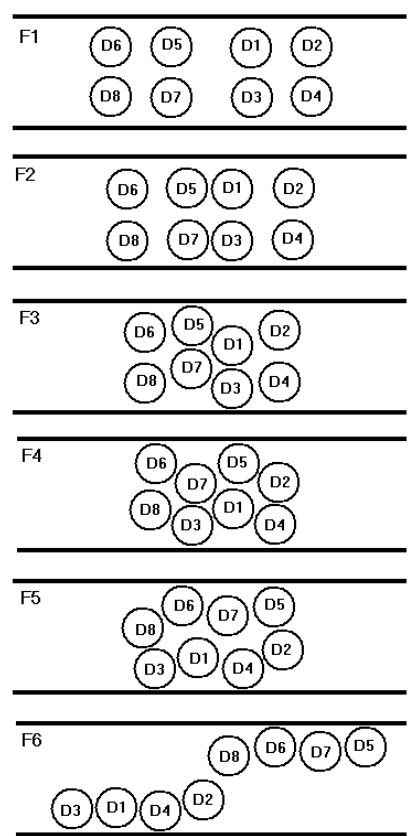
Fig. 34. Two groups of robots passing each other in a confined space

In Figure 35 eight robots are confined in a box with very little room to move. The goal is for D1 to move to C9. The robots collectively reach a solution that efficiently utilizes free space. The robots solve the problem by keeping the center robot stationary, with the remaining robots rotating around it until D1 reaches its target.
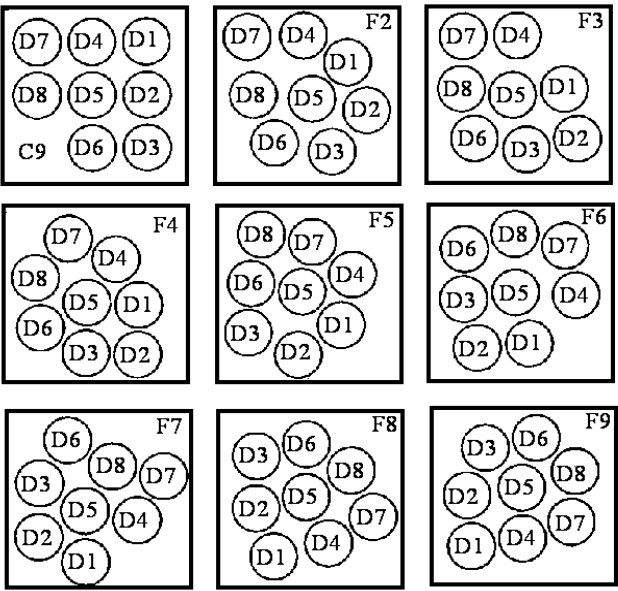
Fig. 35. A group of robots self-organizing to allow D1 to reach C9

5.1.4.5 CRF fields strength and deadlock prevention

In the following example the importance of the circulating fields for conflict resolution is demonstrated. Here a group of eight agents each is required to hold its position, except for D8 which is required to move to C8. No circulating field are used in figure 36. As can be seen, while D8 managed to pass the first group of agents, it got trapped in a deadlock formation when it attempted to pass the second group. In figure 37 circulating fields are added. As can be seen D8 is able to reach its target, and the remaining agents maintained their original positions.
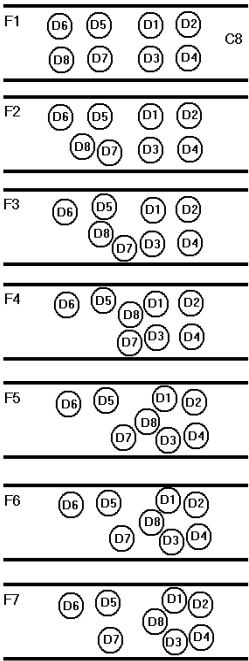
Fig. 36. D1-D7 hold positions, D8 moves to C8, no Circulating fields
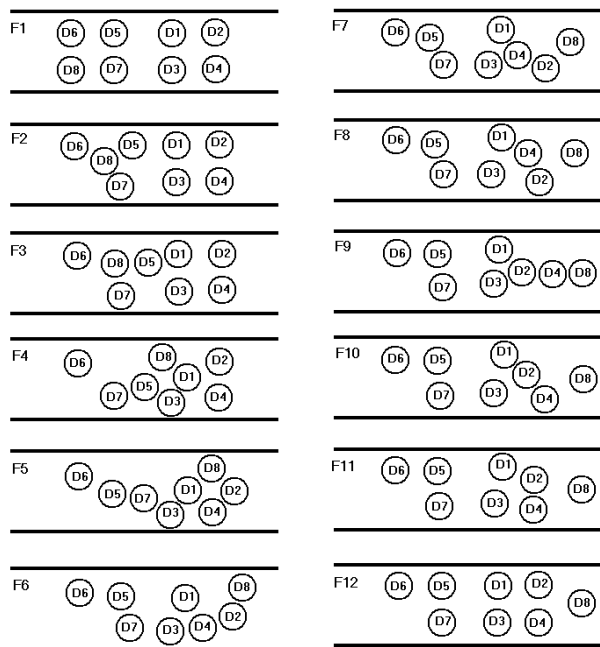
Fig. 37. D1-D7 hold positions, D8 moves to C8, Circulating fields present

5.1.4.6 All purposive agents in a congested space

In Figure 38, the difficult planning task of exchanging positions in a confined area is assigned to the robots. The order of the exchange is as follows $D1 \leftrightarrow D6$, $D2 \leftrightarrow D5$, $D3 \leftrightarrow D8$, $D4 \leftrightarrow D7$. As can be seen, the group successfully carried out the task.
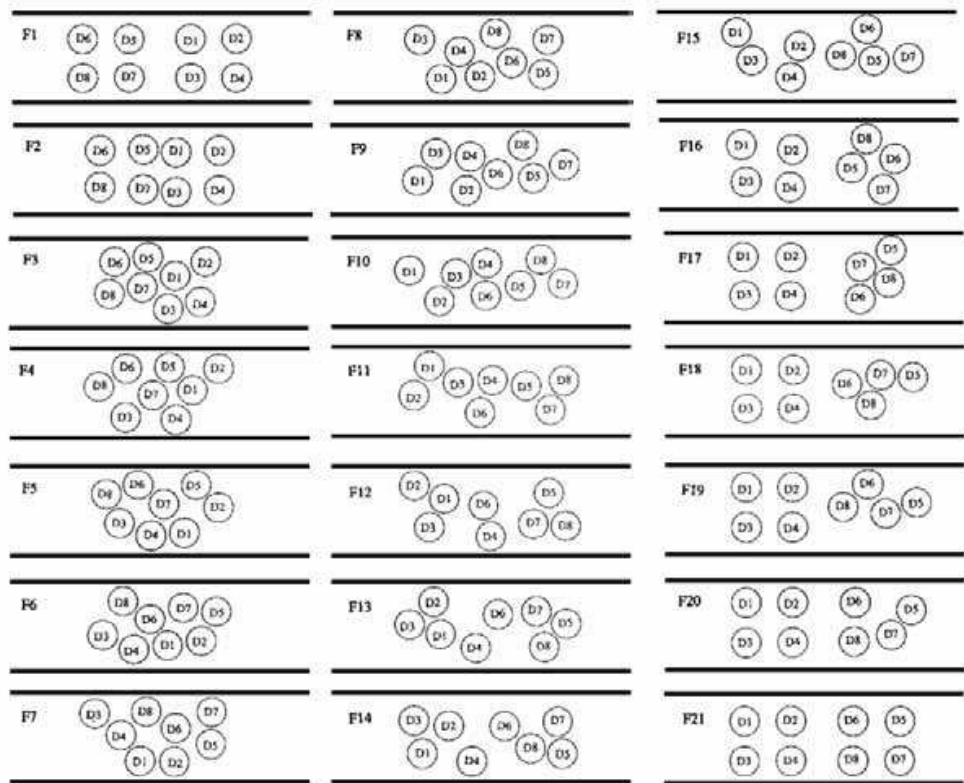
Fig. 38. A group of robots exchanging positions in a confined space

5.1.4.7 Environments with tight passages

While the third condition of (25) is by no means stringent (after all, it is only reasonable for a two-way street to be wide enough to allow two vehicles to pass at the same time), there are, nevertheless, environments with tight passages that have only room for one robot at a time. In such a situation there are no guarantees that the multi-agent planner will function properly. One way to remedy this situation is to mark a tight passage as a one-way street (i.e. constrain motion in such passages to become unidirectional). This may be accomplished by using the NAHPF-based EHPC scheme in (Masoud & Masoud, 2002; Masoud & Masoud, 2000a) for synthesizing the PRF control component of the multi-agent controller. The following example illustrates the use of NAHPFs for such a purpose.

