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# Simplified Human Hand Models for Manipulation Tasks 

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## 1. Introduction

The human hand is the most dexterous and versatile biomechanical device that possesses the human body, this device created by Nature during millions years of evolution represents one of the more distinctive qualities among other animals. Since the 70s and 80s, important contributions have appeared in physiological studies of the human hand (I. A. Kapandji, 1970), (I. A. Kapandji, 1981). Studies about robotic hands have made several contributions e.g. the Stanford/JPL hand (K. S. Salisbury \& B. Roth, 1983), the Utah hand (S. C. Jacobsen et al., 1986), the Okada hand (T. Okada, 1982), the Belgrade/USC hand (G. Bekey et al., 1990), the UB hands (C. Melchiorri and G. Vassura 1992), (C. Melchiorri and G. Vassura 1993), the DLR hands (J. Butterfass et al., 1999), (J. Butterfass et al., 2001), the University of Tokyo hand (Y. K. Lee \& I. Shimoyama, 1999), Barrett Hand (W. T. Townsend, 2000), the Robo-Naut hand by NASA (C. S. Lovchik et al., 2000), the Karlsruhe University ultra-light hand (S. Schulz et al., 2001), the GIFU hand (H. Kawasaki et al., 2001), the Shadow Dextrous Hand (Shadow Robot Company), (F. Röthling et al., 2007), a prosthetic hand (H. Yokoi et al., 2004), the DLR-HIT-Hand (H. Liu et al., 2008) and other. These devices have different kinematic configurations with respect to the number of Degrees of Freedom (DoF) controlled, number of fingers, number of joints, type of actuation, etc. This chapter describes simplified human hand models that properly represent the kinematic behaviour of the human hand in accordance with the precision and application required. The first part describes a human hand model with 24 DoF. This model represents a balance between complexity and realism. Simplified human hand (SHH) models are analyzed using the model with 24 DoF . These SHH models ( 1 to 24 DoF ) are evaluated in accordance with the level of dexterous or power required. A Cyberglove ${ }^{\circledR}$ (Immersion) is used for the experiments carried out in this work. Kinematic constrainswere checked with the information provided by the glove. Also, this glove was used for evaluating the error of the SHH versus the full 24 DoF hand model. Finally, the experiments carried out with SHH and 24 DoF hand model compare the efficiency in grasping for circular and prismatic grasps in accordance with the application.

## 2. Kinematic Human Hand Model

The hand model used for this work is based on the human skeleton. The kinematic model is comprised of 19 links that imitate the corresponding human bones, and 24 degrees of freedom (DoF) that represent the joints. Links and joints are defined by $L_{i, j}$ and $\theta_{i, j}$, where $i$ represents a finger ( $i=$ Thumb, Index, Middle, Ring or Little) and $j$ its corresponding link or joint respectively. Two kinematic configurations are considered in this hand model, one for the thumb and another for the rest of the fingers. Therefore, the same kinematic configuration is used for the index, middle, ring and little fingers. This configuration is defined by 5 joints and 4 links: metacarpal ( $L_{i, M e}$ ), proximal $\left(L_{i, P}\right)$, middle ( $L_{i, M i}$ ) and distal $\left(L_{i, D}\right)$ links. The joints are defined as: carpometacarpal ( $\theta_{i, C M C}$ ), proximal interphalangeal $\left(\theta_{i, P I P}\right)$ distal interphalangeal $\left(\theta_{i, D I P}\right)$ and metacarpophalangeal, which is modelled by a universal joint (2 DoF) that defines the abduction/adduction ( $\theta_{i, M C P \_a a}$ ) and flexion/extension ( $\theta_{i, M C P \_f e}$ ) rotations. The thumb is modelled by 4 DoF and 3 links: metacarpal ( $L_{T, M}$ ), proximal ( $L_{T, P}$ ), and distal $\left(L_{T, D}\right)$. The thumb joints are defined as: metacarpophalangeal $\left(\theta_{T, M C P} f_{e}\right)$, interphalangeal $\left(\theta_{T, I P}\right)$ and trapeziometacarpal, which is also defined by a universal joint that defines the abduction/adduction ( $\theta_{T, T M C \_a}$ ) and flexion/extension ( $\theta_{T, T M C \_f e}$ ) respectively. Figure 1 shows details of both kinematic models. The main features of this model are: the use of 4 DoF for the thumb modelling, the inclusion of the $\theta_{\mathrm{i}, \mathrm{CMC}}$ joint, and the movement concatenation in the $\theta_{i, M C P}$ joint. The MCP joint of the thumb is considered to have $1 \operatorname{DoF}\left(\theta_{T, M C P_{f} e}\right)$. The $\theta_{i, C M C}$ joint allows simulation of the palm arc, the palm deformation when the hand is grasping a ball or similar objects. The MCP abduction/adduction $\left(\theta_{i, M C P \_a a}\right)$ twist is defined before the MCP flexion/extension ( $\theta_{i, M C P_{-} f}$ ) aiming at simulating finger displacements in a better way. These points contribute to the definition of highly realistic hand movements and gestures. In the following sections the forward and inverse kinematic models are developed.

### 2.1 Forward Kinematics

Forward kinematic is used to obtain the finger tip position and orientation according to the finer joint angles. These models are obtained for both cases (thumb and the other fingers). Model equations are calculated by means of the Denavit-Hartenberg (D-H) parameters (M.W. Spong, 2006). This convention is commonly used for mechanism and robotics modelling. Human anatomical terminology has been used to describe the hand model. Direct kinematic equations are required when virtual grasping is simulated. More specifically, a user wearing a glove that provides the human joints and forward kinematic equations determines what the fingertip positions and orientations are. This information will also be used to calculate object contact points, grasping routes, etc.

### 2.1.1 Forward kinematics of the index, middle, ring and little fingers

Table 1 shows the D-H parameters for index, middle, ring and little fingers. Joints represent the variables of this model that are defined by $\theta_{i, j}$. Links are defined by the parameters $a_{i, j}$ that represent the length of the bones. Parameter $d_{i, j}$ is always null since bones are aligned, and parameter $\alpha_{i, j}$ is the angle of separation of the $Z_{i-1}$ axis and the $Z_{i}$ axis, measured in a plane perpendicular to the $X_{i}$ axis, utilizing the rule of the right hand.


