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Lorentzian Wormholes Thermodynamics

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1. Introduction

The term wormhole was initially coined by Misner and Wheeler (Misner & Wheeler, 1957) (see also Wheeler (Wheeler, 1955)) in order to describe the extra connections which could exist in a spacetime, composed by two mouths and a throat, denoting therefore more general structures than that was initially considered by Einstein and Rosen (Einstein & Rosen, 1935). Nevertheless, the study of macroscopic wormholes in general relativity was left in some way behind when Fuller and Wheeler (Fuller & Wheeler, 1962) showed the instability of the Einstein-Rosen bridge.

Although other solutions of the wormhole type, stable and traversable, were studied in those years (Ellis, 1973; Bronnikov, 1973; Kodama, 1978), it was in 1988 when the physics of wormhole was revived by the work of Morris and Thorne (Morris & Thorne, 1988). These authors considered the characteristics that should have a spacetime in order to describe a wormhole, which could be used by a intrepid traveler either as a short-cut between two regions of the same universe or as a gate to another universe. They found that such structure must be generated by a stuff not only with a negative radial pressure, but with a radial pressure so negative that this exotic material violates the null energy condition. Such pathological characteristic could have been suspected through the mentioned previous studies (Ellis, 1973; Bronnikov, 1973; Kodama, 1978), that pointed out the necessity to change the sign of the kinetic term of the scalar field which supports the geometry in order to maintain the stability of the wormhole. Moreover, in the work of Morris and Thorne (Morris & Thorne, 1988) it is included a comment by Page indicating that the exoticity of the material would be not only needed in the static and spherically symmetric case, but in more general cases. Although Morris and Thorne was aware of violations of the null energy condition, both in theoretical examples and in the laboratory, they also studied the possibility to minimize the use of this odd stuff, which they called exotic matter.

Nevertheless, it seems that exotic matter should not longer be minimized since the universe itself could be an inexhaustible source of this stuff. Recent astronomical data (Mortlock & Webster, 2000) indicate that the Universe could be dominated by a fluid which violates the null energy condition, dubbed phantom energy (Caldwell, 2002). In fact, Sushkov and Lobo (Sushkov, 2005; Lobo, 2005), independently, have shown that phantom energy could well be the class of exotic matter which is required to support traversable wormholes. This result could be regarded to be one of the most powerful arguments in favor of the idea that wormholes should no longer be regarded as just describing purely mathematical toy spacetime models with interest only for science fictions writers, but also as plausible physical

realities that could exist in the very spacetemporal fabric of our Universe. The realization of this fact has motivated a renaissance of the study of wormhole spacetimes, the special interest being the consideration of the possible accretion of phantom energy onto wormholes, which may actually cause the growth of their mouths (Gonzalez-Diaz, 2004)

Due to the status at which wormholes have been promoted, it seems that the following step should be the search of these objects in our Universe. Cramer et al. (Cramer et al., 1995) were the first in noticing that a negative mass, like a wormhole, would deflect the rays coming from a luminous source, similarly to a positive mass but taking the term deflection its proper meaning in this case. Considering a wormhole between the source and the observer, the observer would either measure an increase of the intensity or receive no signal if he/she is in a certain umbral region. Following this line of thinking other works have studied the effects of microlensing (Torres et al., 1998a;b; Safonova et al., 2002) or macrolensing (Safonova et al., 2001) that wormholes could originate. Nevertheless, wormholes would affect the trajectory not only of light rays passing at some distance of them, but of rays going through them coming from other universe or other region of the same universe (Shatskiy, 2007; n.d.). In this case, as it could be expected (Morris et al., 1988), the wormhole would cause the divergence of the light rays, which would form an image of a disk with an intensity reaching several relative maxima and minima, and an absolute maximum in the edge (Shatskiy, n.d.). However, if for any reason the intensity in the edge could be much higher than in the interior region (Shatskiy, 2007), then this image may be confused with an Einstein ring, like one generated by the deformation of light due to a massive astronomical object with positive mass situated on the axis formed by the source and the observer, between them[1] (Gonzalez-Diaz, n.d.). In summary, whereas the deformation in the trajectory of light rays passing close to the wormhole could be due to any other astronomical object with negative mass, if it might exist, the observational trace produced by the light rays coming through the hole could be confused with the deformation produced by massive object with positive mass. Therefore, if it would be possible to measure in the future both effects together, then we might find a wormhole.

On the other hand, it is well known that the thermodynamical description of black holes (Bardeen et al., 1973) and other vacuum solutions, as the de Sitter model (Gibbons & Hawking, 1977), has provided these spacetimes with quite a more robust consistency, allowing moreover for a deeper understanding of their structure and properties. Following this spirit, a possible thermodynamical representation of wormholes could lead to a deeper understanding of both, these objects and the exotic material which generates them, which could perhaps be the largest energy source in the universe. Therefore, in the present chapter, we consider the potential thermodynamical properties of Lorentzian traversable wormholes. Such study should necessarily be considered in terms of local concepts, as trapping horizons, since in the considered spacetime the definition of an event horizon is no longer possible.

The importance of the use of trapping horizons in order to characterize the black holes themselves in terms of local quantities has been emphasized by Hayward (Hayward, 1994a; 1996; 1998; 2004), since global properties can not be measured by a real observer with a finite life. In this way, the mentioned author has developed a formalism able to describe the thermodynamical properties of dynamical and spherically symmetric black holes, based

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[1]It must be pointed out that whereas an structure of this kind is obtained in Ref. (Shatskiy, 2007), in Ref. (Shatskiy, n.d.) it is claimed that a correct interpretation of the results would indicate an image in the form of a luminous spot in the case that the number of stars in the other universe would be infinite, which would tend to a situation in which the maxima and minima could be distinguished when the real case tends to separate of the mentioned idealization.

in the existence of trapping horizons. Therefore, the presence of trapping horizons in the wormhole spacetime would also make possible the study of these objects. Moreover, since both objects, black holes and wormholes, can be characterized by outer trapping horizons, which are spacelike or null and timelike, respectively, they could show certain similar properties (Hayward, 1999), in particular, an analogous thermodynamics.

As we will show, the key point in this study will not lie only in applying the formalism developed by Hayward (Hayward, 1994a; 1996; 1998; 2004) to the wormhole spacetime, but in noticing that the results coming from the accretion method, (Babichev et al., 2004; Martin-Moruno et al., 2006) and (Gonzalez-Diaz, 2004; Gonzalez-Diaz & Martin-Moruno, 2008), must be equivalent to those which will be obtained by the mentioned formalism; this fact will allow a univocal characterization of wormholes. Such a characterization, together with some results about phantom thermodynamics (Gonzalez-Diaz & Siguenza, 2004; Saridakis et al., 2009), which concluded that phantom energy would possess a negative temperature, would provide any possible Hawking-like radiation from wormholes with a well defined physical meaning.

