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# Parameter Perturbation Analysis through Stochastic Petri Nets: Application to an Inventory System

Labadi Karim<sup>1</sup>, Darcherif Moumen<sup>1</sup>, Haoxun Chen<sup>2</sup>

<sup>1</sup>*EPMI (ECS EA 3649), Ecole d'Electricité, de Production et de Méthodes Industrielles  
Cergy-Pontoise,*

<sup>2</sup>*UTT (LOSI- FRE CNRS 2848), Université de Technologie de Troyes, Troyes,  
France*

## 1. Introduction

Sensitivity analysis is used to determine how “sensitive” a performance measure of a model is with respect to a change in the value of the parameters of the model. Parameter sensitivity analysis of a model is usually performed as a series of tests in which the model analyst sets different values for the parameters of the model to see how a change in the parameters causes a change in the dynamic behavior of the model. Broadly speaking, this analysis is to see what happens when the value of some crucial parameters of a model changes. If a small change in the value of a parameter leads to a big change in the performance of the model, the parameter needs a closer look. It is a useful tool for performance evaluation of a model as well as its model building. Hence, sensitivity analysis can help the modeler to understand the dynamics of a system.

Sensitivity analysis is often used to estimate the sensitivity of a performance measure of a system with respect to its decision variables (parameters) by evaluating the gradient (derivatives) of the performance measure at each given value of the parameters. With the gradient, the system performance measure can be optimized by using a gradient method. Moreover, sensitivity analysis can be used to identify key parameters of a system by discovering the parameters whose small change in value leads to a big change in the behavior of the system. In model building, sensitivity analysis can be used to validate a model with unknown parameters by studying the uncertainties of the model associated with the parameters.

In the past, sensitivity analysis was usually based on simulation. One of the major research fields in this area is perturbation analysis (PA). The approach firstly applied to an engineering problem was proposed by Ho, Eyler and Chien in 1979. With great efforts made by many researchers in more than one decade, fundamental results for PA have been obtained. Currently, formal sensitivity analysis approaches based on stochastic processes were proposed in the literature. Particularly, efficient algorithms were developed to compute the performance derivatives of Markov processes with respect to infinitesimal changes of their parameters (infinitesimal generators) (Cao *et al.*, 1998, 1997). Besides the fundamental works in developing its theory and algorithms, perturbation analysis has also

been successfully applied to a number of practical engineering problems (Brooks & Varaiya, 1994; Caramanis & Liberopoulos, 1992; Haurie *et al.*, 1994; Xiao *et al.*, 1994; Yan & Zhou 1994). In this chapter, we deal with sensitivity analysis with respect to timing parameters based on stochastic Petri nets. Besides the great scientific and practical interest of the sensitivity analysis, this work is motivated by two reasons:

- Petri nets (Murata, 1989) are a powerful graphical and mathematical formalism which has been gaining popularity as a tool particularly suitable for modelling and analysis of discrete event systems. The literature on Petri nets is ample and their applications in practical manufacturing problems are numerous (Zhou and Kurapati, 1999; Zurawski and Zhou, 1994; Silva and Teruel, 1997). Several books were published in 1990s (Ajmone Marsan *et al.*, 1995; Haas, 2002; Lindeman, 1998; Zhou and DiCesare 1993).
- Although the literature on Petri nets is plentiful, very little work deals with sensitivity analysis or perturbation analysis of Petri net models. Few exceptions are: a perturbation analysis method based on stochastic Petri net models to estimate the derivatives of performance measures with respect to timing parameters can be found in (Xie 1998; Archetti *et al.*, 1993). For Markov regenerative stochastic Petri nets, a mathematical formulation for sensitivity of the steady state probabilities is developed in (Mainkar and *al.* 1993). Furthermore, performance sensitivity formulas are given by exploring structural characteristics of Petri nets (Feng, Desrochers, 1993; Proth *et al.*, 1993).

In this chapter, we try to apply a perturbation analysis method based on stochastic Petri nets for parameter sensitivity analysis to the performance analysis of an inventory system. The remainder of the chapter is organized in two parts as follows:

- The first part of the chapter addresses the sensitivity analysis of stochastic discrete event systems described by Stochastic Petri nets (SPN) as a performance evaluation tool of the systems. By exploring some existing results on perturbation analysis of Markov processes (Cao *et al.*, 1997-1998; Dai, 1995-1996) and by a natural extension of them to the underlying stochastic processes of SPNs, a stochastic Petri net-based sensitivity analysis method with respect to timing parameters is presented. The approach is widely applicable because stochastic Petri nets (Ajmone Marsan *et al.*, 1995; Haas, 2002; Lindeman, 1998) have been proven to be one of the most fundamental models for stochastic discrete-event systems.
- The second part of the chapter is dedicated to a case study on an inventory system. Previously, the modeling and performances evaluation of the system were performed by using Batch stochastic Petri nets recently introduced in the literature (Labadi *et al.*, 2007). In this part, the sensitivity analysis method developed in the first part of the chapter is used to estimate the sensitivity of performance measures with respect to the decision parameters of the inventory system.

## 2. Stochastic Petri nets models

Petri nets (PN), as a graphical and mathematical model, have been used for the study of qualitative properties of discrete event systems exhibiting concurrency and synchronization characteristics. A Petri net may be defined as a particular bipartite directed graph consisting of places, transitions, and arcs. Input arcs are ones connecting a place to a transition, whereas output arcs are ones connecting a transition to a place. A positive weight may be assigned to each arc. A place may contain tokens and the current state (the marking) of the modeled system is specified by the number of tokens in each place. Each transition usually models an

activity whose occurrence is represented by its firing. A transition can be fired only if it is enabled, which means that all preconditions for the corresponding activity are fulfilled (there are enough tokens available in the input places of the transition). When the transition is fired, tokens will be removed from its input places and added to its output places. The number of tokens removed/added is determined by the weight of the arc connecting the transition with the corresponding place. Graphically, places are represented by circles, transitions by bars or thin rectangles (filled or not filled), tokens by dots, respectively.

The use of PN-based techniques for the quantitative analysis of a system may require the introduction of temporal specifications in its basic untimed model. Time is then introduced in Petri nets by associating each transition with a firing delay (time). This delay specifies the duration during which the transition has to be enabled before it can actually be fired. In a stochastic Petri net, the time delays associated with certain transitions are random variables and the underlying marking process (state evolution process) of the net is a stochastic process. There are several variants of this model type, among them we have stochastic Petri net (SPN) models where each transition is associated with an exponentially distributed time delay. Stochastic Petri net models were proposed with the goal of developing a tool which integrates formal description, proof of correctness, and performance evaluation of systems. For what concerns the performance evaluation, many previous proposals aimed at establishing an equivalence between SPN and Markov models. Stochastic Petri net-based Markov modelling is thus a potentially very powerful and generic approach for performance evaluation of a variety of systems such as computer systems, communication networks and manufacturing systems.

## 2.1 The basic SPN model

The SPNs are obtained by associating each transition with an exponentially distributed firing time whose firing rate (average firing time) may be marking dependent.

A formal definition of SPN is thus given by:

$$SPN = (P, T, I, O, M_0, \Lambda) \quad (1)$$

where  $(P, T, I, O, M_0)$  is the marked untimed PN underlying the SPN, which as usual comprises:

$P = (p_1, p_2, \dots, p_n)$  is a finite set of places, where  $n > 0$ ;  $T = (t_1, t_2, \dots, t_m)$  is a finite set of exponentially distributed transitions, where  $m > 0$ , with  $P \cup T \neq \emptyset$  and  $P \cap T = \emptyset$ ;  $I: P \times T \rightarrow N$  is an input function that defines the set of directed arcs from  $P$  to  $T$  where  $N$  is the set of natural numbers;  $O: T \times P \rightarrow N$  is an output function that defines the set of directed arcs from  $T$  to  $P$ ;  $M_0$  is the initial marking of the net whose  $i^{th}$  component represents the number of tokens in the  $i^{th}$  place and  $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$  is an array of firing rates associated with transitions. Each rate is defined as the inverse of the average firing time of the corresponding transition.

