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Parametric Modelling of EEG Data for the Identification of Mental Tasks

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1. Introduction

Electroencephalographic (EEG) data is widely used as a biosignal for the identification of different mental states in the human brain. EEG signals can be captured by relatively inexpensive equipment and acquisition procedures are non-invasive and not overly complicated. On the negative side, EEG signals are characterized by low signal-to-noise ratio and non-stationary characteristics, which makes the processing of such signals for the extraction of useful information a challenging task.

When a person performs specific events, such as cued imagery tasks, left-hand or right-hand movements, imagined motor tasks and auditory tasks, corresponding variations in the characteristics of the person's EEG signal take place. These are typically identified by so-called event-related potentials (ERP). For example, event-related potentials associated with real and imagined motor tasks exhibit frequency-specific characteristics: a decrease in EEG band power occurs on the contra-lateral side, a phenomenon known as Event-Related Desynchronization (ERD), followed some time later by an increase in band power on the ipsi-lateral side, known as Event -Related Synchronization (ERS). Hence the detection and identification of ERD and ERS phenomena would enable the classification of mental activity. Such techniques can find useful application in *brain-computer interface* (BCI) systems where EEG data is measured from the brain and processed by a computer so as to, for example, detect and classify real or imagined left and right-hand movements for the execution of useful tasks such as wheelchair navigation (Pfurtscheller *et al.*, 2006). Several signal processing techniques have been proposed to classify left and right-hand movement from the EEG signal either by detecting ERD and ERS phenomena directly, or by the application of appropriate signal analysis techniques which are characterised by the ERD/ERS phenomena. These include the inter-trial variance approach, the Short-time Fourier Transform, Wavelet Transform methods and Source Localization methods (Pfurtscheller & Lopes da Silva, 1999; Qin *et al.*, 2004).

A different approach, on which this chapter will focus, aims to capture the dynamics of the EEG signal by means of auto-regressive (AR) or auto-regressive moving average (ARMA) parametric models (Pardey *et al.*, 1996). This chapter will specifically address the use of such models for the identification and classification of imagined left and right-hand movements. It will start with a literature review on the use of AR and ARMA parametric models for EEG signals and their practical applications. It then proceeds to report, in a unified manner, a number of novel contributions proposed and published by the authors as summarized below.

The first contribution proposes a novel three-mode classifier which uses the parameters of the AR model as a feature for distinguishing between no hand movement and left-hand or right-hand movement (Cassar *et al.*, 2010b). The quality of the classifier depends on the optimal estimation of the AR model parameters. The Kalman filter presents itself as a versatile tool for on-line estimation of model coefficients. However, the performance of the Kalman filter depends critically on its initialisation and the correct setting of the Kalman filter parameters, which are generally unknown. (Khan & Dutt, 2007) showed that the Expectation Maximization (EM) algorithm could be used to estimate the Kalman filter parameters and hence obtain AR coefficient estimates which improve the identification of ERD. Following the same approach, this chapter shows that the EM algorithm for Kalman filter initialization, together with the proposed three-mode classifier, yields better scores than two-mode classifiers typically proposed in the literature for brain computer interface applications.
Additionally, it is shown that a potentially richer interpretation of the AR parameters can be obtained through their relation with the poles of the model. It is argued that these poles provide explicit dominant frequency information which may be useful to describe, represent and identify ERD/ERS phenomena taking place over time. Such dominant frequencies can be individually tracked, showing that poles in the alpha band represent ERD phenomena which are related to hand movement.
The final contribution focuses on the selection of the AR or ARMA model order; an important aspect of parametric techniques. Several approaches have been proposed for the estimation of model order from the data, each having its own advantages and disadvantages. In order to obtain a more robust model order estimation, this chapter reviews an improved criterion developed by the authors in (Cassar *et al.*, 2010a) which leads to a lower probability of error for model order estimation in the case of multivariate systems. The performance of this criterion is tested by extensive Monte Carlo analysis and it is also used to fit an ARMA model to real EEG data.

2. Literature review

Parametric modelling has long been recognized as a versatile tool for the analysis of EEG data (Bohlin, 1973; Isaksson *et al.*, 1981). Nevertheless, this is still an active area of research and several open problems need to be addressed in order to successfully deploy these techniques for practical applications such as EEG driven, brain-computer interface systems. Linear parametric models have been widely used to fit EEG data. The auto-regressive moving average (ARMA) model structure, or variations based upon this structure, are commonly employed. The tutorial paper by (Pardey *et al.*, 1996) provides good explanations on the use of parametric modelling techniques for time series analysis, with emphasis on EEG data and autoregressive models. The concept of signal stationarity is addressed and the use of adaptive and non-adaptive models is discussed, together with the issues of model complexity and stability.
EEG data is generally considered to be non-stationary because its statistical characteristics change with time, depending upon the mental states that are active at any given time instant. In order to handle this feature, one approach is to assume that over short time intervals the signal remains stationary. A batch processing algorithm is then applied to estimate the optimal parameters which best fit the measurements taken over each of these short time intervals. This approach leads to non-adaptive models, where AR or ARMA structures are normally used. The Burg algorithm or the Levinson-Durbin algorithm are

used to estimate the parameters of the model. Alternatively the model's parameter estimates are updated recursively at every time instant with the arrival of new data samples, typically by means of the Kalman filter algorithm. This approach leads to so-called adaptive models because the model parameters adapt themselves recursively to the characteristics of the data measurements taken along the course of time (Pardey *et al.*, 1996; Isaksson *et al.*, 1981). The term "time varying auto-regressive" (TVAR) is also used to refer to this technique when AR model structures are applied.

Several papers on different aspects of adaptive modelling for EEG signal analysis and applications have been published. For example, (Shloegl *et al.*, 1997) used the Adaptive Autoregressive Method (AAR) together with a linear discriminant classifier to investigate EEG data from two subjects executing imagined left-hand and right-hand movements. The AAR parameters were estimated by a Recursive Least Squares (or Kalman filter) algorithm. The parameters at a specific classification time instant were applied for linear discriminant analysis and an error rate calculated. By repeating this at different classification time instants, the dependence of error rate upon time was determined. It was concluded that left-hand and right-hand movements could be discriminated on the basis of the estimated AAR parameters instead of the more traditional method based upon the analysis of signal energy in specific frequency bands. In order to analyze non-stationary EEG data, TVAR models were used in (Krystal *et al.*, 1999) where the evolution of the models was represented by latent components having a time-varying frequency leading to a representation made up of: damped sinusoid components with amplitude, phase and frequency characteristics that vary temporally; and high-frequency sinusoid components. In this paper, these latent components were plotted and tracked to provide insights into the EEG variation in time. The work of (Li, 2007) investigated how a time varying AR model can be used to estimate the complexity and synchronization of an EEG signal in order to identify epileptic seizures. The onset of a seizure is detected by a change in complexity of the AR model, where complexity is measured by the model order that is required to adequately represent the EEG signal. In (Tarvainen *et al.*, 2001) the Kalman smoother is adopted instead of the more common LMS or RLS algorithms for adaptive ARMA modelling of non-stationary EEG. An ARMA(6,2) model was used to track alpha band characteristics of a subject having eyes opening and closing. The study suggests that the Kalman smoother method gives more reliable tracking compared to the other algorithms. The problem of model order estimation was also pointed out in this paper, indicating methods that could be used to handle this problem. However an order of (6,2) was arbitrarily selected and deemed to be suitable for all the experiments in this work.