In this chapter, we will start by summarizing some previous concepts on the Morris and Thorne solution 2.1 and the Hayward formalism 2.1. This formalism will be applied to Morris-Thorne wormholes in Sec. 3. In Sec. 4 we will introduce a consistent characterization of dynamical wormholes, which will allow us to derive a thermal radiation and formulate a whole thermodynamics in Sec. 5. Finally, in Sec.6, the conclusions are summarized and further comments are added. Throughout this chapter, we use the signature convention $(-,+,+,+)$.

2. Preliminaries

2.1 The Morris-Thorne wormholes

Morris and Thorne (Morris & Thorne, 1988) considered the most general static and spherically symmetric metric able to describe a stable and traversable wormhole. That solution describes a throat connecting two asymptotically flat regions of the spacetime, without any event horizon. This metric is

$$\mathrm{d}s^2 = -e^{2\Phi(l)}\mathrm{d}t^2 + \mathrm{d}l^2 + r^2(l)\left[\mathrm{d}\theta^2 + \sin^2\mathrm{d}\varphi^2\right], \tag{1}$$

where the coordinate $-\infty < l < \infty$ and the function $\Phi(l)$ should be positive definite for any value of l. In order to recover the asymptotic limit, $r(l)/|l| \to 1$ and $\Phi(l) \to$ constant, when $l \to \pm\infty$. On the other hand, the wormhole throat is the minimum of the function $r(l)$, r_0, which we can suppose, without loss of generality, placed at $l = 0$; therefore $l < 0$ and $l > 0$ respectively cover the two asymptotically flat regions connected through the throat at $l = 0$. It is useful to express metric (1) in terms of the Schwarzschild coordinates, which yields

$$\mathrm{d}s^2 = -e^{2\Phi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - K(r)/r} + r^2\left[\mathrm{d}\theta^2 + \sin^2\mathrm{d}\varphi^2\right], \tag{2}$$

where $\Phi(r)$ and $K(r)$ are the redshift function and the shape function, respectively, and it must be pointed out that now two sets of coordinates are needed in order to cover both spacetime regions, both with $r_0 \leq r \leq \infty$. For preserving asymptotic flatness, both such functions[2], $\Phi(r)$ and $K(r)$, must tend to a constant value when the radial coordinate goes to infinity. On the other hand, the minimum radius, r_0, corresponds to the throat, where $K(r_0) = r_0$. Although

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[2]In general, there could be different functions $\Phi(r)$ and $K(r)$ in each region, (Visser, 1995), although, for our present purposes, this freedom is not of interest.

the metric coefficient g_{rr} diverges at the throat, this surface is only an apparent singularity, since the proper radial distance

$$l(r) = \pm \int_{r_0}^{r} \frac{\mathrm{d}r^*}{\sqrt{1 - K(r^*)/r^*}}, \tag{3}$$

must be finite everywhere.

In order to interpret this spacetime, we can use an embedding diagram (Morris & Thorne, 1988) (see also (Visser, 1995) or (Lobo, n.d.))). This embedding diagram, Fig. 1, can be obtained by using the spherical symmetry of this spacetime, which allow us to consider, without lost of generality, a slice defined by $\theta = \pi/2$. Such a slice is described at constant time by

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{1 - K(r)/r} + r^2 \mathrm{d}\varphi^2. \tag{4}$$

Now, we consider the Euclidean three-dimensional spacetime in cylindrical coordinates, i. e.

$$\mathrm{d}s^2 = \mathrm{d}z^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\varphi^2. \tag{5}$$

In this spacetime the slice is an embedded surface described by an equation $z = z(r)$. Therefore, Eq. (5) evaluated at the surface yields

$$\mathrm{d}s^2 = \left[1 + \left(\frac{\mathrm{d}z}{\mathrm{d}r}\right)^2\right] \mathrm{d}r^2 + r^2 \mathrm{d}\varphi^2. \tag{6}$$

Taking into account Eqs. (4) and (6) we can obtain the equation of the embedded surface. This is

$$\frac{\mathrm{d}z}{\mathrm{d}r} = \pm \left[\frac{r}{K(r)} - 1\right]^{-1/2}, \tag{7}$$

which diverges at the throat and tends to zero at the asymptotic limit. The throat must flare out in order to have a wormhole, which is known as the "flaring-out condition". This condition implies that the inverse of the embedded function should have an increasing derivative at the throat and close to it, i. e. $(\mathrm{d}^2 r)/(\mathrm{d}^2 z) > 0$. Therefore, taking the inverse of Eq. (7) and deriving with respect to z, it can be obtained that

$$\frac{K(r) - rK'(r)}{2\left(K(r)\right)^2} > 0, \tag{8}$$

implying that $K'(r_0) < 1$.

On the other hand, considering that the energy-momentum tensor can be written in an orthonormal basis as $T_{\mu\nu} = \mathrm{diag}\left(\rho(r), p_r(r), p_t(r), p_t(r)\right)$, the Einstein equations of this spacetime produce (Morris & Thorne, 1988; Visser, 1995; Lobo, n.d.)

$$\rho(r) = \frac{K'(r)}{8\pi r^2}, \tag{9}$$

$$p_r(r) = -\frac{1}{8\pi} \left[\frac{K(r)}{r^3} - 2\left(1 - \frac{K(r)}{r}\right) \frac{\Phi'(r)}{r}\right], \tag{10}$$

$$p_t(r) = \frac{1}{8\pi} \left[\Phi''(r) + \left(\Phi'(r)\right)^2 + \frac{K - K'(r)r}{2r^3 \left(1 - K(r)/r\right)} \left(r\Phi'(r) + 1\right) + \frac{\Phi'(r)}{r}\right]. \tag{11}$$
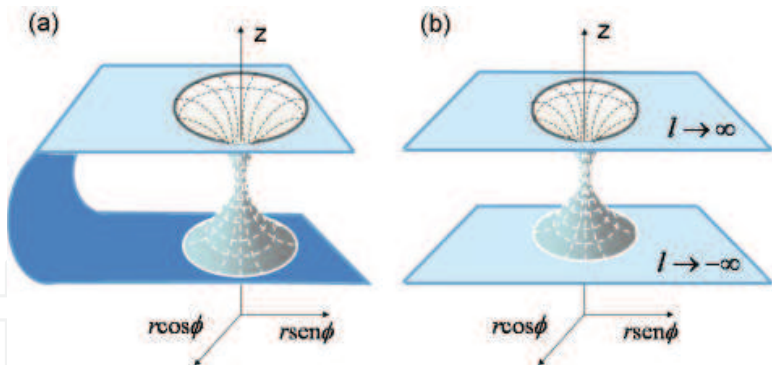
Fig. 1. The embedding diagram of an equatorial slice ($\theta = \pi/2$) of a wormhole at a given time ($t =$const). The wormhole can connect two regions of the same universe (a) or two different universes (b).

Evaluating the condition (8) at the throat and taking into account Eqs. (9) and (10), these equations are seen to entail $\rho(r_0) + p(r_0) < 0$. Therefore, the stuff generating this geometry violates the null energy condition at the throat and close to it. On the other hand, in order to minimize the exoticity of this stuff, it can be required that, at least, the energy density should be positive, leading to $K'(r) > 0$.