Fig. 1. Kinematic configuration of the human hand with 19 links and 24 DoF.
Thumb is defined by 3 links ( $\mathrm{L}_{T, M}, \mathrm{~L}_{T, P}$ and $\left.\mathrm{L}_{T, D}\right)$ and $4 \operatorname{DoF}\left(\theta_{T, T M C \_a a,} \theta_{T, T M C \_f e}, \theta_{T, M C P_{-f e}}\right.$ and $\left.\theta_{T, I P}\right)$ whereas index, middle, ring and little fingers are defined by 4 links ( $\mathrm{L}_{i, M e}, \mathrm{~L}_{i, P}, \mathrm{~L}_{i, M i}$ and $\left.\mathrm{L}_{i, D}\right)$ and $5 \mathrm{DoF}\left(\theta_{i, C M C}, \theta_{i, M C P \_a a}, \theta_{i, M C P \_ \text {fer }}, \theta_{i, \text { PIP }}\right.$ and $\left.\theta_{i, D I P}\right)$.

| Joint | $\boldsymbol{\theta}_{i, j}$ | $\boldsymbol{d}_{i, j}$ | $\boldsymbol{a}_{i, j}$ | $\boldsymbol{\alpha}_{i, j}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $\theta_{i, C M C}$ | 0 | $L_{i, M e}$ | $\pi / 2$ |
| 2 | $\theta_{i, M C P \text { aa }}$ | 0 | 0 | $-\pi / 2$ |
| 3 | $\theta_{i, M C P f e}$ | 0 | $L_{i, P}$ | 0 |
| 4 | $\theta_{i, P I P}$ | 0 | $L_{i, M i}$ | 0 |
| 5 | $\theta_{i, D I P}$ | 0 | $L_{i, D}$ | 0 |

Table 1. D-H parameters for index, middle, ring and little fingers.
The forward kinematics of these fingers is shown in equation 1, which is defined according to the parameters of Table 1.

$$
\begin{equation*}
p_{i}={ }_{0}^{-1} T_{i}\left(u_{i}\right) \cdot{ }_{5}^{0} T_{i}\left(\theta_{i, j}\right)={ }_{0}^{-1} T_{i}\left(u_{i}\right) \cdot{ }_{1}^{0} T_{i}\left(\theta_{i, C M C}\right) \cdot{ }_{2}^{1} T_{i}\left(\theta_{i, M C P_{-} a a}\right) \cdot{ }_{3}^{2} T_{i}\left(\theta_{i, M C P_{-} f e}\right) \cdot{ }_{4}^{3} T_{i}\left(\theta_{i, P I P}\right) \cdot{ }_{5}^{4} T_{i}\left(\theta_{i, D I P}\right) \tag{1}
\end{equation*}
$$

Where terms in this equation are: $p_{i}$ represents a matrix that contains position and orientation of the $i$-finger tip with respect to the center of the wrist, $u_{i}$ represents the vector
between the center of wrist and the corresponding $i$-finger reference frame, ${ }_{5}^{0} T_{i}\left(\theta_{i, j}\right)$ is a matrix that contains the homogeneous matrix between the i-finger reference frame and its corresponding $i$-finger tip. This matrix consists of the concatenation of matrixes that represent the contribution of each $i$-finger joint displacement $\left(\theta_{i, C M C}, \theta_{i, M C P}\right.$ aa, $\theta_{i, M C P}$ fe, $\theta_{i, P I P,}$ $\theta_{i, \text { DIP }}$ ) the translation and rotation contribution of each joint is defined by the matrix ${ }^{j-1} T_{i}\left(\theta_{i, j}\right)$.

$$
{ }_{j-1}^{j} T_{i}\left(\theta_{i, j}\right)=\left(\begin{array}{cccc}
\cos \left(\theta_{i, j}\right) & -\cos \left(\alpha_{i, j}\right) \sin \left(\theta_{i, j}\right) & \sin \left(\alpha_{i, j}\right) \sin \left(\alpha_{i, j}\right) & a_{i, j} \cos \left(\theta_{i, j}\right)  \tag{2}\\
\sin \left(\theta_{i, j}\right) & \cos \left(\alpha_{i, j}\right) \cos \left(\theta_{i, j}\right) & -\sin \left(\alpha_{i, j}\right) \cos \left(\alpha_{i, j}\right) & a_{i, j} \sin \left(\theta_{i, j}\right) \\
0 & \sin \left(\alpha_{i, j}\right) & \cos \left(\alpha_{i, j}\right) & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

2.1.2 Homogeneous matrixes for the index, middle, ring and little fingers.

In order to simplify the equations, some variables are replaced (VR) for the corresponding variable showed in table 2.

| $\boldsymbol{\theta}_{i, j}$ | VR | $\boldsymbol{a}_{i, j}$ | $\mathbf{V R}$ | Link |
| :--- | :--- | :--- | :--- | :--- |
| $\theta_{i, C M C}$ | $\theta_{5}$ | $L_{i, M e}$ | $L_{4}$ | Metacarpal |
| $\theta_{i, M C P \text { aa }}$ | $\theta_{6}$ | 0 | 0 |  |
| $\theta_{i, M C P f e}$ | $\theta_{7}$ | $L_{i, P}$ | $L_{5}$ | Proximal |
| $\theta_{i, \text { PIP }}$ | $\theta_{8}$ | $L_{i, M i}$ | $L_{6}$ | Middle |
| $\theta_{i, \text { DIP }}$ | $\theta_{9}$ | $L_{i, D}$ | $L_{7}$ | Distal |

Table 2.

$$
\begin{aligned}
& c=\cos \\
& s=\sin
\end{aligned}
$$

$$
\begin{gather*}
{ }^{0} T_{1}\left(\theta_{i, C M C}\right)=\left[\begin{array}{cccc}
c \theta_{i, C M C} & 0 & -s \theta_{i, C M C} & L_{i, M e} c \theta_{i, C M C} \\
s \theta_{i, C M C} & 0 & c \theta_{i, C M C} & L_{i, M e} s \theta_{i, C M C} \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{3}\\
{ }^{1} T_{2}\left(\theta_{i, M C P_{-} a a}\right)=\left[\begin{array}{cccc}
c \theta_{i, M C P_{-} a a} & 0 & s \theta_{i, M C P_{-} a a} & 0 \\
s \theta_{i, M C P_{-} a a} & 0 & -c \theta_{i, M C P_{-} a a} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \tag{4}
\end{gather*}
$$