Consider the workspace in figure 39. Two robots D1 and D2 are required to exchange positions. As can be seen, the passages in Ω are not wide enough for the two robots to pass at the same time.
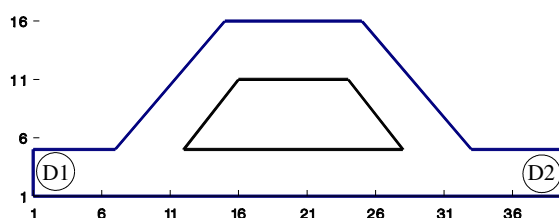
Fig. 39. A workspace with tight passages
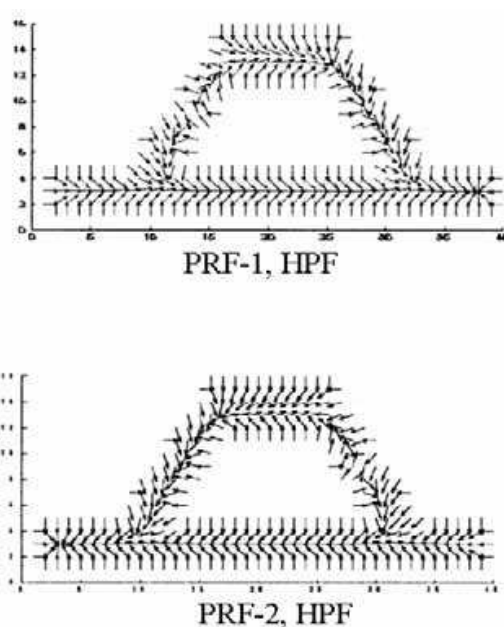
Fig. 40. PRF components

Figure 40 shows the HPF-based PRFs for both D1 and D2. Figure 41 shows, using snapshots, the locations of the robots that are generated by the multi-agent controller at different instants of the solution. As can be seen, an unresolvable conflict arises between D1 and D2. Figure 42 shows the NAHPF-based PRFs for D1 and D2. Figure 43 shows the corresponding locations of the robots at different instants in time. As can be seen, conflict was resolved by marking the tight passages as one-way streets.
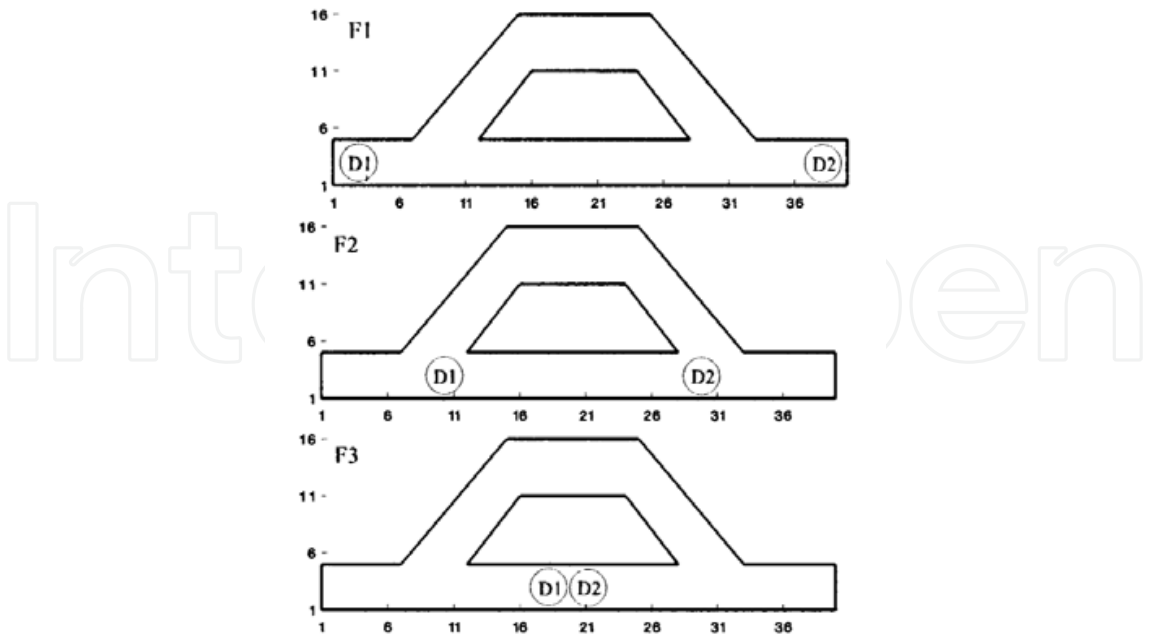
Fig. 41. Deadlock caused by a tight passage
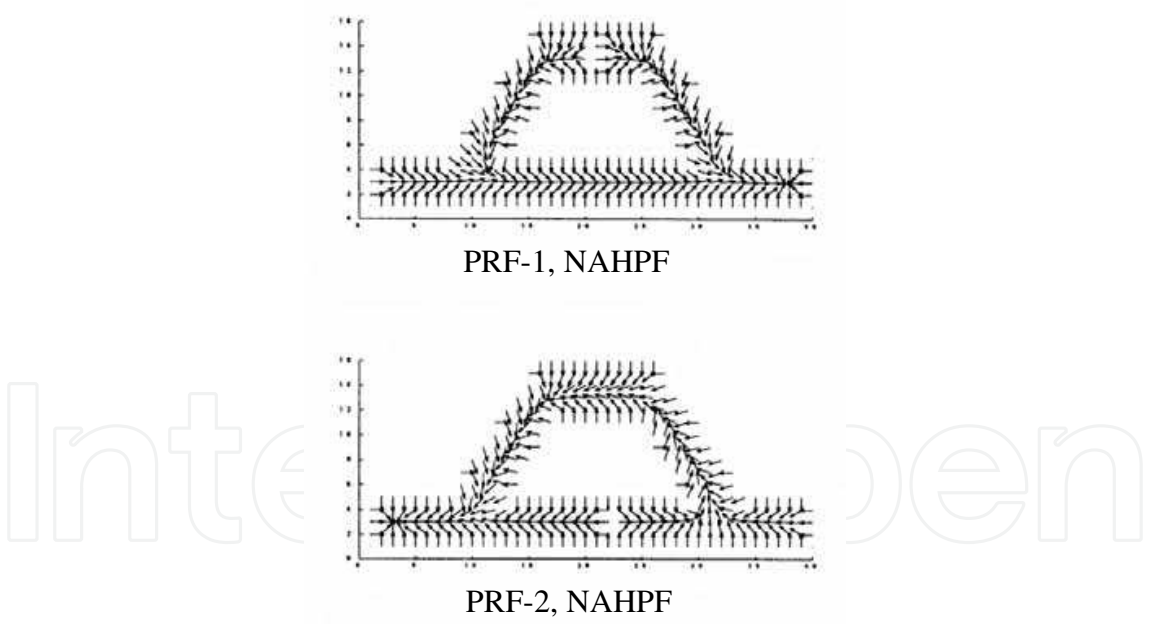