Apart from some quantum effects, as the Casimir effect, which could allow violations of the null energy condition, this violation has gained naturalness with the accelerated expansion of the universe. As we have already mentioned, some studies (Sushkov, 2005; Lobo, 2005; 2006) have extended the notion of phantom energy to inhomogeneous spherically symmetric spacetimes by regarding that the pressure related to the energy density through the equation of state parameter must be the radial pressure, calculating the transverse components by means of the Einstein equations. One can see (Lobo, 2005) that a particular specification of the redshift and shape functions in metric (2) leads to a static phantom traversable wormhole solution (where no dynamical evolution for the phantom energy is considered) which satisfies the traversability conditions (Morris & Thorne, 1988), in particular the outward flaring condition $K'(r_0) < 1$.

2.2 Trapping horizons

As a necessary tool for the development of the following sections, in the present subsection we summarize some concepts and notation of the Hayward formalism, which are based on the null dynamics and applicable to spherically symmetric spacetimes (Hayward, 1998).

First of all, it must be noticed that the metric of a spherically symmetric spacetime can always be written, at least locally, as

$$ds^2 = 2g_{+-}d\xi^+ d\xi^- + r^2 d\Omega^2, \tag{12}$$

where $r > 0$ and $g_{+-} < 0$ are functions of the null coordinates (ξ^+, ξ^-), related with the two preferred null normal directions of each symmetric sphere $\partial_\pm \equiv \partial/\partial\xi^\pm$, r is the so-called areal radius (Hayward, 1998), which is a geometrical invariant, and $d\Omega^2$ refers to the metric on the unit two-sphere. One can define the expansions in the null directions as

$$\Theta_\pm = \frac{2}{r}\partial_\pm r. \tag{13}$$

The sign of $\Theta_+\Theta_-$ is invariant, therefore it can be used to classify the spheres of symmetry. One can say that a sphere is trapped, untrapped or marginal if the product $\Theta_+\Theta_-$ is bigger,

less or equal to zero, respectively. Locally fixing the orientation on an untrapped sphere such that $\Theta_+ > 0$ and $\Theta_- < 0$, ∂_+ and ∂_- will be also fixed as the outgoing and ingoing null normal vectors (or the contrary if the orientation $\Theta_+ < 0$ and $\Theta_- > 0$ is considered). A marginal sphere with $\Theta_+ = 0$ is future if $\Theta_- < 0$, past if $\Theta_- > 0$ and bifurcating[3] if $\Theta_- = 0$. This marginal sphere is outer if $\partial_-\Theta_+ < 0$, inner if $\partial_-\Theta_+ > 0$ and degenerate if $\partial_-\Theta_+ = 0$. A hypersurface foliated by marginal spheres is called a trapping horizon and has the same classification as the marginal spheres.

In spherical symmetric spacetimes a unified first law of thermodynamics can be formulated (Hayward, 1998), by using the gravitational energy in spaces with this symmetry, which is the Misner-Sharp energy (Misner & Sharp, 1964). This energy can be defined by

$$E = \frac{1}{2}r\left(1 - \nabla^a r \nabla_a r\right) = \frac{r}{2}\left(1 - 2g^{+-}\partial_+ r \partial_- r\right), \tag{14}$$

and become $E = r/2$ on a trapping horizon[4].

Two invariants are also needed in order to write the unified first law of thermodynamics. These invariants can be constructed out of the energy-momentum tensor of the background fluid which can be easily expressed in these coordinates:

$$\omega = -g_{+-}T^{+-} \tag{15}$$

and the vector

$$\psi = T^{++}\partial_+ r \partial_+ + T^{--}\partial_- r \partial_-. \tag{16}$$

The first law can be written as

$$\partial_\pm E = A\psi_\pm + \omega\partial_\pm V, \tag{17}$$

where $A = 4\pi r^2$ is the area of the spheres of symmetry and $V = 4\pi r^3/3$ is defined as the corresponding flat-space volume. The first term in the r.h.s. could be interpreted as an energy-supply term, since this term produces a change in the energy of the spacetime due to the energy flux ψ generated by the surrounding material. The second term, $\omega\partial_\pm V$, behaves like a work term, something like the work that the matter content must do to support this configuration.

The Kodama vector plays also a central role in this formalism. This vector, which was introduced by Kodama (Kodama, 1980), can be understood as a generalization from the stationary Killing vector in spherically symmetric spacetimes, reducing to it in the vacuum case. The Kodama vector can be defined as

$$k = \text{rot}_2 r, \tag{18}$$

where the subscript 2 means referring to the two-dimensional space normal to the spheres of symmetry. Expressing k in terms of the null coordinates one obtains

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[3]It must be noted that on the first part of this work we will consider future and past trapping horizons with $\Theta_+ = 0$, implying that ξ^- must be related to the ingoing or outgoing null normal direction for future ($\Theta_- < 0$) or past ($\Theta_- > 0$) trapping horizons, respectively. In Sec. 5, where we will only treat past outer trapping horizons, we will fix $\Theta_- = 0$, without lost of generality, implying that $\Theta_+ > 0$ and ξ^- related to the ingoing null direction.

[4]The reader interested in properties of E may look up Ref. (Hayward, 1994b).

$$k = -g^{+-}\left(\partial_+ r \partial_- - \partial_- r \partial_+\right), \tag{19}$$

where the orientation of k can be fixed such that in a untrapped region it is future pointing. From Eq. (14), it can be noted that the squared norm of the Kodama vector can be written as

$$||k||^2 = \frac{2E}{r} - 1. \tag{20}$$

Therefore, this vector provides the trapping horizon with the additional definition of a hypersurface where the Kodama vector is null. So, such as it happens in the case of static spacetimes, where a boundary can be generally defined as the hypersurface where the temporal Killing vector is null, in the present case we must instead use the Kodama vector. This vector has some special properties[5] similar to those of the Killing vector in static spacetimes with boundaries (Hayward, 1998), such as

$$k^a \nabla_{[a} k_{b]} = \kappa \nabla_b r, \tag{21}$$

which, evaluated on a trapping horizon, implies

$$k^a \nabla_{[a} k_{b]} = \kappa k_b \ \text{on a trapping horizon}, \tag{22}$$

where the square brackets means antisymmetrization in the included scripts and

$$\kappa = \frac{1}{2} \text{div}_2 \text{grad}_2 r. \tag{23}$$

Due to the similarity between Eq. (22) and the corresponding one involving the Killing vector and its horizon[6], κ is known as generalized or geometric surface gravity. From the definition of this quantity (23) and the classification of the trapping horizons introduced at the beginning of this subsection, it can be seen that an outer, degenerate or inner horizon has $\kappa > 0$, $\kappa = 0$ y $\kappa < 0$, respectively. On the other hand, κ can be expressed in terms of the null coordinates as

$$\kappa = g^{+-} \partial_- \partial_+ r. \tag{24}$$

Taking into account Eq. (24), it can be seen that the projection of Eq. (17) along the vector z which generates the trapping horizon yields

$$L_z E = \frac{\kappa L_z A}{8\pi} + \omega L_z V, \tag{25}$$

where $L_z = z \cdot \nabla$ and $z = z^+ \partial_+ + z^- \partial_-$. This expression allows us to relate the geometric entropy and the surface area through

$$S \propto A|_H. \tag{26}$$

Finally, the Einstein equations of interest, in terms of the null coordinates (Hayward, 1998), can be expressed using the expansions (13) as

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[5]In Ref. (Hayward, 1996) other interesting properties of k are also studied.

