We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists



186,000

200M



Our authors are among the

TOP 1% most cited scientists





WEB OF SCIENCE

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us? Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected. For more information visit www.intechopen.com



A Robust Nonlinear Control for Differential Mobile Robots and Implementation on Formation Control

Jie Wan and Peter C. Y. Chen Department of Mechanical Engineering, National University of Singapore Singapore

1. Introduction

The past three decades witness a long-term intensive interest in researching a variety of control methods on wheeled mobile robots (WMRs), although the general nonholonomic systems have been investigated for more than one and half a century (Kolmanovsky & McClamroch, 1995). It is well-known that usually WMRs are characterized by non-integrable kinematic constraints, namely the nonholonomic constraints. The consequence is that those constraints rule out the possibility of direct application of standard control theories, such as linear control system theory in this field. Furthermore, as pointed out in a landmark paper (Brockett, 1983), nonholonomic systems can not be stabilized by continuously differentiable, time invariant, state feedback control laws. To cope with the challenges arising in nonhonomic system control, a great number of approaches have been proposed and some selections of the vast amount of published literature are reflected in the survey paper (Kolmanovsky & McClamroch, 1995) and the book (Dixon et al., 2001) and in chapters (7 - 9) of the book (Wit & Siciliano, 1996). Several controls from experiment perspectives are examined and implemented in the work (Luca et al., 2001). Central to the WMR motion control are the tracking control problems. Normally there are two categories of tracking control: posture tracking and point tracking. The former aims to achieving stably tracking a moving reference posture (i.e., position and orientation) while the latter only concerns about position tracking. Nonlinear feedback control strategies (Wit & Sordalen, 1992), (Godhavn & Egeland, 1997),(Kanayama et al., 1990),(Closkey & Murray, 1997),(Teel et al., 1995) are often favored in dealing with tracking control problem to compensate disturbances and uncertainties although open-loop control laws are also workable (Murray & Sastry, 1990),(Lafferiere & Sussmann, 1991),(Brraquand & Latombe, 1989),(Brockett, 1981). In recent years, more and more attention have been drawn to the robustness of the controller in presence of uncertainties. In (Aguiar et al., 2000), the authors address the regulation control of WMR with parametric modelling uncertainty using Lyapunov functions. A robust new Kalman-based active observer controller for path following was proposed (Coelho & Nunes, 2005) in circumstances of uncertainties and disturbances. And the paper (Dixon et al., 1999) presents a controller robust to parametric uncertainty and additive bounded disturbance in the dynamic model through the use of a dynamic oscillator.

An autonomous multi-robot system comprises a group of (often homogeneous) robots, each has a certain degree of mobility and autonomy. Research interests in unmanned autonomous robots have been growing significantly in recent years, due to the potential that this type of robotic systems will be able to perform a variety of tasks in environments inaccessible or too dangerous to humans. One basic problem concerning multi-root systems is formation control, whereby a group of robots maintain a certain (usually 2D) geometry while in concerted motion. When encountering obstacles (either static or dynamic), the group must maneuver to avoid them while maintaining the overall formation geometry whenever possible.

In this chapter, we studied the robustness of a nonlinear feedback control law based on a kinematic model. A generic kind of feedback control laws which cover some commonly used methods and the associated stability are quickly reviewed in a new perspective by invoking Lyapunov stability theorems. A simple fact is then unveiled that the stability proof actually depends on perfect mathematical manipulation which requires some terms of the differential equations to be cancelled. However, in real world, the perfect will be ruined under some circumstances and the robustness issue has to be investigated. Invariance principles, rather than Lyapunov stability theorems, are the major tools in dealing with the imperfect cases in which no term cancellation can be reached. The stability issue and the robustness of this control law as well is analyzed and the results lead to stable zones for each given set of controller gains. It is found that under certain circumstances the closed-looped system may fail in reaching the desired control objectives and performance. Such insights into the stability zone for a given set of controller gains make it possible to improve the controller by choosing proper controller gains. This new robust control can overcome the drawbacks of the previous commonly used counterpart. Guidelines on designing improved control law are also provided to facilitate real implementation. The merits and benefits of this new control are also highlighted through comparisons with its prototype. The analysis shows that except the more robustness, the new control law is entitled to faster response if the controller gains are properly chosen. Matlab simulation results which show the benefit are presented.

Implementation of the proposed new control law on real robots is another major work of this chapter. An implementation on multiple robot formation control is reported including overview of the whole system structure, description of the robots used in the experiments, the vision system which is used for acquisition of the real-time position information of a group of robots moving on a test bed, and the background noise analysis of the vision system and so on. In the experiments, a group of mobile robots are requested to follow their corresponding visual leaders to form certain geometric patterns. A triangle formation with three robots and a square formation with four robots are demonstrated. The velocities/headings of robots during the formation control and other information are summarized in figures. The downside of robot locomotion mechanism, which is based on step motor and its effects on real implementation, such as misstep and dead zone are discussed. Suggestions and further improvements are discussed at the end of this chapter.

This chapter is organized as follows: in Section 2, the stability problem is reviewed in a new perspective. Section 3 deals with formulation the problem of robustness. In Section 4 robustness analysis and its benefits are addressed while the effectiveness of the new proposed robust control law is verified via simulation in Section 5. Implementation and experiments with multiple robots on the application of formation control are addressed in Section 6, followed by conclusions summarized in Section 7.