Three different model order estimation techniques for fitting an AR model to EEG data were studied by (Palaniappan *et al.*, 2000). The model order selection criteria considered in this work are the Final Prediction Error, Akaike's Information Criterion and Reflection Coefficient. The parameters of the resulting AR models were used to generate the Power Spectral Density which was applied as an input to a neural network classifier. Out of the three criteria, the Reflection Coefficient criterion resulted in models which gave the best classification performance and lower optimum order.

When a combination of signals from multiple EEG channels is used for analysis, a multivariate (vector) model is often used to fit this data. The advantages of multivariate modelling techniques for biomedical signals are explained in (Rezek, 2006), where an example of multivariate AR (MVAR) modelling for sleep EEG is demonstrated. The work of (Anderson *et al.*, 1998) introduced the use of multivariate AR models for mental state classification. The performance of classification of two mental tasks – relaxed state and mental multiplication –

using features derived from univariate (scalar) AR and 6-channel MVAR models was studied. The results showed that the MVAR features have a slightly better classification performance and better consistency. Adaptive on-line MVAR was explored in (Arnold *et al.*, 1998), adapting the use of the Kalman filter from the univariate case to the multivariate model. A trivariate AR model of order 22 was used, from which spectral parameters were extracted yielding relevant information regarding neural communication processes. A multivariate adaptive autoregressive model is proposed in (Ding *et al.*, 2000) for the analysis of non-stationary, multichannel event-related potentials originating from the cerebral cortex. The model parameters, captured over successive time windows, are used to derive spectral quantities by means of which the cortical dynamics can be illustrated. In (Pei & Zheng, 2004) a multivariate AR model is fitted to EEG signals from two channels recorded from a subject performing left-hand and right-hand movements. They show that the parameters of the multivariate AR model can be used to form a feature vector for discriminant analysis based upon the Mahalanobis distance, and that the performance of the multivariate approach surpasses that of a univariate AR model. In (Schloegl & Supp, 2006), multivariate AR models are applied to event-related EEG data for the analysis of multichannel spectral properties of EEG.

Some research studies have investigated whether nonlinear models are more suitable than linear models for fitting EEG data. In (Popivanov *et al.*, 1998), linear (AR) and a non-linear analysis (point-wise dimension, Kolmogorov entropy, largest Lyapunov exponent and non-linear prediction) of EEG data arising from voluntary movement were compared. The study concluded that linear and non-linear components in the EEG of voluntary movement co-exist and that both the linear and the non-linear methods detected EEG transitions prior to the movement. The results of the study also indicate that the two classes of methods do not have temporally coincident measures and were thus supposed to detect different dynamics of the EEG signal. The paper by (Inoue *et al.*, 2004) addresses the issue of nonlinearity by proposing the use of a Quasi-AR model for EEG data during motor tasks. This model has a linear structure, similar to AR, but nonlinear parameters and hence it could capture the nonlinear dynamics of the EEG signal. The model parameters were estimated by a recursive prediction error method. The features obtained from the spectrum of the Quasi-AR model were used for discrimination between left and right-hand movement tasks. This approach showed superior performance when compared with conventional AR models. Two linear and two nonlinear models were compared by (Jain & Deshpande, 2004). Their analysis shows that the Bilinear model structure gives better results for EEG than AR, ARMA and Polynomial AR models. Unfortunately it is also the most computationally demanding of all four. The Bilinear AR model has also been studied by (Poulos *et al.*, 2010) in the context of person identification from EEG. The results of this study also show that the Bilinear model gives superior results than an AR model, indicating the presence of information bearing nonlinearities in the EEG signal, and the capability of the Bilinear model to extract these nonlinear components.

(Atry *et al.*, 2005) address the problem of noise. They propose an EEG signal purification technique on specific channels using the fitted parametric models prior to classification, so as to mitigate the effects of noise and improve classification results by up to 15%. They consider univariate AR, multivariate AR and Box-Jenkins models. (Kelly *et al.*, 2002) investigate the use of an autoregressive model with exogenous input (ARX) to model event-related potentials from the EEG of a subject executing left-hand and right-hand tasks. Classification results derived from the use of features from this model and features from other approaches are compared. It is concluded that the ARX model leads to the best results in the feature extraction stage. This analysis and results are developed further in (Burek *et*

al., 2005). AR modelling has also been applied to the inverse problem of EEG source localisation or current distribution estimation. In (Galka *et al.*, 2004) a new interpretation of the dynamic inverse problem for estimating local current vectors in the brain was proposed by breaking away from the classical constrained least squares formulation and reformulating the problem as a spatiotemporal AR model.
Several other relevant publications can be found in the literature. However, due to space limitations, the rest of the chapter will review in some detail the authors' own contributions to EEG data analysis covering parameter estimation and its effect on classification results, the relation between model poles and EEG characteristics, and the problem of model order estimation. However, due to space limitations, the rest of the chapter will review in some detail the authors' own contributions to EEG data analysis (Cassar *et al.*, 2010a; Cassar *et al.*, 2010b). These concern the parameter estimation problem and its effect on classification results, the relation between model poles and EEG characteristics, and the problem of model order estimation.

3. Classification of imagined hand movements and their effect on AR pole variations

One popular technique for brain-computer interfacing involves the acquisition of EEG signals from a person who interacts with a computer by imagining the movement of his/her left-hand or right-hand. This method opens up the possibility of a communication channel between a person who is subject to serious motor impairments and a computer. The execution of such imagined actions gives rise to a pattern of specific variations in particular frequency bands of the person`s EEG (Penny *et al.*, 1998). Just prior to the imagined event, a decrease in power is typically detected in the alpha band (8-12Hz) of EEG signals captured from the controlateral side of the imagined hand movement. This is called Event Related Desynchronization (ERD) (Pfurtscheller & Lopes da Silva, 1999). When the imagined movement is stopped, an increase in power is typically exhibited in the beta (13-30Hz) band of the EEG signals captured from the ipsilateral side of the relevant hand movement. This is called Event Related Synchronization (ERS). Therefore, if ERD and ERS phenomena are identified from EEG signals, there exists a potential for the classification of the imagined action *i.e.* whether the person imagined a left-hand or a right-hand movement.
Towards this end, one approach is to model the EEG signals by means of an auto-regressive (AR) parametric model (Pardey *et al.*, 1996). The parameters of the model are typically estimated from the EEG signals by using the Yule-Walker equations or a Kalman filter/smoother. These parameters, which are sometimes assumed to be constant and sometimes time-varying, can then be used as a feature vector to classify the underlying event related potential (Huan & Palaniappan, 2005; Anderson *et al.*, 1998). This process is typically cast as a 2-mode classifier, one mode for each of the two possible actions, namely imagined left or right-hand movement. This chapter proposes to identify also the background periods, where no imagined actions are taking place, as a third class that is distinct from the two classes representing left-hand or right-hand movement. It will be shown that this approach, together with a Kalman Smoother algorithm for parameter estimation that is initialized by the Expectation Maximization algorithm, will lead to improved classification results.
It is assumed that the EEG signal is modelled by a linear AR model of order p, characterized by the following difference equation in the discrete-time domain:

$$y_t = -\sum_{k=1}^{p} a_t^{(k)} y_{t-k} + \upsilon_t \tag{1}$$

where y_t represents the EEG signal recorded at time t, $a_t^{(k)}$ is the k^{th} time-varying AR model parameter and υ_t is a random Gaussian process of zero mean and covariance R. The AR model is fitted to EEG data measurements by estimation of parameters $a_t^{(k)}$. In this work, the Kalman Smoother (Maybeck, 1979) is used to find the optimal value of the parameters which fit the data in the least square error sense. Equation (1) is first re-written in state space form:

$$x_t = \Phi x_{t-1} + \omega_t \tag{2}$$

$$y_t = H_t x_t + \upsilon_t \tag{3}$$

where x_t is the vector of AR parameters $[a_t^{(1)} a_t^{(2)} ... a_t^{(p)}]^T$ which requires estimation by the Kalman Smoother. H_t is the vector of past EEG measurements $[y_{t-1} y_{t-2} ... y_{t-p}]$ and ω_t is a random Gaussian process of zero mean and covariance Q which allows a random walk of the parameter vector x_t. Although state matrix Φ is sometimes set to be the identity matrix, this could be detrimental to the estimation of the AR parameters. (Khan & Dutt, 2007) show that the accuracy of the Kalman Smoother algorithm is very much dependent on the values assigned to Φ, the noise covariances Q and R, and the initial conditions of the parameter vector and its covariance. They show that if these are estimated by the Expectation-Maximization algorithm, rather than set randomly to some reasonable values, the Kalman Smoother algorithm yields AR parameter estimates which capture better the dynamics and spectra of event related potentials.

The Kalman Smoother algorithm consists of a set of forward recursion Equations (4) to (8), also known as the Kalman filter equations, followed by the set of backward recursion Equations (9) to (11) as shown here:

$$x_t^{t-1} = \Phi x_{t-1}^{t-1} \tag{4}$$

$$P_t^{t-1} = \Phi P_{t-1}^{t-1} \Phi' + Q \tag{5}$$

$$K_t = P_t^{t-1} H_t' \left(H_t P_t^{t-1} H_t' + R \right)^{-1} \tag{6}$$

$$x_t^t = x_t^{t-1} + K_t \left(y_t - H_t x_t^{t-1} \right) \tag{7}$$

$$P_t^t = P_t^{t-1} - K_t H_t P_t^{t-1} \tag{8}$$

$$J_{t-1} = P_{t-1}^{t-1} \Phi' \left(P_{t-1}^{t-1} \right)^{-1} \tag{9}$$

$$x_{t-1}^n = x_{t-1}^{t-1} + J_{t-1} \left(x_t^n - \Phi x_{t-1}^{t-1} \right) \tag{10}$$

$$P_{t-1}^{n} = P_{t-1}^{t-1} + J_{t-1}\left(P_{t}^{n} - P_{t}^{t-1}\right)J_{t-1}^{'} \tag{11}$$

where the initial conditions x_0^0 and P_0^0 are estimated together with Φ, Q and R by the Expectation-Maximization algorithm as detailed in (Khan & Dutt, 2007; Cassar *et al.*, 2010b).
The following comparative analysis will utilize the above algorithms to analyze EEG data from a subject performing imagined left and right-hand movements. It is shown that the classification results based upon a 3-mode classifier with the Kalman Smoother initial conditions estimated by Expectation-Maximization, are superior to both the 2-mode classifier and to a Kalman Smoother that is initialized with arbitrary values.
The EEG data utilized for this analysis was recorded by Dr. Allen Osman from the University of Pennsylvania which was made available for the Neural Information Processing Systems (NIPS 2001) Brain-Computer Interface Workshop (Sajda *et al.*, 2003). This data consists of 59 channels of EEG signals sampled at 100Hz. The experimental protocol starts off with a blank screen for 2 seconds. This is followed by a fixation point for 500ms indicating the start of the trial. A letter 'E' or 'I', is then displayed for 250ms to indicate whether the subject is requested to perform an explicit or an imaginary movement. The fixation cross is shown again for 1s and in the next 250ms the subject is told whether to act with the left-hand, right-hand, both hands or not at all through the letters 'L', 'R', 'B', 'N' displayed on the screen. After another fixation cross presented for 1s, an 'X' appears for 50ms acting as the synchronization cue to perform the requested response. The trial ends with the fixation cross being displayed for the next 950ms. This analysis will utilize the EEG from channels C3 and C4 (10/20 electrode placement system) arising from the imagined left or right-hand movements of Subject 2, for which 90 left and 90 right-hand trials are available. In order to enhance spatial activity of the EEG, the Hjorth derivation is applied to the signals (Pfurtscheller & Lopes da Silva, 1999).
The Kalman Smoother algorithm is applied on this data so as to estimate the parameters of a 7th order AR model. For the purpose of comparative analysis, the initial conditions Φ, Q, R, x_0^0 and P_0^0 are set in two different ways:

a. an arbitrary initialization, with Φ set to be an identity matrix I, Q = 0.001xI, R = 1, x_0^0 = 0 and P_0^0 = 10xI,
b. using the Expectation-Maximization (EM) code provided by (Khan, 2007). This estimation starts off by using the Kalman Smoother. The EM algorithm is then used to estimate the initial conditions. Rather than working on each individual trial, the EM initialized parameters are found from all trials in the training set and an average is calculated. This allows for smoother estimates of the AR parameters. Once the EM initialized system parameters are obtained, they are fed to the Kalman Smoother to estimate the AR parameters of each trial in the data set.

3.1 A three-mode classification approach

The novel approach presented here is to consider that the process is characterized by three possible modes:

a. Background Mode; effective during the time period prior to 3.75s where the subject is told the type of movement to perform. At 3.75s, the subject is aware of the required task and hence the EEG characteristics are expected to change.

b. Left Movement Mode; effective from 3.75s up to 6s and characterized by ERD activity on the controlateral channel (C4) and no ERD activity on the ipsilateral channel (C3).
c. Right Movement Mode; effective from 3.75s up to 6s and characterized by ERD activity on the controlateral channel (C3) and no ERD activity on the ipsilateral channel (C4).

AR parameters are estimated for the whole 6s time period of each signal from channels C3 and C4, for both cases of left-hand and right-hand movement. For the case of arbitrary setting of initial conditions, these are kept fixed throughout the whole length of data irrespective of whether the EEG signal is in a background or a movement state. For classification purposes, a feature vector is generated by concatenating the AR parameters estimated from signals recorded at C3 and C4 respectively, leading to a 14-element feature vector. To train the classifier, the parameters at each time instant up to 3.75s are considered as Background Mode, whereas the parameters at C3 and C4 for each time instant between 3.75s and 6s for a left task are considered for the Left Movement Mode. Similarly, the parameters at C3 and C4 for each time instant between 3.75s and 6s for a right task are considered for the Right Movement Mode.