[6]Although in the equation which relates the Killing vector with the surface gravity there is no any explicit antisymmetrization, that equation could be written in an equivalent way using an antisymmetrization. This fact is a consequence of the very definition of the Killing vector, which implies $\nabla_{(a} K_{b)} = 0$, where the brackets means symmetrization in the included scripts and K is the Killing vector.

$$\partial_\pm \Theta_\pm = -\frac{1}{2}\Theta_\pm^2 + \Theta_\pm \partial_\pm \ln(-g_{+-}) - 8\pi T_{\pm\pm}, \tag{27}$$

$$\partial_\pm \Theta_\mp = -\Theta_+ \Theta_- + \frac{1}{r^2}g_{+-} + 8\pi T_{+-}. \tag{28}$$

3. 2+2-formalism applied to Morris-Thorne wormholes

The 2+2-formalism was initially introduced by Hayward for defining the properties of real black holes in terms of measurable quantities. Such a formalism can be considered as a generalization that allows the formulation of the thermodynamics of dynamical black holes by using local quantities which are physically meaningful both in static and dynamical spacetimes. In fact, this formalism consistently recovers the results obtained by global considerations using the event horizon in the vacuum static case (Hayward, 1998). Even more, as Hayward has also pointed out (Hayward, 1999), this local considerations can also be applied to dynamic wormholes spacetimes, implying that there exists a common framework for treating black holes and wormholes.

Nevertheless one of the most surprising features of the 2+2-formalism is found when applied to Morris-Thorne wormholes. Whereas in this spacetime it is not possible to obtain any property similar to those obtained in black holes physics by using global considerations, since no event horizon is present, the consideration of local quantities shows similar characteristics to those of black holes. This fact can be better understood if one notices that the Schwarzschild spacetime is the only spherically symmetric solution in vacuum and, therefore, any dynamical generalizations of black holes must be formulated in the presence of some matter content. The maximal extension of the Schwarzschild spacetime (Kruskal, 1960) can be interpreted as an Einstein-Rosen bridge (Einstein & Rosen, 1935), which corresponds to a vacuum wormhole and has associated a given thermodynamics. Nevertheless, the Einstein-Rosen bridge can not be traversed since it has an event horizon and it is unstable (Fuller & Wheeler, 1962). If we consider wormholes which can be traversed, then some matter content must be present even in the static case of Morris-Thorne. So the need of a formulation in terms of local quantities, measurable for an observer with finite life, must be related to the presence of some matter content, rather than with a dynamical evolution of the spacetime.

In this section we apply the results obtained by Hayward for spherically symmetric solutions to static wormholes, showing rigorously their consequences (Martin-Moruno & Gonzalez-Diaz, 2009b), some of which were already suggested and/or indicated by Ida and Hayward himself (Ida & Hayward, 1995).

Defining the coordinates $\xi^+ = t + r_*$ and $\xi^- = t - r_*$, with r_* such that $dr/dr_* = \sqrt{-g_{00}/g_{rr}} = e^{\Phi(r)}\sqrt{1 - K(r)/r}$, and ξ^+ and ξ^- being related to the outgoing and ingoing direction, respectively, the metric (2) can be expressed in the form given by Eq. (12). It can be seen, by the definitions introduced in the previous section, that there is a bifurcating trapping horizon at $r = r_0$. This horizon is outer since the flaring-out condition implies $K'(r_0) < 1$.

In this spacetime the Misner-Sharp energy (14), "energy density" (15) and "energy flux" (16) can be calculated to be

$$E = \frac{K(r)}{2}, \tag{29}$$

$$\omega = \frac{\rho - p_r}{2}, \tag{30}$$

and

$$\psi = -\frac{(\rho + p_r)}{2}e^{-\Phi(r)}\sqrt{1 - K(r)/r}\,(-\partial_+ + \partial_-)\,, \tag{31}$$

where we have taken into account that the components of an energy-momentum tensor which takes the form[7] $T_{\mu\nu}^{(2)} = \mathrm{diag}(\rho, p_r)$ in an orthonormal basis, with the superscript (2) meaning the two-dimensional space normal to the spheres of symmetry, are in this basis $T_{\pm\pm} = e^{2\Phi(r)}(\rho + p_r)/4$ and $T_{+-} = T_{-+} = e^{2\Phi(r)}(\rho - p_r)/4$. The Misner-Sharp energy in this spacetime reaches its limiting value $E = r/2$ only at the wormhole throat, $r = r_0$, which corresponds to the trapping horizon, taking smaller values in the rest of the space which is untrapped. We want to emphasized that, as in the case of the studies about phantom wormholes performed by Sushkov (Sushkov, 2005) and Lobo (Lobo, 2005), any information about the transverse components of the pressure becomes unnecessary.

Deriving Eq. (29) and rising the index of Eq. (31), one can obtain

$$\partial_\pm E = \pm 2\pi r^2 \rho e^{\Phi}\sqrt{1 - K(r)/r} \tag{32}$$

and

$$\psi_\pm = \pm e^{\Phi}(r)\sqrt{1 - K(r)/r}\frac{\rho + p_r}{4}. \tag{33}$$

Therefore, we have all terms[8] of Eq. (17) for the first law particularized to the Morris-Thorne case, which vanish at the throat, what could be suspected since we are considering a wormhole without dynamic evolution. Nevertheless, the comparison of these terms in the case of Morris-Thorne wormholes with those which appear in the Schwarzschild black hole could provide us with a deeper understanding about the former spacetime, based on the exotic properties of its matter content. Of course, the Schwarzschild metric is a vacuum solution, but it could be expected that it would be a good approximation when small matter quantities are considered, which we will assume to be ordinary matter. So, in the first place, we want to point out that the variation of the gravitational energy, Eq. (32), is positive (negative) in the outgoing (ingoing) direction in both cases[9], since $\rho > 0$; therefore, this variation is positive for exotic and usual matter. In the second place, the "energy density", ω, takes positive values no matter whether the null energy condition is violated or not. Considering the "energy supply" term, in the third place, we find the key difference characterizing the wormhole spacetime. The energy flux depends on the sign of $\rho + p_r$, therefore it can be interpreted as a fluid which "gives" energy to the spacetime, in the case of usual matter, or as a fluid "receiving" or "getting" energy from the spacetime, when exotic matter is considered. This "energy removal", induced by the energy flux in the wormhole case, can never reach a value so large to change the sign of the variation of the gravitational energy.

On the other hand, the spacetime given by (2) possesses a temporal Killing vector which is non-vanishing everywhere and, therefore, there is no Killing horizon where a surface gravity can be calculated as considered by Gibbons and Hawking (Gibbons & Hawking, 1977).