2. Problem statement

We consider the *point tracking* problem for a wheeled mobile robot which is depicted in Figure 1. In this scenario, a wheeled mobile robot is supposed to track a series of goal points denoted by the symbol q_g on a segment, which is a smooth curve in the world frame.

72



Fig. 1. Illustration of a wheeled mobile robot and its goal point q_g , which may be moving on a segment (smooth curve) in the world frame.

Referring to this figure, intuitively we refer to notations r and ϕ as "distance to target" and "misalignment angle" respectively.

As shown in Figure 2, in order to facilitate modelling kinematics of the wheeled mobile robot in a polar coordinate, specially we assign the origin to be the goal point (on the segment) for the robot to track. In this figure, a differential mobile robot, together with the associated notations, is illustrated in a polar coordinate. The separation between point (x,y) and center of each wheel is represented by C_d , which is a constant parameter for a given model of real robot. The heading of the robot is θ while its translational velocity and angular velocity are denoted by v and ω respectively. Notice that throughout this chapter, both ϕ and θ are defined in the domain $[-\pi, \pi]$.

Referring to Figure 2, motion of a differential mobile robot can be described by

$$\begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \cos(\phi) & 0 \\ \sin(\phi) & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}.$$
(1)

To link this model with the notations in polar coordinate, we can calculate *r* and ϕ as

$$r = \sqrt{x^2 + y^2},$$

$$\phi = \pi + \theta - \varphi,$$

respectively.

A kinematic model of a differential robots in the polar coordinate can be derived as follows:

$$\begin{bmatrix} \dot{\theta} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\cos(\phi) & 0 \\ \frac{1}{r}\sin(\phi) & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}.$$
 (2)



Fig. 2. Representation of a wheeled mobile robot in the polar coordinate frame with the origin O' (i.e., point q_g on Figure 1) being its goal point to track and the associated notations.

Detailed derivation procedures of Equation (2) can be found in the work (Lee et al., 1999) and are omitted for the sake of brevity in this chapter. This model is similar to the ones used in chapter 3 of (Siegwart & Nourbakhsh, 2004). From this model, specifically we have the relationship between \dot{r} , $\dot{\phi}$ and v, ω as

$$\begin{bmatrix} \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -\cos(\phi) & 0 \\ \frac{1}{r}\sin(\phi) & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}.$$
(3)

Now we consider to derive a general form of control laws which can stabilize the robot in the sense of Lyapunov stability theorems. It requires that both *r* and ϕ tend to zero as time $t \to \infty$. One possible way is to choose control laws which lead to diagonalization of the matrix on the right hand side of Equation (3). To this end, we let *v* and ω being

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} g_1(r,\phi) & 0 \\ 0 & g_2(r,\phi) \end{bmatrix} \begin{bmatrix} r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ -g_1(r,\phi)\sin(\phi) \end{bmatrix},$$
(4)

where $g_1(r,\phi)$ and $g_2(r,\phi)$ are certain unexplicit functions to be determined. Then by substituting the above equations into Equation (3), we can rewrite the Equation (3) into

$$\begin{bmatrix} \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -\cos(\phi) & 0 \\ \frac{1}{r}\sin(\phi) & 1 \end{bmatrix} \begin{bmatrix} g_1(r,\phi) & 0 \\ 0 & g_2(r,\phi) \end{bmatrix} \begin{bmatrix} r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ -g_1(r,\phi)\sin(\phi) \end{bmatrix}.$$
 (5)

A family of possible functions $g_1(r,\phi)$ and $g_2(r,\phi)$ can be chosen as follows:

$$g_{1}(r,\phi) = K_{1}r^{n}\phi^{2q}(\cos(\phi))^{2p+1}, g_{2}(r,\phi) = -K_{2}\phi^{2s},$$
(6)

where $n = 0, 1, 2, \dots, p = 0, 1, 2, \dots, q = 0, 1, 2, \dots$ and $s = 0, 1, 2, \dots$.

Accordingly Equation (4) can be rewritten into the following form:

$$v = K_1 r^{n+1} \phi^{2q} (\cos(\phi))^{2p+1},$$

$$\omega = -K_1 r^n \phi^{2q} \sin(\phi) (\cos(\phi))^{2p+1} - K_2 \phi^{2s+1}.$$
(7)

Proposition 1 The family of control laws given in Equation (7) asymptotically stabilizes a differential robot on its goal point.

Proof: The proof for this proposition is pretty straightforward by constructing a Lyapunov function candidate as 1 1 $V = \frac{1}{2}$ (8)

$$\frac{1}{2}r^2 + \frac{1}{2}\phi^2.$$

Simplifying Equation (5), we reach at

$$\begin{bmatrix} \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -g_1(r,\phi)\cos(\phi) \\ \frac{1}{r}g_1(r,\phi) + g_2(r,\phi) \end{bmatrix} \begin{bmatrix} r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ -g_1(r,\phi)\sin(\phi) \end{bmatrix}$$
$$= \begin{bmatrix} -rg_1(r,\phi)\cos(\phi) \\ g_1(r,\phi)\sin(\phi) + \phi g_2(r,\phi) \end{bmatrix} + \begin{bmatrix} 0 \\ -g_1(r,\phi)\sin(\phi) \end{bmatrix}$$
$$= \begin{bmatrix} -rg_1(r,\phi)\cos(\phi) \\ \phi g_2(r,\phi) \end{bmatrix}.$$
(9)

Based on the results from Equation (9) and Equation (6), the first time derivative of V can be calculated readily as

$$\dot{V} = r\dot{r} + \phi\dot{\phi}
= -r^2 g_1(r,\phi) \cos(\phi) + \phi^2 g_2(r,\phi)
= -K_1 \phi^{2q} r^{n+2} (\cos(\phi))^{2p+2} - K_2 \phi^{2s+2} \le 0,$$
(10)

thus completes the proof.

