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Robust nonlinear control of a 7 DOF model-scale helicopter under wind gusts using disturbance observers

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1. Introduction

Nowadays, high levels of agility, maneuverability and capability of operating in reduced visual environments and adverse weather conditions are the new trends of helicopter design. Helicopter flight control systems should make these performance requirements achievable by improving tracking performance and disturbance rejection capability. Robustness is one of the critical issues which must be considered in the control system design for such high-performance autonomous helicopter, since any mathematical helicopter model, especially those covering large flight domains, will unavoidably have uncertainty due to the empirical representation of aerodynamic forces and moments.
Recently the control problems of unmanned scale helicopter have been attracted extensively attention of control researchers. As the helicopter can hover, it is used to implement many important flight missions such as rescue, surveillance, security operation, traffic monitoring, etc. However, helicopter, which is difficult to hover, is more complicated than other familiar control objects. Helicopter is dynamic unstable when it flights in hover mode at nearly zero forward speed. Moreover, the helicopter is open-loop unstable and most mathematical model contain a moderate-high degree of uncertainty models associated with neglected dynamics and poorly understood aeromechanical couplings. Therefore, it is very important to design a stable controller for unmanned helicopter.
Many previous works focus on (linear and nonlinear, robust, ...) control (Beji and Abichou, 2005) (Frazzoli et al., 2000) (Koo and Sastry, 1998), including a particular attention on the analysis of the stability (Mahony and Hamel, 2004), but very few works have been made on the influence of wind gusts acting on the flying system, whereas it is a crucial problem for out-door applications, especially in urban environment : as a matter of fact, if the autonomous flying system (especially when this system is relatively slight) crosses a crossroads, it can be disturbed by wind gusts and leave its trajectory, which could be critical in a highly dense urban context.
In (Martini et al., 2008), thw controllers (an approximate feedback control AFLC and approximate disturbance observer AADRC) are designed for a nonlinear model of a 7 DOF helicopter using in its approximate minimum phase model. In (Pflimlin et al., 2004), a

control strategy stabilizes the position of the flying vehicle in wind gusts environment, in spite of unknown aerodynamic efforts and is based on robust backstepping approach and estimation of the unknown aerodynamic efforts.
The purpose of this chapter is to present the stabilization (tracking) with motion planning of a 7 DOF disturbed helicopter VARIO Benzin Trainer (Lozano et al. (2005)). As a feedback control, a dynamic decoupling method obtained with an approximate minimum phase model (Koo and Sastry (1998)) is proposed AFLC. To deal with uncertainty and vertical wind gust an approximate disturbance observer is added AADRC (Martini et al. (2008)). Simulations show that AADRC is more effective than the AFLC, ie. the norm of the tracking error are lower in presence of disturbances (small body forces, air resistance and vertical wind gust). However, in the presence of nonlinear disturbances the system after linearization remains nonlinear. The observer used here overcomes easily this nonlinearities by an inner estimation of the external disturbances to impose desired stability and robustness properties on the global closed-loop system. The zero dynamics stabilizes quickly with the two controls. In section 2, a model of a disturbed helicopter is presented. In Section 3 the design and the application of two approaches of robust control for the approximate model are proposed. The study of model stability is carried out in section 4. In section 5, several simulations of the helicopter under vertical wind gust show the relevance of the two controls which are described in this work. Finally some conclusions are presented in section 6.

2. Model of the 7DOF disturbed helicopter

This section presents the nonlinear model of the disturbed helicopter VARIO Benzain Trainer starting from a non disturbed model (Vilchis, 2001). This dynamic model using the formalism of Euler-Lagrange is based on the vector of generalized coordinates q G $\mathbf{R}^7$ that can be defined by (Vilchis, 2001) :

$$q(t) = [x\ y\ z\ \varnothing\ \theta\ \psi\ \gamma]^T$$

where $\xi = [x \quad y \quad z]$ denote the position vector of the center of mass of the helicopter relative to the navigation frame for aircraft attitude problems I (Fig.2). γ is the main rotor azimuth angle. Let $(\varnothing, \theta$ and $\psi)$ denote the three Euler angles(yaw, pitch and roll angles) expressed in the body fixed frame $C = (cm, E_1, E_2, E_3)$ In addition,
we define $\eta = [\varnothing \quad \theta \quad \psi \quad \gamma]^T$, let $\Re : (cm, E_1, E_2, E_3) \rightarrow (O, E_x, E_y, E_z)$ be the rotation matrix representing the orientation of the body fixed frame C with respect to the inertial frame $\mathcal{I}$, where $\Re \in SO(3)$ is an orthogonal matrix :

$$\Re = \begin{bmatrix} C_\theta C_\varnothing & S_\psi S_\theta C_\varnothing - C_\psi S_\varnothing & C_\psi S_\theta C_\varnothing + S_\psi S_\varnothing \\ C_\theta S_\psi & S_\psi S_\theta S_\varnothing + C_\psi C_\varnothing & C_\psi S_\theta S_\varnothing - S_\psi C_\varnothing \\ -S_\theta & S_\psi C_\theta & C_\psi C_\theta \end{bmatrix} \tag{1}$$

The vector :

$$\Omega = \begin{bmatrix} \Omega_\varnothing & \Omega_\theta & \Omega_\psi & \Omega_\gamma \end{bmatrix}^T$$

denotes the angular velocity of the vehicle in the body fixed frame, which can also be written as : $\Omega = J_0\dot{\eta}$, where $\dot{\eta} \in R^4$ denote the generalized velocities. Then :

$$\Omega = \underbrace{\begin{bmatrix} -S_\theta & 0 & 1 & 0 \\ C_\theta S_\psi & C_\psi & 0 & 0 \\ C_\theta C_\psi & -S_\psi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{J_0} \underbrace{\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\gamma} \end{bmatrix}}_{\dot{\eta}} \tag{2}$$

This allows to define :

$$J_R^T = \begin{bmatrix} \Re & 0;0 & J_0^T \end{bmatrix}$$

In the case of VARIO helicopter, the vector of control inputs ***u G*** R^4 is given by :

$$u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T$$

where :

- $u_1(m)$ is the collective pitch angle (swashplate displacement) of the main rotor and the motor power.

F_c and $\mathbf{r_c}$	Analytical Expression
fxc	$T_M u_3$
fyc	$T_M UA + T_T$
fzc	$TM + D_{VI}$
	kbu - $T_M UAZ_M$
$Tyc = T\beta$	$kau + C_T + T_M u3 z_M$
Tzc $Tfj)$	$TTXT$
$\mathbf{T_7}$	$C_m ot + CM$

Table 1. Components of simplified external forces vector (Vilchis (2001))

- $u_2(m)$ is the collective pitch angle (swashplate displacement) of the tail rotor.
- $u_3(rad)$ is the longitudinal pitch angle of the main rotor.
- $u_4(rad)$ is the lateral pitch angle of the main rotor.