$$
\begin{align*}
& { }^{2} T_{3}\left(\theta_{i, M C P_{-} f e}\right)=\left[\begin{array}{cccc}
c \theta_{i, M C P_{-} f e} & -s \theta_{i, M C P \_f e} & 0 & L_{i, p} c \theta_{i, M C P^{f f}} \\
s \theta_{i, M C P_{-} f e} & c \theta_{i, M C P \_f e} & 0 & L_{i, p} s \theta_{i, M C P_{-} f e} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{5}\\
& { }^{3} T_{4}\left(\theta_{i, P I P}\right)=\left[\begin{array}{cccc}
c \theta_{i, P I P} & -s \theta_{i, P I P} & 0 & L_{i, M i} c \theta_{i, P I P} \\
s \theta_{i, P I P} & c \theta_{i, P I P} & 0 & L_{i, M i} s \theta_{i, P I P} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{6}\\
& { }^{4} T_{5}\left(\theta_{i, D I P}\right)=\left[\begin{array}{cccc}
c \theta_{i, D I P} & -s \theta_{i, D I P} & 0 & L_{i, D} c \theta_{i, D I P} \\
s \theta_{i, D I P} & c \theta_{i, D I P} & 0 & L_{i, D} s \theta_{i, D I P} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{7}\\
& { }^{-1} T_{0}\left(u_{i}\right)=\left[\begin{array}{cccc}
1 & 0 & 0 & u_{i, x} \\
0 & 1 & 0 & u_{i, y} \\
0 & 0 & 1 & u_{i, z} \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{8}\\
& { }^{0} T_{5}=\left[\begin{array}{cccc}
n_{2 x} & s_{2 x} & a_{2 x} & P_{2 x} \\
n_{2 y} & s_{2 y} & a_{2 y} & P_{2 y} \\
n_{2 z} & s_{2 z} & a_{2 z} & P_{2 z} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{9}
\end{align*}
$$

Where:

$$
\begin{gather*}
n_{2 x}=\left(\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{s}_{5} \mathrm{~s}_{7}\right) \mathrm{c}_{8}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}-\mathrm{s}_{5} \mathrm{c}_{7}\right) \mathrm{s}_{8}\right) \mathrm{c}_{9}+\left(-\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{s}_{5} \mathrm{~s}_{7}\right) \mathrm{s}_{8}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}-\mathrm{s}_{5} \mathrm{c}_{7}\right) \mathrm{c}_{8}\right) \mathrm{s}_{9}  \tag{10}\\
n_{2 y}=\left(\left(\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~s}_{7}\right) \mathrm{c}_{8}+\left(-\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{s}_{8}\right) \mathrm{c}_{9}+\left(-\left(\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~s}_{7}\right) \mathrm{s}_{8}+\left(-\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{c}_{8}\right) \mathrm{s}_{9}  \tag{11}\\
n_{2 z}=\left(-\mathrm{s}_{6} \mathrm{c}_{7} \mathrm{c}_{8}+\mathrm{s}_{6} \mathrm{~s}_{7} \mathrm{~s}_{8}\right) \mathrm{c}_{9}+\left(\mathrm{s}_{6} \mathrm{c}_{7} \mathrm{~s}_{8}+\mathrm{s}_{6} \mathrm{~s}_{7} \mathrm{c}_{8}\right) \mathrm{s}_{9}  \tag{12}\\
s_{2 x}=-\left(\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{s}_{5} \mathrm{~s}_{7}\right) \mathrm{c}_{8}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}-\mathrm{s}_{5} \mathrm{c}_{7}\right) \mathrm{s}_{8}\right) \mathrm{s}_{9}+\left(-\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{s}_{5} \mathrm{~s}_{7}\right) \mathrm{s}_{8}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}-\mathrm{s}_{5} \mathrm{c}_{7}\right) \mathrm{c}_{8}\right) \mathrm{c}_{9}  \tag{13}\\
s_{2 y}=-\left(\left(\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~s}_{7}\right) \mathrm{c}_{8}+\left(-\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{s}_{8}\right) \mathrm{s}_{9}+\left(-\left(\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~s}_{7}\right) \mathrm{s}_{8}+\left(-\mathrm{s}_{5} \mathrm{c}_{6} \mathrm{~s}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{c}_{8}\right) \mathrm{c}_{9}  \tag{14}\\
s_{2 z}=-\left(-\mathrm{s}_{6} \mathrm{c}_{7} \mathrm{c}_{8}+\mathrm{s}_{6} \mathrm{~s}_{7} \mathrm{~s}_{8}\right) \mathrm{s}_{9}+\left(\mathrm{s}_{6} \mathrm{c}_{7} \mathrm{~s}_{8}+\mathrm{s}_{6} \mathrm{~s}_{7} \mathrm{c}_{8}\right) \mathrm{c}_{9} \tag{15}
\end{gather*}
$$

$$
\begin{align*}
& a_{2 x}=\mathrm{c}_{5} \mathrm{~S}_{6}  \tag{16}\\
& a_{2 y}=\mathrm{S}_{5} \mathrm{~S}_{6}  \tag{17}\\
& a_{2 z}=\mathrm{c}_{6}  \tag{18}\\
& a_{2 z}=\mathrm{c}_{6}  \tag{19}\\
& P_{2 x}=\left(\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{S}_{5} \mathrm{~S}_{7}\right) \mathrm{c}_{8}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~S}_{7}-\mathrm{S}_{5} \mathrm{c}_{7}\right) \mathrm{S}_{8}\right) \mathrm{c}_{9} \mathrm{~L}_{7}+\left(-\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{S}_{5} \mathrm{~S}_{7}\right) \mathrm{S}_{8}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~S}_{7}-\mathrm{S}_{5} \mathrm{c}_{7}\right) \mathrm{c}_{8}\right) \mathrm{s}_{9} \mathrm{~L}_{7}  \tag{20}\\
& +\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{S}_{5} \mathrm{~S}_{7}\right) \mathrm{c}_{8} \mathrm{~L}_{6}+\left(-\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{~S}_{7}-\mathrm{S}_{5} \mathrm{c}_{7}\right) \mathrm{S}_{8} \mathrm{~L}_{6}+\left(\mathrm{c}_{5} \mathrm{c}_{6} \mathrm{c}_{7}-\mathrm{S}_{5} \mathrm{~S}_{7}\right) \mathrm{L}_{5}+\mathrm{c}_{5} \mathrm{~L}_{4} \\
& P_{2 y}=\left(\left(\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~S}_{7}\right) \mathrm{c}_{8}+\left(-\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{~S}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{S}_{8}\right) \mathrm{c}_{9} \mathrm{~L}_{7}+\left(-\left(\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~S}_{7}\right) \mathrm{S}_{8}+\left(-\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{~S}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{c}_{8}\right) \mathrm{S}_{9} \mathrm{~L}_{7} \\
& +\left(\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~S}_{7}\right) \mathrm{c}_{8} \mathrm{~L}_{6}+\left(-\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{~S}_{7}+\mathrm{c}_{5} \mathrm{c}_{7}\right) \mathrm{s}_{8} \mathrm{~L}_{6}+\left(\mathrm{S}_{5} \mathrm{c}_{6} \mathrm{c}_{7}+\mathrm{c}_{5} \mathrm{~S}_{7}\right) \mathrm{L}_{5}+\mathrm{s}_{5} \mathrm{~L}_{4}  \tag{21}\\
& P_{2 z}=\left(-S_{6} c_{7} c_{8}+S_{6} S_{7} S_{8}\right) c_{9} L_{7}+\left(S_{6} c_{7} S_{8}+S_{6} S_{7} c_{8}\right) S_{9} L_{7}-\left(S_{6} c_{7} c_{8}+S_{6} S_{7} S_{8}\right) L_{6}-S_{6} c_{7} L_{5} \tag{22}
\end{align*}
$$