It should be noted that the general control law represented in Equation (7) can theoretically asymptotically stabilize the robot at its goal point. However the term ϕ^{2q} will greatly slow down the system response. Therefore for the sake of practical considerations, q = 0 is preferred. Let us focus on the control with simple structure. Obviously if we let n = p =q = s = 0, then the Equation (7) can be reduced into the simplest form as shown below:

$$v = K_1 r \cos(\phi),$$

$$\omega = -K_1 \sin(\phi) \cos(\phi) - K_2 \phi.$$
(11)

However, it is noted that control in Equation (11) is actually not the "simplest". We can adopt another family of possible functions $g_1(r,\phi)$ and $g_2(r,\phi)$ as: $g_1(r,\phi) = K_1 r^n \phi^{2q}$, and $g_2(r,\phi) = K_1 r^n \phi^{2q}$. $-K_2\phi^{2s}$, where $n = 0, 1, 2, \dots, p = 0, 1, 2, \dots$ and $s = 0, 1, 2, \dots$. Therefore a more simplified control law can be found as follows:

$$v = K_1 r,$$

$$\omega = -K_1 \sin(\phi) - K_2 \phi,$$
(12)

by simply letting n = q = s = 0. The control law presented by Equation (12) is used in (Baillieul, 2005) with preliminary analysis results.

www.intechopen.com

3. Robustness problem

Beneath the control law stated in Equation (11) exists a fundamental challenge, although the stability issue seems to be affirmatively guaranteed by Lyapunov stability theorem. We already noticed that the technique of term cancellation (i.e., $g_1(r,\phi)\sin(\phi)$) is used when simplifying Equation (9). One may be interested in the following question: what if the ideal cancellation fails and the gain K_1 in v and ω does not match with each other? To put it in details, it is to consider an alternative to the control law in Equation (11) as follows:

$$v = K_1 r \cos(\phi),$$

$$\omega = -K_3 \sin(\phi) \cos(\phi) - K_2 \phi,$$
(13)

where K_3 is another independent variable and it may not be equal to K_1 . In other words, it is equivalent to ask: "will the closed-loop system be stable if an alternative control represented in Equation (13) rather than the one in Equation (11) is applied to the system;'.

In real world, there are numerous factors contributing to the such kind of "gain mismatching". Take the digital control for example, truncation error of numerical calculation of triangle functions of ϕ is unavoidable. More than that, in terms of real outputs of physical actuator, this "mismatching gain" phenomena may happen from time to time. To explain it, let v_L , v_R denote the tangent velocities of each wheel about the centers of rotation.

$$v = \frac{v_L + v_R}{2},$$

$$\omega = \frac{v_R - v_L}{2C_d},$$
(14)

where C_d is the displacement from the point (x, y) to each wheel. We can establish the relationship between the vector $[v \ \omega]^T$ and $[v_L \ v_R]^T$ as follows:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ \frac{1}{C_d} & \frac{1}{C_d} \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix},$$
(15)

$$\begin{bmatrix} v_L \\ v_R \end{bmatrix} = \begin{bmatrix} 1 & C_d \\ 1 & -C_d \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}.$$
 (16)

The ideal case of control law Equation (11) is based on the assumption that we can make the equations

$$v_L = K_1 r \cos(\phi) - C_d (K_1 \sin(\phi) \cos(\phi) + K_2 \phi),$$

$$v_R = K_1 r \cos(\phi) + C_d (K_1 \sin(\phi) \cos(\phi) + K_2 \phi),$$

strictly hold for each moment during the operation. However, in the real world, this turns out to be unrealistic. Apart from external disturbances, there are many factors that can ruin the perfect diagnosing shown in aforementioned context. For instance, each motor have different electro-mechanical characteristics. And each motor has its own nonlinearities (e.g. saturation) and so on. So in dynamic scenarios, we only have the real velocities v'_L and v'_R instead of the ideal counterparts v_L and v_R . It means that in the real world, we have the following relationship,

Both v' and ω' can be transformed into uncertainties in K_1 and K_2 . To simplify the analysis, we consider uncertainties of K_1 caused by mismatching of ω with respect to v, which is the case with control in Equation (13). Substituting Equation (13) into Equation (2) in Part I of this chapter, we obtain

$$\begin{bmatrix} \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -\cos(\phi) & 0 \\ \frac{1}{r}\sin(\phi) & 1 \end{bmatrix} \begin{bmatrix} K_1 r \cos(\phi) \\ -K_3 \sin(\phi) \cos(\phi) - K_2 \phi \end{bmatrix}$$
$$= \begin{bmatrix} -K_1 (\cos(\phi))^2 r \\ -K_2 \phi - (\frac{K_3 - K_1}{2}) \sin(2\phi) \end{bmatrix}.$$
(18)

Through studying the stability of the closed-loop system described by Equation (18), we are entitled to investigating the robustness of the alternative control law given in Equation (13).