PRF-1, NAHPF

PRF-2, NAHPF

Fig. 42. PRF components, NAHPF-based EHPCs

Unfortunately, using NAHPF-based EHPCs to avoid conflict in environments with tight passages has some drawbacks. Marking a passage as a one-way street leads to a loss in potentially realizable solutions. Consider for example the environment in figure 44. It is obvious that a solution exist to move D1 to the location of D2 and vice versa. Marking the tight passage as a one way street makes it impossible for D2 to move to the left. To solve the tight passage problem, the planner must jointly manipulate spatial and temporal behavior.
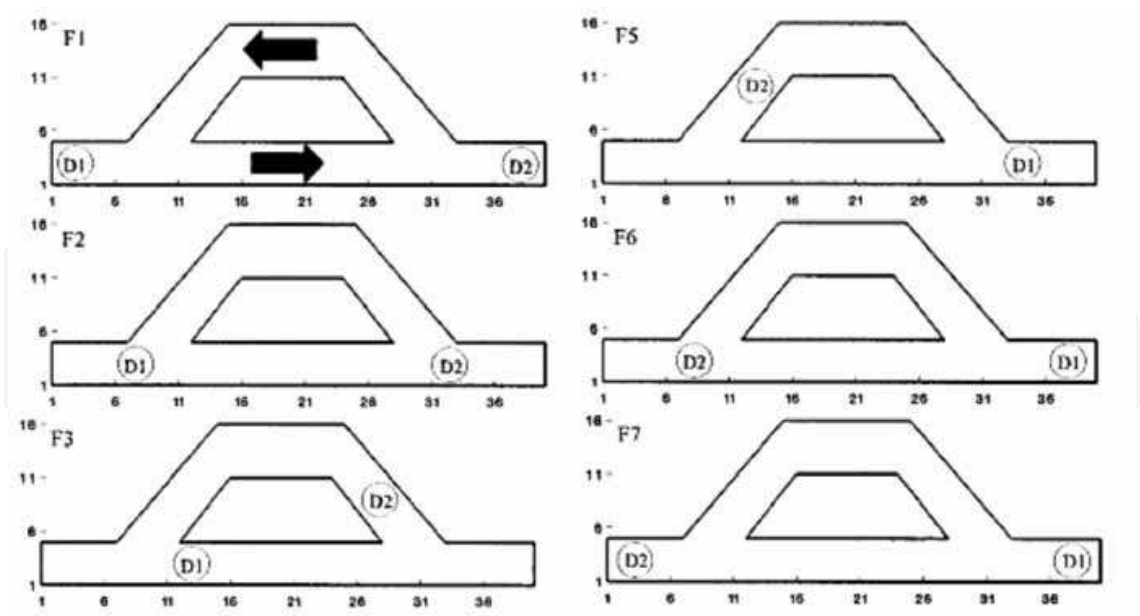
Fig. 43. NAHPF-PRF component can resolve deadlock in spaces with tight passages
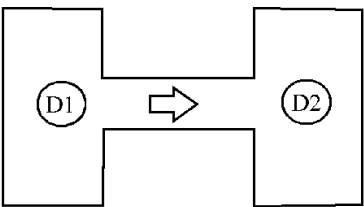
Fig. 44. NAHPF-PRF could lead to a loss in potential solutions

5.2 Individually-motivated, multi-agent, HPF-based planner

The single agent HPF has a social nature that allows it to co-exist with other agents using a similar navigation procedure in the same cluttered space (a proof is also provided in section-5.2.1). This makes a single-agent, HPF planner a valid G-type controller (figure 45) in a multi-agent system. The reason for that is: the HPF approach treats other agents as obstacles to be avoided. Hence the same procedure used to avoid clutter can also be used to accommodate the presence of the other agents.
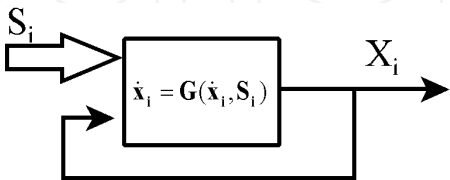

Fig. 45. The G-type controller

The agent could use the basic BVP shown below (45) for generating the self-control action or a BVP corresponding to any of the HPF setting discussed earlier in this chapter. The trajectory can be easily generated by the gradient dynamical system resulting from the computed potential field (figure 46).

$$\nabla^2 V(x_i) = 0 \qquad\qquad x_i \in \Omega \ .$$
$$V(x_i)\big|_{x_i \in S_i} = 1 \quad \& \quad V(x_i)\big|_{x_i \in T_i} = 0 \tag{45}$$
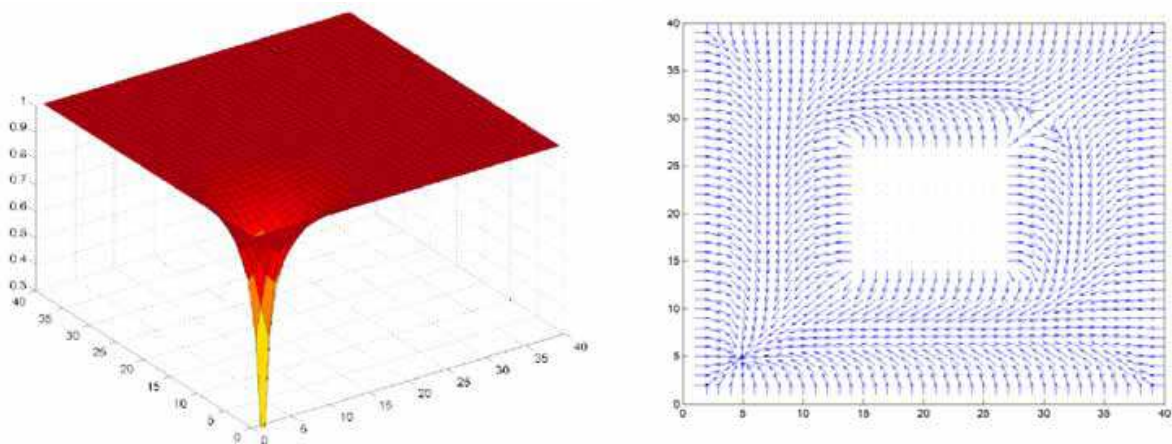$$\dot{x}_i = -\nabla V(x_i) \ .$$

Fig. 46. Potential and gradient guidance field for a goal-oriented, G-type controller

5.2.1 Analysis: the goal-oriented case:

In this section proofs of the ability of the goal-oriented multi-agent controller to avoid obstacles and steer each of its members to its target are provided.

Proposition: If $x_i(0) \in \Omega$, the motion steered by the gradient dynamical system in (45) will always remain inside Ω (i.e. $x_i \cap S_i \equiv \varphi \ \ \forall \ t, \ \forall \ i)$.

Proof: Consider the part of Ω near a forbidden region (S_i). Let $n(x_i)$ be a vector that is normal to the surface of the obstacle. Let S_i' be a region created by infinitesimally expanding the forbidden region S_i such that $S_i \subset S_i'$. The radial derivative of $V(x_i)$ along S_i may be computed as:

$$\frac{\partial V}{\partial n} = \frac{V(x_i) - V(x_i')}{\Delta r} \tag{46}$$

where x_i' is taken as the minimum distance between x and S_i' and Δr is a positive differential element. Since by the maximum principle the value of the potential in Ω is less than 1, and x_i' lies inside Ω, the radial derivative of the potential along $n(x_i)$ is negative, i.e.

$$n(x_i)^T \nabla V(x_i) < 0 \tag{47}$$

Since motion is steered using the negative gradient of V, the agent will be pushed away form S_i and x_i will remain inside Ω.

Definition: Let $V(X)$ be a smooth (at least twice differentiable), scalar function ($V(X)$: $R^N \rightarrow R$). A point Xo is called a critical point of V if the gradient vanishes at that point ($\nabla V(Xo)=0$); otherwise, Xo is regular. A critical point is Morse, if its Hessian matrix ($H(Xo)$) is nonsingular. $V(X)$ is Morse if all its critical points are Morse (Milnor, 1963).

Proposition: If $V(X)$ is a harmonic function defined in an N-dimensional space (R^N) on an open set Ω, then the Hessian matrix at every critical point of V is nonsingular, i.e. V is Morse.

Proof: There are two properties of harmonic functions that are used in the proof:

1.   a harmonic function ($V(X)$) defined on an open set Ω contains no maxima or minima, local or global in Ω. An extrema of $V(X)$ can only occur at the boundary of Ω,

2.   if $V(X)$ is constant in any open subset of Ω, then it is constant for all Ω.

Other properties of harmonic functions may be found in (Axler et al, 1992).