For the EM initialization approach, different initial conditions are estimated for each of the 3 modes of operation. Taking the EEG signal on channel C4 during a left task as an example, two sets of AR parameters are estimated; one obtained by using the initial conditions from the Background Mode and another obtained by using the initial conditions from the Left Movement Mode. Similarly for channel C3 during a right task, one set of AR parameters is estimated assuming Background Mode initial conditions and another set assuming Right Movement Mode initial conditions. These AR parameters are concatenated, leading to a 28-element feature vector in this case. The time periods used to train this classifier are the same as those for the arbitrary initialization.

The available 90-trial data was split into two: the first 45 trials were used for training and the rest for testing, for both imagined left and right-hand movements respectively. For comparative purposes, the classification results using both arbitrary initialization as well as initial conditions estimated by the EM algorithm will be shown next. For Background Mode classification results, the number of trials at each time instant between 0 and 3.75s which were classified as correct have been considered. For Left Movement and Right Movement Mode classification, the total number of correctly classified movement tasks at each time instant between 3.75s and 6s were considered. Figure 1 shows the percentage of correctly classified results for the case of arbitrary initialization. It is clear that for the Background Mode there occur frequent misclassifications as either a left or a right–hand movement. Classifications results improve to just below 80% at around 5s when the imagined movement is actually performed.

A significant improvement is obtained when the initial conditions are estimated with the Expectation-Maximization algorithm, as shown in the classification results of Figure 2. There is a significant improvement in the classification of the Background Mode because the estimated parameters are very smooth and differ from those during the movement periods. Classification is close to 100% during the first 2s but it reduces as the movement period approaches. At around 5s, when the movement is performed, the classification score approaches 90%, as opposed to 80% for the arbitrary initialization case.

Additional insight can be obtained by the construction of a confusion matrix. Table 1 and Table 2 show the confusion matrices for the two different initialization approaches. The arbitrarily initialized case leads to around 64% and 65% correct classification for left and right movement respectively and it performs very poorly when classifying the background

period. Many of these time instances are classified as an imagined right-hand movement instead. The EM initialized case gives superior performance with background periods being identified with a score of 89%, a significant improvement on the 39% score of the previous case. There is also improvement in the classification of right and left-hand movement, where scores of 68% and 80% respectively, are obtained. As in the previous case, a number of left modes (around 22%) are incorrectly classified as right modes. These classification results compare well with the 76% overall classification obtained by Dornhege *et. al.* in the NIPS 2001 workshop, where a 2-mode classifier was used (Sajda *et al.*, 2003).

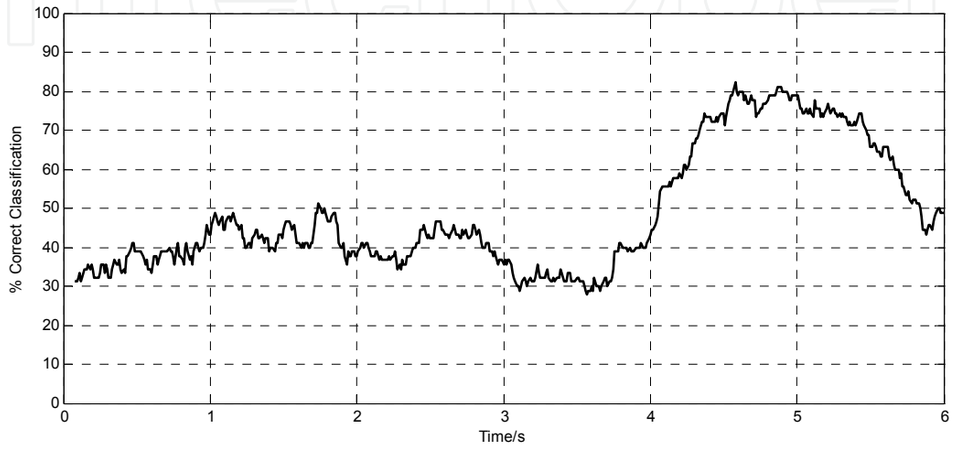


Fig. 1. Percentage correct classification for arbitrary initialization

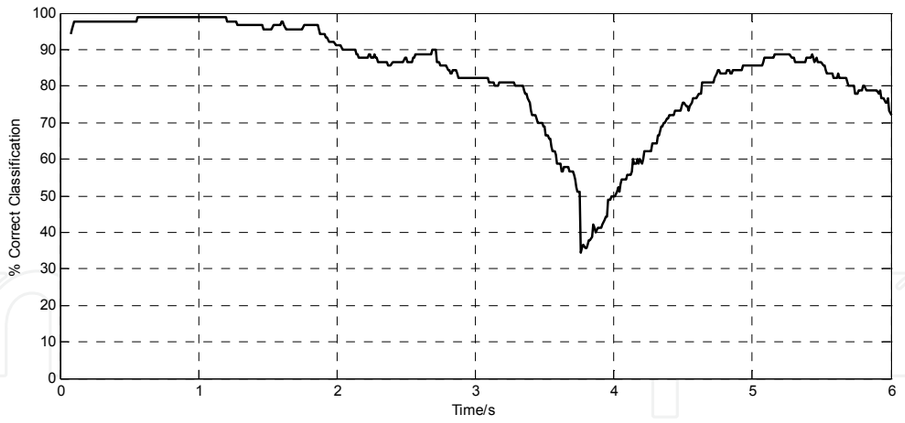


Fig. 2. Percentage correct classification for EM-based initialization

		Actual Mode		
		Background	Left	Right
Predicted Mode	Background	38.68%	14.52%	22.35%
	Left	24.11%	63.87%	12.76%
	Right	37.20%	21.51%	64.89%

Table 1. Confusion matrix for arbitrary initialization

		Actual Mode		
		Background	Left	Right
Predicted Mode	Background	89.32%	9.59%	8.96%
	Left	3.50%	68.12%	11.50%
	Right	7.19%	22.29%	79.89%

Table 2. Confusion matrix for EM-based initialization

3.2 Interpretation in terms of AR pole variation

Another contribution of this work centres on the analysis of the variation of the AR parameter estimates with time, together with the corresponding poles of the model, during the 6s time interval of the trial. The transfer function of the AR model in the z-domain is:

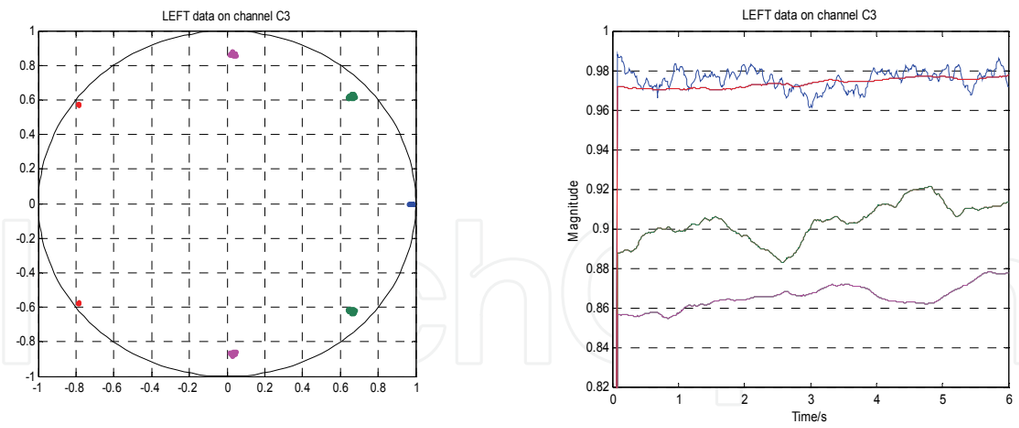
$$G(z) = \frac{1}{1 + \sum_{k=1}^{p} a_t^{(k)} z^{-k}} \tag{12}$$

The poles of the model are given by the roots of the denominator of Equation (12). These poles can therefore be calculated at every time instant from the AR parameter estimates $a_t^{(k)}$ and marked on a pole-zero plot whose x-axis represents the real part of the pole and the y-axis represents the imaginary part. The frequency associated with a given pole is proportional to the angle it subtends with the positive x-axis on the pole-zero plot.

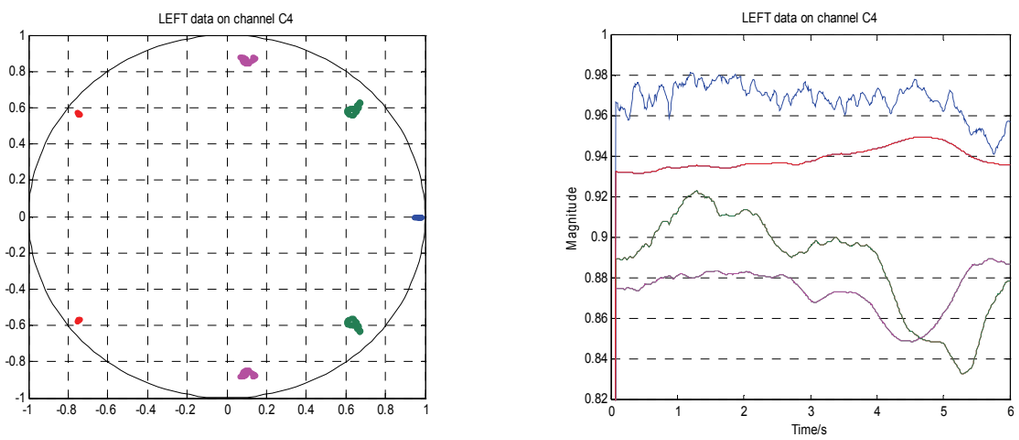
The resulting poles obtained at each time instant for imagined left-hand trials are plotted in Figure 3 and 4, which show both the pole-zero plot and the variation of the magnitude of the poles with time. Note that the latter displays four plots instead of seven because 3 pairs of poles are complex conjugates and their magnitudes are equal. These poles are computed from the average of the AR parameters over the 90 available trails.

Figure 3(a) shows the poles obtained from the model fitted to data from channel C3 when the Kalman Smoother is initialized arbitrarily. Figure 3(b) follows the same pattern, but for channel C4. Figures 4(a) and 4(b) show the corresponding plots for the case of EM-based initialization.

An analysis of the pole variations gives some interesting insights on the EEG data which are otherwise not directly apparent from the AR parameter estimates. In all cases, the angle of the obtained poles indicates that there is concentrated activity at around 12Hz (alpha band), 24Hz (beta band) and 40Hz (gamma band). Activity in the alpha and beta bands is synonymous with this type of data (Pfurtscheller & Lopes da Silva, 1999). All plots also show that on the ipsilateral side (C3 for the case of left-hand trials) the poles do not vary much with time and that their magnitude remains fairly constant. This contrasts with the activity on the contralateral side (C4 for the case of left-hand trials) where, during the imagined hand movement, a significant decrease in the magnitude of the alpha and beta poles occurs. This indicates that on the contralateral side there is a significant change in pole magnitude between background and movement periods, which corresponds to ERD. This characteristic is correspondingly reflected in the trajectories of the poles shown in the pole-zero plots, which are more spread out on the contalateral side (C4) than the pole trajectories on the ipsilateral (C3) side.



(a)



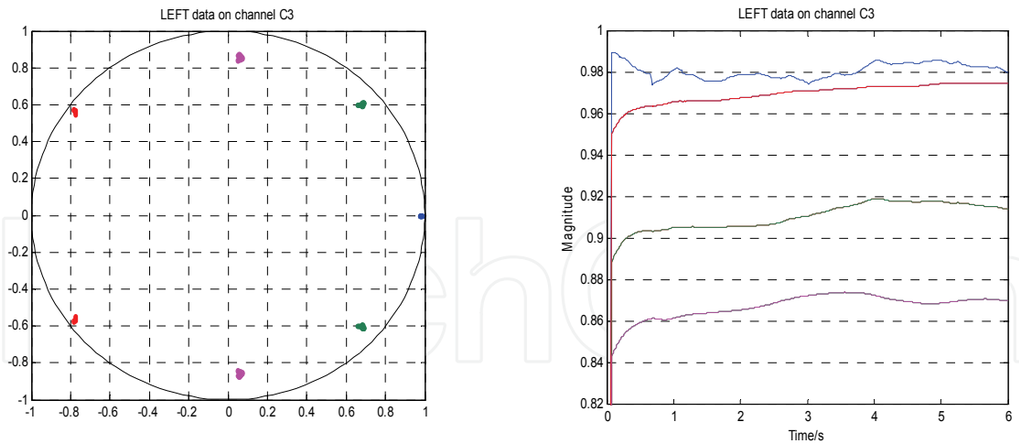
(b)

Fig. 3. Pole trajectories and magnitudes for imagined left-hand movement arising from arbitrary initialization. (a) corresponds to C3 and (b) corresponds to C4

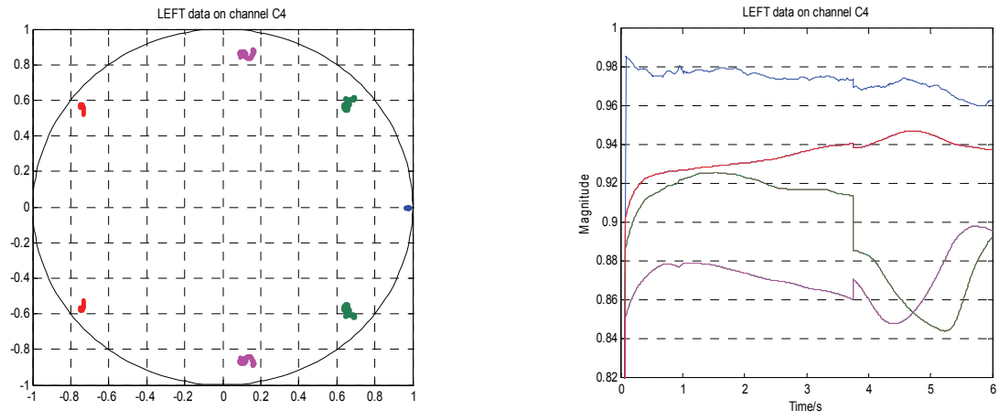
4. Model order estimation

In system identification, the effectiveness of the model that is used to fit a data set depends critically on the determination of a suitable model order. There exist various techniques for the estimation of model order. These can be categorized into three main groups. One group is based on the prior estimation of model parameters using a set of candidate models of different order. Information criteria such as the Akaike Information Criterion or Minimum Description Length are then used to find the best compromise between model complexity and best fit within the candidate models of this set. These methods incur computational complexity due to the consideration of multiple candidate models.