We can now calculate the Euler-Lagrange equation $L = T - U$ and obtain the motion equations of helicopter :

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \frac{\partial L}{\partial q_i} = F_L + F_w \tag{3}$$

where U is the potential energy of helicopter, T is its kinetic energy and $F_L = (F_c, T_C)^T$ represents the aerodynamical forces and torques applied to the helicopter at the center of mass (table 1), $F_w = (F_{ra}f, T_{ra}f)^T$ represents the external aerodynamical forces and torques produced by the vertical wind gust.

TM, TT are the main and tail rotor thrust. Here M stands for main rotor and T for tail rotor and D_{vi} is the amplitude of the drag force created by induced velocity in the disc of the main rotor, CM and CT are the main and tail rotor drag torque, respectively. $C_{mo}t = k_{mot}\ x$ **ui** is the engine torque which is assumed to be proportional to the first control input. $\mathbf{ai_s}$ and bi_s are the longitudinal and lateral flapping angles of the main rotor blades, k is the blades stiffness of main rotor. ZM and XT represent the main and tail rotor center localization with respect to the center of mass, respectively.

The development of equation (3) makes it possible to obtain the following equations Vilchis (001):

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=Q(q,\dot{q},u,v_{raf}) \tag{4}$$

with: $M(q)\in R^{7x7}$ is the inertia matrix. $C(q,\dot{q})\in R^{7x7}$ is the Coriolis and centrifugal forces matrix, $G(q)$ represents the vector of conservative forces. $Q(q,\dot{q},u,v_{raf})=J_R^T(F_L+F_w)$ is the vector of generalized forces. $F_L=D(\dot{q},u)u+A(\dot{q})u+B(\dot{q})$ The induced gust velocity is noted v_{raf} . Let us now introduce our approach based on induced velocity. While returning to the definition of the induced

velocity in three dimensions to the main rotor disc of the helicopter (R.W.Prouty, 1995) and (Vilchis, 2001) :

$$v_1=\sqrt{-\frac{\dot{x}^2+\dot{y}^2}{2}+\sqrt{\frac{(\dot{x}^2+\dot{y}^2)^2}{4}+v_h^4}}-\frac{\dot{z}}{2}+\sqrt{\frac{\dot{z}^2}{4}+v_h^2}-v_h$$

if A is the area of the main rotor disk then :

$$v_h=\sqrt{\frac{mg+D_{vi}}{2\rho A}} \tag{5}$$

The case $X = y = 0$ corresponds to a vertical flight :

$$v_1=v_2=-\frac{\dot{z}}{2}+\sqrt{\frac{\dot{z}^2}{4}+v_h^2} \tag{6}$$

where v_v and vh are respectively the induced velocities in the case of vertical flight and of hovering. Hereafter an ascending vertical flight with a gust velocity vraf is considered (the wind gust is vertical and it has a downward direction as the vertical induced velocity). In this case the total induced velocity becomes : vi = vv + vraf. If it is assumed that the helicopter flies at low speed, then the induced velocity in vertical flight and the induced velocity in hovering are almost the same ones : (vv « vh), so that vi = vh + vraf. Replacing this value in the force and the torque equations Fl provides the contribution STm, STt and SCm of wind on these aerodynamical actions.

$$\begin{aligned}
\delta T_M &= -c_{18}\dot{\gamma}v_{raf_v} \\
\delta T_T &= 0 \\
\delta C_M &= c_{17}\dot{\gamma}v_{raf_v}u_1+c_{36}\|V\|v_{raf_v}u_3+c_{38}\dot{\gamma}v_{raf_v}u_4 \\
&\quad +c_{43}\left(2v_vv_{raf_v}+v_{raf_v}^2\right)+c_{45}\|V\|v_{raf_v}+c_{46}\dot{\gamma}v_{raf_v}
\end{aligned} \tag{7}$$

Table 2 shows the change of thrust and torque of main rotor (parameter variations of the helicopter) acting on the helicopter in the presence of the vertical wind gust. These variations are calculated from a nominal position when the helicopter performs a hover flight : vrafv =0 : Tm = -77, 2 (N) and Cm = 3, 66 (N.m).

$v_{raf}(m/s)$	$\delta T_M(N)$	$\frac{\delta T_M}{T_{M_o}}\%$	$\delta C_M(N.m)$	$\frac{\delta C_M}{C_{M_o}}\%$
0,68	-14	18 %	1	27 %
3	-62	80 %	6,4	160%

Table 2. - Forces and torques variations for different vertical wind gusts

We can decompose the dynamique of (4) into two dynamics, a slowly translational dynamic and a fastly rotational dynamic, where m is the total mass of the helicopter, g is the gravitation constant.

$$\begin{aligned} m\ddot{\xi} &= \Re\left(F_c + F_{raf}\right) - mgE_z \\ M(\eta)\ddot{\eta} &= J_0^T\left(\tau_c + \tau_{raf}\right) - C(\eta,\dot{\eta})\dot{\eta} - G(\eta) \end{aligned} \tag{8}$$

3. Design and implementation of the control

3.1. Approximate feedback linearization control (AFLC)

The expressions of forces and torques (which contained 4 controls $[u_1,u_2,u_3,u_4]$) are very complex and have a strong nonlinearities. Therefore, it is appropriate to consider the main rotor thrust $\mathbf{T_M}$ and the $\tau_{\phi},\tau_{\theta},\tau_{\psi}$ couples as the new vector of control inputs. Then the real controls can be calculated. The objective of the control flight is to design an autopilot $\left(T_M,\tau_{\phi},\tau_{\theta},\tau_{\psi}\right)$ for the miniature helicopter to let the vertical, lateral, longitudinal and yaw attitude dynamics track a desired smooth trajectories $Y_d = \left(x_d, y_d, z_d\right)$ and ϕ_d : the tracking errors $e_{\xi} = \xi - Y_d$ and $e_{\phi} = \phi - \phi_d$ should converge asymptotically to zero in the presence of vertical wind gust. The calculation of the relative degrees gives : $r_1 = r_2 = r_3 = \mathbf{r_4} = 2$, the standard helicopter model have a dimension $n = 14$ and : $\mathbf{r_1} + \mathbf{r_2} + r_3 + \mathbf{r_4} = 8 < n = 14$, this implies the existence of an internal dynamics. In order to ensure system stability, we must analyze the internal stability of the system by studying the zero dynamics. A simulation study of the model (9) shows that the zero dynamics, parameterized by $\left\{\theta,\psi,\gamma,\dot{\theta},\dot{\psi},\dot{\gamma}\right\}$, is not asymptotically stable since the equilibrium point is surrounded by a family of periodic orbits. It can also be shown that exact state-space linearization fails to transforme the system into a linear and controllable system. Hence, it is impossible to fully linearize the nonlinear system (Koo and Sastry, 1998). Neglecting the couplings between moments and forces, we show that the approximated system with dynamic decoupling is full state linearizable.