## 2.2 Stochastic behaviour analysis

According to (Molloy, 1982), the SPNs are isomorphic to continuous time Markov chains (CTMC) due to the memoryless property of the exponential distributions of the firings times of their transitions. The SPN markings correspond to the states of the corresponding Markov chain so that the SPN model allows the calculation of the steady state probabilities of its states. As in Markov analysis, ergodic (irreducible) property of SPN is of special interest. For

an ergodic *SPN*, the steady state probability of the model in any state always exists and is independent of the initial state. If the firing rates of all transitions do not depend upon time, a stationary (homogeneous) Markov chain is obtained. In particular,  $k$  bounded *SPNs* are isomorphic to finite Markov chains.

The reachability graph of an *SPN* is identical to that of the underlying untimed *PN*. The nodes of the graph represent all markings reachable from the initial marking. Each arc is labelled by its corresponding fired transition. The *CTMC* state space  $S = \{S_0, S_1, \dots, S_m\}$  corresponds to the set of all markings in the reachability graph  $M^* = \{M_0, M_1, \dots, M_m\}$ . The transition rate from state  $S_i$  ( $M_i$ ) to state  $S_j$  ( $M_j$ ) is obtained as the sum of the firing rates of the transitions that are enabled in  $M_i$  and whose firing produces the marking  $M_j$ . The steady-state solution of the model is then obtained by solving a system of linear equations:

$$\begin{cases} \pi \times A = 0 \\ \sum_{i=0}^m \pi_i = 1 \end{cases} \quad (2)$$

where:

- $\pi = (\pi_0, \pi_1, \dots, \pi_m)$  denotes the steady-state probability of each marking  $M_i$  (and of state  $S_i$  as well, since there is a one-to-one correspondence between markings and states).
- $A = [a_{ij}]_{(m+1) \times (m+1)}$  is the transition rate matrix of the *CTMC*. For  $i = 0, 1, 2, \dots, m$ , the  $i^{th}$  row, i.e., the elements  $a_{ij}$ ,  $j = 0, 1, 2, \dots, m$ , are obtained as follows:
  - If  $j \neq i$ ,  $a_{ij}$  is the sum of the firing rates of all the outgoing arcs from state  $M_i$  to  $M_j$ .
  - If  $i = j$ ,  $a_{ij}$  represents the sum of the firing rates of all transitions enabled at  $M_i$ .

### 2.3 Performance evaluation

The analysis of an *SPN* model usually aims at the computation of more aggregate performance indices than the steady-state probabilities of individual markings. Several aggregate performance indices are easily obtained from the steady-state distribution of reachable markings.

The required performance estimates of a system modelled by an *SPN* can be computed using a unifying approach in which proper index functions (also called reward functions) are defined over the markings of the *SPN* and an average reward for each reward function is derived using the steady-state probability distribution of the *SPN*. Assuming that  $f$  represents one of such reward functions, its average reward can be computed using the following weighted sum:

$$P = \sum_{i=0}^m \pi_i \cdot f_i = \pi \cdot f \quad (3)$$

where  $f_i$  is incurred per unit time at each reachable marking  $M_i$  of the underlying stochastic process of the *SPN*.

## 3. Parameter sensitivity analysis

### 3.1 Perturbation realization

Consider first the nominal behavior of a system modeled as an *SPN*. Let  $P_\theta$  a performance function defined over the marking process of the net under a nominal parameter vector  $\theta$

which is a set of parameters of the system (here, it represents the firing rates associated with the transitions of the net). That is:

$$P_{\theta} = \sum_{i=1}^m \pi_i \cdot f_i = \pi \cdot f \quad (4)$$

where  $\pi_i$  denotes the steady-state probability of each marking  $M_i$  and  $f_i$  is a measure of the performance function  $f$  incurred at the marking  $M_i$  of the stochastic marking process of the net. Consider now a perturbation  $\delta$  on one or more parameters of the underlying Markov process that is equivalent to a perturbation in the transition rates matrix  $A$ . With the perturbation, the transition matrix  $A$  changes to:

$$A_{\delta} = A + \delta \cdot Q \quad (5)$$

where  $A_{\delta}$  is the transition rate matrix of the perturbed behavior system,  $\delta$  is a very small positive real number and  $Q = [q_{ij}]$  is a matrix representing the direction of the perturbation.

- $q_{ij}$  equals 0 indicates that the matrix entry  $A_{ij}$  is not perturbed.
- $q_{ij}$  equals  $x$  different from 0 indicates that the matrix entry  $A_{ij}$  is perturbed by an amount  $x\delta$ .

The only condition on the structure of  $Q$  is that the matrix  $A_{\delta}$  is also a transition matrix i.e. the sum of each row equals 0.

Under this formulation,  $A_{\delta}$  is also a well-defined infinitesimal generator, and hence its steady state probabilities  $\pi_{\delta} = (\pi_{\delta 0}, \pi_{\delta 1}, \dots, \pi_{\delta m})$ , of the perturbed marking process is also defined. That is:

$$\pi_{\delta} \times A_{\delta} = 0 \quad \text{and} \quad \sum_{i=0}^m \pi_{\delta i} = 1 \quad (6)$$

Then, the stationary performance measure of the perturbed Markov process (that is the Markov process with transition matrix  $A_{\delta}$ ) can be defined as:

$$P_{\delta} = \sum_{i=0}^m \pi_{\delta i} \cdot f_i = \pi_{\delta} \cdot f \quad (7)$$

### 3.2 Computation of sensitivity measures

Many solutions have been proposed in the literature to evaluate sensitivity measures corresponding to partial derivatives.

1. Exact solutions rely on Frank's approach (Frank, 1978): the classical set of differential equations is extended to a bigger set of equations including the sensitivity factor equations. However, this approach is computationally burdensome and almost unusable or highly inefficient on realistic-size systems because the state space dimension is too great. To cope with this problem, some approximate solutions have been proposed (Ou et al., 2003) but applicable to a limited class of systems.
2. Many simulation methods have been also proposed to estimate derivative measure. See for example (Glynn 1990; Glassermann et al., 1992). Concerning Markov process modelling and stationary performance measure, perturbation realization is well adapted (Cao et al., 1997-1998; Dai, 1996). It allows:



- The evaluation of sensitivity of performance measures formulated under the marking process of a stochastic Petri net model:

The sensitivity of a performance measure  $P$  of a system due to the introduced changes in the infinitesimal generator  $A$ , can be analyzed by computing the derivate of  $P_\delta$  in the direction of  $Q$  (noted below by  $SPerf$ ). It can be defined as:

$$SPerf = \frac{dP}{dQ} = \lim_{\delta \rightarrow 0} \frac{P - P_\delta}{\delta} \quad (8)$$

- The evaluation of sensitivity of steady-state probabilities of the marking process:  
The sensitivity of steady state probabilities can also be defined as:

$$SProb = \frac{d\pi}{dQ} = \lim_{\delta \rightarrow 0} \frac{\pi - \pi_\delta}{\delta} \quad (9)$$

### 3.3 Calculation of the performance derivatives

The particular structure of the Chapman-Kolmogorov equations and the linearity of the performance measure lead to the following expression of the measure derivatives (Cao et al., 1997-1998):

$$SPerf = \frac{dP}{dQ} = -\pi \cdot Q \cdot A^\# \cdot f \quad (10)$$

where:

$A^\#$  is defined as:

$$A^\# = (A + e\pi)^{-1} - e\pi \quad (11)$$

where:

- $e = (1, 1, \dots)^T$  is a column vector of size  $(m \times 1)$  with  $e_{i,1} = 1$  for any  $i$ .
- $g = -A^\# \cdot f$  is called the performance potential vector.