### 2.1.3 Forward kinematics of the thumb

Forward kinematics of the thumb is defined in a similar way. Table 3 shows the D-H parameters of the thumb.

| Joint | $\boldsymbol{\theta}_{T, j}$ | $\boldsymbol{d}_{T, j}$ | $\boldsymbol{a}_{T, j}$ | $\boldsymbol{\alpha}_{\boldsymbol{T}, \boldsymbol{j}}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $\theta_{T, T M C \text { aa }}$ | 0 | 0 | $\pi / 2$ |
| 2 | $\theta_{T, T M C ~ f e}$ | 0 | $L_{T, M}$ | 0 |
| 3 | $\theta_{T, M C P} f e$ | 0 | $L_{T, P}$ | 0 |
| 4 | $\theta_{T, I P}$ | 0 | $L_{T, D}$ | 0 |

Table 3. D-H Parameters for the Thumb
Equation 23 represents the direct kinematics for the thumb as follows:

$$
\begin{equation*}
p_{T}={ }_{0}^{-1} T_{T}\left(u_{T}\right)_{4}^{0} T_{T}\left(\theta_{T, j}\right)={ }_{0}^{-1} T_{T}\left(u_{T}\right)_{1}^{0} T_{T}\left(\theta_{T, T M C_{\_} a a}\right)_{2}^{1} T_{T}\left(\theta_{T, T M C_{\_} f e}\right)_{3}^{2} T_{T}\left(\theta_{T, M C P_{-} f e}{ }_{4}^{3} T_{T}\left(\theta_{T, I P}\right)\right. \tag{23}
\end{equation*}
$$

Where terms in this equation are:
$P_{T}$ represents a matrix that contains position and orientation of the thumb finger tip with respect to the center of the wrist, $u_{T}$ represents the vector between the center of wrist and the corresponding thumb reference frame, ${ }_{4}^{{ }_{4} T_{i}\left(\theta_{T, j}\right)}$ is a matrix that contains the homogeneous matrix between the thumb reference frame and its finger tip. This matrix is also a concatenation of the corresponding matrixes; which are obtained by the thumb joints $\left(\theta_{T, T M C \_a a,} \theta_{T, T M C_{-} f e} \theta_{T, M C P} f_{e} \theta_{T, I P}\right)$.

### 2.1.4 Homogeneous matrixes for the Thumb model

In order to simplify the equations, some variables are replaced (VR) for the corresponding variable showed in table 2.

| $\boldsymbol{\theta}_{T, j}$ | VR | $\boldsymbol{a}_{T, j}$ | VR | Link |
| :--- | :--- | :--- | :--- | :--- |
| $\theta_{T, T M C \text { aa }}$ | $\theta_{1}$ | 0 | 0 |  |
| $\theta_{T, T M C ~}$ e | $\theta_{2}$ | $L_{T, M}$ | $L_{1}$ | Metacarpal |
| $\theta_{T, M C P ~ f e}$ | $\theta_{3}$ | $L_{T, P}$ | $L_{2}$ | Proximal |
| $\theta_{T, I P}$ | $\theta_{4}$ | $L_{T, D}$ | $L_{3}$ | Distal |

Table 4.

$$
\begin{aligned}
& c=\cos \\
& s=\sin
\end{aligned}
$$

$$
\begin{align*}
& { }^{0} T_{1}\left(\theta_{T, T M C_{\_} a a}\right)=\left[\begin{array}{cccc}
c \theta_{T, T M C_{\_} a a} & 0 & s \theta_{T, T M C_{\_} a a} & 0 \\
s \theta_{T, T M C_{\_} a a} & 0 & -c \theta_{T, T M C_{\_} a a} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{24}\\
& { }^{1} T_{2}\left(\theta_{T, T M C_{\_} f e}\right)=\left[\begin{array}{cccc}
c \theta_{T, T M C_{\_} f e} & -s \theta_{T, T M C_{\_} f e} & 0 & L_{T, M} c \theta_{T, T M C_{\_} f e} \\
s \theta_{T, T M C_{\_} f e} & c \theta_{T, T M C_{\_} f e} & 0 & L_{T, M} s \theta_{T, T M C_{\_} f e} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{25}\\
& { }^{2} T_{3}\left(\theta_{T, M C P}\right)=\left[\begin{array}{cccc}
c \theta_{T, M C P} & -s \theta_{T, M C P} & 0 & L_{T, P} c \\
s \theta_{T, M C P} \\
\tau \theta_{T, M C P} & c \theta_{T, M C P} & 0 & L_{T, P} s \theta_{T, M C P} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{26}\\
& { }^{3} T_{4}\left(\theta_{T, I P}\right)=\left[\begin{array}{cccc}
c \theta_{T, I P} & -s \theta_{T, I P} & 0 & L_{T, D} c \theta_{T, I P} \\
s \theta_{T, I P} & c \theta_{T, I P} & 0 & L_{T, D} \theta_{T, I P} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{27}\\
& { }^{-1} T_{0}\left(u_{T \text { Tumb }}\right)=\left[\begin{array}{cccc}
1 & 0 & 0 & u_{T, x} \\
0 & 1 & 0 & u_{T, y} \\
0 & 0 & 1 & u_{T, z} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{28}
\end{align*}
$$