4. Robustness analysis

4.1 Stable zone

We refer to the model in Equation (18) as the real closed-loop system model. Then our problem is to analyze the stability and robustness of this real-world model. We can decompose this model into two subsystems as follows.

$$\dot{r} = -K_1 (\cos(\phi))^2 r,$$

$$\dot{\phi} = -K_2 \phi - \left(\frac{K_3 - K_1}{2}\right) \sin(2\phi).$$

Obviously except the special case with $\cos(\phi(t)) \equiv 0$, r(t) is at least asymptotically convergent to zero. As to $\phi(t)$, the situation is more complicated. Let $K_4 = (K_3 - K_1)/2$, then we have

$$\dot{\phi} = -K_2\phi - K_4\sin(2\phi) \tag{19}$$

As to the subsystem denoted by Equation (19), construct a Lyapunov candidate as $V = \frac{1}{2}\phi^2$. The derivative of *V* with respect to time is

$$\dot{V} = \phi \dot{\phi} = -K_2 \phi^2 - K_4 \phi \sin(2\phi).$$
 (20)

As shown by the closed-loop system equation in Equation (18), this system is time invariant indicating that LaSalle's theorem is applicable. Therefore we are motivated to find out the invariant set Σ , which leads to negative \dot{V} represented in Equation (20). The invariant set Σ can be calculated according to the following equation:

$$\Sigma = \{ (K_1, K_2, K_3) | \dot{V} < 0 \}.$$

To this end we let $\dot{V} = 0$, then we have to make either $\phi = 0$ or $\phi = -K_4/K_2 \sin(2\phi)$.

To find out the solution of $\phi = -K_4/K_2 \sin(2\phi)$ for $\phi \in [0, \pi]$, we perform numerical calculation in Matlab environment. There are two scenarios: either $K_4/K_2 \ge 0$ or $K_4/K_2 < 0$. The illustration of different solutions when $K_4/K_2 > 0$ is shown in Figure 3 while the case with $K_4/K_2 < 0$ is shown in Figure 4. The calculation shows that:

- if $K_4/K_2 \ge 0$ when $0 \le K_4/K_2 < c_1$, equation $\phi = -K_4/K_2 \sin(2\phi)$ has only one solution, i.e., $\phi = 0$.

Fig. 3. Illustration of different solutions with $K_4/K_2 > 0$.

- if $K_1/K_2 < 0$ when $c_2 < K_4/K_2 < 0$, equation $\phi = -K_4/K_2 \sin(2\phi)$ has only one solution, i.e., $\phi = 0$.

where c_1 and c_2 are constants. The numerical calculations offer approximation values of c_1 and c_2 as $c_1 \approx 2.30$ and $c_2 \approx -0.50$.

To sum up, the ratio K_4/K_2 should be within the range (c_2, c_1) to make subsystem Equation (19) asymptotically stable. Or in other words, the relationship among K_1, K_2, K_3 to make subsystem Equation (19) stable is: $K_1 + 2c_2K_2 < K_3 < 2c_1K_2 + K_1$ ($K_2 > 0$) or $2c_1K_2 + K_1 < K_3 < K_1 + 2c_2K_2$ ($K_2 < 0$).

In practise, K_2 is usually chosen to be positive. So we can further simplify the conclusions above. In this case, the whole stable range of K_3 is:

$$K_1 + 2c_2K_2 < K_3 < 2c_1K_2 + K_1.$$
⁽²¹⁾

The stable zone is shown in Figure 5. The stable zone is the whole wedge and is separated by a plane with K3 = K1. The upper part of this wedge has the property of K4 > 0 while the lower part with K4 < 0. Compared with the nominal sets of parameters in the plane K3 = K1,

Fig. 4. Illustration of different solutions with $K_4/K_2 < 0$.

78

Fig. 5. Illustration of stable zone with practical concerns in the case when $K_1 > 0$ and $K_2 > 0$. Note that the whole zone is separated by a plane K3 = K1, i.e., K4 = 0.

the difference of those two parts of the zone is that the system response will be different as revealed by Proposition 3.

4.2 A new robust control and its benefits

Proposition 2 If $K_4 \ge 0$, namely, $K_3 - K_1 \ge 0$, r(t) is exponentially convergent to zero for arbitrary *initial* ϕ_0 .

The proof is pretty straightforward and hence it is omitted here for the sake of brevity.

Proposition 3 *A Set of* (*K*1, *K*2, *K*3) *in the upper part of the wedge in Figure 5 expedites the response* of ϕ if $\phi_0 \in (0, \pi/2)$.

Proof: It is noted that equation

$$\dot{\phi} = -K_2\phi - K_4\sin(2\phi)$$

has a unique solution on time interval $[0, t_1]$ for any $t_1 > 0$ because

$$f(\phi) = -K_2\phi - K_4\sin(2\phi),$$

is locally Lipschitz. Let $p(t) = \phi^2(t)$, then

$$\dot{p}(t) = 2\phi\dot{\phi}$$

$$= -2K_2\phi^2 - 2K_4\phi\sin(2\phi)$$

$$\leq -2K_2\phi^2$$

$$= -2K_2p(t).$$

Let q(t) be the solution of the differential equation

$$\dot{q}(t) = -2K_2q(t),$$

where $q(0) = \phi(0)$ then we arrive at

$$q(t) = \phi^2(0)e^{-2K_2t}.$$

According to comparison principle, the solution $\phi(t)$ is defined for all $t \ge 0$ and satisfies

$$|\phi(t)| = \sqrt{p(t)} \le |\phi(0)|e^{-K_2 t}, \forall t \ge 0,$$

thus completes the proof.