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[7]As we will comment in the next section, this energy-momentum tensor is of type I in the classification of Hawking and Ellis (Hawking & Ellis, 1973).

[8]The remaining terms can be easily obtained taking into account that $\partial_\pm r = \pm\frac{1}{2}e^{\Phi(r)}\sqrt{1 - K(r)/r}$.

[9]The factor $e^{\Phi}\sqrt{1 - K(r)/r} \equiv \alpha$, which appears by explicitly considering the Morris-Thorne solution, comes from the quantity $\alpha = \sqrt{-g_{00}/g_{rr}}$, which is a general factor at least in spherically symmetric and static cases; therefore α has the same sign both in Eq. (32) and in Eq. (33).

Nevertheless, the definition of a Kodama vector or, equivalently, of a trapping horizon implies the existence of a generalized surface gravity for both static and dynamic wormholes. In particular, in the Morris-Thorne case the components of the Kodama vector take the form

$$k^{\pm} = e^{-\Phi(r)} \sqrt{1 - K(r)/r}, \tag{34}$$

with $||k||^2 = -1 + K(r)/r = 0$ at the throat. The generalized surface gravity, (24), is

$$\kappa|_H = \frac{1 - K'(r_0)}{4r_0} > 0, \tag{35}$$

where "$|_H$" means evaluation at the throat and we have considered that the throat is an outer trapping horizon, which is equivalent to the flaring-out condition ($K'(r_0) < 1$). By using the Einstein equations (9) and (10), κ can be re-expressed as

$$\kappa|_H = -2\pi r_0 \left[\rho(r_0) + p(r_0) \right]. \tag{36}$$

with $\rho(r_0) + p(r_0) < 0$, as we have mentioned in 2.1.

It is well known that when the surface gravity is defined by using a temporal Killing vector, this quantity is understood to mean that there is a force acting on test particles in a gravitational field. The generalized surface gravity is in turn defined by the use of the Kodama vector, which can be interpreted as a preferred flow of time for observers at a constant radius (Hayward, 1996), reducing to the Killing vector in the vacuum case and recovering the surface gravity its usual meaning. Nevertheless, in the case of a spherically symmetric and static wormhole one can define both, the temporal Killing and the Kodama vector, being the Kodama vector of greater interest since it vanishes at a particular surface. Moreover, in dynamical spherically symmetric cases one can only define the Kodama vector. Therefore it could be suspected that the generalized surface gravity should originate some effect on test particles which would go beyond that corresponding to a force, and only reducing to it in the vacuum case. On the other hand, if by some kind of symmetry this effect on a test particle would vanish, then we should think that such a symmetry would also produce that the trapping horizon be degenerated.

4. Dynamical wormholes

The existence of a generalized surface gravity which appears in the first term of the r.h.s. of Eq. (25) multiplying a quantity which can be identify as something proportional to an entropy would suggest the possible formulation of a wormhole thermodynamics, as it was already commented in Ref. (Hayward, 1999). Nevertheless, a more precise definition of its trapping horizon must be done in order to settle down univocally its characteristics. With this purpose, we first have to summarize the results obtained by Hayward for the increase of the black hole area (Hayward, 2004), comparing then them with those derived from the accretion method (Babichev et al., 2004). Such comparison will shed some light for the case of wormholes.

On the one hand, the area of a surface can be expressed in terms of μ as $A = \int_S \mu$, with $\mu = r^2 \sin\theta d\theta d\varphi$ in the spherically symmetric case. Therefore, the evolution of the trapping horizon area can be studied considering

$$L_z A = \int_S \mu \left(z^+ \Theta_+ + z^- \Theta_- \right), \tag{37}$$

with z the vector which generates the trapping horizon.

On the other hand, by the very definition of a trapping horizon we can fix $\Theta_+|_H = 0$, which provides us with the fundamental equation governing its evolution

$$L_z\Theta_+|_H = \left[z^+\partial_+\Theta_+ + z^-\partial_-\Theta_+\right]|_H = 0. \tag{38}$$

It must be also noticed that the evaluation of Eq. (27) at the trapping horizon implies

$$\partial_+\Theta_+|_H = -8\pi T_{++}|_H, \tag{39}$$

where $T_{++} \propto \rho + p_r$ by considering an energy-momentum tensor of type I in the classification of Hawking and Ellis[10]. (Hawking & Ellis, 1973). Therefore, if the matter content which supports the geometry is usual matter, then $\partial_+\Theta_+|_H < 0$, being $\partial_+\Theta_+|_H > 0$ if the null energy condition is violated.

Dynamic black holes are characterized by outer future trapping horizons, which implies the growth of their area when they are placed in environment which fulfill the null energy condition (Hayward, 2004). This property can be easily deduced taking into account the definition of outer trapping horizon and noticing that, when it is introduced in the condition (38), with Eq. (39) for usual matter, implies that the sign of z^+ and z^- must be different, i.e. the trapping horizon is spacelike when considering usual matter and null in the vacuum case. It follows that the evaluation of $L_z A$ at the horizon, $\Theta_+ = 0$, taking into account that the horizon is future and that z has a positive component along the future-pointing direction of vanishing expansion, $z^+ > 0$, yields[11] $L_z A \geq 0$, where the equality is fulfilled in the vacuum case. It is worth noticing that when exotic matter is considered, then the previous reasoning would lead to a black hole area decrease.

It is well known that accretion method based on a test-fluid approach developed by Babichev et al. (Babichev et al., 2004) (and its non-static generalization (Martin-Moruno et al., 2006)) leads to the increase (decrease) of the black hole when it acreates a fluid with $p + \rho > 0$ ($p + \rho < 0$), where p could be identified in this case with p_r. These results are the same as those obtained by using the $2 + 2$-formalism, therefore, it seems natural to consider that both methods in fact describe the same physical process, originating from the flow of the surrounding matter into the hole.

Whereas the characterization of black holes appears in this study as a natural consideration, a reasonable doubt may still be kept about how the outer trapping horizon of wormholes may be considered. Following the same steps as in the argument relative to dynamical black holes, it can be seen that, since a traversable wormhole should necessarily be described in the presence of exotic matter, the above considerations imply that its trapping horizon should be timelike, allowing a two-way travel. However, if this horizon would be future (past) then, by Eq. (37), its area would decrease (increase) in an exotic environment, remaining constant in the static case when the horizon is bifurcating. In this sense, an ambiguity in the characterization of dynamic wormholes seems to exist.

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[10]In general one would have $T_{++} \propto T_{00} + T_{11} - 2T_{01}$, where the components of the energy-momentum tensor on the r.h.s. are expressed in terms of an orthonormal basis. In our case, we consider an energy-momentum tensor of type I (Hawking & Ellis, 1973), not just because it represents all observer fields with non-zero rest mass and zero rest mass fields, except in special cases when it is type II, but also because if this would not be the case then either $T_{++} = 0$ (for types II and III) which at the end of the day would imply no horizon expansion, or we would be considering the case where the energy density vanishes (type IV)

[11]It must be noticed that in the white hole case, which is characterized by a past outer trapping horizon, this argument implies $L_z A \leq 0$.