Starting from (8), and neglecting the small body forces ($f_{xc} = f_{yc} = 0$) and for $F_{raf} = T_{raf} = 0$, an approximate model of the dynamics of translation is obtained :

$$\ddot{\xi}_a = \frac{1}{m}\left(\Re\begin{bmatrix}0 & 0 & T_M\end{bmatrix}^T - \begin{bmatrix}0 & 0 & mg\end{bmatrix}^T\right) \tag{9}$$

In order to make the approximate model (9) completely linearizable, we will use a dynamic extension procedure. This is done by adding two integrators for the thrust control input T_M. To simplify the expressions, we propose the change of input variables :

$$\tilde{\tau} = -M(\eta)^{-1}C(\eta,\dot{\eta})\dot{\eta} - M(\eta)^{-1}G_7 + M(\eta)^{-1}J_0^T\tau_c \Rightarrow \ddot{\eta} = \tilde{\tau}$$

we will thus consider as the control inputs the vector :

$$\bar{w}_e = \left[\bar{T}_M \quad \tilde{\tau}_{\not{y}} \quad \tilde{\tau}_\theta \quad \tilde{\tau}_\varnothing\right]^T$$

where $\bar{T}_M = \ddot{T}_M$.

Using the input-output feedback linearization procedure of the position $\pounds_a$, we take the third time derivative of (9):

$$\begin{bmatrix} x_a^{(3)} \\ y_a^{(3)} \\ z_a^{(3)} \\ \ddot{\varnothing} \end{bmatrix} = \begin{bmatrix} * \\ * \\ * \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A_{e_1}} \begin{bmatrix} \dot{T}_M \\ \tilde{\tau}_{\not{y}} \\ \tilde{\tau}_\theta \\ \tilde{\tau}_\varnothing \end{bmatrix} \tag{10}$$

The decoupling matrix Aei has rank 2 only, and therefore is not invertible. Hereafter, we propose to use dynamic decoupling algorithm, and continue differentiating the position £a. At last, the iteration ends, since the decoupling matrix Ae(X) has full rank and is invertible (if Tm = 0). The extended system is in the following form:

$$\underbrace{\begin{bmatrix} x_a^{(4)} \\ y_a^{(4)} \\ z_a^{(4)} \\ \ddot{\varnothing} \end{bmatrix}}_{v_a} = \underbrace{\begin{bmatrix} b^P \\ b^\varnothing \end{bmatrix}}_{b^e} + \underbrace{\begin{bmatrix} A^p \\ A_\varnothing \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} \ddot{T}_M \\ \tilde{\tau}_{\not{y}} \\ \tilde{\tau}_\theta \\ \tilde{\tau}_\varnothing \end{bmatrix}}_{\ddot{w}_e} \tag{11}$$

for which the vector relative degree is { 4 4 4 2 }. The state vector of our extended system can be writhen as following:

$$X = \left[x \quad y \quad z \quad \varnothing \quad \theta \quad \not{y} \quad \gamma \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad \dot{\varnothing} \quad \dot{\theta} \quad \dot{\not{y}} \quad T_M \quad \dot{T}_M\right]^T$$

Its order is 16 and its control vector is $\bar{w}_e$. We can rewrite the true system in normal form using $(\zeta, \tilde{\eta})$, where:

$$\tilde{\eta} = (\tilde{\eta}_1(X) \quad \tilde{\eta}_2(X))$$

is such that the transformation $I(\zeta, \tilde{\eta})$ **-->** X define a coordinated change with the particularity that $\mathbf{n}$ depend only on Z and $\boldsymbol{n}$ (Isidori (1995)). Define Zi = x_a, Z^2 = $ya, Zi = za, Z^4 = P,$ and : $Zi = [\ Zi\ Zi\ Zi\ ]^T, Z\boldsymbol{i} = Z^4,$ We have a representation of the full state model helicopter (see (8)) :

$$\begin{aligned} \dot{\zeta}_1^p &= \zeta_2^p & \dot{\zeta}_1^\varnothing &= \dot{\zeta}_2^\varnothing \\ \dot{\zeta}_2^p &= \zeta_3^p + H\left(X, u, v_{raf}\right) & \dot{\zeta}_2^\varnothing &= b^\varnothing + A^\varnothing \bar{w}_e \\ \dot{\zeta}_3^p &= \zeta_4^p & \dot{\tilde{\eta}} &= f\left(\zeta, \tilde{\eta}, v_{raf}\right) \\ \dot{\zeta}_4^p &= b^p + A^p \bar{w}_e \end{aligned} \tag{12}$$

in which $H\left(X, u, v_{raf}\right)$ represents the small body forces and the vertical wind gust forces and torques:

$$H\left(X, u, v_{raf}\right) = \ddot{\xi} - \ddot{\xi}_a = \frac{T_M}{m} \Re\left[u_3 \quad u_4 + \quad \frac{T_T}{T_M} \quad \frac{F_{raf}}{T_M}\right]^T$$

It appears that the sum of relative degree of our extended system is 14, while its size is n_e = $dim(X)$ = 16. There is a difference of 2, which corresponds to the dynamics of the main rotor which is free and which creates a dynamics of order 2, but it is stable (by simulation). It may be noted that this persistent zero dynamics does not exist in the helicopter studied in (Koo and Sastry, 1998) (Mahony et al., 1999) ((Frazzoli et al., 2000) because our helicopter has 7DOF. We can then use the approximate system, the following control who linearizes it for the new controls v_a. We obtain the following equations : $\bar{w}_e = -A_e^{-1}b_e + A_e^{-1}v_a$ where:

$$v_a = \left[v_\xi^p \ \ v_\phi \right]^T$$

We can then apply the following tracking control law $\boldsymbol{v_a}$ for the approximate system (11), and the true system (12) :

$$\begin{aligned} v_\xi^p &= Y_d^{(4)} - \lambda_3\left(\zeta_4^p - Y_d^{(3)}\right) - \lambda_2\left(\zeta_3^p - \ddot{Y}_d\right) - \lambda_1\left(\zeta_1^p - Y_d\right) \\ v_\phi &= \phi_d^2 - \lambda_5\left(\zeta_2^\phi - \dot{\phi}_d\right) - \lambda_4\left(\zeta_1^\phi - \phi_d\right) \end{aligned} \tag{13}$$