According to the proposition above, we can deliberately choose $K_3 \ge K_1$ to make the system more robust. Specifically we can design control laws according to guidelines as follows:

- 1. As revealed in Figure 5, *K*₂ should not be too close to zero as the bigger *K*₂, the wider zone between upper bound and lower bound.
- 2. To maximize the stability zone for a given set of (K_1, K_2, K_3) , it is desirable to choose $K_3 = K_1 + (c_1 + c_2)K_2$. In other words, (K_1, K_2, K_3) is within the plane in the middle of upper bound and lower bound as illustrated in Figure 5.
- 3. To obtain comparatively large stability zone for a given set of (K_1, K_2, K_3) while keep the converging rate from being negatively affected, it is desirable to choose $K_3 = K_1 + c_1 K_2$.

5. Simulation study

5.1 Mismatching K_3 and K_1

A simulation in Matlab is designed to show two cases of mismatching K_3 and K_1 . In case one, initial conditions are set to be $\phi_0 = 1$ rad and $r_0 = 1$ and in case two $\phi_0 = \pi$ rad and $r_0 = 1$. The nominal gains are chosen as $K_1 = 20$ and $K_2 = 1$. Suppose there is -6% deviation of K_3 with respect to K_1 , i.e., $K_3 = 18.8$ in case one and a positive 24% deviation of K_3 , i.e., $K_3 = 24.8$ in case two.

The simulation results of system response are depicted in Figure 6 with (a), (b) for case one and (c), (d) for case two. From this figure, it is obvious that in those two cases $\phi(t)$ fails to approach to zero due to -6% and 24% deviation of K_3 respectively. In other words, this is because both cases break the constraint described by Equation (21).

Fig. 6. Illustration of mismatching K_3 and K_1 . In (a) and (b) initial conditions are $\phi_0 = 1$ rad and $r_0 = 1$ and gain $K_1 = 20$, K2 = 1 and K3 = 18.8 (i.e., -6% deviation). And in (c) and (d) initial conditions are $\phi_0 = \pi$ rad and $r_0 = 1$ and gain $K_1 = 20$, K2 = 1 and K3 = 24.8 (i.e., 24% deviation).

www.intechopen.com

Fig. 7. Illustration of the effects of mismatching K_3 on system response. Initial conditions are $\phi_0 = 1$ rad and $r_0 = 1$ and gain $K_1 = 20$, $K_2 = 1$ and $K_3 = 19.2, 20, 21.80, 22.30$ respectively.

5.2 Effects of K₃ on system response

In this simulation we investigate the effects of mismatching K_3 on system response through simulation. Initial conditions are set to be $\phi_0 = 1$ rad and $r_0 = 1$ and gain $K_1 = 20$ and K2 = 1. We vary the value of K_3 with respect to K_1 . Refer to the stable zone illustrated in Figure 5, we deliberately choose several sets of (K1, K2, K3) from upper part, separation plane $(K_1 = K_3)$ and lower part respectively. According to design guidelines, in this experiment, we choose $K_3 = 22.30, 21.8$ from upper part and $K_3 = 20$ for the nominal case and $K_3 = 19.2$ from the lower part. The results are depicted in Figure 7. From this figure, it is noticed that compared with $K_3 = 20$, a set in upper part of the wedge in Figure 5 contributes to expediting the system's response while a set in lower part of the wedge will negative affect the system's response. The most significant effects of mismatching K_3 is on the converging rate of term $\phi(t)$. Since they are all within the stable wedge, both r(t) and $\phi(t)$ approach to zero as time $t \to 0$.

To compare the energy needed for each controller, we define a function J_n which is describe by

$$J_n = \int_0^t \left(v^2(\tau) + \omega^2(\tau) \right) d\tau.$$

In this simulation, the integral of the norm squared of the actual velocity signals for each controller is shown in Table 1. From the figures shown in this table, the control laws recommended by design guidelines seem to be more efficient than the nominal case with $K_3 = K_1$ and the one with negative deviation ($K_3 = 19.2$). And there is not significant difference between the two recommended control laws, i.e., $K_3 = 21.80$ and $K_3 = 22.30$ respectively.

$K_3 = 19.2$	$K_3 = 20$	$K_3 = 21.80$	$K_3 = 22.30$
331.8	129.8	69.78	63.87

Table 1. Comparison of the integral of the norm squared of the velocity input signals $\int_0^{30} (v^2(t) + \omega^2(t)) dt$.

6. Implementation and experiment

6.1 Overview of implementation

One picture of the real implementation is presented by Figure 8. In this figure, a 4m by 2.8m wooden test bed offers the field for a group of mobile robots. The MRKIT mobile robots presented in Figure 8 with on-board infrared sensors and compass, which are used in the experiments consist of the main platform to verify algorithms. Each robot has two independently controlled wheels driven by stepper motors. A GPS system is simulated by a vision system comprising vision frame grabber, CCD color camera with lens, a working station, and wireless communication modules. Two web-cam are mounted on the ceiling and can be used for robot tracking or video recording and only one is showed in Figure 8. The main parts of this implementation are connected as shown by Figure 10.