$$
{ }^{0} T_{4}=\left[\begin{array}{cccc}
n_{1 x} & s_{1 x} & a_{1 x} & P_{1 x}  \tag{29}\\
n_{1 y} & s_{1 y} & a_{1 y} & P_{1 y} \\
n_{1 z} & s_{1 z} & a_{1 z} & P_{1 z} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Where:

$$
\begin{gather*}
n_{1 x}=\left(\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{c}_{4}+\left(-\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{c}_{3}\right) \mathrm{s}_{4}  \tag{30}\\
n_{1 y}=\left(\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{c}_{4}+\left(-\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{c}_{3}\right) \mathrm{s}_{4}  \tag{31}\\
n_{1 z}=\left(\mathrm{s}_{2} \mathrm{c}_{3}+\mathrm{c}_{2} \mathrm{~s}_{3}\right) \mathrm{c}_{4}+\left(-\mathrm{s}_{2} \mathrm{~s}_{3}+\mathrm{c}_{2} \mathrm{c}_{3}\right) \mathrm{s}_{4}  \tag{32}\\
s_{1 x}=-\left(\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{s}_{4}+\left(-\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{c}_{3}\right) \mathrm{c}_{4}  \tag{33}\\
s_{1 y}=-\left(\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{s}_{4}+\left(-\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{c}_{3}\right) \mathrm{c}_{4}  \tag{34}\\
s_{1 z}=-\left(\mathrm{s}_{2} \mathrm{c}_{3}+\mathrm{c}_{2} \mathrm{~s}_{3}\right) \mathrm{s}_{4}+\left(-\mathrm{s}_{2} \mathrm{~s}_{3}+\mathrm{c}_{2} \mathrm{c}_{3}\right) \mathrm{c}_{4}  \tag{35}\\
a_{1 x}=s_{1}  \tag{36}\\
a_{1 y}=-\mathrm{c}_{1}  \tag{37}\\
a_{1 z}=0  \tag{38}\\
P_{1 x}=\left(\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{c}_{4} \mathrm{~L}_{3}+\left(-\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{c}_{3}\right) \mathrm{s}_{4} \mathrm{~L}_{3}+\left(\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{c}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{L}_{2}+\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{~L}_{1}  \tag{39}\\
P_{1 y}=\left(\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{c}_{4} \mathrm{~L}_{3}+\left(-\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{c}_{3}\right) \mathrm{s}_{4} \mathrm{~L}_{3}+\left(\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{c}_{3}-\mathrm{s}_{1} \mathrm{~s}_{2} \mathrm{~s}_{3}\right) \mathrm{L}_{2}+\mathrm{s}_{1} \mathrm{c}_{2} \mathrm{~L}_{1}  \tag{40}\\
P_{1 z}=\left(\mathrm{s}_{2} \mathrm{c}_{3}+\mathrm{c}_{2} \mathrm{~s}_{3}\right) \mathrm{c}_{4} \mathrm{~L}_{3}+\left(-\mathrm{s}_{2} \mathrm{~s}_{3}+\mathrm{c}_{2} \mathrm{c}_{3}\right) \mathrm{s}_{4} \mathrm{~L}_{3}+\left(\mathrm{s}_{2} \mathrm{c}_{3}+\mathrm{c}_{2} \mathrm{~s}_{3}\right) \mathrm{L}_{2}+\mathrm{s}_{2} \mathrm{~L}_{1} \tag{41}
\end{gather*}
$$

### 2.2 Inverse Kinematics

The model of the two finger types described in the forward kinematics section can generate a combination of movement with flexions, extensions, abductions, adductions and redundant cases. Figure 2 shows examples of flexion, extension and abductions.
The solution of the inverse kinematics can be derived from geometric methods (J.M. Selig, 2005), such as the relation of triangles. The hand can reproduce positive or negative movements with regard to a reference line for some joints. The movements of the fingers that can be in two different quadrants are: flexion/extension and abduction/adduction. The inverse kinematics is solved in all these cases of movement. In addition, the kinematic
behaviour of the kinematic chain depends on the MCP abduction/adduction for the model of the index, middle, ring and little fingers, because this joint can cause situations such as redundancy (Figure 3) if the value of $\theta_{i, M C P \_a a}=0$. Also, this joint can produce a more complex situation if the adduction is high. The most stable behaviour of the kinematic chain is when there are only abductions.


Fig. 2. a) Extension in the MCP joint of the index finger b) Flexion in all joints c) Index and ring fingers show abduction while index and middle finger show abduction in the MCP joint.


Fig. 3. Example of a redundant case of the index finger model when $\theta_{i, M C P \_a a}=0$.
When $\theta_{i, M C P \_a a}=0$, the model of the human hand is a redundant case and several solutions therefore exist. In this type of situation, the inverse kinematics can be solved by means of iterative methods such as the Newton-Raphson method that uses the Jacobean matrix (W.A.

Wolovich \& H. Elliot, 1984) and (A. Balestino et al., 1984). To solve this redundant case correctly, constraints have been implemented to solve with a convergent solution.

### 2.2.1 Inverse kinematics for the index, middle, ring and little fingers

To solve the inverse problem for each finger, the first step to take is to measure the orientation and end position of the fingertip.


Fig. 4. Vectorial representation of the inverse kinematic model.
With this, it is possible to obtain a homogeneous matrix [n s a P]. Algebraically, for the first degree of freedom of the CMC joint the following is obtained:

$$
\begin{equation*}
\theta_{i, C M C}=a \tan \left[\frac{a_{2 y}}{a_{2 x}}\right] \tag{42}
\end{equation*}
$$

In the same way, the value of the CMC joint was obtained algebraically, the MCP abduction/adduction joint as

$$
\begin{equation*}
\theta_{i, M C P_{-} a a}=a \tan \left[\frac{a_{2 x}}{\frac{\cos \left(\theta_{i, C M C}\right)}{a_{2 z}}}\right] \tag{43}
\end{equation*}
$$

Dependency exists among the following joints: flexions for MCP_fe, PIP and DIP. They are solved by means of geometric methods. With vector $P_{2}$ it is possible to obtain a point $J_{2}$ with the following expression:

$$
\begin{equation*}
J_{2}=P_{2}-\left[L_{7} * n_{2}\right] \tag{44}
\end{equation*}
$$