3.2. Active and Approached Disturbance Rejection Control (AADRC)

In this work, a methodology of generic design is proposed to treat the disturbance. A second order system described by the following equation is considered (Han, 1999; Hou et al., 2001)

$$\ddot{y} = f\left(y, \dot{y}, d\right) + bu, \tag{14}$$

where f (.) represents the dynamics of the model and the disturbance, d is the input of unknown disturbance, u is the control input, and y is the measured output. An alternative method is presented by (Han, 1999) as follows. The system in (14) is initially increased:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_3 + bu, \quad \dot{x}_3 = \dot{f} \tag{15}$$

where xi = y, xi = y, x3 = f (y,y,d). f $(.)$ is treated as an increased state. Here f and $\boldsymbol{f}$ are unknown. By considering $f\left(y, \dot{y}, d\right)$ as a state, it can be estimated with a state estimator. Han in (Han, 1999) proposed a nonlinear observer for (15):

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + Lg\left(e, \alpha, \delta\right), \\ \hat{y} &= C\hat{x}, \end{aligned} \tag{16}$$

where:

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{17}$$

and:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}; L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \tag{18}$$

The observer error is $\boldsymbol{e} =$ $\boldsymbol{y}$ **-** $\boldsymbol{xx}\,\boldsymbol{i}$ and:

$$g_i\left(e, \alpha_i, \delta\right)_{|i=1,2,3} = \begin{cases} |e|\alpha_i \, sign(e) & |e| > \delta \\ \dfrac{e}{\delta^1 - \alpha_i} & |e| \le \delta \end{cases} \quad \delta > 0. \tag{19}$$

The observer is reduced to the following set of state equations, and is called extended state observer $(\boldsymbol{ESO})$:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1 g_1(e,\alpha,\delta), \\ \dot{\hat{x}}_2 = \hat{x}_3 + L_2 g_2(e,\alpha,\delta) + bu \\ \dot{\hat{x}}_3 = L_3 g_3(e,\alpha,\delta). \end{cases} \tag{20}$$

The dynamics of translation of the approximate helicopter has an order four (11) : $Y_a^{(4)} = \hat{v}_{\hat{\zeta}_1^p}$, with:

$$Y_a = \begin{bmatrix} \zeta_1^1 & \zeta_1^2 & \zeta_1^3 \end{bmatrix}^T$$

While the full system (12) can be put in the following form:

$$Y_a^{(4)} = f\left(Y_a, \dot{Y}_a, Y_a^{(2)}, Y_a^{(3)}, d\right) + \hat{v}_{\hat{\zeta}_1^p} \tag{21}$$

Here d represents $H(X,u,v_{ra}f)$ and the model uncertainty. Then the system (21) is estimated with the observer:

$$\begin{aligned} \dot{\hat{\zeta}}_1^p &= \hat{\zeta}_2^p + L_{1i} g_{1i}(\hat{e}_{1i}, \alpha_{1i}, \delta_{1i}) \\ \dot{\hat{\zeta}}_2^p &= \hat{\zeta}_3^p + L_{2i} g_{2i}(\hat{e}_{1i}, \alpha_{2i}, \delta_{2i}) \\ \dot{\hat{\zeta}}_3^p &= \hat{\zeta}_4^p + L_{3i} g_{3i}(\hat{e}_{1i}, \alpha_{3i}, \delta_{3i}) \\ \dot{\hat{\zeta}}_4^p &= \hat{\zeta}_1^p + \hat{\zeta}_5^p + L_{4i} g_{4i}(\hat{e}_{4i}, \alpha_{4i}, \delta_{4i}) \\ \dot{\hat{\zeta}}_5^p &= L_{5i} g_{5i}(\hat{e}_{5i}, \alpha_{5i}, \delta_{5i}) \end{aligned} \tag{22}$$

and for Ø-dynamics:

$$\begin{cases} \dot{\hat{\phi}}_1 = \hat{\phi}_2 + L_{16} g_{16}(\hat{e}_4, \alpha_4, \delta_4) \\ \dot{\hat{\phi}}_2 = \hat{\phi}_3 + L_{17} g_{17}(\hat{e}_4, \alpha_4, \delta_4) + \hat{v}_{\phi} \\ \dot{\hat{\phi}}_3 = L_{18} g_{18}(\hat{e}_4, \alpha_4, \delta_4) \end{cases} \tag{23}$$

where $(i = x, y, z)$, and:

$$\hat{v}_a = \begin{bmatrix} \hat{v}_{\hat{\zeta}_1^p} & \hat{v}_{\phi} \end{bmatrix}^T$$

The observer error is $\hat{e}_{1x} = \zeta \int_1^1 - \hat{\zeta} \int_1^1, \hat{e}_{1z} = \zeta \int_1^2, \hat{e}_{1z} = \zeta \int_1^3 - \hat{\zeta} \int_1^3 and \hat{e}_4 = \zeta \int_1^{\phi} - \hat{\phi}_1$ and *(4 = (* − 4>i*. The controls are then defined as follows (PD control):

$$\hat{v}_{\hat{\zeta}_1^p} = Y_d^{(4)} - \lambda_3\left(\hat{\zeta}_4^p - Y_d^{(3)}\right) - \lambda_2\left(\hat{\zeta}_3^p - \ddot{Y}_d\right) - \lambda_1\left(\hat{\zeta}_2^p - \dot{Y}_d\right) - \lambda_0\left(\hat{\zeta}_1^p - Y_d\right) - \hat{\xi}_5^p$$

$$\hat{v}_{\phi} = \phi_d^2 - \lambda_5\left(\hat{\phi}_2 - \dot{\phi}_d\right) - \lambda_4\left(\hat{\phi}_1 - \phi_d\right) - \hat{\phi}_3$$

The control signal $\hat{v}_a$takes into account of the terms which depend on the observer $\hat{\zeta}_1^1 \ldots \hat{\phi}_1$ The fifth part $\left(\hat{\zeta}_5^1, \hat{\zeta}_5^2, \hat{\zeta}_5^3\right)$ and $\hat{\phi}_3$ which also comes from the observer, is added to eliminate the effect of disturbance in this system (21) and in ϕ – dynamics.