Then, according to the equation (8),  $SPerf$  can be written as:

$$SPerf = \frac{dP}{dQ} = -\pi \cdot Q \cdot A^\# \cdot f = \pi \cdot Q \cdot g \quad (12)$$

Similarly, according to the equation (9), the steady state derivate,  $SProb$ , can be computed using the following formula:

$$SProb = \frac{d\pi}{dQ} = -\pi_\delta \cdot Q \cdot A^\# \quad (13)$$

## 4. Inventory system modelling and performance analysis

This part of the chapter is dedicated to a case study on an inventory system represented in Fig. 1. The presented approach in the previous section is then applied to estimate the sensitivity of performance measures with respect to some parameters of the inventory system.

#### 4.1 Modelling of the inventory system

Consider the continuous review  $(s, S)$  inventory system shown in Fig. 1. In this application, it is assumed that the system has the following characteristics: the inventory replenishment time is subject to an exponential distribution; customer demand is Poisson and in batch; and the system has no backorder.

The modeling and performance evaluation of the system will be performed by using Batch stochastic Petri nets introduced in the literature as a powerful modelling tool for both analysis and simulation of logistic systems. The capability of the model to meet real needs is shown through applications dedicated to modelling and performance optimization of inventory control systems (see Labadi et al., 2007) and a real-life supply chain (see Chen et al., 2005; Amodeo et al., 2007).

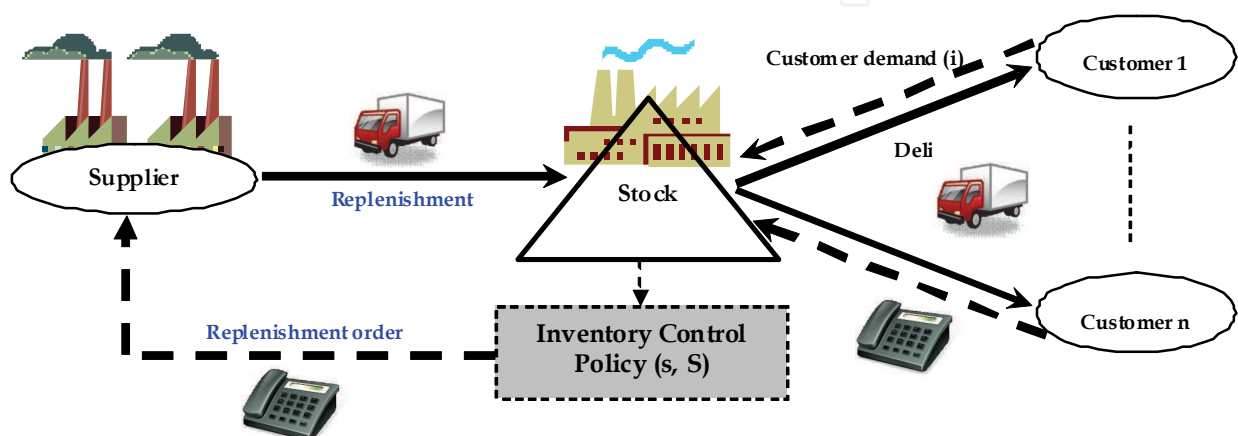


Fig. 1. A continuous  $(s, S)$  inventory system

We model the dynamics of the above mentioned supply chain by using a Batch Stochastic Petri net represented in Fig. 2. In the model, discrete place  $p1$  represents the on-hand inventory of the considered stock and place  $p3$  represents outstanding orders. Discrete place  $p2$  represents the on-hand inventory of the stock plus its outstanding orders (the orders that are placed by stock  $p1$  but not filled yet), that is,  $M(p2) = M(p1) + M(p3)$ . The operations of the system such as generation of replenishment orders ( $t3$ ), inventory replenishment ( $t2$ ), and order delivery ( $t1$ ) are performed in a batch way because of the batch nature of customer orders (generated by the batch place  $p4$  and the batch transition  $t1$ ) and the batch nature of the outstanding orders recorded in batch place  $p3$ . The fulfillment of a customer order will decrease on-hand inventory of the stock as well as its inventory level. This is described by the arcs from places  $p1$ ,  $p4$  and  $p2$  to transition  $t1$ . Batch customer demand is assumed to be a Poisson process, which is specified by the batch transition  $t_1$  whose firing time is subject to an exponential distribution. We assume that transition  $t_1$  generates randomly with the same probability batch customer orders of two different sizes (1 or 2) available in batch place  $p4$  (i.e.;  $\mu(p4) = \{1, 2\}$ ). The inventory control policy used in the system is a continuous review  $(s, S)$  policy specified by the immediate transition  $t3$ . It is assumed that the reorder point and the order-up-to-level of the policy are taken as  $s = 4$  and  $S = 6$  respectively, and the initial  $\mu$ -marking of the net is  $\mu_0 = (6, 6, \emptyset, \{1, 2\})$ . Furthermore, the firing delays of batch transitions  $t1$  and  $t2$  (the demand rate and the inventory replenishment rate) are assumed to be exponentially distributed with rates  $\lambda_{1[q]} = \lambda_1$  and  $\lambda_{2[q]} = \lambda_2$  respectively for any feasible batch firing index  $q$ .



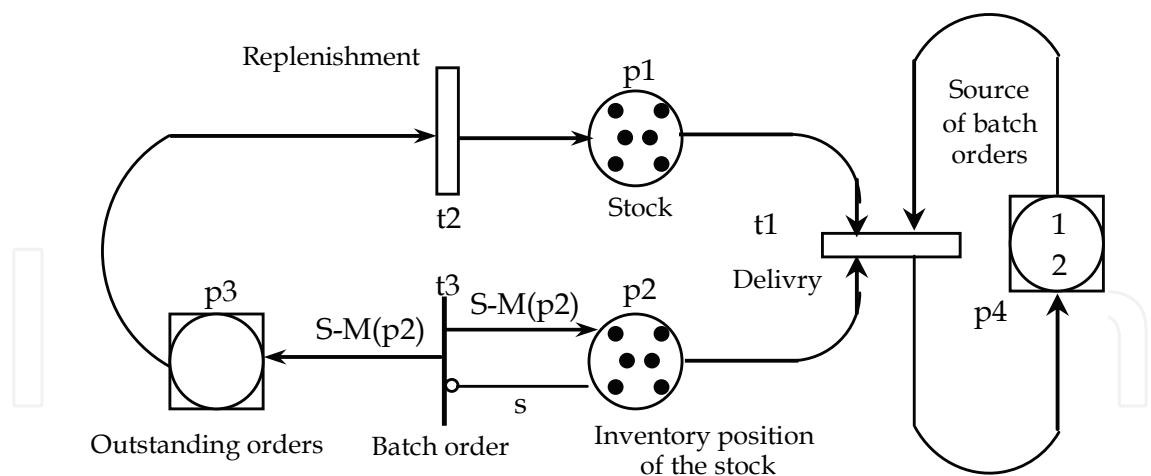


Fig. 2. Batch stochastic Petri net model of the supply chain with  $(s, S)$  inventory control policy

4.2 Dynamic behaviour analysis

The state space of the Petri net is represented by its  $\mu$ -reachability graph shown in Fig. 3. In the graph, each directed edge is associated with a label representing the transition whose firing generates the successor  $\mu$ -marking. Each batch transition is marked by its corresponding batch firing index  $q$ .