Nevertheless, this ambiguity is only apparent once noticed that this method is studying the same process as the accretion method, in this case applied to wormholes (Gonzalez-Diaz & Martin-Moruno, 2008), which implies that the wormhole throat must increase (decrease) its size by accreting energy which violates (fulfills) the null energy condition. Therefore, the outer trapping horizons which characterized dynamical wormholes should be past (Martin-Moruno & Gonzalez-Diaz, 2009a;b). This univocal characterization could have been suspected from the very beginning since, if the energy which supports wormholes should violate the null energy condition, then it seems quite a reasonable implication that the wormhole throat must increase if some matter of this kind would be accreted.

In order to better understand this characterization, we could think that whereas dynamical black holes would tend to be static as one goes into the future, being their trapping horizon past, white holes, which are assumed to have born static and then allowed to evolve, are characterized by a past trapping horizon. So, in the case of dynamical wormholes one can consider a picture of them being born at some moment (at the beginning of the universe, or constructed by an advanced civilization, or any other possible scenarios) and then left to evolve to they own. Therefore, following this picture, it seems consistent to characterize wormholes by past trapping horizons.

Finally, taking into account the proportionality relation (26), we can see that the dynamical evolution of the wormhole entropy must be such that $L_z S \geq 0$, which saturates only at the static case characterized by a bifurcating trapping horizon.

5. Wormhole thermal radiation and thermodynamics

The existence of a non-vanishing surface gravity at the wormhole throat seems to imply that it can be characterized by a non-zero temperature so that one would expect that wormholes should emit some sort of thermal radiation. Although we are considering wormholes which can be traversed by any matter or radiation, passing through it from one universe to another (or from a region to another of the same single universe), what we are refereeing to now is a completely different kind of radiative phenomenon, which is not due to any matter or radiation following any classically allowed path but to thermal radiation with a quantum origin. Therefore, even in the case that no matter or radiation would travel through the wormhole classically, the existence of a trapping horizon would produce a semi-classical thermal radiation.

It has been already noticed in Ref. (Hayward et al., 2009) that the use of a Hamilton-Jacobi variant of the Parikh-Wilczek tunneling method led to a local Hawking temperature in the case of spherically symmetric black holes. Nevertheless, it was also suggested (Hayward et al., 2009) that the application of this method to past outer trapping horizon could lead to negative temperatures which, therefore, could be lacking of a well defined physical meaning. In this section we show explicitly the calculation of the temperature associated with past outer trapping horizons (Martin-Moruno & Gonzalez-Diaz, 2009a;b), which characterizes dynamical wormholes, applying the method considered in Ref. (Hayward et al., 2009). The rigorous application of this method implies a wormhole horizon with negative temperature. This result, far from being lacking in a well defined physical meaning, can be interpreted in a natural way taking into account that, as it is well known (Gonzalez-Diaz & Siguenza, 2004; Saridakis et al., 2009), phantom energy also possesses negative temperature.

We shall consider in the present study a general spherically symmetric and dynamic wormhole which, therefore, is described through metric (12) with a trapping horizon

characterized by $\Theta_- = 0$ and[12] $\Theta_+ > 0$. The metric (12) can be consequently written in terms of the generalized retarded Eddington-Finkelstein coordinates, at least locally, as

$$\mathrm{d}s^2 = -e^{2\Psi}C\mathrm{d}u^2 - 2e^{\Psi}\mathrm{d}u\mathrm{d}r + r^2\mathrm{d}\Omega^2, \tag{40}$$

where $\mathrm{d}u = \mathrm{d}\xi^-$, $\mathrm{d}\xi^+ = \partial_u\xi^+\mathrm{d}u + \partial_r\xi^+\mathrm{d}r$, and Ψ expressing the gauge freedom in the choice of the null coordinate u. Since $\partial_r\xi^+ > 0$, we have considered $e^{\Psi} = -g_{+-}\partial_r\xi^+ > 0$ and $e^{2\Psi}C = -2g_{+-}\partial_u\xi^+$. It can be seen that $C = 1 - 2E/r$, with E defined by Eq. (14). The use of retarded coordinates ensures that the marginal surfaces, characterized by $C = 0$, are past marginal surfaces.

From Eqs. (18) and (23), it can be seen that the generalized surface gravity at the horizon and the Kodama vector are

$$\kappa|_H = \frac{\partial_r C}{2} \tag{41}$$

and

$$k = e^{-\Psi}\partial_u, \tag{42}$$

respectively.

Now, similarly to as it has been done in Ref. (Hayward et al., 2009) for the dynamical black hole case, we consider a massless scalar field in the eikonal approximation, $\phi = \phi_0\exp(iI)$, with a slowly varying amplitude and a rapidly varying action given by

$$I = \int \omega_\phi e^{\Psi}\mathrm{d}u - \int k_\phi\mathrm{d}r, \tag{43}$$

with ω_ϕ being an energy parameter associated to the radiation. In our case, this field describes radially outgoing radiation, since ingoing radiation would require the use of advanced coordinates.

The wave equation of the field which, as we have already mentioned, fulfills the eikonal equation, implies the Hamilton-Jacobi one[13]

$$\gamma^{ab}\nabla_a I\nabla_b I = 0, \tag{44}$$

where γ^{ab} is the metric in the 2-space normal to the spheres of symmetry. Now, taking into account $\partial_u I = e^{\Psi}\omega_\phi$ and $\partial_r I = -k$, Eq. (44) yields

$$k_\phi^2 C + 2\omega_\phi k_\phi = 0. \tag{45}$$

One solution of this equation is $k_\phi = 0$, which must corresponds to the outgoing modes, since we are considering that ϕ is outgoing. On the other hand, the alternate solution, $k_\phi = -2\omega_\phi/C$, should correspond to the ingoing modes and it will produce a pole in the action integral 43, because C vanishes on the horizon. Expanding C close to the horizon, one can express the second solution in this regime as $k_\phi \approx -\omega_\phi/[\kappa|_H(r-r_0)]$. Therefore the action has an imaginary contribution which is obtained deforming the contour of integration in the lower r half-plane, which is

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[12]We are now fixing, without loss of generality, the outgoing and ingoing direction as ∂_+ and ∂_-, respectively.