4. Stability Analysis

In this section, the stability of the perturbed helicopter (12) controlled using observer based control law (24) is considered. To simplify this study, the demonstration is done to one input and one output (Hauser et al., 1992) and the result is applicable for other outputs. By defining the trajectory error

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{bmatrix} - \begin{bmatrix} Y_d \\ \dot{Y}_d \\ \ddot{Y}_d \\ Y_d^{(3)} \end{bmatrix} \tag{25}$$

we can write:

$$\begin{aligned} \dot{e} &= Ae + B\hat{e} + e\varphi\left(X,\hat{\zeta},v_{raf}\right) \\ \dot{\eta} &= f(\zeta,\eta) \end{aligned} \tag{26}$$

Where $\boldsymbol{A}$ is a stable matrix determined by pole placement:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 \end{bmatrix}$$

and:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 & 1 \end{bmatrix}$$

$\hat{e}$ is the observer error:

$$\begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 & \hat{e}_4 & \hat{e}_5 \end{bmatrix}^T = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 \end{bmatrix}^T - \begin{bmatrix} \hat{\zeta}_1 & \hat{\zeta}_2 & \hat{\zeta}_3 & \hat{\zeta}_4 & \hat{\zeta}_5 \end{bmatrix}^T$$

Morever:

$$e\varphi\left(X,\hat{\zeta},v_{raf}\right) = \begin{bmatrix} 0 & H\left(X,\hat{\zeta},v_{raf}\right) & 0 & 0 \end{bmatrix}^T$$

In this equation:

$$H\left(X,\hat{\zeta},v_{raf}\right) = e\rho\left(X,v_{raf}\right)u_a\left(X,\hat{\zeta}\right)$$

where:

$$u_a(X) = \begin{bmatrix} \ddot{T}_M & \tilde{\tau}_{\psi} & \tilde{\tau}_{\theta} & \tilde{\tau}_{\phi} \end{bmatrix}$$

$\boldsymbol{e}$ is a positive coupling constant. To simplify the study, we consider the case of a linear observer:

$$\begin{bmatrix}\dot{\hat{e}}_1\\ \dot{\hat{e}}_2\\ \dot{\hat{e}}_3\\ \dot{\hat{e}}_4\\ \dot{\hat{e}}_5\end{bmatrix}=\begin{bmatrix}-L_1 & 1 & 0 & 0 & 0\\ -L_2 & 0 & 1 & 0 & 0\\ -L_3 & 0 & 0 & 1 & 0\\ -L_4 & 0 & 0 & 0 & 1\\ -L_5 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\\ \hat{e}_4\\ \hat{e}_5\end{bmatrix}+\underbrace{\begin{bmatrix}0\\ H\left(X,\hat{\zeta},v_{raf}\right)\\ 0\\ 0\\ p\left(X,\hat{\zeta},v_{raf}\right)\end{bmatrix}}_{e\hat{\varphi}\left(X,\hat{\zeta},v_{raf}\right)} \tag{27}$$

Where $\boldsymbol{p}\left(X,\hat{\zeta},v_{raf}\right)=\ \dot{\zeta}_5$ represents the disturbance that comes from the system error and the coupling term $\boldsymbol{H}\left(X,\hat{\zeta},v_{raf}\right)$ we can write: $\dot{\hat{e}}=\hat{A}\hat{e}+e\hat{\varphi}\left(X,\hat{\zeta},v_{raf}\right)$ Where $\boldsymbol{A^V}$ is a stable matrix determined by pole placement.

Theorem : Suppose that:

- The zero dynamics of the approximate system (12) are locally exponentially stable and:
- The functions $\boldsymbol{H}\left(X,\hat{\zeta},v_{raf}\right)$ and $\boldsymbol{p}\left(X,\hat{\zeta},v_{raf}\right)$ are locally Lipschitz and continuous.
- Then for $\boldsymbol{e}$ small and for desired trajectories with sufficiently small values and derivatives $\left(Y_d,\dot{Y}_d,\ldots,Y_d^{(4)}\right)$ the states of the system (12) and the states of the observer (22, 23) will be bounded and the tracking error:

$$\|e\|=\|\zeta-Y_d\|\le ke \ \text{ pour } k<\infty \tag{28}$$

Proof : Since the zero dynamics of approximate model are assumed to be exponentially stable, a conservative Lyapunov theorem implies the existence of a Lyapunov function $V_1(\tilde{\eta})$ for the system $\dot{\tilde{\eta}}=f(0,\tilde{\eta})$ satisfying:

$$\begin{aligned}&k_1\|\tilde{\eta}\|^2\le V_1(\tilde{\eta})\le k_2\|\tilde{\eta}\|^2\\ &\frac{\partial V_1}{\partial\tilde{\eta}}f(0,\tilde{\eta},0)\le -k_3\|\tilde{\eta}\|^2\\ &\left\|\frac{\partial V_1}{\partial\tilde{\eta}}\right\|\le k_4\|\tilde{\eta}\|\end{aligned} \tag{29}$$

for some positive constants k_1,k_2,k_3 and k_4. We first show that $e,\hat{e},\tilde{\eta}$ are bounded. To this end, consider as a Lyapunov function for the error system ((26) and (27)):

$$V(e,\hat{e},\tilde{\eta})=e^TPe+\delta\hat{e}^T\hat{P}\hat{e}+\mu V_1(\tilde{\eta}) \tag{30}$$

where P, $\hat{P}$ > 0 are chosen so that : A^T P + PA = -I and $\hat{A}^T\hat{P}+\hat{P}\hat{A}=-I$ (possible since A and A are Hurwitz), p and S are a positive constants to be determined later. Note that, by assumption, Yd and its first derivatives are bounded : $\|\zeta\|\le\|e\|+b_d,\|\hat{\zeta}\|\le\|\hat{e}\|+\|e\|+b_d$ where $\|Y_d\|\le b_d$. The functions, $f(\zeta,\eta)$ and $\hat{\varphi}\left(X,\hat{\zeta},v_{raf}\right)$ are locally Lipschitz with $\hat{\varphi}(0,0,0)=0$ *et* $\hat{\varphi}_i(0,0,0)\hat{v}_a(0)=0$, we have:

$$\begin{aligned}&\left\|f\left(\zeta^1,\tilde{\eta}^1,v_{raf}{}^1\right)-f\left(\zeta^2,\tilde{\eta}^2,v_{raf}{}^2\right)\right\|\le l_q\left(\left\|\zeta^1-\zeta^2\right\|+\left\|\tilde{\eta}^1-\tilde{\eta}^2\right\|+\left\|v_{raf}^1+v_{raf}^2\right\|\right)\\ &\left\|2P\varphi\left(X,\hat{\zeta},v_{raf}\right)\right\|\le l_u\left(\|X\|+\|\hat{\zeta}\|+\|v_{raf}\|\right)\\ &\left\|2\hat{P}\hat{\varphi}\left(X,\hat{\zeta},v_{raf}\right)\right\|\le\hat{l}_u\left(\|X\|+\|\hat{\zeta}\|+\|v_{raf}\|\right)\end{aligned}$$