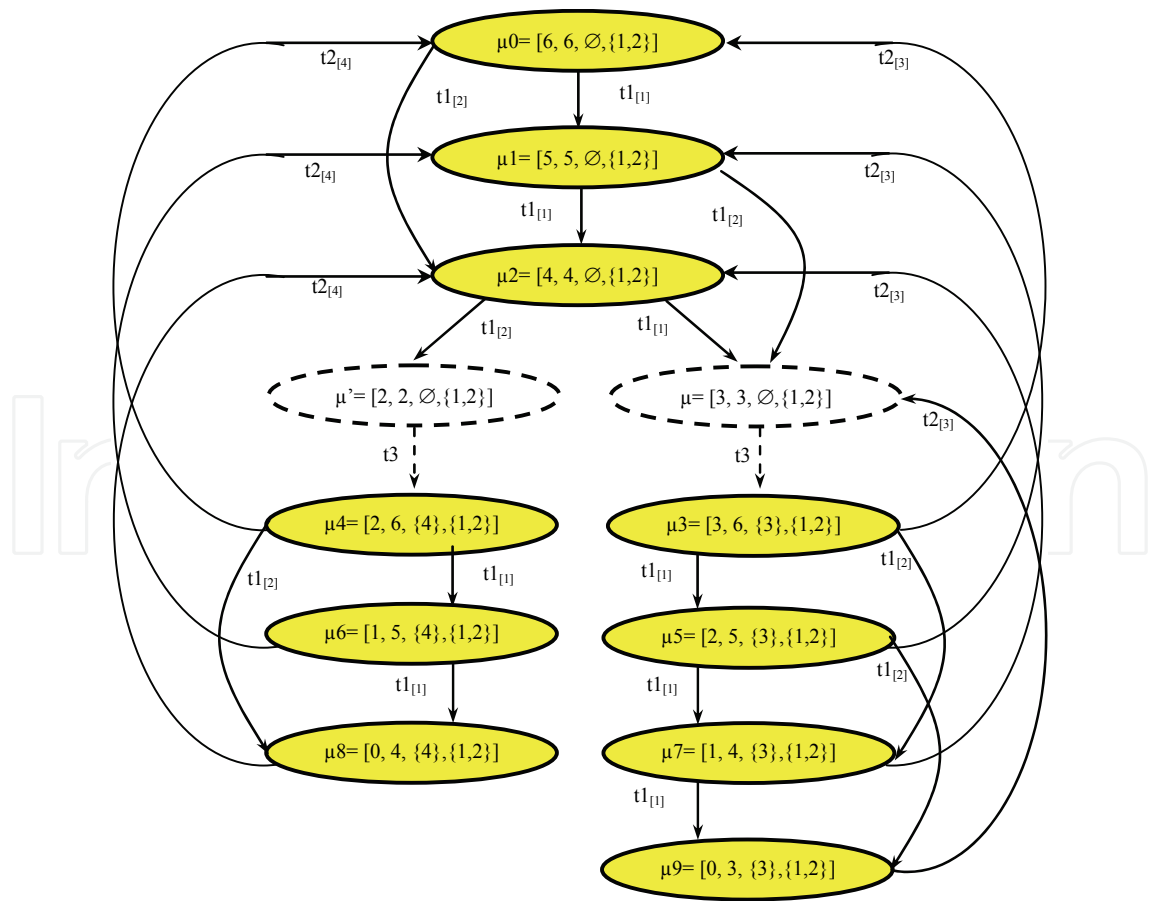


Fig. 3. The  $\mu$ -reachability graph of the batch stochastic Petri net model shown in Fig. 2

The  $\mu$ -markings obtained can be classified into *vanishing* and *tangible*  $\mu$ -markings. A vanishing  $\mu$ -marking is one in which at least one immediate transition is enabled, and a tangible  $\mu$ -marking is one in which no immediate transition is enabled. In the  $\mu$ -reachability graph, the vanishing  $\mu$ -markings  $\mu_i$  ( $i = 0$  to  $9$ ) are represented by rectangles and two tangible  $\mu$ -markings  $\mu$  and  $\mu'$  are represented by dotted rectangles. After eliminating the vanishing  $\mu$ -markings by merging them with their successor tangible  $\mu$ -markings and converting the reduced  $\mu$ -reachability graph to its corresponding stochastic process, we get a continuous timed Markov chain (CTMC) represented in Fig. 4.

By solving the linear equations system (14) where  $A$  is the infinitesimal generator matrix (transition rate matrix) of the CTMC, the steady-state probabilities  $\pi = (\pi_1, \pi_2, \dots, \pi_9)$  can be explicitly obtained as functions of parameters  $\lambda 1$  and  $\lambda 2$  given in Table 1.

$$\begin{cases} \pi \times A = 0 \\ \sum_{i=0}^9 \pi_i = 1 \end{cases} \tag{14}$$

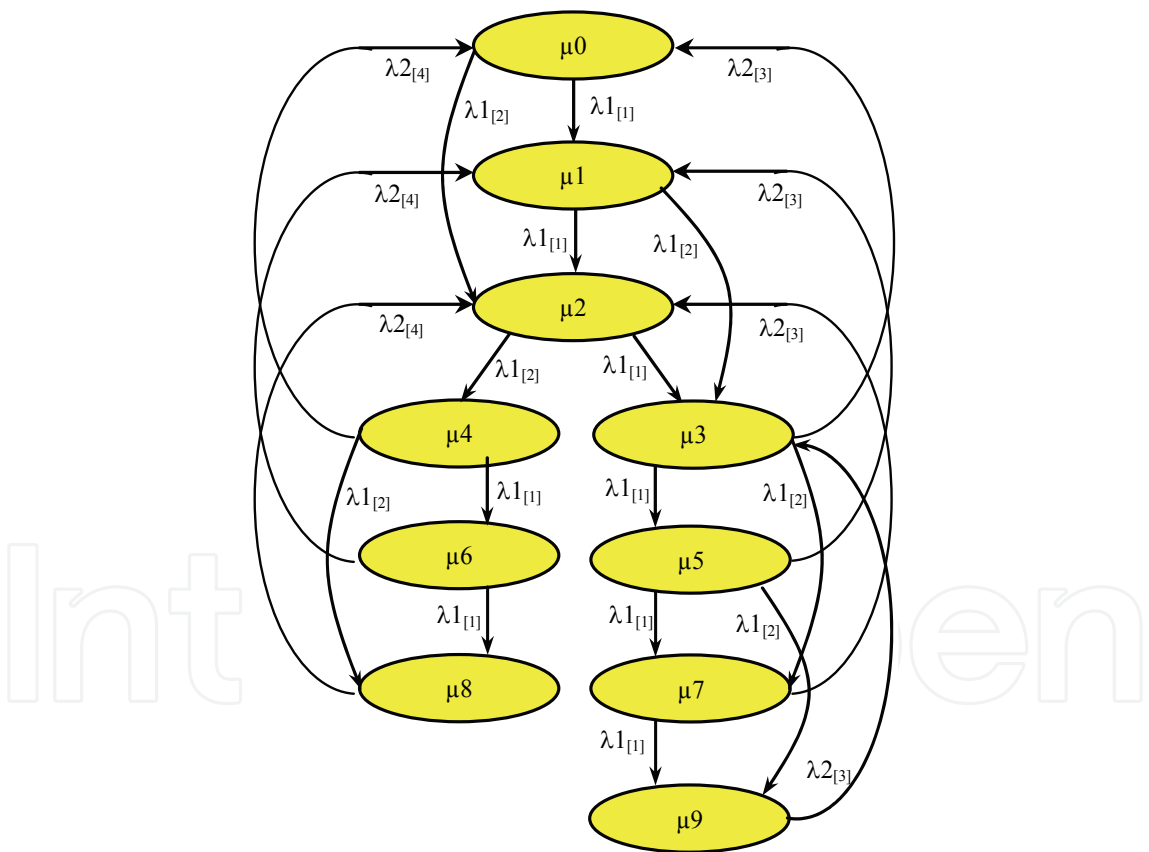


Fig. 4. The underlying Markov chain of the batch stochastic Petri net model shown in Fig. 2

4.3. Performance analysis of the system

With the steady state probabilities  $\pi = (\pi_1, \pi_2, \dots, \pi_9)$ , we can easily compute several important performance measures of the supply chain such as the average inventory level, the stockout rate, the average inventory turnover, etc.