Let X_0 be a critical point of $V(X)$ inside Ω. Since no maxima or minima of V exist inside Ω, X_0 has to be a saddle point. Let $V(X)$ be represented in the neighborhood of X_0 using a second order Taylor series expansion:

$$V(X) = V(X_0) + \nabla V(X_0)^T (X - X_0) +$$
$$\frac{1}{2}(X - X_0)^T H(X_0)(X - X_0) \qquad |X\text{-}X0| << 1. \tag{48}$$

Since X_0 is a critical point of V, we have:

$$V' = V(X) - V(X_0) =$$
$$\frac{1}{2}(X - X_0)^T H(X_0)(X - X_0) \qquad |X\text{-}X0| << 1. \tag{49}$$

Notice that adding or subtracting a constant from a harmonic function yields another harmonic function , i.e. $V`$ is also harmonic. Using eigenvalue decomposition:

$$V' = \frac{1}{2}(X - X_0)^T U^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} U(X - X_0)$$

$$= \frac{1}{2}\xi^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} \xi \quad = \quad \frac{1}{2}\sum_{i=1}^{N} \lambda_i \xi_i^2 \tag{50}$$

where U is an orthonormal matrix of eigenvectors, λ's are the eigenvalues of $H(X_0)$, and $\xi = [\xi_1 \ \xi_2 \ ..\xi_N]^T = U(X-X_0)$. Since $V`$ is harmonic, it cannot be zero on any open subset Ω; otherwise, it will be zero for all Ω, which is not the case. This can only be true if and only if all the λ_i's are nonzero. In other words, the Hessian of V at a critical point X_0 is nonsingular. This makes the harmonic function V also a Morse function.

Proposition: If the G-type controller of the multi-agent system in (1) is selected as the HPF planner in (45) then every agent is guaranteed to converge to the target (T_i),

$$\lim_{t \to \infty} x_i = T_i \qquad i=1,2, ..N \tag{51}$$

Proof: Since $V_i(x_i)$ is shown to be a valid Liapunov function candidate (LFC) (Vidyasagar, 1987). i.e.

$$V_i(x_i) = 0 \quad \text{at and only at } x_i = T_i \ \& $$
$$V_i(x_i) > 0 \quad \text{elsewhere,} \tag{52}$$

Their summation is also an LFC:

$$V(X) = \sum_{i=1}^{N} V_i(x_i)$$
$$V(X) = 0 \quad \text{at and only at X=T} \quad \& \quad (53)$$
$$V(X) > 0 \quad \text{elsewehre,}$$

where $X=[x_1 \ x_2 \ ... \ x_N]^t$, and $T=[T_1 \ T_2 \ ...T_N]^t$. The time derivative of V(X) is:

$$\dot{V}(X) = -\sum_{i=1}^{N} \nabla V_i(x_i)^t \dot{x}_i$$
$$\dot{x}_i = -\nabla V_i(x_i) \quad (54)$$
$$\dot{V}(X) = -\sum_{i=1}^{N} \left| \nabla V_i(x_i) \right|^2$$

Since harmonic functions are Morse, the stable equilibrium, target points (T_i's) are the only points in the minimum invariant set of the system. By applying the LaSalle invariance principle (LaSalle, 1960) it can be easily shown that each agent will converge to its respective target.

5.2.2 Results
The ability of agents equipped with an HPF-based, G-type controller to cooperatively solve the planning problem while treating space as a scarce resource is tested. Five agents positioned opposite to each other are required to move to a specified target from a starting point selected so that a high probability of conflict scenario is established. In figure 47 the five agents utilizing a full communication graph attempt to solve the planning problem they are faced with. As can be seen, all agents reach their destination safely maintaining at all time a nonzero, minimum inter-agent distance (DM).
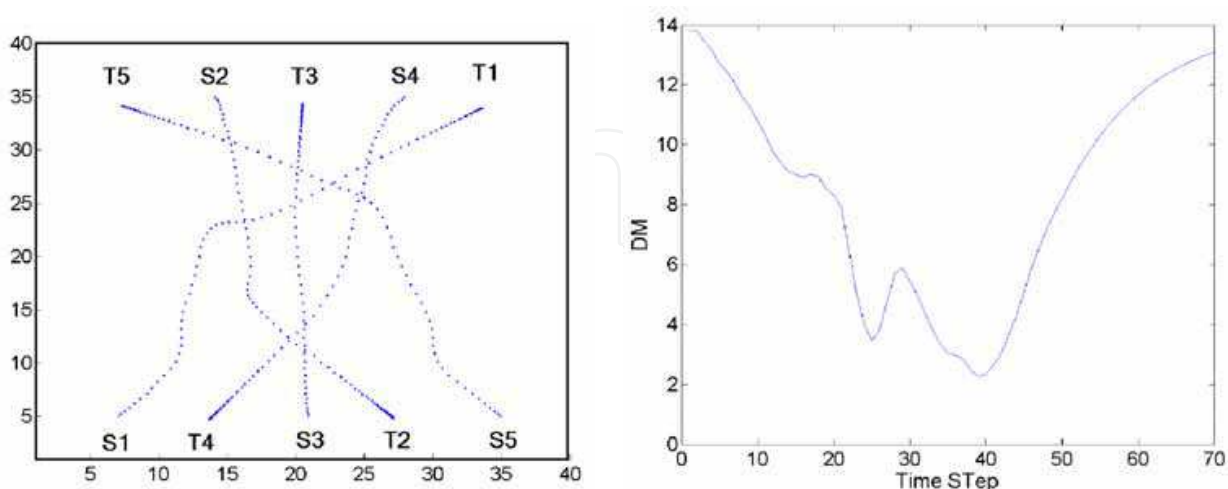

Fig. 47. Goal-oriented mode, full communication: a. trajectories, b. minimum distance

In figure 48 the agents attempt to deal with the same situation. However, this time instead of using a full communication graph, each agent only communicates its position to its closest

neighbor. Again the agents were able to safely resolve the conflict and arrive at their destination.
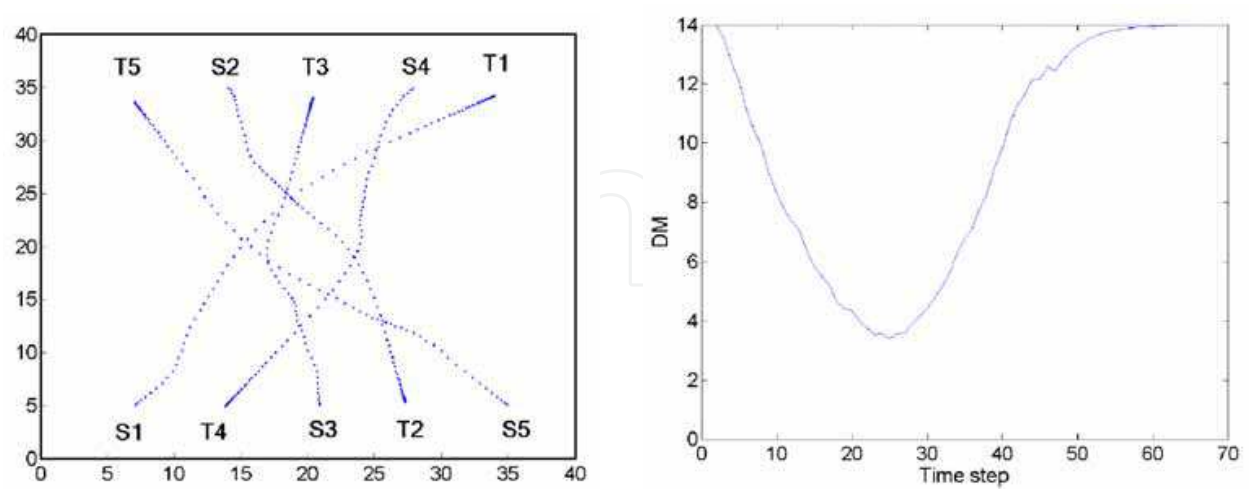

Fig. 48. Goal-oriented mode, nearest neighbor communication: a. trajectories, b. minimum distance