Another group of methods, which do not require prior estimation of model parameters, use eigendecomposition of the input/output data covariance matrix to estimate the model order. This approach, based on the Minimum Description Length criterion, was applied to univariate ARMA and ARX models by Liang et al. (1993). The method was shown to be able



(a)



(b)

Fig. 4. Pole trajectories and magnitudes for imagined left-hand movement arising from EM-based initialization. (a) corresponds to C3 and (b) corresponds to C4

to estimate the correct model order even in the presence of limited noise conditions. These methods are more computationally efficient than the previous group. The third group of methods estimate the model order and model parameters simultaneously. They utilize a Bayesian approach and are normally very demanding computationally.

The work presented in this section, which is based on the second group of methods, proposes a modified criterion to that of (Liang *et al.*, 1993) which leads to a lower probability of error for model order estimation. As in (Lardies & Larbi, 2001), which extended Liang's method to multivariate AR models, this work will not be restricted to univariate models. However in (Lardies & Larbi, 2001), the effects of different model order, model parameters and data lengths are not investigated. In this work, an extensive Monte Carlo analysis will be applied in order to evaluate such effects. Finally the approach is applied to real EEG data recorded from a subject performing motor imagery tasks.

A multivariate ARMA model is given by the difference equation

$$y_t = -A_1 y_{t-1} - \ldots - A_p y_{t-p} + B_1 e_{t-1} + \ldots + B_q e_{t-q} + e_t + \upsilon_t \tag{13}$$

where $y_t \in R^n$ represents the output data on n channels measured at time t, A_i and $B_i \in R^{n\times n}$, represent the AR and MA parameter matrices respectively, p and q represent the AR and MA orders, $e_t \in R^n$ represents the input to the model which is a white Gaussian noise process having zero mean and covariance C, and $\upsilon_t \in R^n$, which represents observation or modeling error, is also a random white noise process having covariance Q_υ.
Extending the approach of (Liang *et al.*, 1993) to the multivariate case, assuming that N time samples of the output data are available, Equation (13) can be written as follows:

$$D_{p,q}\theta_{p,q} = \upsilon \tag{14}$$

where $D_{p,q}, \theta_{p,q}, \upsilon$ are defined as follows:

$$D_{p,q} = \begin{pmatrix} y_1^T & 0 & \dots & 0 & e_1^T & 0 & \cdots & 0 \\ \vdots & y_1^T & \cdots & 0 & \vdots & e_1^T & & 0 \\ \vdots & \vdots & \cdots & & \vdots & \vdots & \cdots & \\ y_N^T & y_{N-1}^T & \cdots & y_{N-p}^T & e_N^T & e_{N-1}^T & \cdots & e_{N-q}^T \end{pmatrix} \tag{15}$$

$$\theta_{p,q} = \begin{pmatrix} A_0^T & \cdots & A_p^T & -B_0^T & \cdots & -B_q^T \end{pmatrix}^T \tag{16}$$

$$\upsilon = \begin{pmatrix} \upsilon_1^T & \cdots & \upsilon_N^T \end{pmatrix}^T \tag{17}$$

with A_0 and B_0 assumed to be identity matrices.
In order to compose the input/output matrix $D_{p,q}$, the unknown input signal e_t needs to be estimated. This could be done by preliminarily fitting a high order AR model to the output data y_t using a least squares approach to estimate the corresponding AR parameters. As explained in (Cassar *et al.*, 2010a), these AR parameters are next used to generate an estimate for e_t which, together with the output data, is used to compose $D_{p,q}$ as defined in Equation (15). This matrix is then used to generate the covariance data matrix $R_{p,q}$ defined as:

$$R_{p,q} = D_{p,q}^T D_{p,q} \tag{18}$$

The relevance of this matrix becomes evident when the Minimum Description Length criterion is used for the estimation of model order. This criterion strikes a compromise between model complexity and the maximum likelihood estimator of the parameters, by minimizing the following cost function:

$$J_{MDL}(p,q) = -\log f(\upsilon_1,\cdots,\upsilon_N) + 0.5n^2(p+q+1)\log N \tag{19}$$

where $f(\upsilon_1,\cdots,\upsilon_N)$ is the probability density function of noise υ. For a multivariate Gaussian model, this density function is given by the normal distribution equation:

$$f(\upsilon_1,\cdots,\upsilon_N) = \frac{1}{(2\pi)^{N/2}\left|Q_\upsilon\right|^{N/2}} \exp\left(-\frac{1}{2\left|Q_\upsilon\right|}\theta^T R_{p,q}\theta\right) \tag{20}$$

which, after substitution in (20), yields

$$J_{MDL}(p,q,\theta) = \frac{N}{2}\log 2\pi + \frac{N}{2}\log|Q_\upsilon| + \frac{1}{2Q_\upsilon}\theta^T R_{p,q}\theta + \frac{1}{2}n^2(p+q+1)\log N \tag{21}$$

Given a fixed model order (*p*,*q*), the covariance matrix Q_υ which minimizes Equation (21) is $Q_\upsilon = \theta^T R_{p,q}\theta$. As shown by (Lardis & Larbi, 2001), the minimum value of the determinant of Q_υ is obtained from an eigendecomposition of the covariance data matrix $R_{p,q}$ as follows:

$$R_{p,q} = \begin{bmatrix} V_L & V_S \end{bmatrix} \begin{bmatrix} \lambda_L & 0 \\ 0 & \lambda_S \end{bmatrix} \begin{bmatrix} V_L^T \\ V_S^T \end{bmatrix} \tag{22}$$

where λ_L is a diagonal matrix whose terms consist of the first $(p+q)n$ largest eigenvalues of $R_{p,q}$, the columns of V_L contain the corresponding eigenvectors, λ_S is a diagonal matrix whose terms are the $2n$ smallest eigenvalues and the columns of V_S contain the corresponding eigenvectors of $R_{p,q}$.

If an orthogonality constraint is imposed on θ, then the value of θ which minimizes (21) is the matrix of eigenvectors V_S which correspond to the smallest eigenvalues of $R_{p,q}$ (Lardis & Larbi, 2001). Due to the orthonormality of the eigenvectors, it turns out that $Q_\upsilon = \lambda_s$ and $|Q_\upsilon| = \prod_{i=1}^{n} \lambda_S(i)$. Following substitution in (21) and some manipulation, the following result is obtained

$$\frac{2}{N} J_{MDL}(p,q) = \log\left(\prod_{i=1}^{n} \lambda_S(i) \right) N^{n^2(p+q)/N} \tag{23}$$

An analysis of matrix J_{MDL} shows that the eigenvalues of $R_{p,q}$ are large for model orders less than the true order. By contrast, they decrease significantly for orders which are greater than the true order (Liang *et al.*, 1993). Hence if $J_{MDL}(p,q)$ is organized such that the AR order *p* increases along the columns and the MA order *q* increases along the rows, as shown in Figure 5, it will be possible to identify a 'corner` which corresponds to the true order of the multivariate model.