6.2 Parameters of MRKIT mobile robots

Each wheel of MRKIT robot is driven independently with step motor being controlled by on-board micro-controller. The velocity of wheel is controlled via PWM waveform and is determined by an internal time interval T in the micro-controller. The relationship between the velocity V of a wheel and T can be represented as

$$V = \frac{D\pi}{NP},$$

where D = 54 mm is the diameter of the wheel; N = 400 is the step of motor per revolution; P is the time (second) per step and

Fig. 8. Picture of real robots, test bed(on the floor), CCD color camera with wide-angle lens and one web-cam (mounted on the ceiling).

Fig. 9. Picture of one MRKIT mobile robot with on-board color pads.

Finally we arrive at

$$V = \frac{1060.288}{T} m/s,$$

and *T* is a 16-bit integer which can be set in micro-controller. Due to the finite length of *T* and physical limitations of motor, *V* has a minimum $V_{min} = 0.0162$ m/s and a maximum $V_{max} = 0.3$ m/s.

Fig. 10. Illustration of connection of the whole implementation.

6.3 Vision system: resolution and noise analysis

A vision system, comprising of CCD camera, lens, frame grabber and application program as shown in Figure 10, is developed for the tracking of mobile robots and detecting position/orientation of them. Its resolution is largely determined by the resolution of the CCD camera and the optical system. In the experiment, the CCD camera is mounted on a bracket fixed on the ceiling. Due to the limitation of ceiling's height, the viewable area on the test bed is of 1800mm by 2480mm. The CCD camera has a resolution of 576 by 768 pixels. We designate the x and y coordinate to the vision system and therefore we can calculate the real resolution of the vision system. The calculation results show that on the x axis, the resolution is 3.229mm per pix while on the y axis, 3.177mm per pix.

To identify the robot's position and orientation, a color pad is attached on the top of a robot as shown in Figure 9. Each color pad has two different color circles aligned in a line. Each color circle has a diameter of 65mm. One circle is painted blue and another one is yellow. The center of each circle can be calculated through the image processing hardware and software, namely the frame grabber and the corresponding vision processing software running on working station. We can use coordinates of the centers of the two color circles to calculate the position of the robot's center and its orientation as well. Let (x_a, y_a) and (x_b, y_b) denote the measured coordinates of the center of yellow circle and blue circle respectively. Hence, the coordinate of the robot's center can be represented as $(\frac{x_a+x_b}{2}, \frac{y_a+y_b}{2})$. Meanwhile the orientation of the robot can be calculated as

$$\theta = \cos^{-1}\left(\frac{x_a - x_b}{\rho}\right),\tag{22}$$

where

$$\rho = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}.$$

The measurement of the position of each color circle is a resultant of its real position and the error signal together with noise. The position error is incurred by the hardware of the system. For instance the field is not even and can end up with position error. Another sample of the source of the error signal can be the optical system. The distortion of the lens on the margin of the viewable area is relatively salient and such distortion in fact affects the accuracy of the measurement. Roughly the measurement of position can be expressed in the following equation:

$$X_m = x_r + x_e + x_n,$$

where x_r is the real position; x_e is the system error and x_n the noise. It is of interest and practically importance to know the noise level of the measured signal. For any static robot on the test bed, its real position and system error are always constant and contribute no variation to the mean value of X_m and to the variance either. From this observation, it helps to sample the measurement for a certain period and then use the spectrum analysis tools to get the information of the noise signal. One convenient way is to use the FFT technique. It is well known that Microsoft Windows is not a real time operation system. For the purpose of FFT, it is required to evenly sample the data. To solve this conflict, a high resolution timer without accumulation error is in need. In this experiment, the multi-media timer is used. It is a high resolution timer with high accuracy and resolution while demands of the system resource are relatively low. We set the sampling rate to be 500 Hz. A period of 2 minutes is used to sample the data and the error signal along x-axis is presented in Figure 11. Figures of error signal along y-axis and the associated orientation error signal are shown in Figure 12 and 13 respectively. Then we apply the FFT technique to analyze its frequency components. It turns

Fig. 11. Position error signal along x-axis with sample rate f = 500Hz.

out that the noise signals on x, y and orientation all show on the feature of Gaussian noise. The analysis results show that $\delta_x = 0.142$ pix, $\delta_y = 0.154$ pix $\delta_\theta = 0.0122$ radius. Obviously compared with vision system's resolution, the noise level of position signal is relatively low.

6.4 Experiment-1: triangle formation of three robots

A scalable formation control scheme is introduced in (Ge & Fua, 2005). The idea is that, instead of being attracted to a predetermined point, each robot is to be attracted to the corresponding segment, and once there, move along the segment to distribute themselves along the trench in order to form a formation by maintaining the desired position in relation to other robots. To briefly review this idea, Figure 14 is presented to show the segments and the robots which are supposed to fall into certain assigned segment. Usually a segment *S* is a curve defined by some smooth (i.e., at least twice-differentiable) function in R^3 that passes through one or two formation vertices. And a robot will arrive at the nearest point on the segment and then move along the curve of the segment.

In this experiment, suppose that assignment mechanism of segment is known and initially all robots are static. Three straight lines are assigned to three robots respectively. For the first

Fig. 12. Position error signal along y-axis with sample rate f = 500Hz.