with l_q, l_u and l_u 3 positive reals. X is a locally diffeomorphism of $(\zeta, \tilde{\eta})$ namely that exists l_x is such that:

$$\|X\| \le l_x \left(\|\zeta\| + \|\tilde{\eta}\|\right)$$

Using these bounds and the properties of $V_1(.)$, we have:

$$\begin{aligned}
&\frac{\partial V_1}{\partial \tilde{\eta}} f\left(\zeta, \tilde{\eta}, v_{raf}\right) = \frac{\partial V_1}{\partial \tilde{\eta}} f\left(0, \tilde{\eta}, 0\right) + \frac{\partial V_1}{\partial \tilde{\eta}} f\left(\zeta, \tilde{\eta}, v_{raf}\right) - \frac{\partial V_1}{\partial \tilde{\eta}} f\left(0, \tilde{\eta}, 0\right) \\
&\le -k_3 \|\tilde{\eta}\|^2 + k_4 l_q \|\tilde{\eta}\| \left(\|e\| + b_d + \|v_{raf}\|\right)
\end{aligned} \tag{32}$$

Taking the derivative of $V(.,.)$ along the trajectory, we find:

$$\begin{aligned}
V &= e^T P e + \delta \hat{e}^T \hat{T} \hat{e} +_1 (\tilde{\eta}) \\
\dot{V} &= \dot{e}^T P e + e^T P \dot{e} + \delta \dot{\hat{e}}^T \hat{P} \hat{e} + \delta \hat{e}^T \hat{P} \dot{\hat{e}} + \mu \frac{\partial V_1}{\partial \eta} f(\zeta, \tilde{\eta}) \\
&\le -[\tfrac{3}{8} - \epsilon\{l_u(2 + l_x) - \tfrac{\delta}{2} \hat{l}_u (1 + l_x)\}] ||e||^2 + \tfrac{1}{2} - \delta + \\
&+ \epsilon\{\delta l_x (\tfrac{1}{2} l_u + 1) + \tfrac{5}{2} \hat{l}_u\} + 2(||B|| ||P||)^2 \\
&-[\tfrac{3}{4} \mu k_3 - \tfrac{1}{2} \delta \epsilon l_u l_x - 4(\epsilon l_u l_x + \mu k_4 l_q)^2] ||\eta||^2 - \\
&+ (2\epsilon l_u (1 + l_x) b_d)^2 + \mu \tfrac{(k_4 l_q b_D)^2}{k_3} + \tfrac{1}{2} [\delta \epsilon b_d (l_u l_x + \hat{l}_u)]^2 + \\
&+ \epsilon (l_u + \delta \hat{l}_u) ||v_{raf}||^2 + \tfrac{1}{2} \mu k_4 l_q ||v_{raf}||^2
\end{aligned}$$

Define:

$$\begin{aligned}
\mu_0 &= \frac{k_3}{32\left(l_u l_x + k_4 l_q\right)^2} \\
\delta &= 1 + 4\left(\|B\| \|P\|\right)^2 \\
e_1 &\le \frac{1}{8 l_u (2 + l_x)} \\
e_2 &\le \frac{-\frac{1}{2} + \frac{\delta}{2} - 2\left(\|B\| \|P\|\right)^2}{\delta \left[\frac{1}{2} l_u l_x + l_x + 2\hat{l}_u + l_u\right]} \\
e_3 &\le \frac{\mu}{4 \delta l_u l_x} \left[k_3 + k_4 l_q\right]
\end{aligned} \tag{33}$$

Then, for all $\mu \le \mu_0$ and all $e < min(\boldsymbol{\mu}, ei, ei, e\text{-}l)$ and for $\delta \ge \delta_0$, we have:

$$\begin{aligned}
\dot{V} \le &-\frac{\|e\|^2}{4} - \frac{\delta}{2} \|\hat{e}\|^2 - \frac{\mu k_3 \|\eta\|^2}{2} + \left[\mu \frac{\left(k_4 l_q\right)^2}{k_3} + \left[2 e l_u (1 + \right.\right. \\
&\left.\left. + l_x)\right]^2 + \frac{1}{2} \left[\delta e \left(l_u l_x + \hat{l}_u\right)\right] b_d^2 + e\left(l_u + \delta \hat{l}_u\right) \|v_{raf}\|^2 \\
&+ \frac{1}{2} \mu k_4 l_q \|v_{raf}\|^2
\end{aligned}$$

Thus, $\dot{V} < 0$ whenever $\|e\|, \|\hat{e}\|$ and $\|\tilde{\eta}\|$ is large which implies that $\|\hat{e}\|, \|e\|$ and $\|\tilde{\eta}\|$ and, hence, $\|\zeta\|, \|\hat{\zeta}\|$ and $\|X\|$ are bounded. The above analysis is valid in a neighborhood of the origin. By choosing b_d and $\boldsymbol{e}$ sufficiently small and with appropriate initial conditions, we can guarantee the state will remain in a small neighborhood, which implies that the effect of the disturbance on the closed-loop can be attenuated.

5. Results in simulation

In this section, simulations are presented to illustrate the performance and robustness of proposed controls laws when applied to the full helicopter model with the small body forces, air resistance and with the influence of vertical wind gust in the case of stabilization and trajectory tracking. The regulations parameters values used concern the dynamics model of VARIO 23cc helicopter Vilchis (2001). The initial conditions are:
$\xi_0 = \begin{pmatrix} 2 & 3 & -0.2 \end{pmatrix}, \dot{\xi}_0 = 0, \phi_0 = \frac{\pi}{30}$ and $\dot{\phi}_0 = 0$. The initial value adopted for the main rotor thrust force is $T_M = -73.5N$. It is exactly equal to the main rotor thrust force required for the helicopter to perform a hover flight:

- for ***AFLC***: The gains values $(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ and (λ_4, λ_5) are calculated by pole placement, 4 poles in -10 for the translational dynamics and 2 poles in -5 to **p.**
- for the AADRC: The bandwidths chosen to the observer: $w_{0x} = w_{0y} = w_{0z} = 50\text{rad/s}$, $w_{0\phi} = 25\text{rad/s}$. The choice of $\alpha = 0.5$ and $\delta = 0.1$ for x, y and z, and $\alpha = 0.5$ and $\boldsymbol{\delta} = 0.02$ for ϕ.