5.3 Multi-agent harmonic separation maintenance planner

Sometimes a group of agents are required to operate in a flexible formation mode where the members distribute themselves within a confine whose shape and motion are determined by the leader agent (figure 49). The overall trajectory of the agent is constructed by superimposing the trajectory supplied by the leader on the trajectory the agents generate to avoid collision with the components of the environment and stay within the specified geometric confines.
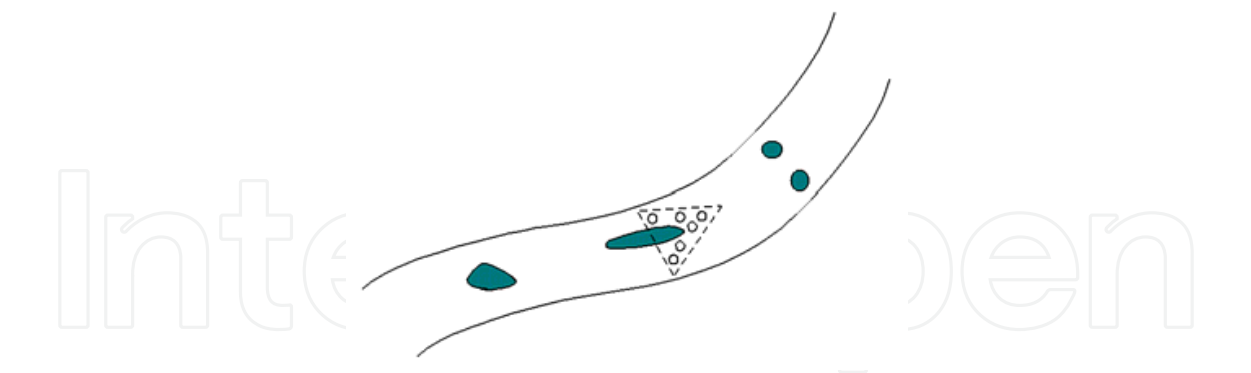
Fig. 49. Agents in separation mode

Harmonic potential fields can provide the G-Type controller for a multi-agent formation separation mode controller. Unlike the goal-oriented mode where the target point is given and the group need only to lay a conflict-free path to it, the separation mode requires the group to jointly generate the target point for each agent as well as lay a safe trajectory to that point. The HPF approach may still be used to generate a self-controller for this case. The BVP generating the potential is similar to the one in (45) with no target point having a potential preset to zero. The control action that dynamically distribute the agent in specified space may be derived from the BVP:

$$\nabla^2 V(\mathbf{x}_i) = 0 \qquad \mathbf{x}_i \in \Omega .$$
$$V(\mathbf{x}_i)\big|_{\mathbf{x}_i \in S_i} = 1$$

(55)

The above BVP may appear to be of little use since by the maximum principle, the solution of V in Ω is a constant. This means that the gradient field degenerates everywhere in Ω. The potential field from an environment similar to the one in figure 46 is shown in figure 50.
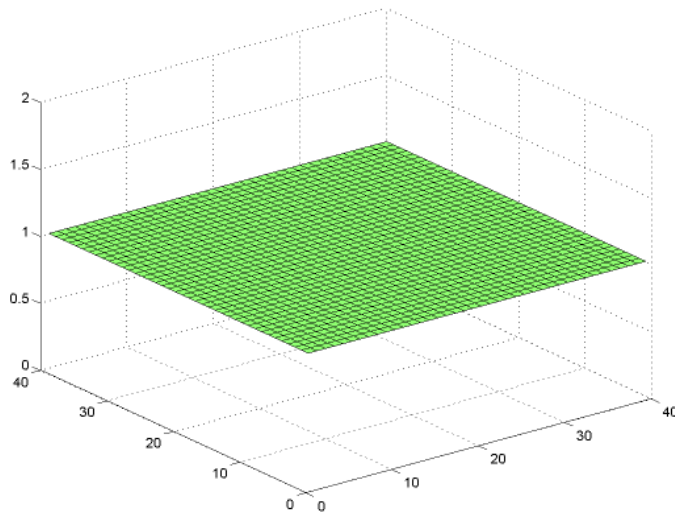

Fig. 50. Potential field degenerates in the formation case

A careful examination of the solution of (55) reveals that only the magnitude of the gradient field ($A(x_i)$) degenerates while the phase field ($Q(x_i)$) remains stable and computable. The component of the BVP in (55) that corresponds to the phase field may be derived as follows:

$$\nabla V(x_i) = A(x_i)Q(x_i) ,$$
$$\nabla^2 V(x_i) = \nabla \cdot \nabla V(x_i)$$
$$= \nabla A(x_i)^T Q(x_i) + A(x_i)\nabla \cdot Q(x_i) .$$

(56)

The gradient of the magnitude of ∇V in (4) drops to an infinitesimally small positive constant ε while A converges to unity. In this case the laplacian becomes:

$$\nabla^2 V(\mathbf{x}_i) = \nabla \cdot Q(x_i) .$$

(57)

Since the potential is restricted to a constant value at S_i, Q will have no component tangent to S_i (i.e. n×$Q(x_i)$=0, $x_i \in S_i$), where n is a unit vector normal to S_i. Therefore, the boundary value problem that may be used to generate Q is:

$$\nabla \cdot Q(x_i) \equiv 0 \qquad\qquad x_i \in \Omega$$
$$n \times Q(x_i) = 0, \qquad n^T G(x_i) = 1. \qquad x_i \in S_i.$$
$$\dot{x}_i = \alpha \cdot Q(x_i)$$

(58)

where α is a positive constant. The field, Q, generated by solving the above BVP is observed to possess field lines that emanate normal to S_i and move into Ω meeting at

critical points inside the region (figure 51). Among other things these points show the tendency to form far from clutter and other agents occupying Ω. This makes it possible to utilize Q as the G-type separation control. As can be seen stable equilibrium points spontaneously form equally far from the obstacles in the environment. Figure 52 shows the separation field for another environment. The reason equilibrium points form inside Ω has to do with the fact that all the flows at the boundary are forced to be inside Ω, the continuity condition ($\nabla \cdot Q = 0$) will fail at some areas in Ω. This results in stable and unstable equilibrium points being formed. A Quantitative study of these points in terms of how far from the closest object they will form is expected to be mathematically involved and will be kept for future work. However, a qualitative examination (figure 53) show that these points are comparable to maximizing the minimum distance from the obstacles.
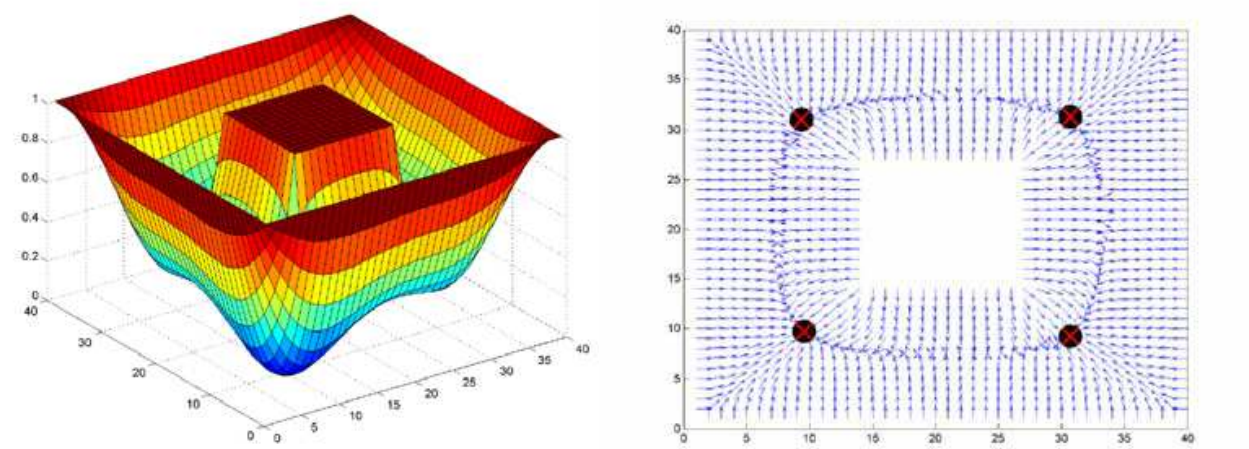