Another vector H is calculated with the CMC information and length $L_{4}$ of the metacarpal link such as:

$$
\begin{gather*}
H_{x}=L_{4} * \cos \left(\theta_{i, C M C}\right) \\
H_{y}=L_{4} * \sin \left(\theta_{i, C M C}\right) \\
H_{z}=0  \tag{45}\\
H=\left[\begin{array}{lll}
H_{x} & H_{y} & H_{z}
\end{array}\right]^{T}
\end{gather*}
$$

With information $J_{2}$ and $H$ it is possible to obtain the vectors $u, r_{2}$ and $r_{3}$ as shown in Figure 4.

$$
\begin{gather*}
u=J_{2}-H  \tag{46}\\
r_{2}=\operatorname{norm}\left(J_{2}\right)  \tag{47}\\
r_{3}=\operatorname{norm}(u) \tag{48}
\end{gather*}
$$

With vectors $u, r_{2}, r_{3}$ and lengths $L_{4}, L_{5}, L_{6}$ and using the law of cosines the angles $\beta_{4}, \beta_{5}$ and $\beta_{6}$ are obtained.

$$
\begin{align*}
& \beta_{4}=a \cos \left(\frac{L_{4}^{2}+r_{3}^{2}-r_{2}^{2}}{2 L_{4} r_{3}}\right)  \tag{49}\\
& \beta_{5}=a \cos \left(\frac{L_{5}^{2}+r_{3}^{2}-L_{6}^{2}}{2 L_{5} r_{3}}\right)  \tag{50}\\
& \beta_{6}=a \cos \left(\frac{L_{6}^{2}+L_{5}^{2}-r_{3}^{2}}{2 L_{6} L_{5}}\right) \tag{51}
\end{align*}
$$

MCP flexion/extension is obtained as:

$$
\begin{equation*}
\theta_{i, M C P-f e}=\pi-\beta_{5}-\beta_{4} \tag{52}
\end{equation*}
$$

The extension \& hyperextension for the MCP joint is obtained as:

$$
\begin{equation*}
\theta_{i, M C P}{ }_{\text {hyp }}=\beta_{4}-\pi-\beta_{5} \tag{53}
\end{equation*}
$$

The PIP joint is obtained as:

$$
\begin{equation*}
\theta_{i, P I P}=\pi-\beta_{6} \tag{54}
\end{equation*}
$$

Finally, the DIP joint is obtained by the algebraic method.

$$
\begin{align*}
& \mathrm{k}_{1}=\left(\mathrm{c}_{2} \mathrm{c}_{4} \mathrm{c}_{3} \mathrm{c}_{1}-\mathrm{c}_{4} \mathrm{~s}_{1} \mathrm{~s}_{3}-\mathrm{c}_{2} \mathrm{~s}_{4} \mathrm{c}_{1} \mathrm{~s}_{3}-\mathrm{s}_{4} \mathrm{c}_{3} \mathrm{~s}_{1}\right) \mathrm{n}_{\mathrm{x}}+ \\
& \left(\mathrm{c}_{2} \mathrm{c}_{4} \mathrm{c}_{3} \mathrm{~s}_{1}+\mathrm{c}_{4} \mathrm{c}_{1} \mathrm{~s}_{3}-\mathrm{c}_{2} \mathrm{~s}_{4} \mathrm{~s}_{1} \mathrm{~s}_{3}+\mathrm{s}_{4} \mathrm{c}_{3} \mathrm{c}_{1}\right) \mathrm{n}_{\mathrm{y}}+  \tag{55}\\
& \left.\left(\mathrm{s}_{4} \mathrm{~s}_{3}-\mathrm{c}_{4} \mathrm{c}_{3}\right) \mathrm{s}_{2}\right) \mathrm{n}_{\mathrm{z}} \\
& \mathrm{k}_{2}=\left(-\mathrm{s}_{4} \mathrm{c}_{2} \mathrm{c}_{1} \mathrm{c}_{3}+\mathrm{s}_{4} \mathrm{~s}_{1} \mathrm{~s}_{3}-\mathrm{c}_{4} \mathrm{c}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}-\mathrm{c}_{4} \mathrm{c}_{3} \mathrm{~s}_{1}\right) \mathrm{n}_{\mathrm{x}}+ \\
& \left(-\mathrm{s}_{4} \mathrm{c}_{2} \mathrm{~s}_{1} \mathrm{c}_{3}-\mathrm{s}_{4} \mathrm{c}_{1} \mathrm{~s}_{3}-\mathrm{c}_{4} \mathrm{~s}_{1} \mathrm{c}_{2} \mathrm{~s}_{3}+\mathrm{c}_{4} \mathrm{c}_{3} \mathrm{c}_{1}\right) \mathrm{n}_{\mathrm{y}}+ \\
& \left.\left(\mathrm{c}_{4} \mathrm{~s}_{3}+\mathrm{s}_{4} \mathrm{c}_{3}\right) \mathrm{s}_{2}\right) \mathrm{n}_{\mathrm{z}}
\end{align*}
$$

$$
\begin{equation*}
\theta_{i, D I P}=\operatorname{atan} 2\left[\mathrm{k}_{2}, \mathrm{k}_{1}\right] \tag{57}
\end{equation*}
$$

### 2.2.2 Inverse kinematics of the thumb

For the thumb the inverse kinematic is obtained by means of algebraic method.
TMC_aa is obtained as:

$$
\begin{equation*}
\theta_{T, T M C_{-} a a}=a \tan \left[\frac{P_{1 y}}{P_{1 x}}\right] \tag{58}
\end{equation*}
$$

$J_{1}$ and vectors $r_{1}, x_{1}$ and $x_{2}$ are obtained as:

$$
\begin{align*}
& J_{1}=P_{1}-\left[L_{3} n_{1}\right] \\
& r_{1}=\operatorname{norm}\left(J_{1}\right) \\
& x_{1}=\frac{r_{1}-L_{1}^{2}-L_{2}^{2}}{2 L_{1} L_{2}} \tag{59}
\end{align*}
$$

$$
x_{2}=\sqrt{1-x_{1}^{2}}
$$

MCP is obtained as:

$$
\begin{equation*}
\theta_{T, M C P}=a \tan 2\left[x_{2}, x_{1}\right] \tag{60}
\end{equation*}
$$