[13]For a deeper understanding about the commonly used approximations of this method, as the eikonal one, it can be seen, for example, Ref. (Visser, 2003).

$$\text{Im}\,(I)\,|_H = -\frac{\pi\omega_\phi}{\kappa|_H}. \tag{46}$$

This expression can be used to consider the particle production rate as given by the WKB approximation of the tunneling probability Γ along a classically forbidden trajectory

$$\Gamma \propto \exp\left[-2\text{Im}\,(I)\right]. \tag{47}$$

Although the wormhole throat is a classically allowed trajectory, being the wormhole a two-way traversable membrane, we can consider that the existence of a trapping horizon opens the possibility for an additional traversing phenomenon through the wormhole with a quantum origin. One could think that this additional radiation would be somehow based on some sort of quantum tunneling mechanism between the two involved universes (or the two regions of the same, single universe), a process which of course is classically forbidden. If such an interpretation is accepted, then (47) takes into account the probability of particle production rate at the trapping horizon induced by some quantum, or at least semi-classical, effect. On the other hand, considering that this probability takes a thermal form, $\Gamma \propto \exp\left(-\omega_\phi/T_H\right)$, one could compute a temperature for the thermal radiation given by

$$T = -\frac{\kappa|_H}{2\pi}, \tag{48}$$

which is negative. At first sight, one could think that we would be safe from this negative temperature because it is related to the ingoing modes. However this can no longer be the case as even if this thermal radiation is associated to the ingoing modes, they characterize the horizon temperature. Even more, the infalling radiation getting in one of the wormhole mouths would travel through that wormhole following a classical path to go out of the other mouth as an outgoing radiation in the other universe (or the other region of universe). Such a process would take place at both mouths producing, in the end of the day, outgoing radiation with negative temperature in both mouths.

Nevertheless, it is well known that phantom energy, which is no more than a particular case of exotic matter, is characterized by a negative temperature (Gonzalez-Diaz & Siguenza, 2004; Saridakis et al., 2009). Thus, this result could be taken to be a consistency proof of the used method, as a negative radiation temperature simply express the feature to be expected that wormholes should emit a thermal radiation just of the same kind as that of the stuff supporting them, such as it also occurs with dynamical black holes with respect to usual matter and positive temperature.

Now, Eq. (25) can be re-written, taking into account the temperature expressed in Eq. (48), as follows

$$L_z E = -T L_z S + \omega L_z V, \tag{49}$$

defining univocally the geometric entropy on the trapping horizon as

$$S = \frac{A|_H}{4}. \tag{50}$$

The negative sign appearing in the first term in the r.h.s. of Eq. (49) would agree with the consideration included in Sec. 3 and according to which the exotic matter supporting this spacetime "removes" energy from the spacetime itself. Following this line of thinking we can then formulate the first law of wormhole thermodynamics as:

First law: The change in the gravitational energy of a wormhole equals the sum of the energy removed from the wormhole plus the work done in the wormhole.

This first law can be interpreted by considering that the exotic matter is responsible for both the energy removal and the work done, keeping the balance always giving rise to a positive variation of the total gravitational energy.

On the other hand, as we have pointed out in Sec. 5, $L_z A \geq 0$ in an exotic environment, implying $L_z S \geq 0$ through Eq. (50), which saturates only at the static case. Thus, considering that a real, cosmological wormhole must be always in an exotic dynamical background, we can formulate the second law for wormhole thermodynamics as follows:

Second law: The entropy of a dynamical wormhole is given by its surface area which always increases, whenever the wormhole accretes exotic material.

Moreover, a wormhole is characterized by an outer trapping horizon (which must be past as has been argued in Sec. 4) which, in terms of the surface gravity, implies $\kappa > 0$. Therefore, we can formulate the third law of thermodynamic as:

Third law (first formulation): It is impossible to reach the absolute zero for surface gravity by any dynamical process.

It is worth noticing that if some dynamical process could change the outer character of a trapping horizon in such a way that it becomes an inner horizon, then the wormhole would converts itself into a different physical object. If this hypothetical process would be possible, then it would make no sense to continue referring to the laws of wormhole thermodynamics, being the thermodynamics of that new object which should instead be considered. Following this line of thinking, it must be pointed out that whenever there is a wormhole, $\kappa > 0$, its trapping horizon is characterized by a negative temperature by virtue of the arguments showed. Thus, we can re-formulate the third law of wormhole thermodynamic as:

Third law (second formulation): In a wormhole it is impossible to reach the absolute zero of temperature by any dynamical process.

It can be argued that if one could change the background energy from being exotic matter to usual one, then the causal nature of the outer trapping horizon would change[14] (Hayward, 1999). Even more, we could consider that as caused by such a process, or by a subsequent one, a past outer trapping horizon (i. e. a dynamical wormhole) should change into a future outer trapping horizon (i.e. a dynamical black hole), and vice versa. If such process would be possible, then it could be expected the temperature to change from negative (wormhole) to positive (black hole) in a way which is necessarily discontinuous due to the holding of the third law, i. e. without passing through the zero temperature, since neither of those objects is characterized by a degenerate trapping horizon.

In the hypothetical process mentioned in the previous paragraph the first law of wormholes thermodynamics would then become the first law of black holes thermodynamics, where the energy is supplied by ordinary matter rather than by the exotic one and the minus sign in Eq. (49) is replaced by a plus sign. The latter implication arises from the feature that a future outer trapping horizon should produce thermal radiation at a positive temperature. The second law would remain then unchanged since it can be noted that the variation of the horizon area, and hence of the entropy, is equivalent for a past outer trapping horizon surrounded by exotic matter and for a future outer trapping horizon surrounded by ordinary matter. And, finally, the two formulations provided for the third law would also be the same,

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[14]This fact can be deduced by noticing that both, the material content and the outer property of the horizon, fix the relative sign of z^+ and z^- through Eq. (38).

but in the second formulation one would consider that the temperature takes only on positive values.

6. Conclusions and further comments

In this chapter we have first applied results related to a generalized first law of thermodynamics (Hayward, 1998) and the existence of a generalized surface gravity (Hayward, 1998; Ida & Hayward, 1995) to the case of the Morris-Thorne wormholes (Morris & Thorne, 1988), where the outer trapping horizon is bifurcating. Since these wormholes correspond to static solutions, no dynamical evolution of the throat is of course allowed, with all terms entering the first law vanishing at the throat. However, the comparison of the involved quantities (such as the variation of the gravitational energy and the energy-exchange so as work terms as well) with the case of black holes surrounded by ordinary matter actually provide us with some useful information about the nature of this spacetime (or alternatively about the exotic matter), under the assumption that in the dynamical cases these quantities keep the signs unchanged relative to those appearing outside the throat in the static cases. It follows that the variation of the gravitational energy and the "work term", which could be interpreted as the work carried out by the matter content in order to maintain the spacetime, have the same sign in spherically symmetric spacetimes supported by both ordinary and exotic matter. Notwithstanding, the "energy-exchange term" would be positive in the case of dynamical black holes surrounded by ordinary matter (i. e. it is an energy supply) and negative for dynamical wormholes surrounded by exotic matter (i. e. it corresponds to an energy removal).

That study has allowed us to show that the Kodama vector, which enables us to introduce a generalized surface gravity in dynamic spherically symmetric spacetimes (Hayward, 1998), must be taken into account not only in the case of dynamical solutions, but also in the more general case of non-vacuum solutions. In fact, whereas the Kodama vector reduces to the temporal Killing in the spherically symmetric vacuum solution (Hayward, 1998), that reduction is no longer possible for the static non-vacuum case described by the Morris-Thorne solution. That differentiation is a key ingredient in the mentioned Morris-Thorne case, where there is no Killing horizon in spite of having a temporal Killing vector and possessing a non degenerate trapping horizon. Thus, it is possible to define a generalized surface gravity based on local concepts which have therefore potentially observable consequences. When this consideration is applied to dynamical wormholes, such an identification leads to the characterization of these wormholes in terms of the past outer trapping horizons (Martin-Moruno & Gonzalez-Diaz, 2009a;b).