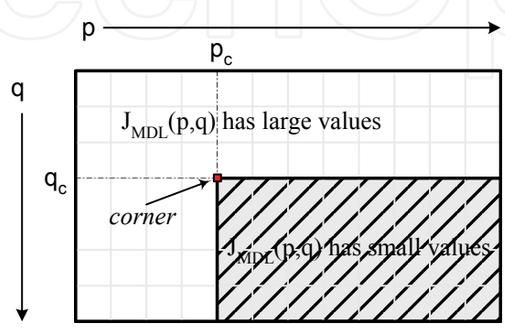


Fig. 5. The values of J_{MDL} for a range of *p* and *q* values. The corner gives the model order

For the univariate case, (Liang *et al.*, 1993) generate a column ratio (CR) and a row ratio (RR) table to identify the transition between the two regions shown in Figure 5. The CR entry at (p,q) is constructed by dividing $J_{MDL}(p-1,q)$ by $J_{MDL}(p,q)$. Similarly, the RR table is determined by dividing $J_{MDL}(p,q-1)$ by $J_{MDL}(p,q)$. The optimal model order is then chosen by taking the value of the p column which corresponds to the largest value in the CR table, and the value of q row which corresponds to the largest value in the RR table.

In the multivariate ARMA case, the situation becomes complicated because additional spurious peaks tend to appear in the CR and RR tables, thereby making it difficult to select the peak which actually corresponds to the corner that identifies the true model order. This phenomenon often gives rise to incorrect estimation of the model order. In order to address this problem, a modified criterion is proposed to enhance the value of the corner, as described next.

The CR and RR tables are constructed as described previously, but then an element-by-element product of these two tables is performed so as to generate a product matrix P_M. This has the tendency of enhancing the value of the true peak in matrix P_M, because it is reinforced by peaks appearing in the same location within both the CR and RR tables. On the other hand, the contribution of incorrect peaks is diminished because their location will not be consistent between the two tables. Nevertheless, experimental analysis has shown that there may be situations where the product matrix P_M exhibits a number of comparable peaks in close proximity, with the maximum peak not necessarily corresponding to the correct corner. Given that a significant change in value is expected between the point corresponding to the correct corner and its neighbours, the modified criterion also checks whether neighbouring locations exhibit a value that is relatively much smaller than the peak being analyzed. A relative difference corresponding to a factor of 5 was found to be suitable from Monte Carlo analysis. Hence the modified criterion can be described by the following algorithm:

1. Form the CR and RR tables by computing $J_{MDL}(p-1,q)/J_{MDL}(p,q)$ and $J_{MDL}(p,q-1)/J_{MDL}(p,q)$ respectively.
2. Perform an element by element product of CR and RR to generate the product matrix P_M.
3. Find the largest value in P_M and check whether its neighbours, corresponding to a one step increase in p and q satisfy the following conditions:
   a. $P_M(p,q) \geq 5P_M(p,q+1)$
   b. $P_M(p,q) \geq 5P_M(p+1,q)$
4. If the conditions in step 3 are satisfied, (p, q) are taken to be the model order. If the conditions in step 3 are not satisfied, recursively go through the other values of P_M in descending order until a (p, q) pair that satisfies both conditions (a) and (b) is found. This is chosen as the optimal model order.
5. If none of the values in P_M satisfy the above, recursively reduce the factor of 5 appearing on the right-hand side of the equations in conditions (a) and (b) above, until the correct corner, and hence optimal model order, is identified.

4.1 Performance analysis

Probability arguments were applied in (Cassar *et al.*, 2010a) to show that the modified criterion proposed in the previous section has less chance of model order estimation error

than the criterion proposed by (Liang *et al.*, 1993). Cassar *et al.* showed that if Liang's criterion estimates q correctly and the proposed criterion does not, then Liang's criterion would always estimate p incorrectly except for one specific exceptional condition. Similarly, if Liang's criterion estimates p correctly and the proposed criterion does not, then Liang's criterion would always estimate q incorrectly except for a second, very specific exceptional condition. Extensive Monte Carlo trials demonstrated that the probability of occurrence of the above-mentioned two exceptional conditions is close to zero *i.e.* there were no cases during the Monte Carlo trials where the proposed criterion gave an error while Liang's criterion did not. This led to the conclusion that the probability of error of the proposed criterion is bounded above by the probability of error of Liang's criterion (Cassar *et al.*, 2010a).

In order to evaluate the proposed order estimation technique, Monte Carlo analysis was performed. One hundred different two-channel, minimum phase ARMA models with $p = 2$ and $q = 2$ were generated. The magnitude of the poles and zeros of these models was randomly chosen within the range of 0.6 to 0.99, and their phase between 1 degree and 180 degrees. Each of these models was subjected to different random realizations of e_t, υ_t. The Monte Carlo trials were conducted with different signal-to-noise ratios (SNR). 2000 samples were considered for each model. The order of the preliminary AR model used to estimate the input signal e_t was set to 10. The proposed modified criterion and Liang's criterion were applied and, in both cases, the percentage amount of correctly estimated model orders was noted.

The results are shown in Table 3. Note the improvement in performance obtained by using the proposed criterion, where the percentage of correct hits always exceeds Liang's criterion, even with low SNR. For every model, the mean square error between the true and estimated model order was calculated across the 2000 samples. The mean and standard deviation of these errors across the 100 different models were then calculated and are shown in the last two columns of Table 3. Once again, note the improved performance of the modified criterion with order estimation error being consistently lower on average, than that of Liang's criterion. The difference between the errors obtained from the two criteria was tested for statistical significance by means of a *t*-Test. With a significance level of 0.05, the results from the no noise case up to the case with SNR of 10dB, indicate that the difference between the errors of the two criteria is indeed statistically significant.

SNR/dB	**Percentage of Correct Hits**		**Mean Error ± Std. Dev.**	
	Liang's Criterion	Modified Criterion	Liang's Criterion	Modified Criterion
no noise	32	97	0.50± 0.35	0.03± 0.17
25	44	94	0.43± 0.39	0.06± 0.24
20	39	90	0.49± 0.41	0.09± 0.29
15	38	83	0.55± 0.45	0.16± 0.35
10	28	59	0.69± 0.44	0.39± 0.48
5	15	29	0.83± 0.36	0.81± 0.76

Table 3. Results of Monte Carlo analysis utilizing 100 models

4.2 Results with EEG data

The proposed technique was applied on a real EEG measurements obtained from data set 3a of the BCI Competition III where a subject performed a cued motor imagery task while EEG

was recorded from 60 channels at a sampling frequency of 250Hz. The data was bandpass filtered between 1 and 50Hz by a notch filter. The subject, facing a computer screen, was asked to perform one of four possible imaginary movements: left-hand, right-hand, foot or tongue. For this analysis, only the trials for left-hand and right-hand movement were considered. 36 left-hand and 36 right-hand artifact-free trials were averaged over the 4s period during which movement is performed, utilizing data from channels C3 and C4.