Fig. 51. Separation G-type control, harmonic phase field and gradient guidance field
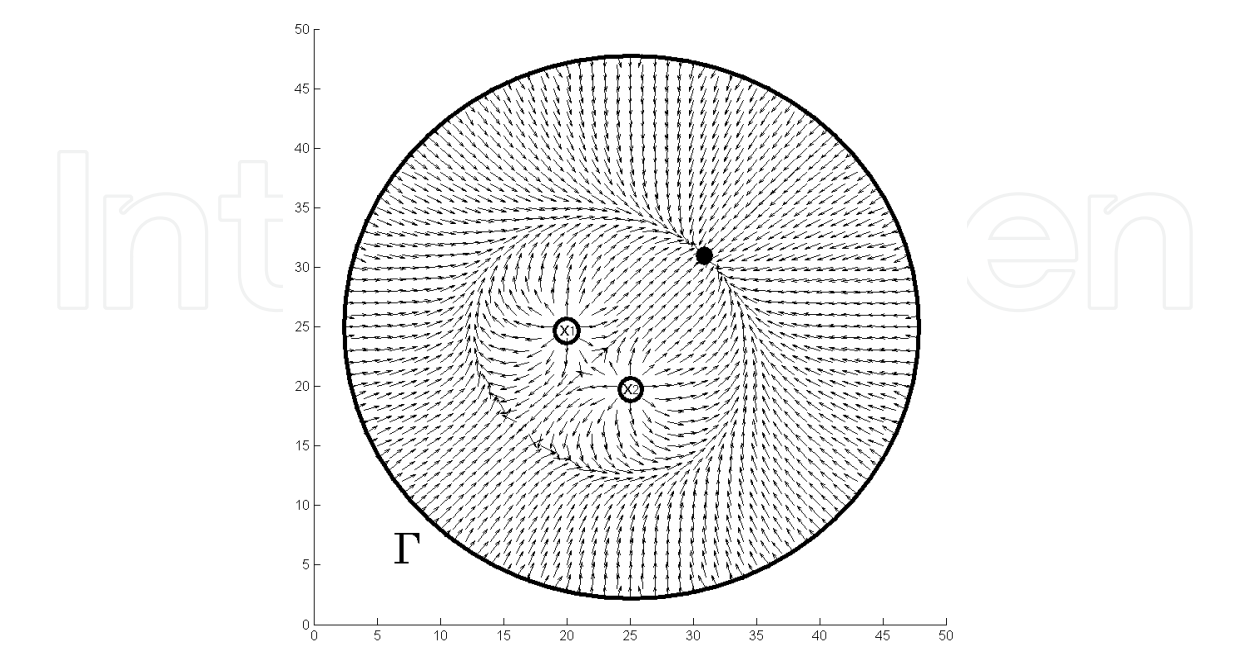

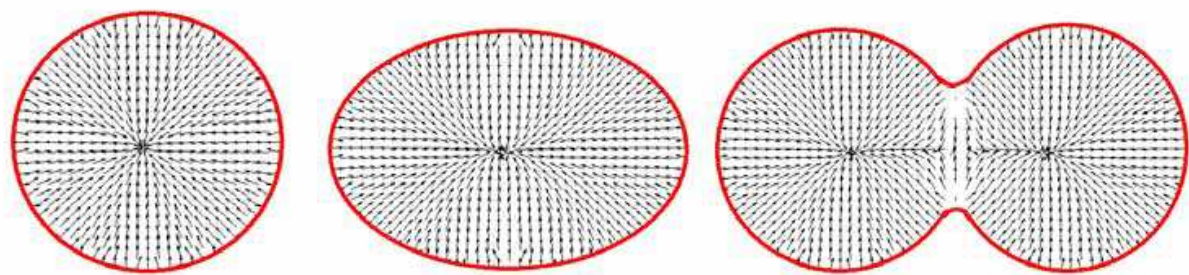Fig. 52. Guidance field, G-type controller, separation mode

Fig. 53. Workspace geometry versus stable equilibrium points, separation mode

5.3.1 Results

The planner is tested in the separation mode for both full communication graph (figure 54) and nearest neighbor communication (figure 55). The five agents in the previous example were not provided with target points. As can be seen, in both cases, the agents managed to generate goal points that places them in a well-separated final configuration (better results were obtained in the case of the full communication graph). In addition to that, the decentralized controller was able to safely drive the agents from their initial positions to their respective target points, practically achieving a strictly increasing time - minimum separation distance profile.
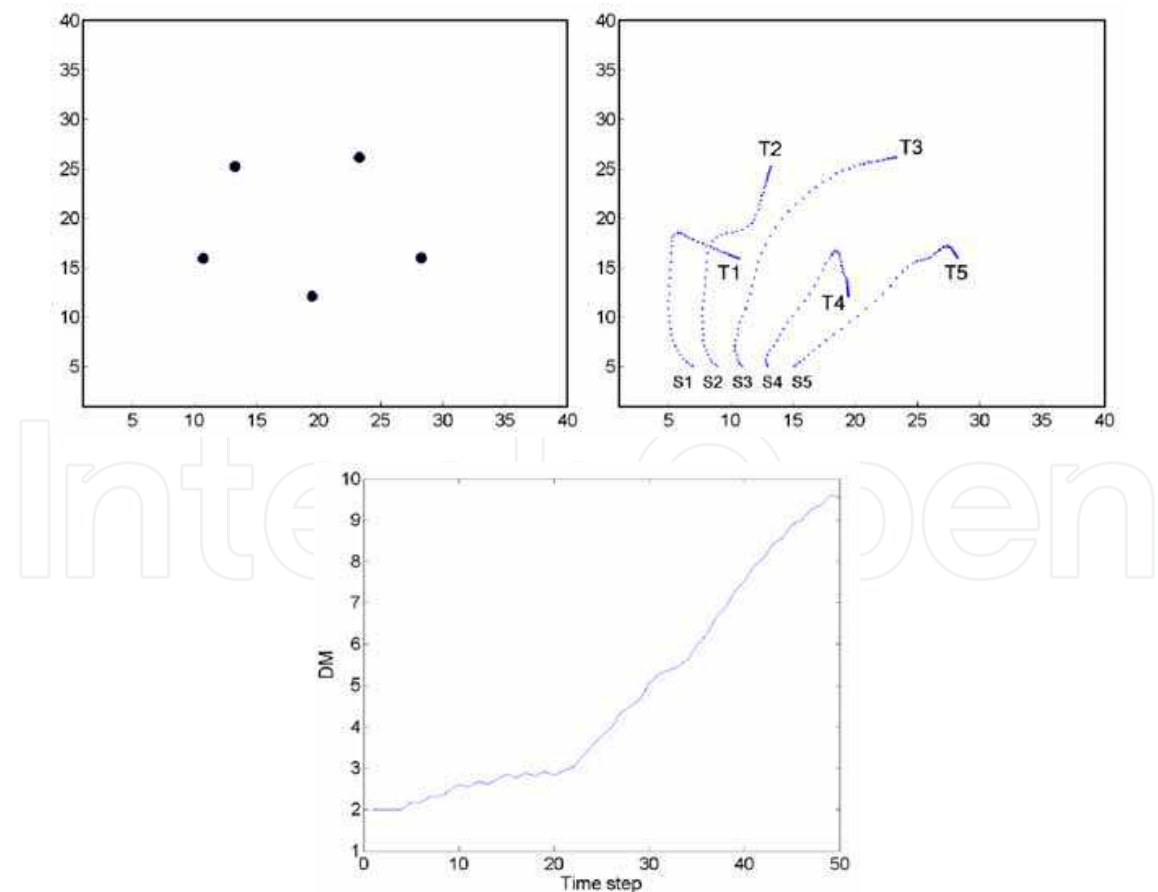