Auxiliary variables are calculated such as:

$$
\begin{gather*}
x_{3}=\left(L_{1}+\left(L_{2} x_{1}\right)\right) \\
x_{4}=L_{2} x_{2} \\
J_{x y}=\sqrt{J_{1 x}^{2}+J_{1 y}^{2}} \\
x_{5}=\frac{J_{1 z} x_{4}+J_{x y} x_{3}}{x_{3}^{2}+x_{4}^{2}}  \tag{61}\\
x_{6}=\frac{J_{1 z} x_{3}+J_{x y} x_{4}}{x_{3}^{2}+x_{4}^{2}}
\end{gather*}
$$

TMC_fe is obtained as:

$$
\begin{align*}
& \theta_{T, T M C_{-} f e}=a \tan 2\left[x_{6}, x_{5}\right]  \tag{62}\\
& \theta=a \tan 2\left[n_{1 z}, \sqrt{n_{1 x}^{2}+n_{1 y}^{2}}\right] \tag{63}
\end{align*}
$$

Finally, IP is obtained as:

$$
\begin{equation*}
\theta_{T, I P}=\theta-\theta_{T, T M C_{-} f e}-\theta_{T, M C P} \tag{64}
\end{equation*}
$$

## 3. Main Constraints of Finger Movements

Joint finger movements are limited to a specific range because of static constraints, intrafinger constraints and inter-finger constraints. Intra-finger and inter-finger constraints are often called kinematic constraints, and these are the ones responsible for producing natural movements both statically and kynematically. However, this range of movement is somewhat ambiguous because the range depends on various factors involving human hand biomechanics.

### 3.1 Inter-finger constraints

This type of constraint refers to some dependency among fingers while they are in motion (J. Lin et al., 2000). The inter-finger constraints were obtained by using the hand model and Cyberglove. These types of constraints are coupled movements among the index, middle,
ring and little fingers. The relationship among angles with the middle, ring and little fingers has been measured to represent real movements of the hand model.
Middle, ring and little fingers share common flexor tendons. It implies involuntary movements due to this strong coupling. These cases are defined by:

- Flexion in the ring finger $\theta_{R, M C P \_f e}$ due to a flexion in the middle finger and no flexions of the index and little fingers. When this unique flexion is generated, the flexion of the ring finger is equal to a percentage of the middle finger flexion, as described in equation (65).

$$
\begin{equation*}
\theta_{R, M C P-f e} \approx \frac{2}{3} \theta_{M, M C P-f e} \tag{65}
\end{equation*}
$$

- Flexion in the ring finger $\theta_{R, M C P_{f} f e}$ is produced when there is flexion solely in the little finger. It is described in equation (66). A small flexion is also produced in the middle when flexion in the little is high.

$$
\begin{equation*}
\theta_{R, M C P-f e} \approx \frac{7}{12} \theta_{L, M C P-f e} \tag{66}
\end{equation*}
$$

- Flexion of middle and little fingers $\theta_{M, M C P_{-} f e} \theta_{L, M C P_{-} f e}$ are produced when there is flexion solely in the ring finger. Involuntary flexions are similar and proportional to the ring flexion as described in equation (67) and (68).

$$
\begin{align*}
& \theta_{R, M C P-f e}-\theta_{M, M C P-f e}<60^{\circ}  \tag{67}\\
& \theta_{R, M C P-f e}-\theta_{L, M C P-f e}<50^{\circ} \tag{68}
\end{align*}
$$

- Ring and little abduction/adductions are similar. In most cases is very difficult to change the abduction of a finger without changes in the other. Equation (69) represents this constraint.

$$
\begin{equation*}
\theta_{R, M C P_{-} a a} \approx \theta_{L, M C P_{-} a a} \tag{69}
\end{equation*}
$$

- Finally, another involuntary movement appears in the middle flexion when a unique flexion is done in the index finger. This relation is described in equation (70).

$$
\begin{equation*}
\theta_{M, M C P-f e} \approx \frac{1}{5} \theta_{I, M C P-f e} \tag{70}
\end{equation*}
$$

### 3.2 Intra-finger constraints

Intra-finger constraints refer to the dependency of internal joints. New intra-finger have been obtained according to the hand gesture. Gestures have been classified as prismatic and circular regardless of power or precision grasping (M.R. Cutkosky, 1989). Some experiments have been carried out in order to obtain these intra-finger relations.
Circular grasping implies a strong relation among finger joints. The most accepted intrafinger constraint is:

$$
\begin{equation*}
\theta_{D I P} \approx \frac{2}{3} \theta_{P I P} \tag{71}
\end{equation*}
$$

It has been analyzed by several researchers, such as (C.S. Fahn \& H. Sun, 2005) and has been efficiently checked in our experiments.
Equations (72) and (73) show two constraints used for index, middle, ring and little fingers in circular grasping.

$$
\begin{gather*}
\theta_{i, D I P} \approx \frac{2}{3} \theta_{i, P I P}  \tag{72}\\
\theta_{i, M C P-f e} \approx \frac{4}{3} \theta_{i, P I P} \tag{73}
\end{gather*}
$$

The following equations show intra-finger constraints used for circular grasping:

$$
\begin{align*}
& \theta_{T, T M C_{-} f e} \approx \frac{11}{10} \theta_{T, M C P}  \tag{74}\\
& \theta_{T, M C P} \approx \frac{4}{5} \theta_{T, I P}  \tag{75}\\
& \theta_{I, C M C} \approx \theta_{M, C M C}  \tag{76}\\
& \theta_{M, C M C} \approx \frac{1}{2} \theta_{R, C M C}  \tag{77}\\
& \theta_{M, M C P_{-} a a} \approx \frac{1}{5} \theta_{I, M C P_{-} a a}  \tag{78}\\
& \theta_{R, C M C} \approx \frac{2}{3} \theta_{L, C M C}  \tag{79}\\
& \theta_{R, M C P_{-} a a} \approx \frac{1}{2} \theta_{L, M C P_{-} a a}  \tag{80}\\
& \theta_{R, D I P} \approx \frac{1}{2}\left(\theta_{M, D I P}+\theta_{L, D I P}\right) \tag{81}
\end{align*}
$$

Also, equations (76), (77), (78), (79) and (80) are used for prismatic trajectories. Equations (82) and (83) show two constraints used for index, middle, ring and little fingers in prismatic grasping.