$\pi_0$	$\frac{2(\lambda_2)^2[45(\lambda_2)^2(\lambda_1)^2+25\lambda_2(\lambda_1)^3+32(\lambda_2)^3\lambda_1+8(\lambda_2)^4+4(\lambda_1)^4] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_1$	$\frac{(\lambda_2)^2[69(\lambda_2)^2(\lambda_1)^2+46\lambda_2(\lambda_1)^3+40(\lambda_2)^3\lambda_1+8(\lambda_2)^4+8(\lambda_1)^4] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_2$	$\frac{(4(\lambda_1+\lambda_2)^2(\lambda_2)^2) \cdot [9\lambda_1\lambda_2+7(\lambda_1)^2+3(\lambda_2)^2] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_3$	$\frac{2\lambda_1\lambda_2[29(\lambda_2)^2(\lambda_1)^2+18\lambda_2(\lambda_1)^3+20(\lambda_2)^3\lambda_1+5(\lambda_2)^4+4(\lambda_1)^4] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_4$	$\frac{(2(\lambda_1+\lambda_2)(\lambda_2)^2\lambda_1) \cdot [9\lambda_1\lambda_2+7(\lambda_1)^2+3(\lambda_2)^3] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_5$	$\frac{\lambda_2(\lambda_1)^2[15(\lambda_2)^2\lambda_1+14\lambda_2(\lambda_1)^2+5(\lambda_2)^3+4(\lambda_1)^3] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_6$	$\frac{(\lambda_2)^2(\lambda_1)^2 \cdot [9(\lambda_2\lambda_1+7(\lambda_1)^2+3(\lambda_2)^2)] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_7$	$\frac{0.5 \cdot (12(\lambda_1)^3+38\lambda_2(\lambda_1)^2+35(\lambda_2)^2\lambda_1+10(\lambda_2)^3) (\lambda_1)^2\lambda_2 \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_8$	$\frac{\lambda_2(\lambda_1)^2(9\lambda_2\lambda_1+7(\lambda_1)^2+3(\lambda_2)^2)(2\lambda_1+\lambda_1) \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$
$\pi_9$	$\frac{0.5(\lambda_1)^3[52\lambda_1(\lambda_1)^2+15(\lambda_2)^3+50(\lambda_2)^2\lambda_1+16(\lambda_1)^3] \div}{[351(\lambda_2)^4(\lambda_1)^2+180(\lambda_2)^5(\lambda_1)+342(\lambda_2)^3(\lambda_1)^3+184(\lambda_1)^4(\lambda_2)^2+58(\lambda_1)^5(\lambda_2)+8(\lambda_1)^6+36(\lambda_2)^6]}$

Table 1.  $\mu$ -markings (states) probabilities

- The average inventory level of the system  $S_{avg}(\lambda_1, \lambda_2)$  which corresponds to the mean number of tokens in discrete place  $p_1$  can be calculated by applying the formula:

$$S_{avg}(\lambda_1, \lambda_2) = \overline{\mu(p_1)} = \sum_{i=0}^9 \pi_i \times \mu(p_1) \quad (15)$$

$$S_{avg}(\lambda_1, \lambda_2) =$$

$$\frac{0.5\lambda_2 \times [368(\lambda_2)^5 + 1732(\lambda_1)(\lambda_2)^4 + 780(\lambda_1)^4(\lambda_2) + 3038(\lambda_1)^2(\lambda_2)^3 + 2385(\lambda_1)^3(\lambda_2)^2 + 76(\lambda_1)^5]}{8(\lambda_1)^6 + 58(\lambda_1)^5(\lambda_2) + 184(\lambda_1)^4(\lambda_2)^2 + 342(\lambda_2)^3(\lambda_1)^3 + 351(\lambda_1)^2(\lambda_2)^4 + 180(\lambda_2)^5(\lambda_1) + 36(\lambda_2)^6}$$

- The stockout rate is the probability of the emptiness of the stock. In the  $\mu$ -marking graph of the Petri net model in Fig. 3, the marking of the discrete place  $p_1$  is equal to zero ( $M(p_1) = 0$ ) in two  $\mu$ -markings which are  $\mu_8$  and  $\mu_9$ . Thus the stock-out rate of the inventory system  $Prob_{S=0}(\lambda_1, \lambda_2)$  is given by the formula:

$$Prob_{S=0}(\lambda_1, \lambda_2) = 0 = Prob[\mu(p_1) = 0] = \pi_8 + \pi_9 \quad (16)$$

$$Prob_{S=0}(\lambda_1, \lambda_2) =$$

$$\frac{0.5(\lambda_1)^2 \times [80(\lambda_1)^3(\lambda_2) + 100(\lambda_1)^2(\lambda_2)^2 + 45(\lambda_1)(\lambda_2)^3 + 6(\lambda_2)^4 + 16(\lambda_1)^4]}{8(\lambda_1)^6 + 58(\lambda_1)^5(\lambda_2) + 184(\lambda_1)^4(\lambda_2)^2 + 342(\lambda_2)^3(\lambda_1)^3 + 351(\lambda_1)^2(\lambda_2)^4 + 180(\lambda_2)^5(\lambda_1) + 36(\lambda_2)^6}$$

- The average replenishment frequency of the stock,  $FA_{avg}(\lambda1,\lambda2)$  corresponds to the average firing frequency of batch transition  $t2$ . Thus, it is the sum of the average firing frequencies  $F(t2_{[3]})$  and  $F(t2_{[4]})$  of the transitions  $t2_{[3]}$  and  $t2_{[4]}$  respectively. These transitions are generated by batch transition  $t2$  with two different batch firing indexes.  $FA_{avg}(\lambda1, \lambda2)$  is thus given as follows:

$$FA_{avg}(\lambda1,\lambda2) = F(t2_{[3]}) + F(t2_{[4]}) = \sum_{i|\mu_i \in S(t2_{[3]})} (\lambda2_{[3]} \times \pi_i) + \sum_{i|\mu_i \in S(t2_{[4]})} (\lambda2_{[4]} \times \pi_i) \tag{17}$$

where  $S(t2_{[3]}) = \{\mu_3, \mu_5, \mu_7, \mu_9\}$  is the set of the  $\mu$ -markings in which batch transition  $t2$  is fired with index 3 (firing of  $t2_{[3]}$ ) and  $S(t2_{[4]}) = \{\mu_4, \mu_6, \mu_8\}$  is the set of the  $\mu$ -markings in which batch transition  $t2$  is fired with index 4 (firing of  $t2_{[4]}$ ). Since  $\lambda2_{[3]} = \lambda2_{[4]} = \lambda2$ , we obtain that:

$$FA_{avg}(\lambda1,\lambda2) = F(t2_{[3]}) + F(t2_{[4]}) = \sum_{i|\mu_i \in S(t2_{[3]}) \cup S(t2_{[4]})} \pi_i \times \lambda2 = (\pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 + \pi_9) \times \lambda2 \tag{18}$$

$$FA_{avg}(\lambda1,\lambda2) = \frac{2(\lambda_1) \times [8(\lambda_2)^5 + 4(\lambda_1)^5 + 77(\lambda_2)^3(\lambda_1)^2 + 70(\lambda_2)^2(\lambda_1)^3 + 40(\lambda_2)^4(\lambda_1) + 29(\lambda_2)(\lambda_1)^4]}{8(\lambda_1)^6 + 58(\lambda_1)^5(\lambda_2) + 184(\lambda_1)^4(\lambda_2)^2 + 342(\lambda_2)^3(\lambda_1)^3 + 351(\lambda_1)^2(\lambda_2)^4 + 180(\lambda_2)^5(\lambda_1) + 36(\lambda_2)^6}$$

The average inventory level, the stock-out rate, and the average replenishment frequency of the stock as functions of parameters  $\lambda1$  and  $\lambda2$  are depicted in Fig. 5, Fig. 6, and Fig. 7, respectively.