Fig. 54. Separation mode, full communication: final constellation, trajectories, minimum distance
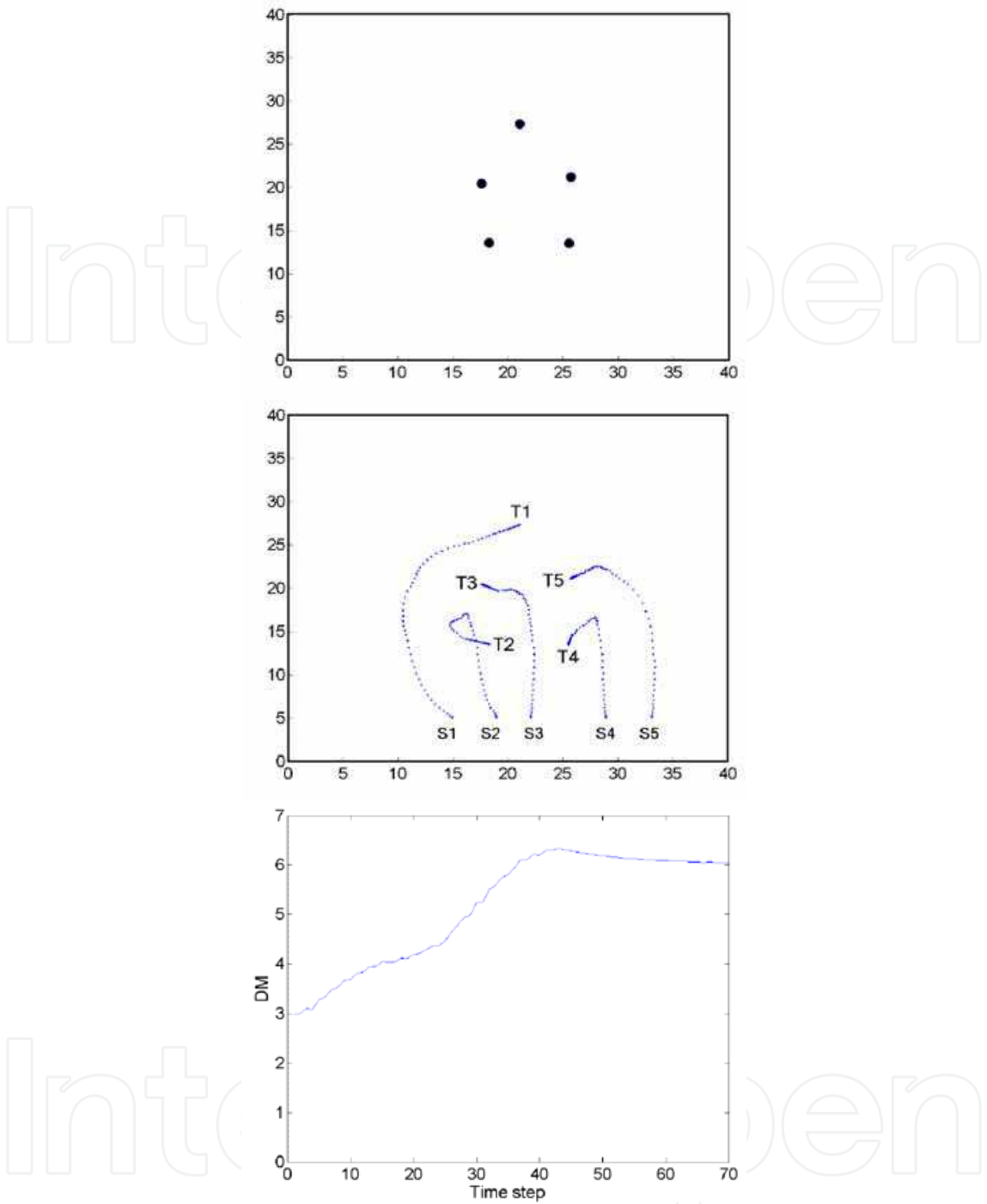
Fig. 55. Separation mode, nearest neighbour communication: final constellation, trajectories, minimum distance

In figure 56, the computational effort needed by the planner is examined in terms of the time needed to complete the steering process in the separation mode. The number of agents (Na) is varied from two to five and time needed to complete the steering process is recorded for each. Figure 16 shows the time needed to perform the steering process versus the number of agents. The time is normalized using that of the case Na=2. As can be seen, the computational time linearly grows with the number of agents. A full communication graph is used.
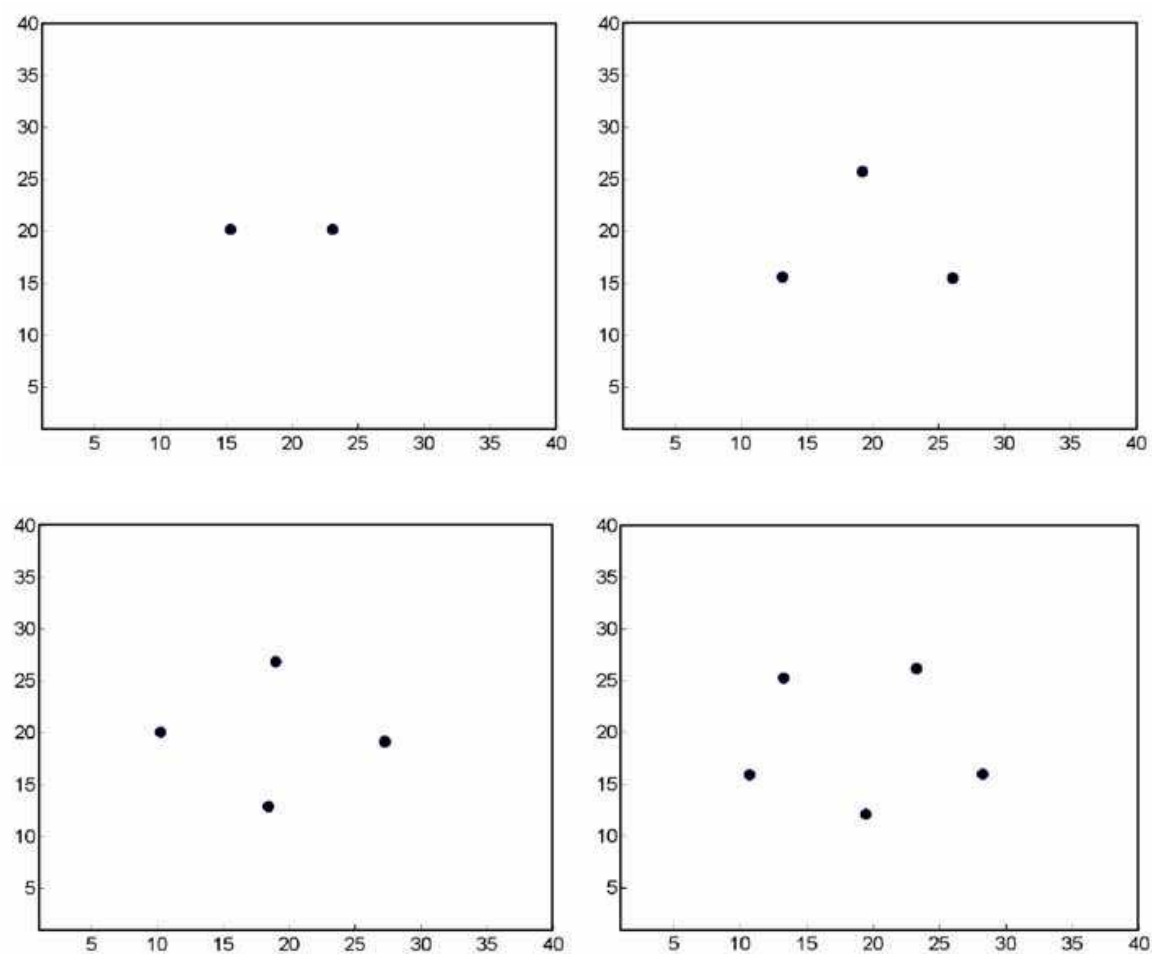
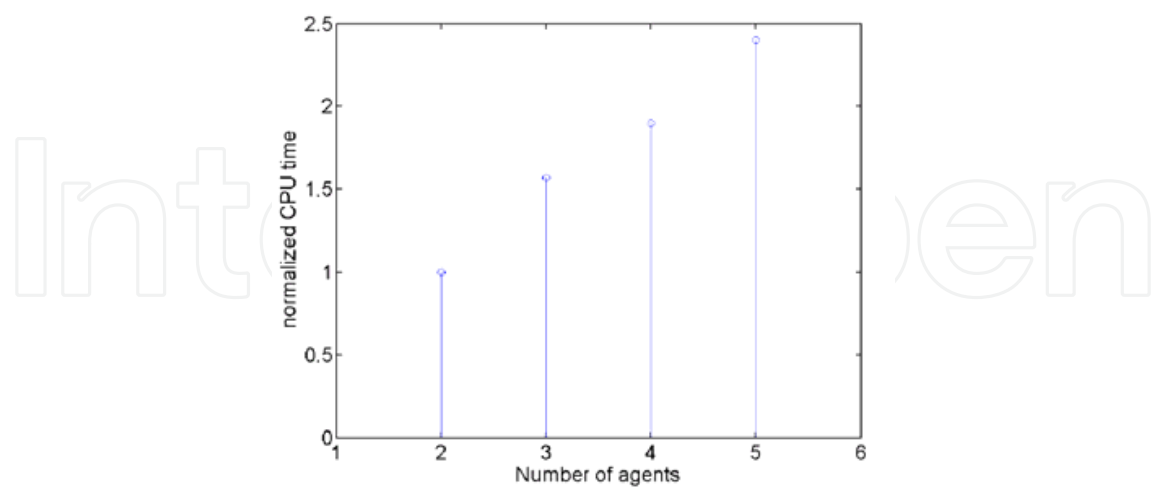Fig. 56. Separation mode, final constellation versus number of agents