Fig. 13. Angular error signal with sample rate f = 500Hz.

8 seconds, each robot will try to approach the nearest point on the segment and then three virtual points moving along segments are assigned to each robot. Those three virtual points form a triangle pattern and will stop at the vertices of segments. The velocity of virtual points are set to be 20 pix per second. During the process of formation, velocities and headings of each robot are depicted in Figure 15 - 18. Snapshots of video (taken by web-cam) are shown in Figure 19-22. The controller parameters are set to be $K_1 = 0.1$, $K_3 = 0.12$ and $K_2 = 1.0$. From those figures that all the robots are attracted to the segment for the first 8 seconds and later on form the triangular pattern while moving forward.

6.5 Experiment-2: square formation of four robots

In this experiment, four robots which are initially randomly scattered are required to form a square patter. Two straight lines are assigned. For the first 3 seconds, each robot will try to approach the corresponding nearest point on the segment and then try to approach to four virtual points moving along segments are assigned to each robot. Those three virtual points form a triangle pattern and will stop at the vertices of segments. The velocity of virtual points are set to be 20 pix per second. During the process of formation, velocities and headings of each robot are depicted in Figure 23 - 27. Snapshots of video (taken by web-cam) are shown in Figure 28-30. The controller parameters are set to be $K_1 = 0.1$, $K_3 = 0.12$ and $K_2 = 1.0$. From those figures that all the robots are attracted to the segment for the first 3 seconds and later on form the square pattern while moving forward.

Fig. 14. Illustration of segments and robots falling into assigned segments.

Fig. 15. Velocity of robot 1 during 3-robot triangle formation control.

6.6 Limitation of locomotion

Each wheel of MRKIT utilizes an independent step motor for locomotion. There are two outstanding drawbacks which impede implementation. As indicated by Equation (11), the desired velocity is proportional to the distance to goal point (i.e., r). If a robot is initially placed far away from its goal point, the desired velocity will be relatively high. However, step motor usually is weak on its maximum starting speed and starting torque. If the gain (K_1, K_2, K_3) is too high, a robot initially at standstill will immediately miss its step at the very beginning of formation control. The other shortcoming arises from the minimum speed of step motor. Due to the limitation of minimum speed, a wheeled mobile robot in fact can not reach a fixed goal point. Instead it will stop moving once it enters certain range with respect to its goal point. It results in a dead zone to which the robot is prohibited. To reduce dead zone, higher gain is demanded and thus increases the risk of missing steps. Trade-off has to be done for real implementation. To overcome such downside of locomotion, other motors with high starting torque such as permanent magnet brushless DC motor cater for real implementation.

Fig. 16. Velocity of robot 2 during 3-robot triangle formation control.

Fig. 17. Velocity of robot 3 during 3-robot triangle formation control.

7. Conclusion

In this chapter we first consider the stability issue of a generic form of nonlinear feedback control based on kinematic model in polar coordinate in a novel perspective. Some commonly used controls can be derived from this general form of a family of stable control laws. In addition to the commonly used stability analysis based on Lyapunov stability theorems, in this chapter we employe LaSalle's invariance theorem to investigate the robustness of a point tracking controller. Then the robustness problem of the control law (Lee et al., 1999) is investigated and successfully solved. Thanks to LaSalle's invariance theorem and Lyapunov's stability theorem as well, we are able to unveil the whole stable range of controller gains when velocity and angular velocity of each driving motor are not exactly what they are supposed to be in the real world. This study yields some exciting conclusions on converging rate than the counterpart. Based on the robustness analysis, we can propose useful and handy guidelines in determining controller gains and consequently a new robust control law is proposed. Improvement to this control law is achieved from the analysis results and is verified in Matlab

Fig. 18. Headings of all robots during 3-robot triangle formation control.

Fig. 19. Snapshot of initial conditions of 3-robot triangle formation control at t = 0.

simulation. Implementation with real robots has been done to demonstrate the application to multiple robot formation control. An implementation of multi-robot formation control has been done and discussed in details.

8. References

- Kolmanovsky, I. & McClamroch, N.(1995). Developments in nonholonomic control problems, In: *IEEE Control Systems Magazine*, Vol.15, page numbers (20-36).
- Brockett,R.W. (1983). Asymptotic stability and feedback stabilization, pp. 181-191, Boston, U.S., 1983, Birkhauser.
- Dixon, W.E., Dawson, D.M., & Behal, A. (2001). Nonlinear control of wheeled mobile robots, Springer.
- Wit, C., & Siciliano, B. (1996). Theory of robot control, Springer.
- Luca, A.D., Oriolo,G., & M.Vendittelli, M. (2001). Control of Wheeled Mobile Robots: An Experimental Overview. Lecture Notes in Control and Information Sciences 270, Springer-Verlag, Berlin Heidelberg.

Fig. 20. Snapshot of 3-robot triangle formation control at (t = 8s).

Fig. 21. Snapshot of 3-robot triangle formation control at (t = 16s).