The induced gust velocity operating on the principal rotor is chosen as:
$v_{raf} = v_{gm} \sin(2.1 t_d)$ *if* $28 \le t \le 33$, where $t_d = t - 28$, the value of 2.1 represents $\frac{2\pi V}{L_u}$ where $\boldsymbol{V}$ in ***m/s*** is the height rise speed of the helicopter (V = 0.5 m/s) and $v_{gm} = 0.3\ m/S$ is the gust density. This density corresponds to an average vertical wind gust, and $L_u = 1.5m$ is its length (see Fig.4). We propose a simple trajectory to verify the applicability of designed controls, the trajectories are presented in Fig.2.
Now, we show some results that we have obtained through simulations. The both controls (with or without an observer) manage to stabilize $\theta, \dot{\theta}, \phi, \dot{\phi}$ and $\dot{\gamma}$, which are free. The difference between the two controls appears in Fig.3 where the tracking errors are less significant by using the *AADRC* than *AFLC.* The *AADRC* compensates quickly the effect of the disturbance and the small body forces that destabilize the system.

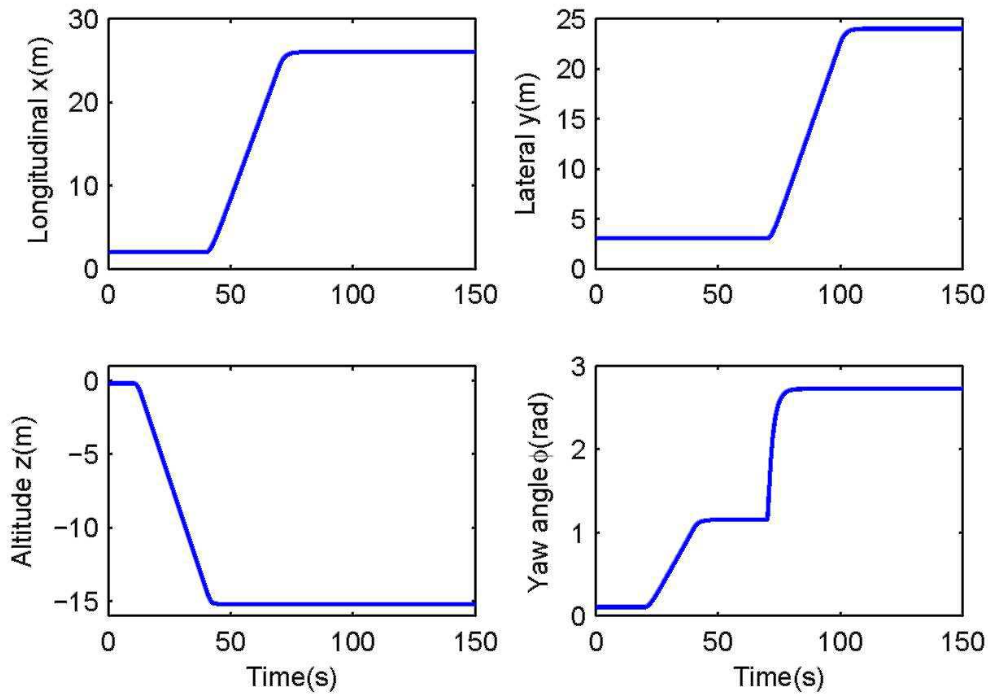


Fig. 2. Desired trajectories

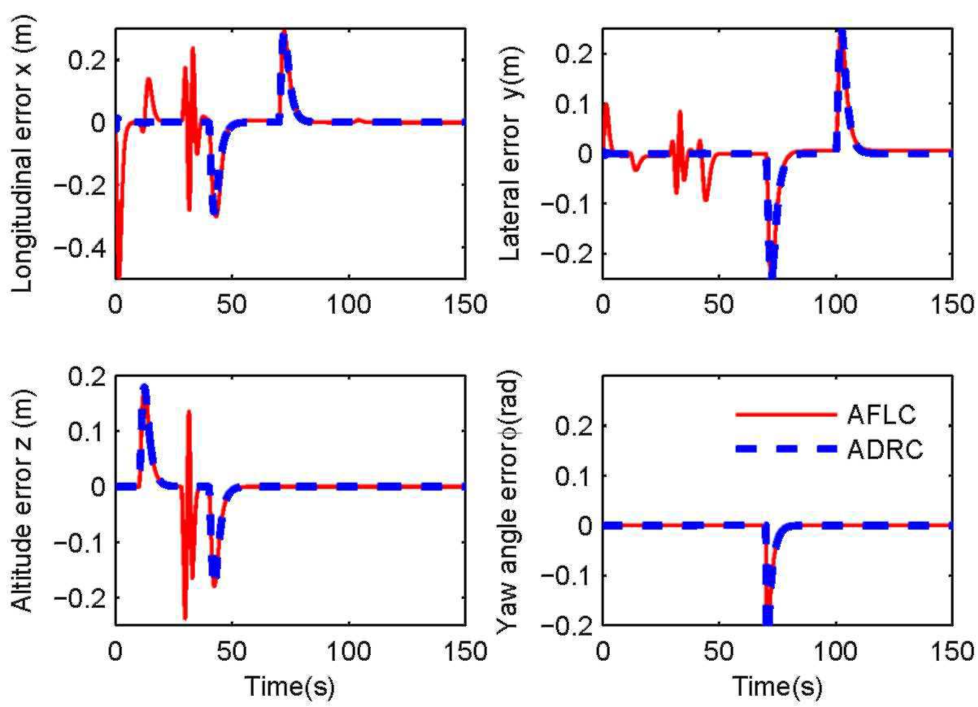


Fig. 3. Tracking errors

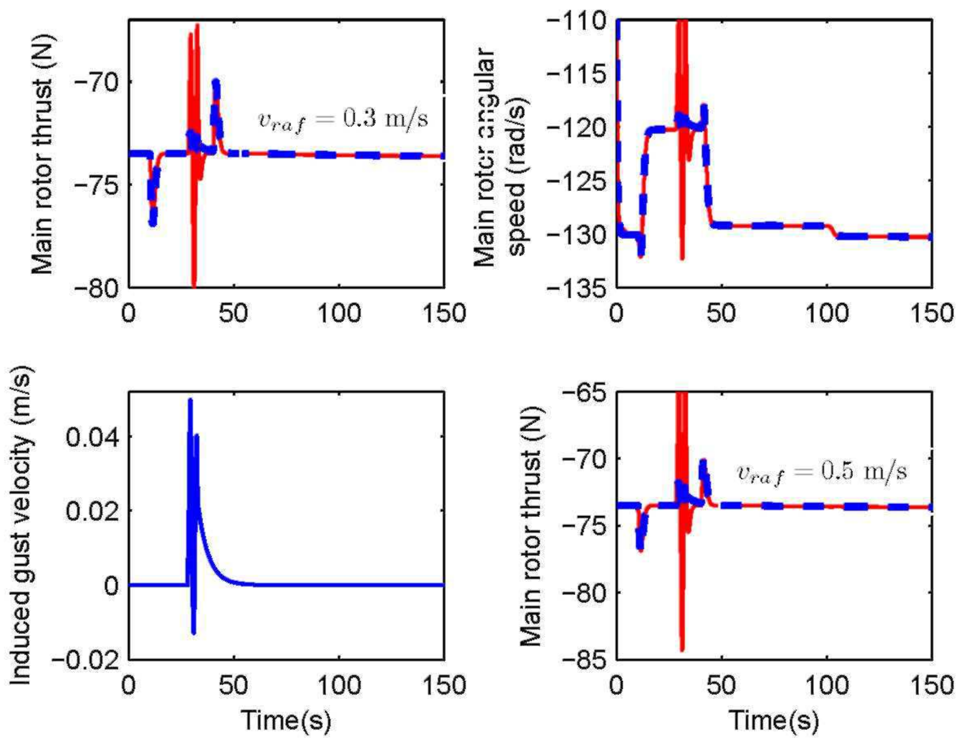


Fig. 4. Variations of $\boldsymbol{T_M}$, $\dot{\gamma}$ and $\boldsymbol{v_{raf}}$

One can observe that $\dot{\gamma} \to -130 rad/s$ (which correspond to hover flight). One can also notice that the main rotor angular speed is similar for the two controls as illustrated in Fig.4, but the $AADRC$ compensates quickly the effect of the disturbance.
One can see in Fig.4 that the main rotor thrust converges to values that compensate the helicopter weight, the drag force and the effect of the disturbance on the helicopter. The $AADRC$ allows the main rotor thrust T_M to be less away from its balance position than the other control. If one keeps the same parameters of adjustment for the two controls and using a larger vertical wind gust $\left(v_{raf} = 0.5m/s\right)$ (we have a PD controls), we find that the $AADRC$ give better results than the $AFLC$ (see Fig.4).

6. Conclusion

In this chapter, we presented a perturbed nonlinear model of a 7DOF helicopter. As a feedback control, a dynamic decoupling method obtained with an approximate minimum phase model is proposed $AFLC$. To deal with uncertainty and vertical wind gust a disturbance observer is added $AADRC$. Simulations show that the $AADRC$ is more effective than the $AFLC$, ie the tracking error are less important in presence of disturbance (small body forces, air resistance and vertical wind gust). However, in the presence of nonlinear disturbances the system after linearization remains nonlinear. The observer used here overcomes easily this nonlinearities by an inner estimation of the external disturbances to impose desired stability and robustness properties on the global closed-loop system. The zeros dynamics stabilizes quickly with the two controls.