A multivariate ARMA model was set up to capture the dynamics of this data. The proposed modified criterion was then applied, which estimated a model order of $p = 2$ and $q = 1$. The product matrix P_M which gave rise to this model order is shown in Figure 6, exhibiting a distinctive peak at the location corresponding to the recommended model order. For comparison purposes, the classical Akaike Information Criterion was also applied to estimate the model order for this data. The Akaike criterion estimated the same model order as the proposed criterion ($p = 2$ and $q = 1$). However, the computational efficiency of the proposed criterion is superior to that of the Akaike criterion. The former took 0.6 seconds of processing time to generate the model order result, whereas the latter required approximately 4 seconds (both tests performed on an Intel Core 2 Duo PC having a 2GHz processor and 2 GB RAM).

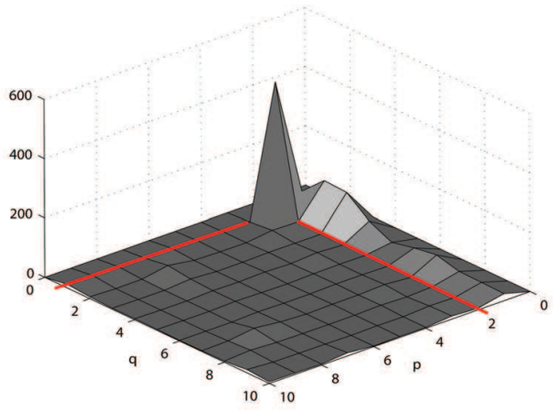


Fig. 6. The product matrix P_M for the EEG data obtained from the modified criterion

Using this recommended model order, the parameters of the ARMA model were then estimated using a Kalman filter initialized by the EM algorithm. This model was used to reconstruct the data within a frequency range of 0 to 45Hz. Auto and cross spectra of the original data and the reconstructed data were then calculated for model evaluation purposes. These are shown in Figure 7, indicating clearly that the recommended model order captures well the dynamics of the EEG data.

5. Conclusion

This chapter has investigated the use of parametric models for the identification of mental tasks from EEG data, with specific emphasis on linear modelling techniques. Following a brief review of relevant papers which appear in the literature, it was shown how improved classification results can be obtained from EEG data recorded during imagined hand movement trials. This improvement is the result of a 3-mode classifier which makes use of AR parameters estimated during periods of left movement, right movement and inactivity, together with a Kalman Smoother initiated by the Expectation-Maximization algorithm.

In addition it was shown how the trajectories of the poles of the AR model relate with ERD phenomena pertaining to imagined hand movements, in a more direct way than the model parameters. Finally a new model order selection criterion was proposed for the multivariate ARMA case, which is more accurate than alternative criteria developed for univariate ARMA. The efficacy of the proposed criterion was exhibited with extensive Monte Carlo analysis under different conditions of signal-to-noise ratio.

The use of EEG data for the identification of mental tasks carries with it several interesting possibilities for future applications. From these possibilities, brain-computer interfacing is one of the leading and most challenging ones. One hopes that the improvements documented in this chapter, together with other techniques which are continuously being developed by the research community, would contribute to the advancement of brain-computer interface technology so as to make it more practical, robust and realistic.

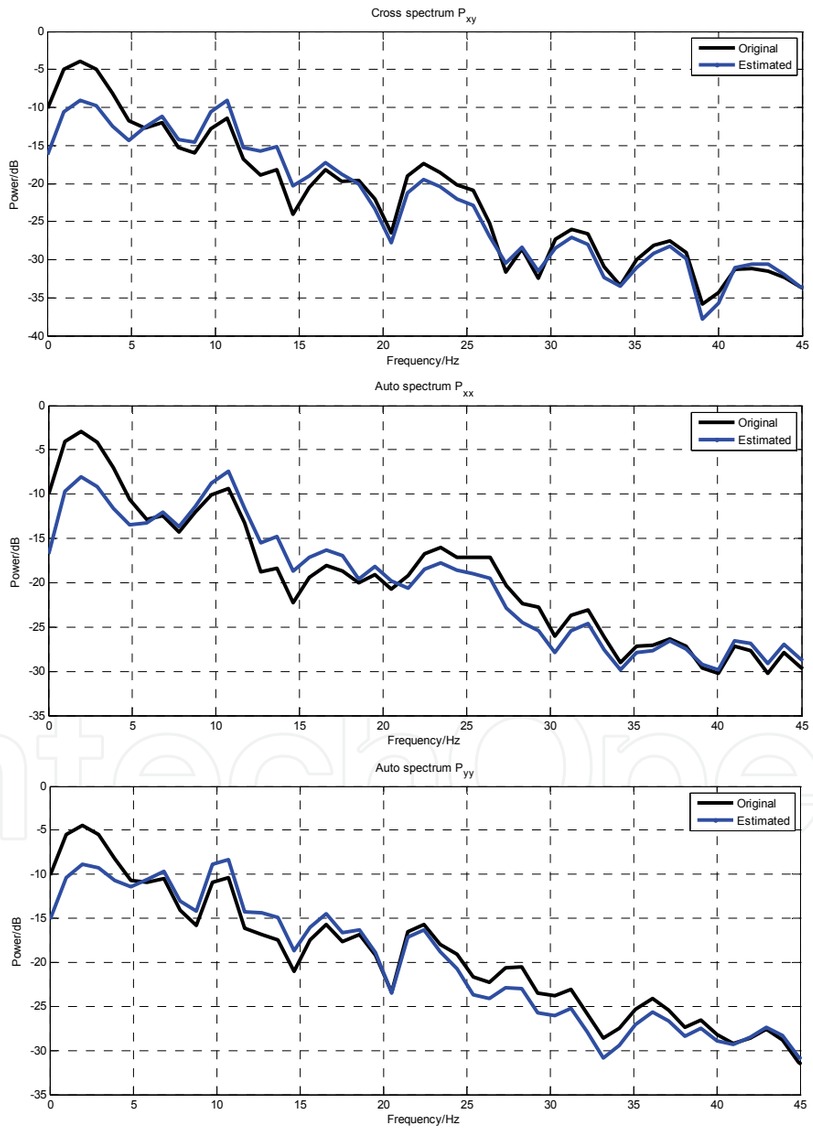


Fig. 7. Cross and auto spectra of the original EEG averaged data and the reconstructed data from the estimated model

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Rapid technological developments in the last century have brought the field of biomedical engineering into a totally new realm. Breakthroughs in materials science, imaging, electronics and, more recently, the information age have improved our understanding of the human body. As a result, the field of biomedical engineering is thriving, with innovations that aim to improve the quality and reduce the cost of medical care. This book is the first in a series of three that will present recent trends in biomedical engineering, with a particular focus on applications in electronics and communications. More specifically: wireless monitoring, sensors, medical imaging and the management of medical information are covered, among other subjects.

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