The univocal characterization of dynamical wormholes implies not only that the area (and hence the entropy) of a dynamical wormhole always increases if there are no changes in the exoticity of the background (second law of wormhole thermodynamics), but also that the hole appears to thermally radiate. The results of the studies about phantom thermodynamics (Gonzalez-Diaz & Siguenza, 2004; Saridakis et al., 2009) allow us to provide this possible radiation with negative temperature with a well-defined physical meaning. Therefore, wormholes would emit radiation of the same kind as the matter which supports them (Martin-Moruno & Gonzalez-Diaz, 2009a;b), such as it occurs in the case of dynamical black hole evaporation with respect to ordinary matter.

These considerations allow us to consistently re-interpret the generalized first law of thermodynamics as formulated by Hayward (Hayward, 1998) in the case of wormholes, noting that in this case the change in the gravitational energy of the wormhole throat is

equal to the sum of the energy removed from the wormhole and the work done on the wormhole (first law of wormholes thermodynamics), a result which is consistent with the above mentioned results obtained by analyzing of the Morris-Thorne spacetime in the throat exterior.

At first sight, the above results might perhaps be pointing out to a way through which wormholes might be localized in our environment by simply measuring the inhomogeneities implied by phantom radiation, similarly to as initially thought for black hole Hawking radiation (Gibbons & Hawking, 1977). However, we expect that in this case the radiation would be of a so tiny intensity as the originated from black holes, being far from having hypothetical instruments sensitive and precise enough to detect any of the inhomogeneities and anisotropies which could be expected from the thermal emission from black holes and wormholes of moderate sizes.

It must be pointed out that, like in the black hole case, the radiation process would produce a decrease of the wormhole throat size, so decreasing the wormhole entropy, too. This violation of the second law is only apparent, because it is the total entropy of the universe what should be meant to increase.

It should be worth noticing that there is an ambiguity when performing the action integral in the radiation study, which depends on the r semi-plane chosen to deform the integration path. This ambiguity could be associated to the choice of the boundary conditions. Thus, had we chosen the other semi-plane, then we had obtained a positive temperature for the wormhole trapping horizon. The supposition of this second solution as physically consistent implies that the thermal radiation would be always thermodynamically forbidden in front of the accretion entropicaly favored process, since the energy filling the space has negative temperature (Gonzalez-Diaz & Siguenza, 2004; Saridakis et al., 2009) and, therefore, "hotter" than any positive temperature. Although this possibility should be mentioned, in our case we consider that the boundary conditions, in which it is natural to take into account the sign of the temperature of the surrounding material, imply that the horizon is characterized by a temperature with the same sign. However, it would be of a great interest the confirmation of this result by using an alternative method where the mentioned ambiguity would not be present.

On the other hand, we find of special interest to briefly comment some results presented during/after the publication of the works in which are based this chapter (Martin-Moruno & Gonzalez-Diaz, 2009a;b), since it could clarify some considerations adopted in our development. First of all, in a recent work by Hayward (Hayward, 2009), in which some part of the present work was also discussed following partly similar though somewhat divergent arguments, the thermodynamics of two-types of dynamic wormholes characterized by past or future outer trapping horizon was studied. Although these two types are completely consistent mathematical solutions, we have concentrated on the present work in the first one, since we consider that they are the only physical consistent wormholes solution. One of the reasons which support the previous claim has already been mentioned in this work and is based on the possible equivalence of the results coming from the 2+2 formalism and the accretion method, at least qualitatively. On the other hand, a traversable wormhole must be supported by exotic matter and it is known that it can collapse by accretion of ordinary matter. That is precisely the problem of how to traverse a traversable wormhole finding the mouth open for the back-travel, or at least avoiding a possible death by a pinched off wormhole throat during the trip. If the physical wormhole could be characterized by a future outer trapping horizon, by Eqs. 37), (38) and (39), then its size would increase (decrease) by accretion of

ordinary (exotic) matter and, therefore, it would not be a problem to traverse it; even more, it would increase its size when a traveler would pass through the wormhole, contrary to what it is expected from the bases of the wormhole physics (Morris & Thorne, 1988; Visser, 1995).

In the second place, Di Criscienzo, Hayward, Nadalini, Vanzo and Zerbini Ref. (Di Criscienzo et al., 2010) have shown the soundness of the method used in Ref. (Hayward et al., 2009) to study the thermal radiation of dynamical black holes, which we have considered valid, adapting it to the dynamical wormhole case; although, of course, it could be other methods which could also provide a consistent description of the process. Moreover, in this work (Di Criscienzo et al., 2010) Di criscienzo et al. have introduced a possible physical meaning for the energy parameter ω_ϕ, noticing that it can be expressed in terms of the Kodama vector, which provides a preferred flow, as $\omega_\phi = -k^\alpha \partial_\alpha I$; thus, the authors claim that ω_ϕ would be the invariant energy associated with a particle. If this could be the case, then the solution presented in this chapter when considering the radiation process, $k_\phi = -2\omega_\phi/C$, could imply a negative invariant energy for the radiated "particles", since it seems possible to identify k_ϕ with any quantity similar to the wave number, or even itself, being, therefore, a positive quantity. This fact can be understood thinking that the invariant quantity characterizing the energy of "the phantom particles" should reflect the violation of the null energy condition.

Finally, we want to emphasize that the study of wormholes thermodynamics introduced in this chapter not only have the intrinsic interest of providing a better understanding of the relation between the gravitational and thermodynamic phenomena, but also it would allow us to understand in depth the evolution of spacetime structures that could be present in our Universe. We would like to once again remark that it is quite plausible that the existence of wormholes be partly based on the possible presence of phantom energy in our Universe. Of course, even though in that case the main part of the energy density of the universe would be contributed by phantom energy, a remaining 25% would still be made up of ordinary matter (dark or not). At least in principle, existing wormhole structures would be compatible with the configuration of such a universe, even though a necessarily sub-dominant proportion of ordinary matter be present, provided that the effective equation of state parameter of the universe be less than minus one.

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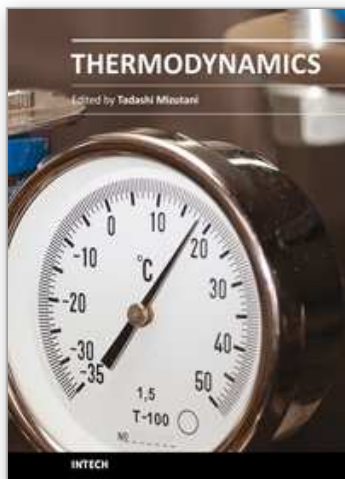
Thermodynamics

Edited by Prof. Mizutani Tadashi

Progress of thermodynamics has been stimulated by the findings of a variety of fields of science and technology. The principles of thermodynamics are so general that the application is widespread to such fields as solid state physics, chemistry, biology, astronomical science, materials science, and chemical engineering. The contents of this book should be of help to many scientists and engineers.

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