Fig. 57. Simulation time versus number of agents

In figure 58 the performance of the controller is examined in the presence of clutter for the separation mode. It is observed that all the attributes of the controller in the free space case were preserved when clutter is present. The agents distributed themselves in a final configuration that seems to maximize the minimum inter-agent distance as well as the

distance to the nearest obstacle. Also, a strictly increasing with time minimum separation distance profile is observed.
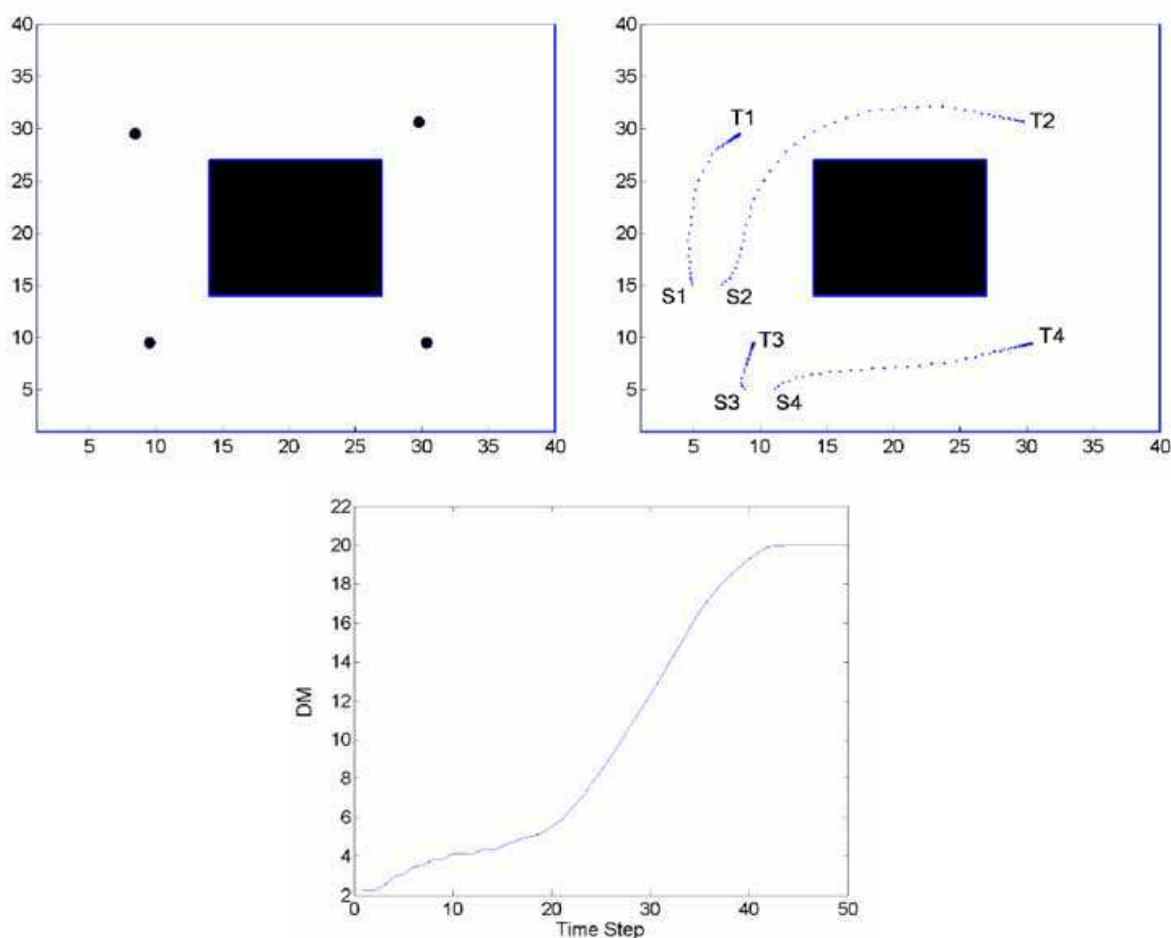

Fig. 58. Separation mode in the presence of clutter: final constellation, trajectories, minimum distance

6. Conclusions

This chapter demonstrates an important feature of harmonic potential field-based planners, that is: the social nature of such planners. This feature allows an agent steered by such a method to share, in a conflict-free manner, the same space with other agents using the same planner. Constructing a multi-agent controller in this manner has many advantages. While the system can operate in an asynchronous, decentralized mode, it can also operate in a centralized, synchronous mode that has a computational effort linear in the number of agents being controlled. The controller does exhibit an excellent ability to self-organize as well as a noncommittal planning action. This enables it to online generate the additional information needed to execute a successful action. It is also noted that the controller exhibits intelligent dispatching capabilities that enables it to redistribute the task of conflict evasion on the properly functioning agents. This property provides significant robustness in the case of sensor, or actuator failure. The controller employs an idea from the artificial life approach to behavior synthesis that is of central importance for the controller to achieve the above capabilities: i.e. the ability to project global useful activities through simple, local interacting

activities without the agents, necessarily, being aware of the generated global behavior. The artificial life G-type and P-type control modes do support such a behavior synthesis paradigm and may be considered as the backbone for building effective decentralized controllers. The HPF-based examples provided in this chapter are only a demonstration of the capabilities of this approach. The author believes that an HPF-based multi-agent controller does serve as a good basis for developing other multi-agent controllers that can effectively tackle challenging problems in many other areas such as decentralized routing in an ad hoc network (figure 59) that was suggested by (Masoud, 2008).
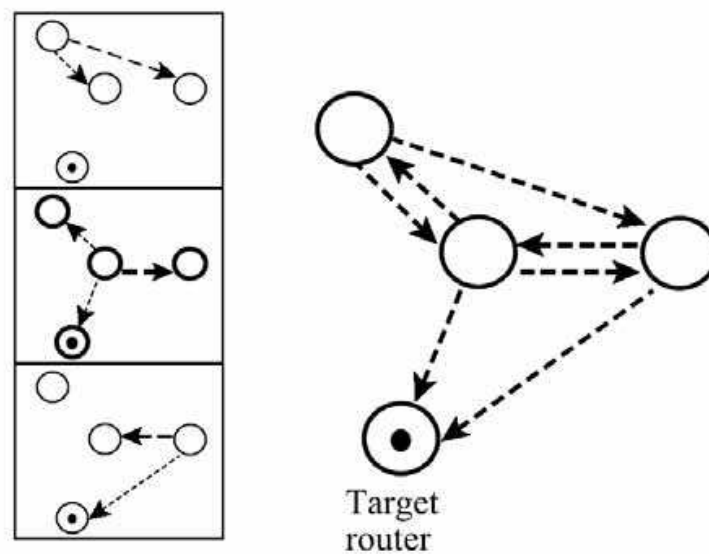

Fig. 59. Discrete harmonic potential field for decentralized routing on a graph

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8. References

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Self Organizing Maps - Applications and Novel Algorithm Design

Edited by Dr Josphat Igadwa Mwasiagi

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