Another investigation is planned by using the *AADRC* to stabilize the model of yaw angular displacement of a Tiny CP3 mini-helicopter mounted on experiment platform. We began by the identification setup and we finished with the experimental validation of the AADRC to stabilize the helicopter in presence of lateral wind gust. We find that the identified model is very close to measured model, and the validation results of the AADRC shows the efficiency of proposed control.

7. References

Beji, L. and A. Abichou (2005). Trajectory generation and tracking of a mini-rotorcraft. ***Proceedings of the 2005 IEEE International Conference on Robotics and Automation, Spain***, 2618-2623.

Frazzoli, E., M. Dahleh, and E. Feron (2000). Trajectory tracking control design for autonomous helicopters using a backstepping algorithm. ***Proceedings of the American Control Conference, Chicago, Illinois***, 4102-4107.

Han, J. (1999). Nonlinear design methods for control systems. Beijing, China : *The Proc of the 14th IF A C World Congress.*

Hauser, J., S. Sastry, and G. Meyer (1992). Nonlinear control design for slightly non-minimum phase systems : Applications to v/stol aircraft. ***Automatica 28***(4), 665-679.

Hou, Y., F. J. Z. Gao, and B. Boulter (2001). Active disturbance rejection control for web tension regulation. *Proceedings of the 40th IEEE Conference on Decision and Control, Orlando, Florida USA,* 4974-4979.

Isidori, A. (1995). ***Nonlinear Control Systems*** (3rd edition ed.). Berlin : SpringerVerlag.

Koo, T. and S. Sastry (1998). Output tracking control design of a helicopter model based on approximate linearization. ***The 37th Conference on Decision and Control, Florida, USA. Vol. 4,*** 3636-3640.

Lozano, R., P. Castillo, and A. Dzul (2005). ***Modeling and control of mini-flying machines.*** Springer Verlag London.

Mahony, R. and T. Hamel (2004). Robust trajectory tracking for a scale model autonomous helicopter. *Int. J. Robust Nonlinear Control 14,* 1035-1059.

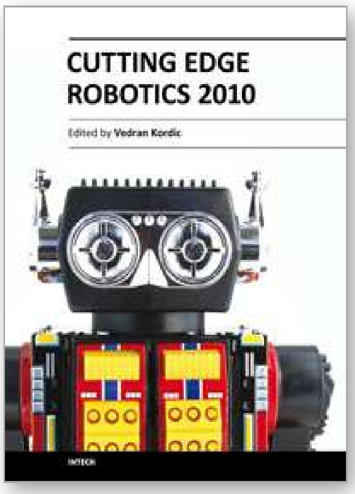
Mahony, R., T. Hamel, and A. Dzul (1999). Hover control via approximate lyapunov control for a model helicopter. ***The Conference on Decision and Control, Phoenix, Arizona, USA,*** 533-544.

Martini, A., F. Léonard, and G. Abba (2008). Robust nonlinear control and stability analysis of a 7dof model-scale helicopter under wind gust. In ***IEEE/RSJ, IROS, International Conference of Intelligent Robots and Systems,*** pp. 354-359. NICE, France.

Pflimlin, J., P. Soures, and T. Hamel (2004). Hovering flight stabilization in wind gusts for ducted fan uav. ***Proc. 43 rd IEEE Conference on Decision and Control, Atlantis, Paradise Island, The Bahamas. Vol. 4,*** 3491- 3496.

R. W. Prouty (1995). ***Helicopter Performance, Stability, and Control.*** Krieger.

Vilchis, A. (2001). ***Modélisation et Commande d'Hélicoptère.*** Ph. D. thesis, Institut National Polytechnique de Grenoble.



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