- Wit, C. & Sordalen, O.J. (1992). Exponential stabilization of mobile robots with nonholonomic constraints. *IEEE Transcations on Automatic Control*, Vol.37, (Nov. 1992.) page numbers (1791-1797).
- Godhavn, J. & Egeland, O. (1997). A lyapunov approach to exponentially stabilization of nonholonomic systems in power form. *IEEE Transcations on Automatic Control*, Vol.42, no. 7 page numbers (1028-1032).
- Kanayama,Y., Kimura,Y., Miyazaki,F. & Noguchi, T. (1997). A stable tracking control method for an autonomous mobile robot. *in Proc. IEEE Conf. Robotics Automation*, page numbers (384-389).
- Closkey,R.M and Murray, R. (1997). Exponentially stabilization of driftless nonlinear control systems using homogeneous feedback. *IEEE Transactions on Automatic Control*, Vol.42, (May 1997) page numbers (614-628).
- Teel,A., Murray,R., & Walsh, C. (1995). Nonholonomic control systems: from steering to stabilization with sinusoids. *Int. Journal Control*, Vol.62,No. 4, page numbers (849-870).

Fig. 23. Velocity of robot 1 during 4-robot square formation control.

IEEE Conf. Decision and Control, Vol.62, No. 4, page numbers (2097-2101). Honolulu, HI, Dec. 1990.

- Lafferiere, G. & Sussmann, H.J. (1991). Motion planning for controllable system without drift. in Proc. IEEE Conf. Robotics and Automation, page numbers (1148-1153). Sacramento, CA, Apr. 1991.
- Brraquand, J. & Latombe, J. (1989). On nonholonomic mobile robots and optimal maneuvering. *in Proc. IEEE Conf. Intelligent Control*, page numbers (340-347). Albany, NY, Sep. 1989.
- Brockett,R.W. (1981). Control theory and singular riemannian geometry. *in New directions in applied mathematics*, NY,Springer-Verlag,1981.
- Aguiar,A.P. Atassi,A. & Pascoal,A.M. (2000). Regulation of a nonhonomic dynamic wheeled mobile robot with parametric modelling uncertainty using lyapunov functions. *in Proc. IEEE Conf. Decision Control.* page numbers (2995-3000). Svdnev. Australia, Dec.

Fig. 24. Velocity of robot 2 during 4-robot square formation control.

Fig. 25. Velocity of robot 3 during 4-robot square formation control.

2000.

- Coelho,P. & Nunes,U. (2005). Path-following control of mobile robots in presence of uncertainties. *IEEE Transactions on Robotics*, Vol.21 (Apr. 2005) page numbers (252-261).
- Dixon,W.E. Dawson,D.M. Zergeroglu,E. & Zhang,F. (1999). Robust tracking and regulation control for mobile robots. *in Proc. IEEE Conf. Control Applications*, page numbers (1015-1020). Kohala Coast-Island of Hawaii, U.S., Aug. 1999.
- Lee,S.O. Cho,Y.J. B.,M.H. You,B.J. & Oh,S.R. (1999). A stable targettracking control for unicycle mobile robots. *in Proc. IEEE/RSJ Conf. Intelligent Robots and Systems*, page numbers (1822-1827). Takamatsu, Japan, 2000.
- Siegwart, R. & Nourbakhsh, I. (2004). *Introduction to Autonomous Mobile Robots*, page numbers (13-46). MIT Press, 2004.
- Baillieul, J. (1999). The Geometrv of Sensor Information Utilization in Nonlinear Feedback

Fig. 26. Velocity of robot 4 during 4-robot square formation control.

92

Fig. 27. Headings of all robots during 4-robot square formation control.

Control of Vehicle Formations. *Lecture Notes in Control and Information Sciences 309: Cooperative Control*, Berlin Heidelberg: Springer-Verlag, 2005.

Ge,S. S. and Fua,C.-H. (2005). Queues and artificial potential trenches for multi-robot formations. *IEEE Transactions on Robotics*, Vol.21(4) (Aug. 2005) page numbers (646-656).

Fig. 28. Snapshot of 4-robot square formation control at (t = 0s).

Fig. 29. Snapshot of 4-robot square formation control at (t = 6s).

Fig. 30. Snapshot of 4-robot square formation control at (t = 12s).

Multi-Robot Systems, Trends and Development Edited by Dr Toshiyuki Yasuda

ISBN 978-953-307-425-2 Hard cover, 586 pages **Publisher** InTech **Published online** 30, January, 2011 **Published in print edition** January, 2011

This book is a collection of 29 excellent works and comprised of three sections: task oriented approach, bio inspired approach, and modeling/design. In the first section, applications on formation, localization/mapping, and planning are introduced. The second section is on behavior-based approach by means of artificial intelligence techniques. The last section includes research articles on development of architectures and control systems.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Jie Wan and Peter Chen (2011). A Robust Nonlinear Control for Differential Mobile Robots and Implementation on Formation Control, Multi-Robot Systems, Trends and Development, Dr Toshiyuki Yasuda (Ed.), ISBN: 978-953-307-425-2, InTech, Available from: http://www.intechopen.com/books/multi-robot-systems-trends-and-development/a-robust-nonlinear-control-for-differential-mobile-robots-and-implementation-on-formation-control

InTech Europe

University Campus STeP Ri Slavka Krautzeka 83/A 51000 Rijeka, Croatia Phone: +385 (51) 770 447 Fax: +385 (51) 686 166 www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai No.65, Yan An Road (West), Shanghai, 200040, China 中国上海市延安西路65号上海国际贵都大饭店办公楼405单元 Phone: +86-21-62489820 Fax: +86-21-62489821 © 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the <u>Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License</u>, which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