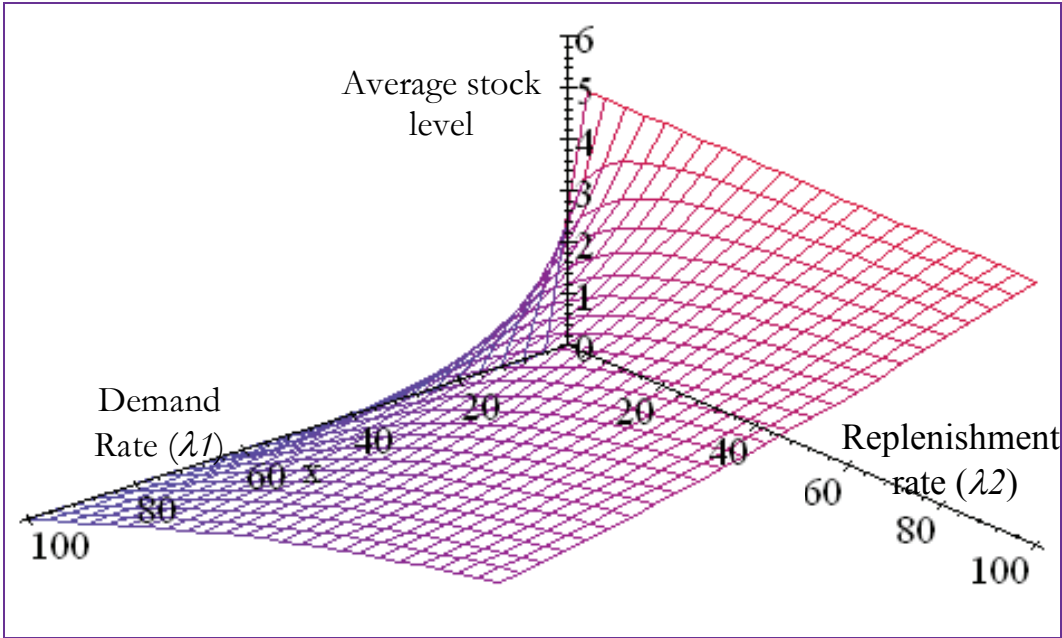


Fig. 5. Average inventory level of the stock

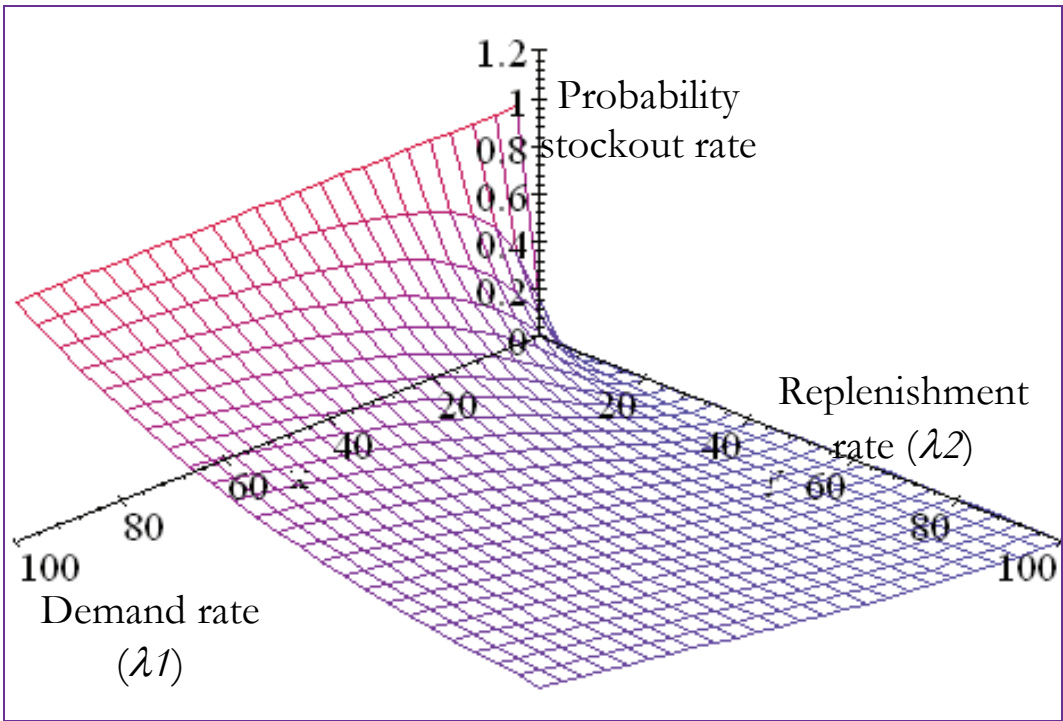


Fig. 6. Stock-out rate of the stock

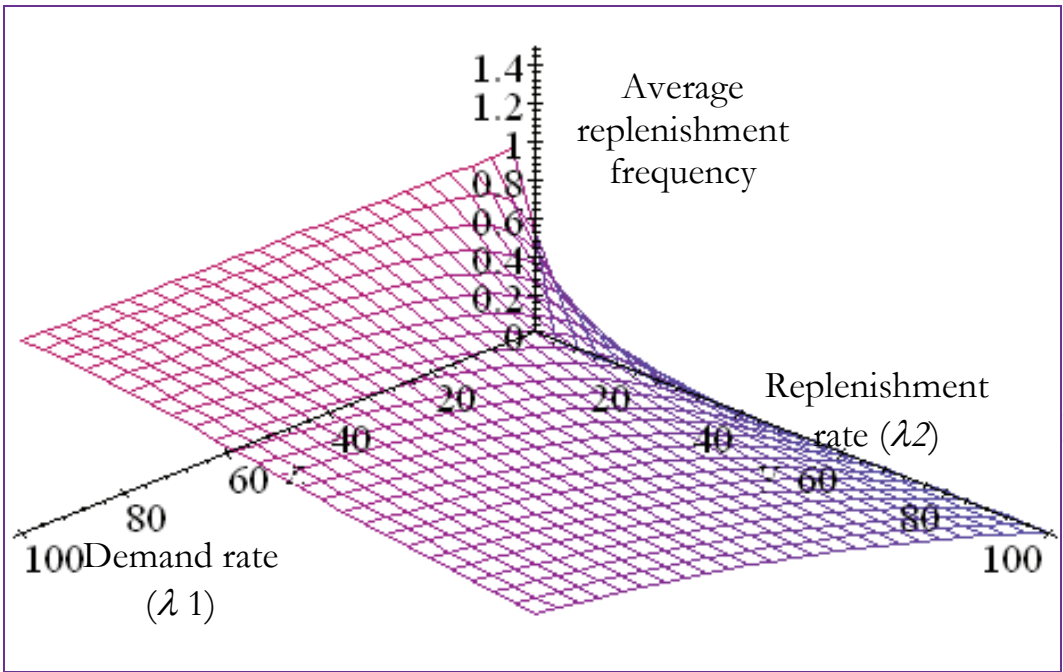


Fig. 7. Average frequency replenishment of the stock

**4.4 Parameter sensitivity analysis of the system**

This section is dedicated to sensitivity analysis of the inventory system. We consider the following parameters for it:  $\lambda1_{[1]} = \lambda1_{[2]} = \lambda1 = 0.5$  and  $\lambda2_{[3]} = \lambda2_{[4]} = \lambda2 = 0.5$ . Its transition rate matrix is thus as follows:

$$A = \begin{pmatrix} -1 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & -1.5 & 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & -1.5 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & -1.5 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & -1 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & -1 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & -0.5 \end{pmatrix}$$

In this case, the steady state probabilities, denoted by  $\pi_i$ ,  $i = 0, \dots, 9$ , obtained by solving the corresponding equations system (14) is :

$$\pi = (0.1025 \quad 0.09075 \quad 0.1915 \quad 0.1411 \quad 0.0638 \quad 0.0471 \quad 0.0320 \quad 0.0942 \quad 0.0958 \quad 0.1411)$$

#### 4.4.1 Sensitivity analysis with respect to one parameter

- **Sensitivity analysis of the average inventory level of the stock with respect to the customer demand rate**

In the Petri net model, the stock is modeled by the discrete place  $p1$ . In other words, the inventory level of the stock, in each state, corresponds to  $M(p1)$ , the marking of the place  $p1$ . Thus, the corresponding performance function is:

$$f = (6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0)$$

where  $f(i)$ ,  $i = 0, 1, \dots, 9$ , corresponds to the marking of the place  $p1$ ,  $M(p1)$ , (the inventory level) at the state  $M_i$  (see the marking graph of the Petri net model in Fig. 3).