$$
\begin{equation*}
\theta_{i, M C P-f e} \approx \frac{3}{2} \theta_{i, P I P} \tag{82}
\end{equation*}
$$

$$
\begin{equation*}
\theta_{i, P I P} \approx 2 \theta_{i, D I P} \tag{83}
\end{equation*}
$$

Finally, the following equations show intra-finger constraints for prismatic grasping.

$$
\begin{align*}
\theta_{T, T M C_{-} f e} & \approx \frac{10}{11} \theta_{T, M C P}  \tag{84}\\
\theta_{T, M C P} & \approx \frac{6}{5} \theta_{T, I P} \tag{85}
\end{align*}
$$

## 4. Simplified Human Hand Models

This section describes simplified human hand models that properly represent the kinematic behaviour of the human hand in accordance with the precision and application required.
The human hand model of 24 DoF is used as a basis for comparison among simplified hand models with fewer degrees of freedom than the 24 DoF of the hand model described in section 2. Kinematic constraints are used in order to obtain simplified hand models, which allow for reducing the number of independent variables or joints in the original model. In other words, with few independent variables is possible to reconstruct a gesture of 24 degrees of freedom with an acceptable error with respect to the original gesture of 24 degrees of freedom not reconstructed. In previous works, two simplified hand models with 6 and 9 DoF have been evaluated in (S. Cobos et al., 2008a) and (S. Cobos et al. 2008b). Simplified human hand models are obtained using dependent and independent variables, theses dependent variables or dependent joints are calculated using kinematic constraints such as are showed in equations (65) to (85).
Table 3 shows simplified hand models from 1 to 24 degrees of freedom, and the independent variables used by each simplified hand model. Models 1 to 6 DoF are appropriate for circular power grasps. To control a gesture with one degree of freedom has been demonstrated previously in a robotic hand e.g. The Tuat/Karlsruhe Hand (N. Fukaya et al., 2000) is designed with 24 joints and 1 DoF controlled, this type of device is able to do circular power grasps. In this category the models are capable of performing power grasps with security and stability in the grip without achieving a great precision and skill in handling for precision grasps. Greater precision and dexterity is derived from 9 degrees of freedom, thus is possible to carry out precision grasps. Simplified hand models of 9 to 13 DoF are more precise for both types of grasp: precision and power grasps. Finally, a higher level of realism and sensitivity is achieved with models from 15 to 24 DoF. Table 3, the model from 1 to 9 DoF shows all independent variables that are used. From the model with 10 to 24 DoF uses the same variables from the previous model plus an additional variable e.g. the model with 10 DoF uses the same variables of the model with 9 DoF plus an additional variable.
The reduction of elements from 13 to 1 DoF leads to increasingly rely on interpolations and constraints associated with an increased error in the grip trajectory when the dependent variables are obtained. Somehow, this technique depends on optimizing the functionality of a particular inter-finger or intra-finger constraint.


Table 3. Simplified hand models reconstructed with kinematic constraints.
Many manipulations involve similar movements among fingers, e.g., a gesture done with the information of five fingers can be simplified by using only the information provided for three fingers, in this case these three fingers can be thumb, index and ring, creating the same
movement for the middle finger through the information of the index and the little finger through the information of the ring finger. The simplified hand models should be used depending on the relation between the number of degrees of freedom and the allowed error in the application.
The degree of dexterity that can be achieved depends largely on the largest number of independent variables having thumb and index finger inside the SHH. The abduction of the thumb and index fingers are very important because at least one degree of freedom from these fingers are considered as independent variables in all the simplified hand models, thus the flexion of the DIP joint of the index finger is included in all the simplified hand models. The abduction/adduction of the thumb TMC joint is important because with a flexion of the IP joint can produce the opposition of the thumb with the other fingers. It's from SHH with 4 DoF where the abduction of the MCP joint takes importance because allows better positioning of the finger, not SHH with 3 DoF that contain just a flexion in the index finger. In summary the universal joints of the thumb and index fingers are very important for obtaining simplified hand models because of the information they provide.

### 4.1 Error of simplified hand models

The original posture of the 24 DoF is considered the "ideal posture" or "ideal trajectory" to determine the final position of the fingertip with the use of the forward kinematics; the same forward kinematics is used to obtain the final position with the reconstructed vectors. The relative error $\left(\delta_{i}\right)$ is obtained with the original position $\left(o p_{i}\right)$ and the reconstructed position $\left(r p_{i}\right)$ such as:

$$
\begin{equation*}
\delta_{i}=\frac{\left|r p_{i}-o p_{i}\right|}{\left|r p_{i}\right|} \times 100 \% \tag{86}
\end{equation*}
$$

The trajectory is conformed of $n$ positions; thus the average error for each finger is:

$$
\begin{equation*}
\overline{\delta_{i}}=\frac{\sum_{k=1}^{n} \delta_{i}}{n} \tag{87}
\end{equation*}
$$

Where $n$ is the number of positions and $i=$ Thumb, Index, Middle, Ring, Little. Finally, the reconstruction error is calculated using the following expression.

$$
\begin{equation*}
\text { error }=\frac{\sum_{k=1}^{n} \overline{\delta_{i}}}{5}+\Delta c \tag{88}
\end{equation*}
$$

Where $\Delta c$ is a parameter of calibration this can vary $1-2 \%$ among users according to their hand size.

## 5. Conclusion

In this chapter, the forward and inverse kinematic models of the human hand with 24 DoF have been presented in detail. Thanks to the kinematic model and with the use of the cyberglove several kinematic constraints were developed for consequently obtained simplified human hand models. This simplified human hand models are useful for diverse applications that require security, stability, sensitivity, dexterity, realism or velocity in calculate kinematic parameters.
These simplified human hand models represent a significant reduction in the number of independent variables needed to describe a hand gesture, taking into account the opportunity to perform a specific manipulation with fewer elements to control for applications where control with many degrees of freedom is complex or computationally expensive. Finally, the human hand model with 24 DoF serves for applications that requires a greater realism, sensitivity in handling or description of a human hand gesture.

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Robotics research，especially mobile robotics is a young field．Its roots include many engineering and scientific disciplines from mechanical，electrical and electronics engineering to computer，cognitive and social sciences． Each of this parent fields is exciting in its own way and has its share in different books．This book is a result of inspirations and contributions from many researchers worldwide．It presents a collection of a wide range of research results in robotics scientific community．We hope you will enjoy reading the book as much as we have enjoyed bringing it together for you．

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