Consider now the perturbation on a parameter  $\lambda 1$  (customer demand rate associated with the transition  $t1$ ) and its influence on the directional matrix  $Q$  given as follows:

$$Q = \begin{pmatrix} -2\delta & \delta & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\delta & \delta & \delta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\delta & \delta & \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2\delta & 0 & \delta & 0 & \delta & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\delta & 0 & \delta & 0 & \delta & 0 \\ 0 & 0 & 0 & 0 & 0 & -2\delta & 0 & \delta & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & 0 & -\delta & 0 & \delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where  $\delta$  is a very small positive real number corresponding to an infinitesimal change of the parameter  $\lambda 1$ . The perturbation is illustrated in Fig. 8.



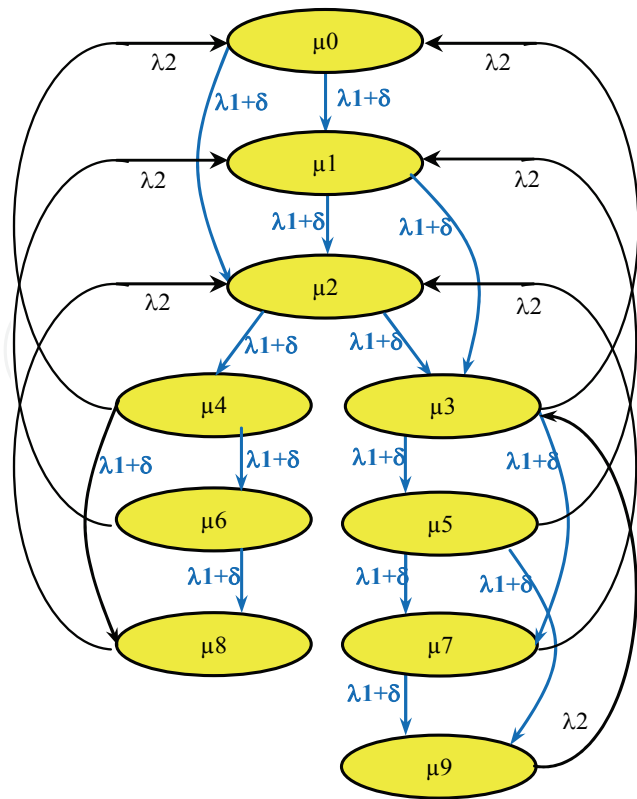


Fig. 8. Perturbation on customer demand rate ( $\lambda 1$ )

By applying the equation (12), the derivate of the considered performance (average inventory level of the stock) with respect to the parameter  $\lambda 1$  is given by the following linear function represented in Fig. 9.

$$SPerf = - 3.2381.\delta$$

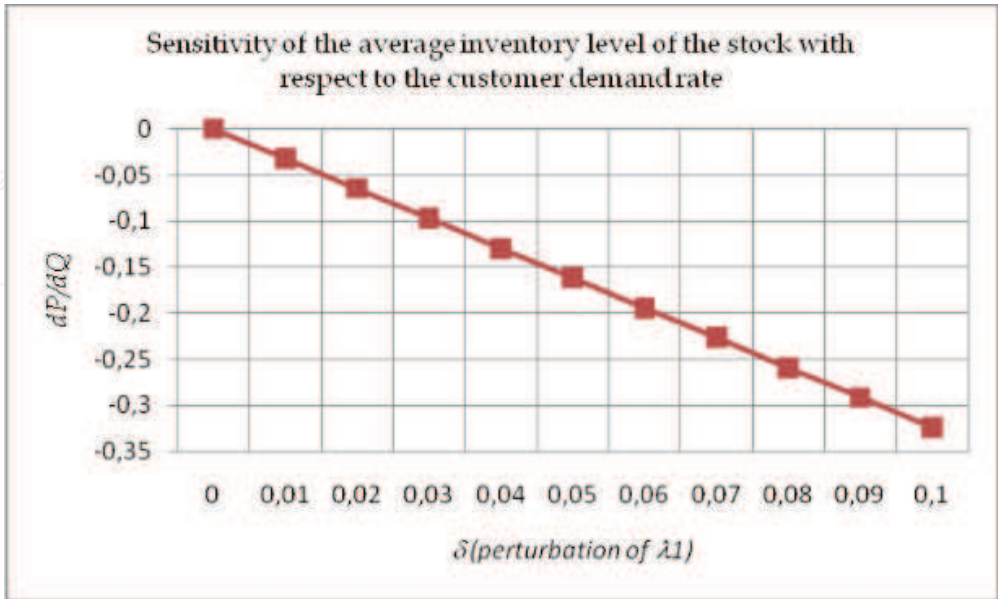


Fig. 9. Sensitivity of the average inventory level of the stock with respect to the customer demand rate ( $\lambda 1$ )

Clearly, this derivate means that if the customer demand rate  $\lambda 1$  of the inventory system is increased by an amount  $\delta$ , then average inventory level of the stock will decrease by an amount  $3,2381 \cdot \delta$ .

- **Sensitivity analysis of the stockout rate with respect to the supplier replenishment rate**

The stockout rate is defined in the previous section as the probability of the emptiness of the stock. In the  $\mu$ -marking graph of the Petri net model in Fig. 3, the marking of the discrete place  $p_1$  is equal to zero ( $M(p_1) = 0$ ) in two markings which are  $\mu_8$  and  $\mu_9$ . Thus, the corresponding performance function is:

$$f = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1)$$

where  $f(i) = 1$  for only  $i = 8$  and  $i = 9$  corresponding to the two  $\mu$ -markings  $\mu_8$  and  $\mu_9$  where the marking of the place  $p_1$  (the inventory level),  $M(p_1)$  is equal to zero (see the  $\mu$ -marking graph of the Petri net model in Fig. 3).

Consider now the perturbation on a parameter  $\lambda 2$  (replenishment rate associated with the transition  $t_2$ ) and its influence on the directional matrix  $Q$  given as follows:

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & 0 & 0 & -\delta & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & 0 & 0 & 0 & -\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta & 0 & 0 & 0 & -\delta & 0 & 0 & 0 & 0 \\ 0 & \delta & 0 & 0 & 0 & 0 & -\delta & 0 & 0 & 0 \\ 0 & 0 & \delta & 0 & 0 & 0 & 0 & -\delta & 0 & 0 \\ 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & -\delta \end{pmatrix}$$

By applying the equation (12), the derivate of the considered performance (stockout rate) with respect to the parameter  $\lambda 2$  is given by the following linear function represented in Fig. 10.

$$SPerf = -5.5736 \cdot \delta$$

Clearly, this derivate means that if the supplier replenishment rate  $\lambda 2$  of the supply chain is increased by an amount  $\delta$ , then the stockout out rate will decrease by an amount  $5,5736 \cdot \delta$ .

#### 4.4.2 Sensitivity analysis with respect to a group of parameters

Here, the perturbations on a group of parameters are illustrated. The sensitivity level in these directions can be used to identify the relative importance of each parameter in the group. For instance in our system, the directional matrix  $Q$  corresponding to the perturbation of the parameters  $\lambda 1$  (the customer demand rate) and  $\lambda 2$  (the supplier replenishment rate) at the same time is:

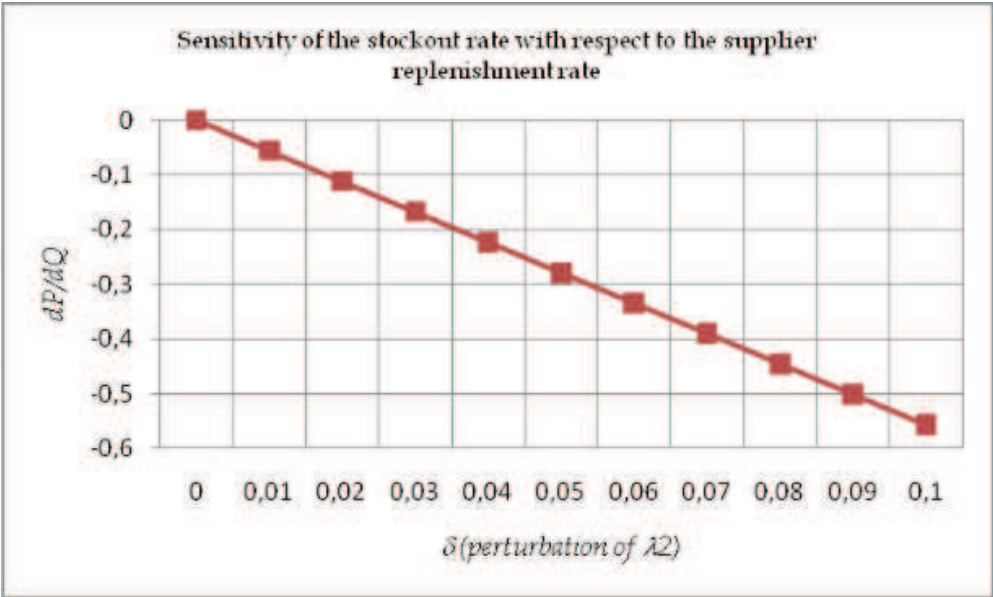


Fig. 10. Sensitivity of the stockout rate with respect to the supplier replenishment rate ( $\lambda_2$ )

$$Q = \begin{pmatrix} -2\delta_1 & \delta_1 & \delta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\delta_1 & \delta_1 & \delta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\delta_1 & \delta_1 & \delta_1 & 0 & 0 & 0 & 0 & 0 \\ \delta_2 & 0 & 0 & -2\delta_1 - \delta_2 & 0 & \delta_1 & 0 & \delta_1 & 0 & 0 \\ \delta_2 & 0 & 0 & 0 & -2\delta_1 - \delta_2 & 0 & \delta_1 & 0 & \delta_1 & 0 \\ 0 & \delta_2 & 0 & 0 & 0 & -2\delta_1 - \delta_2 & 0 & \delta_1 & 0 & \delta_1 \\ 0 & \delta_2 & 0 & 0 & 0 & 0 & -\delta_1 - \delta_2 & 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 & 0 & -\delta_1 - \delta_2 & 0 & \delta_1 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 & 0 & 0 & -\delta_2 & 0 \\ 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & 0 & 0 & -\delta_2 \end{pmatrix}$$

By using this directional matrix, and the function  $f$  expressed in the previous sub-section, the stockout rate derivate with respect to the two parameters  $\lambda_1$  and  $\lambda_2$  can be expressed as the following linear function:

$$SPerf = 5.57 (\delta_1 - \delta_2)$$

Note that because of the linear structure of equation (12), if a perturbation matrix  $Q$  is a linear function of elementary perturbations matrixes  $Q_i$ , it is possible to evaluate the multidirectional sensitivity measures related to  $Q$  on the basis of the elementary perturbation measures related to the  $Q_i$ .

In our example, it is clear that:

$$Q_{(\lambda_1, \lambda_2)} = Q_{(\lambda_1)} + Q_{(\lambda_2)}$$

Then, we can write that:

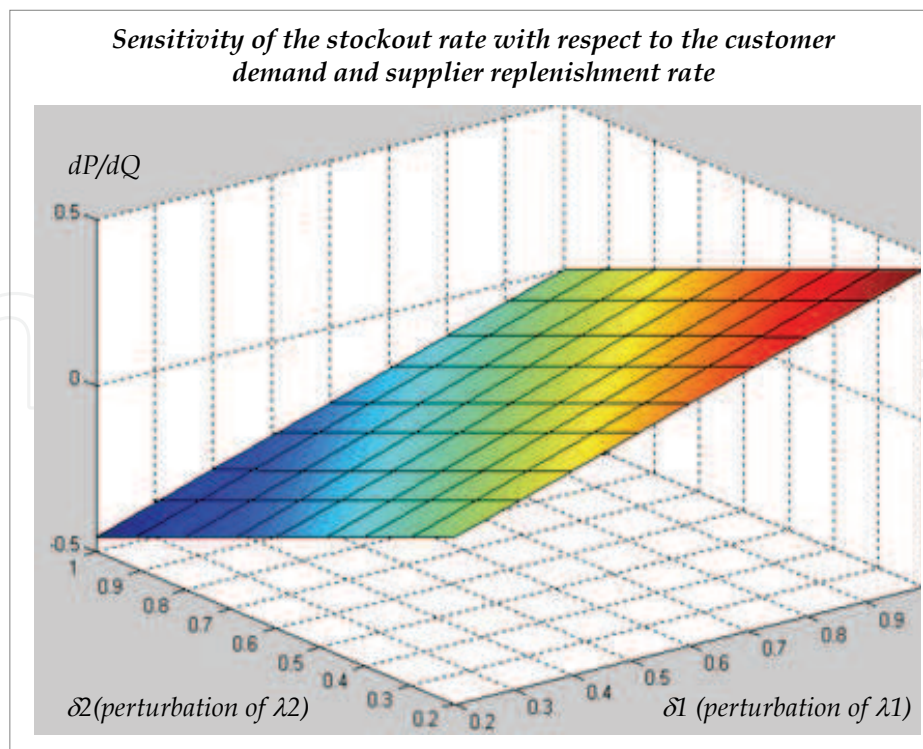


Fig. 11. Sensitivity of the stockout rate with respect to two parameters  $\lambda_1$  (the customer demand rate) and  $\lambda_2$  (the supplier replenishment rate).

$$\begin{aligned} SPerf &= \frac{dP}{dQ_{(\lambda_1, \lambda_2)}} = -\pi \cdot Q_{(\lambda_1, \lambda_2)} \cdot A^\# \cdot f \\ &= \frac{dP}{dQ_{(\lambda_1)}} + \frac{dP}{dQ_{(\lambda_2)}} = -\pi \cdot A^\# \cdot f \cdot (Q_{(\lambda_1)} + Q_{(\lambda_2)}) \end{aligned}$$

Previously, we obtained that  $SPerf = 5.57 (\delta_1 - \delta_2) = 5.57\delta_1 - 5.57\delta_2$ . In this function, we can identify two terms which are:

- The term  $(-5.57\delta_2)$  correspond to the sensitivity measure with respect to the parameter  $\lambda_2$  computed in the previous sub-section.
- The term  $(5.57\delta_1)$  correspond to the sensitivity measure with respect to the parameter  $\lambda_1$ .

## 6. Conclusion

The theoretical results presented in this chapter are a natural extension of the recent development on sensitivity analysis of stochastic processes. The main idea is to obtain the derivatives of a performance measure of a discrete event dynamic system based on its stochastic Petri model. In this work, the SPN model is studied. A Parameter Sensitivity analysis approach for the model is developed and an application to a supply chain is studied. We note that the proposed methodology is also applicable to GSPN (Generalized Stochastic Petri Nets) models since the marking process of a bounded GSPN is also a Markov process. The development of sensitivity analysis methods for non Markovian

stochastic Petri nets and the application of perturbation realization to the sensitivity analysis of dynamic systems with unbounded stochastic processes are two important research issues. Simulation methods based on Petri nets models are also worthy to be studied further.

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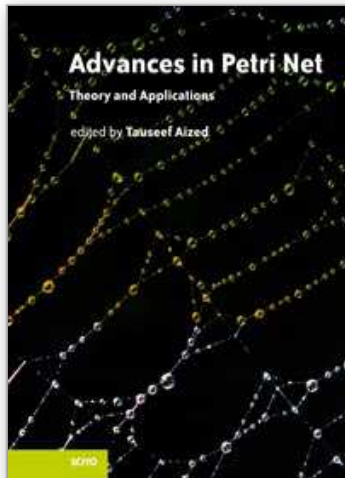
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The world is full of events which cause, end or affect other events. The study of these events, from a system point of view, is very important. Such systems are called discrete event dynamic systems and are of a subject of immense interest in a variety of disciplines, which range from telecommunication systems and transport systems to manufacturing systems and beyond. There has always been an intense need to formulate methods for modelling and analysis of discrete event dynamic systems. Petri net is a method which is based on a well-founded mathematical theory and has a wide application. This book is a collection of recent advances in theoretical and practical applications of the Petri net method and can be useful for both academia and industry related practitioners.

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